Simulations of the Universe using MOdified Newtonian Dynamics (MOND)

1 Summary

Despite the generally accepted success of the cold dark matter cosmology the model still inhibits a number of serious deviations from observations. Moreover, none of the putative dark matter particle candidates have yet been detected. Modified Newtonian dynamics (MOND) is a modification to Newton's second law of motion capable of explaining most of the observations without the need for dark matter. The main objective of this proposal is to perform fully self-consistent cosmological simulations of the formation and evolution of structures in the Universe under the influence of this alternative theory.

Despite the remarkable success of the concordance cosmological model (ΛCDM, Λ-Cold-Dark-Matter) including about 28% dark matter, 4% baryonic matter and 68% dark energy there appear to be quite distinct deviations between predictions of this standard model and observations. The problems frequently summarized as "cold dark matter (CDM) crisis" still persist: the debate about the exact value of the logarithmic inner slope of the density profiles of (galactic) halos continues and the discrepancy about the abundance of satellite galaxies has not been resolved, too. While the general belief amongst the astronomical community is to modify and fine-tune the parameters of the dark mater model in order to move towards an agreement, little effort is spent investigating alternative solutions to this CDM crisis.

In this application another possibility is put forward which is based upon a simple yet ground shaking adjustment to Newton's second law of motion, namly MOdified Newtonian Dynamics or MOND. The recent progress in the development of MOND and the embedding of it into a general relativistic framework made this theory a highly promising alternative the dark matter. First steps of applying MOND to cosmological N-body-simulations of the structure formation in the Universe indicated rather reassuring results speaking in favour of this radical approach. However, to gain full insight into this unusual theory and to better understand its predictive power it appears unavoidable to revisit the subject taking into account the latest improvements in its formulation.

2 Cosmological MOND

Introduction

Although the currently favoured ΛCDM model has proven to be exceptionally successful on large scales (cf. WMAP results, Spergel et al. 2003), high-resolution N-body simulations seem to be in contradiction with observation on sub-galactic scales: the "CDM crisis" is far from being over. The problem with the steep central densities of galactic halos, for instance, is still unsolved as the most recent state-of-the-art simulations (still) favor a cusp with a logarithmic inner slope for the density profile of approximately -1.2 (Diemand, Moore & Stadel 2005; Fukushige, Kawai & Makino 2004; Power et al. 2003), whereas high resolution observations of low surface brightness galaxies are best fit by halos with a core of constant density (Simon et al. 2005; Swaters et al. 2003; de Block & Bosma 2002). Further, CDM simulations predict a great abundance of satellite galaxies orbiting within a galactic halo while orders of magnitudes fewer such systems are actually being observed (Reed et al. 2005; Klypin et al. 1999; Moore et al. 1999).

Suggested solutions to these problems include the introduction of self-interactions into collisionless N-body simulations (e.g. Spergel & Steinhardt 2000; Bento et al. 2000), replacing cold dark matter with warm dark matter (e.g. Knebe et al. 2002; Bode, Ostriker & Turok 2001; Colin, Avila-Reese & Valenzuela 2000) or non-standard modifications to an otherwise unperturbed CDM power spectrum (e.g. bumpy power spectra; Little, Knebe & Islam 2003; tilted CDM; Bullock 2001). Some of the problems, as for instance the overabundance of satellites, can be resolved with such modifications but none of the proposed solutions have been able to rectify *all* shortcomings of CDM simultaneously. We still lack a credible alternative to the dark matter model!

