Non-Thermal Radiation

Nataly Ospina *(Istituto Nazionale di Fisica Nucleare)*

"GALAXIES, PARTICLES, RAYS, WAVES... I'VE NEVER ACTUALLY SEEN ANYTHING I'M REALLY INTERESTED IN."

Non-Thermal Radiation

- Non-Thermal radiation: hadronic processes (recap) vs. leptonic processes
- Electron distributions & normalizations
- Leptonic processes:
    - Synchrotron emission
    - Inverse Compton scattering
    - Non-thermal bremsstrahlung

Non-Thermal Radiation

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Non-thermal radiation:

- Radiation produced **out of thermal equilibrium**
- Not described by Planck spectrum
- Typically: **power-law energy distributions** of particles
- Depends on **acceleration processes**.
- Common in **high-energy astrophysics**

Produced when particles are accelerated (it is not determined simply by temperature). It arises from physical processes such as shocks, turbulence, and strong electromagnetic fields

Hadronic processes are interactions of hadrons (strongly interacting particles such as protons and neutrons) that produce secondary particles, gamma rays, and neutrinos

Hadronic processes:

- Proton-proton collisions
- Pion production & decay
- Gamma-ray and neutrino emission

What are Leptonic Processes?

Leptonic processes are interactions of leptons (electrons and positrons) that produce radiation through electromagnetic mechanisms such as synchrotron emission or inverse Compton scattering

Leptonic processes:

- Synchrotron emission
- Inverse Compton scattering
- Non-thermal bremsstrahlung

Leptonic processes* = interactions of relativistic electrons and positrons with matter

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*Distinct from *leptonic processes* (driven by protons/neutrons)

Non-Thermal Radiation

Leptonic processes* = interactions of relativistic electrons and positrons with matter

→ leptonic interactions are a natural source of *broadband radiation (from radio to γ-rays)*

PSR B1509-58, Chandra X-ray Observatory. *Credit: NASA/CXC/SAO/P.Slane, et al.*

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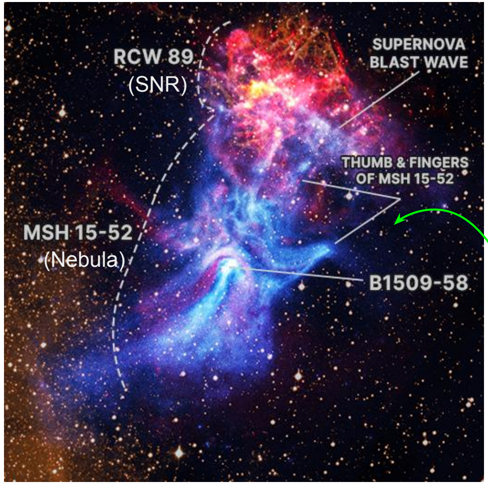
Leptonic processes - definition

Leptonic processes* = interactions of relativistic electrons and positrons with matter

→ leptonic interactions are a natural source of *broadband radiation (from radio to γ-rays)*

Importance:

- Relativistic electrons are common in astrophysical sources
- Electrons radiate very efficiently (short cooling times)
- Produce synchrotron emission → explains radio to X-rays
- Produce inverse Compton γ-rays → often seen in PWNe, AGN jets, SNR shocks, GRN afterglows
- Generate bremsstrahlung γ-rays in dense environments
- Reveal conditions in sources: magnetic fields, ambient radiation, gas density
- Provide diagnostics complementary to hadronic channels (spectral slopes, polarization, cooling breaks)



Credit: **X-ray:** NASA/CXC/Univ. of Hong Kong/S. Zhang et al.; **Radio:** ATNF/CSIRO/ATCA; **H-alpha:** UK STFC/Royal Observatory Edinburgh; **Image Processing:** NASA/CXC/SAO/N. Wolk

Cloud of HE charged particles accelerated by the intense electromagnetic field of the pulsar

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- **Electron distributions & normalizations**
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Non-Thermal Radiation

In *non-thermal* radiation we need to describe how particles are distributed in energy because that distribution controls the emitted radiation

From Momentum distribution to Energy Distribution: $f(p) \longrightarrow N(E)$

Non-Thermal Radiation

Particle energy distribution

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Spectrum is determined by the underlying energy distribution of the particles

=> **particle (electron) energy distribution**

$$N_e(\gamma)\,d\gamma$$

Number of electrons per unit volume in $[\gamma,\gamma+d\gamma]$

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*Why **electron** energy distribution?*

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*Why **electron** energy distribution?*

- Formally: *particle/lepton distribution* (electrons + positrons)
- In astrophysical environments, e^- dominate in number
- Radiative processes (synchrotron, IC, bremsstrahlung) depends on square of the charge (q^2) is the same for e^- and e^+ with the same γ and emit *identical radiation*
- Differences only appear in specific channels (e.g. pair annihilation e^+e^-)

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=> **particle (electron) energy distribution**

$$N_e(\gamma)\,d\gamma$$

Number of electrons per unit volume in [γ,γ+dγ]

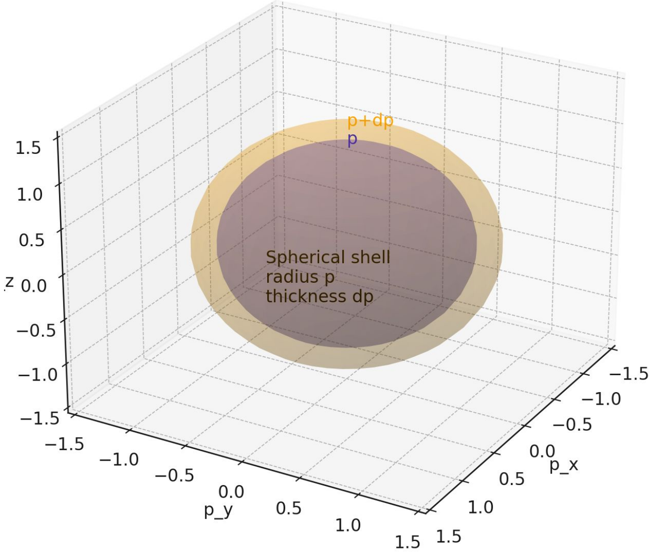
Non-thermal distributions are power laws, controlled by acceleration physics

