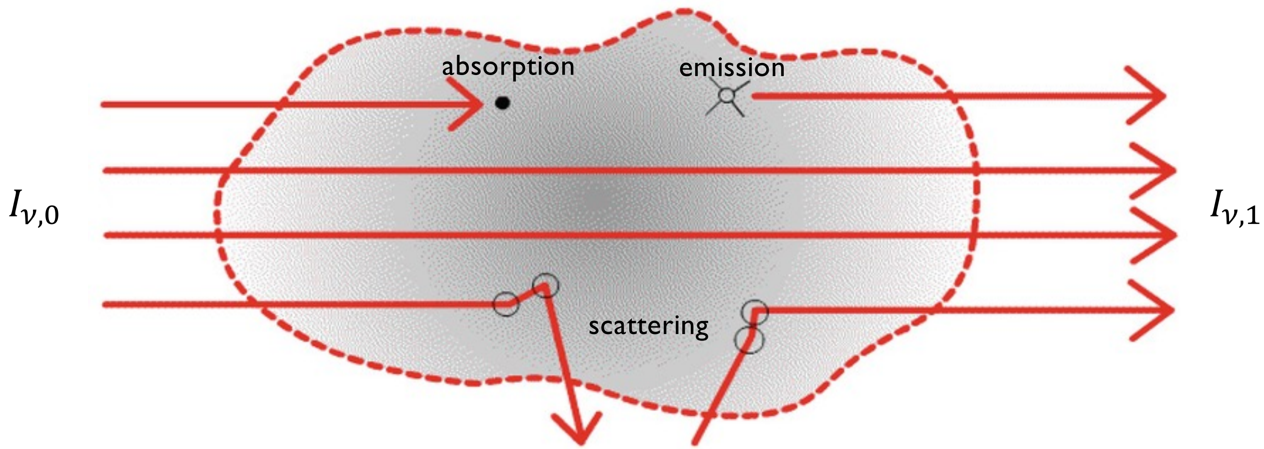
Radiative Transitions

Alexander Knebe (*Universidad Autonoma de Madrid*)

"Sure it's beautiful, but I can't help thinking about all that interstellar dust out there."

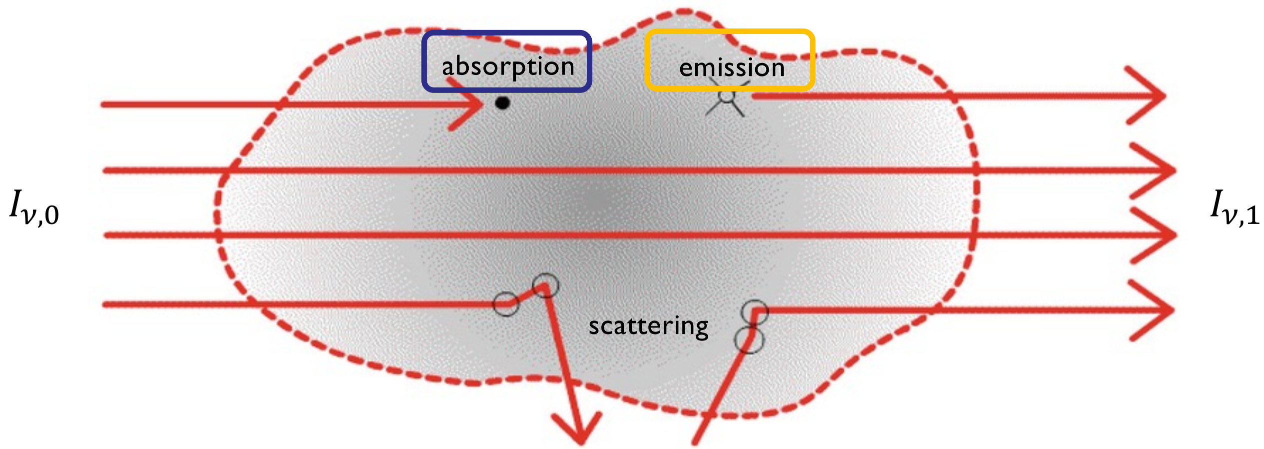
Radiative Transitions



$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$$S_\nu = \frac{j_\nu}{\alpha_\nu}$$

Radiative Transitions

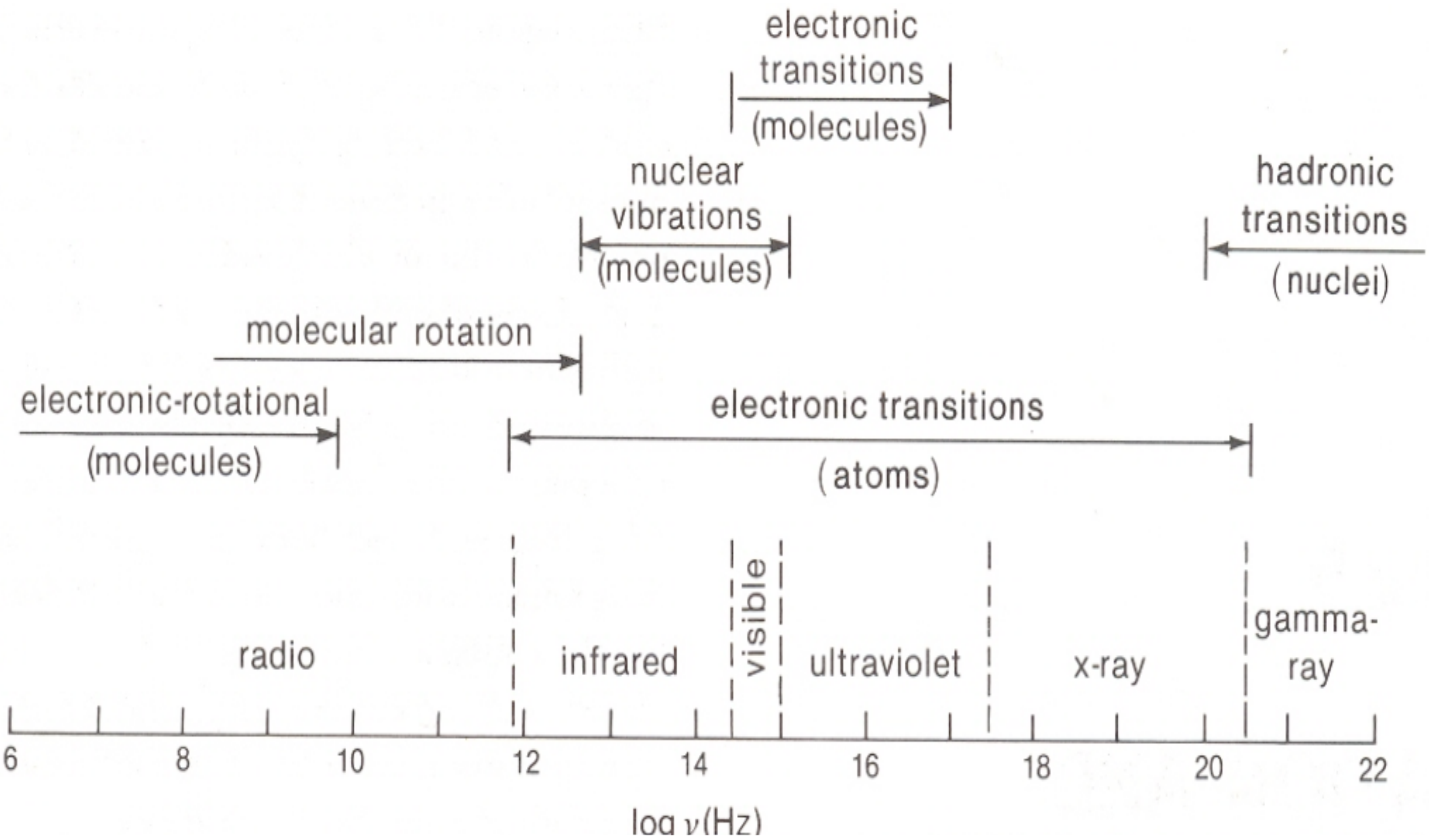


$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$$S_\nu = \frac{j_\nu}{\alpha_\nu}$$

how to calculate?

the nature and origin of light produced in transitions



- thermal excitation
- atomic transitions
- Einstein coefficients
- line broadening

- **thermal excitation**
- atomic transitions
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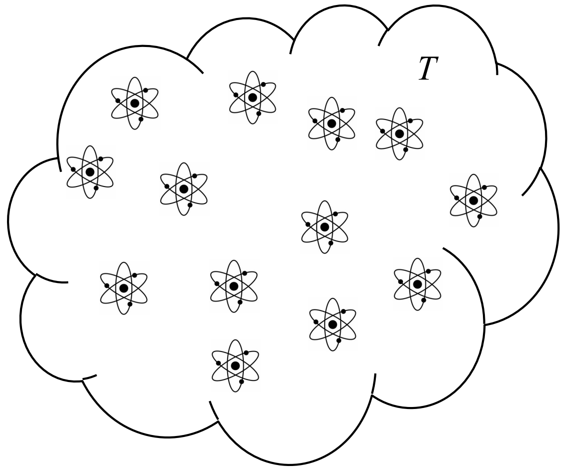
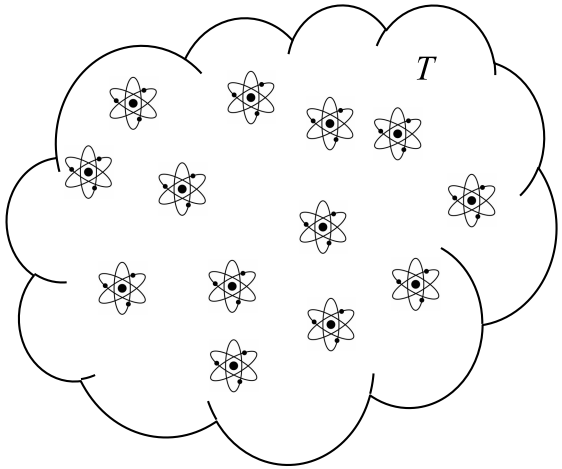
- Boltzmann distribution
- Saha equation

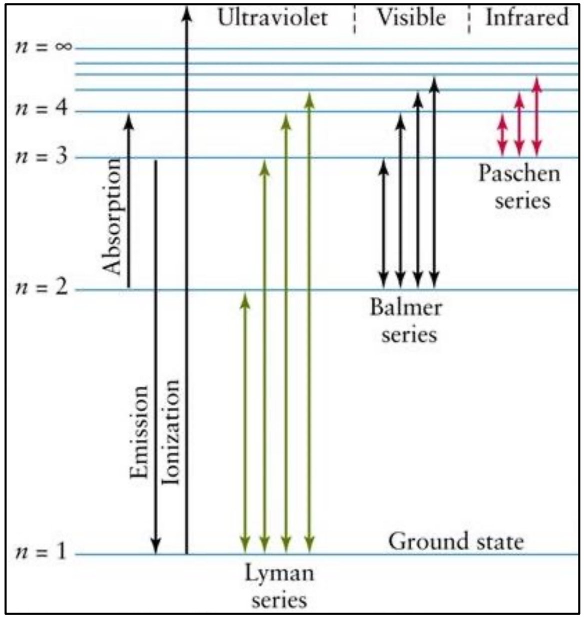
- **Boltzmann distribution**
- Saha equation

- Boltzmann distribution
    - cloud of atoms at temperature T



- Boltzmann distribution
    - cloud of atoms at temperature T
    - each atom has certain energy levels





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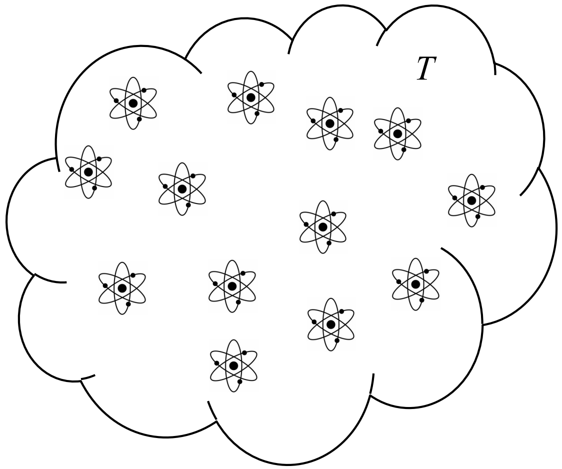

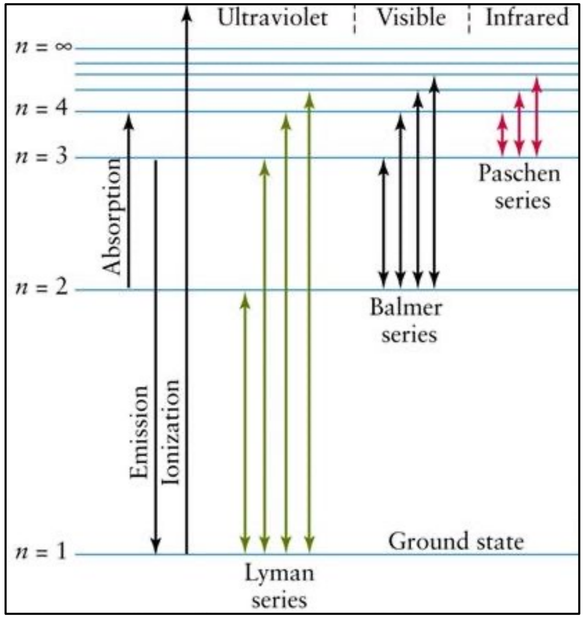

how many atoms N_n are at level n?
(compared to the ground state)

- Boltzmann distribution
    - cloud of atoms at temperature T
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    - Boltzmann distribution:

$$\frac{N_n}{N_1} = \frac{g_n}{g_1} e^{-\frac{(E_n - E_1)}{k_B T}}$$

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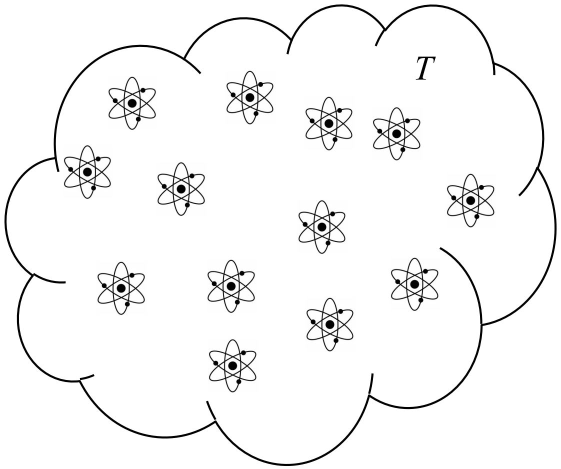

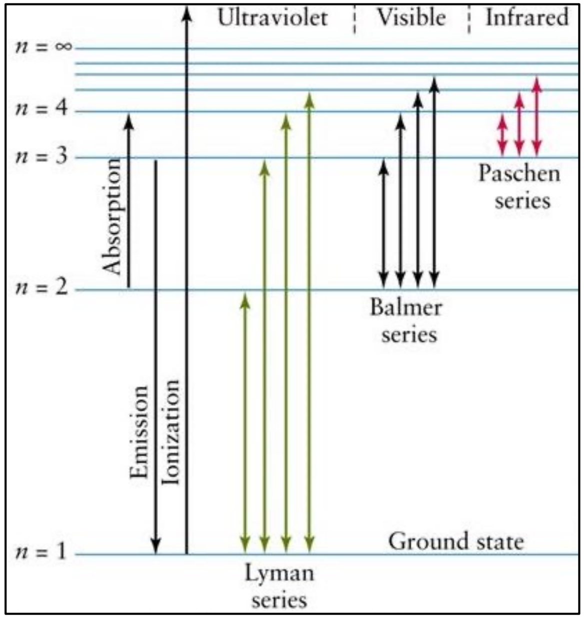

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how many atoms N_n are at level n?
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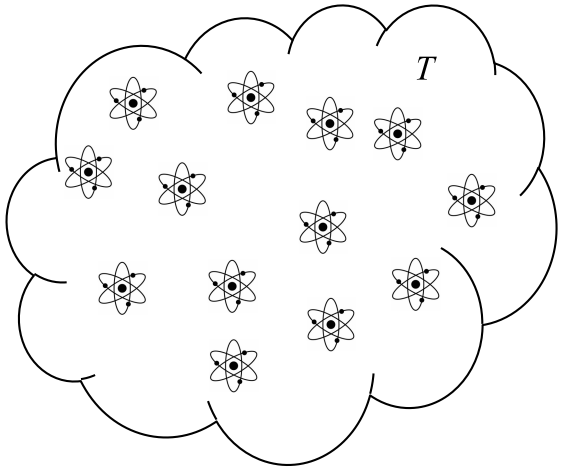

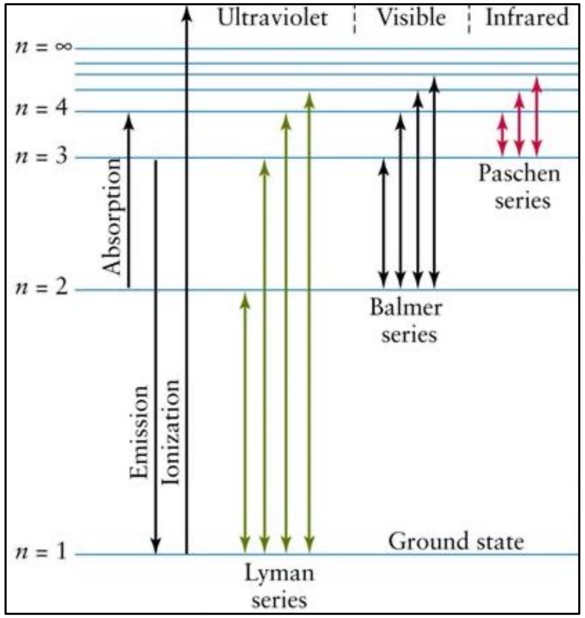

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$$\frac{N_n}{N_1} = \frac{g_n}{g_1} e^{-\frac{(E_n - E_1)}{k_B T}}$$

$$N = \sum_{n=1}^{\infty} N_n$$

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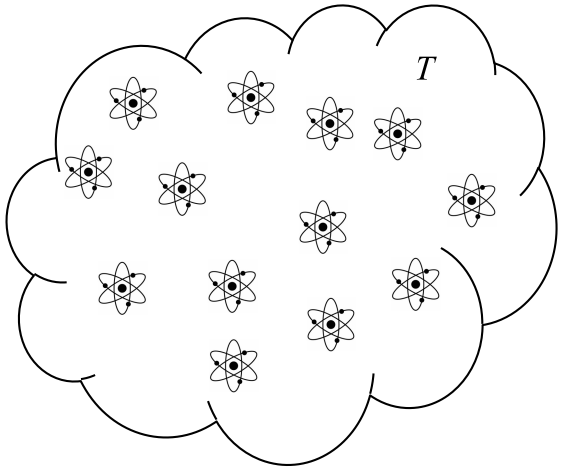

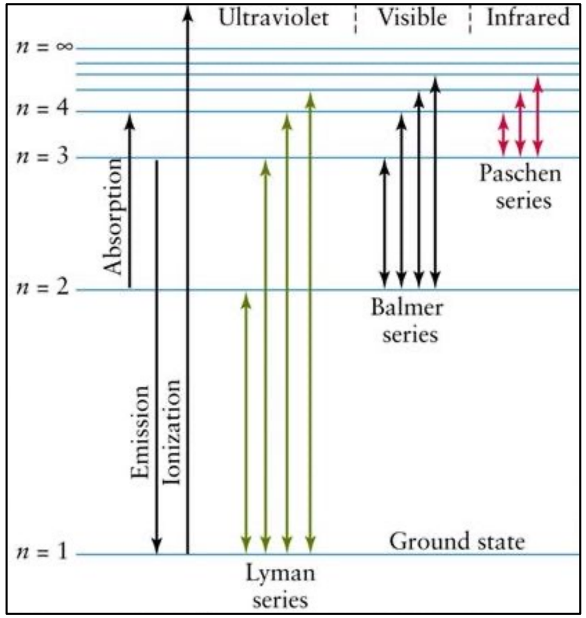

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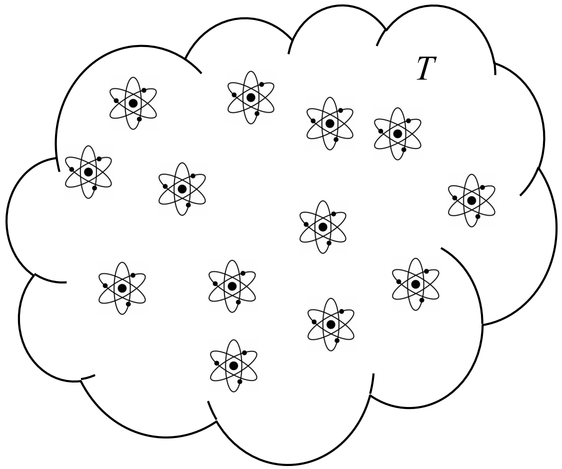

