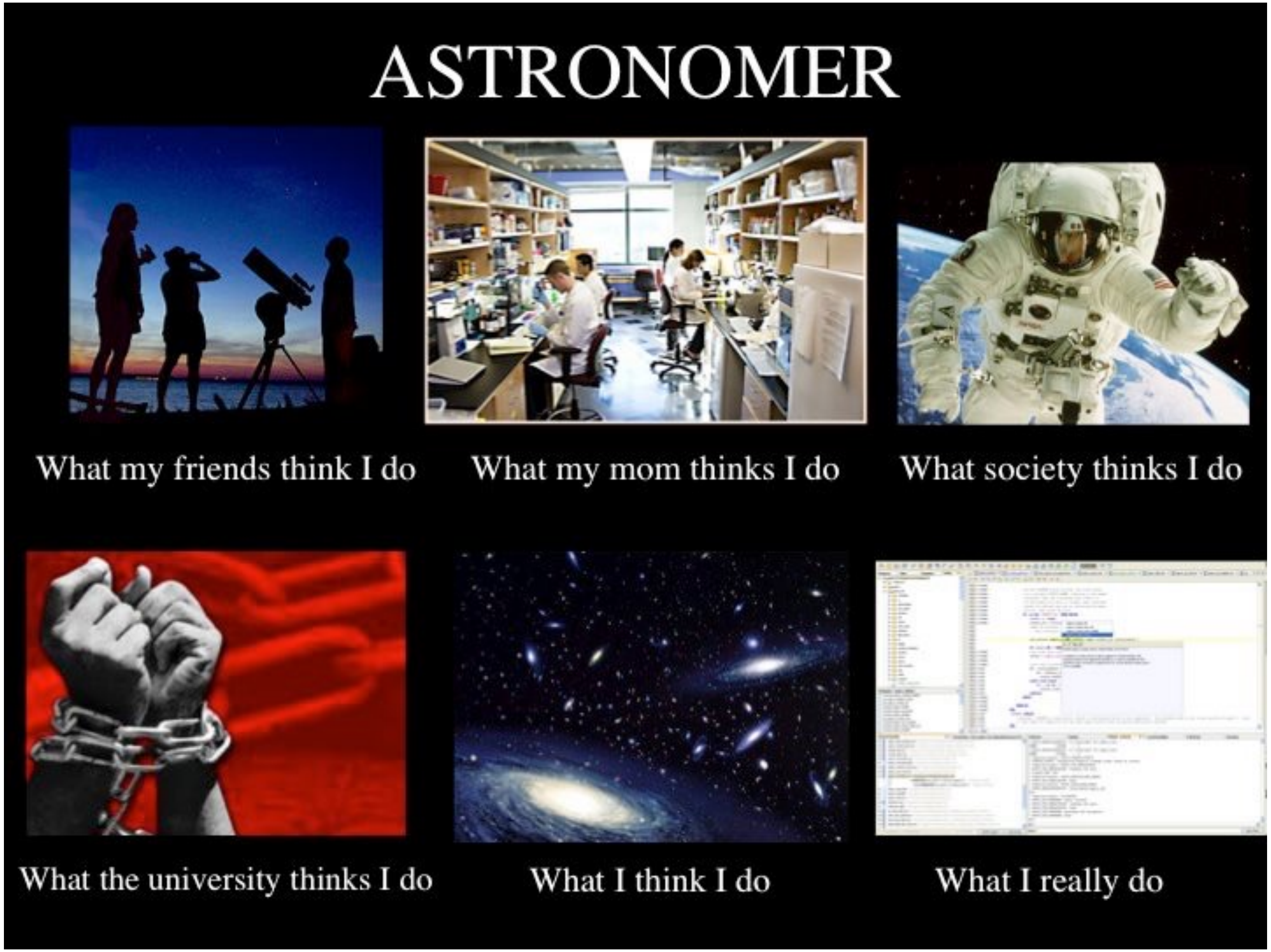
Alexander Knebe (*Universidad Autonoma de Madrid*)

"How else would I keep my solar panels in the sun all day?"

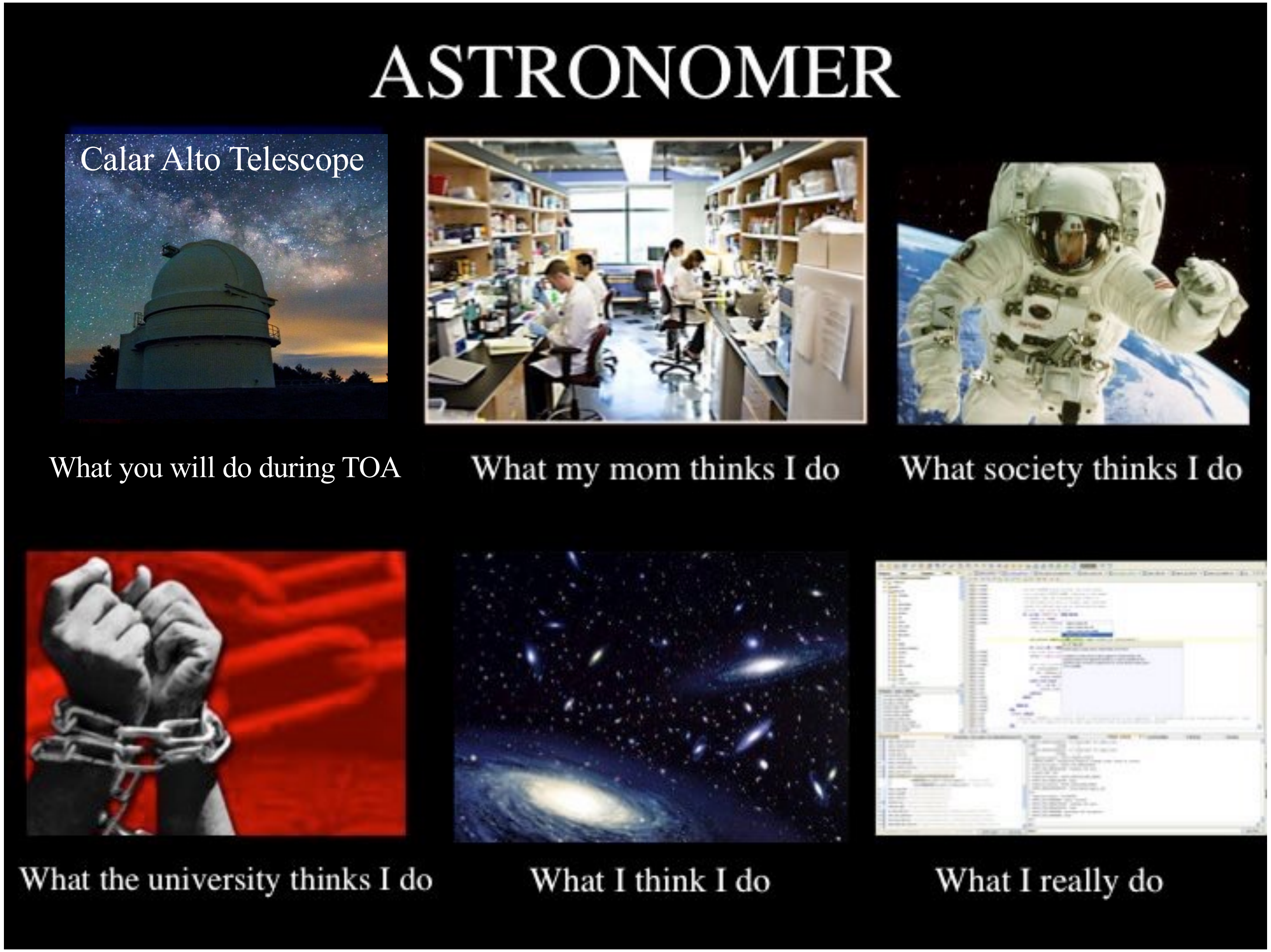
- electromagnetic spectrum
- description of a radiation field
- radiative transfer equation

- **electromagnetic spectrum**
- description of a radiation field
- radiative transfer equation

