Alexander Knebe (*Universidad Autonoma de Madrid*)

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Can we find the intensity $I_\nu(T, \Omega)$ for some simple example?

- black-body radiation
- thermodynamics of black-body radiation
- Planck spectrum
- local thermal equilibrium

- **black-body radiation**
- thermodynamics of black-body radiation
- Planck spectrum
- local thermal equilibrium

- formally we need to distinguish...
  - <u>thermal radiation</u>
  - <u>black-body radiation</u>

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  - thermal radiation
    - generated by thermal motion in matter
    - all matter with $T > 0$ emits thermal radiation
    - described by $I_\nu(T, \Omega)$

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    - generated by matter in thermal equilibrium ($T = \text{const.}$)
    - fully isotropic
    - described by $B_\nu(T) = I_\nu(T, \Omega)$

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  - <u>thermal radiation</u>
    - generated by thermal motion in matter
    - all matter with $T > 0$ emits thermal radiation
    - described by $I_\nu(T, \Omega)$
    - becomes black-body radiation for optically thick media

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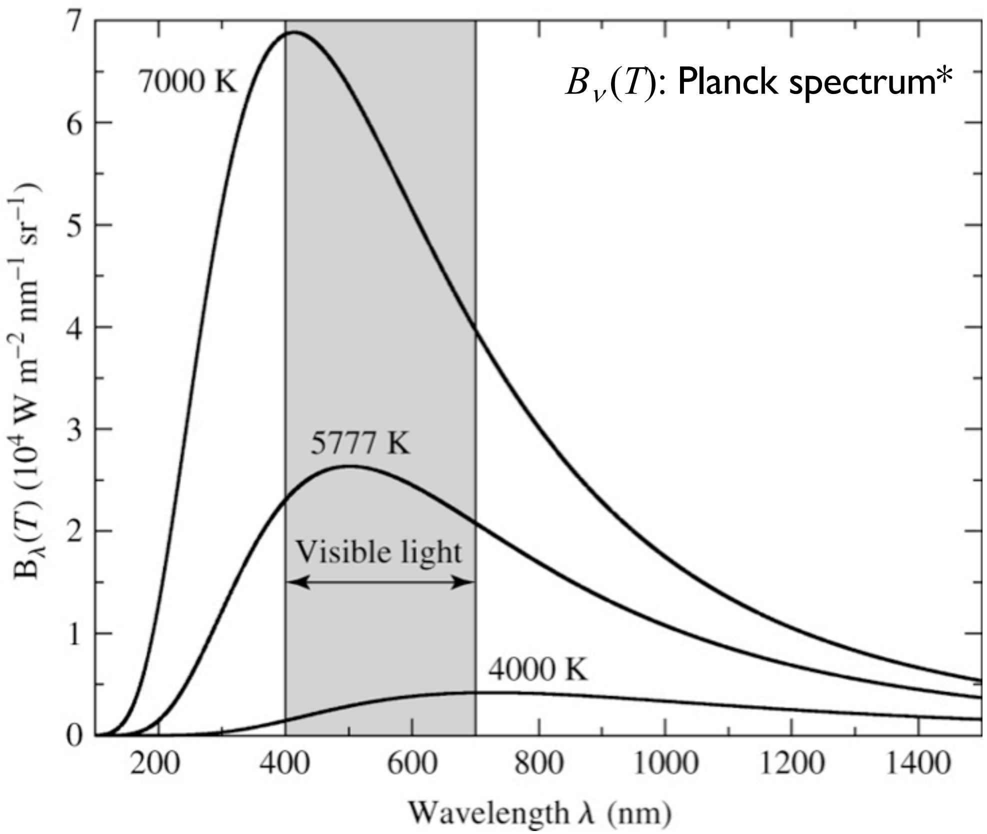

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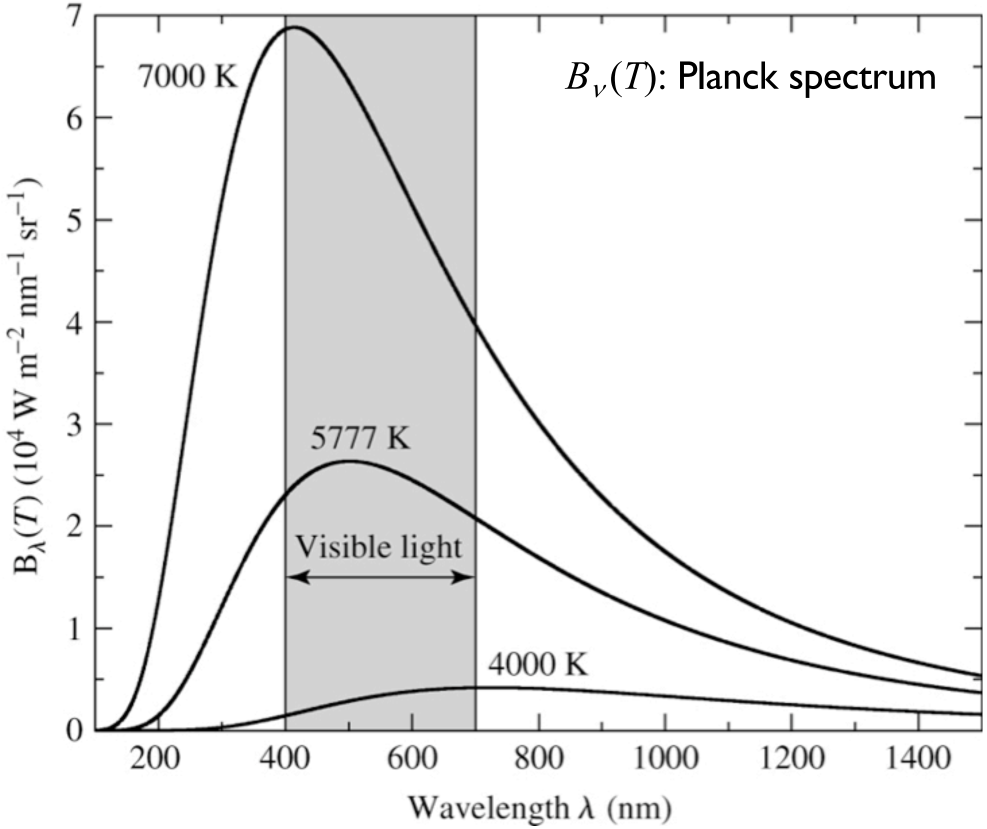
*derivation comes later...



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emitted by an idealized opaque, non-reflective body

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idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence

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examples for black-bodies...

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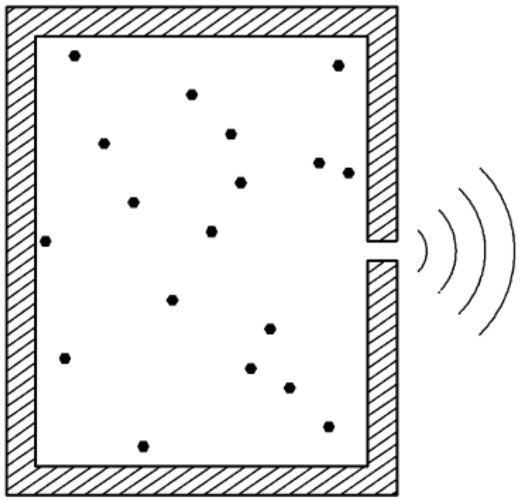

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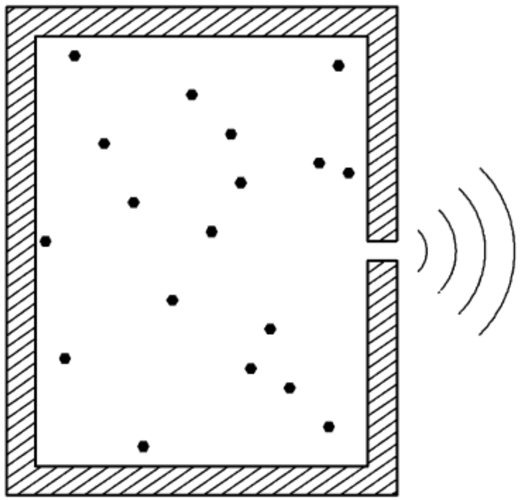

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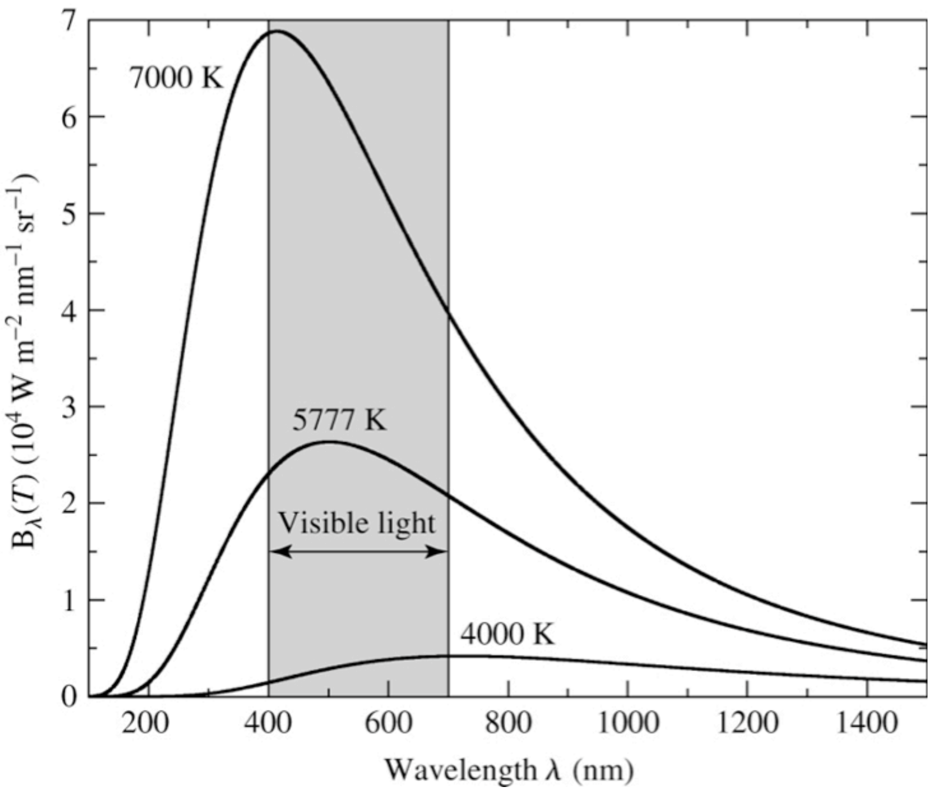

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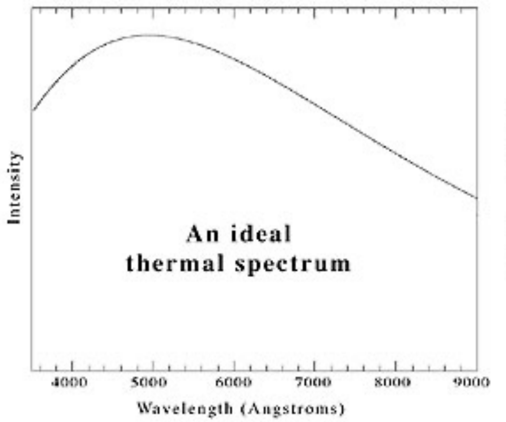

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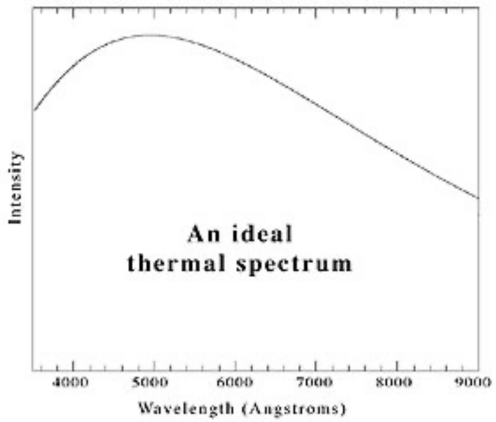

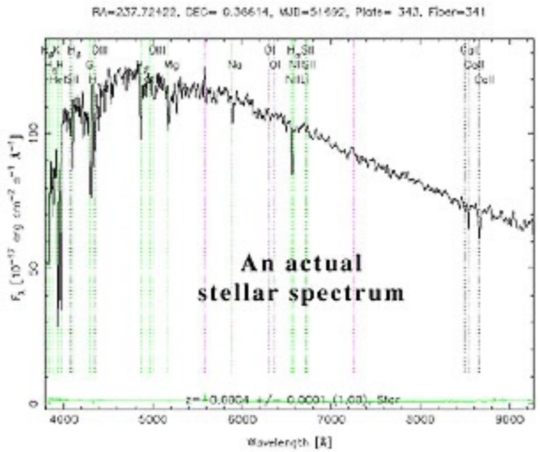

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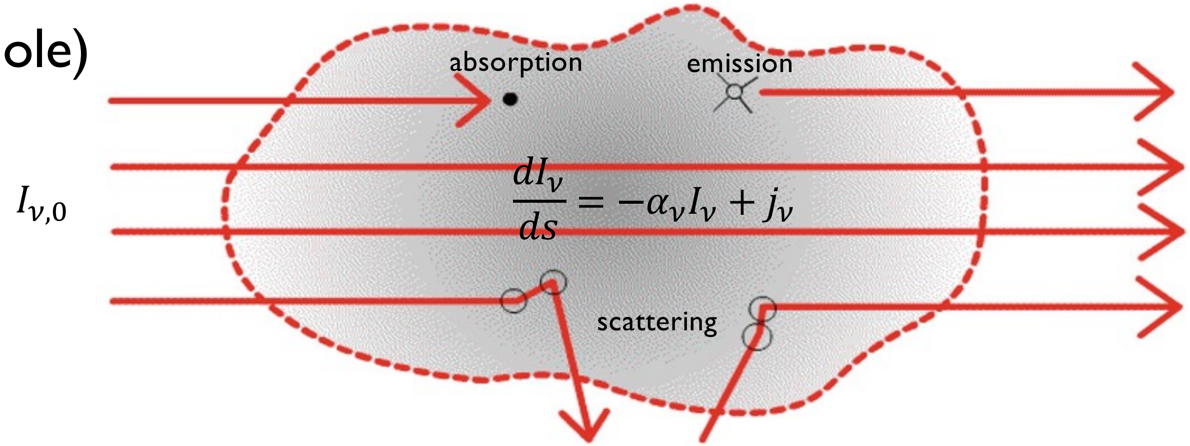

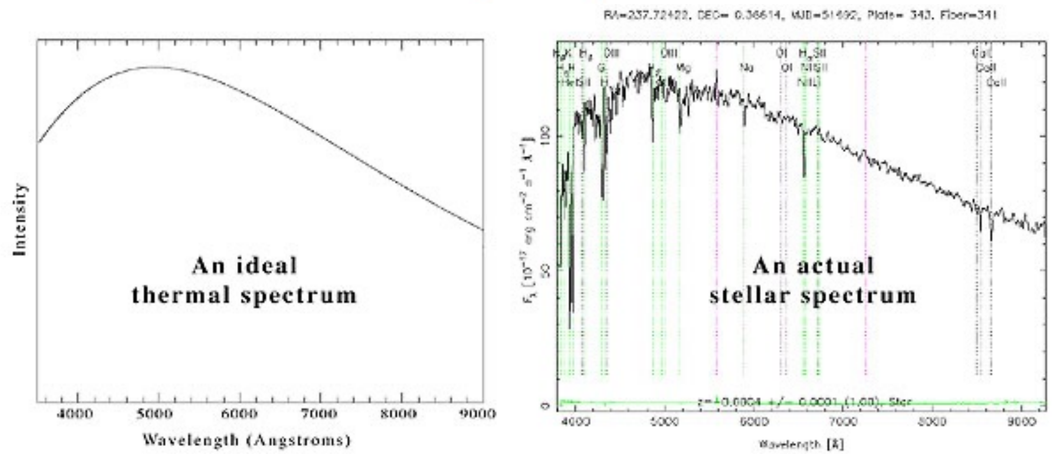

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- black holes?

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  - they absorb all the radiation that falls on them
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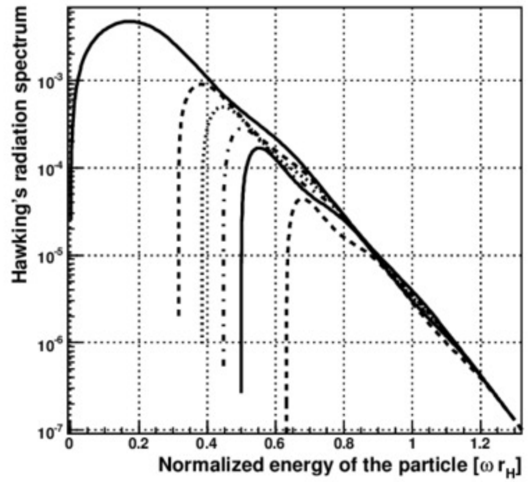

(Grain & Barau 2007)

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- the most perfect black-body in the Universe?

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*Cosmic Microwave Background Radiation: all details in Cosmology course

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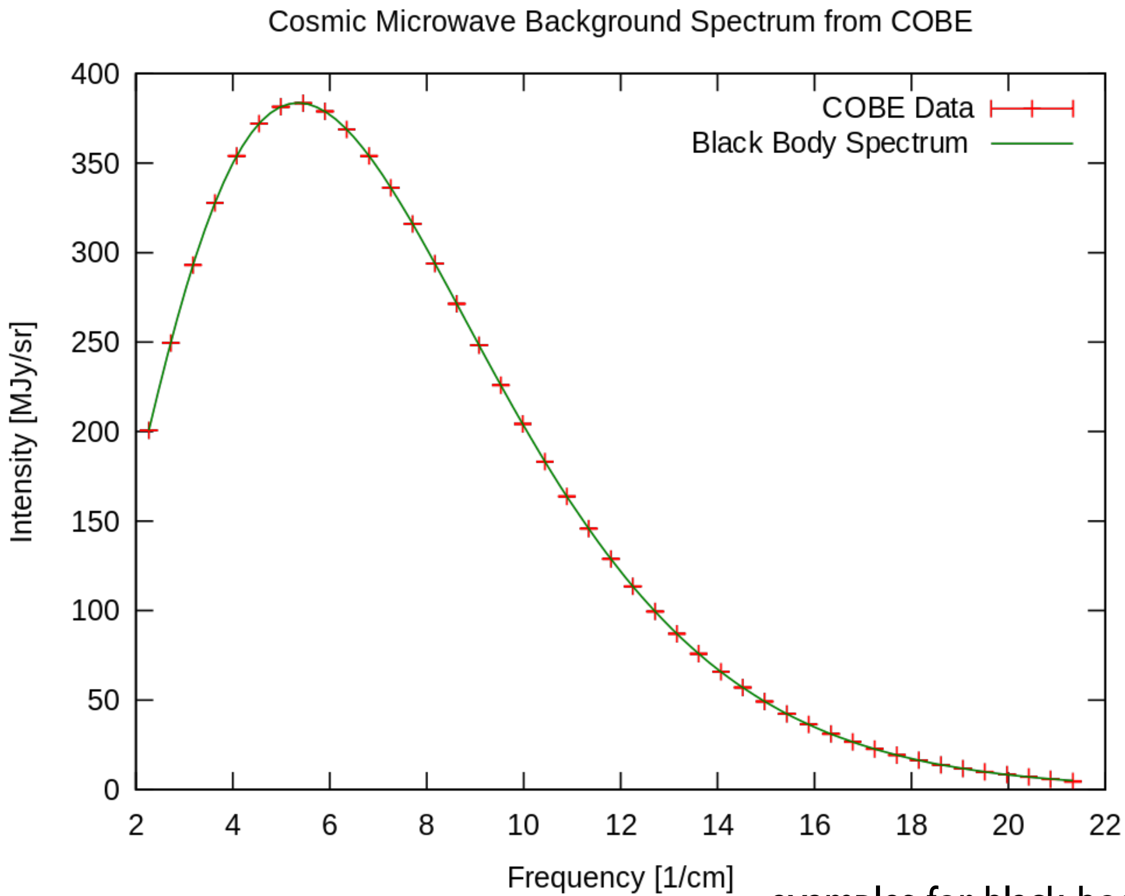

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  thermal radiation of a (black-)body in thermodynamic equilibrium with its environment

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thermal radiation of a (black-)body in ***thermodynamic equilibrium*** *with its environment*

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thermal radiation of a (black-)body in ***thermodynamic equilibrium*** *with its environment*

- populations **described by Saha-Boltzmann statistics***

$$N_i = N e^{-\frac{E_i}{k_B T}}$$

N_i : number of atoms/ions/molecules with energy E_i

*we'll make use of that later when deriving $B_\nu(T)$

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in ***thermal equilibrium*** *the intensity must be spatially constant*

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if material absorbs well at a certain wavelength, it will also radiate well at the same wavelength.