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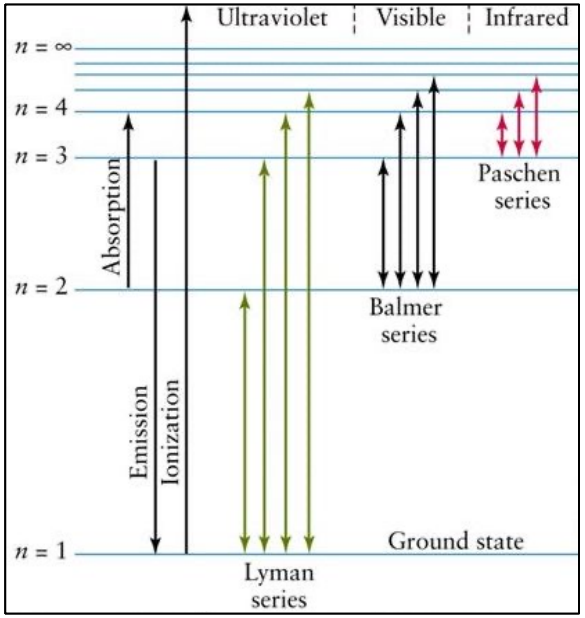

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$$Z(T) = g_1 \frac{N}{N_1} = g_1 \frac{\sum_{n=1}^{\infty} N_n}{N_1} = g_1 \sum_{n=1}^{\infty} \frac{N_n}{N_1} = g_1 + g_2 e^{-\frac{E_2}{k_B T}} + g_3 e^{-\frac{E_3}{k_B T}} + \ldots$$

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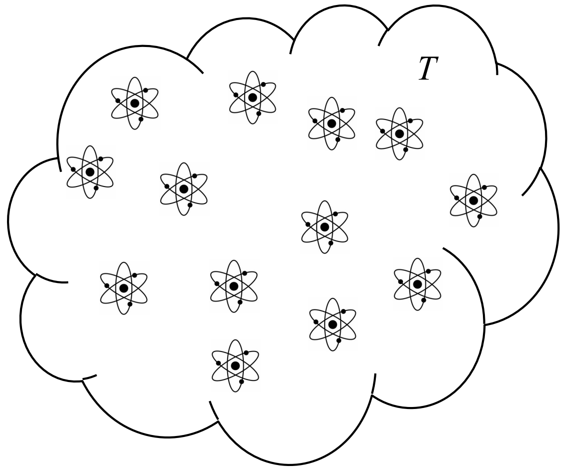

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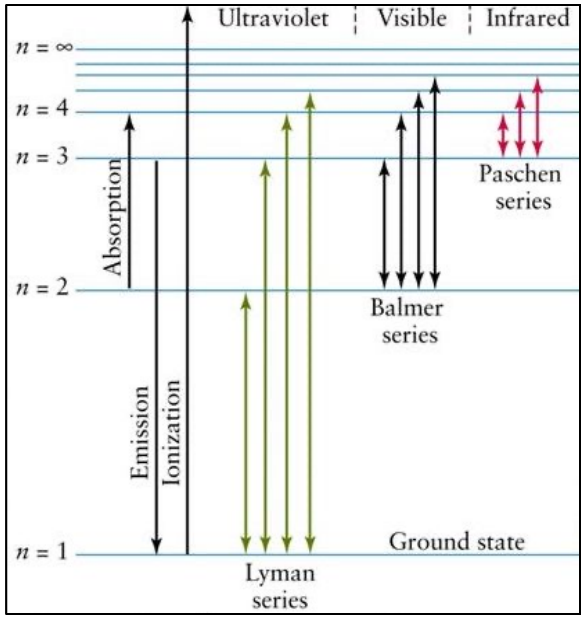

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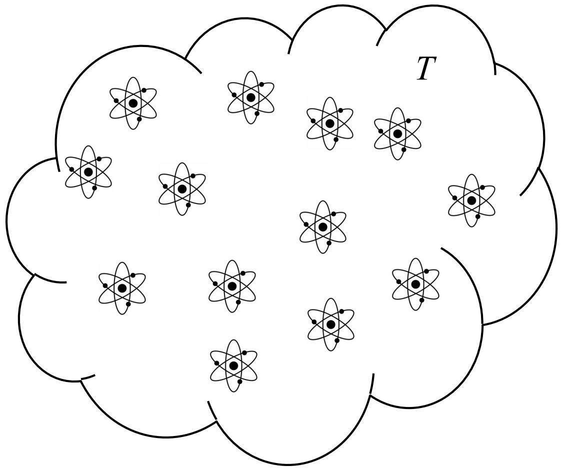
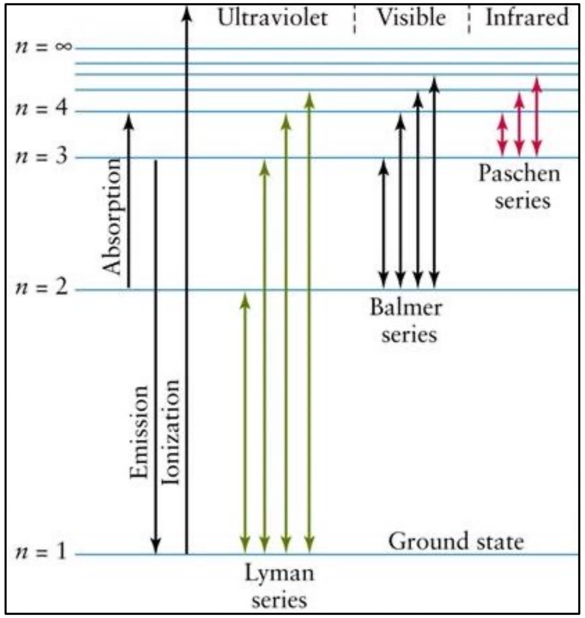
description of statistical properties

- Boltzmann distribution
- **Saha equation**

- Saha equation
    - cloud of atoms at temperature T





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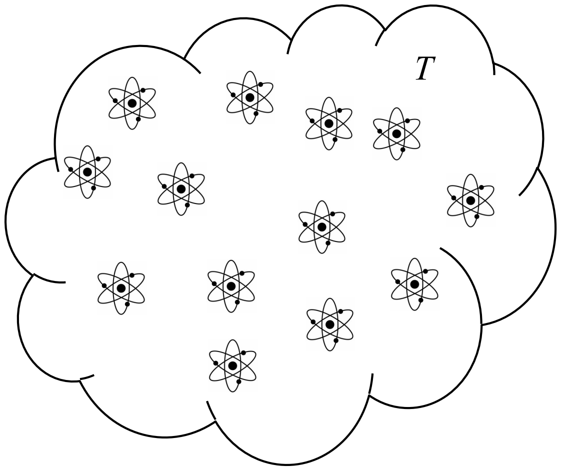
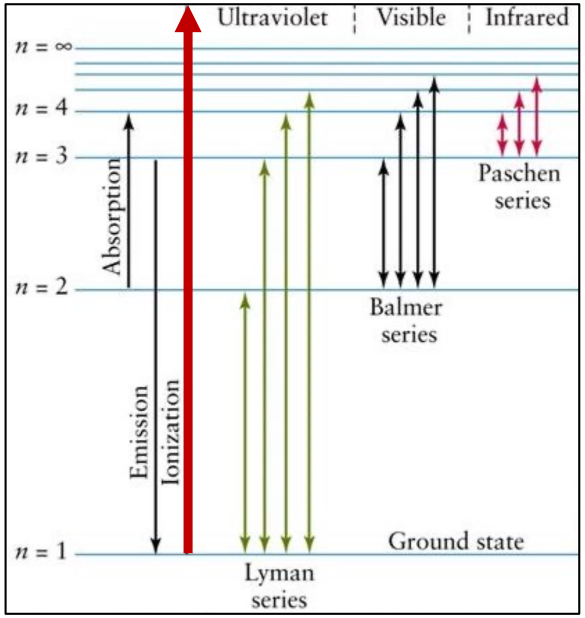
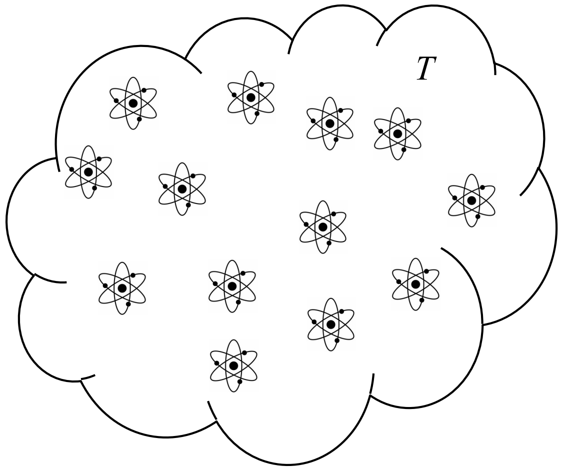
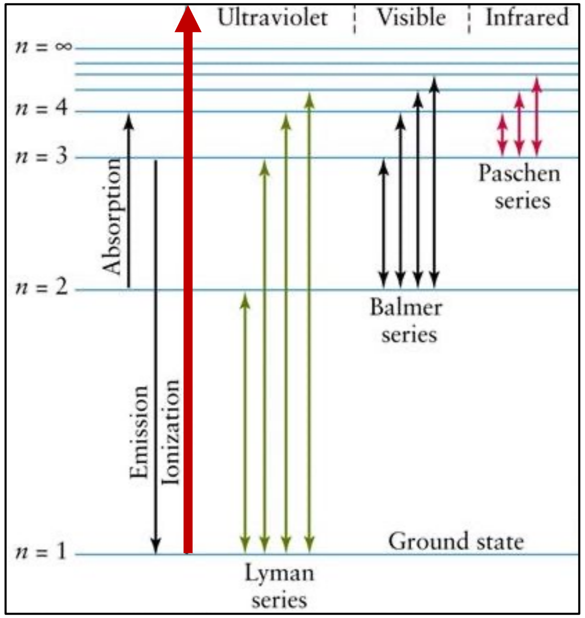




how many atoms are ionized?

- Saha equation
  - cloud of atoms at temperature T





how many atoms are ionized?

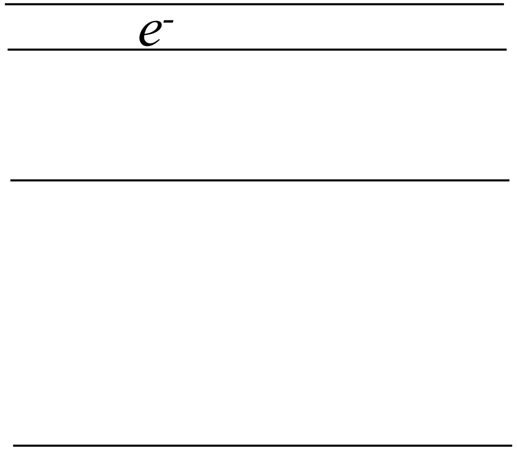
important for calculation of stellar spectra and the early Universe (where we need to know the number of free electrons)

▪ Saha equation

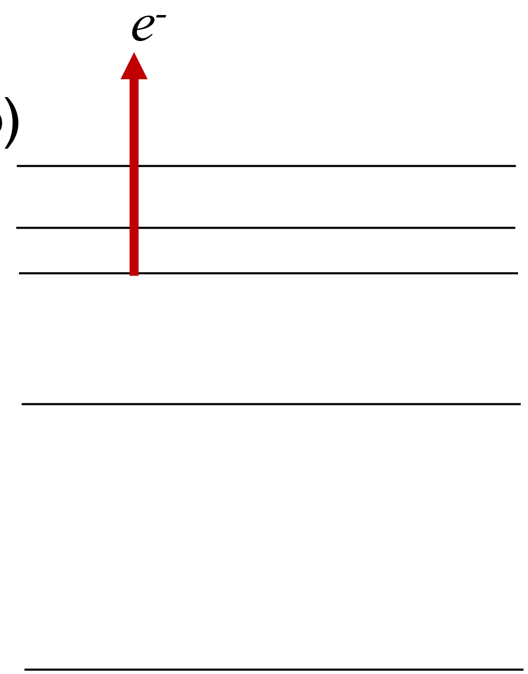
- cloud of atoms at temperature T - *how many atoms are ionized?*

$$A \leftrightharpoons A^{+} + e^{-}$$

a)



b)



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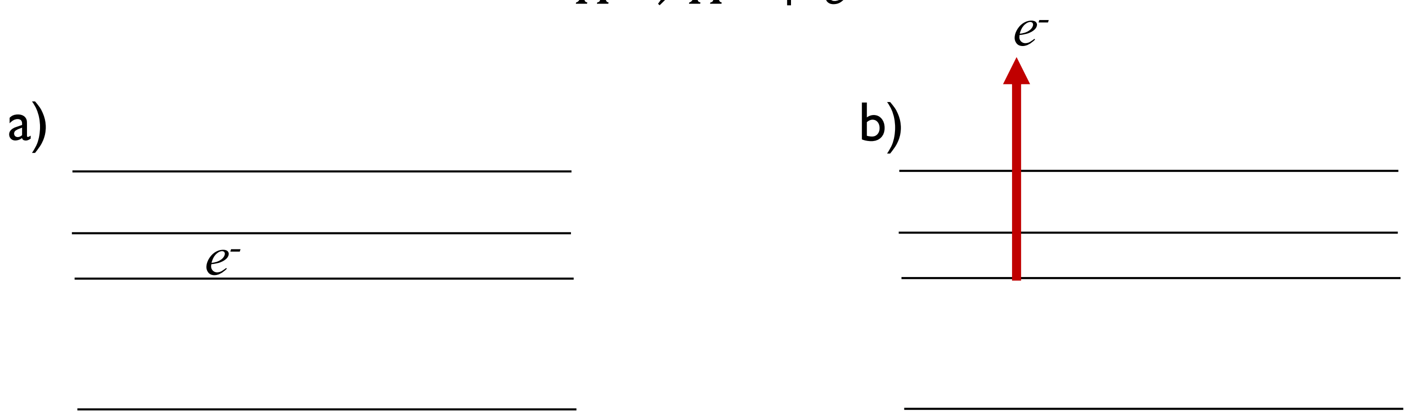

N_i atoms in ionisation state i

g_i statistical weight of state i

N_{i+1} atoms ionisation state $i+1$

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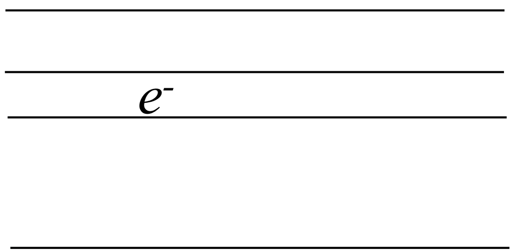

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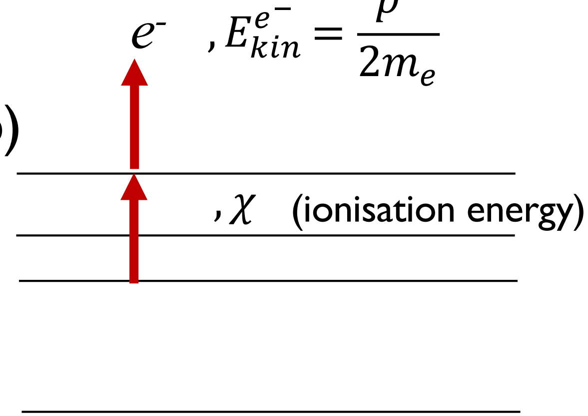

χ ionisation energy

$E = E_{kin}^{e^-} + \chi$ total electron energy

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$$E_{kin}^{e^-} = \frac{p^2}{2m_e}$$ free electron!

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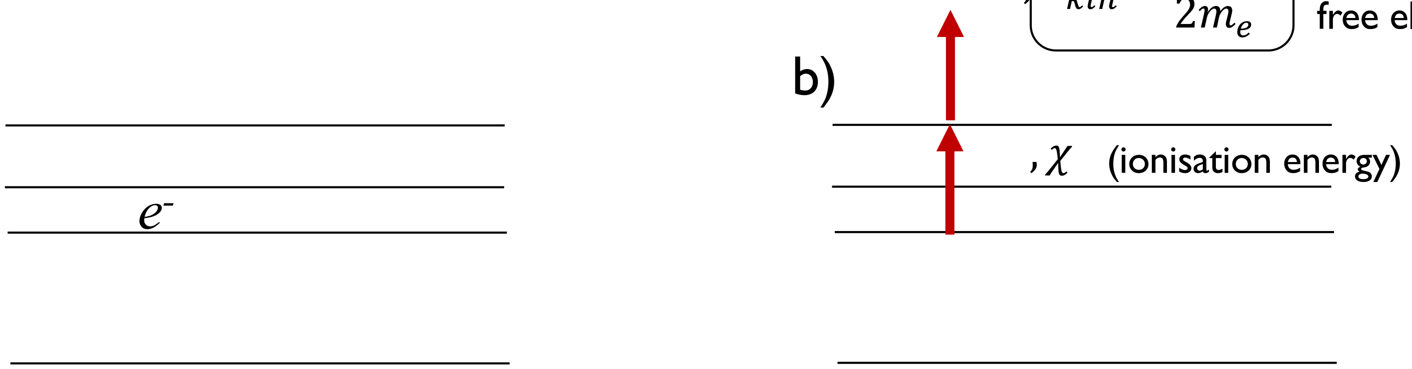

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g_e^{free} all possible states of electron with $[p, p + dp]$

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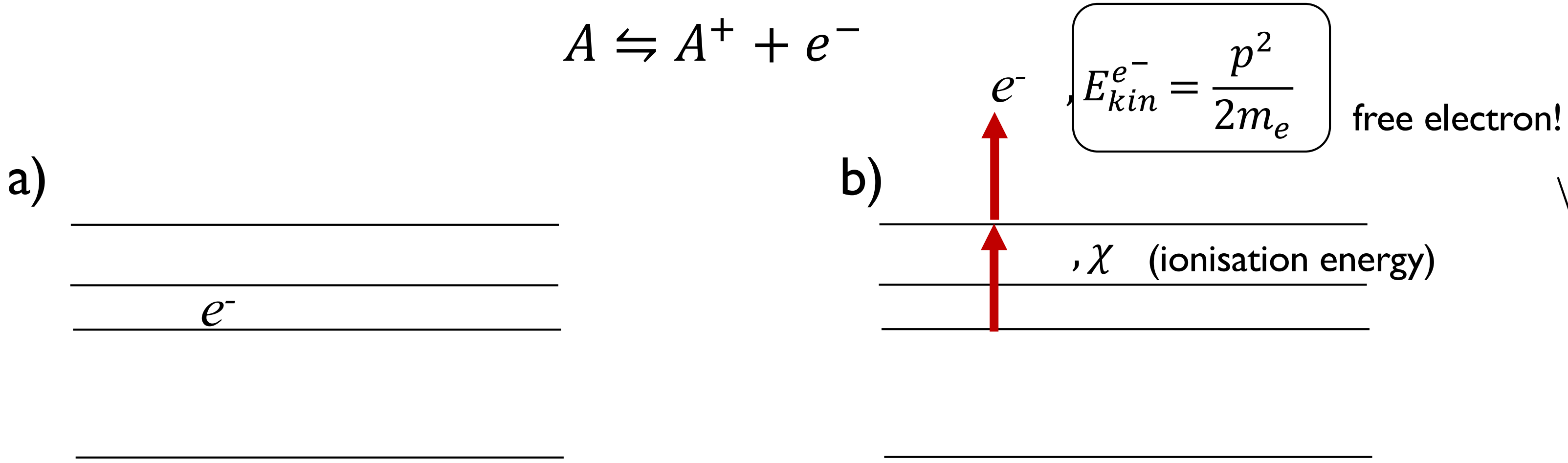

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describes the state of the (ionized) atom

g_e^{free} all possible states of electron with $[p, p+dp]$

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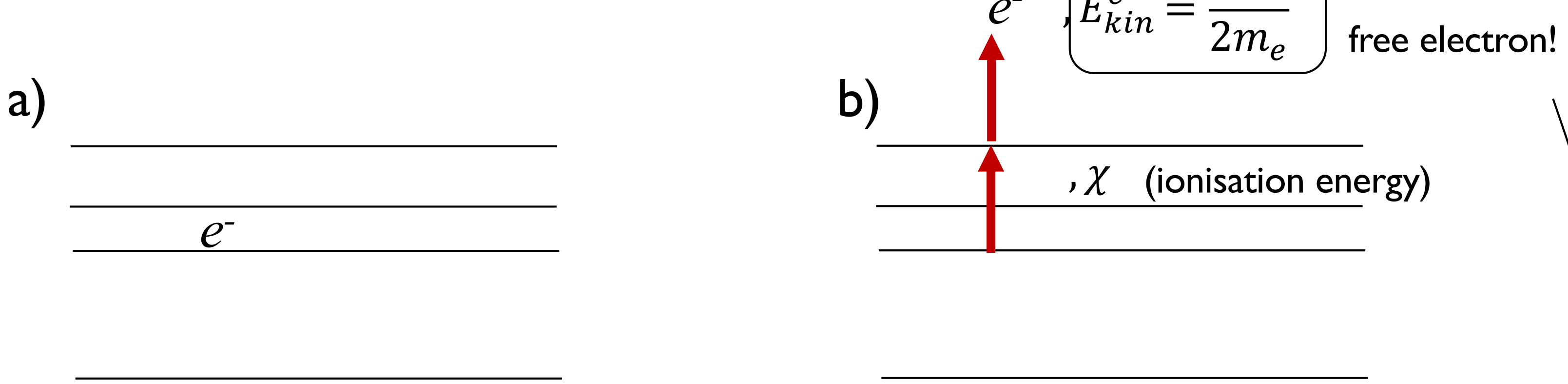

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describes the state of the free electron

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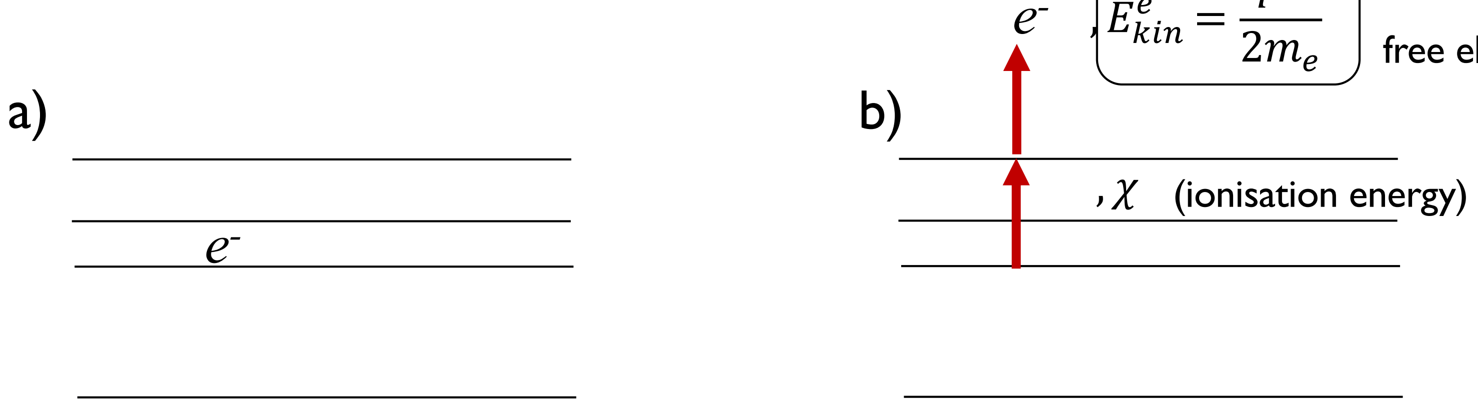

$E_{kin}^{e^-} = \frac{p^2}{2m_e}$ free electron!

putting all into Boltzmann law...