Typical power-law distribution: $N_e(\gamma) = K_e\,\gamma^{-p}$ $\gamma_{\min} \leq \gamma \leq \gamma_{\max}$

Non-Thermal Radiation

Particle energy distribution

From Momentum distribution to Energy Distribution: $f\,(\,p\,) \longrightarrow N(\,E\,) \dashrightarrow \boxed{N_e(\gamma)}$



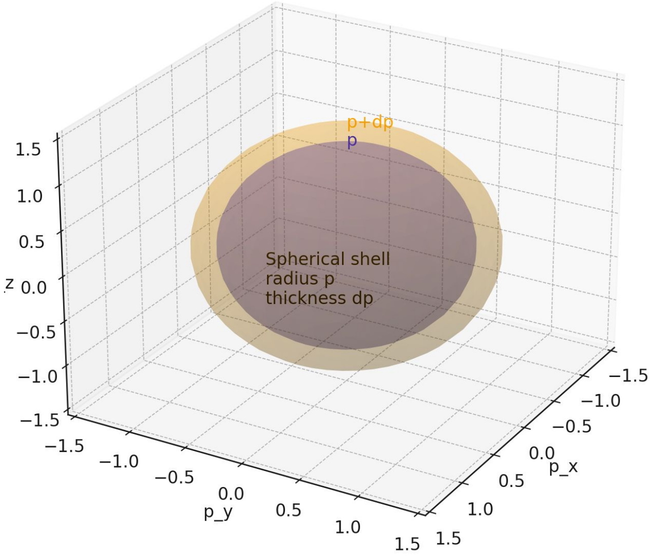
Shell volume: $dV_p = 4\pi p^2\,dp$

Non-Thermal Radiation

Particle energy distribution

From Momentum distribution to Energy Distribution: $f(p) \longrightarrow N(E) \dashrightarrow N_e(\gamma)$

Isotropic momentum distribution $f(p)$



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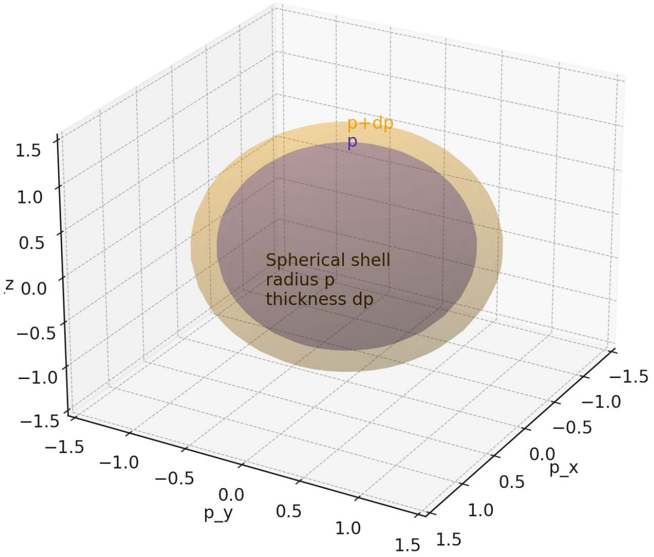

Shell volume: $dV_p = 4\pi p^2\,dp$

Isotropic momentum distribution $f(p)$

Number of particles per unit volume in $[p, p + dp]$:

$$dn = f(p)\,dV_p = 4\pi p^2 f(p)\,dp$$

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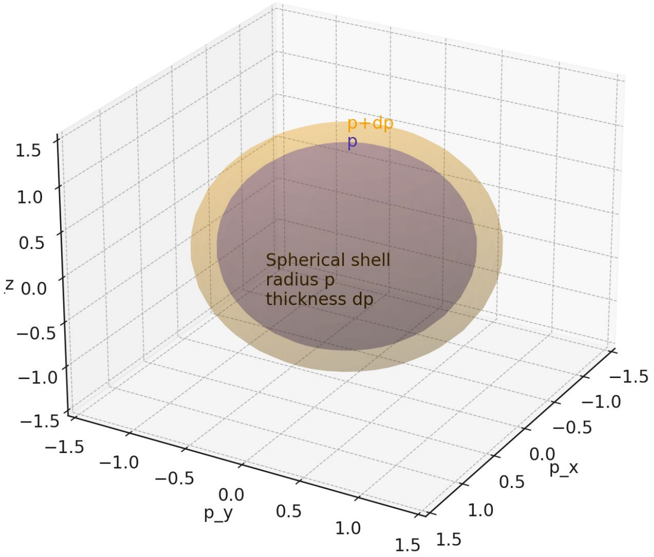