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at thermal equilibrium, the power radiated must be equal to the power absorbed

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- Planck spectrum
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any chance to obtain

energy density, intensity, and flux

of the radiation field

as a function of temperature?

- thermodynamics

any chance to obtain

energy density*, intensity, and flux*

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as a function of temperature*?*

?

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any chance to obtain

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$$u(T) = \frac{4}{c}\sigma_B\, T^4$$ energy density

$$B(T) = \frac{1}{\pi}\sigma_B T^4$$ intensity

$$F(T) = \sigma_B\, T^4$$ flux

$$S(T) = \frac{16}{3c}\,\sigma_B\, T^3 V$$ entropy

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$B_\nu(T)$ comes later...

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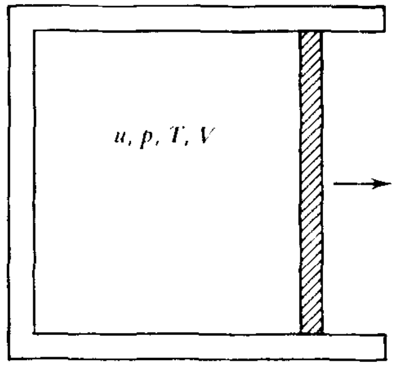
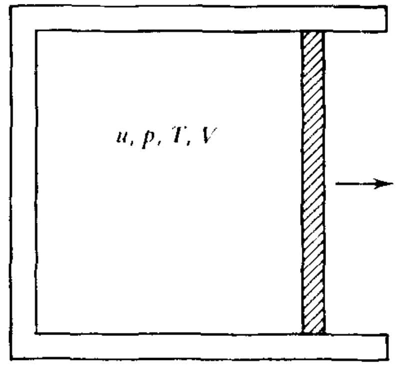


cavity that can be manipulated

▪ thermodynamics

- first law of thermodynamics

$$dQ = dU + pdV$$



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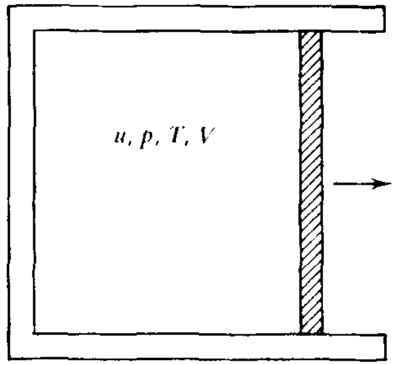

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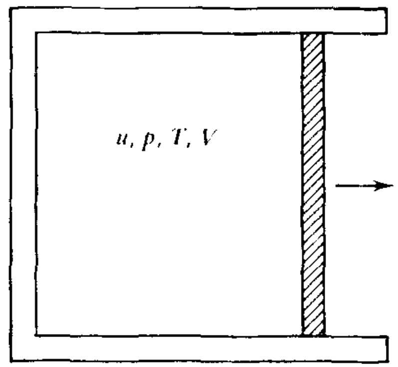
$$U = u\,V, \qquad p = \frac{u}{3}$$



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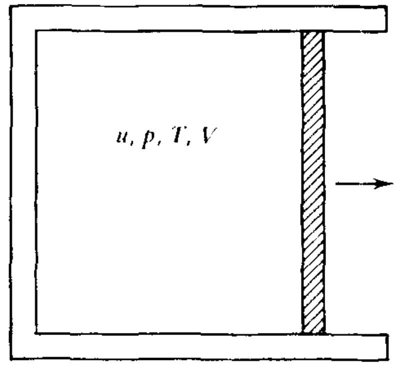

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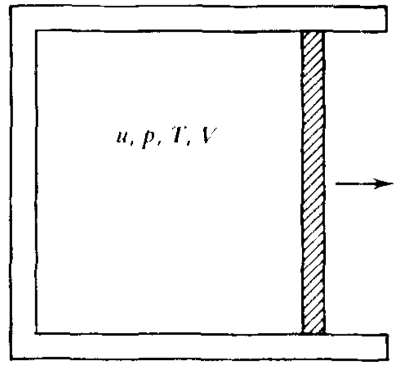

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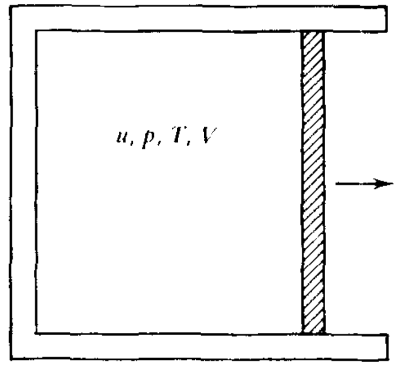

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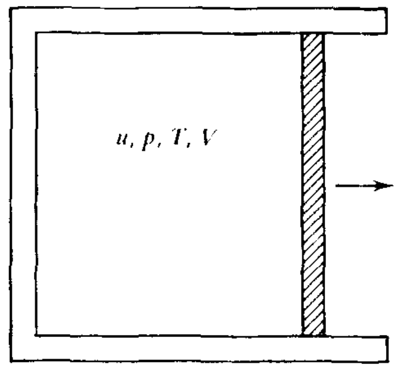

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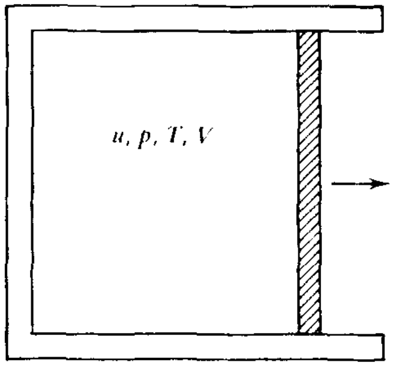

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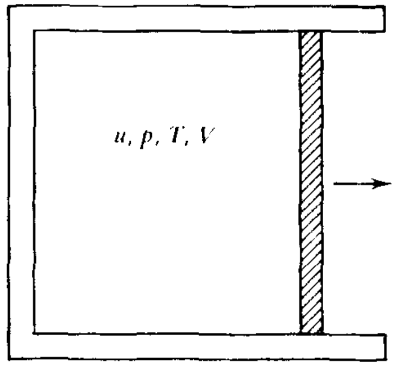

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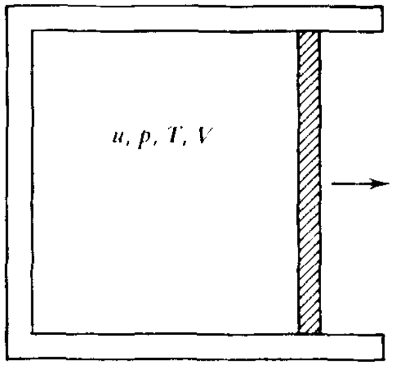

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$$= \boxed{\left(\frac{\partial S}{\partial T}\right)_V} dT + \boxed{\left(\frac{\partial S}{\partial V}\right)_T} dV$$

$S = S(T,V)$

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T}$$

▪ thermodynamics

- first law of thermodynamics

$$dQ = dU + pdV$$

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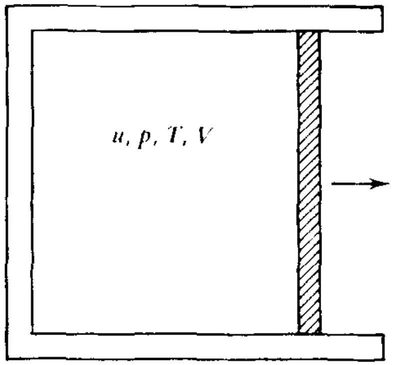
$$dS = \frac{dQ}{T}$$

- radiation field

$$U = u\,V, \qquad p = \frac{u}{3}$$

$$\frac{1}{T}\frac{du}{dT} = -\frac{4}{3}\frac{u}{T^2} + \frac{4}{3T}\frac{du}{dT}$$



cavity that can be manipulated

U : total energy of cavity
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(exercise)

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T}$$

$$\boxed{\frac{V}{T}\frac{du}{dT}}\,dT + \boxed{\frac{4}{3}\frac{u}{T}}\,dV = \boxed{\left(\frac{\partial S}{\partial T}\right)_V}\,dT + \boxed{\left(\frac{\partial S}{\partial V}\right)_T}\,dV$$

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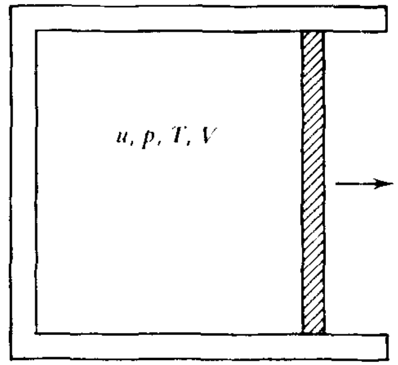

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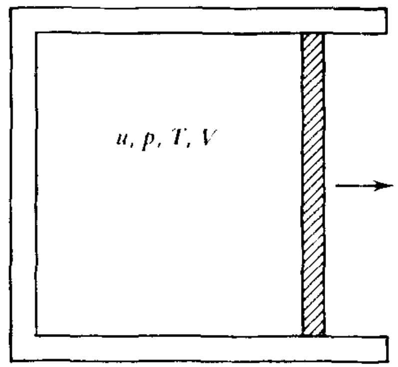
$$\frac{4u}{T} = \frac{du}{dT}$$



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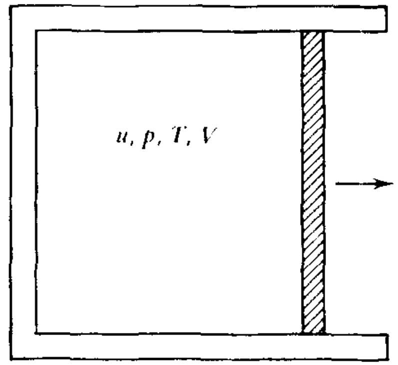
$$\frac{4u}{T} = \frac{du}{dT} \qquad \rightarrow \qquad \frac{du}{u} = 4\frac{dT}{T}$$



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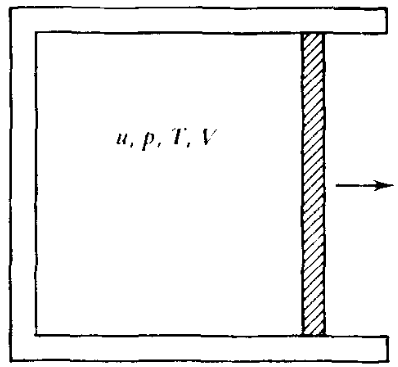
$$\frac{4u}{T} = \frac{du}{dT} \quad \rightarrow \quad \frac{du}{u} = 4\,\frac{dT}{T} \quad \rightarrow \quad u(T) = a\,T^4$$



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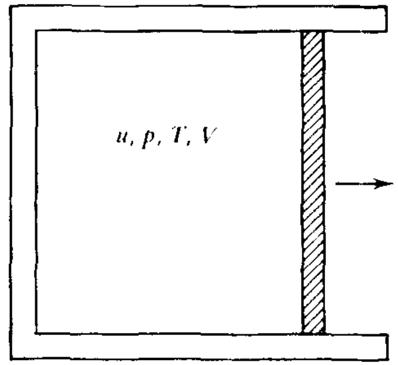
Stefan-Boltzmann law



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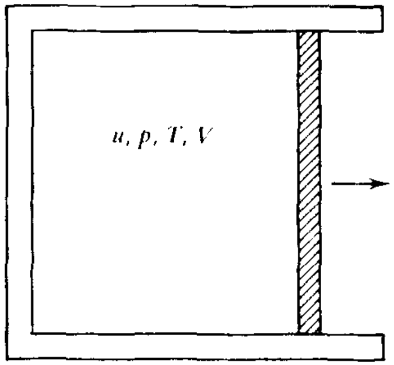
energy density



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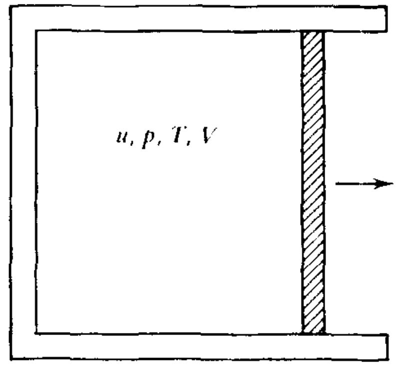
...relation to intensity*

$$u(T) = \int u_\nu\, d\nu = \iint \frac{I_\nu(T)}{c}\, d\Omega d\nu = \frac{4\pi}{c} \int I_\nu(T)\, d\nu$$



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*see Fundamentals lecture...

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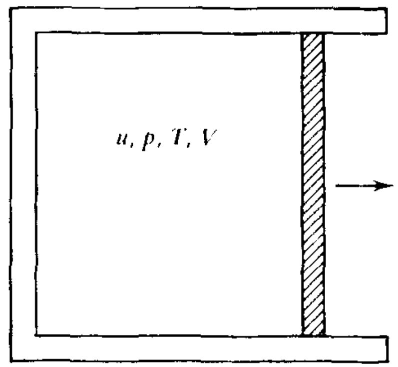

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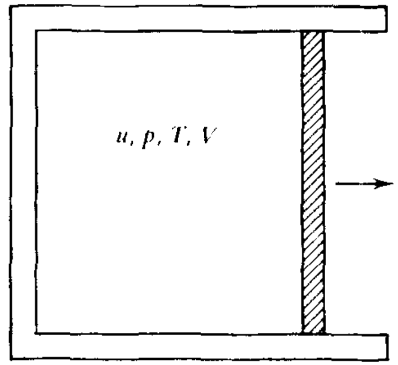

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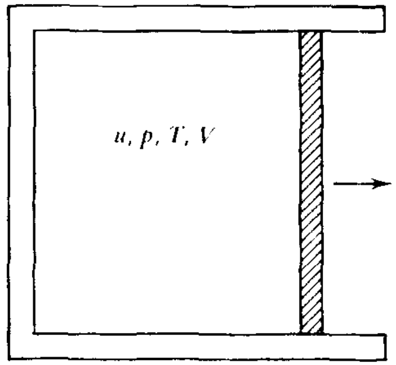

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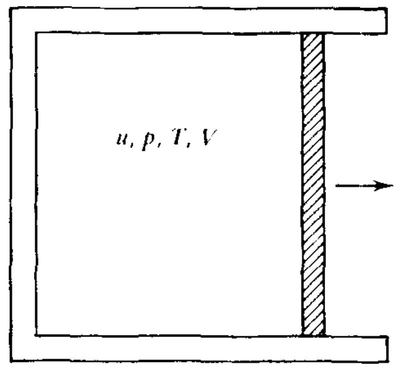

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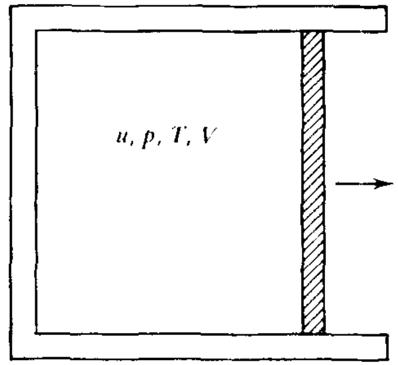

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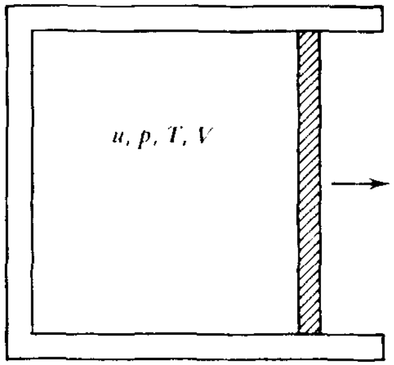

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...relation to flux*

$$F = \int F_\nu\,d\nu = \iint I_\nu(\Omega)\,cos\theta\,d\Omega d\nu = \iint B_\nu\,cos\theta\,d\Omega d\nu$$

$$= \int B_\nu\,d\nu \int cos\theta\,d\Omega = \int B_\nu\,d\nu \int_0^{2\pi} d\varphi \int_0^{\pi/2} cos\theta\,sin\theta\,d\theta = \pi \int B_\nu\,d\nu = \pi B(T)$$

*see Fundamentals lecture...

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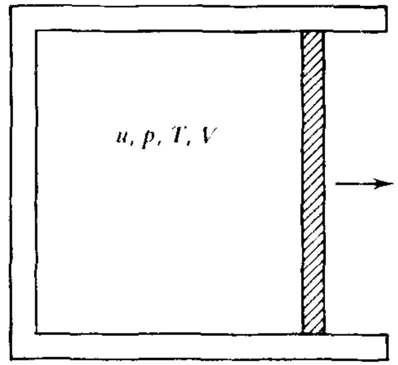
$$F(T) = \frac{ac}{4}\ T^4$$ emergent flux



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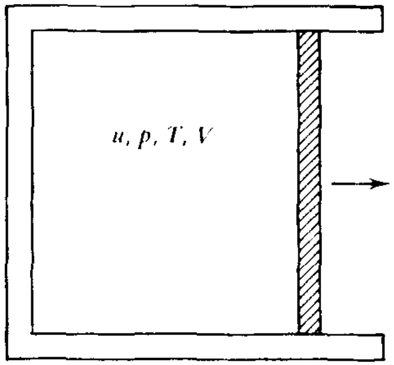
a ?



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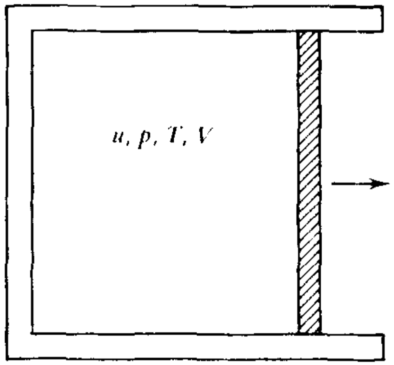
$$a = \frac{4}{c}\sigma_B$$

σ_B : Stefan-Boltzman constant



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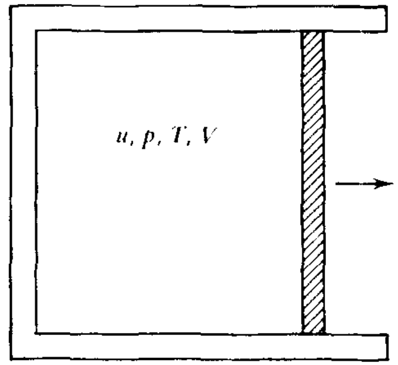

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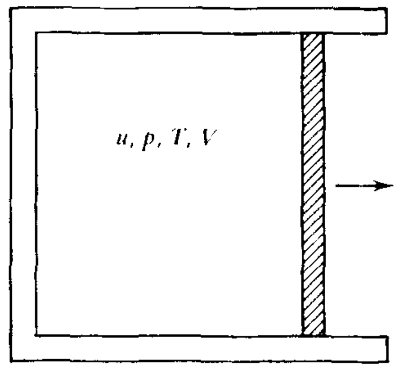
this factor – and its relation to a – will be derived later...