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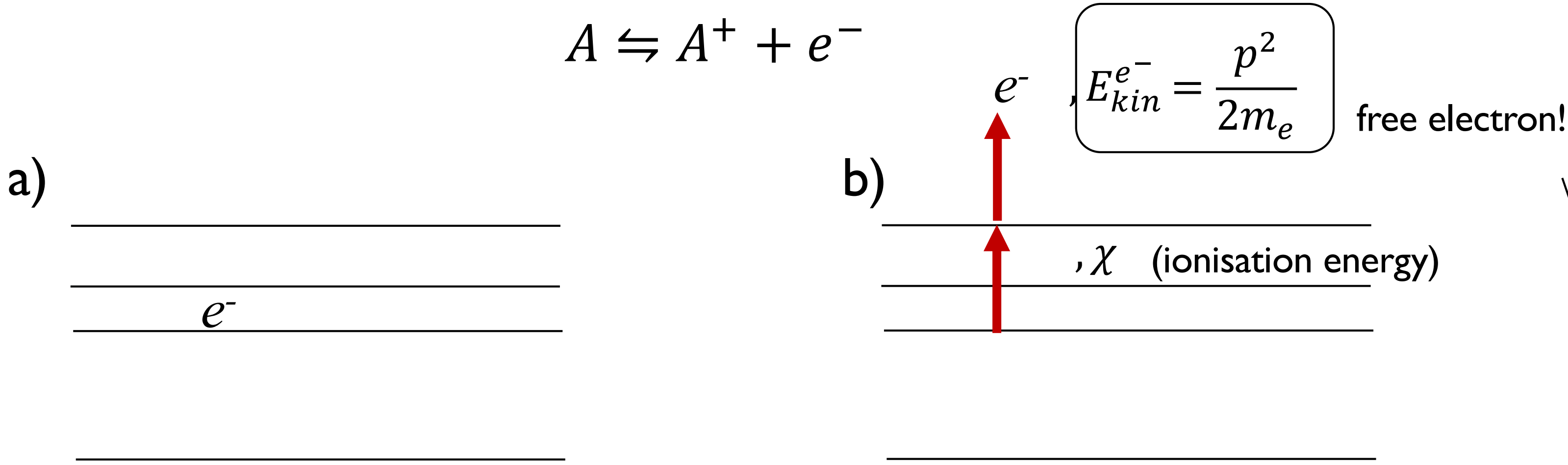

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$g_{i+1}\, g_e^{free}$ is relevant!

- g_{i+1} statistical weight of state $i+1$
- g_e^{free} all possible states of electron with $[p, p + dp]$

Saha equation

- cloud of atoms at temperature T - *how many atoms are ionized?*

$$\frac{N_{i+1}}{N_i} = \frac{g_{i+1} g_e^{free}}{g_i} \; e^{-\frac{\chi}{k_B T}}$$

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g_e: 2 spin states

$\frac{1}{h^3}$: smallest phase-space volume element

V: configuration space volume of electron

d^3p: momentum space volume of electron

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n_e free electrons

n_{i+1} atoms ionisation state i+1

g_{i+1} statistical weight of state i+1

g_e^{free} all possible states of electron with $[p, p + dp]$

- Saha equation

  - cloud of atoms at temperature T - *how many atoms are ionized?*

$$\frac{n_{i+1}}{n_i} = \frac{g_{i+1}}{g_i}\frac{g_e}{h^3\,n_e}(2\pi m_e k_B T)^{3/2} e^{-\frac{\chi}{k_B T}}$$

χ ionisation energy

n_i atoms in ionisation state i

g_i statistical weight of state i

$E = E_{kin}^{e^-} + \chi$ total electron energy

n_e free electrons

n_{i+1} atoms ionisation state i+1

g_{i+1} statistical weight of state i+1

g_e^{free} all possible states of electron with $[p, p + dp]$

- Saha equation

  - cloud of atoms at temperature T

$$\frac{n_{i+1}}{n_i} n_e = \frac{g_{i+1} g_e}{g_i} \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi}{k_B T}}$$

n_i atoms in ionisation state i

n_{i+1} atoms ionisation state $i+1$

n_e free electrons ($g_e = 2$)

χ ionisation energy

g_i statistical weight of state i

g_{i+1} statistical weight of state $i+1$

- Saha equation

  - cloud of atoms at temperature T

$$\frac{n_{i+1}}{n_i} n_e = \frac{g_{i+1} g_e}{g_i} \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi}{k_B T}}$$

n_i atoms in ionisation state i

any relation? { n_{i+1} atoms ionisation state $i+1$; n_e free electrons ($g_e = 2$) }

χ ionisation energy

g_i statistical weight of state i

g_{i+1} statistical weight of state $i+1$

- Saha equation

  - cloud of atoms at temperature T

$$\frac{n_{i+1}}{n_i} n_e = \frac{g_{i+1} g_e}{g_i} \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi}{k_B T}}$$

n_i atoms in ionisation state i

n_{i+1} atoms ionisation state $i+1$

n_e free electrons ($g_e = 2$)

χ ionisation energy

g_i statistical weight of state i = partition function of state i

g_{i+1} statistical weight of state $i+1$ = partition function of state $i+1$

- Saha equation
  - cloud of atoms at temperature T

$$\frac{n_{i+1}}{n_i}n_e = \frac{g_{i+1}g_e}{g_i}\frac{(2\pi m_e k_B T)^{3/2}}{h^3}e^{-\frac{\chi}{k_B T}}$$

example for hydrogen...

n_i atoms in ionisation state i

n_{i+1} atoms ionisation state $i+1$

n_e free electrons ($g_e = 2$)

χ ionisation energy

g_i statistical weight of state i = partition function of state i

g_{i+1} statistical weight of state $i+1$ = partition function of state $i+1$

- Saha equation

  - hydrogen atoms at temperature T

$$\frac{n_{HII}}{n_I} n_e = \frac{g_{HII} g_e}{g_{HI}} \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi_{HI}}{k_B T}}$$

n_{HI} neutral hydrogen

n_{HII} ionized hydrogen

n_e free electrons ($g_e = 2$)

χ_{HI} binding energy of hydrogen

g_{HI} partition function HI

g_{HII} partition function HII

- Saha equation

  - hydrogen atoms at temperature T

$$\frac{n_{HII}}{n_I} n_e = \frac{g_{HII} g_e}{g_{HI}} \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi_{HI}}{k_B T}}$$

$n_H = n_{HI} + n_{HII}$

$n_e = n_{HII}$

n_{HI} neutral hydrogen

n_{HII} ionized hydrogen

n_e free electrons ($g_e = 2$)

χ_{HI} binding energy of hydrogen

g_{HI} partition function HI

g_{HII} partition function HII

▪ Saha equation

- hydrogen atoms at temperature T

$$\frac{n_{HII}}{n_I} n_e = \frac{g_{HII} g_e}{g_{HI}} \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi_{HI}}{k_B T}}$$

$n_H = n_{HI} + n_{HII}$

$n_e = n_{HII}$

n_{HI} neutral hydrogen

n_{HII} ionized hydrogen

n_e free electrons ($g_e = 2$)

χ_{HI} binding energy of hydrogen

$g_{HI} = 2$

$g_{HII} = 1$

g_{HI} partition function HI : ground state of hydrogen

g_{HII} partition function HII : no bound electrons anymore

▪ Saha equation

• hydrogen atoms at temperature T

$$\frac{n_{HII}^2}{n_I} = \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi_{HI}}{k_B T}}$$

$n_H = n_{HI} + n_{HII}$

$n_e = n_{HII}$

n_{HI} neutral hydrogen

n_{HII} ionized hydrogen

n_e free electrons ($g_e = 2$)

χ_{HI} binding energy of hydrogen

$g_{HI} = 2$

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- Saha equation

  - hydrogen atoms at temperature T

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- Saha equation

  - hydrogen atoms at temperature T

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$n_H = n_{HI} + n_{HII}$

$n_e = n_{HII}$

n_{HI} neutral hydrogen

n_{HII} ionized hydrogen

n_e free electrons ($g_e = 2$)

χ_{HI} binding energy of hydrogen

$x = \frac{n_{HII}}{n_H}$ fraction of ionized atoms

- Saha equation

  - hydrogen atoms at temperature T

$$\frac{x^2}{1-x} = \frac{1}{n_H}\frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi_{HI}}{k_B T}}$$

χ_{HI} binding energy of hydrogen

$x = \frac{n_{HII}}{n_H}$ fraction of ionized atoms

n_H total hydrogen density

- Saha equation

  - hydrogen atoms at temperature T

$$\frac{x^2}{1-x} = \frac{1}{n_H}\frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi_{HI}}{k_B T}}$$

has to be solved numerically for $x(T)$...

χ_{HI} binding energy of hydrogen

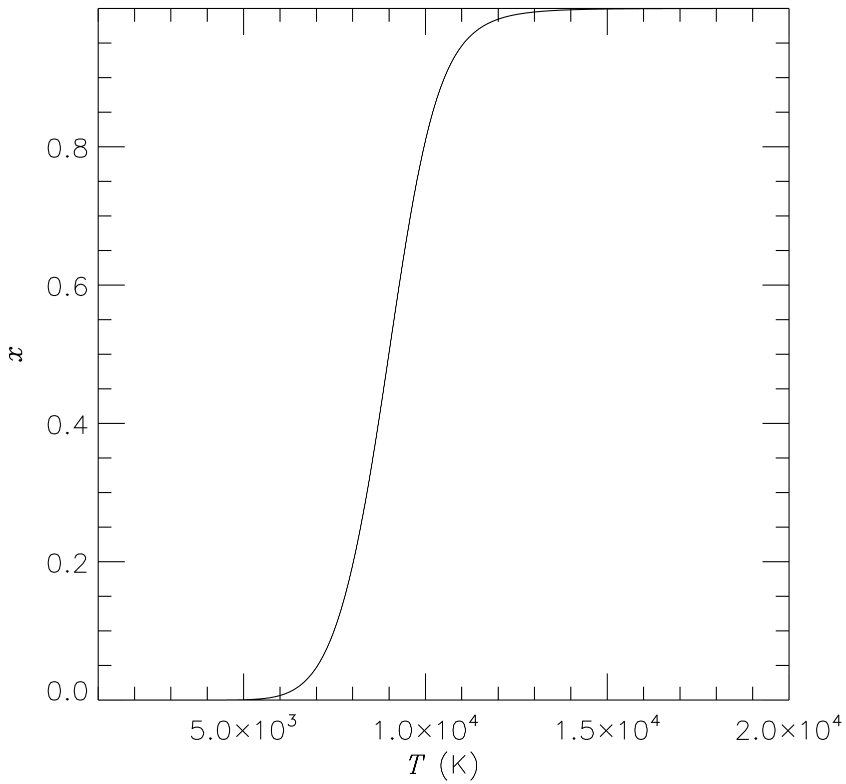
$x = \frac{n_{HII}}{n_H}$ fraction of ionized atoms

n_H total hydrogen density

Saha equation

- hydrogen atoms at temperature T

$$\frac{x^2}{1-x} = \frac{1}{n_H}\frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi_{HI}}{k_B T}}$$



χ_{HI} binding energy of hydrogen

$x = \frac{n_{HII}}{n_H}$ fraction of ionized atoms

$n_H = 10^{20}\ \mathrm{m}^{-3}$ total hydrogen density (typical value for stellar atmosphere)

- Saha equation

  - hydrogen atoms at temperature T

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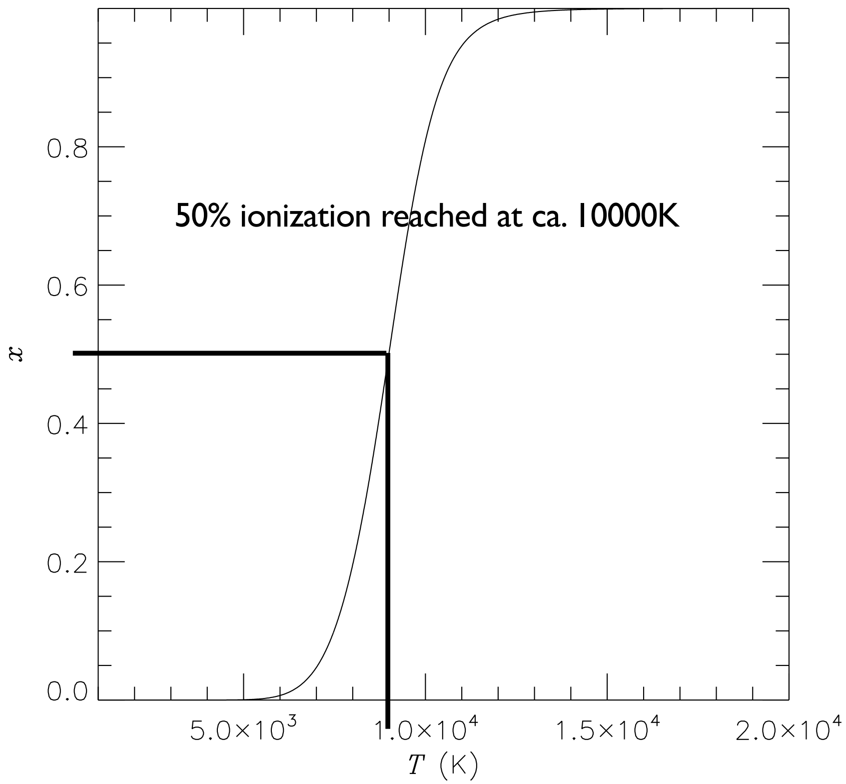

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  - hydrogen atoms at temperature T

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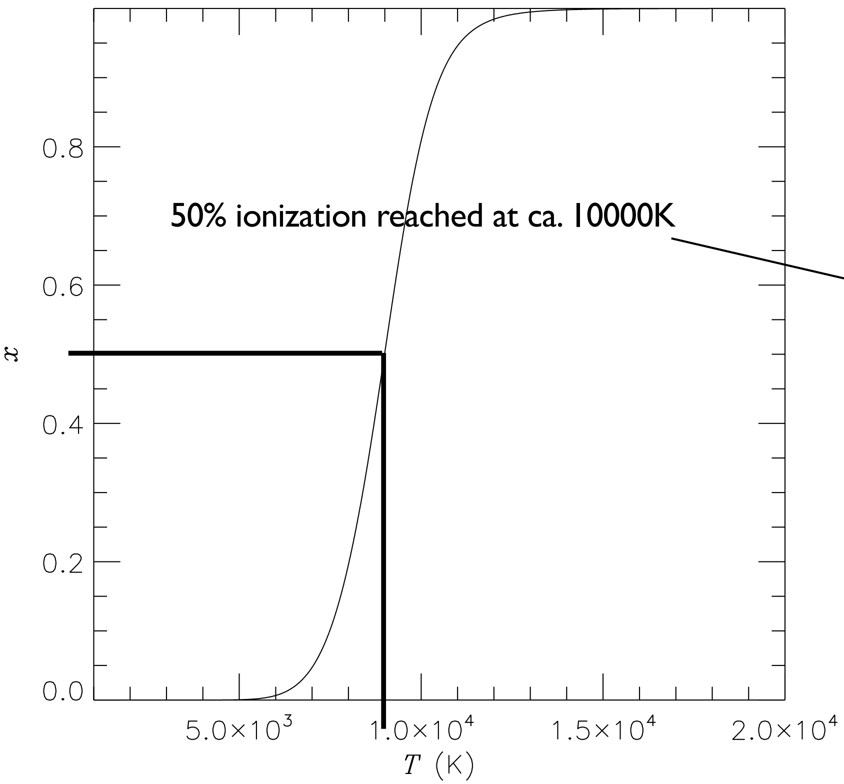
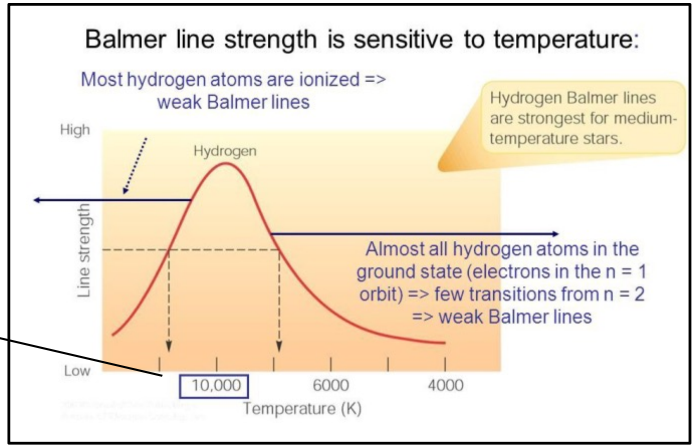




χ_{HI} binding energy of hydrogen

$x = \frac{n_{HII}}{n_H}$ fraction of ionized atoms

$n_H = 10^{20}$ m^{-3} total hydrogen density
(typical value for stellar atmosphere)

▪ summary

- cloud of atoms at temperature T

Boltzmann distribution: *number of atoms in state n*

$$\frac{N_n}{N} = \frac{g_n}{Z(T)} e^{-\frac{E_n}{k_B T}}$$

$$Z(T) = g_1 \frac{N}{N_1} = g_1 \frac{\sum_{n=1}^{\infty} N_n}{N_1} = g_1 \sum_{n=1}^{\infty} \frac{N_n}{N_1} = g_1 + g_2 e^{-\frac{E_2}{k_B T}} + g_3 e^{-\frac{E_3}{k_B T}} + \ldots$$

Saha equation: *number of ionized atoms*

$$\frac{n_{i+1}}{n_i} n_e = \frac{g_{i+1} g_e}{g_i} \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\frac{\chi}{k_B T}}$$

n_i atoms in ionisation state i

n_{i+1} atoms ionisation state $i+1$

n_e free electrons ($g_e = 2$)

χ ionisation energy

g_i statistical weight of state i = partition function of state i

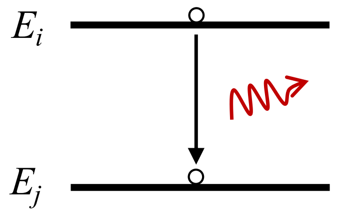
g_{i+1} statistical weight of state $i+1$ = partition function of state $i+1$

- thermal excitation
- **atomic transitions**
- Einstein coefficients
- line broadening

- bound-bound
- bound-free
- free-free

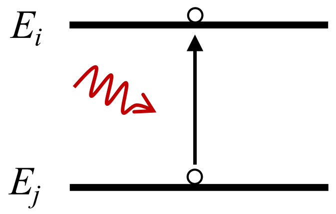
- **bound-bound**
- bound-free
- free-free