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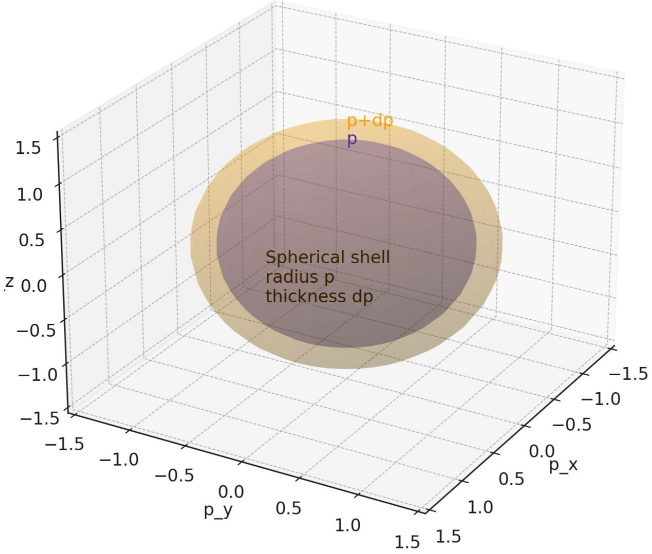

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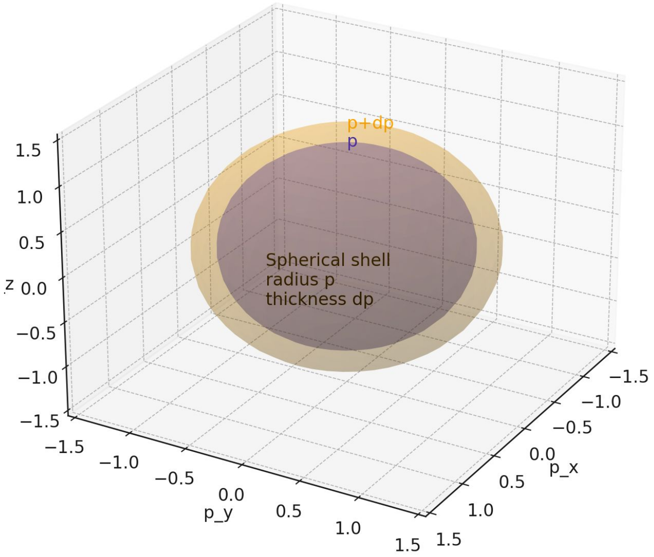

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$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Non-Thermal Radiation

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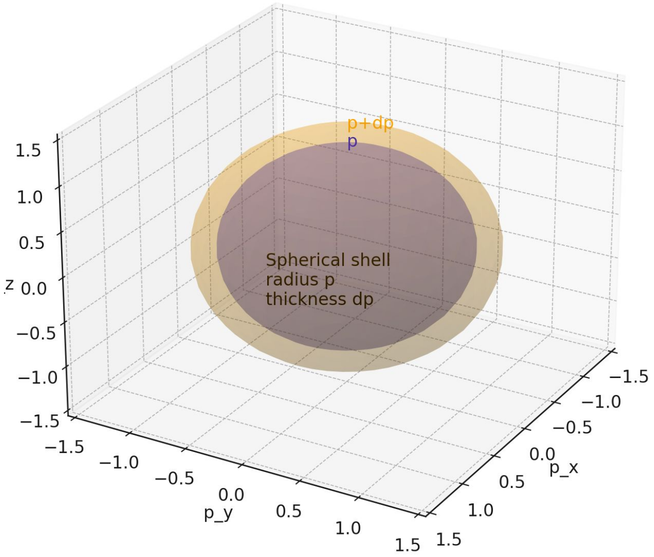

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If $v \ll c$: $\gamma \approx 1$: The particle is almost non-relativistic

If $v \to c$: $\gamma \gg 1$: The particle is ultra-relativistic

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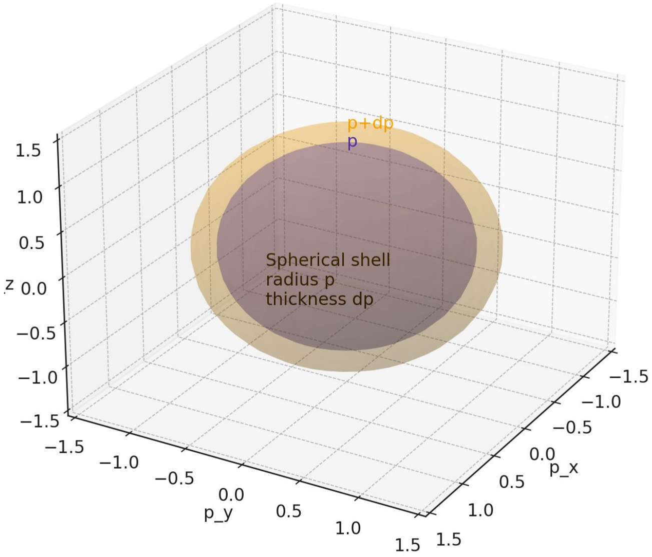
Almost all leptonic radiation comes from e^- with $\gamma \gg 1$



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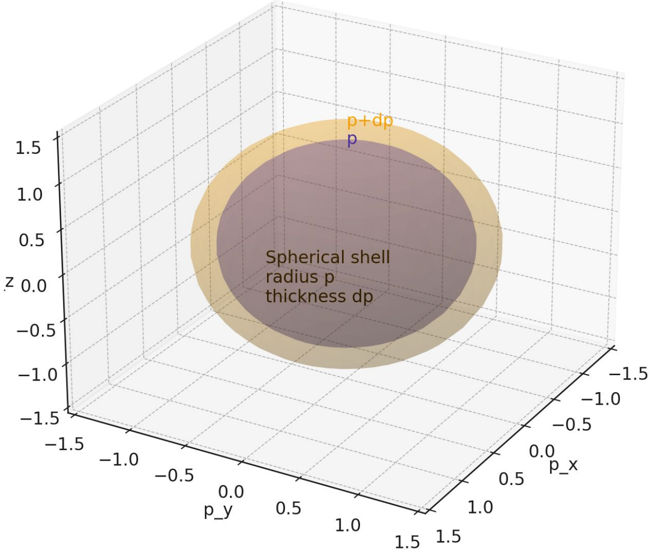