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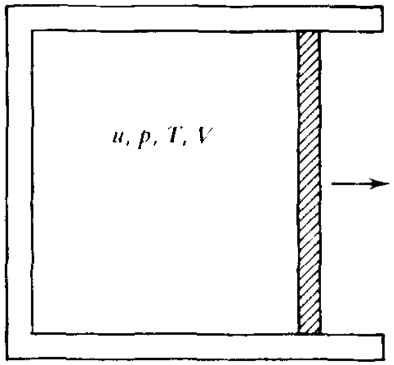

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$$S(T) = \frac{16}{3c}\,\sigma_B\, T^3 V$$ entropy ***(exercise)***

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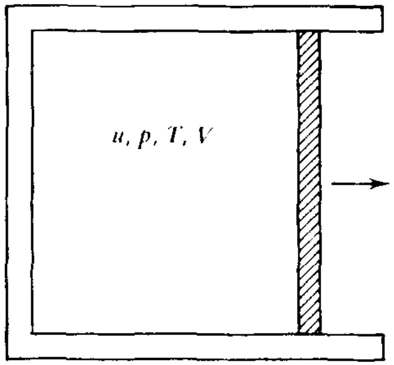


cavity that can be manipulated

- Stefan-Boltzmann law

$$u(T) = \frac{4}{c} \sigma_B\, T^4$$ energy density

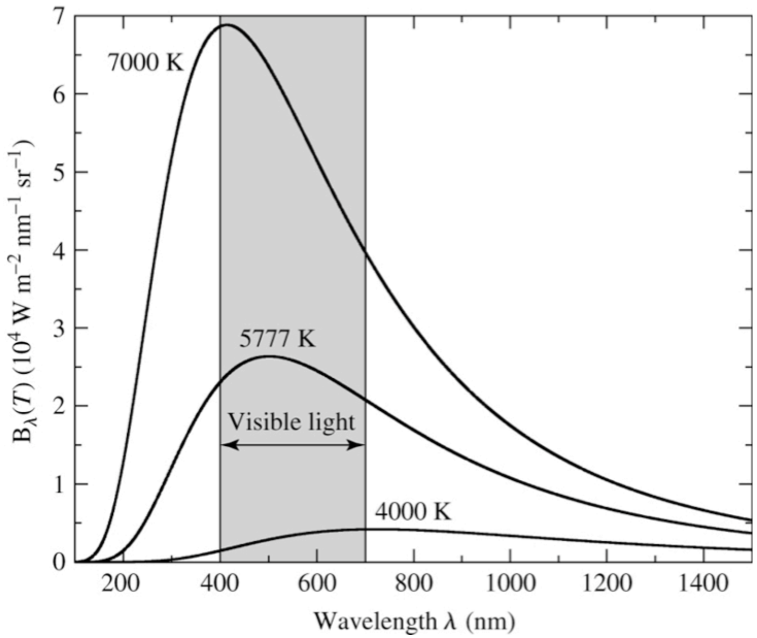
dependency on wave-length?

$B_\nu(T)$?

$$B(T) = \frac{1}{\pi} \sigma_B T^4$$ integrated intensity

$$F(T) = \sigma_B\, T^4$$ emergent flux $(\sigma_B$

$$S(T) = \frac{16}{3c}\, \sigma_B\, T^3 V$$ entropy ***(exercise)***



- black-body radiation
- thermodynamics of black-body radiation
- **Planck spectrum**
- local thermal equilibrium

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- **Planck spectrum:**
  - derivation
  - properties
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  - **derivation**
  - properties
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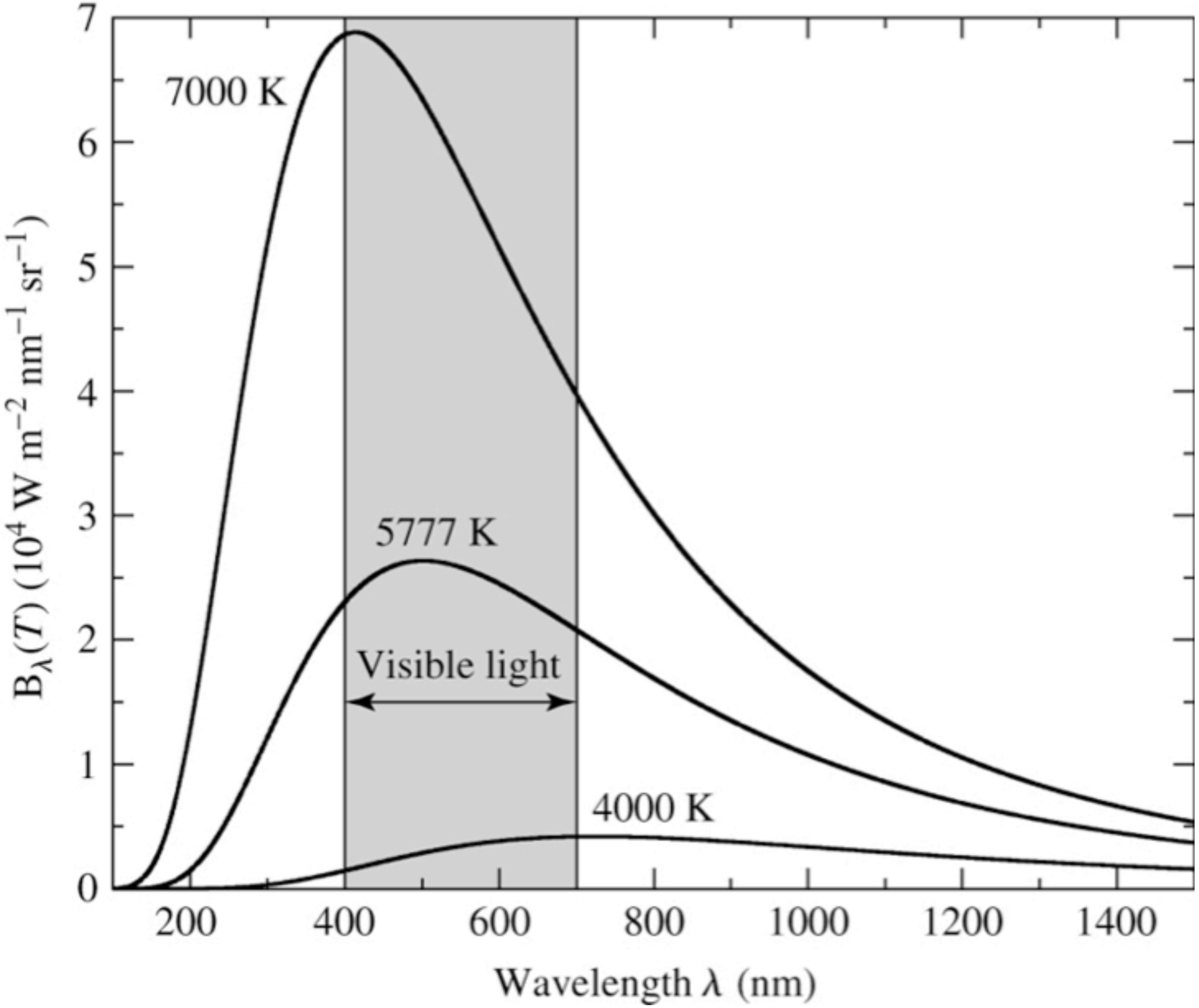
$$B_\nu(T) = ?$$


7000 K
5777 K
4000 K
Visible light
B_λ(T) (10^4 W m^-2 nm^-1 sr^-1)
Wavelength λ (nm)
0
1
2
3
4
5
6
7
200
400
600
800
1000
1200
1400

$$B_\nu(T) = \frac{c}{4\pi}\ u_\nu(T)$$

$$u_\nu(T)d\nu = ?$$

remember: $u_\nu = 4\pi/c\ I_\nu$ for isotropic radiation, and $I_\nu = B_\nu$ for black-body radiation

$$B_\nu(T) = \frac{c}{4\pi}\, u_\nu(T)$$

$$u_\nu(T)d\nu = \frac{dN(\nu)}{dV}\,\langle E\rangle$$

$$B_\nu(T) = \frac{c}{4\pi}\, u_\nu(T)$$

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number density of possible photon states

average energy per state

$$B_\nu(T) = \frac{c}{4\pi}\, u_\nu(T)$$

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number density of possible photon states

average energy per state:

- quantum mechanics → Planck spectrum
- classical thermodynammics → Rayleigh-Jeans law

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average energy per state:

- quantum mechanics → Planck spectrum
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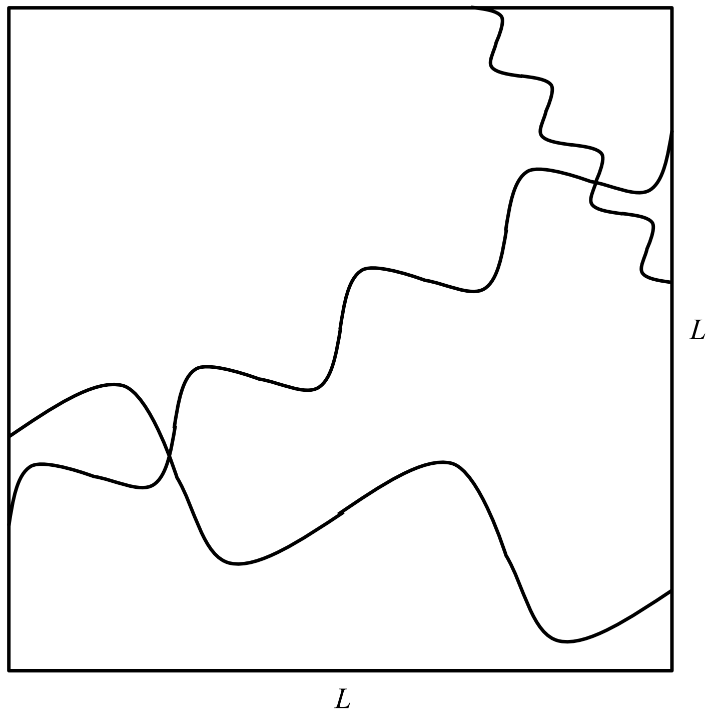
- number density of possible photon states

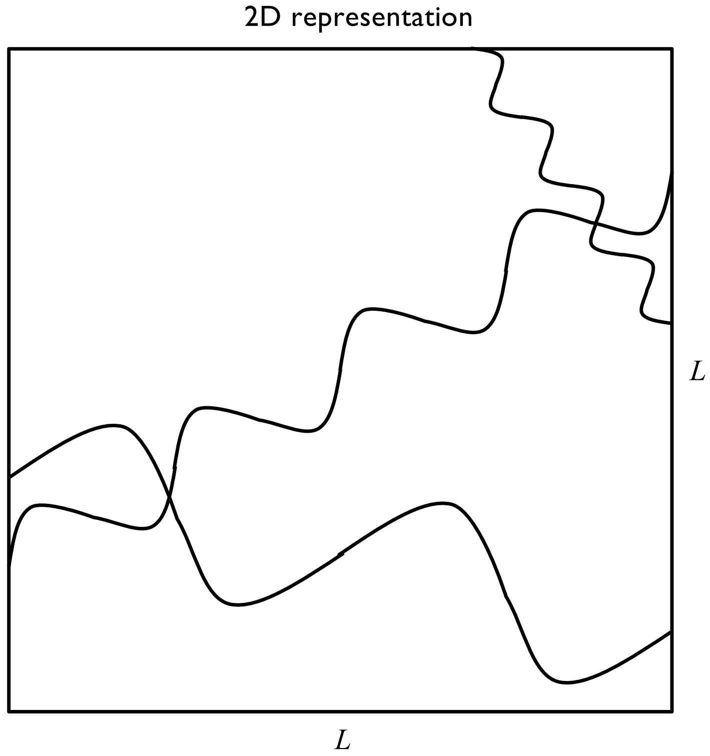
$$\frac{dN(\nu)}{dV} = ?$$

- number density of possible photon states

2D representation



- number density of possible photon states
    - standing wave

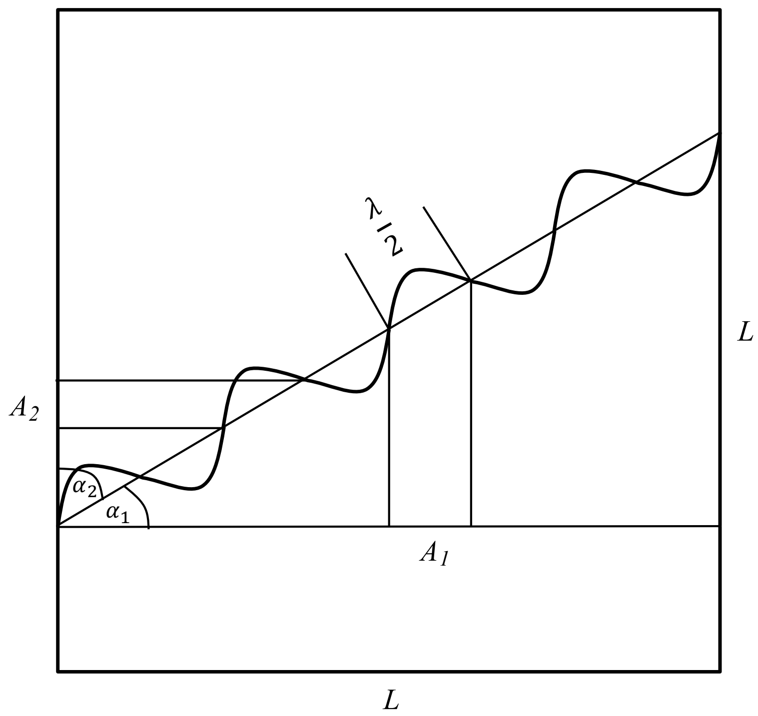


- number density of possible photon states
  - standing wave

$$n_i\, A_i = L \qquad n_i \in \mathbb{N}, i = 1,2,3$$

$$\frac{\lambda/2}{A_i} = \cos(\alpha_i)$$

2D representation



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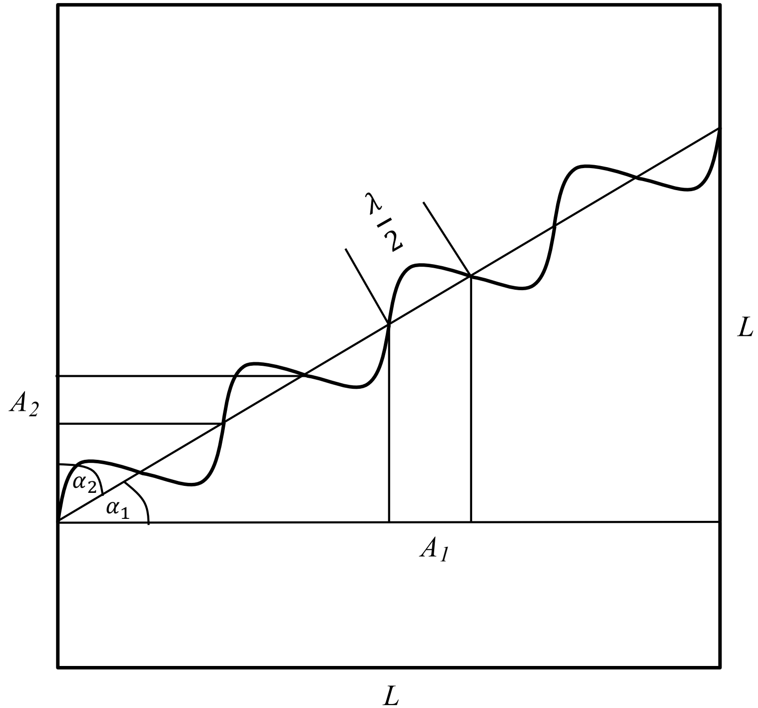
$$\frac{\lambda/2}{A_i} = \cos(\alpha_i)$$

  - direction cosine (3D)

$$1 = cos^2(\alpha_1) + cos^2(\alpha_2) + cos^2(\alpha_3)$$

2D representation

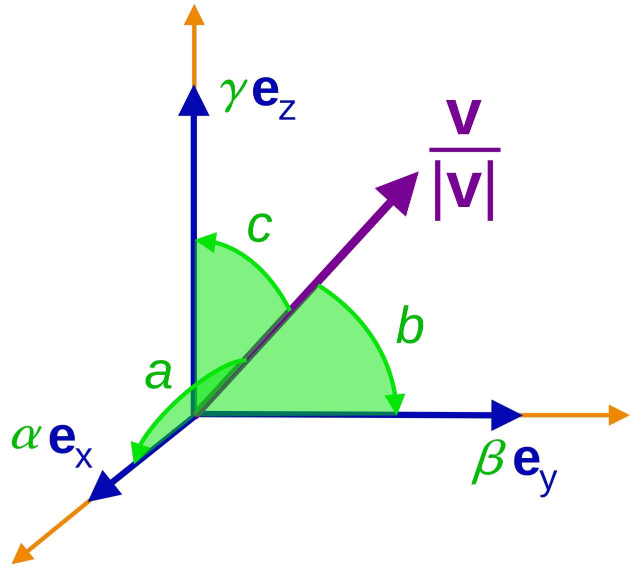


- direction cosine

$$\cos(a) = \frac{\vec{v} \cdot \vec{e}_x}{|\vec{v}|}$$

$$\cos(b) = \frac{\vec{v} \cdot \vec{e}_y}{|\vec{v}|}$$

$$\cos(c) = \frac{\vec{v} \cdot \vec{e}_z}{|\vec{v}|}$$



$$\rightarrow \; 1 = \left(\frac{\vec{v}\cdot\vec{e}_x}{|\vec{v}|}\right)^2 + \left(\frac{\vec{v}\cdot\vec{e}_y}{|\vec{v}|}\right)^2 + \left(\frac{\vec{v}\cdot\vec{e}_z}{|\vec{v}|}\right)^2 = cos^2(a) + cos^2(b) + cos^2(c)$$

- number density of possible photon states
  - standing wave

$$n_i \, A_i = L \qquad n_i \in \mathbb{N}, i = 1,2,3$$

$$\frac{\lambda/2}{A_i} = \cos(\alpha_i)$$

  - direction cosine (3D)

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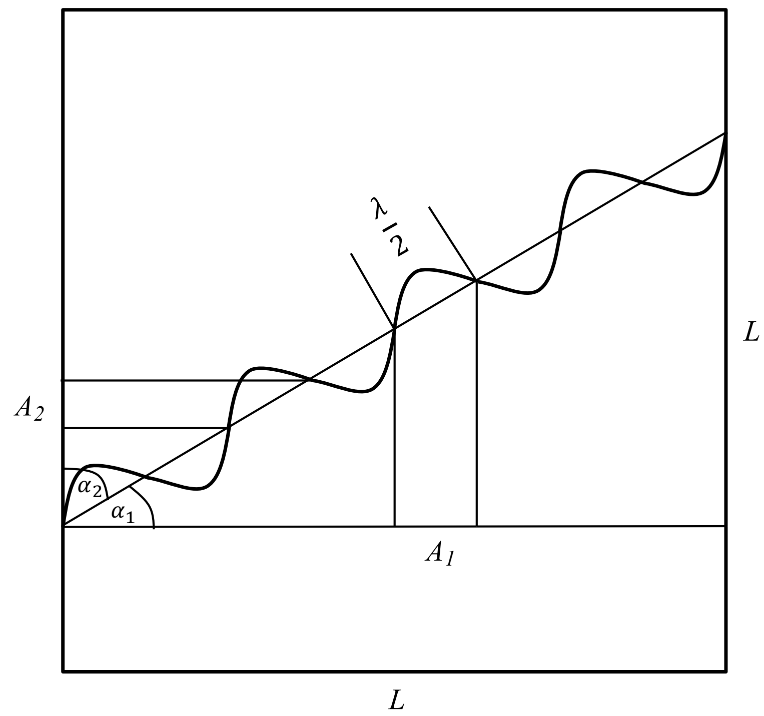
2D representation



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$$\left(\frac{2L}{\lambda}\right)^2 = n_1^2 + n_2^2 + n_3^2$$