- bound-bound



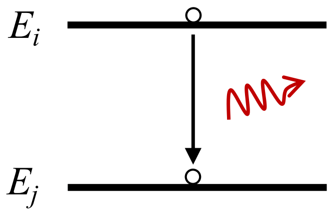
emission

$$h\nu_{ij} = |E_i - E_j|$$



absorption

- bound-bound



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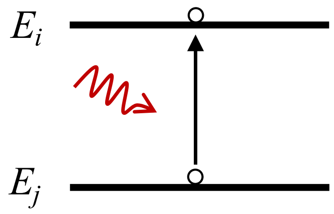
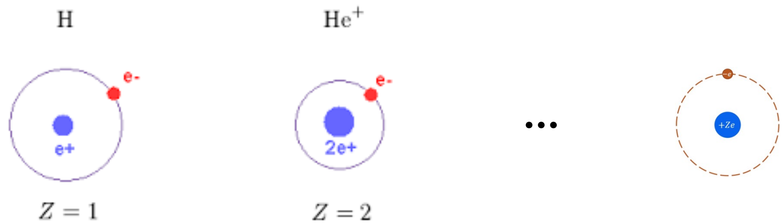


absorption

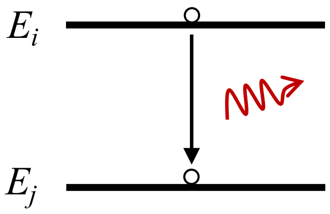
- hydrogen-like atoms

$$E_n = \frac{m_e e^4}{2\hbar^2} \frac{Z^2}{n^2}$$



Z: number of protons

- bound-bound



emission

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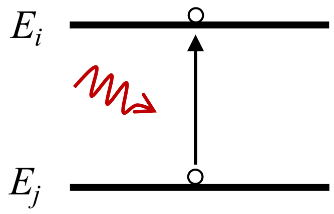
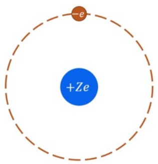
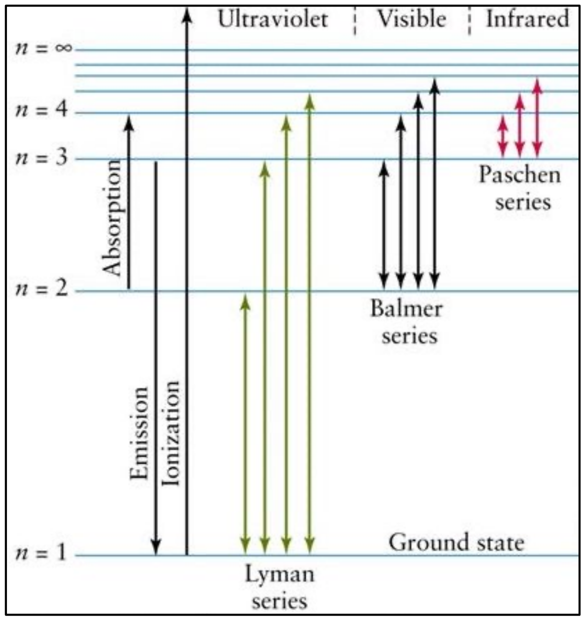


absorption

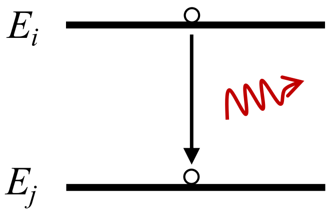


- hydrogen-like atoms

$$E_n = \frac{m_e e^4}{2\hbar^2} \frac{Z^2}{n^2}$$

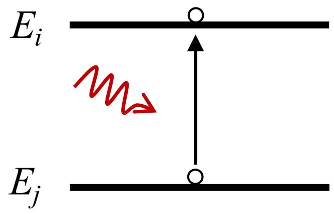


- bound-bound

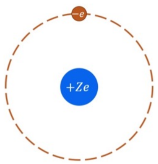


emission

$$h\nu_{ij} = |E_i - E_j|$$



absorption



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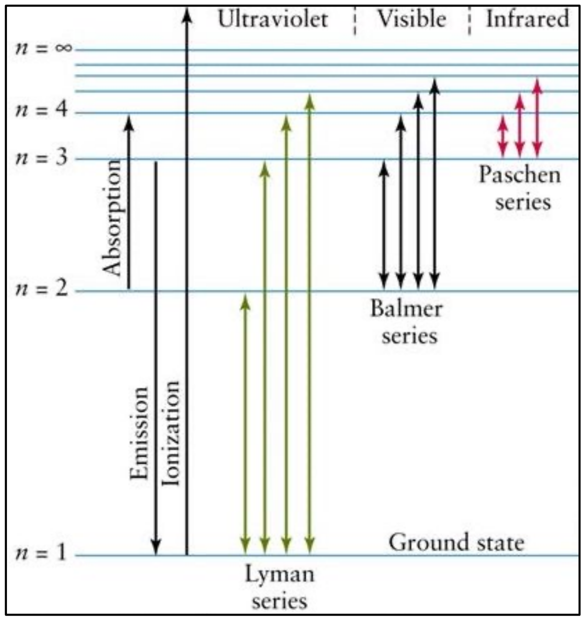
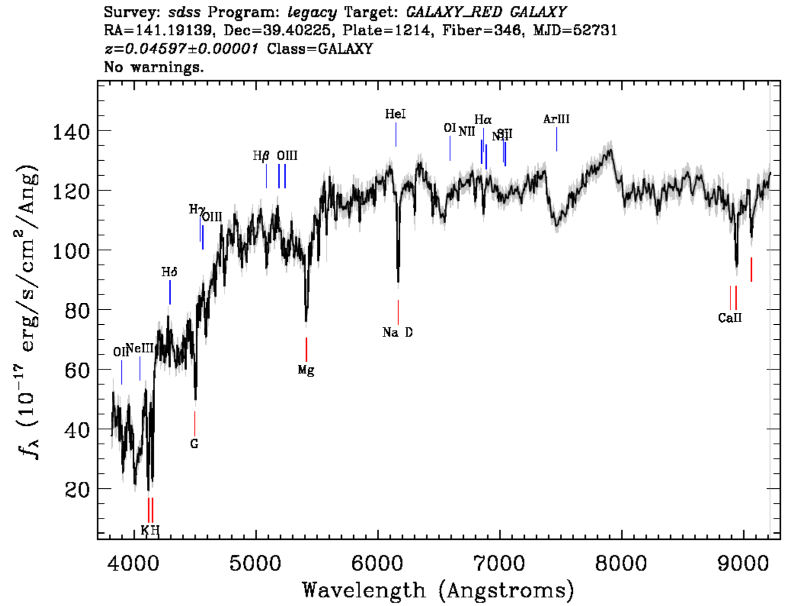
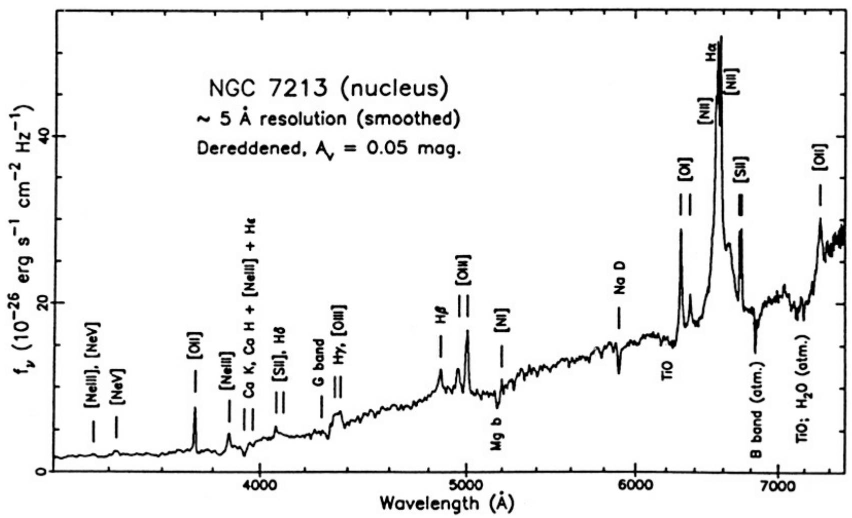
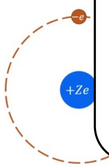


- transitions give...
  - spectral lines (either absorption or emission),
  - that are not sharp though (Heisenberg uncertainty principle)

galaxy spectrum *AGN spectrum*







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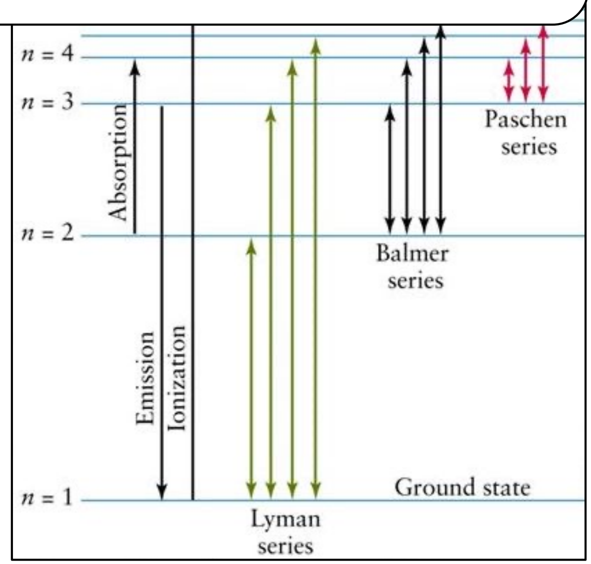
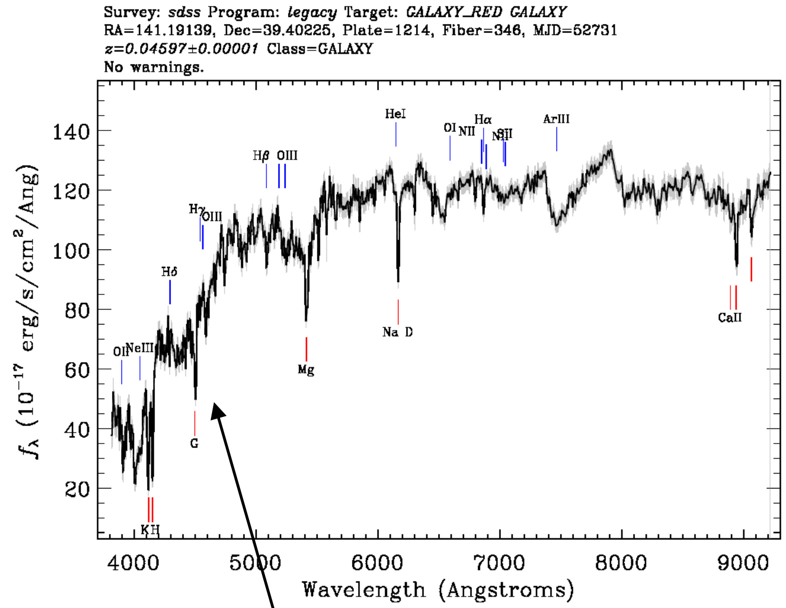
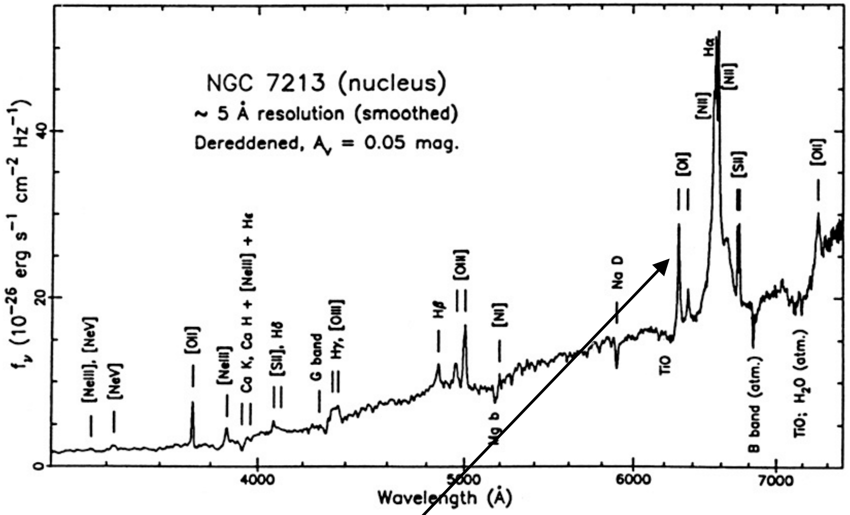
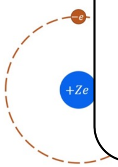


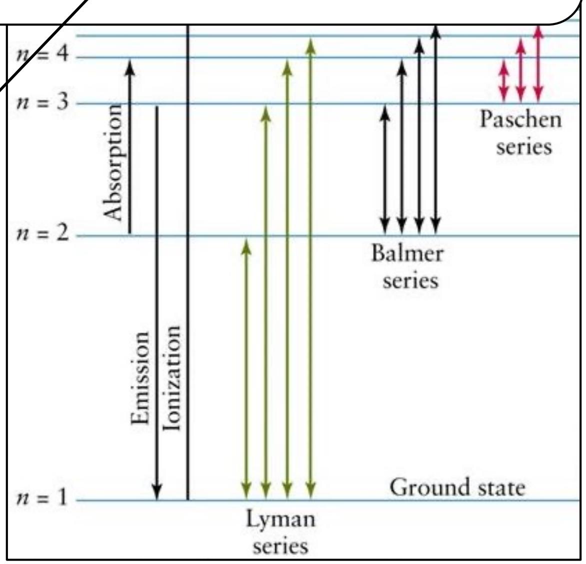
galaxy spectrum

AGN spectrum









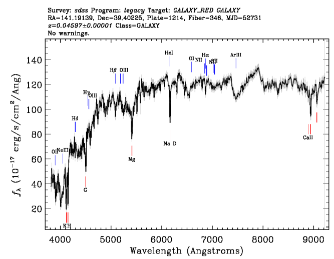
- transitions give...
  - **spectral lines (either absorption or emission),**
  - that are not sharp though (Heisenberg uncertainty principle)

when do we get absorption and when emission?

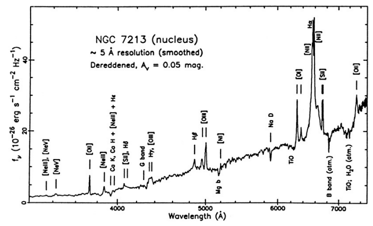
- bound-bound
  - absorption vs. emission lines:

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

absorption



emission



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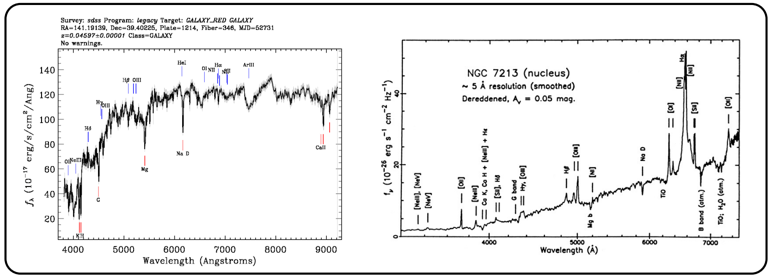
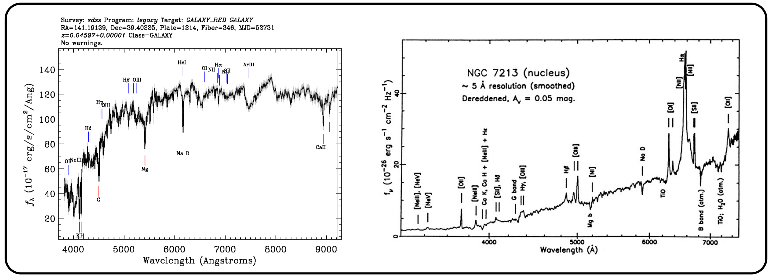
constant source term: $I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$

absorption *emission*



absorption *emission*



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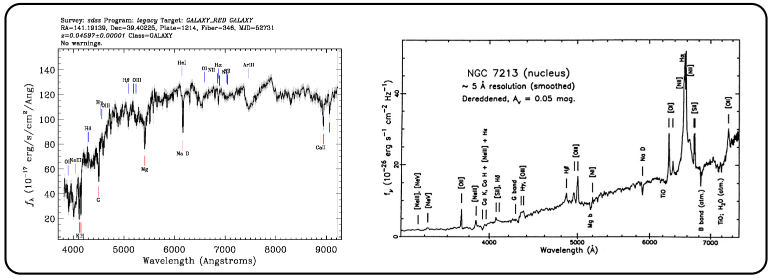
absorption *emission*



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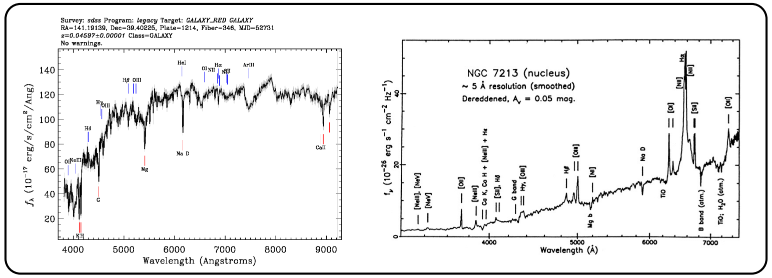
○ optically thin medium $\tau_\nu \ll 1$: $\quad I_\nu(\tau_\nu) = I_\nu(0)\,(1 - \tau_\nu) + \tau_\nu\, S_\nu$

absorption *emission*



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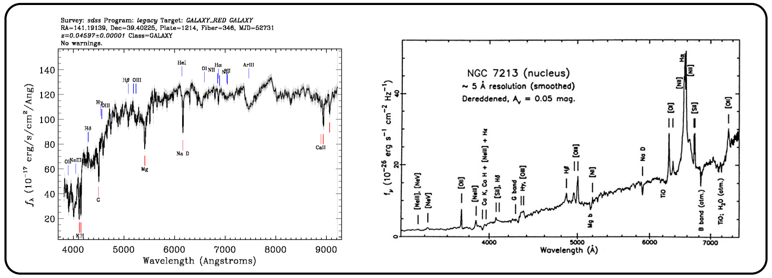
thermal equilibrium: $\tau_\nu\, B_\nu$

absorption *emission*



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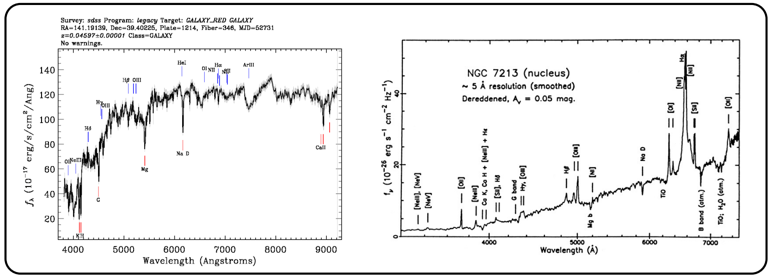
thermal equilibrium: $\tau_\nu\, B_\nu \propto \alpha_\nu\, B_\nu$

definition of τ_ν

absorption *emission*



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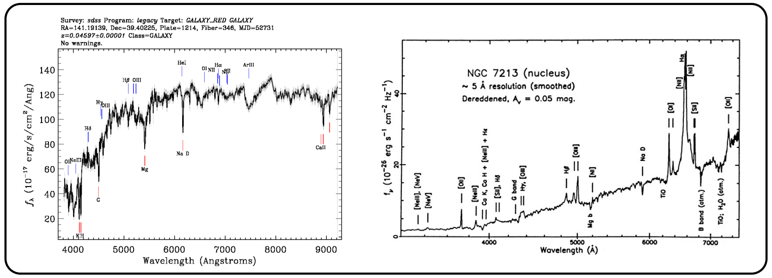
thermal equilibrium: $\tau_\nu\, B_\nu \propto \alpha_\nu\, B_\nu$

the emergent radiation is proportional to the absorption coefficient!

absorption *emission*



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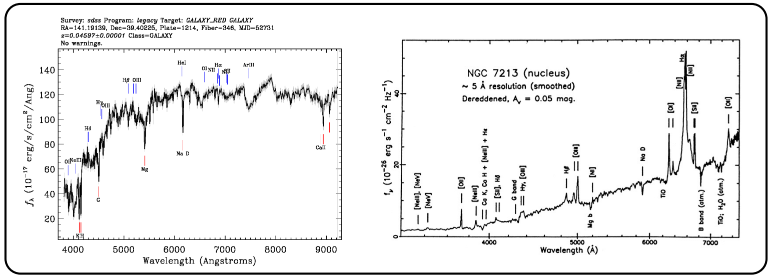
thermal equilibrium: $\tau_\nu\, B_\nu \propto \alpha_\nu\, B_\nu$

the emergent radiation is proportional to the absorption coefficient!
the absorption coefficient is large at the frequencies of the corresponding energy levels!