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In ***astrophysics****, γ is the natural variable to connect relativistic particles with the radiation they produce*

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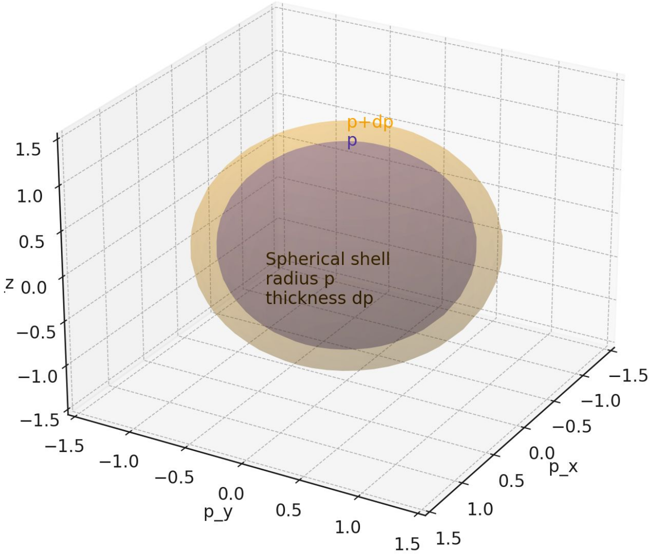
Hadronic processes expressed in terms of E (not γ) → why?



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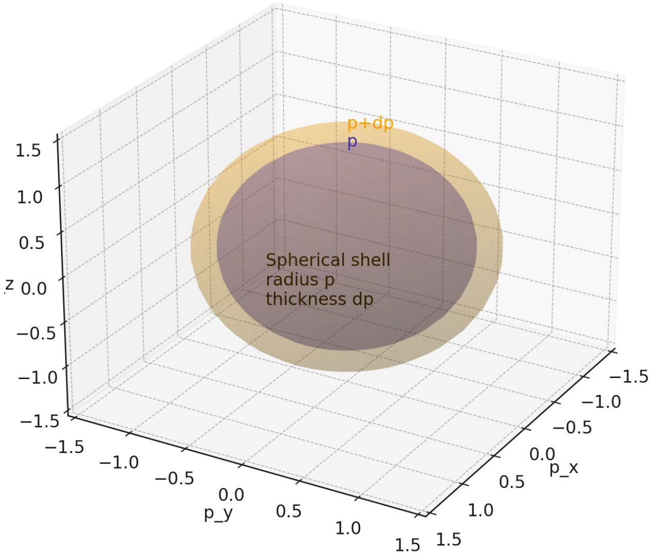

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Change variable to *Lorentz factor* γ:

- Protons are much heavier than e^- => not always ultra-relativistic
- In astrophysical collisions (GeV–TeV), protons are relativistic but not as ultra-relativistic as e^- → It is more natural to work with their kinetic or total energy
- Nuclear physics cross sections are tabulated as functions of proton lab energy E_p, not γ
- ***Proton synchrotron radiation*** is negligible compared to e^- → What matters is energy available to produce pions, best described by E_p or T_p

=> hadronic interactions are conventionally expressed in terms of proton energy

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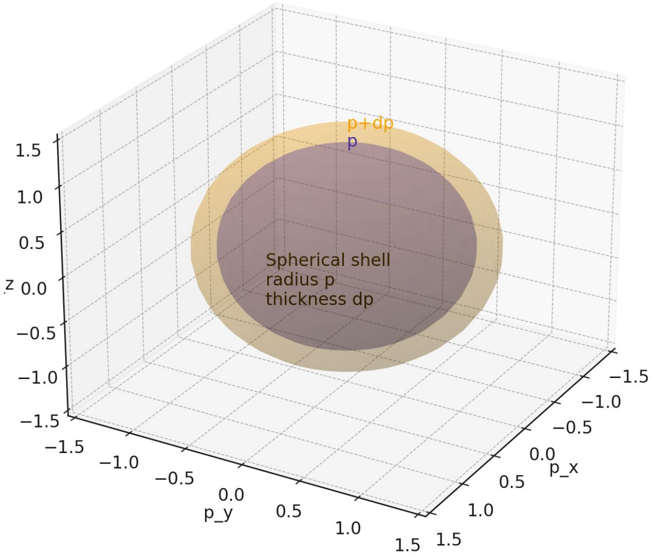

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In terms of energy:

$$E = \gamma m_e c^2 \longrightarrow$$

γ measures the total energy in units of the rest energy $m_e c^2$

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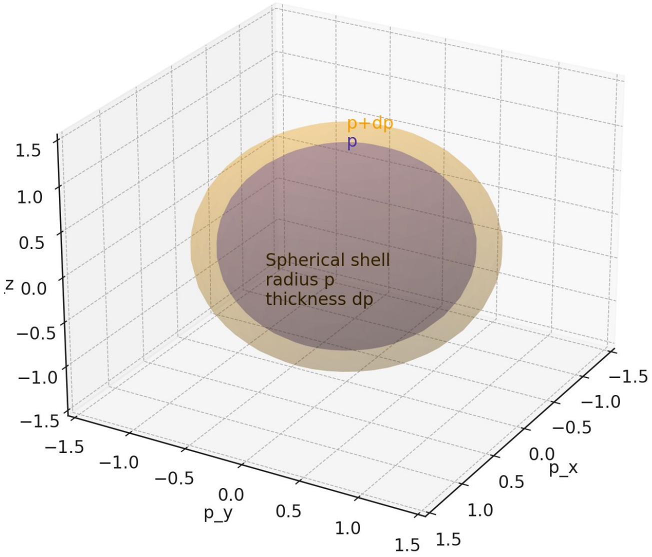