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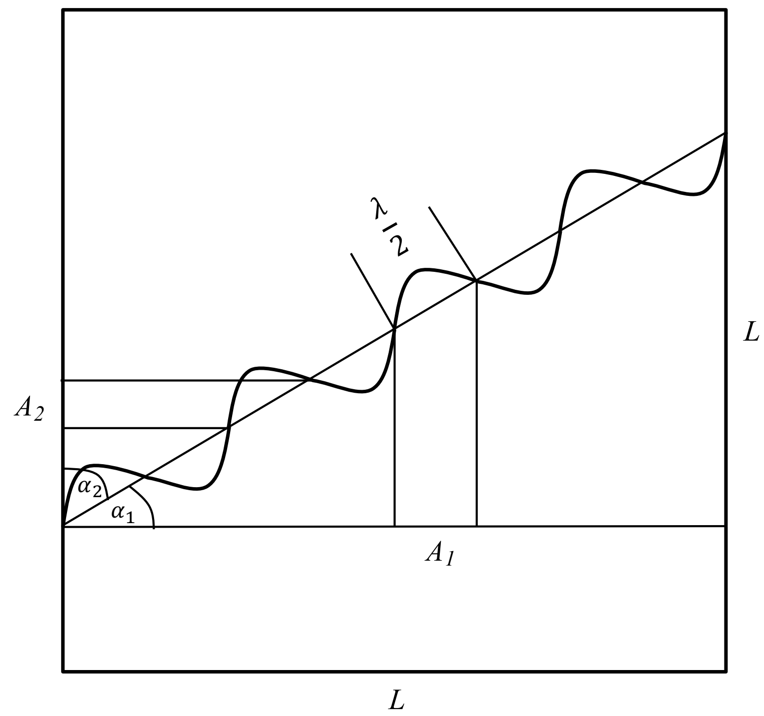

$$1 = \frac{\lambda^2}{4A_1} + \frac{\lambda^2}{4A_2} + \frac{\lambda^2}{4A_2} = \left(\frac{\lambda}{2}\right)^2 \left(\left(\frac{n_1}{L}\right)^2 + \left(\frac{n_2}{L}\right)^2 + \left(\frac{n_3}{L}\right)^2\right) = \left(\frac{\lambda}{2L}\right)^2 (n_1^2 + n_2^2 + n_3^2)$$

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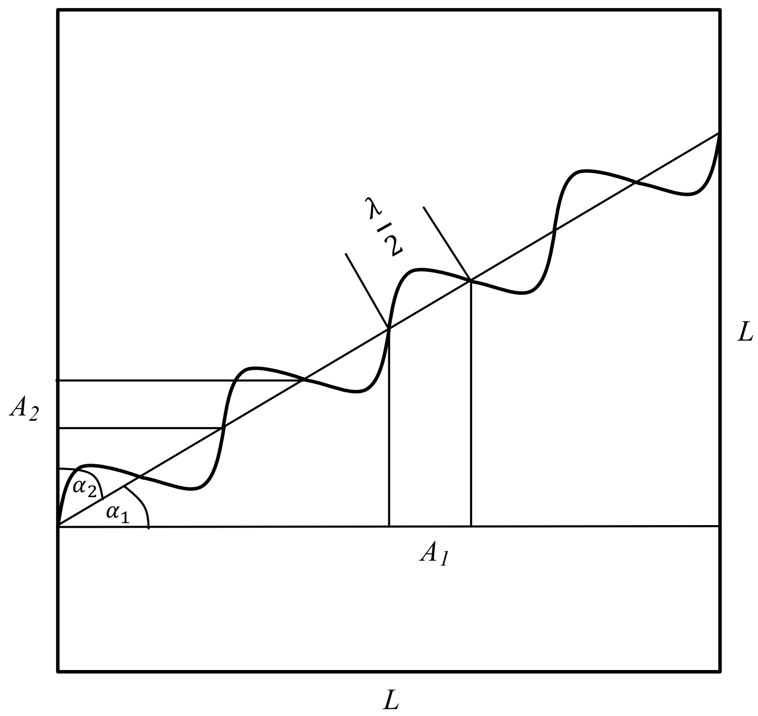

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how many standing waves fit into octant $n_i > 0$ *?*

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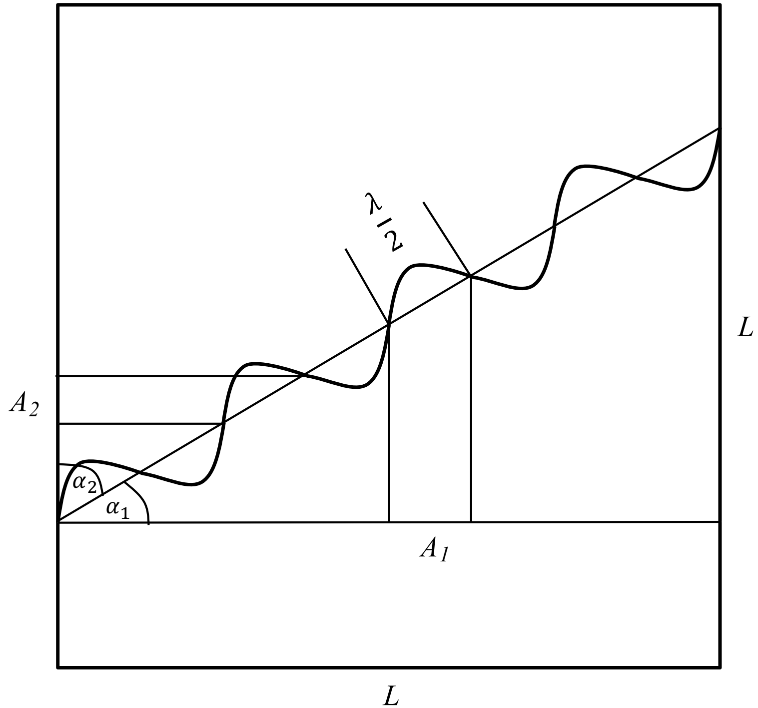

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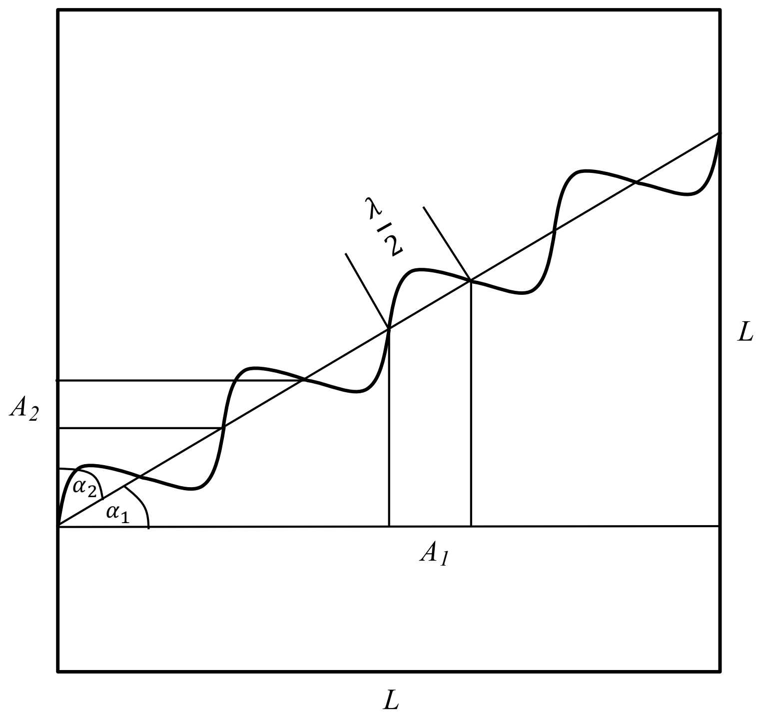

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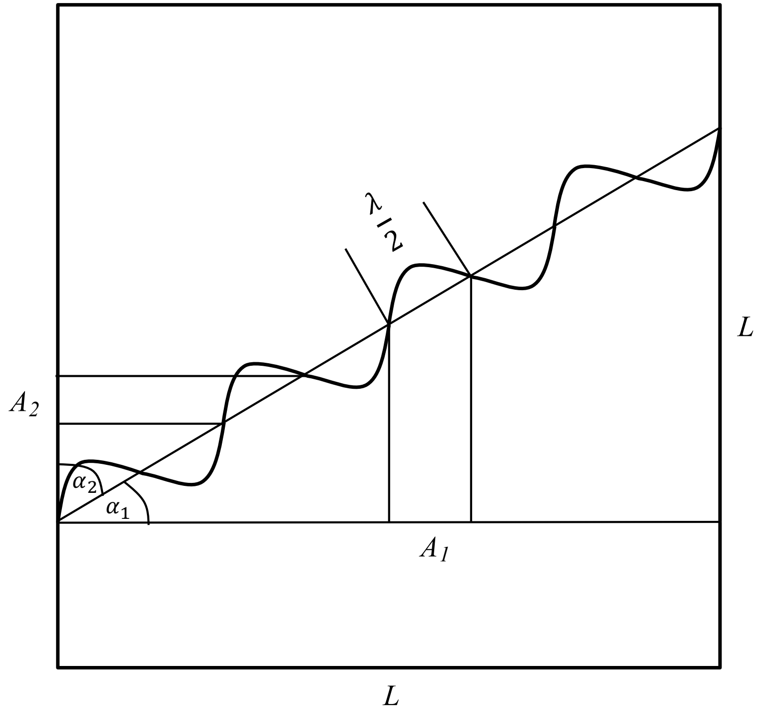

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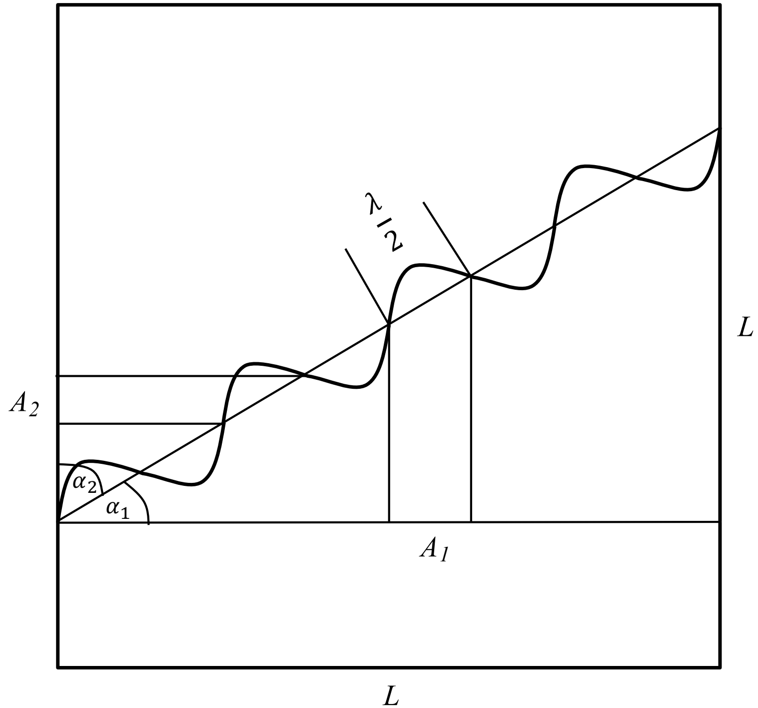

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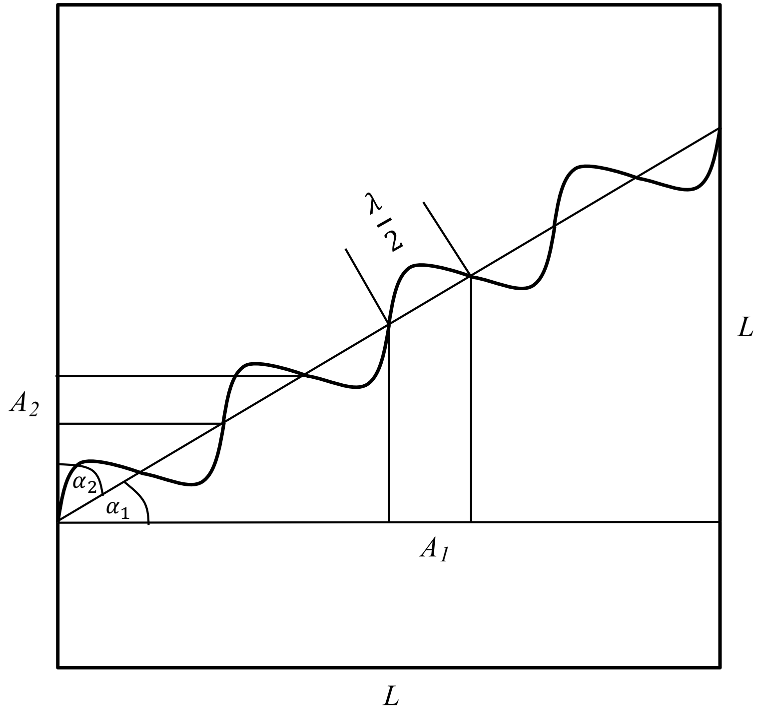

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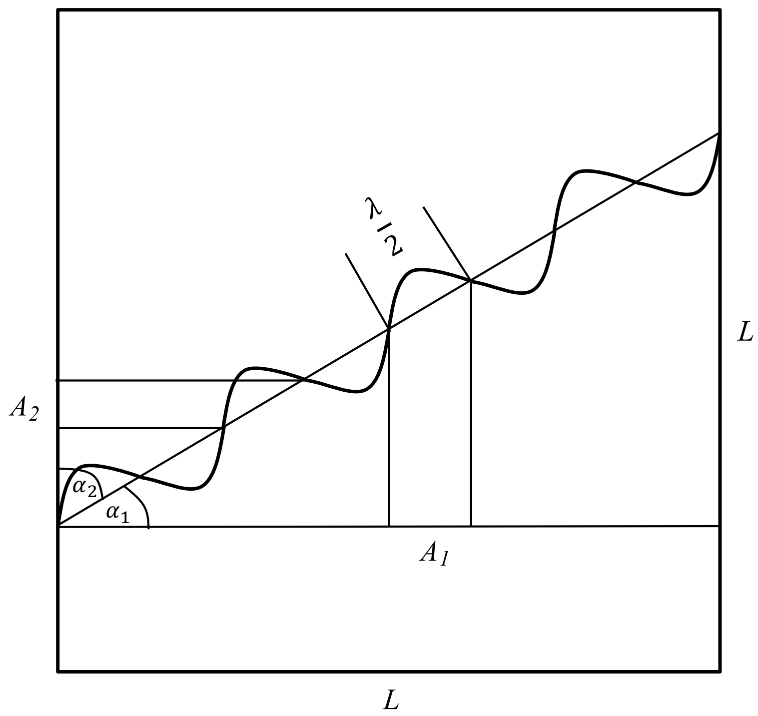

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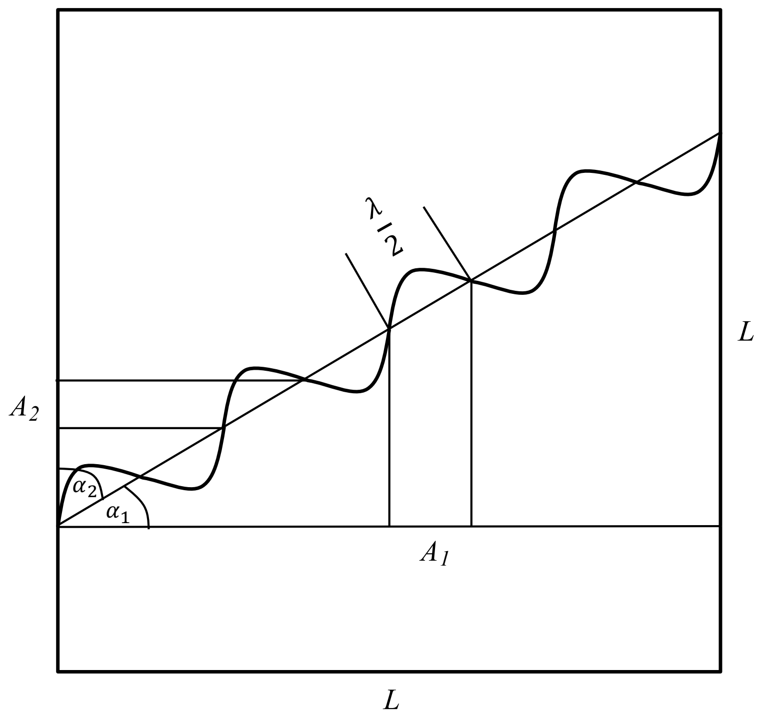

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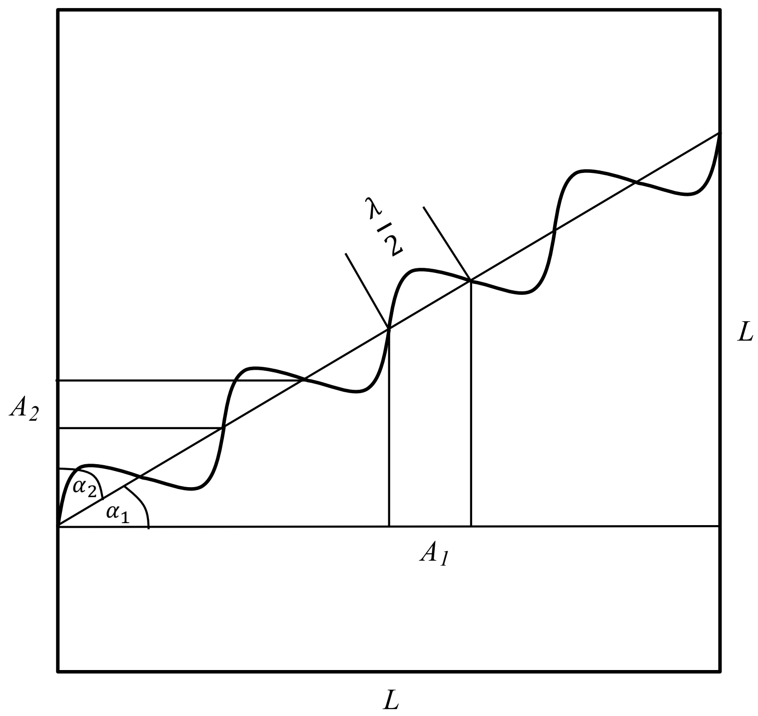

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2 polarisations!

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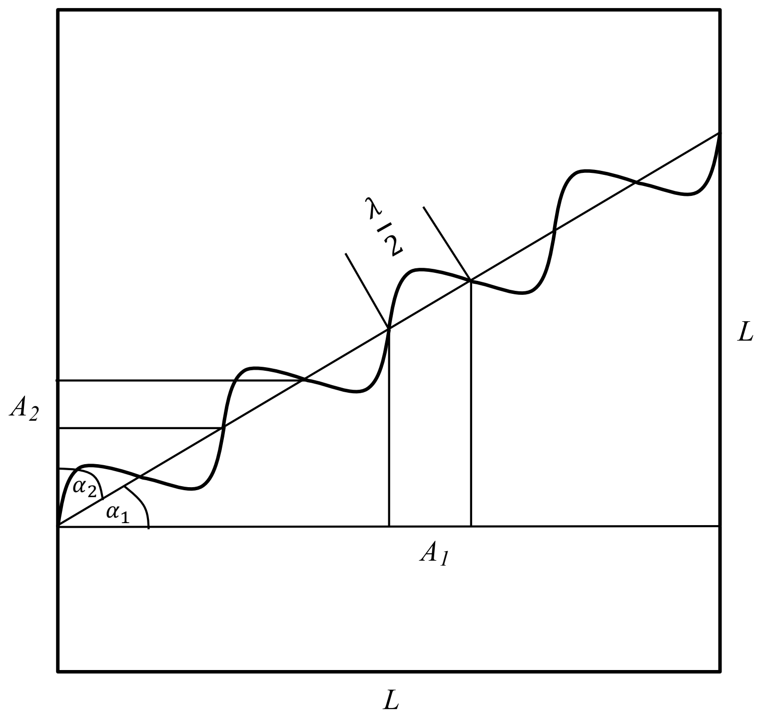

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$$\frac{dN(\nu)}{dV} = 2\frac{4\pi}{c^3}\nu^2 d\nu$$

$$B_\nu(T) = \frac{c}{4\pi}\, u_\nu(T)$$

$$u_\nu(T)d\nu = \frac{dN(\nu)}{dV}\,\langle E\rangle$$

number density of possible photon states:

$$\frac{dN(\nu)}{dV} = \frac{8\pi}{c^3}\nu^2 d\nu$$

average energy per state:

- quantum mechanics → Planck spectrum
- classical thermodynammics → Rayleigh-Jeans law

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quantum mechanical approach!

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  - each state has n discrete photons of energy $h\nu$: $E_n = n\,h\,\nu$
  - population of states* described by Boltzmann statistics: $p(E_n) \propto e^{-\frac{E_n}{k_B T}}$

*this is not the number density of photons

- number density of possible photon states

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$\beta = (k_B T)^{-1}$

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simple geometric series...

$$\sum_{n=0}^{\infty} e^{-\beta E_n} = \sum_{n=0}^{\infty} e^{-\beta n h\nu} = \left(1 - e^{-\beta n h\nu}\right)^{-1}$$

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classical thermodynammics?

