Radiative Processes

1. Optical astronomers detect rocky bodies in the solar system by virtue of their reflected sunlight. The received flux is proportional to aA, where a is the albedo (reflectivity) of the object in some optical passband, and A is the area of the object. Typically, albedos of minor bodies in planetary systems range from ~0.01 to ~0.7. The proportionality of the flux to aA presumes that all of the optical light received from an object is from reflected sunlight. Of course, that is not quite true; there is some contamination at optical frequencies from the object's thermal emission (the fact that it is warm).
Give a quantitative explanation as to why this contamination is not worth troubling over.

hints:
If you wish, you can adopt as your "case study" a Kuiper belt object of optical albedo 0.07 and heliocentric distance 40 AU. What is the temperature of this object and at what wavelength peaks its own thermal radiation? Compare its specific intensity (measured at the sun's peak wavelength) to the reflected specific intensity (where for the latter you can assume $I_\nu^{reflected} = fB_\nu(T_\odot)$, using energy conservation to find f).

(2 points)

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2. Practice with j_ν, α_ν, S_ν, B_ν, I_ν
(a) A plane-parallel slab of uniformly dense gas is known to be in LTE (local thermodynamic equilibrium) at a uniform temperature T. Its thickness normal to its surface is s. Its absorption coefficient is $\alpha_{\nu,\mathrm{gas}}$. Write down the specific intensity, I_ν, viewed normal to the slab, in terms of the variables given.
(b) The same slab is now filled uniformly with non-emissive dust having absorption coefficient $\alpha_{\nu,\mathrm{dust}}$. The dust is non-emissive, so its emissivity $j_{\nu,\mathrm{dust}}$ = 0. Write down I_ν viewed normal to the slab, in terms of all variables given so far.
(c) The slab of gas and dust is further mixed with a third component: an emissive, non-absorptive uniform medium having emissivity $j_{\nu,\mathrm{med}}$ and absorption coefficient $\alpha_{\nu,\mathrm{med}}$ = 0. Write down I_ν viewed normal to the slab, in terms of all variables given.
What could be the analog to this example in the Universe?

(2 points)

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3. A sphere with radius R has constant emissivity j_ν, absorption coefficient α_ν, and source function S_ν. Calculate I_ν at the surface of the sphere and show that the flux is

$$F_\nu = \pi S_\nu \left(1 - \frac{2}{\tau^2}(1 - e^{-\tau}) + \frac{2e^{-\tau}}{\tau}\right)$$

where $\tau = 2\alpha_\nu R$. Sketch the angular dependence of I_ν for different values of τ. Check that I_ν and F_ν make sense in the limits τ<<1 and τ>>1.

(1 points)

general remarks:

- 'show' means that you should derive the equations rather than verifying their correctness; this also means that you have to give every single step of calculation and not just say 'as we can see'.
- if you refer to some calculations or results from somewhere else you must provide the proper reference.
- be clear about what you do and write

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Please also check the calculations marked '***(exercise)***' in the lecture notes as we also plan to discuss them. As a reminder, they were as follows (**1 point for each exercise**):

L1) $$\frac{dI_\nu(\Omega)}{ds} = 0$$ intensity is conserved along a ray s

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L2) $$\frac{dI_\nu}{d\tau_\nu} = -I_\nu(\tau_\nu) + S_\nu(\tau_\nu)$$

=> general solution: $$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu-\tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

L3) $$\frac{V}{T}\frac{du}{dT}\, dT + \frac{4}{3}\frac{u}{T}\, dV = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV \longrightarrow \frac{1}{T}\frac{du}{dT} = -\frac{4}{3}\frac{u}{T^2} + \frac{4}{3T}\frac{du}{dT}$$

L4) $$S(T) = \frac{16}{3c}\, \sigma_B\, T^3 V$$

L5) $$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1} \quad \leftrightarrow \quad B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$$