absorption *emission*



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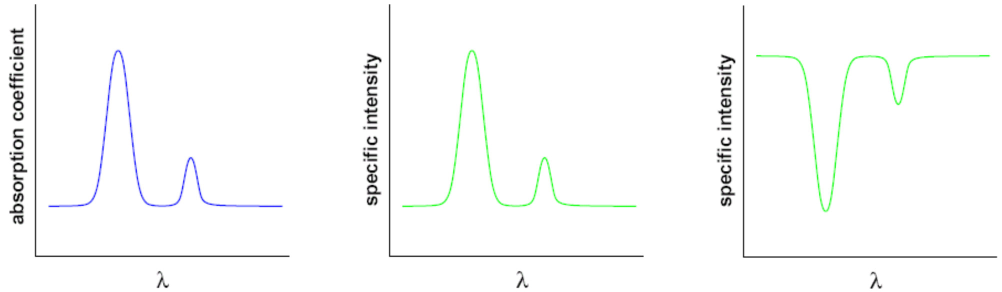
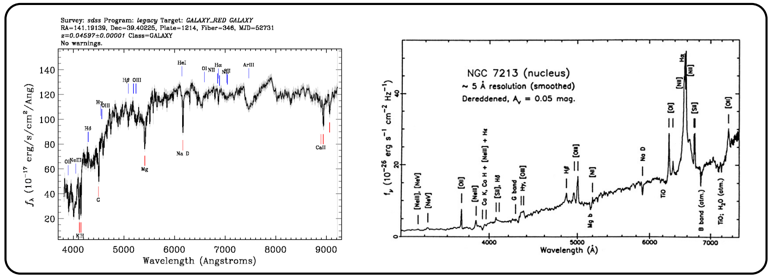
thermal equilibrium: $\tau_\nu\, B_\nu \propto \alpha_\nu\, B_\nu$

the emergent radiation is proportional to the absorption coefficient!
the absorption coefficient is large at the frequencies of the corresponding energy levels!

if we get emission/absorption, it will be at the peaks of α_ν !



absorption *emission*



- bound-bound

  - absorption vs. emission lines:

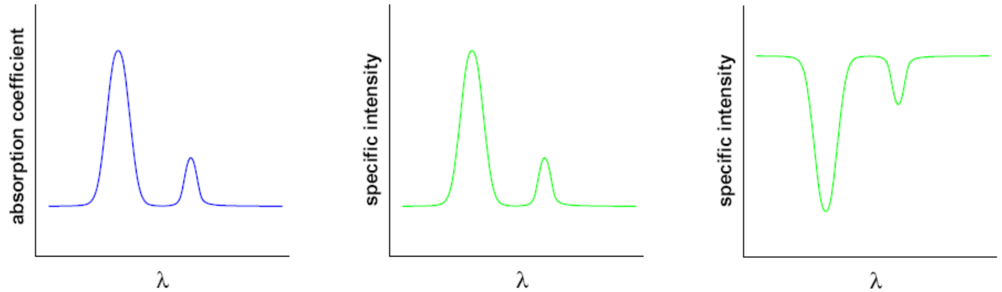
$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

  - optically thin medium $\tau_\nu \ll 1$: $I_\nu(\tau_\nu) = I_\nu(0)\,(1 - \tau_\nu)\, \underbrace{+\,\tau_\nu\, S_\nu}$

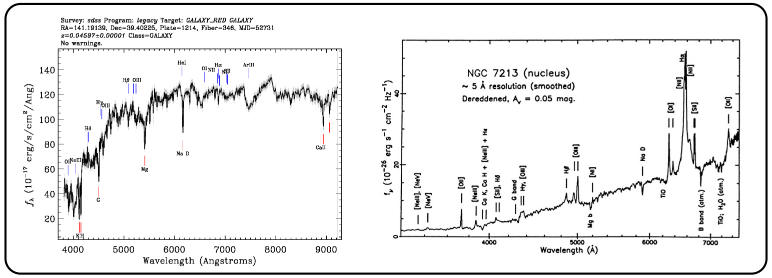
thermal equilibrium: $\tau_\nu\, B_\nu \propto \alpha_\nu\, B_\nu$

the emergent radiation is proportional to the absorption coefficient!
the absorption coefficient is large at the frequencies of the corresponding energy levels!

!?
if we get emission/absorption,
it will be at the peaks of α_ν !



absorption *emission*



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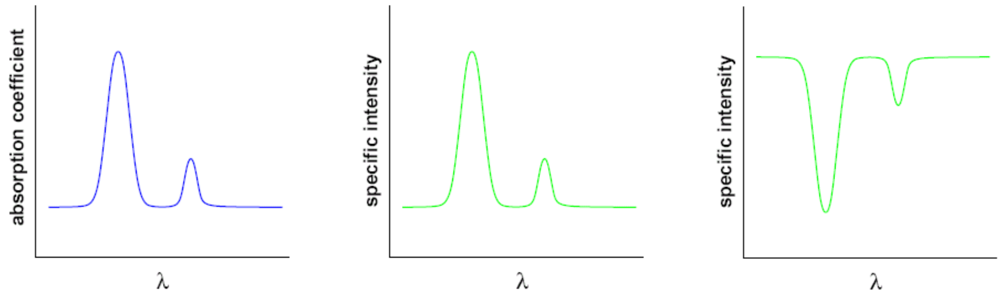
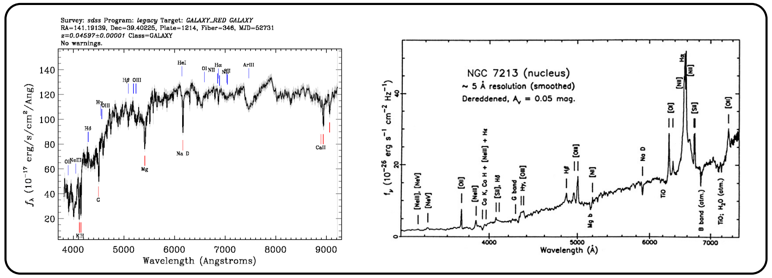


absorption *emission*



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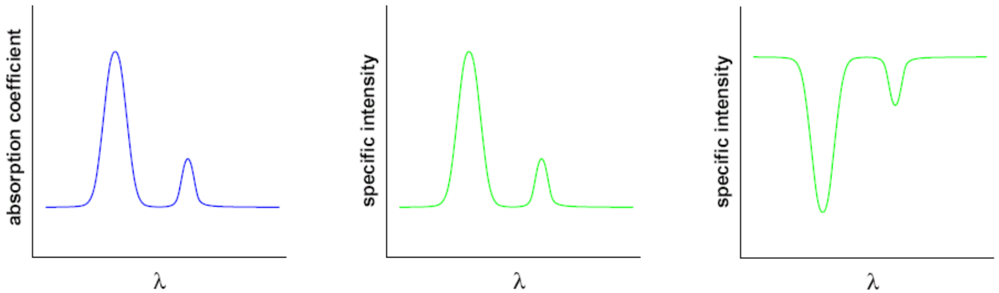
  $S_\nu > I_\nu(0)$: emission lines on top of continum

  $S_\nu < I_\nu(0)$: absorption lines on top of continum



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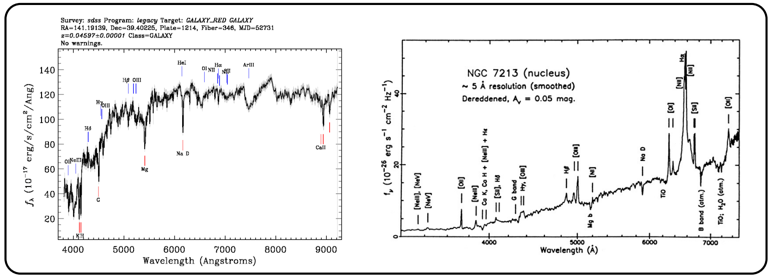
absorption *emission*



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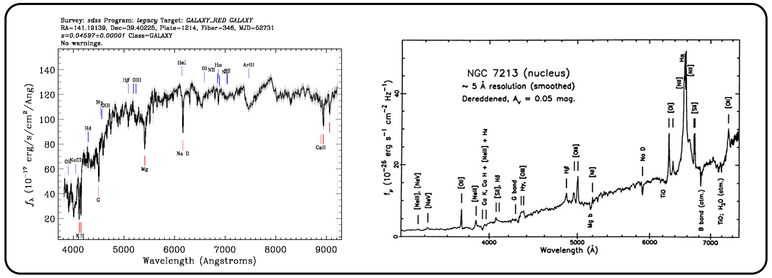

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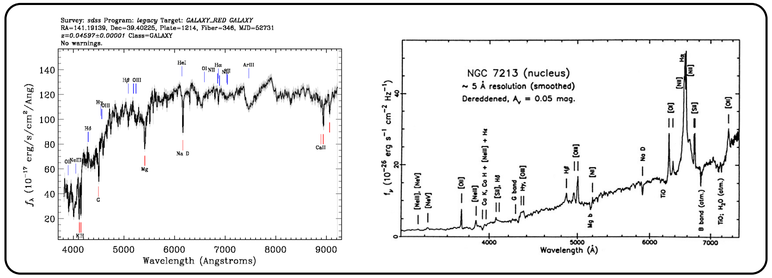
absorption *emission*



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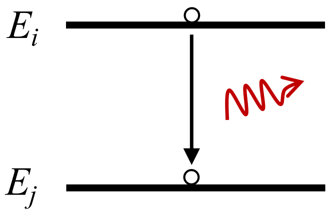
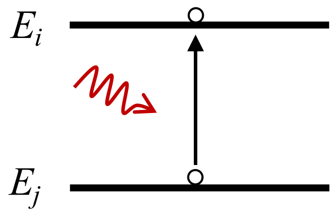
continous radiation like a black-body

- bound-bound

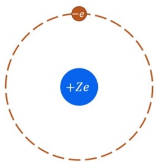


emission

$$h\nu_{ij} = |E_i - E_j|$$

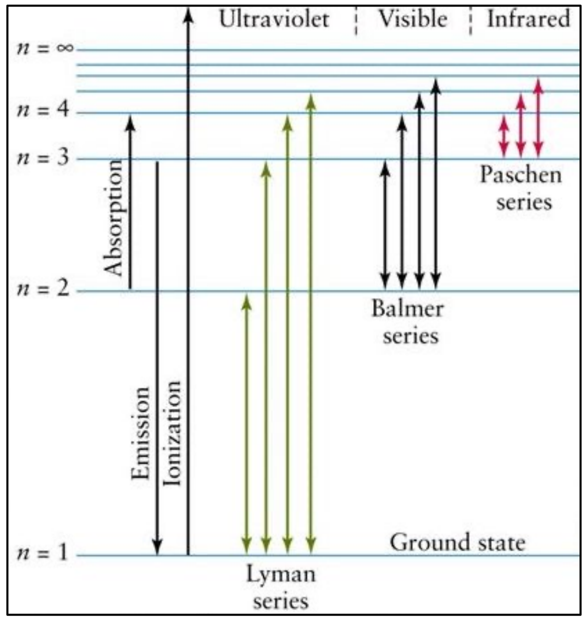


absorption



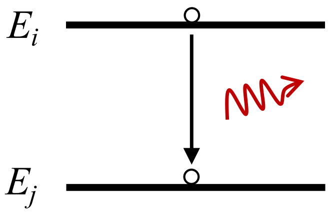
- hydrogen-like atoms

$$E_n = \frac{m_e e^4}{2\hbar^2}\,\frac{Z^2}{n^2}$$



- transitions give...
  - **spectral lines (either absorption or emission),** ✓
  - that are not sharp though (Heisenberg uncertainty principle)

- bound-bound



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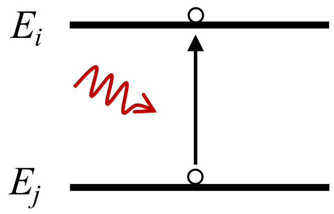
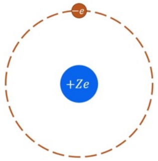


absorption



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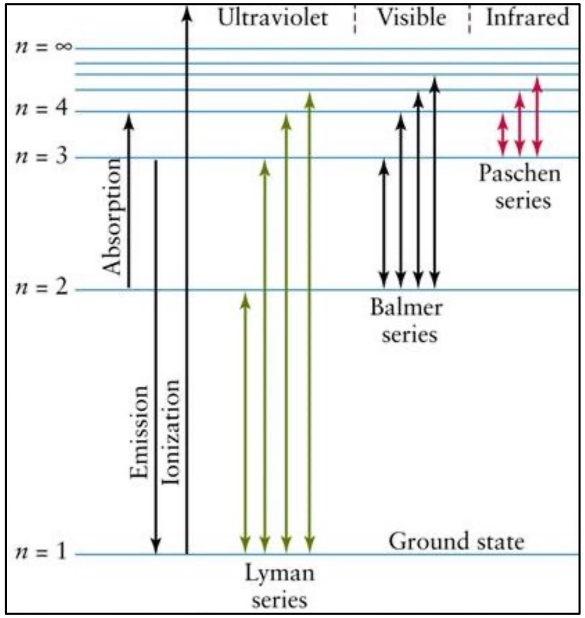
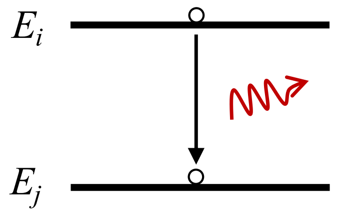


- transitions give…
  - spectral lines (either absorption or emission),
  - **that are not sharp though** (Heisenberg uncertainty principle)

more later in "line broadening"…

- bound-bound



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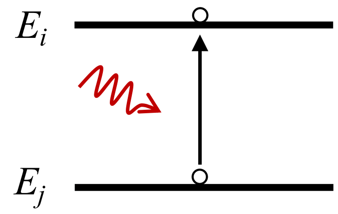
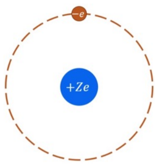


absorption



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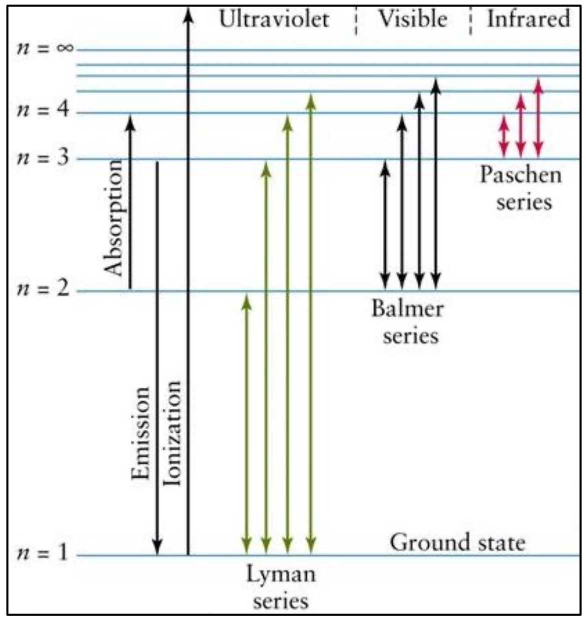
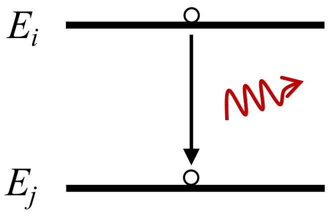


- transitions give…
  - spectral lines (either absorption or emission),
  - that are not sharp though (Heisenberg uncertainty principle)

- electron spin can be coupled to…
  - e-angular momentum $\rightarrow$ fine-structure splitting
  - nucleus spin $\rightarrow$ hyperfine-structure splitting

- bound-bound



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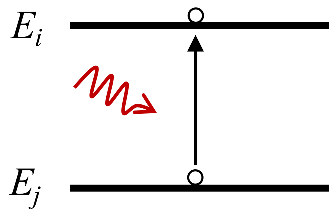
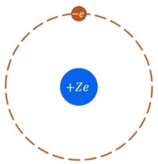


absorption



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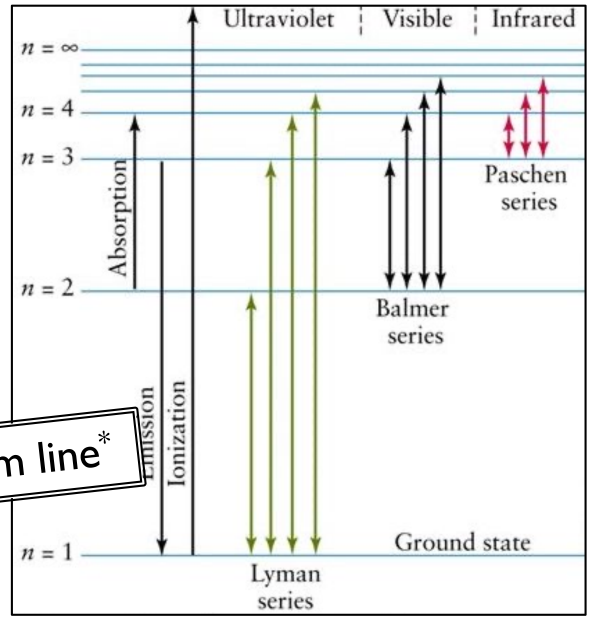

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  - spectral lines (either absorption or emission),
  - that are not sharp though (Heisenberg uncertainty principle)

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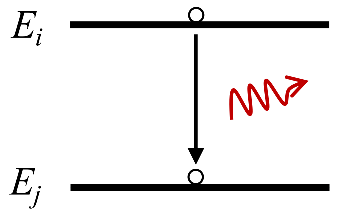
  - *e*-angular momentum → ...e splitting
  - nucleus spin → hyperfine-structure splitting

the (in-)famous and forbidden 21cm line*

*more in 21cm Cosmology lecture

- bound-bound



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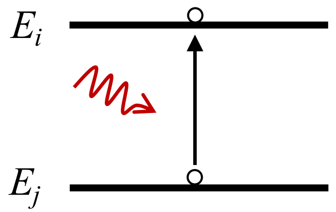
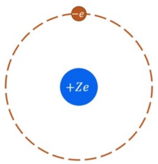


absorption



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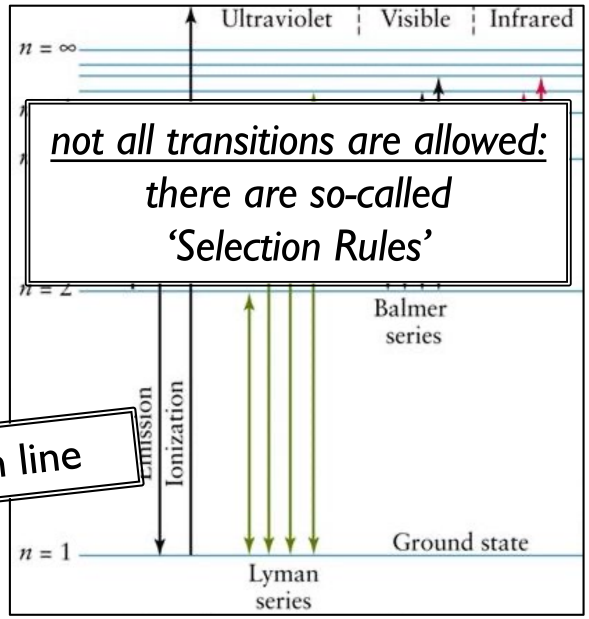


not all transitions are allowed:
there are so-called
'Selection Rules'

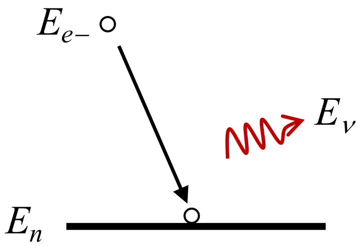
- transitions give...
  - spectral lines (either absorption or emission),
  - that are not sharp though (Heisenberg uncertainty principle)

- electron spin can be coupled to...
  - e-angular momentum ... splitting
  - nucleus spin → hyperfine-structure splitting

the (in-)famous and forbidden 21cm line

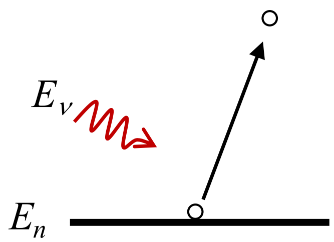
- bound-bound
- **bound-free**
- free-free

- bound-free



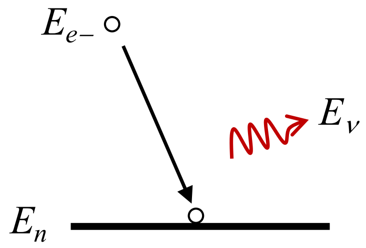
(radiative) recombination

$$h\nu > \frac{m_e e^4}{2\hbar^2} \frac{Z^2}{n^2}$$



(photo-)ionisation

- bound-free



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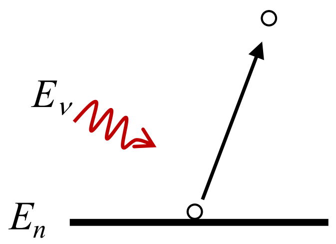
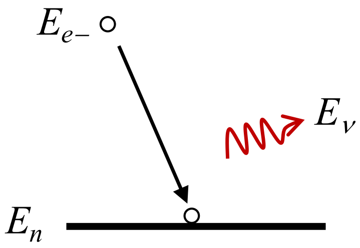