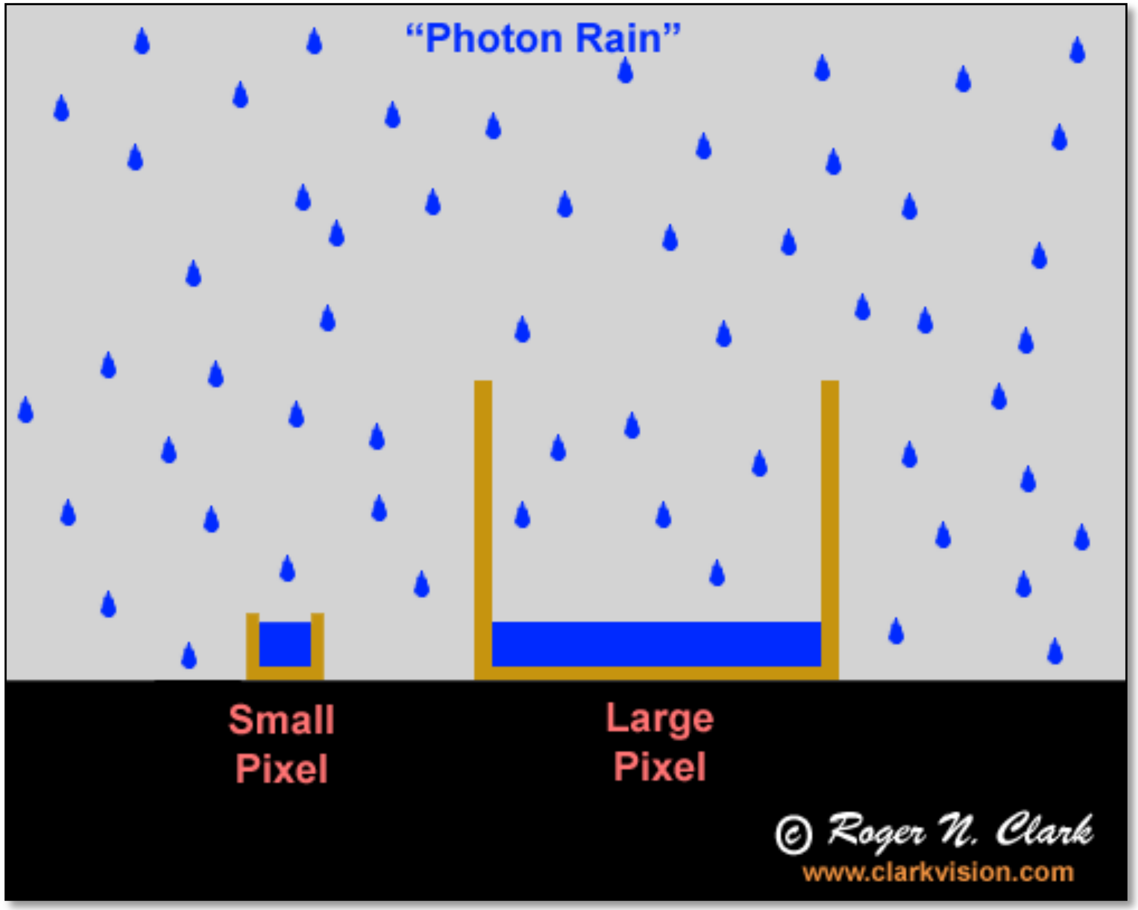
- astronomy is...



- astronomy is...

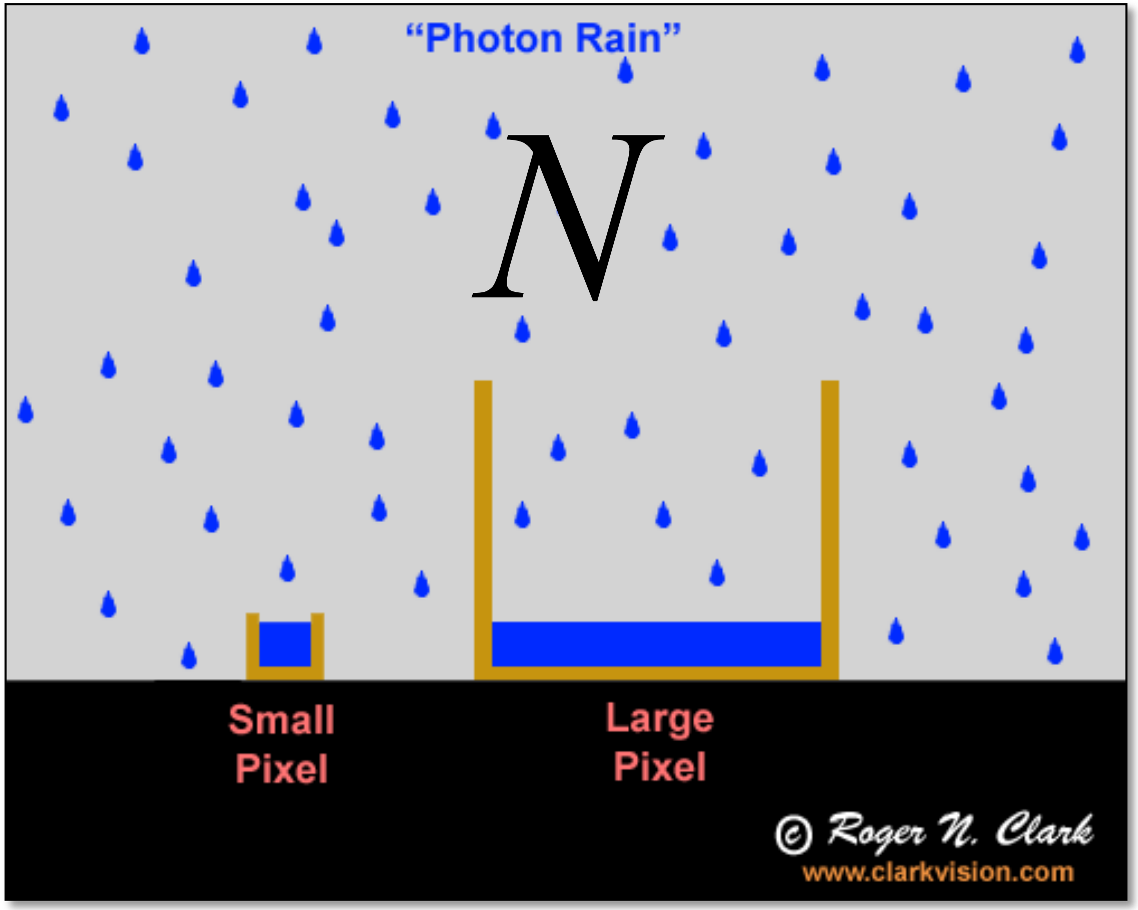


- astronomy is...



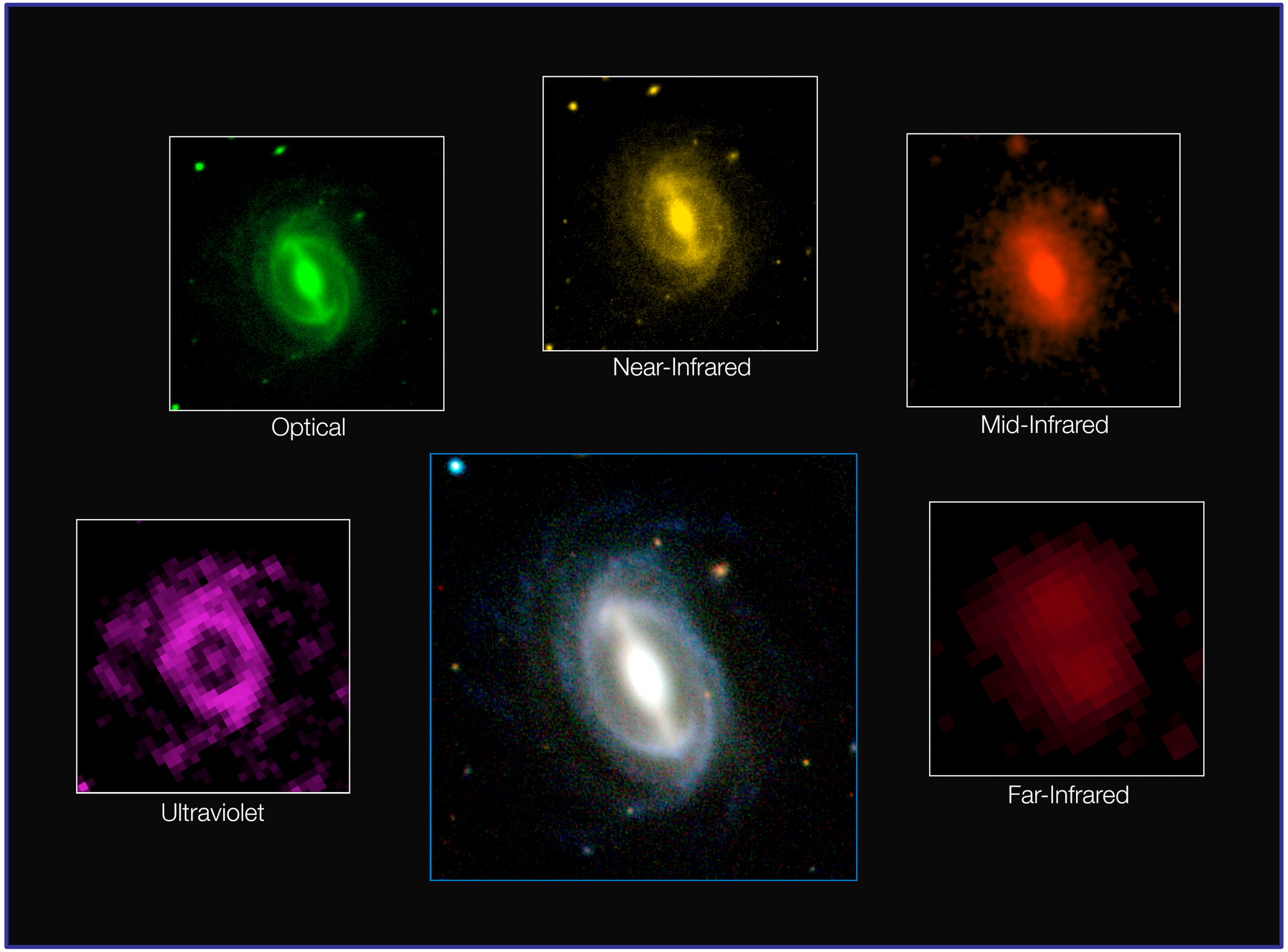
...collecting and counting photons

- astronomy is...