Therefore, other theories are unquestionably worthy of exploration, one of which is to abandon dark matter completely and to adopt the equations of MOdified Newtonian Dynamics (MOND; Milgrom 1983). MOND has been introduced to explain the flattening of rotation curves beyond the luminous extent of disk galaxies without the need for dark matter. MOND is originally an adjustment of Newton's second law of motion introducing a fundamental acceleration a_0 and hence not to be understood as a modification of the law of gravity ab initio; it though can be interpreted as the latter (see below). And until recently it was merely a heuristic theory tailored to fit rotation curves with little (if any) predictive power for cosmological structure formation. One of the most severe problems for the general appreciation and acknowledgment of MOND as a "real" theory (and a conceivable replacement for dark matter) was the lack of success to formulate the theory in a general relativistic manner. Bekenstein (2004) now presented a relativistic gravitation theory whose non-relativistic weak acceleration limit accords with MOND while its non-relativistic strong acceleration regime is Newtonian. Using Bekenstein's formulation Skordis et al. (2005) have very recently provided a framework in which to do MONDian cosmology. One of their main results is the reproduction of the observed fluctuations in the cosmic microwave background which so far have been one of the cornerstones for the dark matter model and a unique test bench for all alternative theories. Further, they formulate an analogon to the Friedmann equations describing the expansion of a MONDian Universe. Due to this very recent progress we are finally in a situation where MOND has matured to a theory that can be used in cosmology without the need for unjustifiable assumptions and unwarranted tweaking!

The main objective of this application is to refine the implementation of MOND in the N-body code `MLAPM` building upon the basis of the recent advance in the formulation of MOND (Bekenstein 2004; Skordis et al. 2005). Only when MOND has been thoroughly and "properly" studied and tested against ΛCDM can we safely either rule it out for once and always or confirm this rather venturesome theory. The theory has become a valid competitor to dark matter and it therefore only appears natural – if not mandatory – to (re-)consider its implications. We propose to develop a tool and the subsequently necessary analysis apparatus allowing to test and discriminate cosmological structure formation in a MONDian Universe from the standard dark matter paradigm. We like to study where MOND deviates from the predictions of the ΛCDM model and what observational tests can be designed to unambiguously distinguish these two competing theories.

2.1 COSMOLOGY - WHERE DO WE STAND, WHERE DO WE GO ?

Cosmology - the study of the formation and ultimate fate of structures and galaxies throughout the Universe - is without a doubt the dominant field of astrophysics today. And in the last few years theoretical and observational studies have begun to converge as we have entered the era of "Precision Cosmology". A picture has emerged in which contemporary structures have evolved by gravitational amplification of seed inhomogeneities that are likely of quantum origin. This picture ties together measurements of the cosmic background radiation, estimates of the primordial abundances of the light elements, measurements of the clustering of galaxies and, to a more limited extent, the characteristic properties of individual galaxies. The interpretation of the high-quality observational data available (SDSS[1], 2dF[2], BOOMERanG[3], WMAP[4], etc.) depends heavily on (extremely) high-resolution numerical simulations of structure formation and evolution, a practice which might be referred to as "Precision Modeling".

Our current understanding of the Universe though rests on some important assumptions and one of them is that luminous matter (i.e. baryons) contributes only a small fraction of the mean density in the Universe, the bulk being made up of some combination of vacuum energy and dark matter (cf. Spergel et al. 2003). Ever since Zwicky's seminal work in the 1930's it has been known that there is a disparity between the mass of galaxies as measured dynamically and the mass inferred from the visible light. The standard solution to this "missing matter problem" was to invoke the existence of some putative dark matter particles. But since none of the well motivated candidate particles have yet been detected, it is important not to reject credible alternatives also providing an explanation for the observed deficiency of matter[5].

WHAT IS MOND ?

One such alternative to the alleged "dark" matter is Milgrom's MOdified Newtonian Dyanmics (Milgrom 1983), also referred to as MOND. Before MOND many theoreticians already toyed with the idea that a change in the $1/r^2$ force law at large length scales can serve as an explanation

[1] http://www.sdss.org

[2] http://www.mso.anu.edu.au/2dFGRS

[3] http://cmb.phys.cwru.edu/boomerang/

[4] http://map.gsfc.nasa.gov/

[5] The reasoning for introducing "dark" matter is solely based upon the assumption that there are no modification to either the law of gravity or Newton's axioms.

for the mass discrepancy. But as Milgrom first realized, any modification attached to a length scale would cause the larger galaxies to exhibit the larger discrepancy. Milgrom hence suggested to rather introduce a universal acceleration scale and modifying Newton's second law of motion, respectively:

$$\vec{F} = m\mu(a/a_0)\vec{a} \tag{1}$$

where a_0 is a free parameter to be determined by fitting the observational data and found to be of the order 10^{-8}cm/s^2. The (interpolation) function $\mu(x)$ is only constrained by requiring $\mu(x) \approx x$ for $x << 1$ and $\mu(x) \to 1$ for $x >> 1$ to assure to retrieve Newtonian physics for large accelerations and an analytical expression is given by Milgrom (1983), too.

