Stellar atmospheres

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Radiative transport

Transfer equation

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu - \alpha I_\nu$$

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau} = S_\nu - I_\nu$$

- emiss. coeff.: $j_\nu = j_{sp} + j_{in} + j_{sc}$
- specific emissivity: $j_\nu = \rho\epsilon_\nu$
- source function: $S_\nu = \frac{j_\nu}{\alpha}$

- extinction coeff.: $\alpha = \alpha_{ab} + \alpha_{sc}$
- opacity: $\alpha = \rho\kappa$
- optical depth: $\tau = \int \alpha \, \mathrm{d}s$

Local thermodynamic equilibrium (LTE)

Kirchhoff law

$$\epsilon_\nu = \kappa B_\nu(T)$$

Transport equation

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau} = \frac{\alpha_{ab}}{\alpha_{ab}+\alpha_{sc}} B_\nu(T) + \frac{\alpha_{sc}}{\alpha_{ab}+\alpha_{sc}} J_\nu - I_\nu$$

$T \simeq T_0$, $\alpha_{sc} \simeq 0$ o $J_\nu \simeq B_\nu$

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau} = B_\nu(T) - I_\nu$$

$$I_\nu(\tau) \simeq I_0 e^{-\tau} + B_\nu(T_0)(1 - e^{-\tau})$$

Stellar interiors

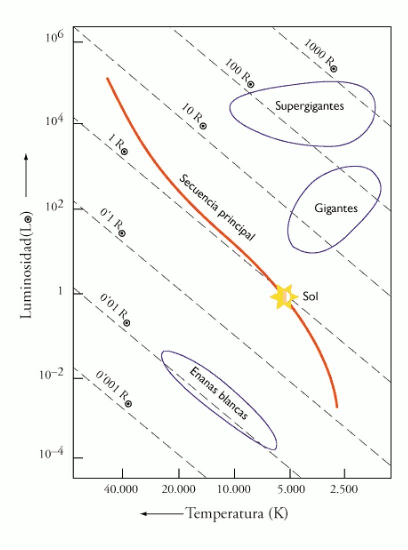
Internal structure

1. mass conservation: $$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \rho$$
2. hydrostatic equilibrium: $$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM}{r^2}$$
3. thermal equilibrium: $$\frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi r^2 \rho\varepsilon$$
4. energy transport:
   - radiative $$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{16\sigma}\frac{\rho\kappa_{\mathrm{r}}}{T^3}\frac{L}{4\pi r^2}$$
   - convective $$\frac{\mathrm{d}T}{\mathrm{d}r} = \frac{\gamma-1}{\gamma}\frac{T}{P}\frac{\mathrm{d}P}{\mathrm{d}r}$$

Composition

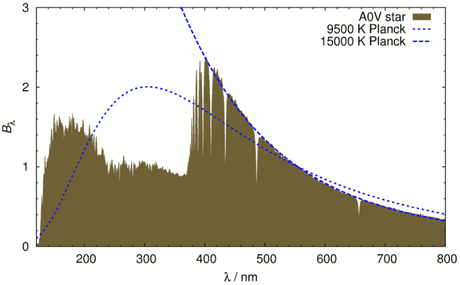
$$P(\rho, T)$$
$$\varepsilon(\rho, T)$$
$$\kappa_{\mathrm{r}}(\rho, T)$$

Hertzsprung-Russell diagram



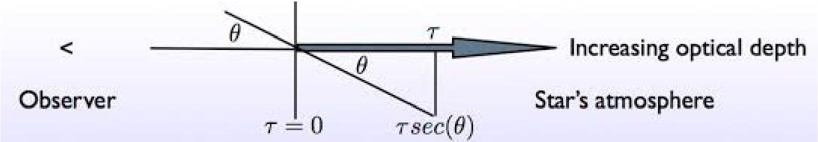
Black body

$$L = 4\pi R^2\,\sigma\,T_{eff}^4$$



Stellar atmospheres

Radiative transfer equation



Spherical symmetry

$$\frac{1}{\alpha(\nu)}\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = \frac{\mathrm{d}I_\nu}{\mathrm{d}r}\frac{\cos\theta}{\alpha(\nu)} - \frac{\mathrm{d}I_\nu}{\mathrm{d}\theta}\frac{\sin\theta}{r\ \alpha(\nu)} = S_\nu - I_\nu$$