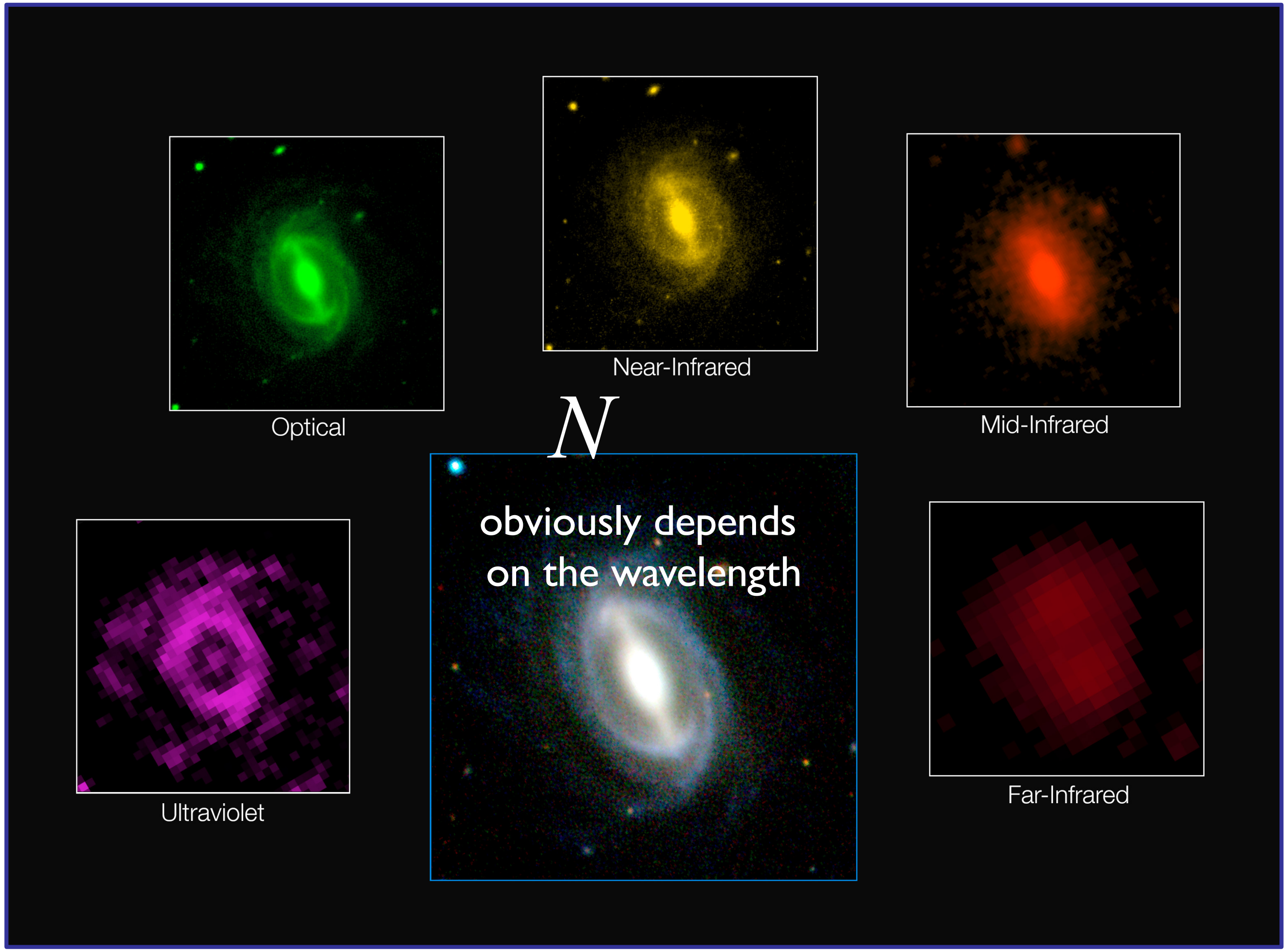
...collecting and counting photons

- astronomy is...



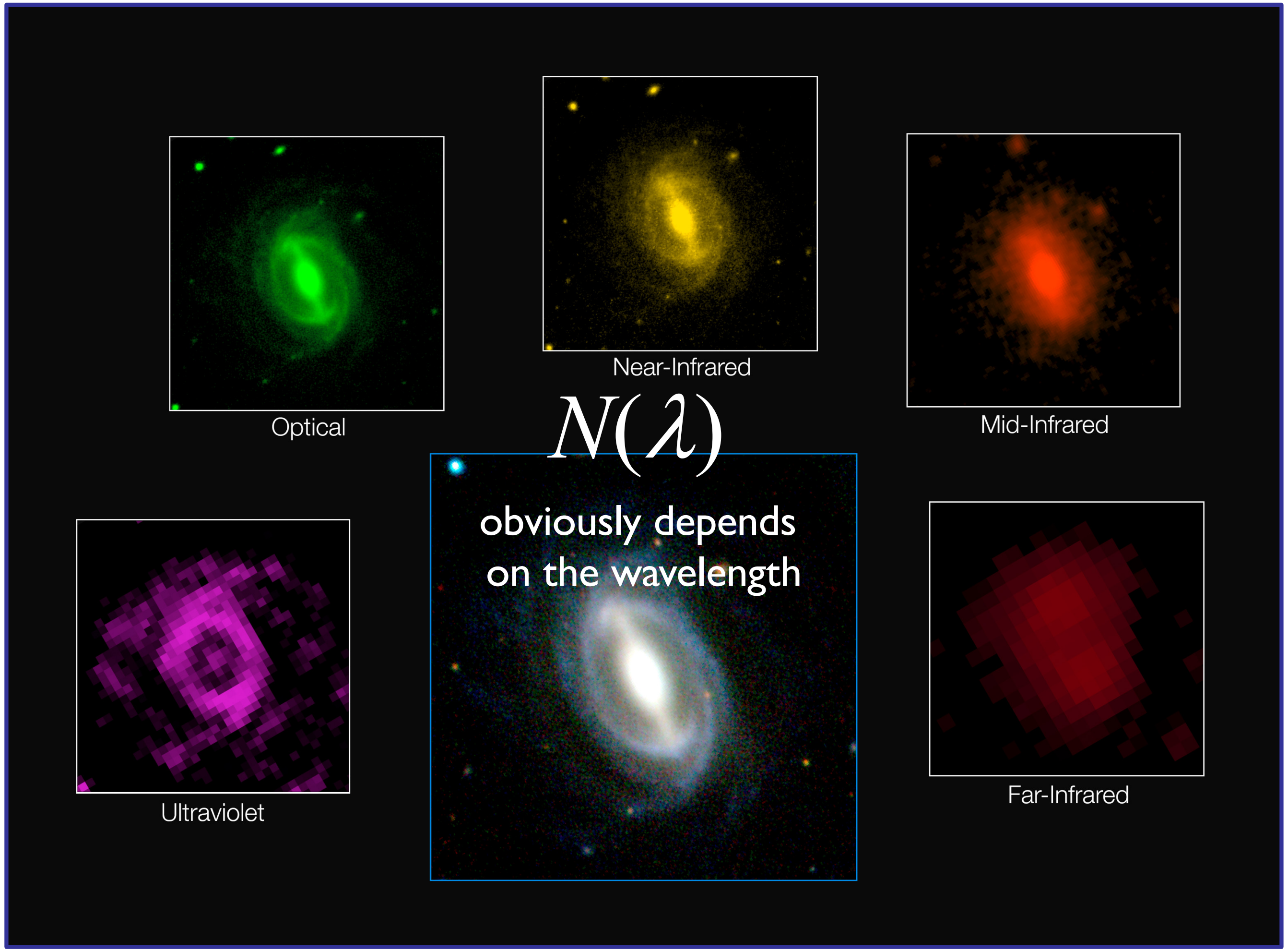
...collecting and counting photons

- astronomy is...