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classical thermodynammics → Rayleigh-Jeans law

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$$u(T) = \frac{4}{c}\sigma_B\, T^4$$ finite energy density

- Planck spectrum

$$u_\nu(T) = \frac{8\pi}{c^3} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$B_\nu(T) = \; ?$$

- Planck spectrum

$$u_\nu(T) = \frac{8\pi}{c^3} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$u_\nu(T) = \frac{4\pi}{c} B_\nu(T)$$

$$B_\nu(T) = \frac{2}{c^2} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

▪ Planck spectrum

- energy density: $$u_\nu(T) = \frac{8\pi}{c^3} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

- intensity: $$B_\nu(T) = \frac{2}{c^2} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

- black-body radiation
- thermodynamics of black-body radiation
- **Planck spectrum:**
  - derivation
  - **properties**
- local thermal equilibrium

- properties

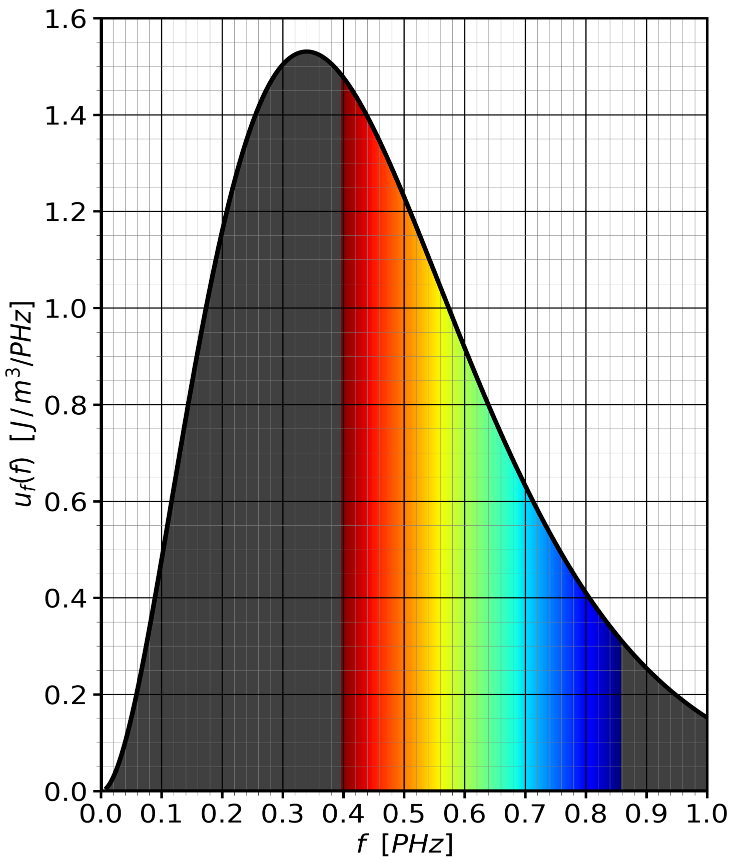
- properties $$B_\nu(T) = \frac{2}{c^2} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

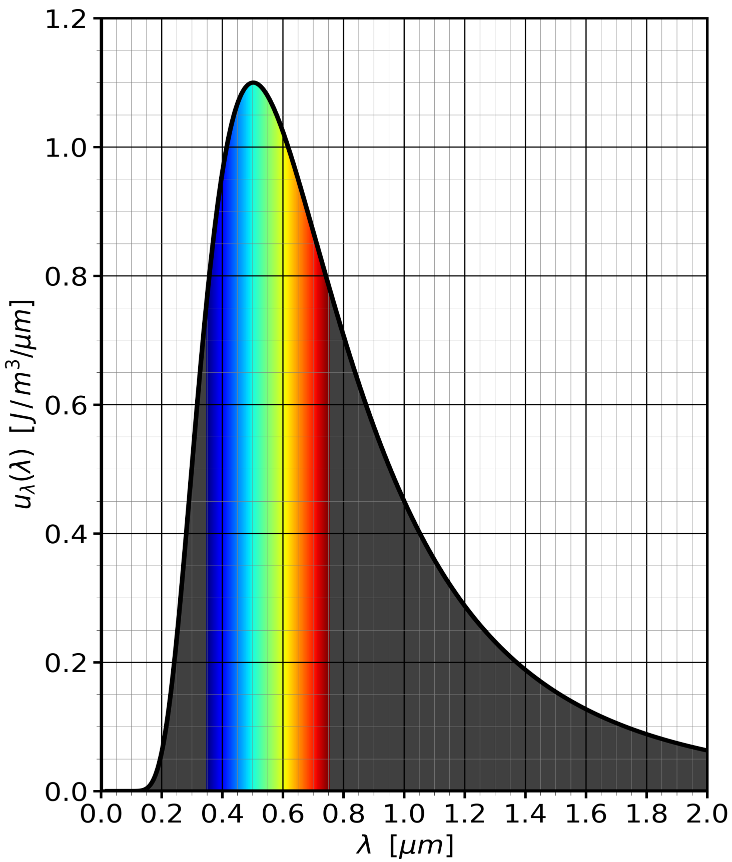
- properties $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

- properties $B_\nu(T) = \frac{2}{c^2} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $\leftrightarrow$ **(exercise)** $B_\lambda(T) = 2c^2 \frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

- **properties** $$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$ $$B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

  - fixed temperature T





- **properties** $$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$ $$B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

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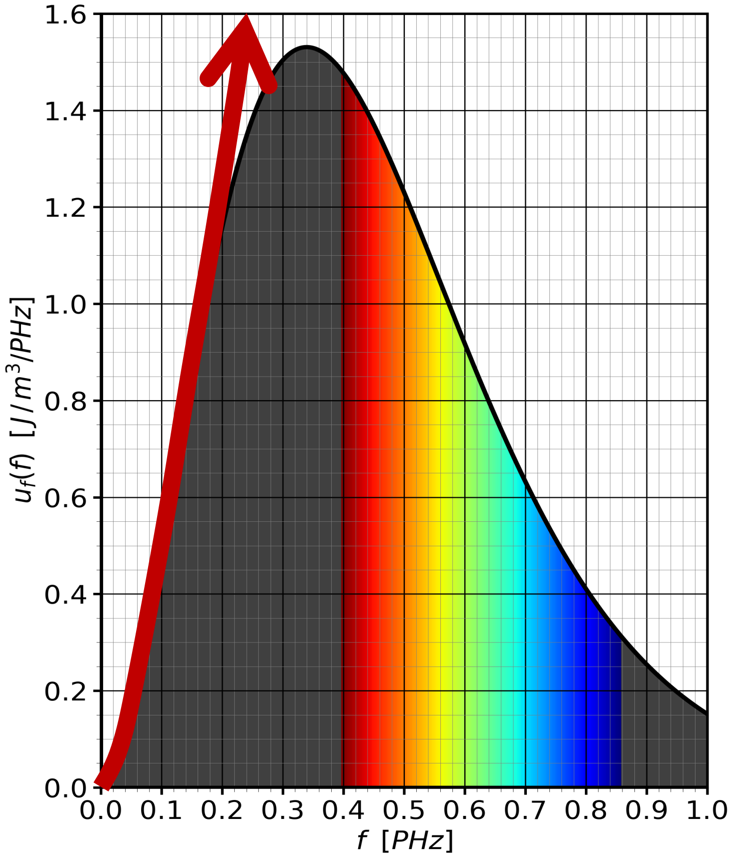


$h\nu \ll k_B T$: Rayleigh-Jeans law

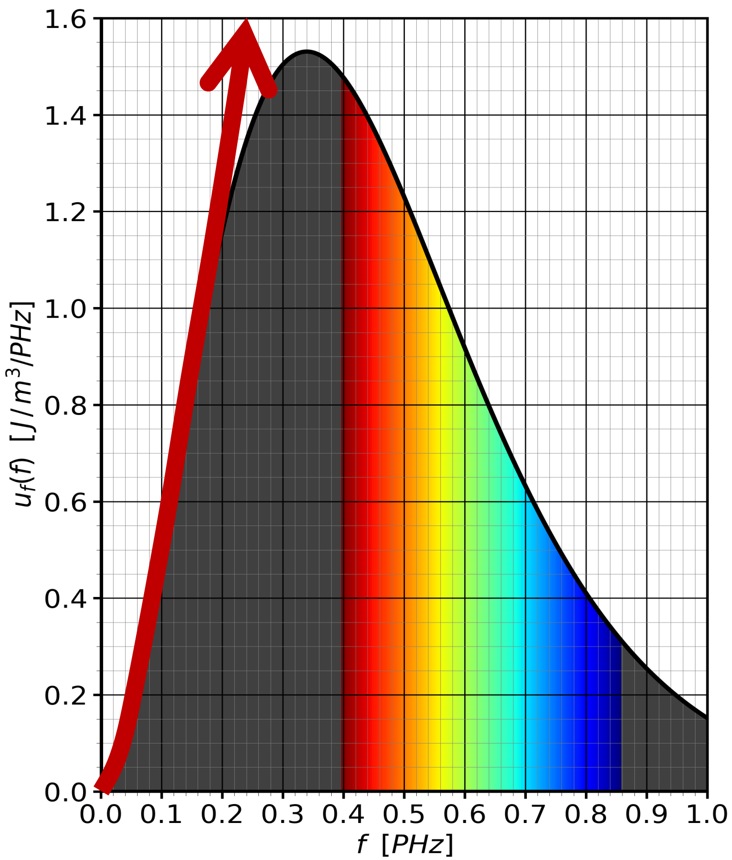
$$e^{\frac{h\nu}{k_B T}} - 1 \approx \frac{h\nu}{k_B T} \quad \rightarrow B_\nu(T) = \frac{2}{c^2}\,\nu^2 k_B T$$

- properties

$$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}}-1}$$

$$B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}}-1}$$

- fixed temperature T



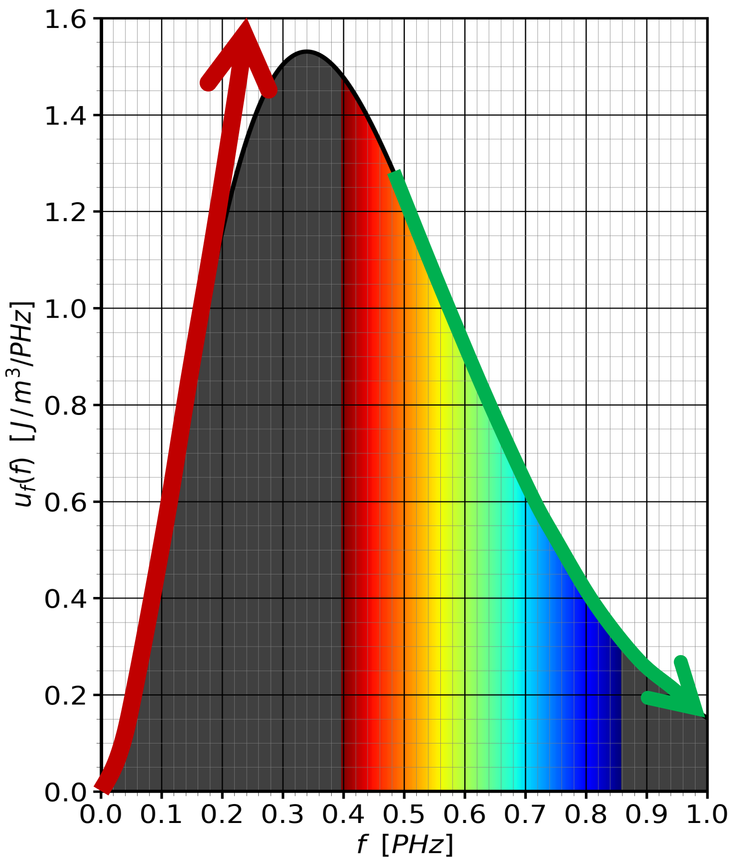
$h\nu \ll k_B T$: *Rayleigh-Jeans law*

$$e^{\frac{h\nu}{k_B T}} - 1 \approx \frac{h\nu}{k_B T} \quad \rightarrow B_\nu(T) = \frac{2}{c^2}\nu^2 k_B T$$

- previously derived using classical thermodynamics $\langle E \rangle = k_B T$
- no Planck constant involved

- **properties** $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}}-1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}}-1}$

  - fixed temperature T



$h\nu \ll k_B T$: *Rayleigh-Jeans law*

$$e^{\frac{h\nu}{k_B T}} - 1 \approx \frac{h\nu}{k_B T} \quad \rightarrow B_\nu(T) = \frac{2}{c^2}\nu^2 k_B T$$

- ➢ previously derived using classical thermodynamics $\langle E \rangle = k_B T$
- ➢ no Planck constant involved

$h\nu \gg k_B T$: *Wien law*

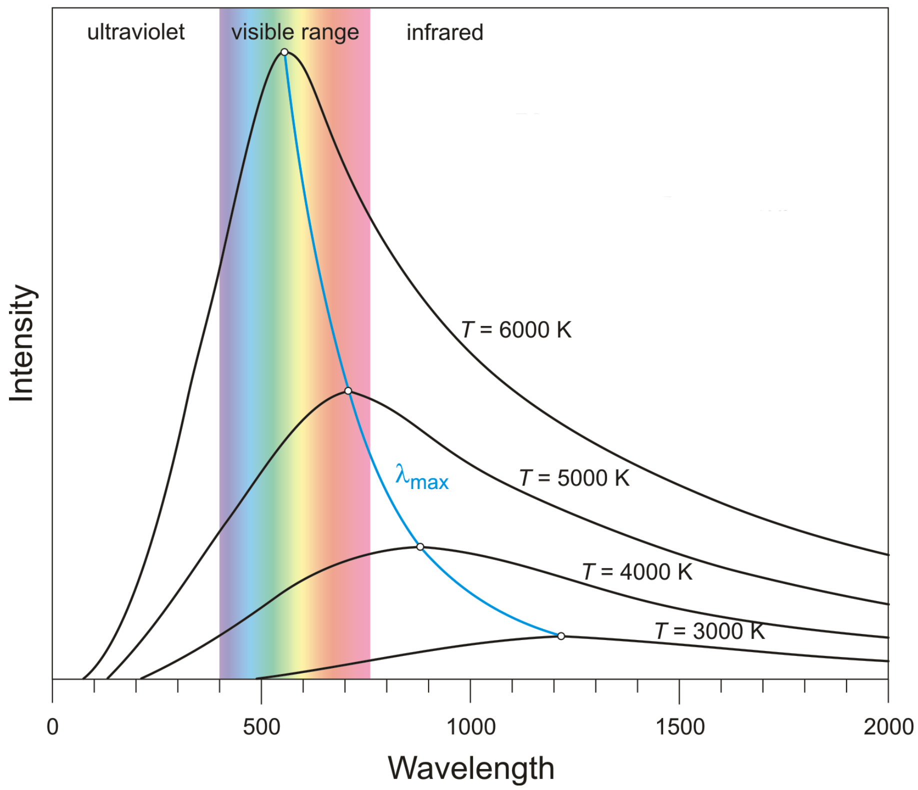
$$e^{\frac{h\nu}{k_B T}} - 1 \approx e^{\frac{h\nu}{k_B T}} \quad \rightarrow B_\nu(T) = \frac{2}{c^2}h\,\nu^3 e^{-\frac{h\nu}{k_B T}}$$

- properties

$$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

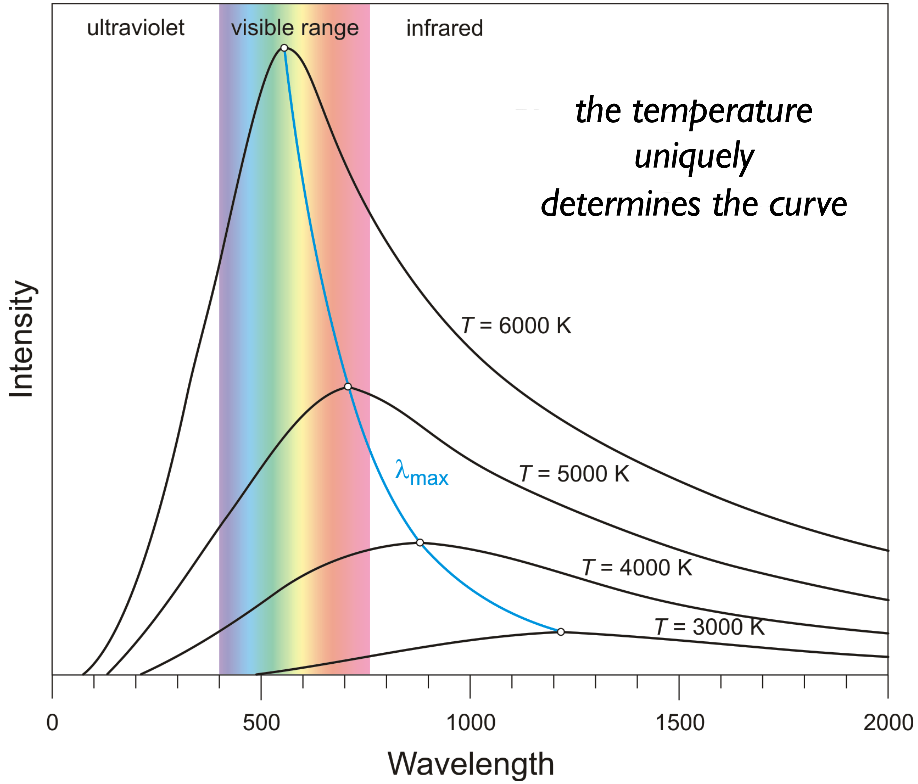
  - varying temperature T



- **properties** $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2 \frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - varying temperature T:

    - curves never cross



- **properties** $B_\nu(T) = \frac{2}{c^2}\frac{h\nu^3}{e^{\frac{h\nu}{k_B T}}-1}$ $B_\lambda(T) = 2c^2 \frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}}-1}$

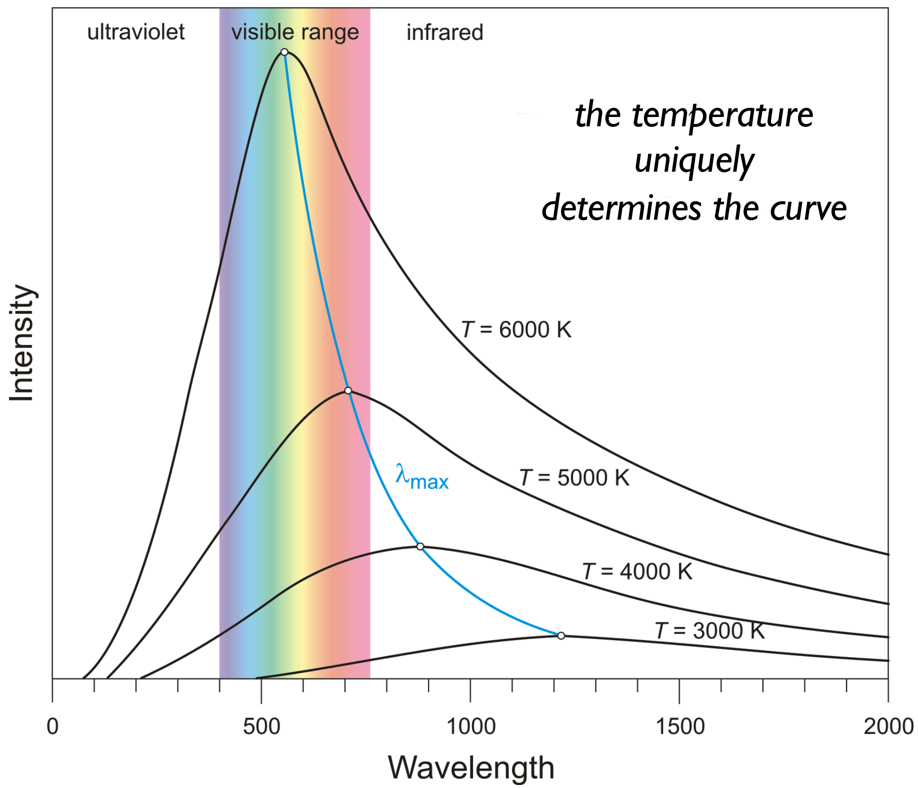
  - varying temperature T:

    - curves never cross
    - maximum shifts with T

Wien's displacement law

$$\lambda_{max} T = 0.290\ cm\ K$$

$$h\nu_{max} = 2.82\ k_B\ T$$



the temperature uniquely determines the curve

- properties

$$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$B_\lambda(T) = 2c^2 \frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

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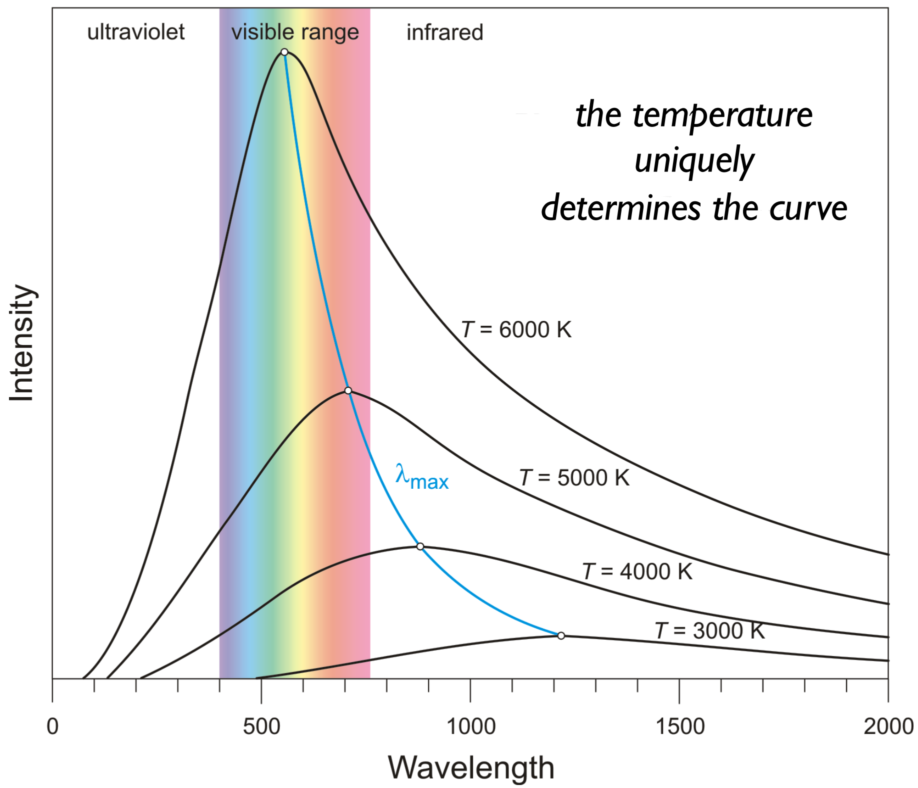

the temperature uniquely determines the curve

$\lambda \gg \lambda_{max}$: *Rayleigh-Jeans law*

$\lambda \ll \lambda_{max}$: *Wien law*

- properties

$$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

- varying temperature T:
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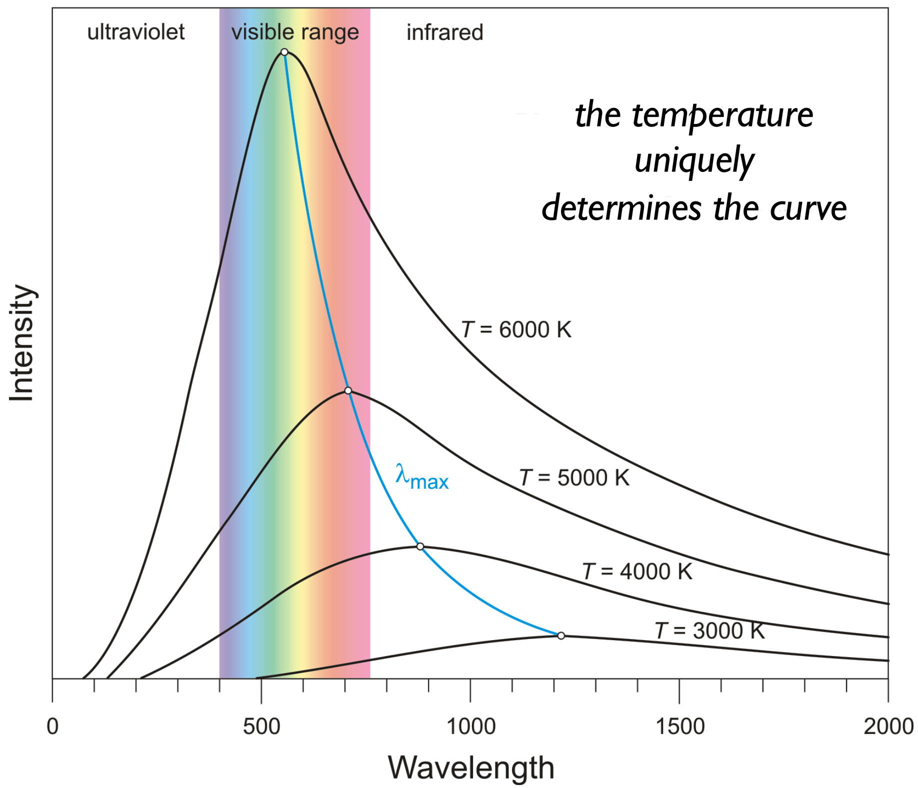
  - intensity increases with T



$\lambda \gg \lambda_{max}$: *Rayleigh-Jeans law*

$\lambda \ll \lambda_{max}$: *Wien law*

- properties

$$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_BT}} - 1}$$

$$B_\lambda(T) = 2c^2 \frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

- varying temperature T:
  - curves never cross
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Wien's displacement law

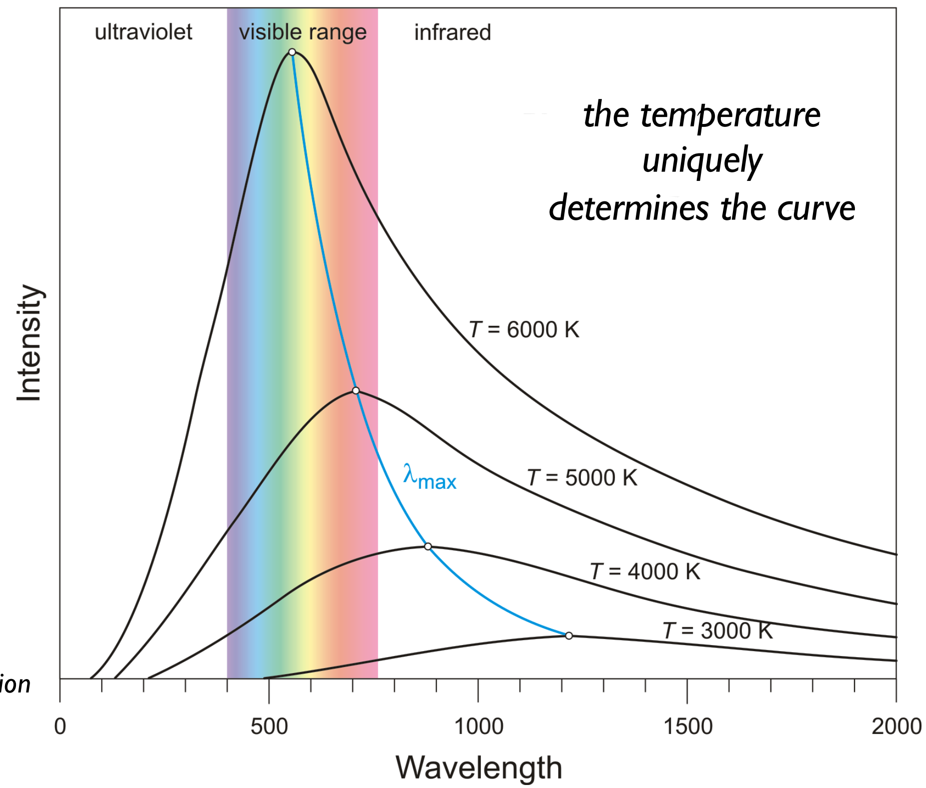
$$\lambda_{max} T = 0.290\; cm\; K$$

$$h\nu_{max} = 2.82\; k_B\; T$$

  - intensity increases with T

$$B(T) = \int B_\nu(T)\, d\nu = \frac{ac}{4\pi} T^4$$

result from thermodynamics of black-body radiation



the temperature uniquely determines the curve

$\lambda \gg \lambda_{max}$: *Rayleigh-Jeans law*

$\lambda \ll \lambda_{max}$: *Wien law*

- properties

$$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1} \qquad B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

- varying temperature T:
  - curves never cross
  - maximum shifts with T

Wien's displacement law

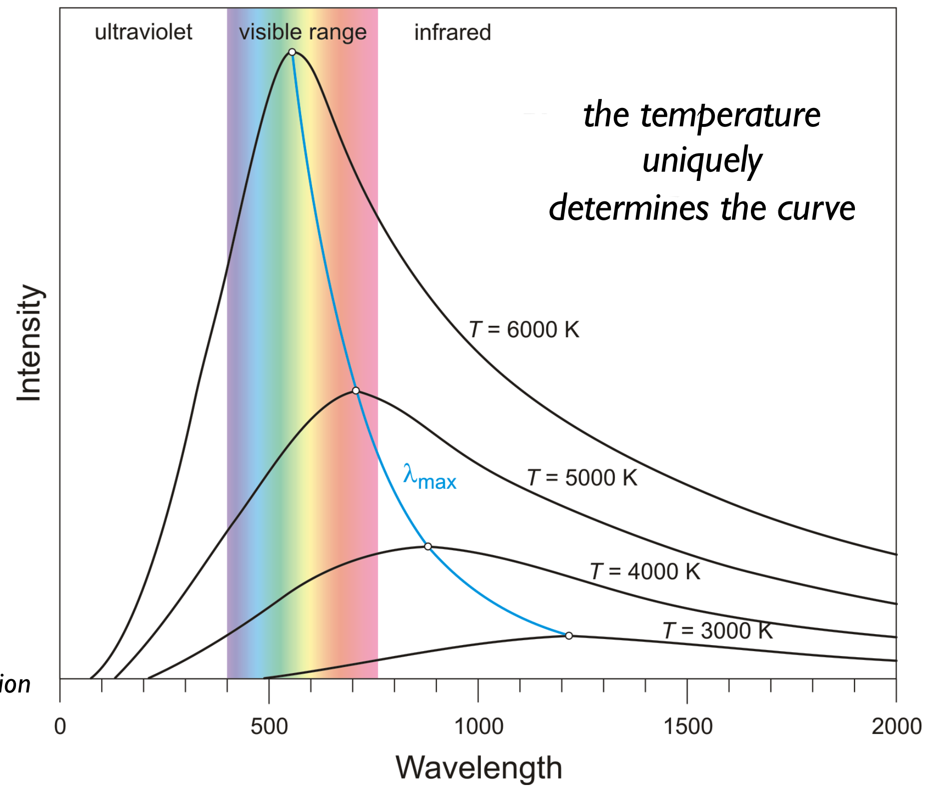
$$\lambda_{max}T = 0.290\ cm\ K$$

$$h\nu_{max} = 2.82\ k_B\ T$$

  - intensity increases with T

$$B(T) = \int B_\nu(T)\,d\nu = \frac{ac}{4\pi}T^4$$

result from thermodynamics of black-body radiation (let's hope that $\int B_\nu(T)\,d\nu$ will also give T^4)



the temperature uniquely determines the curve

$\lambda \gg \lambda_{max}$: *Rayleigh-Jeans law*

$\lambda \ll \lambda_{max}$: *Wien law*

- **properties** $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}}-1}$

$$B_\lambda(T) = 2c^2 \frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}}-1}$$

  - varying temperature T:

    - ○ curves never cross

    - ○ maximum shifts with T

Wien's displacement law

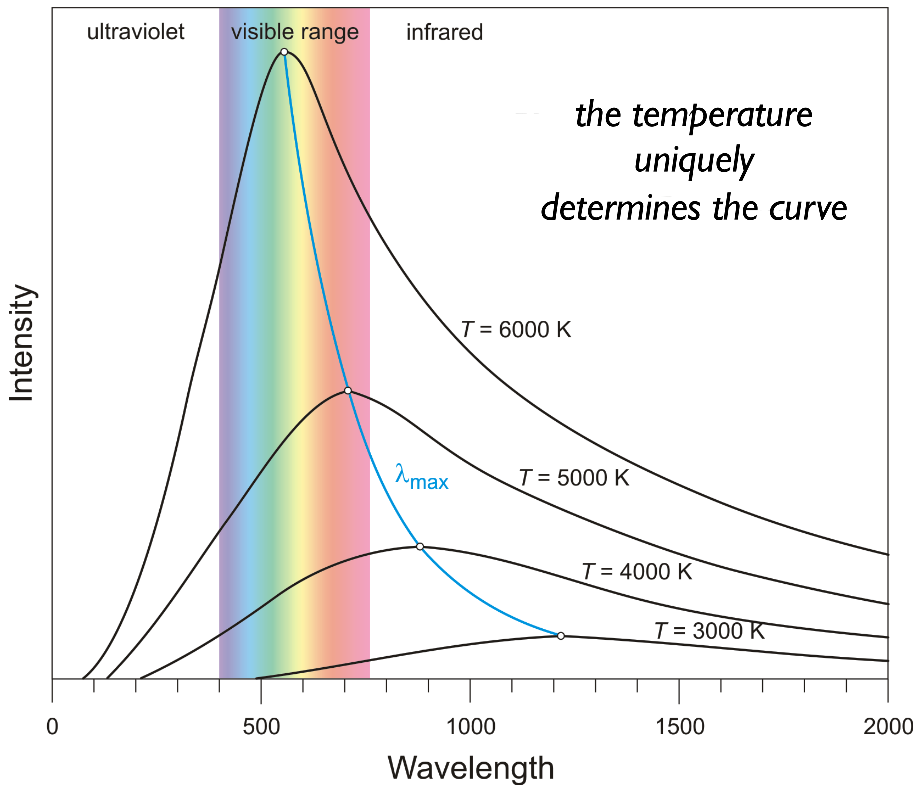
$$\lambda_{max} T = 0.290\; cm\; K$$

$$h\nu_{max} = 2.82\; k_B\; T$$

    - ○ intensity increases with T

$$B(T) = \int B_\nu(T)\, d\nu = \frac{ac}{4\pi} T^4$$

?



the temperature uniquely determines the curve

$\lambda \gg \lambda_{max}$: *Rayleigh-Jeans law*

$\lambda \ll \lambda_{max}$: *Wien law*

- **properties** $B_\nu(T) = \frac{2}{c^2}\frac{h\nu^3}{e^{\frac{h\nu}{k_B T}}-1}$ $B_\lambda(T) = 2c^2 \frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}}-1}$

  - varying temperature T:

let's integrate...

    - curves never cross
    - maximum shifts with T

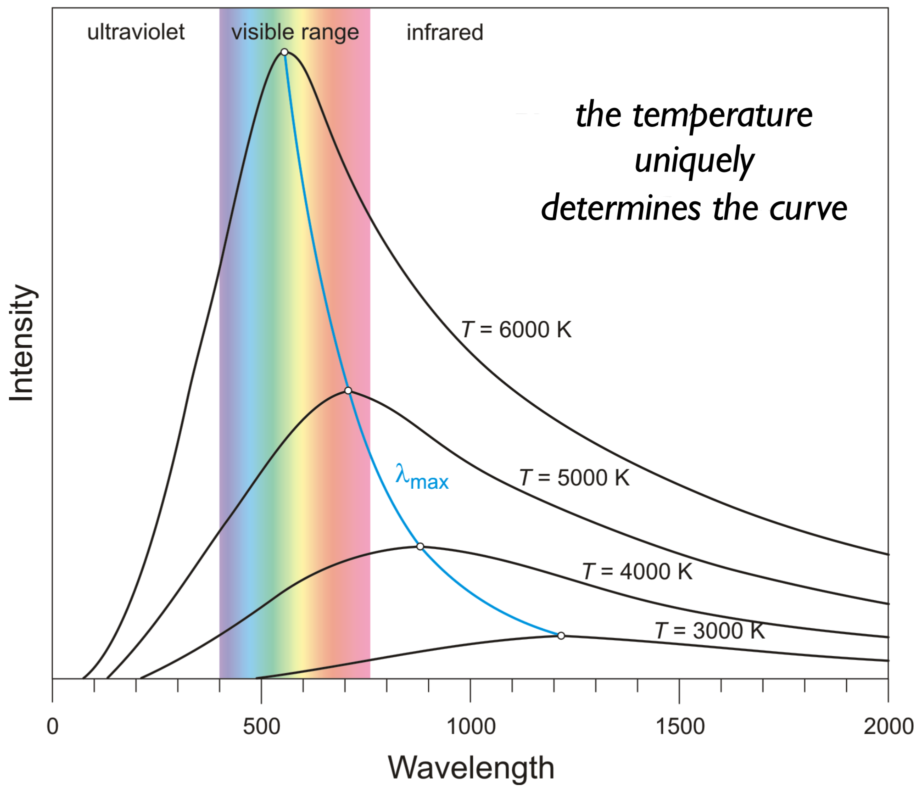
$$\lambda_{max} T = 0.290\ cm\ K$$

Wien's displacement law

$$h\nu_{max} = 2.82\ k_B\ T$$

    - intensity increases with T

$$B(T) = \int B_\nu(T)\, d\nu = \frac{ac}{4\pi} T^4$$



$\lambda \gg \lambda_{max}$: *Rayleigh-Jeans law*

$\lambda \ll \lambda_{max}$: *Wien law*

- **properties** $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - varying temperature T:

    - ○ curves never cross
    - ○ maximum shifts with T

$$\lambda_{max} T = 0.290\ cm\ K$$

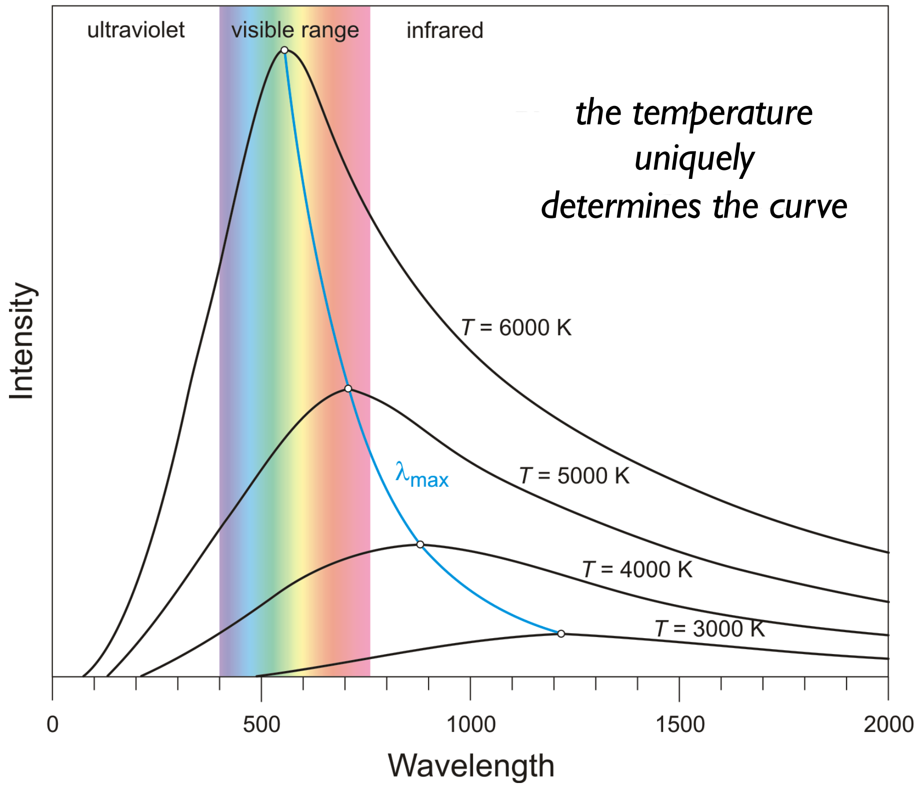
Wien's displacement law

$$h\nu_{max} = 2.82\ k_B\ T$$

    - ○ intensity increases with T

$$B(T) = \int B_\nu(T)\,d\nu = \frac{ac}{4\pi}T^4$$

$$\frac{ac}{4\pi}T^4 = \int \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}\ d\nu$$