Plane-parallel atmosphere

$$\cos\theta\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau(\nu)} = I_\nu - S_\nu$$

Analytical solution

Plane-parallel atmosphere

$$I_\nu^{in}(\tau, \theta > \frac{\pi}{2}) = -\int_0^\tau S_\nu(t)\ e^{-(t-\tau)\sec\theta}\ \sec\theta\ \mathrm{d}t$$

$$I_\nu^{out}(\tau, \theta < \frac{\pi}{2}) = -\int_\infty^\tau S_\nu(t)\ e^{-(t-\tau)\sec\theta}\ \sec\theta\ \mathrm{d}t$$

Thermal equilibrium

$$\int_0^\infty F_\nu(\tau)\ \mathrm{d}\nu = F_0 = \frac{L}{4\pi R^2} \equiv \sigma T_{eff}^4$$

Analytical solution

Schwarzschild-Milne equations

1. $J_\nu = \frac{1}{4\pi} \int I_\nu \, \mathrm{d}\Omega \quad = \frac{1}{2} \int_0^\infty S_\nu(t)\, E_1(|t-\tau|)\, \mathrm{d}t$
2. $H_\nu = \frac{1}{4\pi} \int I_\nu \cos\theta \, \mathrm{d}\Omega \quad = \frac{1}{2} \int_\tau^\infty S_\nu(t)\, E_2(t-\tau)\, \mathrm{d}t - \frac{1}{2} \int_0^\tau S_\nu(t)\, E_2(\tau - t)\, \mathrm{d}t$
3. $K_\nu = \frac{1}{4\pi} \int I_\nu \cos^2\theta \, \mathrm{d}\Omega \quad = \frac{1}{2} \int_0^\infty S_\nu(t)\, E_3(|t-\tau|)\, \mathrm{d}t$

$$E_n(x) \equiv \int_1^\infty \frac{e^{-wx}}{w^n}\, \mathrm{d}w$$

Approximate solution

Gray atmosphere

$$\cos\theta \frac{\mathrm{d}I}{\mathrm{d}\tau} = I - \frac{\sigma T^4}{\pi}$$

Moments

0. $0 = J - \frac{\sigma T^4}{\pi}$
1. $4\pi \frac{\mathrm{d}K}{\mathrm{d}\tau} = \sigma T_{eff}^4 - 0$

Eddington approximation

$$I(\tau, \theta < \pi/2) = I(\tau, 0) \equiv I^{out}(\tau)$$

$$I(\tau, \theta > \pi/2) = I(\tau, \pi) \equiv I^{in}(\tau)$$

0. $J = \frac{1}{2}\left(I^{in} + I^{out}\right) = \frac{\sigma T^4}{\pi}$
1. $F = \pi\left(I^{out} - I^{in}\right) = \sigma T_{eff}^4$
2. $K = \frac{1}{6}\left(I^{in} + I^{out}\right) = J/3$

Thermal structure

$$T^4 = T_{eff}^4 \left(\frac{1}{2} + \frac{3}{4}\tau\right)$$

Observable spectrum

Limb darkening

Eddington-Barbier approximation

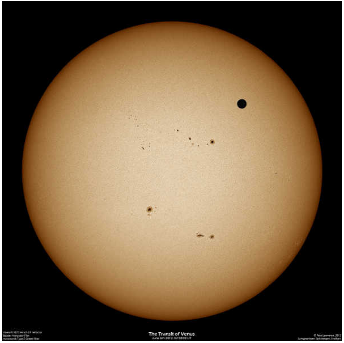
$$S_\nu(\tau) \approx S_\nu(\tau_0) + (\tau - \tau_0)\,\frac{\mathrm{d}S_\nu}{\mathrm{d}\tau}(\tau_0)$$

$$I_\nu(0,\theta) \approx S_\nu(\tau_0) + (\cos\theta - \tau_0)\,\frac{\mathrm{d}S_\nu}{\mathrm{d}\tau}(\tau_0)$$
$$\approx B_\nu[\,T(\tau = \cos\theta)\,]$$