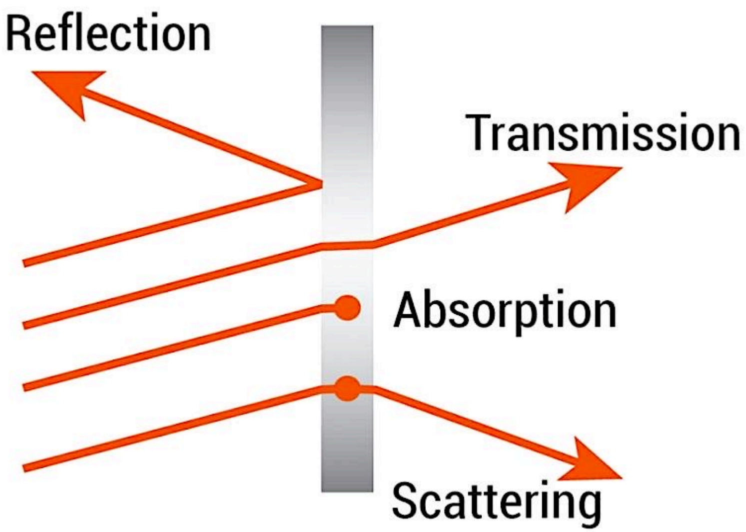
...collecting and counting photons

- astronomy is...



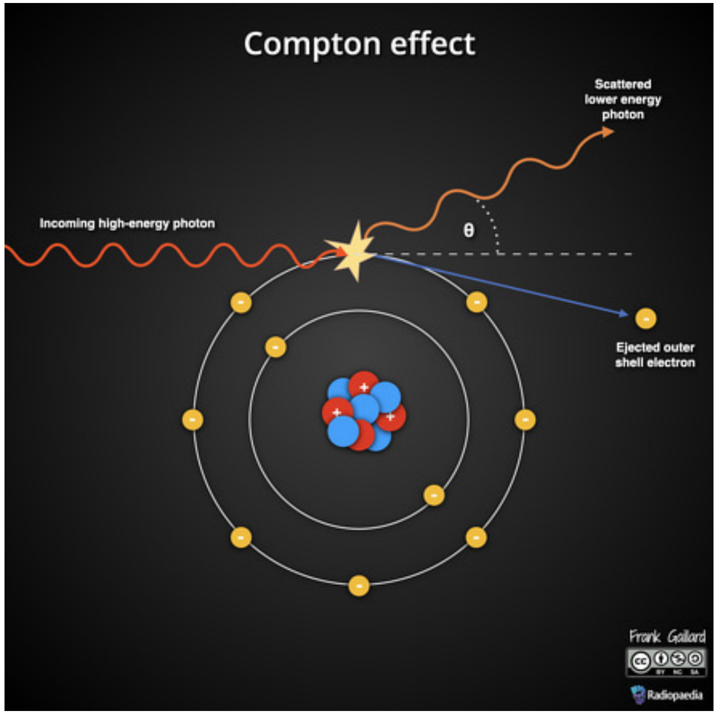
...collecting and counting photons

- astronomy is…



$$N(\lambda)$$

obviously depends on the wavelength



and interactions with matter (radiative processes)