$\lambda \gg \lambda_{max}$: *Rayleigh-Jeans law*

$\lambda \ll \lambda_{max}$: *Wien law*

- **properties** $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - varying temperature T:

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Wien's displacement law

$$\lambda_{max} T = 0.290\ cm\ K$$

$$h\nu_{max} = 2.82\ k_B\ T$$

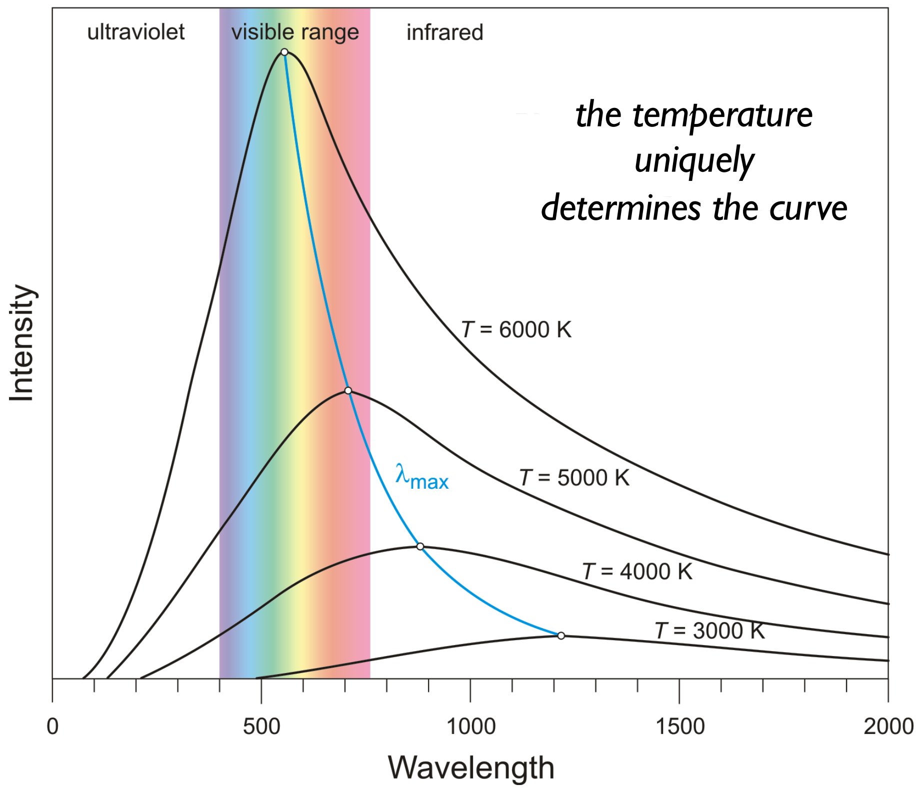
    - intensity increases with T

$$B(T) = \int B_\nu(T)\, d\nu = \frac{ac}{4\pi} T^4$$

$$\frac{ac}{4\pi} T^4 = \int \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}\ d\nu$$

$$= \frac{2h}{c^2}\int \frac{\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}\ d\nu$$

$$= \frac{2\pi^4 k_B^4 T^4}{15 c^2 h^3}$$



$\lambda \gg \lambda_{max}$: *Rayleigh-Jeans law*

$\lambda \ll \lambda_{max}$: *Wien law*

- **properties**

$$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_BT}}-1} \qquad B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_BT}}-1}$$

- varying temperature T:
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Wien's displacement law

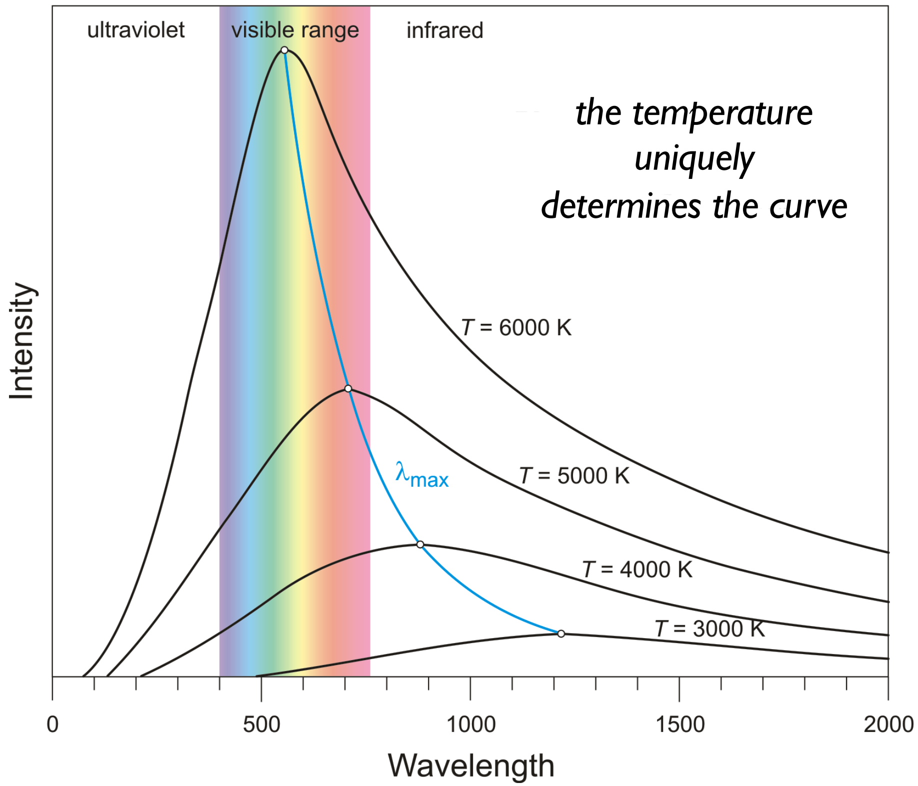
$$\lambda_{max}T = 0.290\ cm\ K$$

$$h\nu_{max} = 2.82\ k_B\ T$$

  - intensity increases with T

$$B(T) = \int B_\nu(T)\,d\nu = \frac{ac}{4\pi}T^4$$

$$a = \frac{8\pi^5 k_B^4}{15c^3h^3} \qquad \begin{cases} \frac{ac}{4\pi}T^4 = \int \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_BT}}-1}\ d\nu \\ = \frac{2h}{c^2}\int \frac{\nu^3}{e^{\frac{h\nu}{k_BT}}-1}\ d\nu \\ = \frac{2\pi^4 k_B^4 T^4}{15c^2h^3} \end{cases}$$



the temperature uniquely determines the curve

$\lambda \gg \lambda_{max}$: *Rayleigh-Jeans law*

$\lambda \ll \lambda_{max}$: *Wien law*

- **properties** $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - varying temperature T:

    - curves never cross
    - maximum shifts with T

      Wien's displacement law

      $$\lambda_{max}T = 0.290\ cm\ K$$

      $$h\nu_{max} = 2.82\ k_B\ T$$

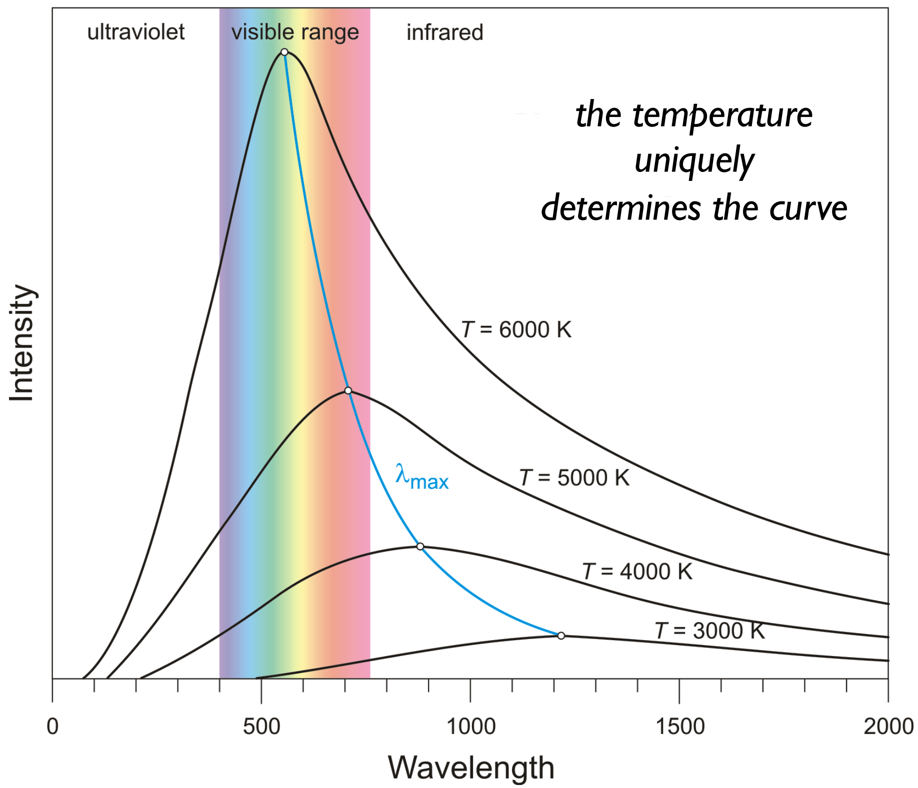
    - intensity increases with T

$$B(T) = \frac{ac}{4\pi}T^4 \qquad a = \frac{8\pi^5 k_B^4}{15c^3h^3}$$

$\sigma_B = ac/4$

$$B(T) = \frac{\sigma_B}{4}T^4 \qquad \sigma_B = \frac{2\pi^5 k_B^4}{15c^2h^3}$$

radiation constants
(expressed in terms of fundamental constants)



the temperature uniquely determines the curve

$\lambda \gg \lambda_{max}$: *Rayleigh-Jeans law*

$\lambda \ll \lambda_{max}$: *Wien law*

- properties $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - characteristic temperatures

- properties $B_\nu(T) = \frac{2}{c^2} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2 \frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - characteristic temperatures:

    - brightness temperature T_b
    - color temperature T_c
    - effective temperature T_{eff}

- properties $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - characteristic temperatures:

    - ○ brightness temperature T_b

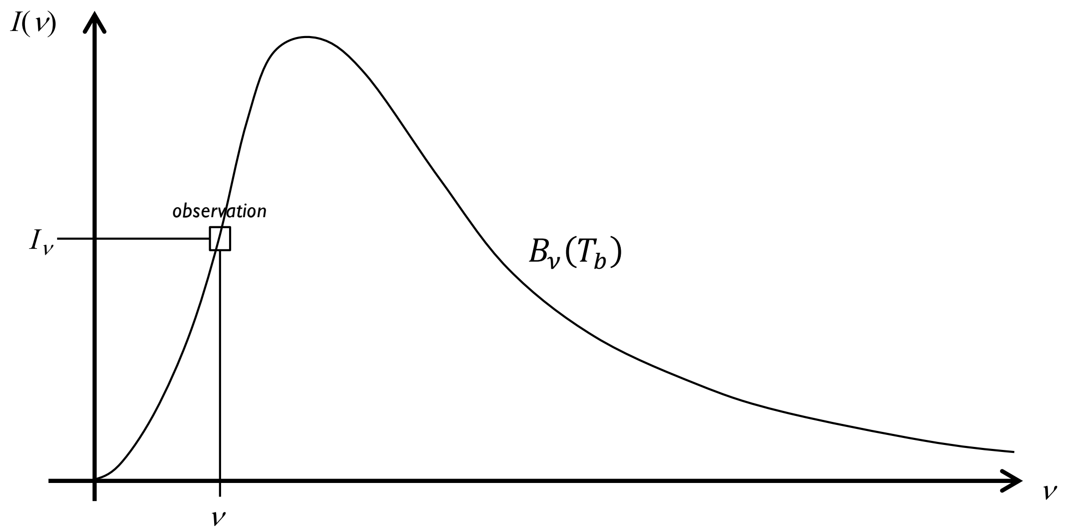
      - *we observe I_ν for fixed ν and use it to define T_b*

- properties $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

    - characteristic temperatures:

        - brightness temperature T_b

            - *we observe I_ν for fixed ν and use it to define T_b*



***remember**, Planck curves do not cross!*

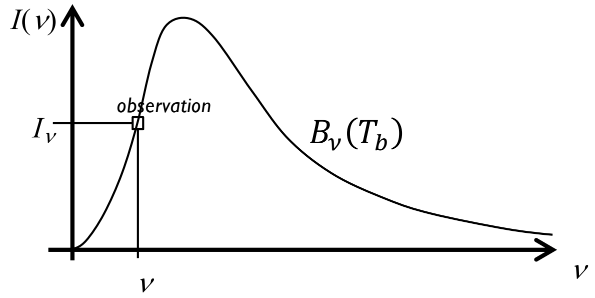
- properties $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - characteristic temperatures:

    - brightness temperature T_b

      - *we observe I_ν for fixed ν and use it to define T_b*
      - *frequently used in radio-astronomy**:

$$I_\nu(T_b) = \frac{2}{c^2}\,\nu^2\,k_B T_b$$



*where one can safely use the Rayleigh-Jeans law for $B_\nu(T)$

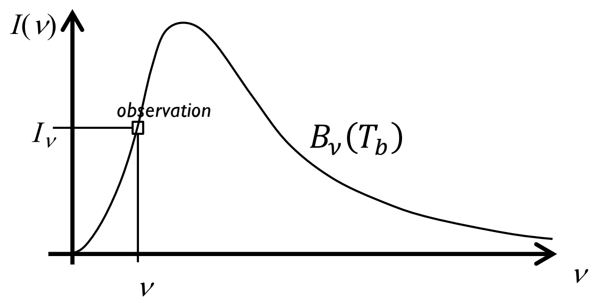
- **properties** $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - characteristic temperatures:

    - brightness temperature T_b

      - *we observe I_ν for fixed ν and use it to define T_b*
      - *frequently used in radio-astronomy* *(Rayleigh-Jeans limit)*:

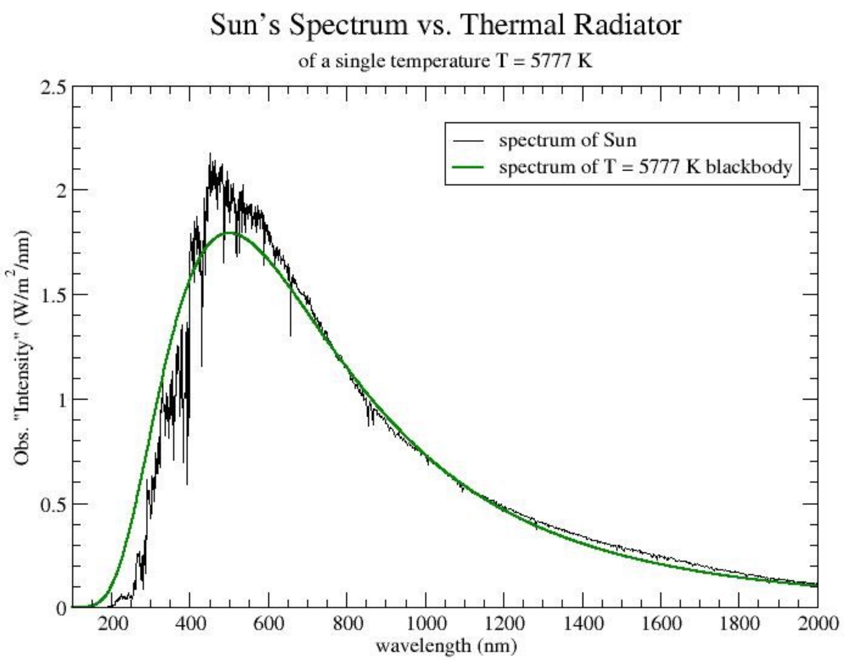
$$T_b = \frac{c^2}{2k_B}\,\nu^{-2} I_\nu$$



- **properties** $$B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}}-1}$$ $$B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}}-1}$$

  - characteristic temperatures:

    - ○ brightness temperature T_b
    - ○ color temperature T_c
      - *temperature of best-fit Planck curve to observed spectrum*



- **properties** $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

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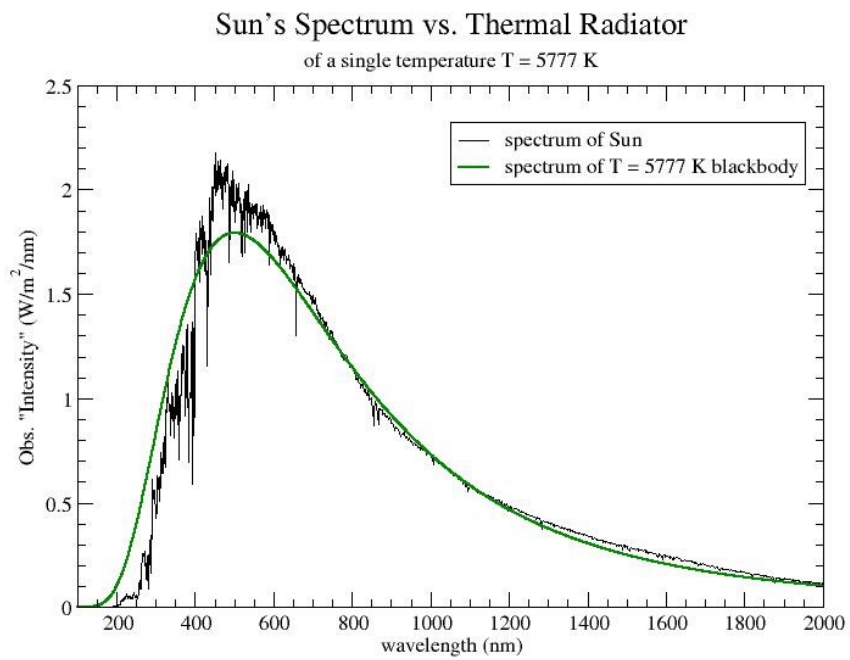

<u>in practice:</u>

- ➢ find maximum
- ➢ use Wien's displacement law to get T_c

- properties $B_\nu(T) = \frac{2}{c^2} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2 \frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - characteristic temperatures:

    - brightness temperature T_b
    - color temperature T_c
    - effective temperature T_{eff}
      - *we only have bolometric, but no frequency information, e.g. total flux* F

$$F = \int I_\nu(\Omega)\, cos\theta\; d\Omega\; d\nu = \sigma_B\, T_{eff}^4$$

- properties $B_\nu(T) = \frac{2}{c^2}\frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2\frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - characteristic temperatures:

    - brightness temperature T_b
    - color temperature T_c
    - effective temperature T_{eff}
      - *we only have bolometric, but no frequency information, e.g. total flux* F

$$F = \int I_\nu(\Omega)\, cos\theta\; d\Omega\, d\nu = \sigma_B\, T_{eff}^4$$

observed (F) derived (T_{eff})

- properties $B_\nu(T) = \frac{2}{c^2} \frac{h\,\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}$ $B_\lambda(T) = 2c^2 \frac{h/\lambda^5}{e^{\frac{hc}{\lambda k_B T}} - 1}$

  - characteristic temperatures:

    - brightness temperature T_b - *we observe I_ν for fixed ν and use it to define T_b*
    - color temperature T_c - *temperature of best-fit Planck curve to observed spectrum*
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*depends on magnitude of the source
#depends on spectral shape only

- black-body radiation
- thermodynamics of black-body radiation
- Planck spectrum
- **local thermal equilibrium**

- global thermal equilibrium
  - the whole system of interest has one well defined temperature T

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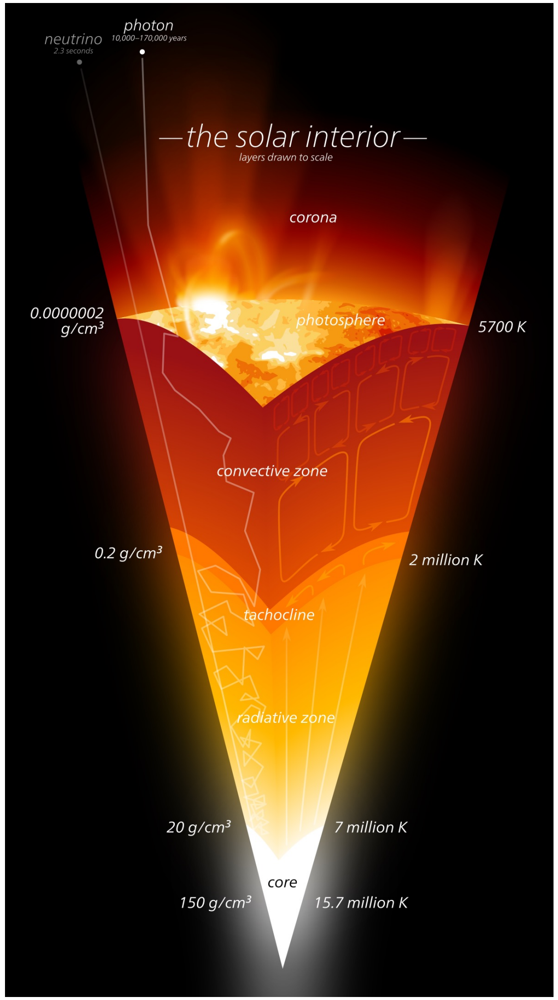
but what about this system?