Gray atmosphere + Eddington-Barbier

$$\frac{I(0,\theta)}{I(0,0)} = \frac{3}{5}\,\left(\cos\theta + \frac{2}{3}\right)$$

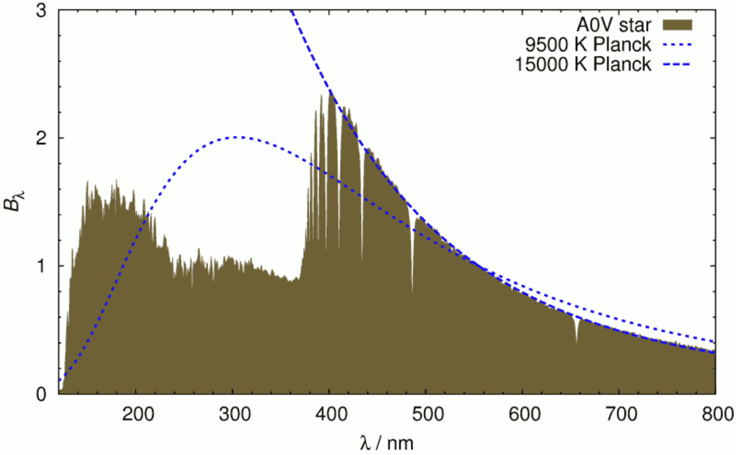
Limb darkening



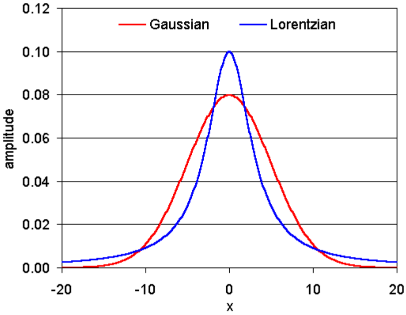
Gray atmosphere + Eddington-Barbier

$$\frac{I(0,\theta)}{I(0,0)} = \frac{3}{5}\left(\cos\theta + \frac{2}{3}\right)$$

Absorption lines



Absorption lines



$G(x;\sigma) \equiv \frac{e^{-x^2/(2\sigma^2)}}{\sigma\sqrt{2\pi}}$ $\quad L(x;\gamma) \equiv \frac{\gamma}{\pi(x^2+\gamma^2)}$

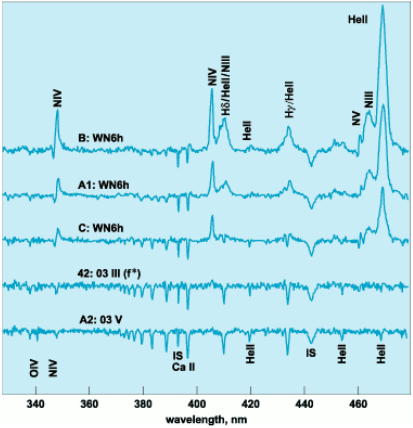
Voigt profile

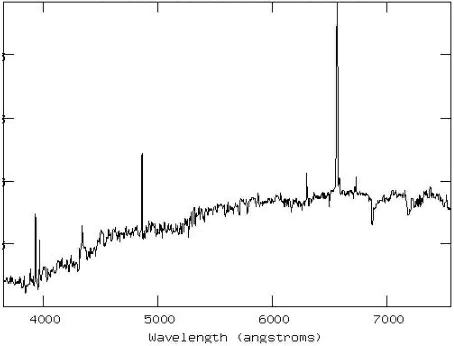
$$V(x) = \int_{-\infty}^{\infty} G(x')\, L(x-x')\, \mathrm{d}x'$$

FWHM

- density, temperature, chemical composition
- velocity (thermal, turbulent, rotation)
- surface gravity
- magnetic field

Emission lines





P-Cygni profiles