…collecting and counting photons

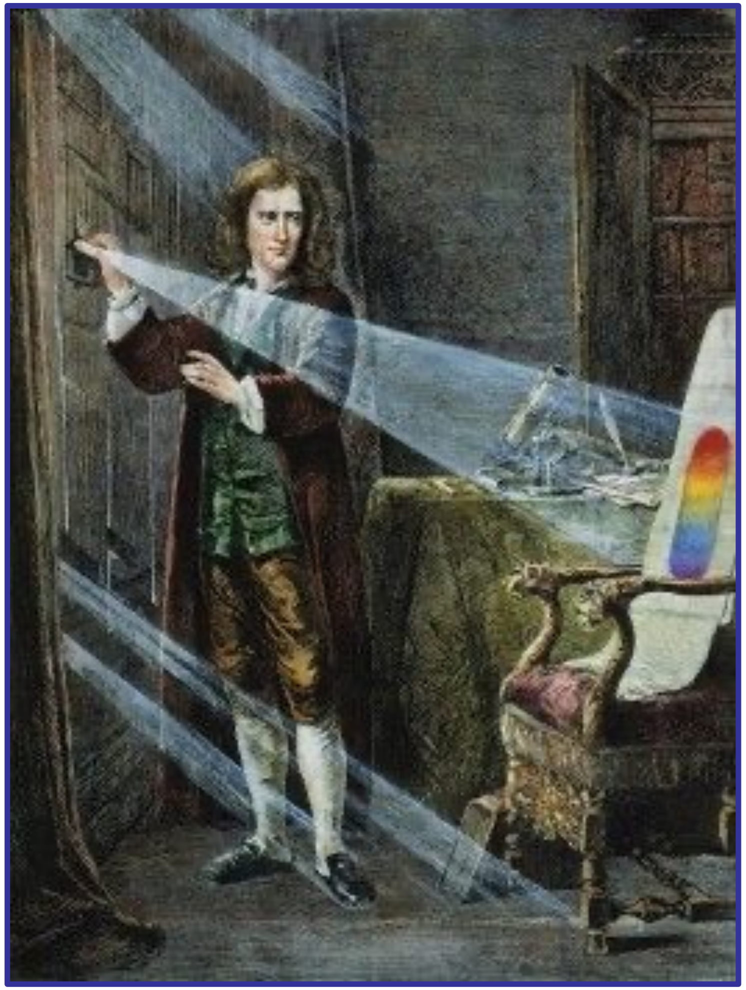
- electromagnetic spectrum

$$N(\lambda)$$

- electromagnetic spectrum

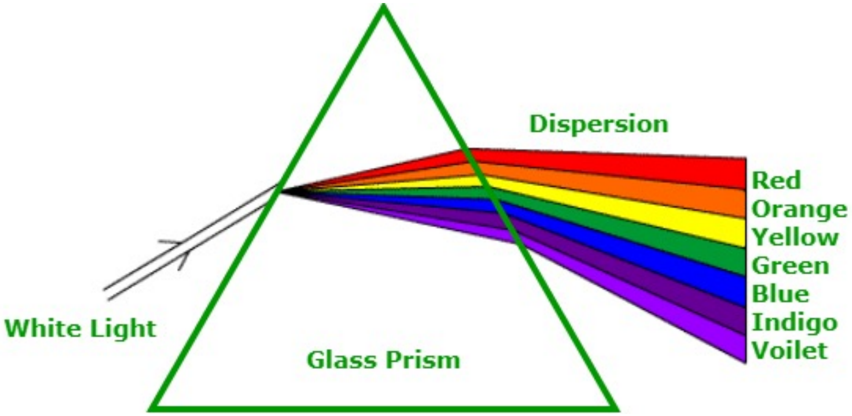
$$N(\lambda)$$

first discovered by Newton in 1672



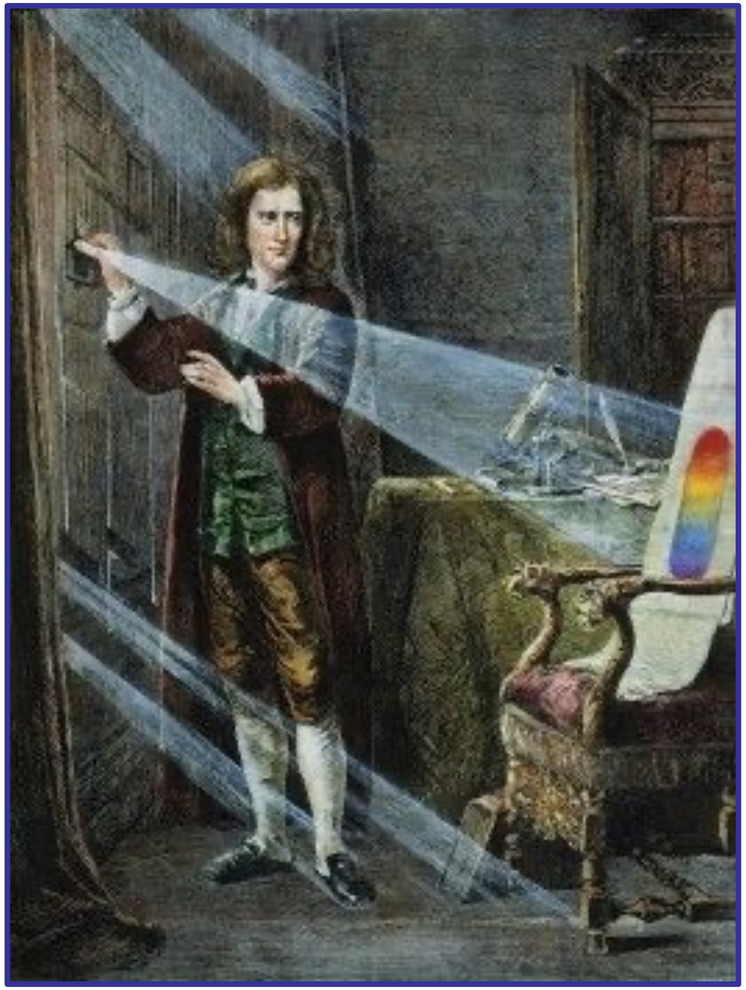
- electromagnetic spectrum

caused by dispersion



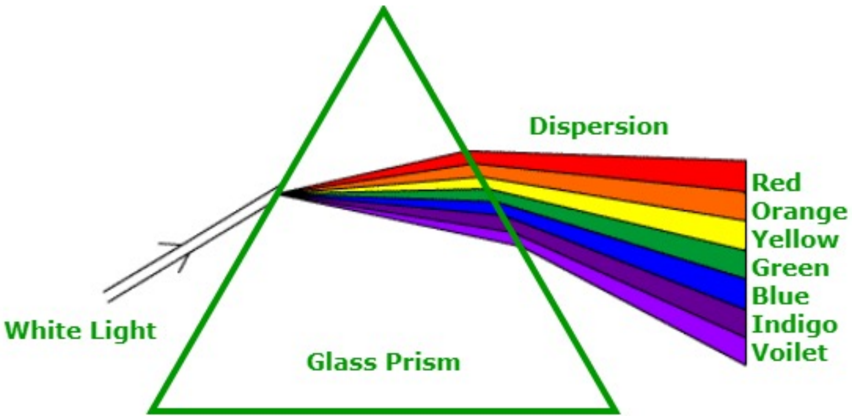
$$N(\lambda)$$

first discovered by Newton in 1672



- electromagnetic spectrum

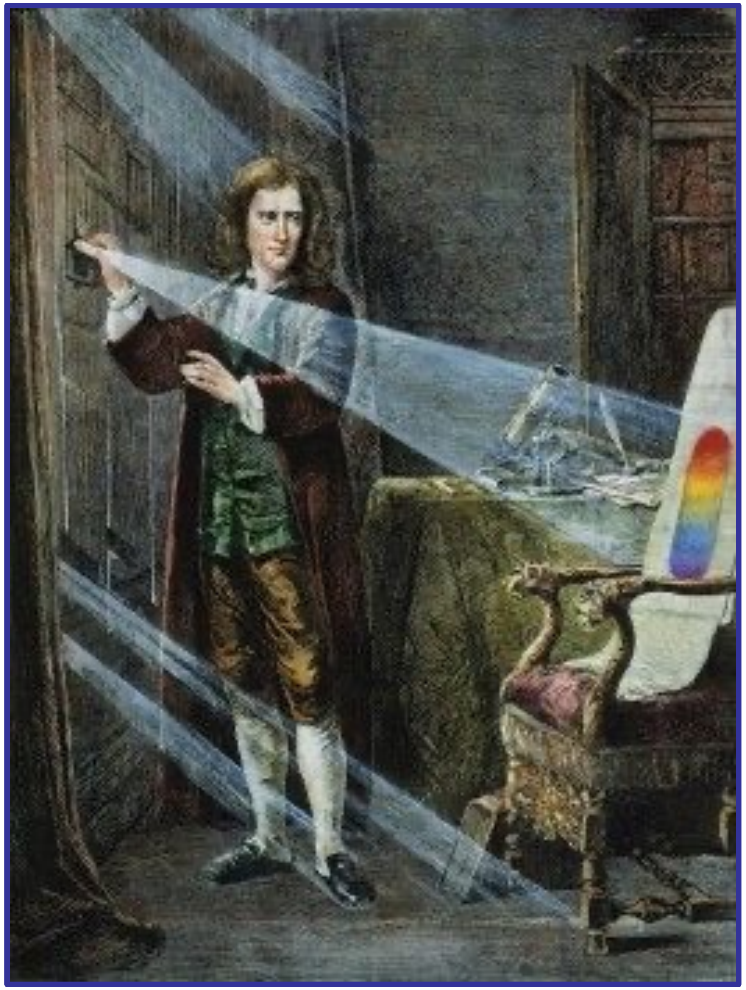
caused by dispersion



$$N(\lambda)$$

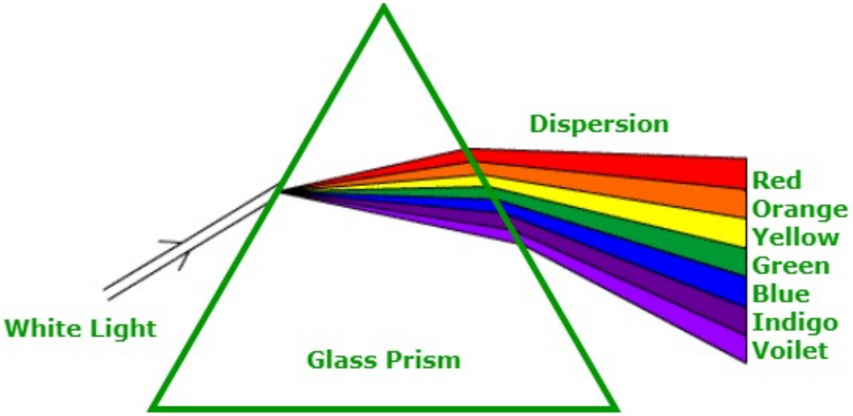
photon interaction with matter!

first discovered by Newton in 1672



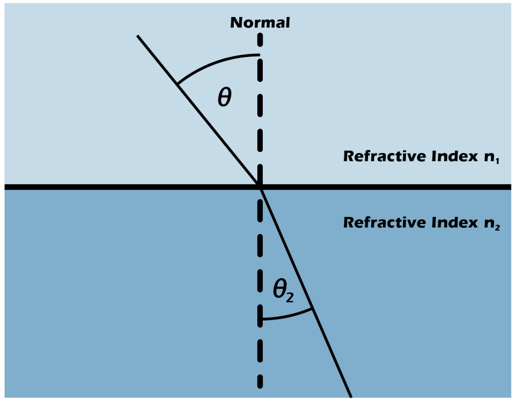
- electromagnetic spectrum

caused by dispersion...