It has been shown that the simple idea of MOND can explain many properties of galaxies without the need of non-baryonic (dark) matter (e.g. Scarpa 2003; McGaugh & de Blok 1998; Sanders 1996; Begeman, Broeils & Sanders et al. 1991). MOND is also successful in describing the dynamics of galaxy groups and clusters (Sanders 1999; Milgrom 1998), globular clusters (Scarpa, Marconi & Gilmozzi 2003) and, to a limited extent[6], gravitational lensing (Mortlock & Turner 2001; Qin, Wu & Zou 1995). A recent review of MOND is given by Sanders & McGaugh (2002) which also summarizes (most of) the successes alluded to above.

Even though MOND has been introduced as a modification to accelerations it can also be re-interpreted as a modification to the law of gravity. Bekenstein & Milgrom (1984) proposed the non-relativistic field equation for the gravitational potential that generates an appropriately modified gravitational acceleration $\vec{a}$ of a test particle:

$$\vec{\nabla} \cdot \left(\mu\left(|\vec{a}|/a_0\right)\vec{a}\right) = 4\pi G\rho(\vec{r}) \ , \tag{2}$$

Comparing this to the usual Poisson equation $\vec{\nabla} \cdot \vec{a}_N = 4\pi G\rho(\vec{r})$ we find the relation between MONDian acceleration $\vec{a}$ and Newtonian acceleration $\vec{a}_N$ to be:

$$\vec{a}_N = \mu(\vec{a}/a_0)\vec{a} + \vec{\nabla} \times \vec{h} \ . \tag{3}$$

Eq. (2) is substantially different from Poisson's equation, highly non-linear and difficult to solve in general and the relation between Newtonian accelerations and MONDian accelerations given by Eq. (3) is non-trivial, too. But it has also been shown by Bekenstein & Milgrom (1984) that the term $\vec{\nabla} \times \vec{h}$ decreases like $\boldsymbol{O}(r^{-3})$ with increasing scale. Moreover, in cases of spherical, planar, or cylindrical symmetry the curl-term $\vec{\nabla} \times \vec{h}$ vanishes completely. In that case the relation between Newtonian and MONDian acceleration simplifies to

$$\vec{a}_N = \mu(\vec{a}/a_0)\vec{a} \tag{4}$$

which can be used in conjunction with a "normal" Newtonian Poisson solver to easily obtain MONDian accelerations. Under the assumptions mentioned above we would not need to solve Eq. (2) but can rather adhere to an existing Poisson solver mapping a_N to a using Eq. (4). This prescription has been realized by Knebe & Gibson (2004) and led to (preliminary) predictions about the self-consistent formation of gravitationally bound objects in cosmological simulations which will be elaborated upon later in Section 2.2.

[6]simply due to the fact that until recently there was no formulation of MOND compatible with General Relativity

Conventional Poisson solvers used throughout (computational) cosmology are no longer applicable to Eq. (2). All of the standard methods such as Fast-Fourier-Transform (FFT) based Particle Mesh (PM) codes, tree codes, expansion codes, and their variants rely on the linearity of Poisson's equation. The corresponding MONDian Poisson equation (2) though is a non-linear partial differential equation which requires more subtle and refined techniques to be solved numerically.