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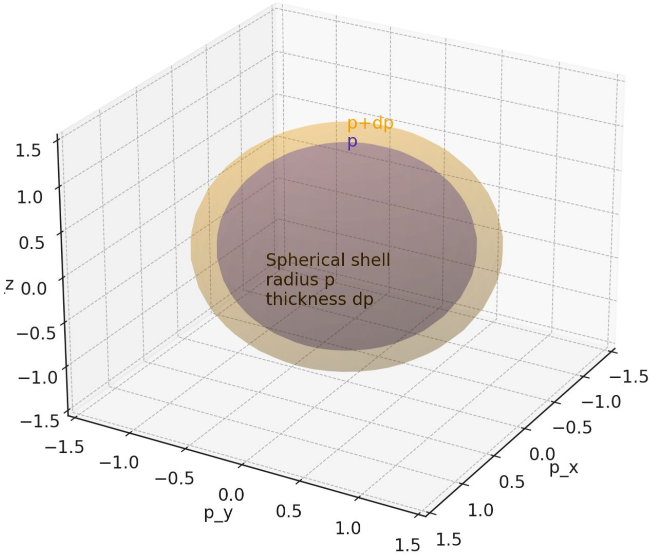

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Relativistic energy–momentum relation:

$$E^2 = (pc)^2 + (m_e c^2)^2$$

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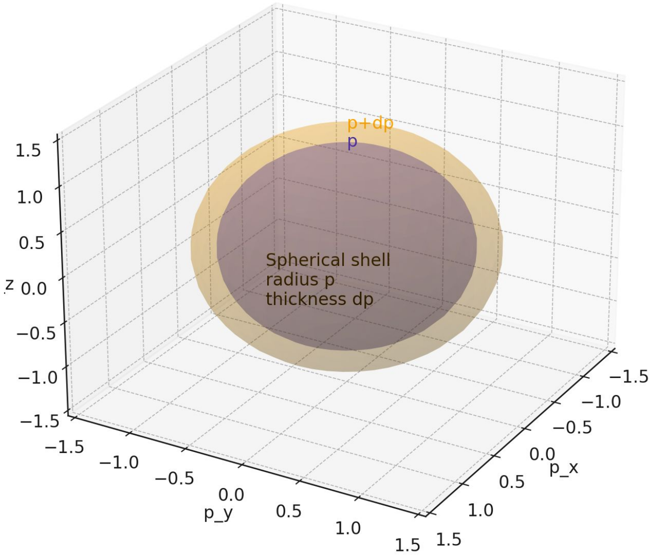

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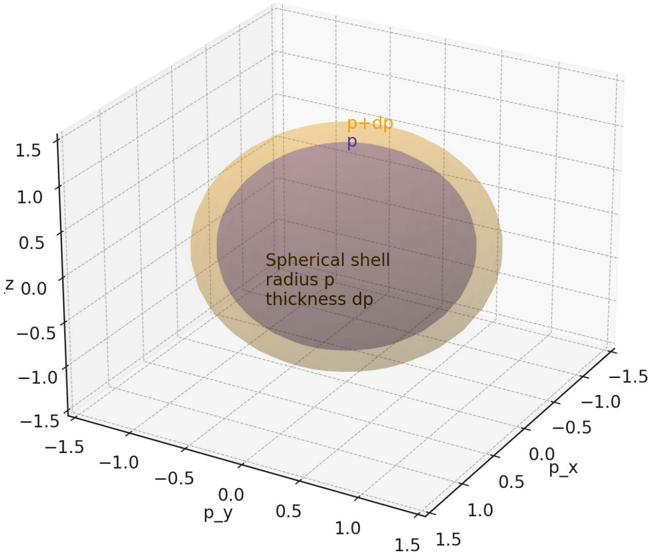

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=> $(\gamma m_e c^2)^2 = (pc)^2 + (m_e c^2)^2 \;\Rightarrow\; pc = m_e c^2\sqrt{\gamma^2 - 1}$ => $\boxed{p(\gamma) = m_e c\sqrt{\gamma^2 - 1}}$

Non-Thermal Radiation

Differentiate to get the Jacobian *dp/dγ:*

$$\boxed{\frac{dp}{d\gamma} = m_e c \, \frac{\gamma}{\sqrt{\gamma^2 - 1}}}$$

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Ultra-relativistic simplifications:

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Non-Thermal Radiation

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In the ultra-relativistic limit ($E \gg m_e c^2$, $v \simeq c$):

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If we want the number density of particles/electrons between $[\gamma, \gamma + d\gamma]$, we define:

$$N_e(\gamma) \, d\gamma \equiv dn$$

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Using:

$$dn = f(p) \, dV_p = 4\pi p^2 f(p) \, dp$$

Non-Thermal Radiation

Particle energy distribution

Therefore:

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We obtain (general equation):

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Ultra-relativistic simplification:

$$p(\gamma) \simeq m_e c\,\gamma \qquad \frac{dp}{d\gamma} \simeq m_e c \qquad \longrightarrow \qquad \boxed{N_e(\gamma) = 4\pi(m_e c)^3\,\gamma^2 f(\gamma m_e c)}$$

Non-Thermal Radiation

Therefore:

$$N_e(\gamma) = 4\pi p^2 f(p)\,\frac{dp}{d\gamma}$$

Using:

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BUT: this is a general relation: no spectral shape yet! ⟶ Power law distribution