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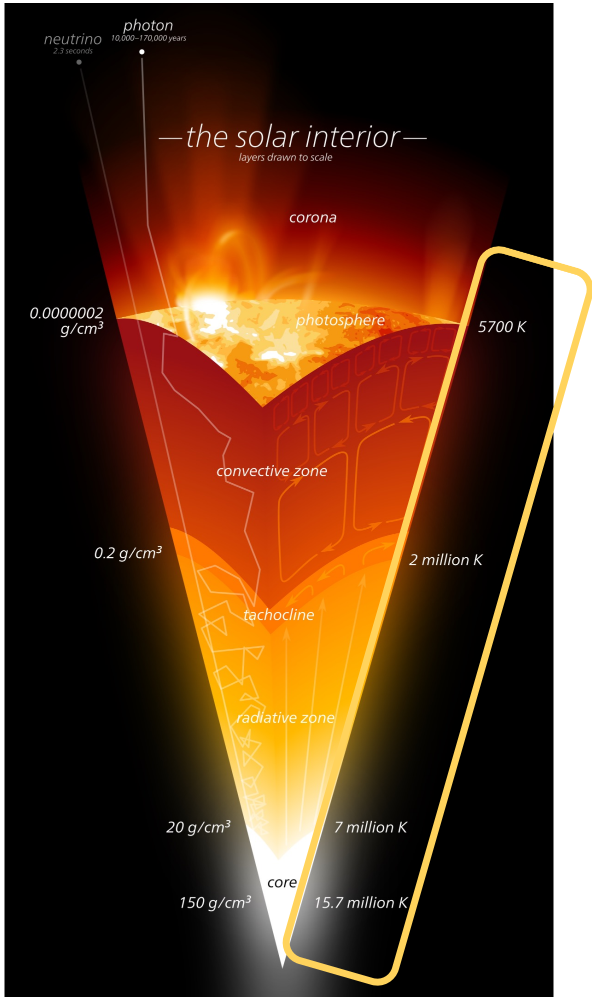
for which the temperature various 3 orders of magnitude!



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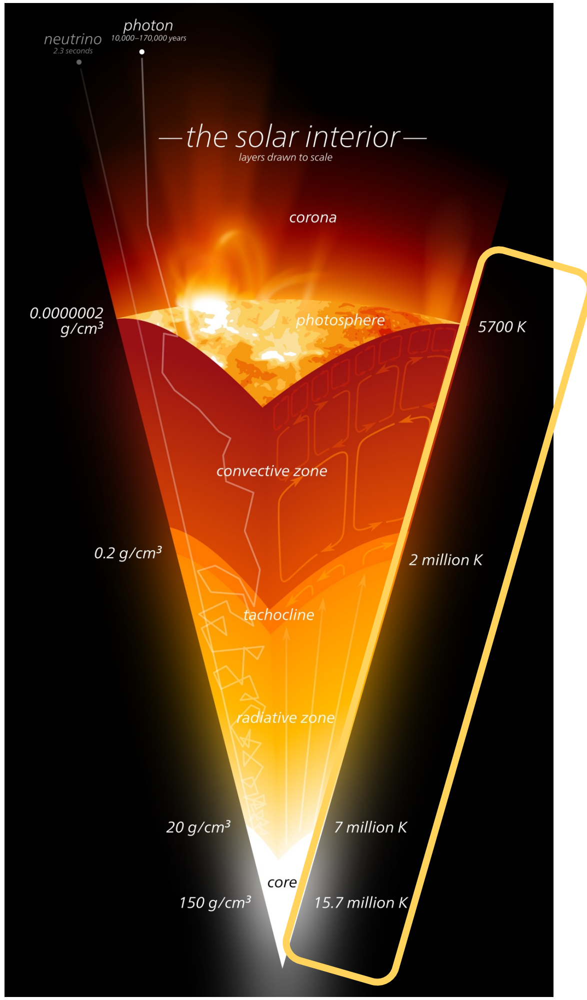

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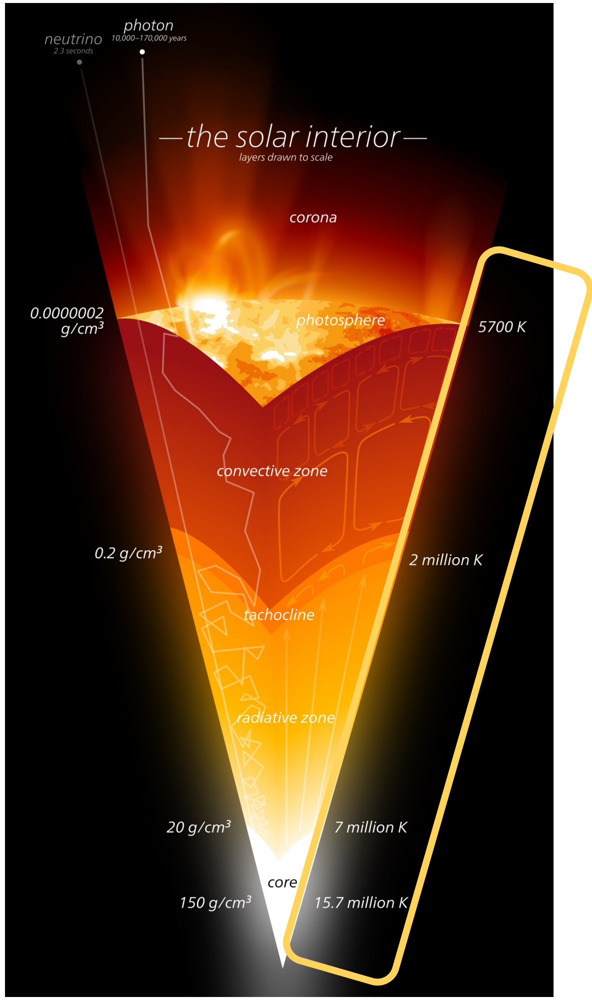

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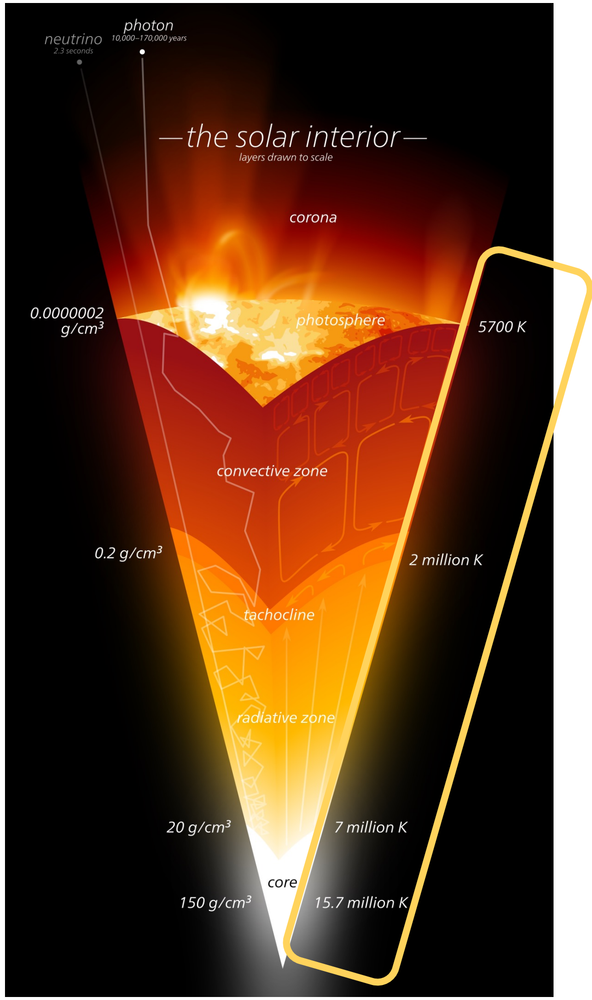

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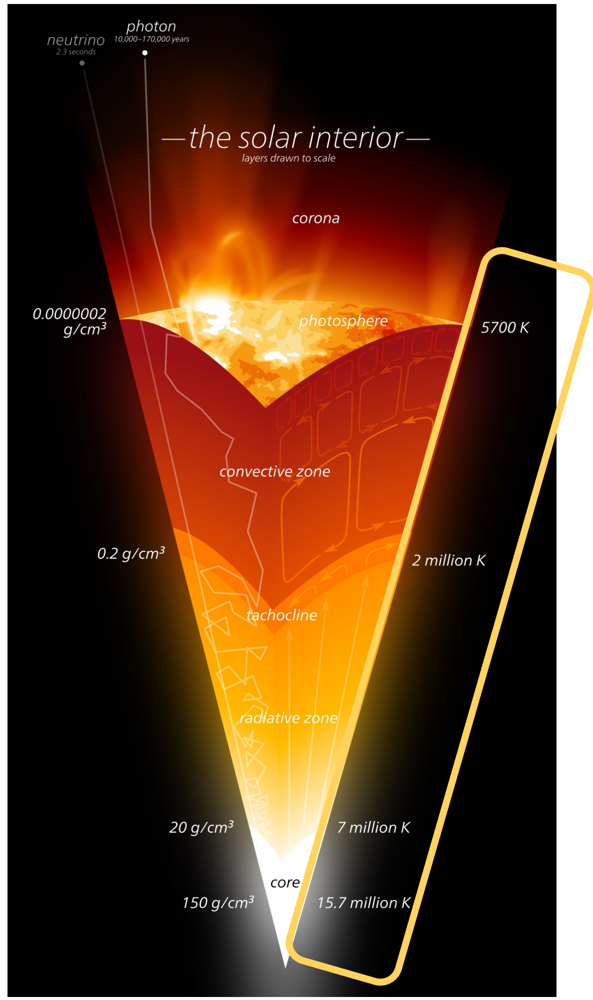

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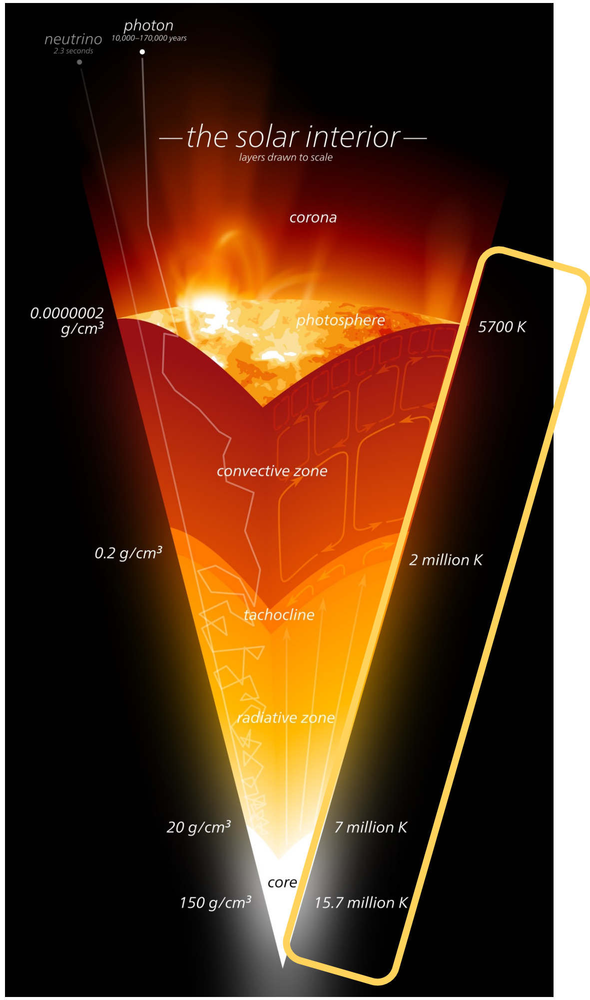

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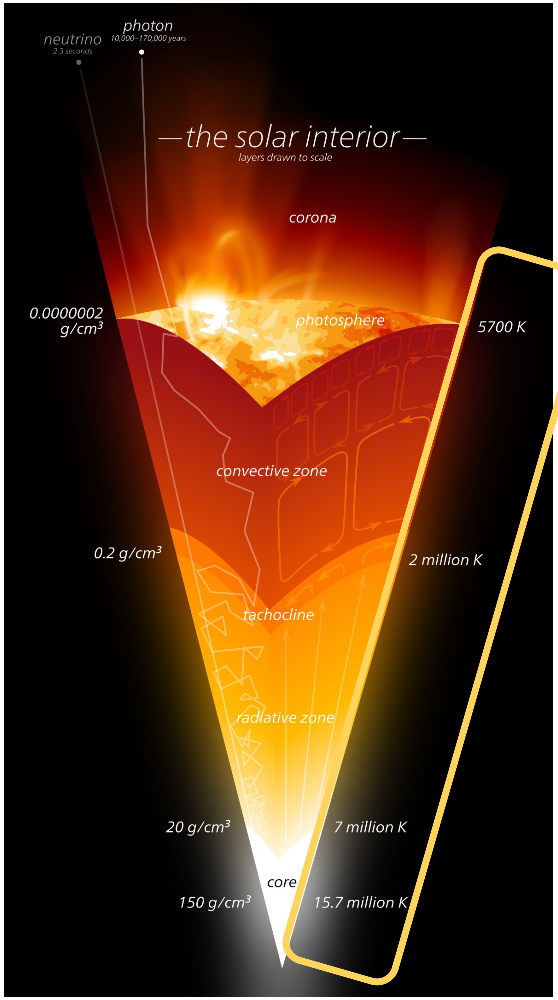
the radiation in a layer is considered to be at a local thermal equilibrium



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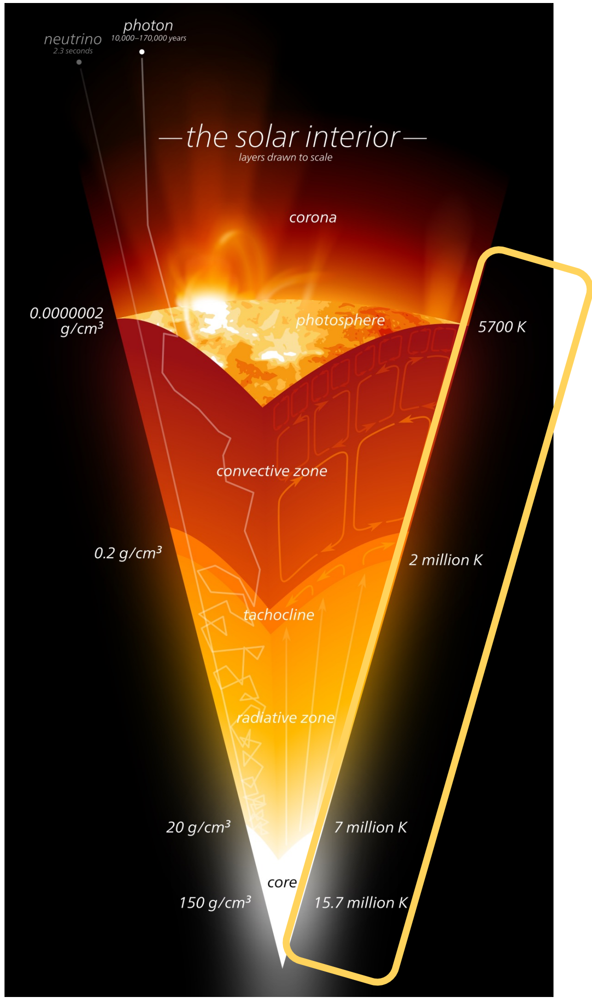

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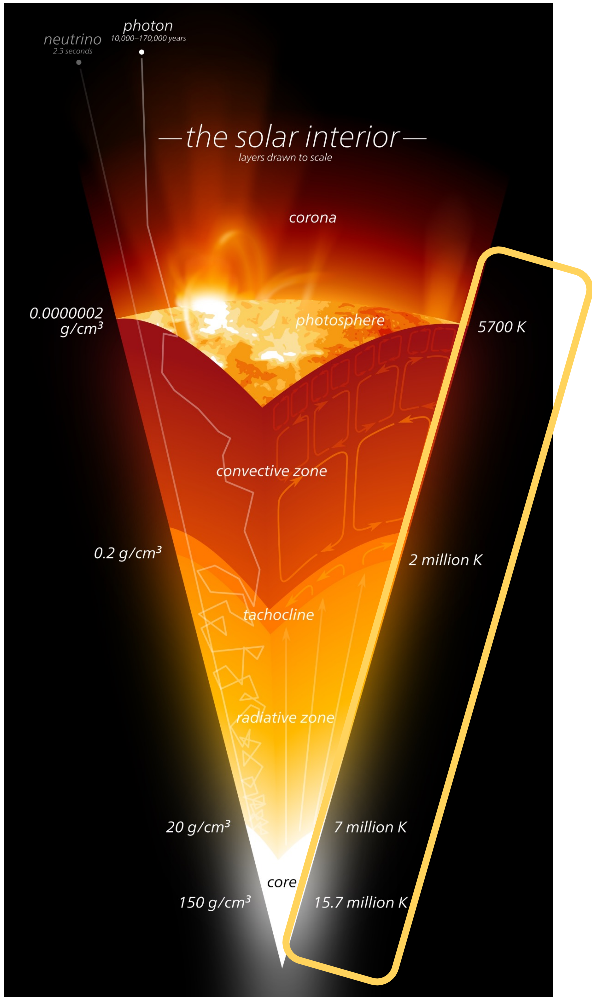

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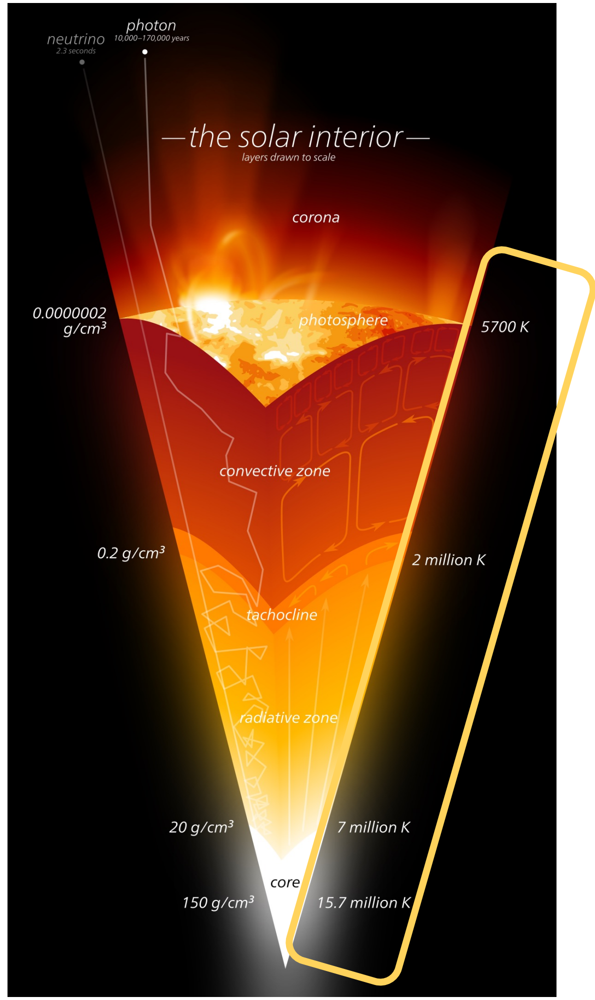
$\longrightarrow$ the radiation locally follows a Planck curve



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