(photo-)ionisation

$$\rightarrow \quad \nu_n^{crit} = \frac{1}{h} \frac{m_e e^4}{2\hbar^2} \frac{Z^2}{n^2}$$

- bound-free



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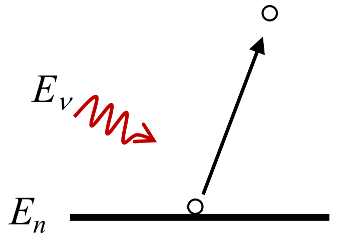
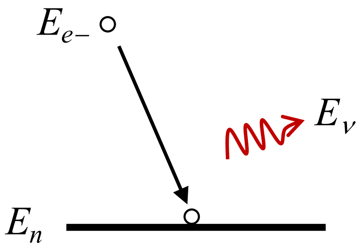


(photo-)ionisation

$$\rightarrow \quad \nu_n^{crit} = \frac{1}{h} \frac{m_e e^4}{2\hbar^2} \frac{Z^2}{n^2}$$

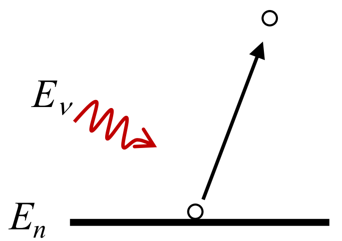
$$\rightarrow \quad \lambda_n^{crit} = \frac{2\pi^2 m_e e^4 c}{h^3 Z^2} n^2$$

- bound-free



(radiative) recombination

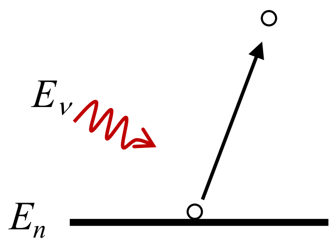
$$\lambda_n^{crit} = \frac{2\pi^2\, m_e e^4 c}{h^3\, Z^2}\; n^2$$



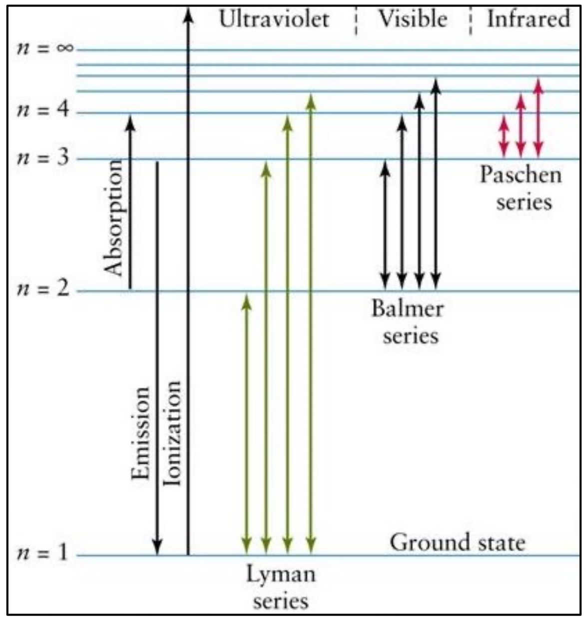
(photo-)ionisation

- bound-free
  - (photo-)ionisation

$$\lambda_n^{crit} = \frac{2\pi^2 \, m_e e^4 c}{h^3 \, Z^2} \, n^2$$



(photo-)ionisation



- bound-free

  - (photo-)ionisation
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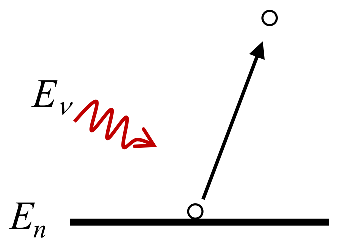
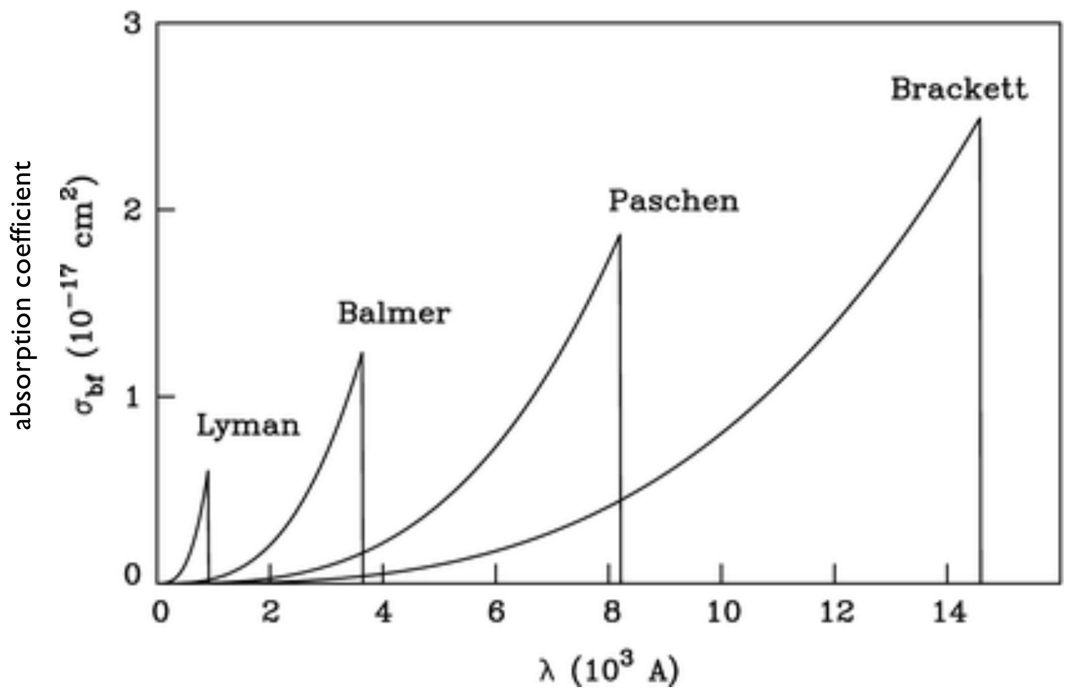
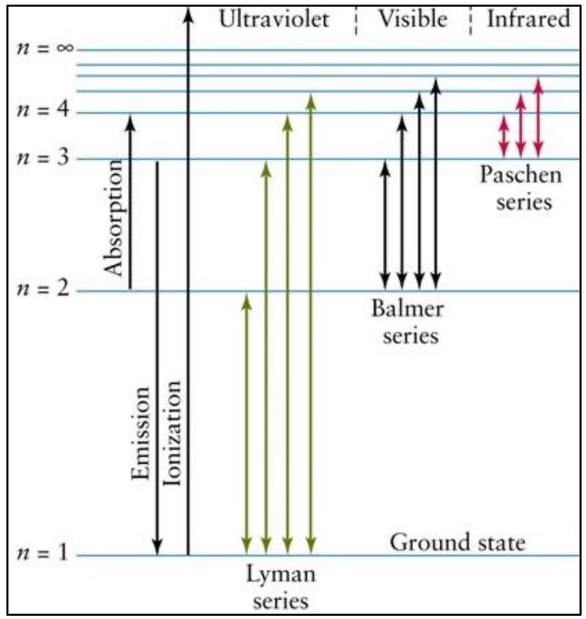


(photo-)ionisation





- bound-free

  - (photo-)ionisation

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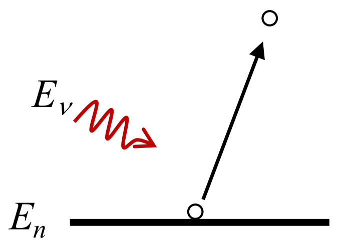
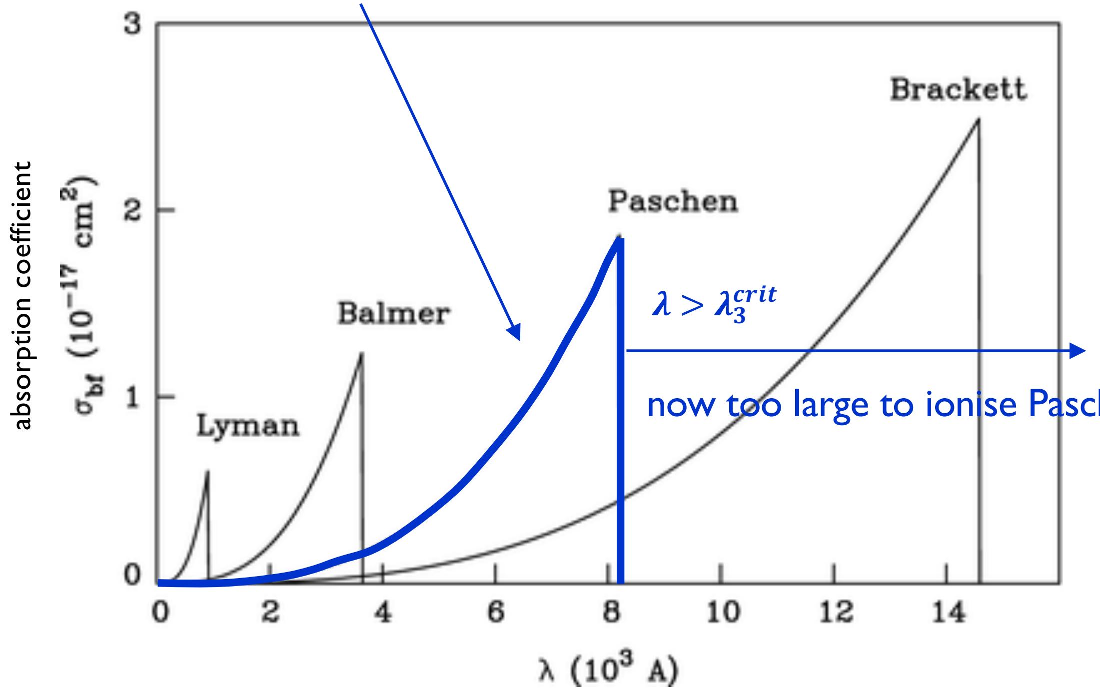
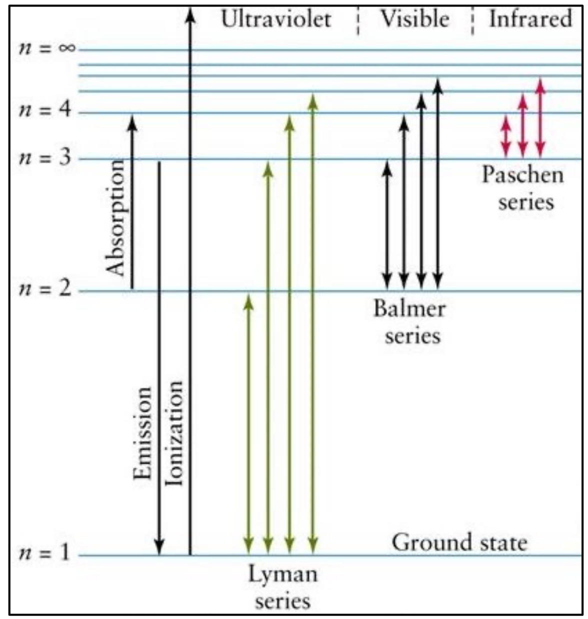


(photo-)ionisation

photon wavelengths start to favour Paschen series



now too large to ionise Paschen series



- **bound-free**

  - (photo-)ionisation

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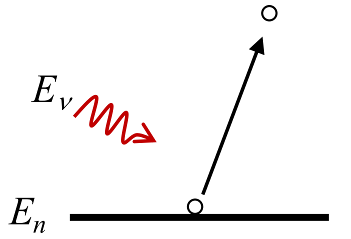
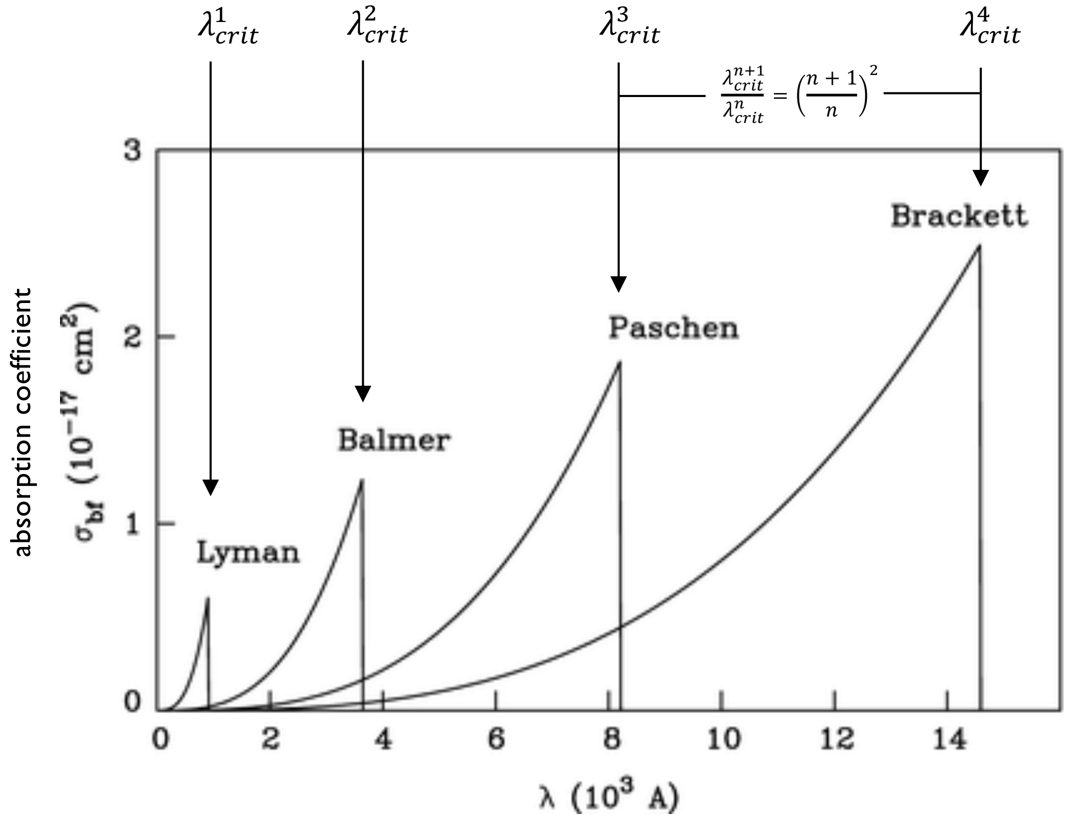
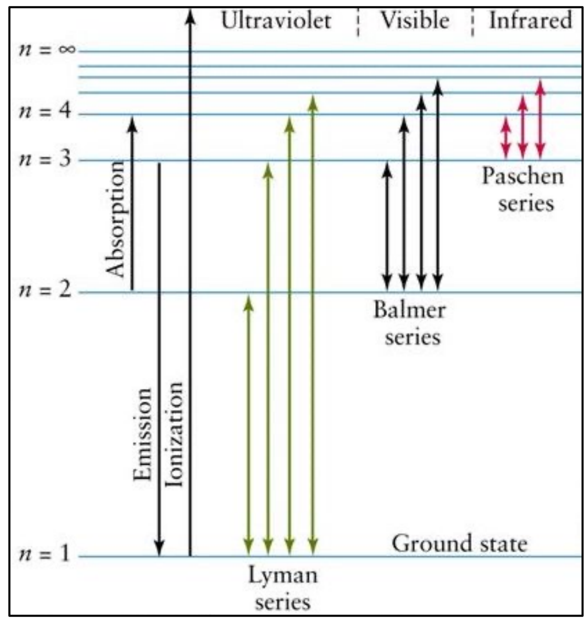
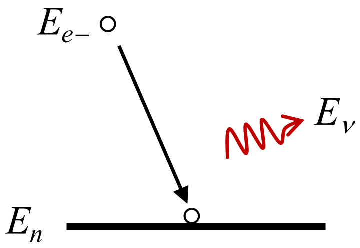


(photo-)ionisation



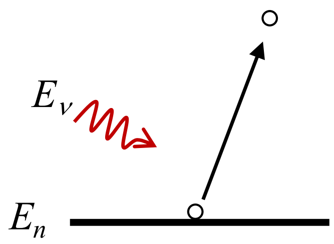


- bound-free



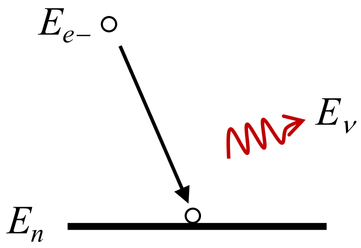
(radiative) recombination

$$\lambda_n^{crit} = \frac{2\pi^2\, m_e e^4 c}{h^3\, Z^2}\; n^2$$



(photo-)ionisation

- bound-free
  - (radiative) recombination

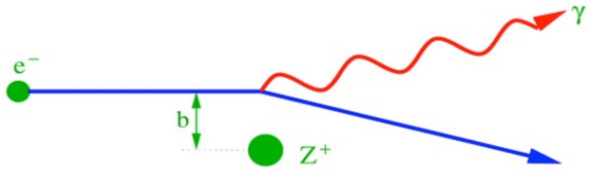


(radiative) recombination

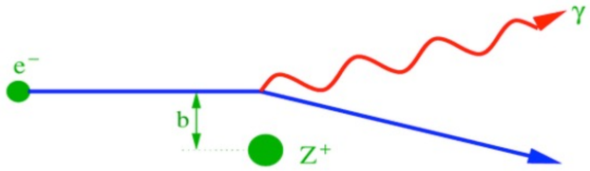
contributes to emission coefficient...

- bound-bound
- bound-free
- **free-free**

- free-free



- free-free



  - collisions/scattering process with ionized atom
  - leads to both continuous emission and absorption
  - is especially important at very high temperatures (i.e. fully ionized plasma)
  - depends on temperature, charge, electron density, and ion density:

$$\epsilon \propto n_e n_i Z^2 T^{1/2}$$

- summary

name	atomic transition	spectral feature	LTE distribution
bound-bound	excitation de-excitation	absorption line emission line	Boltzmann
bound-free	(photo-)ionisation (radiative) recombination	absorption edge emission edge	Saha
free-free	collisions	continuum	Maxwell-Boltzmann

- summary

name	atomic transition	spectral feature	LTE distribution
bound-bound	excitation de-excitation	absorption line emission line	Boltzmann
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$T \leq 10^4 K$: forbidden transitions dominate

$10^4 K \leq T \leq 10^7 K$: allowed transitions dominate

$10^7 K \leq T$: free-free emission dominates

- thermal excitation
- atomic transitions
- **Einstein coefficients**
- line broadening

- Kirchoff's law

$$j_\nu = \alpha_\nu \, B_\nu(T)$$

"if material absorbs well at a certain wavelength,
it will also radiate well at the same wavelength."