Non-Thermal Radiation

Shock acceleration (Diffusive Shock Acceleration), DSA* results in:

$$f(p) \propto p^{-s}$$

Non-Thermal Radiation

Particle energy distribution

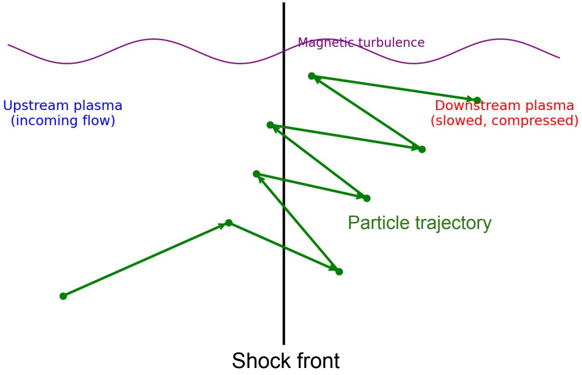
Shock acceleration (Diffusive Shock Acceleration), DSA* results in:

$f(p) \propto p^{-s}$ ← spectral index in momentum and, it is determined by the shock compression ratio

with s~4 for strong shocks

****DSA** is the standard mechanism for accelerating charged particles at astrophysical shocks:*

- Particles scatter on magnetic turbulence and cross the shock multiple times
- Each crossing gives a systematic energy gain
- Produces power-law energy distributions



Particles gain energy by repeatedly crossing the shock
→ Diffusive Shock Acceleration (DSA) → power-law spectrum

for relativistic particles ($E \approx pc$):

$$N(E)\,dE \;\propto\; f(p)\,4\pi p^2 dp \;\propto\; E^{-p}\,dE \quad \text{where:} \quad p = s - 2$$

for s=4:

$$N(E) \propto E^{-p}, \quad p \simeq 2$$

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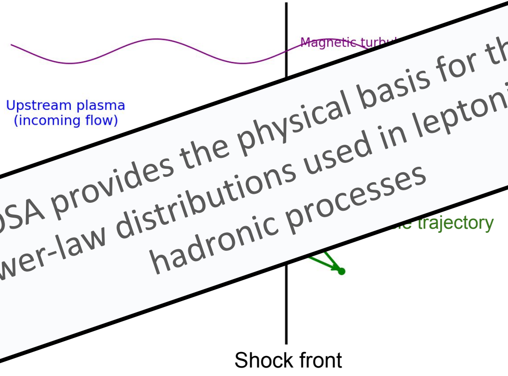

DSA provides the physical basis for the power-law distributions used in leptonic and hadronic processes

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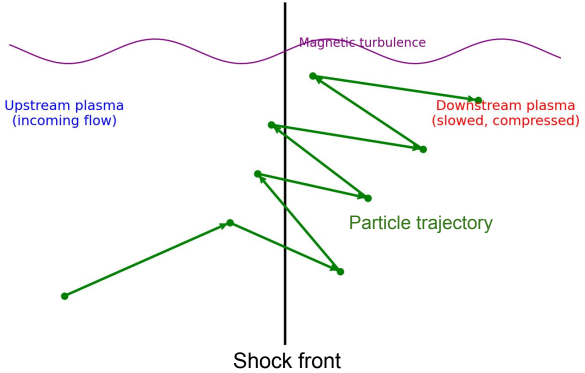

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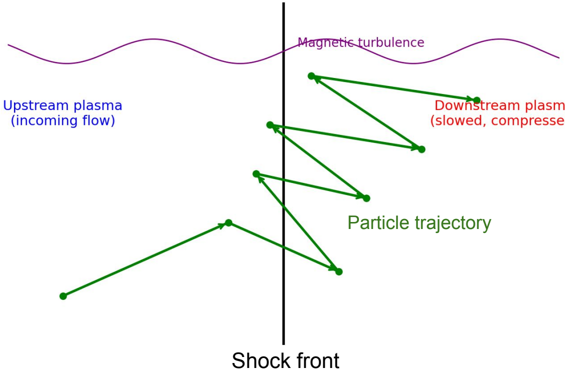

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$\longrightarrow$

Shock acceleration predicts $p \simeq 2$; in real sources, spectra are steeper $p \sim 2.2 - 3$

→ due to *energy losses and particle escape and propagation*

Non-Thermal Radiation

Particle energy distribution

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→

- Energy distribution depends on slope:
  - for *p > 2* low-energy particles dominate
  - For *p < 2* high-energy particles dominate

Non-Thermal Radiation

When we have a power-law spectrum:

- The shape of the spectrum (its slope) is determined by the *spectral index in energy/Lorentz factor p*
- K_e fixes the overall scale, and is determined by the *total number (or energy) of particles*

→ The normalization constant K_e must be determined!

Normalization

There are two common ways:

- Normalize to the ***total number density*** of particles *n*
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Non-Thermal Radiation

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This allows us to solve for K_e:

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Non-Thermal Radiation

$$n_e = K_e \int_{\gamma_{\min}}^{\gamma_{\max}} \gamma^{-p} \, d\gamma$$

- If $p \leq 1$: the integral diverges at the high-energy end ($\gamma \to \infty$), unless we impose a finite γ_{max}
- If $p \geq 1$: the low-energy end dominates, requiring a finite γ_{min}

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If instead of knowing the number of particles, what we know is how much *energy* is available to accelerate them → *total energy density:*

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And the power law $N_e(\gamma) = K_e\, \gamma^{-p}$:

$$U_e = m_e c^2 K_e \int_{\gamma_{\min}}^{\gamma_{\max}} \gamma^{1-p}\, d\gamma$$

Non-Thermal Radiation

Normalization

- Evaluating the integral for $p \neq 2$:

$$U_e = m_e c^2 \, K_e \, \frac{\gamma_{\max}^{2-p} - \gamma_{\min}^{2-p}}{2-p}$$

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if $p < 2$:

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if $p = 2$:

- the contribution comes equally from both γ_{min} and γ_{max}, giving a logarithmic dependence $\ln(\gamma_{max}/\gamma_{min})$

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We study electron distributions $N_e(\gamma)$, but in astrophysics we observe **radiation**: ***photon spectra***

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Electron spectrum: $N_e(\gamma) = K_e\,\gamma^{-p}$ **,** $\gamma_{\rm min} \leq \gamma \leq \gamma_{\rm max}$

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- A relativistic electron with Lorentz factor γ radiates a power spectrum at frequency ν:

$$P_\nu(\nu, \gamma)$$

Non-Thermal Radiation

Emissivity

- The electron population is described by the differential number density $N_e(\gamma)$ in γ and $\gamma+d\gamma$:

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If electrons radiated at a single frequency, the photon spectrum would directly follow N_e*(γ).* Each electron radiates over a broad frequency range (synchrotron, IC, bremsstrahlung*), so the observed spectrum is a convolution of the electron distribution and the single-particle emission physics

---

Synchrotron emission spreads over many frequencies, IC scatters photons into a wide energy range, and bremsstrahlung also covers a spectrum. This means the photon spectrum we measure is not just N_e(γ),* but the result of combining it with the physics of the emission mechanism

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Non-thermal leptonic sources often produce photon spectra that follow power laws, directly reflecting the ***underlying electron distribution***

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- Non-Thermal radiation: hadronic processes (recap) vs. leptonic processes
- Electron distributions & normalizations
- Leptonic processes:
    - Synchrotron emission
    - Inverse Compton scattering
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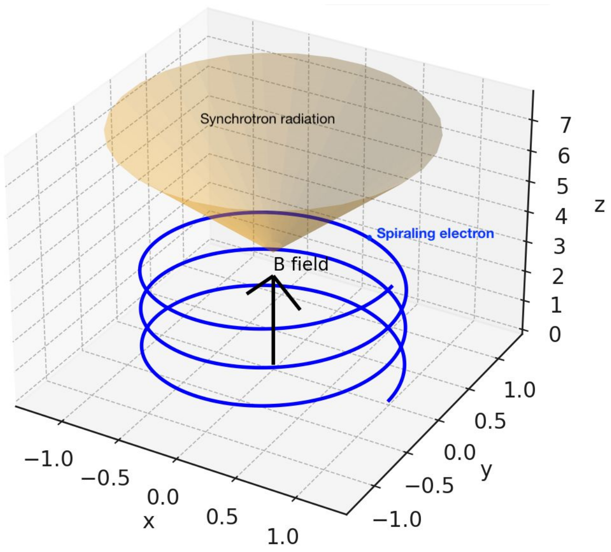
Synchrotron emission

Synchrotron emission is one of the key leptonic processes in astrophysics

- Produced by relativistic *electrons* (or *positrons*) spiraling around magnetic field lines



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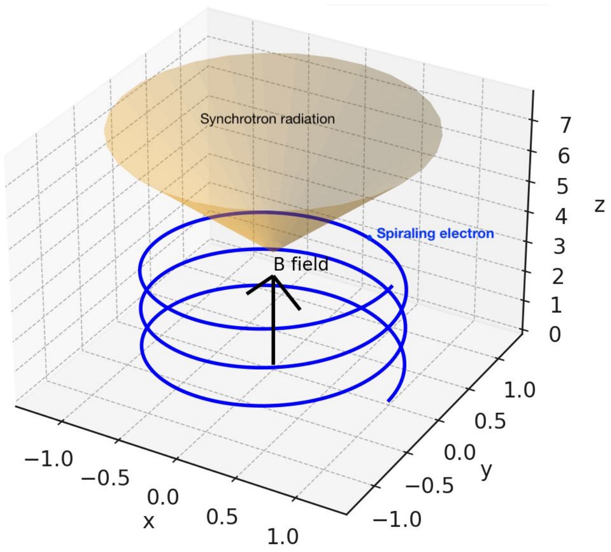

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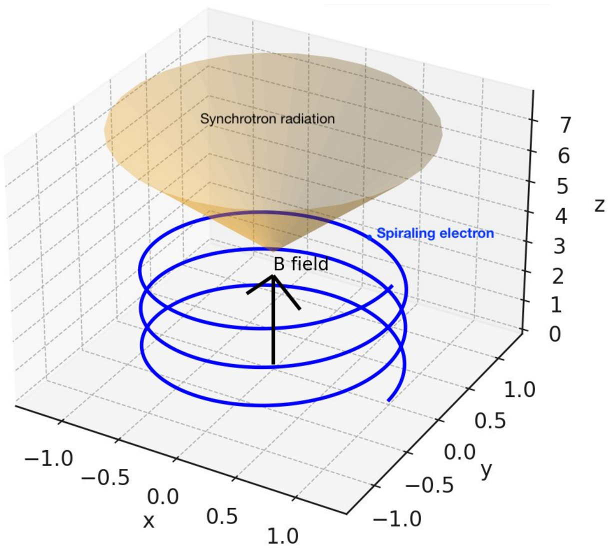

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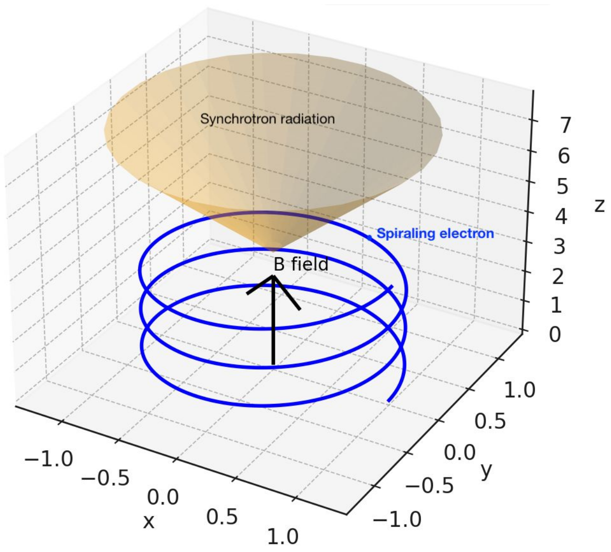