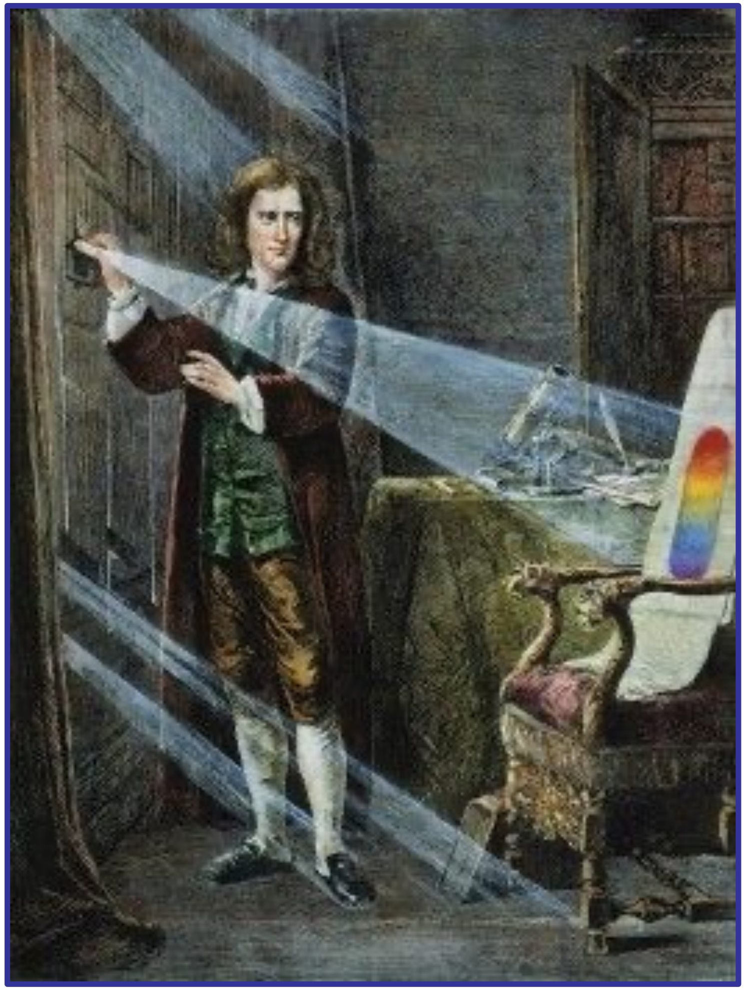


$$N(\lambda)$$

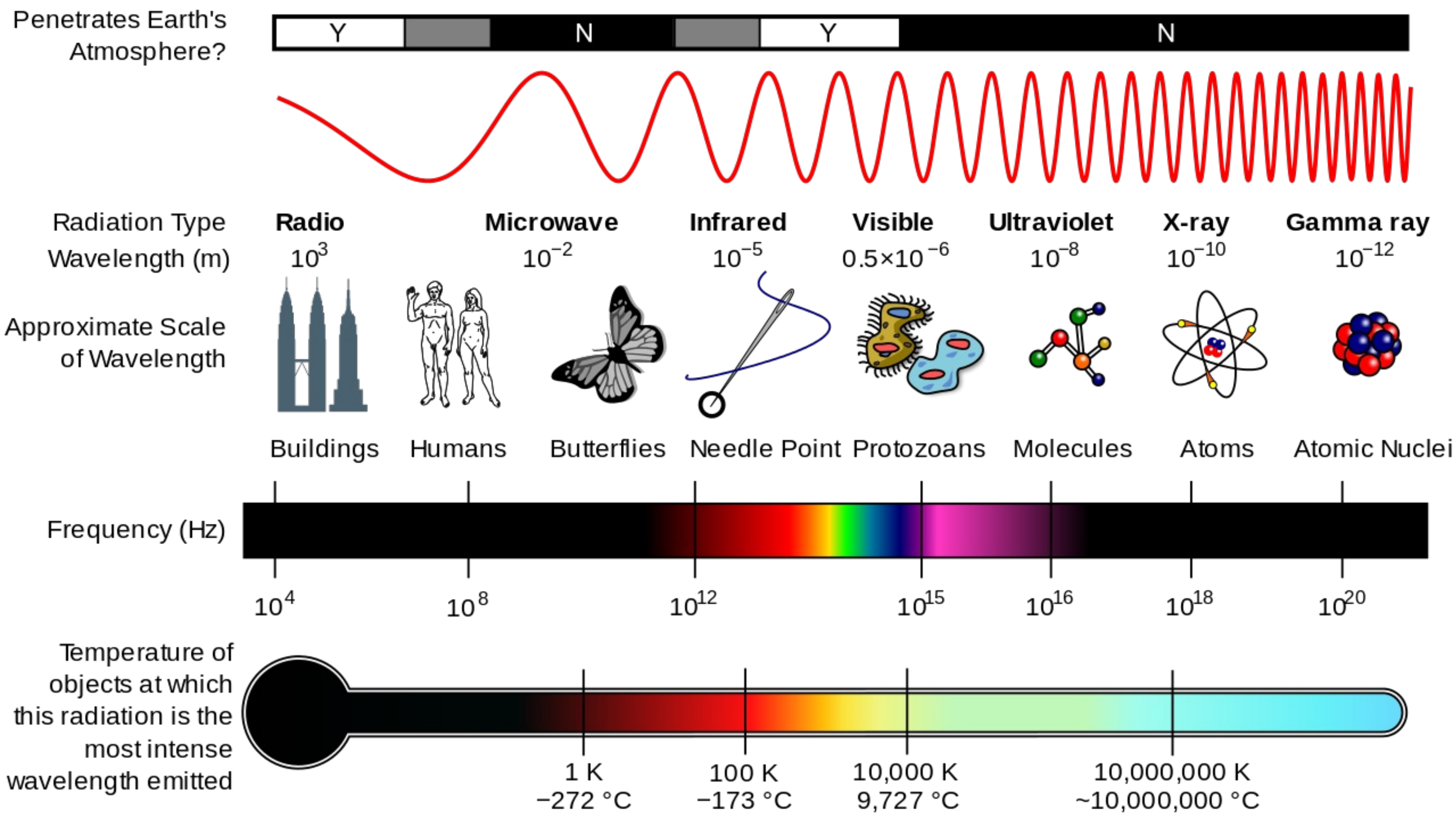
photon interaction described by Snell's law:



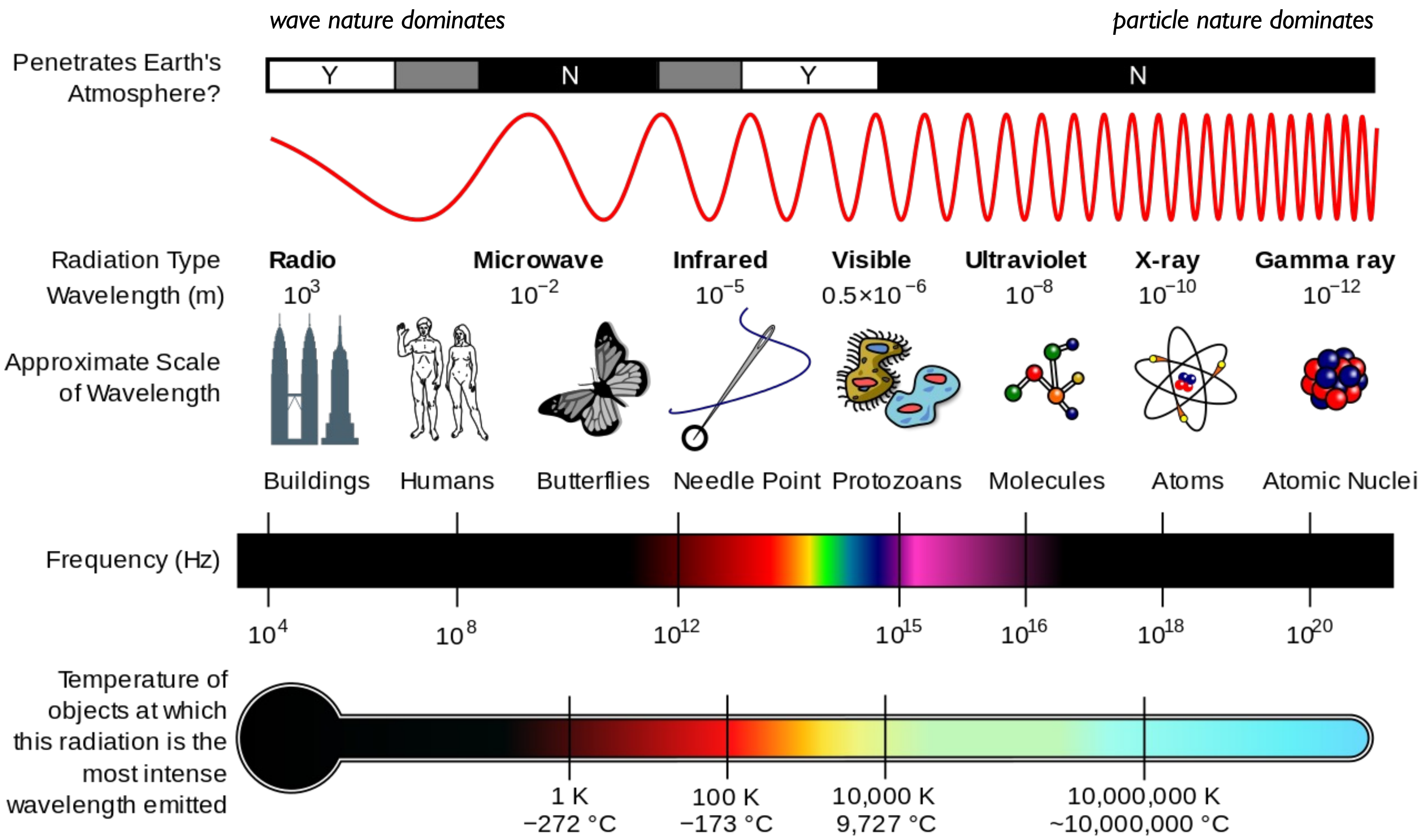
first discovered by Newton in 1672



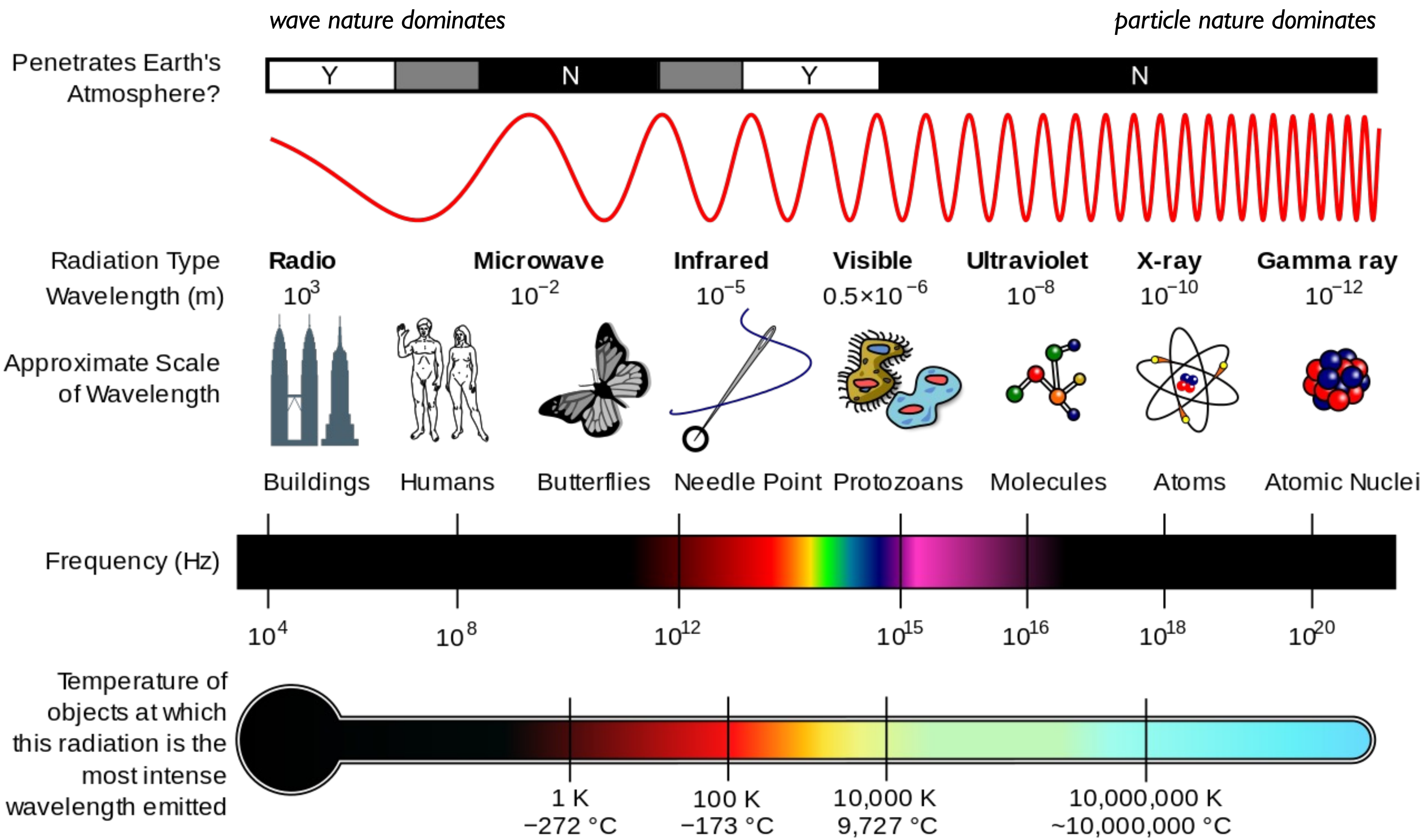
- electromagnetic spectrum



- electromagnetic spectrum



- electromagnetic spectrum



$$\nu = \frac{c}{\lambda}$$

$$E = h\nu$$

$$T = \frac{E}{k_B}$$

$$p = h\lambda$$

- electromagnetic spectrum

wave nature dominates

particle nature dominates

"radiative transfer"

↑

every photon *carries* energy and momentum and interacts with matter in various ways...

$$\nu = \frac{c}{\lambda}$$

$$E = h\nu$$

$$T = \frac{E}{k_B}$$

$$p = h\lambda$$

- electromagnetic spectrum

wave nature dominates

particle nature dominates

every photon carries energy and momentum
and *interacts with matter* in various ways...