Before elaborating on how we intend to deal with Eq. (2) we like to recapitulate the approach taken by Knebe & Gibson (2004). As mentioned before, we already modified the open source cosmological N-body code `MLAPM` to take into account the theory of MOND. However, these adjustements were based upon the two following of assumptions: (1) MOND only affects peculiar accelerations $\vec{a}_{\rm pec}$, and (2) Eq. (4) can be used to relate $\vec{a}_N$ to $\vec{a}$. The first assumption is equivalent to presuming that MOND has no effect on the Hubble expansion of the Universe as the "proper" velocity can be split into two terms

$$\vec{v} = \dot{\vec{r}} = R\dot{\vec{x}} + \dot{R}\vec{x} = R\dot{\vec{x}} + H\vec{r} = \vec{v}_{\rm pec} + \vec{v}_{\rm Hubble} \ , \tag{5}$$

when introducing comoving coordinates $\vec{r} = R\vec{x}$ (where R, the expansion factor of the Universe, has been normalized to unity at today's time).[7] By taking the time derivative of $\vec{v}$ and using the second Friedmann equation[8] one finds the appropriate definition for peculiar accelerations to be

$$\vec{a}_{\rm pec} = \vec{F}_{\rm pec}/R^2 \ , \tag{6}$$

where $\vec{F}_{\rm pec}$ is derived by solving the un-MONDified Poisson equation in comoving coordinates

$$\vec{\nabla} \cdot \vec{F_{\rm pec}} = 4\pi G \left(\rho(\vec{r}) - \overline{\rho}\right) \ , \tag{7}$$

and marks out the force field responsible for peculiar motions alone. For a more detailed discussion of the relevant formulae we refer to Knebe & Gibson (2004) where a thorough derivation of the MONDification of `MLAPM` is presented.

We note in Eq. (6) that at early times in the Universe (i.e. $R << 1$) peculiar accelerations tend to be exceedingly large and hence not affected by MOND. Only at later times, when structures emerge, does MOND influence the dynamics and evolution of the Universe. This naturally leads to assumption (2): when MOND becomes affective we will already be in the regime of gravitationally bound objects which (to a greater or lesser extent) show some kind of symmetry. And as Bekenstein & Milgrom (1984) showed that in this case $\vec{\nabla} \times \vec{h} = 0$, Eq. (4) applies for relating $\vec{a}_N$ to $\vec{a}$.

While we gained great insight into the cosmological structure formation in a MOND model by running cosmological simulations utilizing the modifications outlined above, the results obtained are (unfortunately) far from being conclusive and only to be understood as preliminary. The first assumption can only be dropped when correctly modeling the effects of MOND on the expansion of the Universe, namely with the use of the MONDian Friedmann equations given

[7] R needs to be derived from the Friedmann equation describing the expansion of the Universe $H = \dot{R}/R = H_0(\Omega_{m,0}R^{-3} + \Omega_{\lambda,0})$ where $\Omega_{m,0}$ is a measure for the total matter content and $\Omega_{\lambda,0}$ the contribution from dark energy.

[8] $\ddot{R}/R = (-4\pi G\overline{\rho} + \Lambda c^2)/3$

by Skordis et al. (2005). Further, with our grid-based N-body code `MLAPM` we are in the unique situation that we need not rely on Eq. (4) and the assumptions made during its derivation! Or in other words, `MLAPM` has the potential to directly integrate the MONDian Poisson equation Eq. (2): `MLAPM` uses a variant of the multigrid technique (Brandt 1977; Press et al. 1992) to solve Poisson's equation. Such a method is well suited for solving partial differential equations of the type

$$\vec{\nabla} \cdot \vec{f}(\vec{a}(r)) = S(r) \tag{8}$$

as it discretizes the equation on a grid and iteratively solves for $\vec{a}(r)$.

We now plan to upgrade the `MLAPM` by adjusting the Poisson solver to allow for a numerical integration of Eq. (2). Then, and only then, will we be able to understand the formation and evolution of structures induced and affected by MOND in a fully self-consistent manner.

2.2 WHAT HAS BEEN DONE BEFORE (BY OTHER AUTHORS) ?

MOND has yet been greatly ignored if not argued against, especially when it comes to its implications for cosmology. Little efforts have been put into modeling the formation of structures under the influence of MOND which is clearly reflected by the fact that there are a mere 3 (three!) papers in total based upon more or less approximative numerical potential solvers incorporating the effects of MOND: the work on galactic dynamics by Rafi Brada during his PhD in 1996, the low-resolution cosmological simulations by Adi Nusser (2002) and – most recently – the high-resolution analgon to Nusser's work by Knebe & Gibson (2004).