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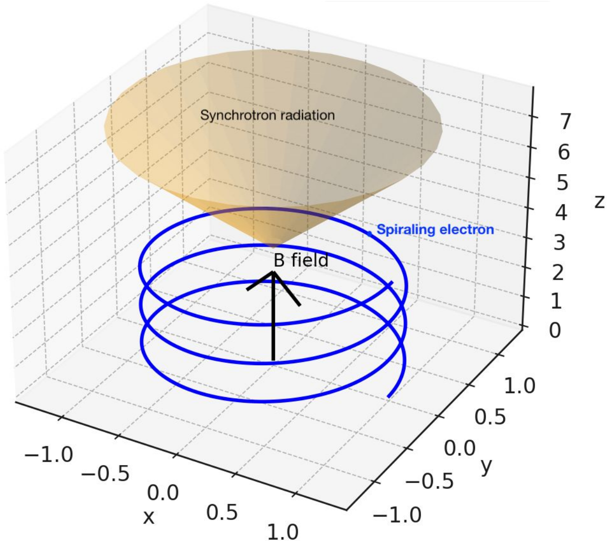
- Very common in SNRs, AGN jets, pulsar wind nebulae



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- The exact synchrotron spectrum of one electron is mathematically complex (involves Bessel functions)
- However, most of the emitted power is concentrated around a typical frequency, ν_c
- In astrophysics, we first define ν_c : it provides the characteristic scale of emission before dealing with the full spectrum

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Let's derive the expression for the *synchrotron characteristic frequency* ν_c *:*

Non-Thermal Radiation

In the classical (cyclotron) regime, an electron spiraling in a magnetic field B emits radiation at the frequency v_B

Classical (Cyclotron):

- Non-relativistic electrons ($v \ll c$)
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- Radiation at a single frequency (the gyrofrequency):

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- Ultra-relativistic electrons ($\gamma \gg 1$)
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α: pitch angle (between velocity and B)

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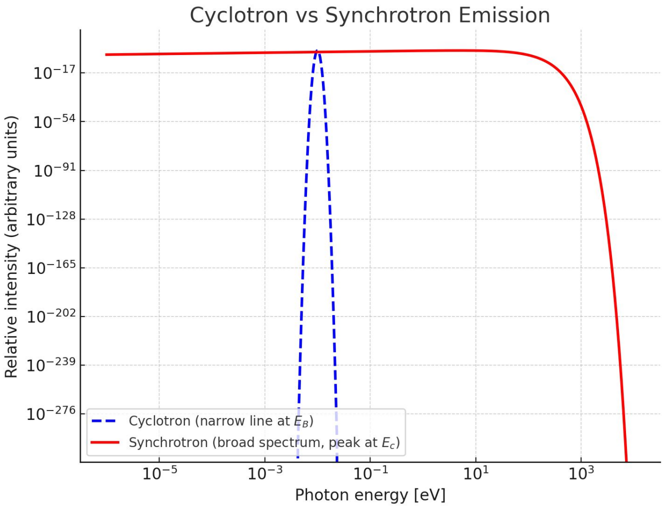

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Universal function

Modified Bessel function of
the second kind of order 5/3

Non-Thermal Radiation

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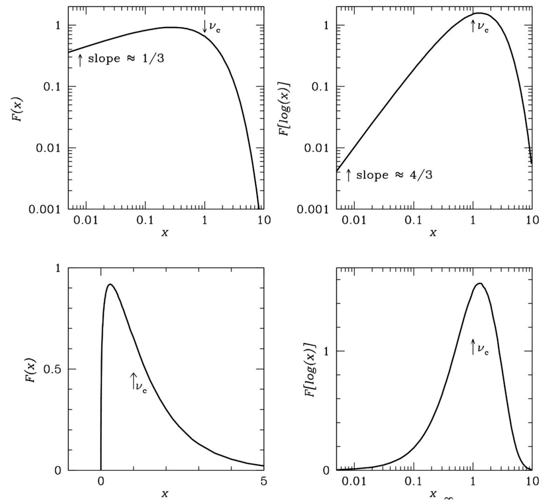

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A single electron produces a spectrum – rising as $\nu^{1/3}$ at low frequencies, peaking around $0.3\nu_c$, and falling exponentially at high frequencies

Non-Thermal Radiation

Synchrotron emission

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Total synchrotron emissivity is obtained by integrating over all electrons:

$$j_\nu = \int_{\gamma_{\min}}^{\gamma_{\max}} N_e(\gamma)\, P_\nu(\nu, \gamma)\, d\gamma$$

Non-Thermal Radiation

For large ranges in γ, the integral yields:

$$j_\nu \;\propto\; \nu^{-\alpha}, \quad \alpha = \frac{p-1}{2}$$

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- With telescopes, we measure the ***photon spectral index α*** (the slope of the spectrum in radio/X/γ)
- From acceleration theory (e.g. DSA), we predict the ***electron index p***

Supernova remnants: p~2 → radio spectra with *α*~0.5

Radio galaxies/jets: p~2.5–3 → steeper spectra with *α*~0.8–1

Non-Thermal Radiation

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⇒ Cooling modifies the electron spectrum over time, producing ***cooling breaks*** in the observed synchrotron spectrum (a change in slope between low and high frequencies)

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Synchrotron emission