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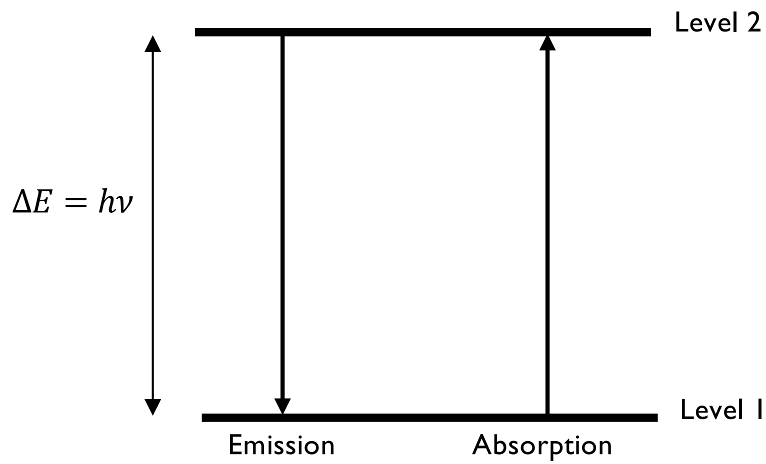

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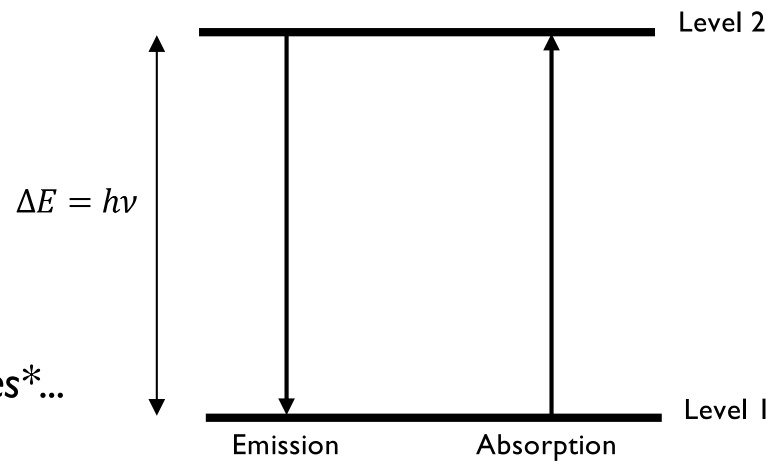

Einstein identified ***three*** *relevant processes*...*

*remember Fundamentals lecture

- spontaneous emission
- stimulated emission
- absorption

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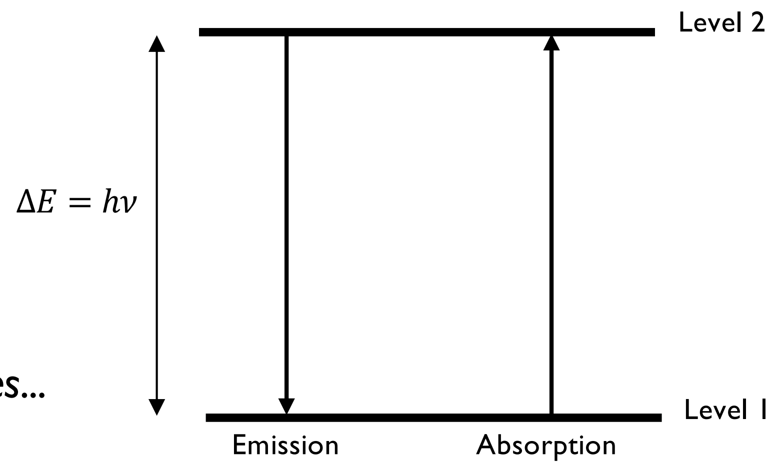
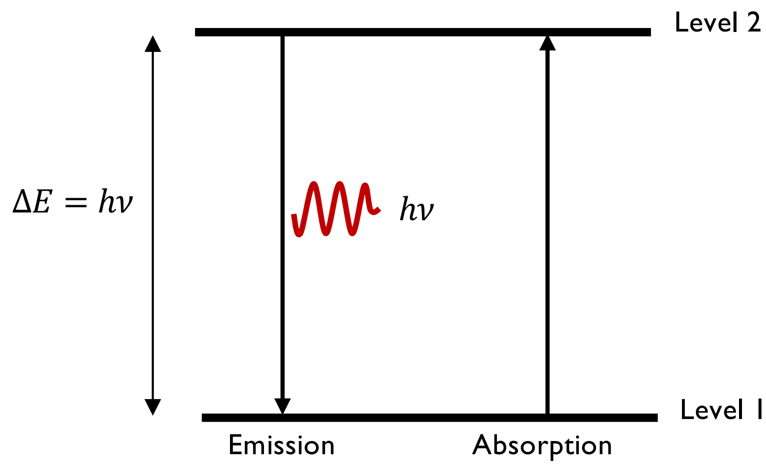


- spontaneous emission

drop from Level 2 to 1, even in absence of radiation field



▪ spontaneous emission

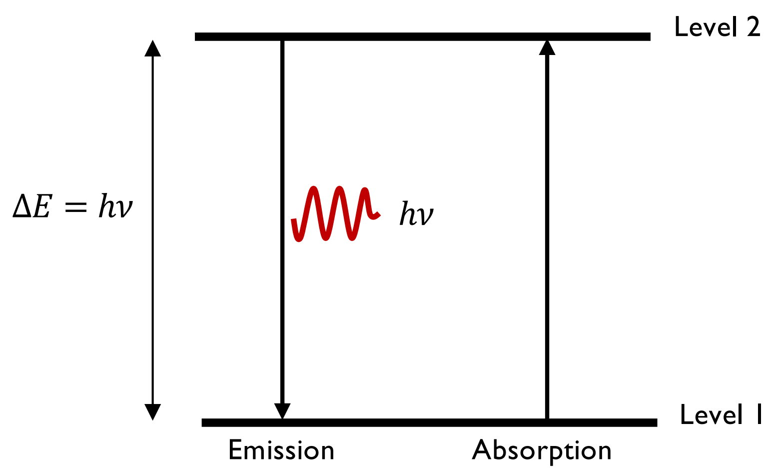
drop from Level 2 to 1, even in absence of radiation field

- Einstein A-coefficient:

A_{21} = transition probability per unit time

$$\frac{dn_2}{dt} = -A_{21}n_2$$

$$\frac{dn_1}{dt} = A_{21}n_1$$



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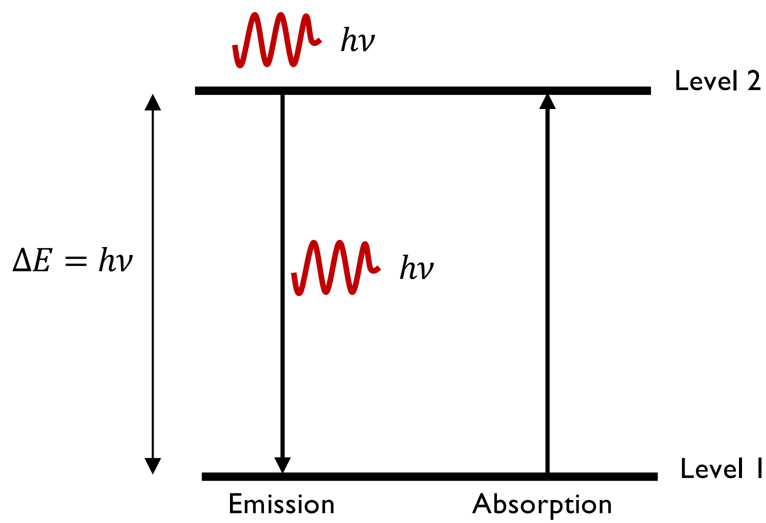
external photon with correct energy triggers emission



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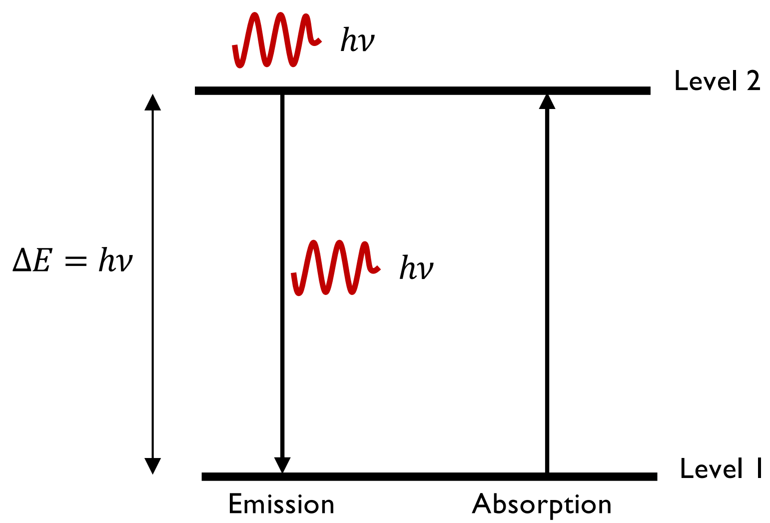
- Einstein B_{21}-coefficient:

B_{21} = transition probability per unit time, unit energy of ext. field, and unit frequency

$$\frac{dn_1}{dt} = B_{21}\, n_2\, u_\nu(T)$$



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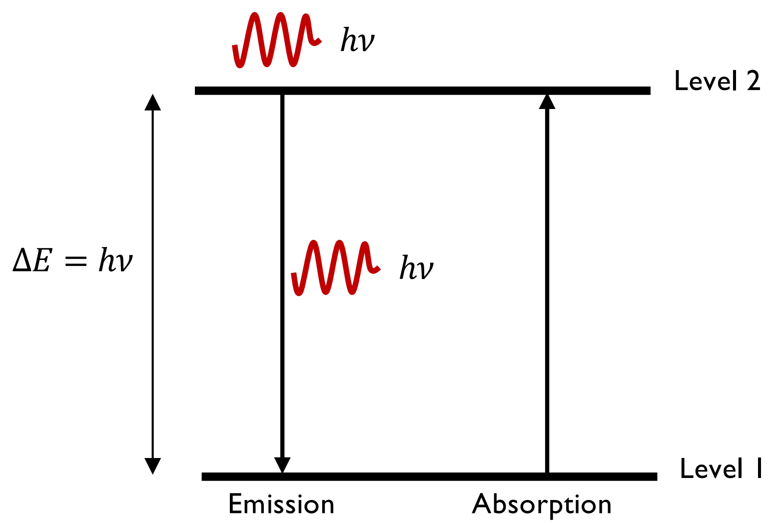
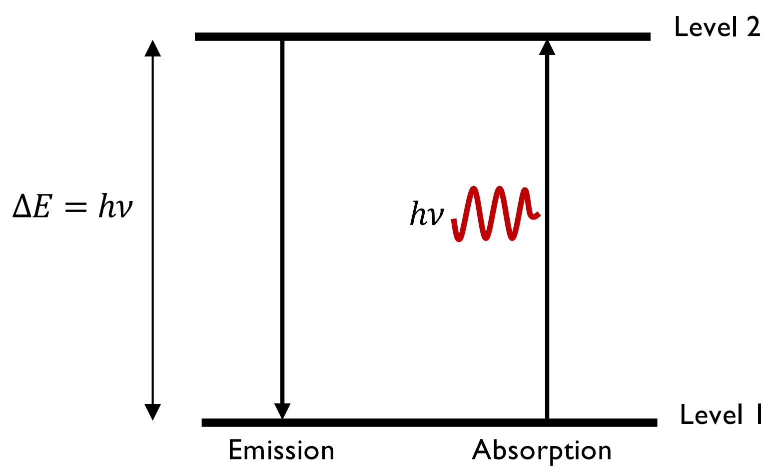


Note, some books use $B_\nu(T)$ instead of $u_\nu(T)$, but that just involves a constant factor $4\pi/c$

- absorption

external photon is absorbed



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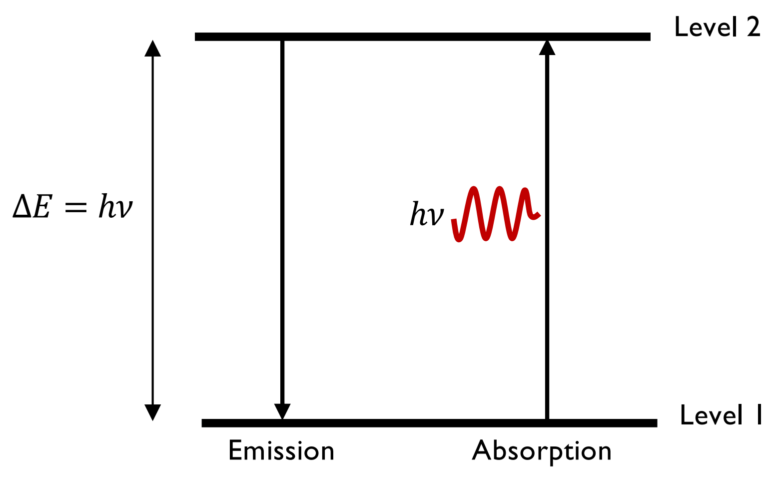

- spontaneous emission $$\frac{dn_1}{dt} = A_{21} n_1 \qquad \frac{dn_2}{dt} = -A_{21} n_2$$

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downwards transitions rate = upwards transition rates: $\frac{dn_2}{dt} = \frac{dn_1^s}{dt} + \frac{dn_1^a}{dt} \Rightarrow -A_{21}n_2 = B_{21}n_2u_\nu(T) - B_{12}n_1u_\nu(T)$

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Boltzman statistics:
$$\frac{n_1}{n_2} = \frac{g_1}{g_2} \frac{e^{-(E/k_B T)}}{e^{-(E+h\nu/k_B T)}} = \frac{g_1}{g_2} e^{-(h\nu/k_B T)}$$

thermal radiation:
$$u_\nu(T) = \frac{8\pi}{c^3} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

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true for any temperature →

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A_{21}, B_{21}, and B_{12} are related to oscillator strengths $\left|f_{nlj\to n'l'j'}\right| \sim \left|\langle\psi_{nlj}|\vec{r}|\psi_{n'l'j'}\rangle\right|^2$

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any relation to our 'usual' emission and absorption coefficients?

- Einstein coefficients vs. α_ν and j_ν

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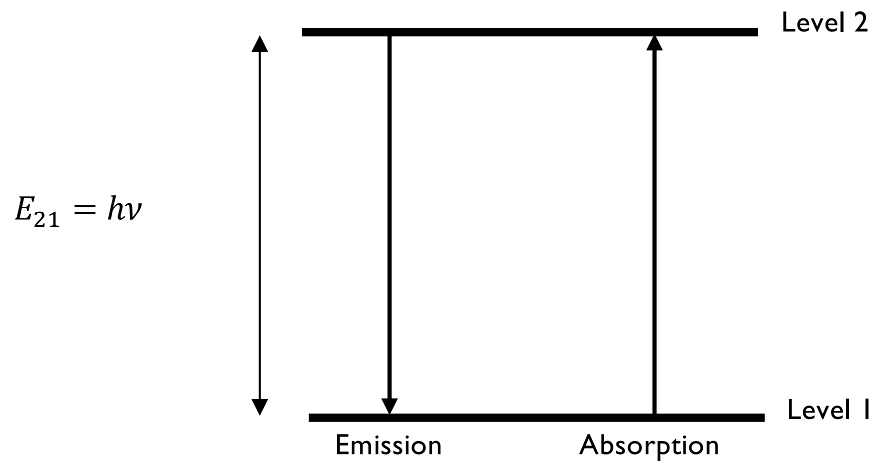

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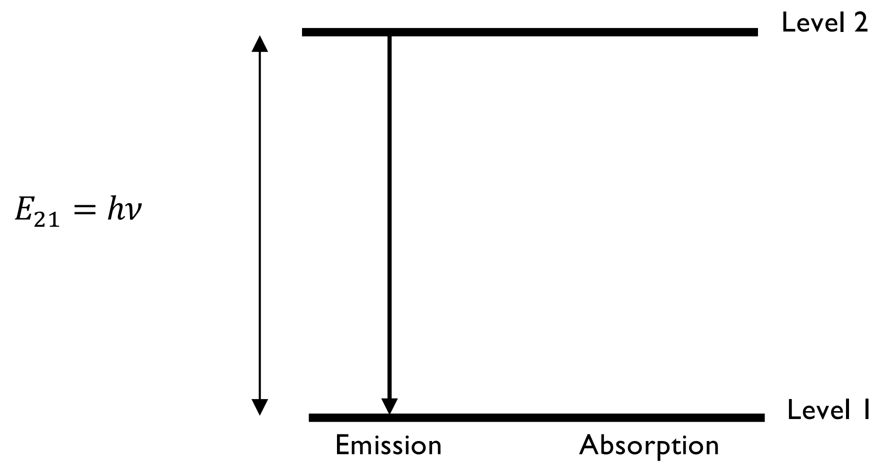
$$\frac{dn_2}{dt} = -\, n_2\, A_{21}$$

Level 2 population decrease…



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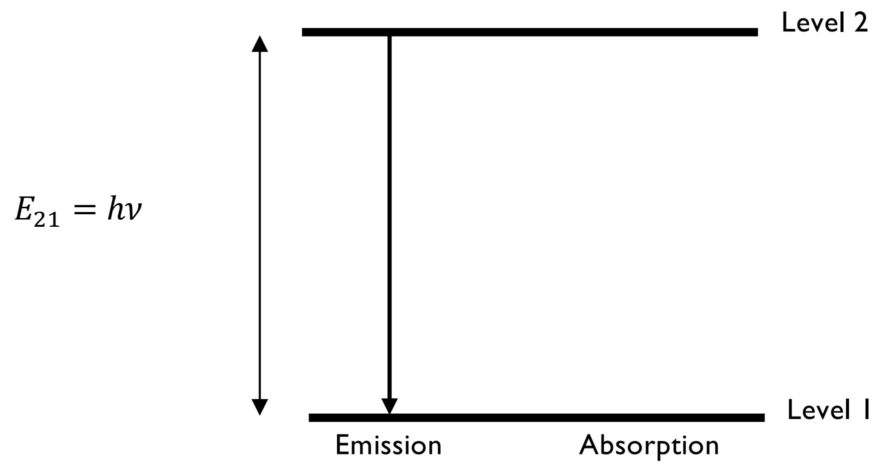

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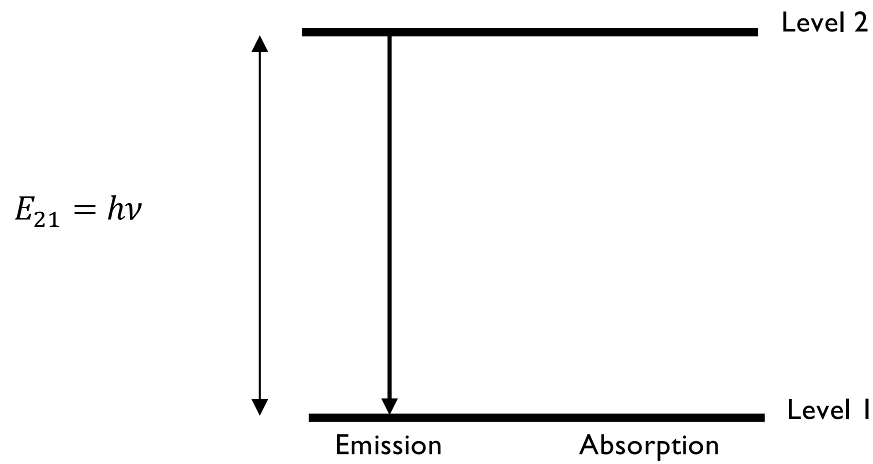

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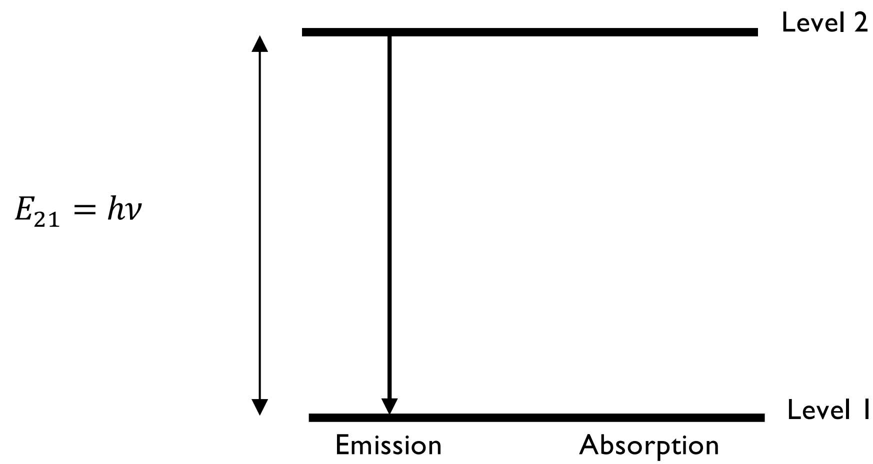

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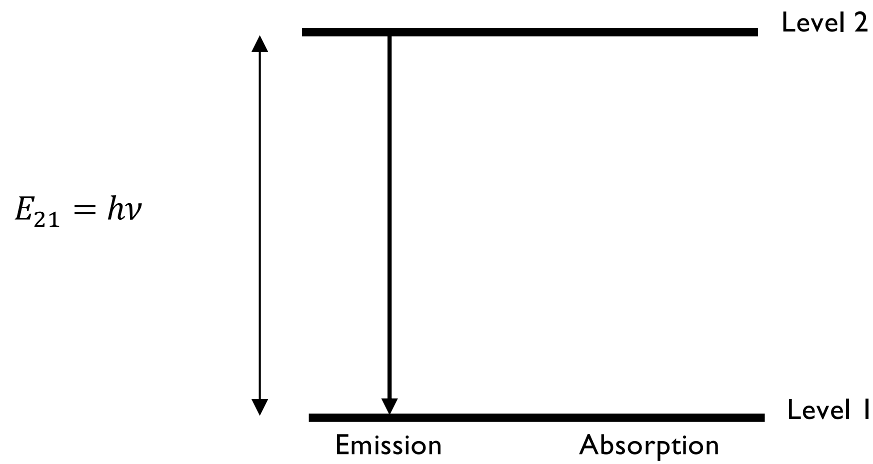
same logic for α_ν and B_{12} …



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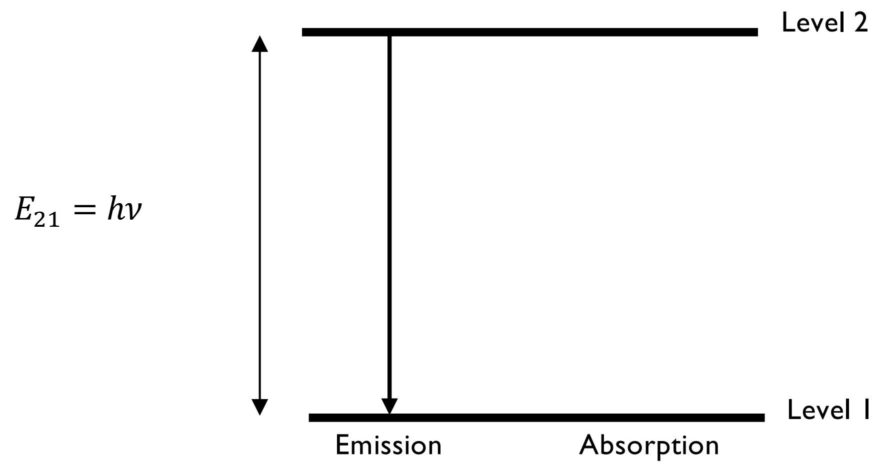

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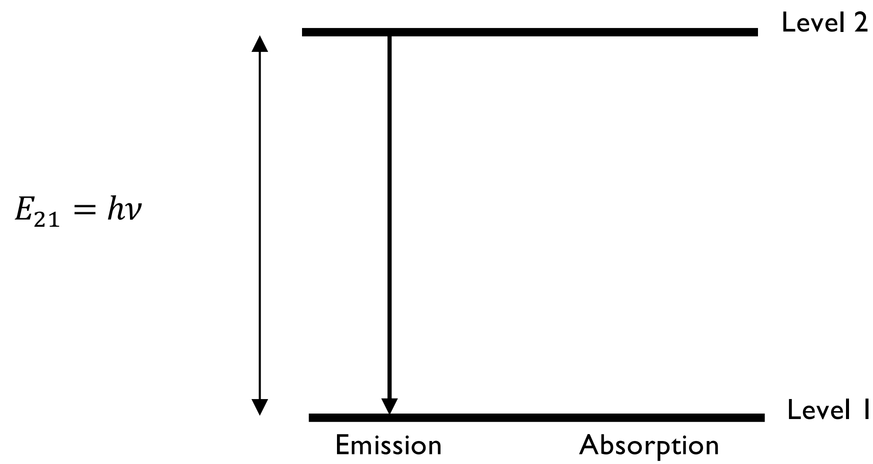
what about stimulated emission?