Specifically, Brada (1996) applied MOND to galactic dynamics by developing a grid based potential solver similar to the one put forward in this proposal. However, there are (of course) subtle yet very important differences to the approach detailed in this application. The code developed by Brada (1996) was solely designed to study the evolution of isolated galaxies (i.e. no cosmological framework) and did not include adaptive mesh refinement (i.e. high spatial resolution). Their main science driver was the stability of disk galaxies (Brada & Milgrom 1999) and the appearance of galactic warps (Brada & Milgrom 2000a). However, for a self-consistent treatment of the formation and evolution of cosmological structures the embedding within an expanding Universe as well as a high spatial dynamic range are necessary requirements. Whereas the former is given by transferring the equations to comoving coordinates and adopting periodic rather than isolated boundary conditions, the latter is best realized using adaptive meshes that automatically adapt to the underlying density field and particle distribution, respectively. This is an integral part of the N-body code `MLAPM`.

More recently, Nusser (2002) also investigated modified Newtonian dynamics of the large-scale structure using the N-body approach. His simulations, however, are of very low resolution, both in terms of spatial and mass resolution, making a study of individual objects difficult. He therefore was restricted to analysing the (non-linear) evolution of large-scale structures. But he could only match the clustering properties in his MONDian simulations to those of an ΛCDM model when adjusting the fundamental acceleration a_0 to values lower than those needed to fit galactic rotation curves.

Knebe & Gibson (2004) improved upon this study and adopted a similar implementation of MOND in cosmological simulations. Their resolution allowed for a more detailed study of individual objects revealing some rather astonishing results: there is little difference between

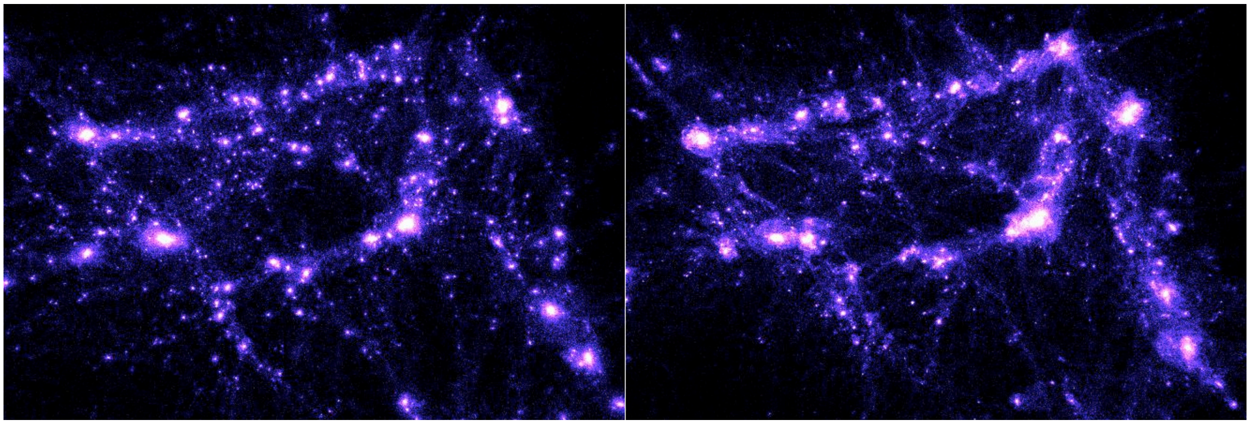

Figure 1: ΛCDM cosmological simulation at redshift $z = 0$ (left panel) vs. a simulation incorporating MOND and using the same initial conditions (right panel).

a ΛCDM and a MOND model at redshift $z = 0$. This is illustrated in Fig. 1 where we confront the final output at redshift $z = 0$ of the fiducial ΛCDM simulation with the corresponding data of the MOND run. We see a projection (at a certain angle to the otherwise regular cubic box) of the whole simulation with the density at each particle position colour-coded: dark is low and white high density. A detailed investigation of the evolution of the structures seen in both models indicates that severe difference emerge the further into the past we look and there are hardly any objects present in the MOND simulation beyond redshift $z > 4 - 5$ which is in stark contrast to observations where galaxies are found out to redshifts of $z = 6$ and beyond (Shioya et al. 2005). The question remains if this result is an implication of MOND itself or just of our realisation of it.

WHAT WILL WE DO BETTER ?

The implementation of MOND into `MLAPM` as detailed in Knebe & Gibson (2004) was highly instructive yet based upon approximative assumptions. Most importantly, the treatment was not self-consistent at all as we discarded the influence and effects of the (unknown) curl-field $\vec{\nabla} \times \vec{h}$. The deficiency of objects at high redshift may very well be a mere reflection of the (unjustified) symmetry assumption. We also need to clarify that we had to lower the amplitude of the primordial density fluctuations in order to get the rather intriguing agreement of the MOND model with ΛCDM at redshift $z = 0$ (cf. Fig. 1); only then could we accommodate the accelerated structure formation of MOND at late times (cf. Fig.1 in Sanders, 2001). But this drop in the normalisation of the initial matter fluctuations naturally explains the lack of objects prior to the MOND regime. However, it also needs to be stated that all previous (cosmological) simulations of structure formation in MOND ignored the influence of the cosmological constant Ω_λ. Both Nusser (2002) and Knebe & Gibson (2004) adopted flat and open Universes, respectively. While MOND leads to an increased clustering once accelerations fall below the universal a_0 value, the Λ-term in the Friedmann equations actually acts in the opposite direction obliterating density contrasts. Can the (unjustifyable) low normalisation of the primordial density fluctuations be balanced by the inclusion of "dark energy"? Even though we were the first to perform high-resolution simulations of the formation of galaxies in a MONDian cosmology, they are only to be understood as a glimpse of results yet to come.

We intend to rectify and clarify this situation by not only taking Ω_λ into consideration but also to adopt the MONDian analogon of the Friedmann equations as given by Skordis et al. (2005). Further, as sketched in Section 2.1 (and detailed in Section 3.1 below) we plan to implement a fully self-consistent treatment of MOND into the N-body code `MLAPM`. This involves a modification to the already existing Poisson solver to allow for a direct numerical integration of Eq. (2). And as shown by the peerless work of Rafi Brada (1996) this is a feasible and manageable undertaking!

3 APPROACH

3.1 AIMS

The most important part of the work is the adaption of the already existing N-body code `MLAPM` to allow for a numerical solution of Eq. (2). We intend to numerically integrate Eq. (2) and the only approach to tackle this task is to discretize the equation on a grid and iteratively relax a trial potential ϕ (where $\vec{a} = -\vec{\nabla}\phi$) until convergence. This technique is already implemented in the N-body code for solving the (Newtonian) Poisson equation and hence `MLAPM` appears to be perfectly suited for this modification. The difference to solving the (Newtonian) Poisson equation though lies in the fact that one can no longer solve directly for the new value of ϕ at a given grid point. ϕ is implicitly defined due to the non-linear nature of the MONDian analogon to Poisson's equation. We plan to carry out a single iteration of, for instance, the Newton-Raphson method for finding the root of a non-linear equation with ϕ at the given grid point as the variable and keeping all other values fixed. For the initial trial potential we intend to use the Newtonian potential returned by `MLAPM`'s usual Poisson solver.