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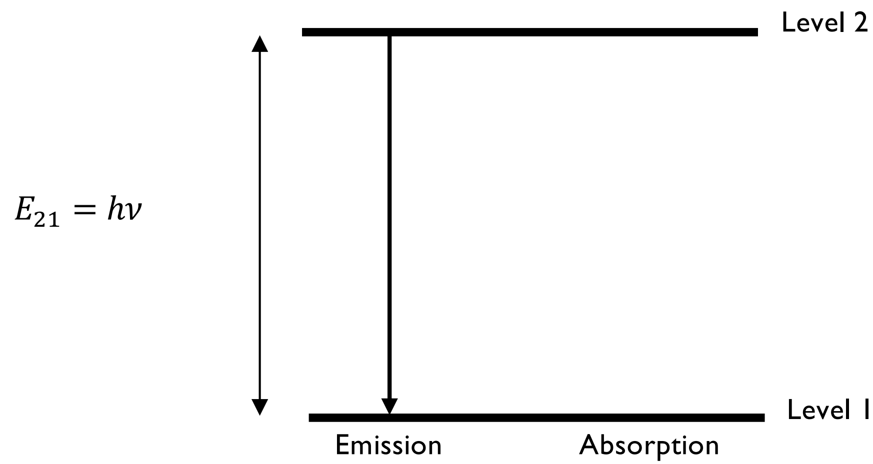

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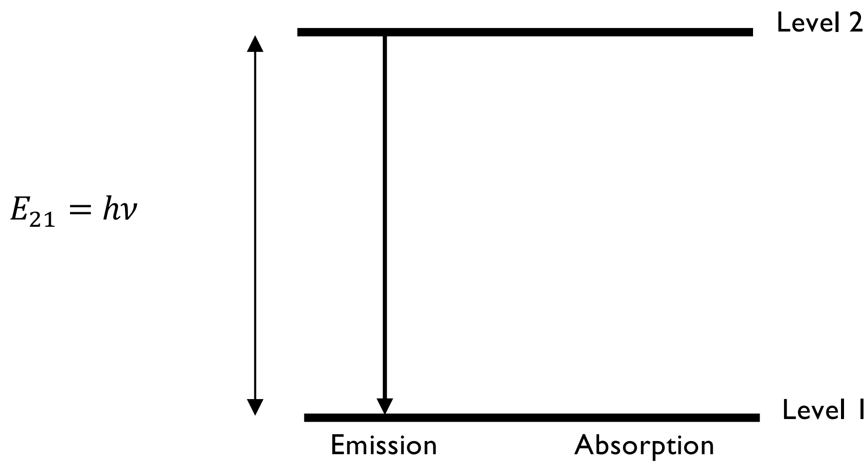
connection
between macroscopic description
and
quantum mechanical properties



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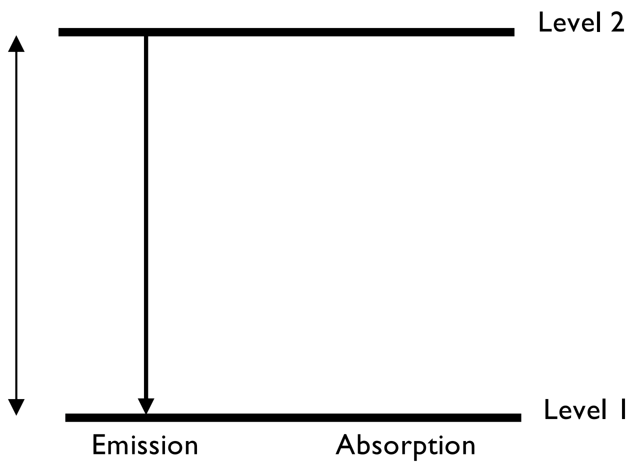
what about stimulated emission?
include as negative absorption...

$E_{21} = h\nu$



connection between macroscopic description and quantum mechanical properties

but there is "one more thing"...

- Einstein coefficients vs. α_ν and j_ν

$$\frac{dn_2}{dt} = -\, n_2\, A_{21}$$

$$dE = \frac{h\nu}{4\pi}\, n_2 A_{21}\, dA\, ds\, dt\, d\Omega\, d\nu$$

$$dE = \qquad j_\nu \qquad dA\, ds\, dt\, d\Omega\, d\nu$$

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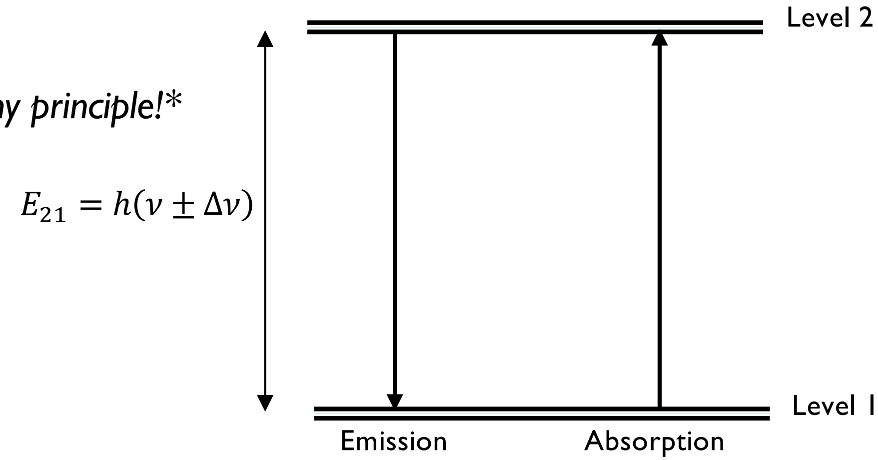
*the energy levels are not sharp due to Heisenberg's uncertainty principle!**



*see 'line broadening' later...

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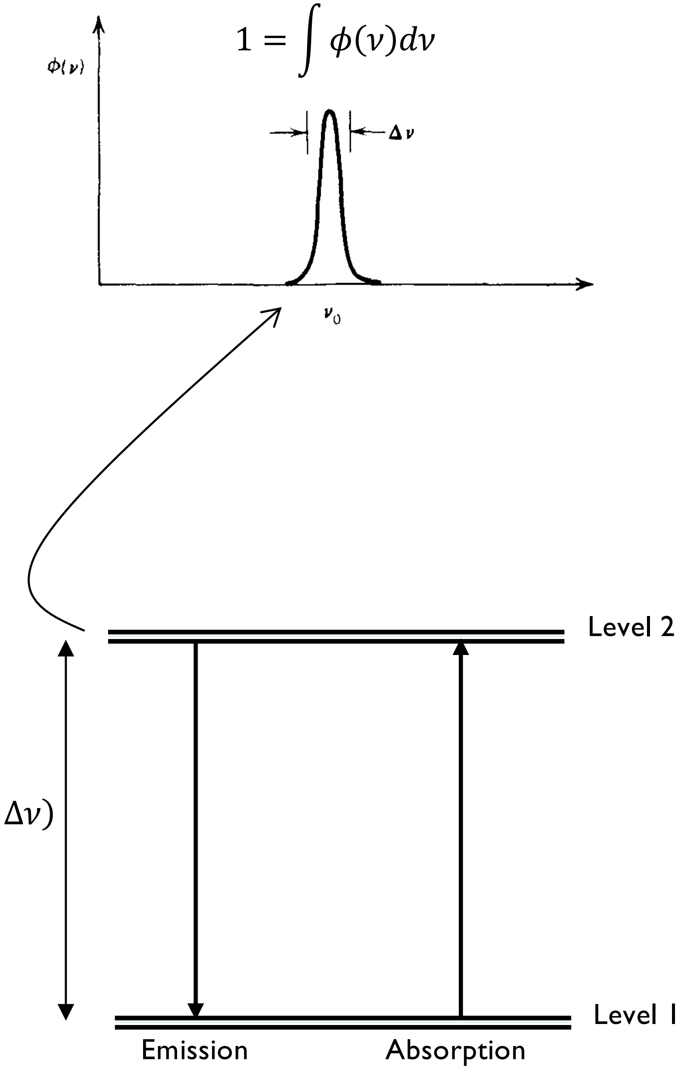
effective line profile $\phi(\nu)$

$$1 = \int \phi(\nu) d\nu$$

$$E_{21} = h(\nu \pm \Delta\nu)$$



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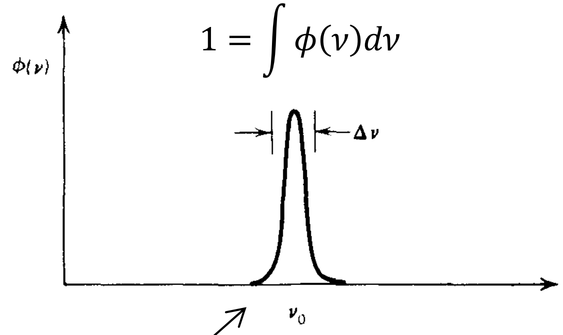

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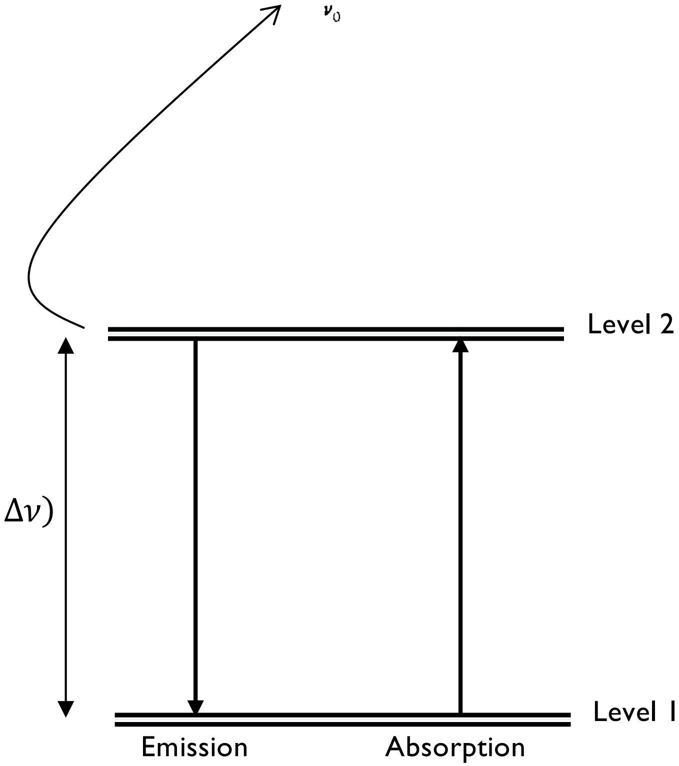

- radiative transfer equation using Einstein coefficients

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

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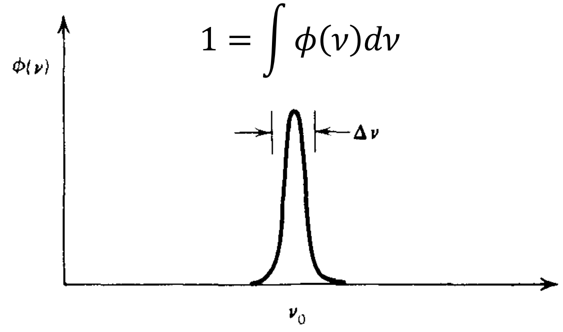
- source function $$S_\nu = \frac{n_2 A_{21}}{n_1 B_{12} - n_2 B_{21}}$$

- thermal excitation
- atomic transitions
- Einstein coefficients
- **line broadening**

- what effects lead to $\phi(\nu)$?



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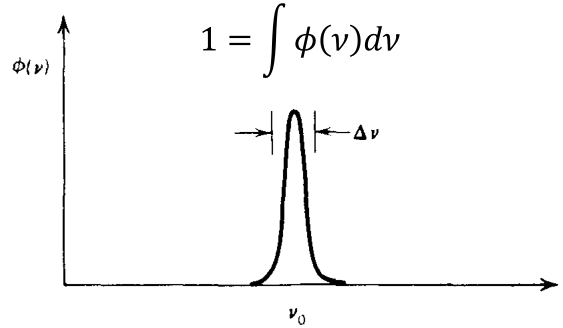
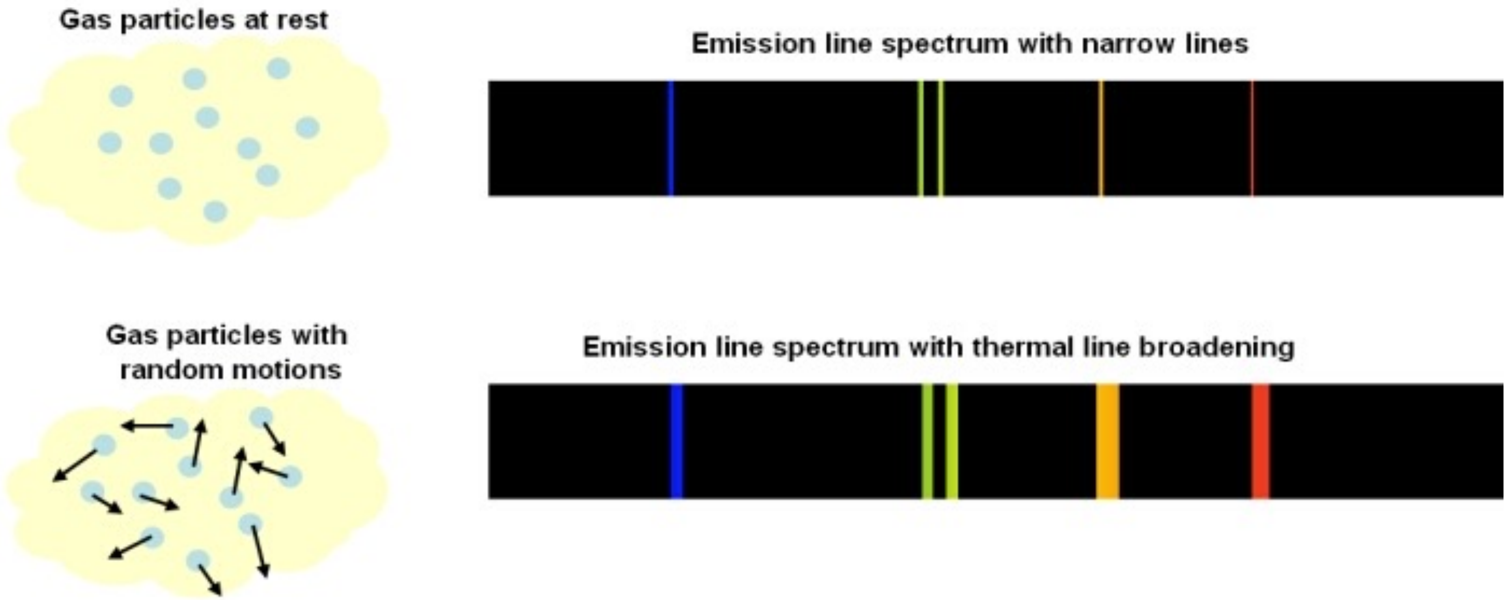


  - Doppler broadening
  - natural broadening
  - collisional broadening

- what effects lead to $\phi(\nu)$?

  - Doppler broadening
    - atoms (with mass m_a) in thermal motion
    - absorption/emission frequency ν differs by $\Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2k_BT}{m_a}}$
    - net effect spreads out line, but not its strength



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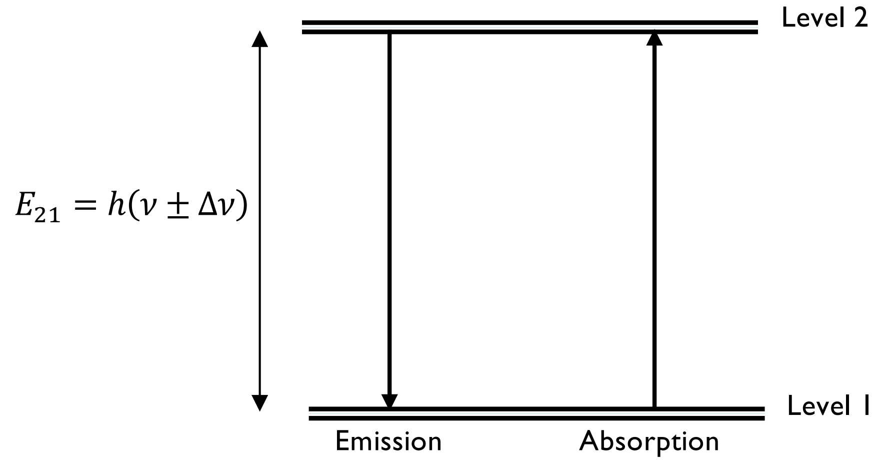
    - energy levels are not sharp which leads to the so-called
    - Lorentz line profile $\phi(\nu) = \frac{\gamma^2}{(\nu-\nu_0)^2-\gamma^2}$ $\quad \gamma = A_{21}\left(=\sum_{m<n} A_{nm}\right)$
    - broadening usually very small



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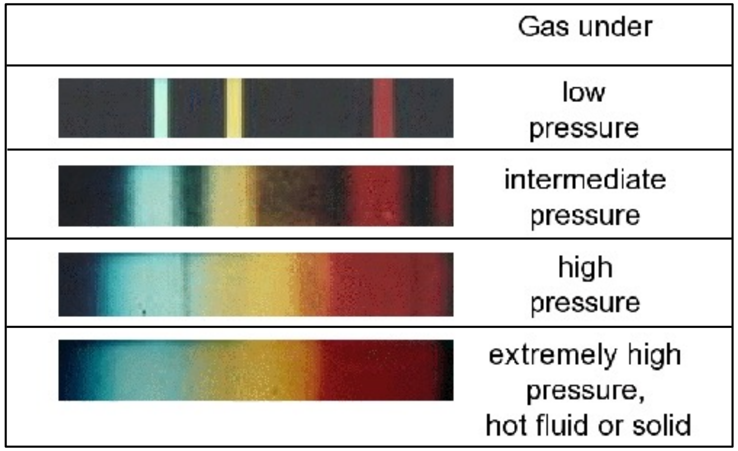
  - collisional broadening
    - broadening due to effects of nearby atoms
    - collisions supply/remove small amounts of energy
    - Lorentz line profile $\phi(\nu) = \frac{\alpha^2}{(\nu-\nu_0)^2 - \alpha^2}$ $\quad \alpha \propto P\,T^{-0.5}$



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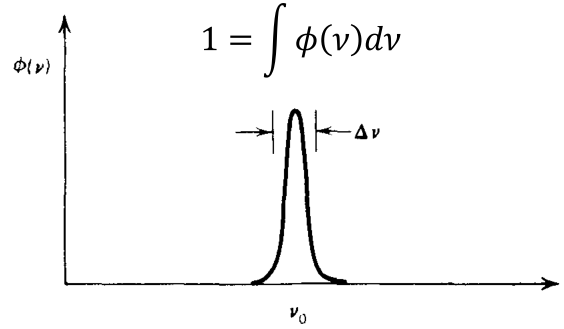

- relation between macro- and micro-physics

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu}$$

$$j_\nu = \frac{h\nu}{4\pi}\, n_2\, A_{21}$$

$$\alpha_\nu = \frac{h\nu}{4\pi}\,(n_1\, B_{12} - n_2\, B_{21})$$