To make this approach more transparent, we outline the procedure in greater detail for a 1-dimensional situation. Eq. (2) then simplifies to

$$\frac{d}{dx}\left[\mu\left(\frac{1}{a_0}|\frac{d\Phi}{dx}|\right)\frac{d\Phi}{dx}\right] = 4\pi G\rho \ , \tag{9}$$

Discretizing the relevant values Φ and ρ on a grid and using mid-point approximations for the differential operator leads to the following set of equations

$$\begin{aligned} \frac{S_{i+1/2}-S_{i-1/2}}{\Delta x} &= 4\pi G\rho_i \\ S_{i\pm1/2} &= \mu\left(\frac{1}{a_0}|\frac{\Phi_{i+1}-\Phi_i}{\Delta x}|\right)\frac{\Phi_{i+1}-\Phi_i}{\Delta x} \ . \end{aligned} \tag{10}$$

For $\mu \equiv 1$ (Newtonian Poisson's equation) we could use simple algebra to reorder the terms to

$$\Phi_i = \frac{1}{2}(\Phi_{i+1}-\Phi_{i-1}) - 2\pi G\rho_i\Delta x^2 \ , \tag{11}$$

which allows to iteratively obtain a solution for Φ (cf. relaxation methods, Press et al. 1992). However, this approach is no longer valid for $\mu \neq 1$ and in order to obtain a value for Φ_i we need to solve for the root of

$$f(\Phi_i) = 0 \ , \text{with } f(\Phi_i) = \frac{S_{i+1/2}-S_{i-1/2}}{\Delta x} - 4\pi G\rho_i \ , \tag{12}$$

where we keep all other values Φ_{i+1}, Φ_{i-1}, and ρ_i fixed and $S_{i\pm1/2}$ are defined by Eq. (10).

The time frame for upgrading MLAPM's Poisson solver and familiarising with the subject, respectively, is expected to be less than a year. The student can most certainly draw upon the in-house expertise and the distinct opportunity to be supervised by the author of MLAPM, respectively. The remaining two years are then dedicated to the analysis of the simulations and the interpretation of the results.

3.2 ASTROPHYSICAL APPLICATIONS

The main science driver for this project is the accurate modeling of structure formation in a MONDian Universe. We like to verify MOND against the standard paradigm of dark matter by performing simulation that are based upon a solid implementation of the just recently developed cosmological MOND formulation. Imminent questions revolve about the confirmation of the preliminary findings of Knebe & Gibson (2004). Were the simplifying assumptions made in that study reasonable? What additional predictions will the revised treatment of MOND lead us to? What unique observational test can be designed allowing for a differentiation from the standard dark matter model?

Our previous investigations showed, for instance, that the density profiles of the most massive (and hence best resolved) object could still be fitted by the universal profile advocated for CDM halos by Navarro, Frenk & White (1997). While the general shape of the density profile appeared to be in agreement with CDM predictions, the object though showed a substantially smaller concentration. Besides of that, a visual impression led to the conclusion that MOND objects contain less substructure. While CDM actually overpredicts the number of satellites galaxies MOND may act in favour of the observed abundance of, for instance, the Milky Way companions. Even though the object under consideration in Knebe & Gibson (2004) contained of order 10^5 particles, the simulation was insufficient to resolve the central parts still under great debate in the astrophysical community. With better resolved simulations we hope to be able to shed light on this subject and improve the statistics by not restricting the analysis to only one halo; we intend to run a series of simulations. On the one hand we are going to create different random realizations of the same cosmological box in order to study equivalent objects and get a handle on the cosmic/sampling variance of the results. On the other hand we will gradually increase the computational volume. This aims at quantifying the influence of the box size and the inclusion of large scale density perturbations on the internal properties of objects, respectively. It is yet unclear how large-scale fluctuations affect galactic halos (cf. Bagla & Ray 2005), especially in MOND where gravitational potentials of individual objects do not add up to a common potential of the combined mass distribution. Both of these approaches are not only interesting for the study of MOND; even the fiducial ΛCDM simulations will provide us with an improved understanding of the interplay between large and small scale structure. This point also ties in with the findings of Knebe & Gibson (2004) that in a MONDian Universe objects are more strongly clustered on small scales. Will we find convergence for this result when varying the box size and how is it related to the lack of substructure found for MOND objects?

Another major focus of the analysis will be the evolution of satellite galaxies orbiting about a common host. We expect noticeable differences to classical Newtonian theory as in MOND the superposition of density fields does not entail the superposition of the individual potentials! The study of tidal debris of disrupting dwarf galaxies around the Milky Way has provided great insight into the formation history of our own galaxy (e.g. Helmi, 2004) but will most certainly lead to different conclusions once modeled with MOND (Read & Moore 2005). Brada & Milgrom (2000b) also studied the influence of MOND on the orbits and tidal destruction of galactic satellites showing that the size and velocity dispersion of the satellite vary as the external field induced by the host galaxy varies along its orbit. A notable outcome of this is a vulnerability to eventual tidal destruction that is higher than under Newtonian dynamics and the presence of dark matter. Great efforts are currently being undertaken to decipher the past of our Milky Way by utilizing such tidal streams. This new branch of astrophysics is labeled "galactic archeology" and the RAVE[9] project, for instance, aims at measuring more than 1mio. stars with an accuracy of the velocity determination down to 1 km/sec. The project laid out in this application is well poised to provide a theoretical counterpart of such observational ventures. And the validity of the analysis of our models is not only limited to the MOND scenario as conventional ΛCDM simulations are a mandatory gauge for comparison.

CONCLUDING REMARKS

It is worth noting that despite the daringness of modifiying gravity the MOND theory is being taken more and more seriously. While it may be regarded as highly venturesome to replace a simple solution such as dark matter with a modified law of gravity (i.e. Eq. (2)), one needs to bear in mind that *none* of these (theoretically predicted) dark matter particles have yet been discovered. Moreover, models based upon dark matter may describe the Universe on scales larger than the typical size of galaxies, but they still face serious problems on sub-galactic scales. And with the most recent advances in the formulation and understanding of MOND this theory has become a promising alternative to dark matter. We basically propose to *correctly* (re-)consider a cosmological MONDian structure formation scenario and devise (observational) tests to falsify this theory.

One of the major criticisms against MOND are the high-precision measurements of the temperature fluctuations in the cosmic microwave background radiation. The predictions of the CDM model for these anisotropies match the observations exceedingly well and hence are interpreted as a great success of the dark matter theory. However, the recent calculations by Skordis et al. (2005) show that using Bekenstein's (2004) relativistic formulation of MOND the agreement with the WMAP data is equally good. This has been acknowledged by both theoreticians and observers in July 2005 at a conference held at the IAP in Paris. The program of that meeting focused on exactly this issue mainly addressing mass profiles and shapes of cosmological structures[10]. There was an ever increasing interest in possible explanations other than CDM at that meeting with a full day of talks solely dedicated to the presentation of alternative theories putting a special emphasis on MOND.

Even though the actual work of the PhD student will revolve around a highly provocative theory the student also has the opportunity to perform not only MONDian cosmological simulations but also high-resolution ΛCDM runs. Besides of developing sophisticated coding abilities he will further gain all the necessary skills required for analysing state-of-the-art data produced by cosmological simulations. This in itself is highly instructive and not an easy task given the amounts of data involved in such studies. The student will hence be given the opportunity

[9] http://www.rave-survey.aip.de/rave/

[10] http://www.iap.fr/ActivitesScientifiques/SeminairesEtColloques/ColloqueIAP2005/index.html

to establish himself amongst the highly competitive community of computational cosmology which appears to be one the most dominant and quickly evolving fields of astrophysics today.

Further, the project proposed in this application ties nicely together with the already DFG-funded Emmy Noether Program, KN-755/1. An integral part of the science outlined in the Emmy Noether application is the study of satellite galaxies orbiting within a common, enveloping (dark matter) host halo. Because of MOND's nonlinearity, a system's internal dynamics can be altered by an external field in which it is immersed (even when this field, by itself, is constant in space). As a result, the size and velocity dispersion of the satellite vary as the external field varies along its orbit. A notable outcome of this is a substantial increase in a halo's vulnerability to eventual tidal disruption. The funding of the project outlined in this application can therefore be understood complementary to the research already under investigation and there will be a mutual benefit for both the new PhD student as well as the already appointed students within the Emmy Noether Research Group. Despite the unconventional science to be undertaken by the student he will find himself in a vibrant environment composed of "standard" cosmologists and exposed to "traditional" theories of structure formation, respectively.