↓

for instance, telescopes

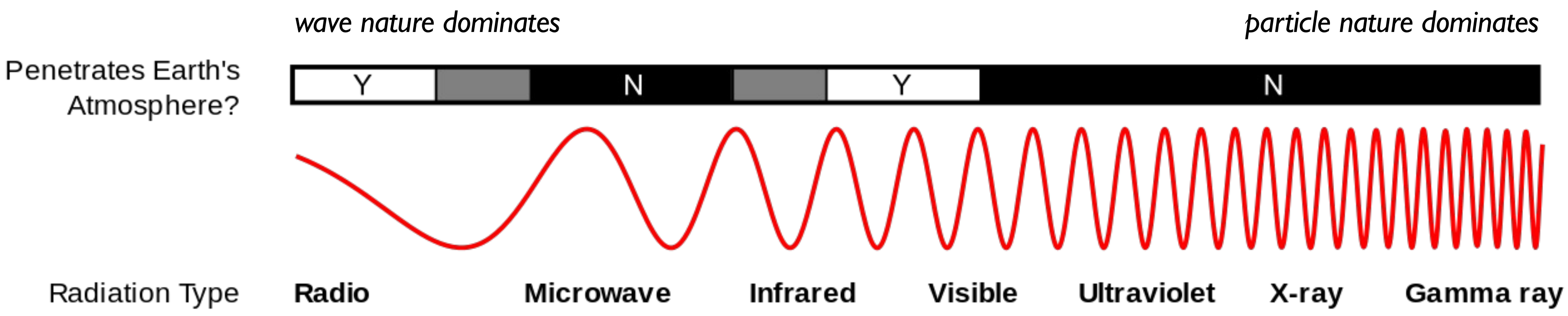
$$\nu = \frac{c}{\lambda}$$

$$E = h\nu$$

$$T = \frac{E}{k_B}$$

$$p = h\lambda$$

- electromagnetic spectrum

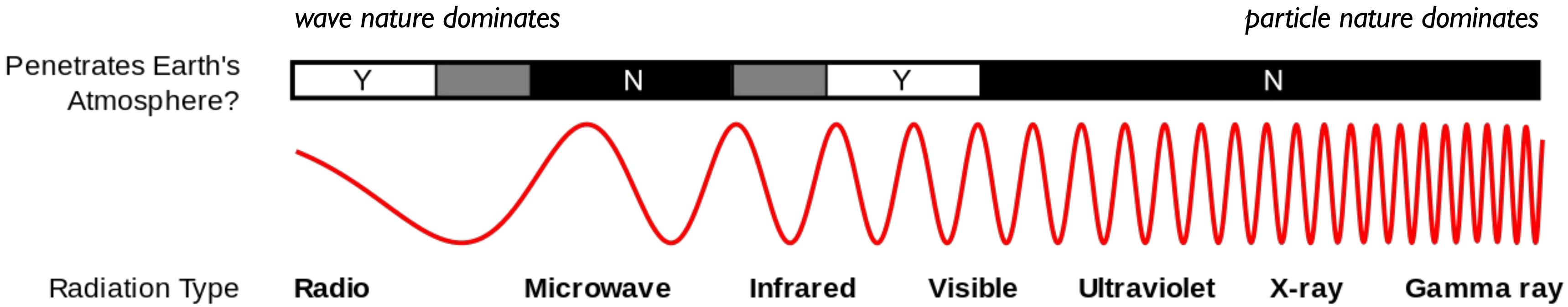


FAST

Gran Telescopio Canarias

Fermi satellite

- electromagnetic spectrum



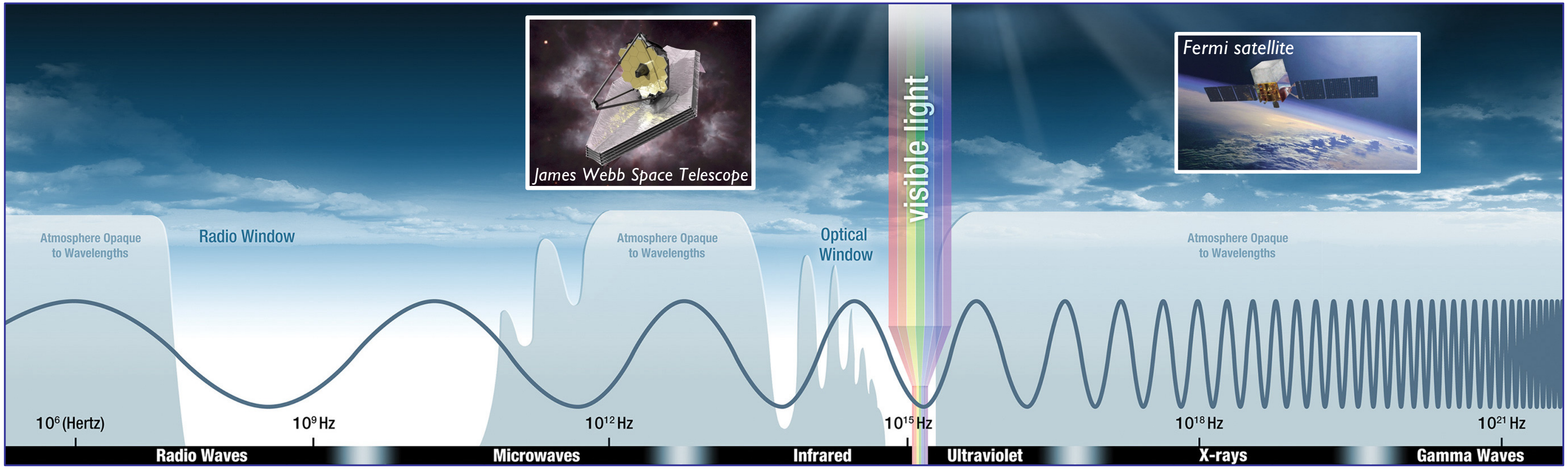

FAST


Gran Telescopio Canarias


Fermi satellite

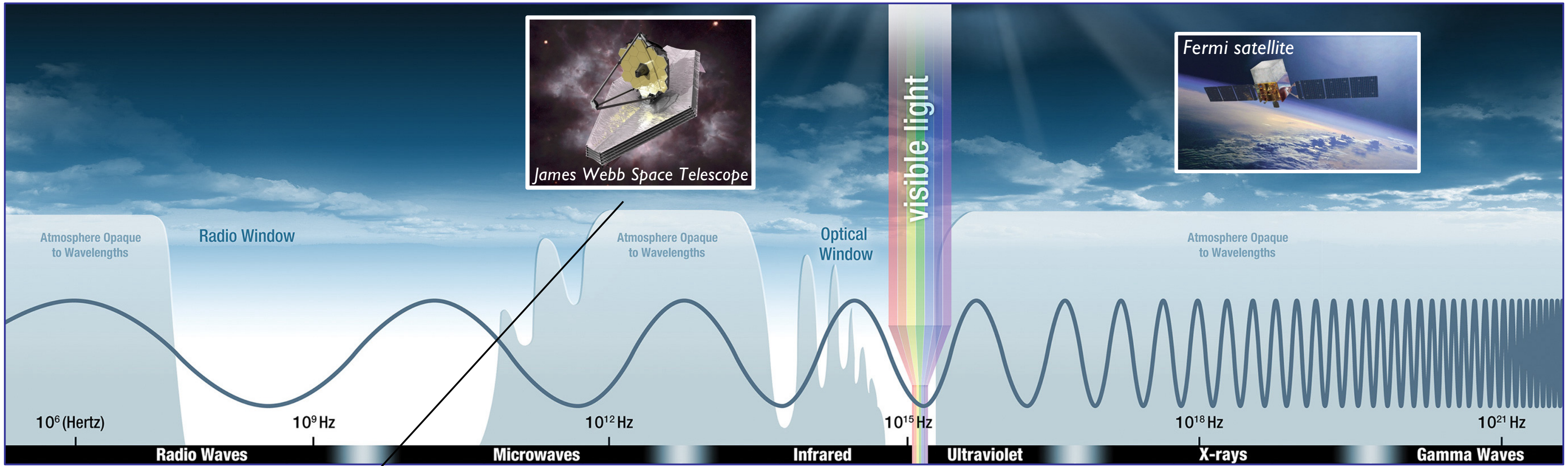
different parts of the EM spectrum
need to be observed with different types of telescopes

- electromagnetic spectrum



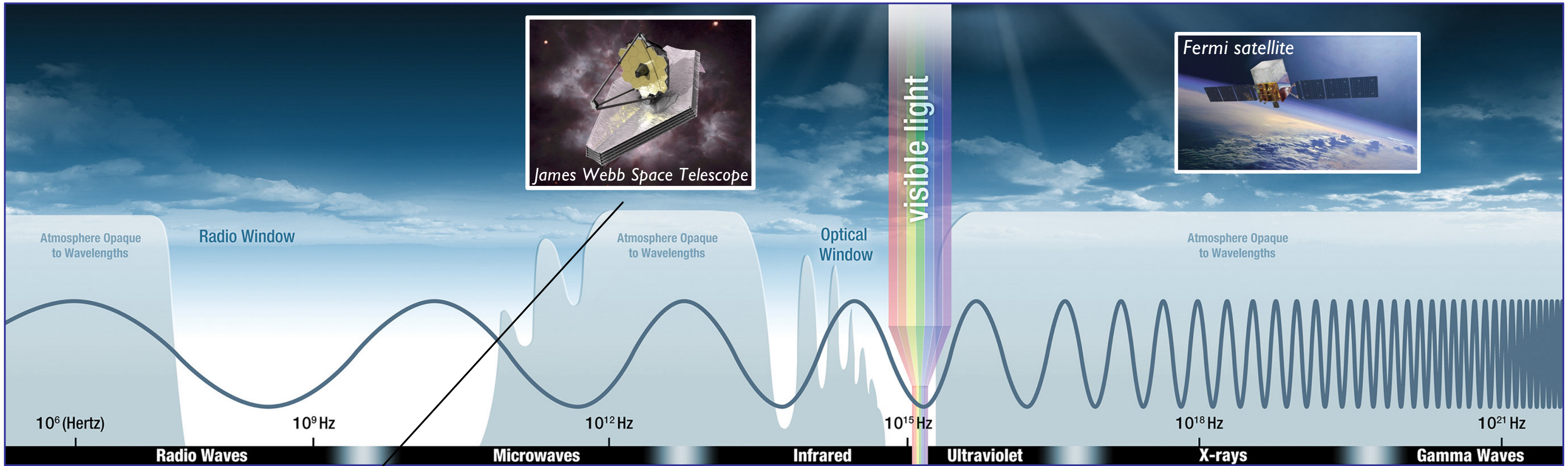
certain parts of the EM spectrum
need to be observed from space

- electromagnetic spectrum



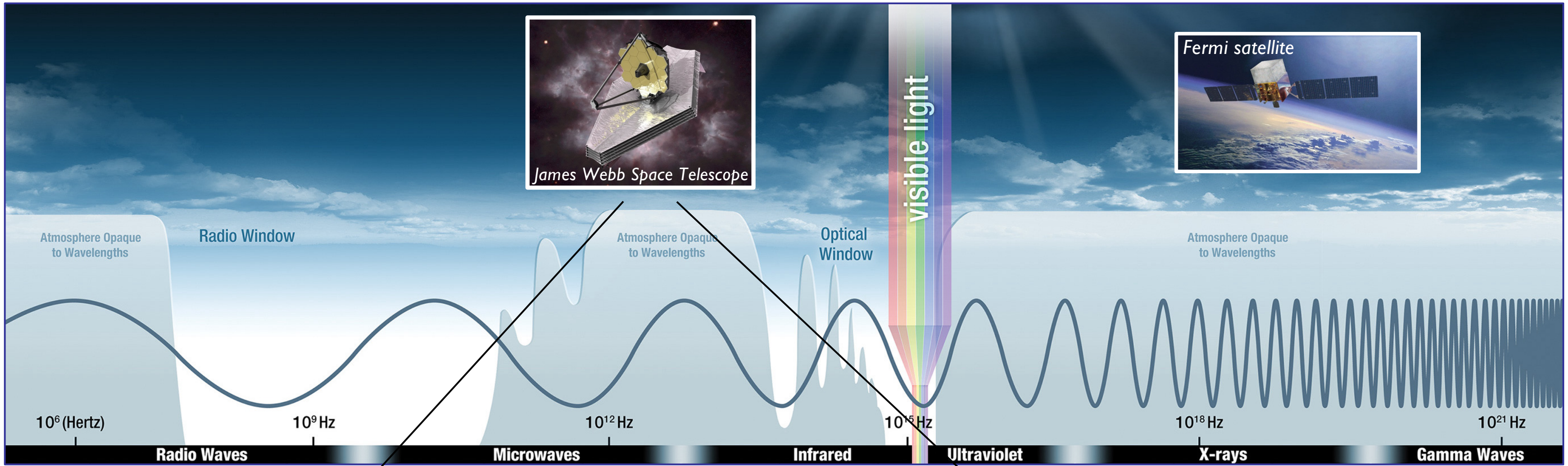
certain parts of the EM spectrum need to be observed from space

- electromagnetic spectrum

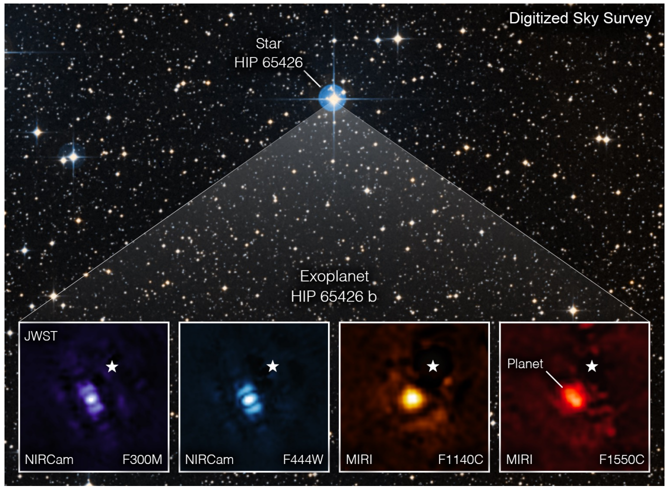


certain parts of the EM spectrum
need to be observed from space

- electromagnetic spectrum

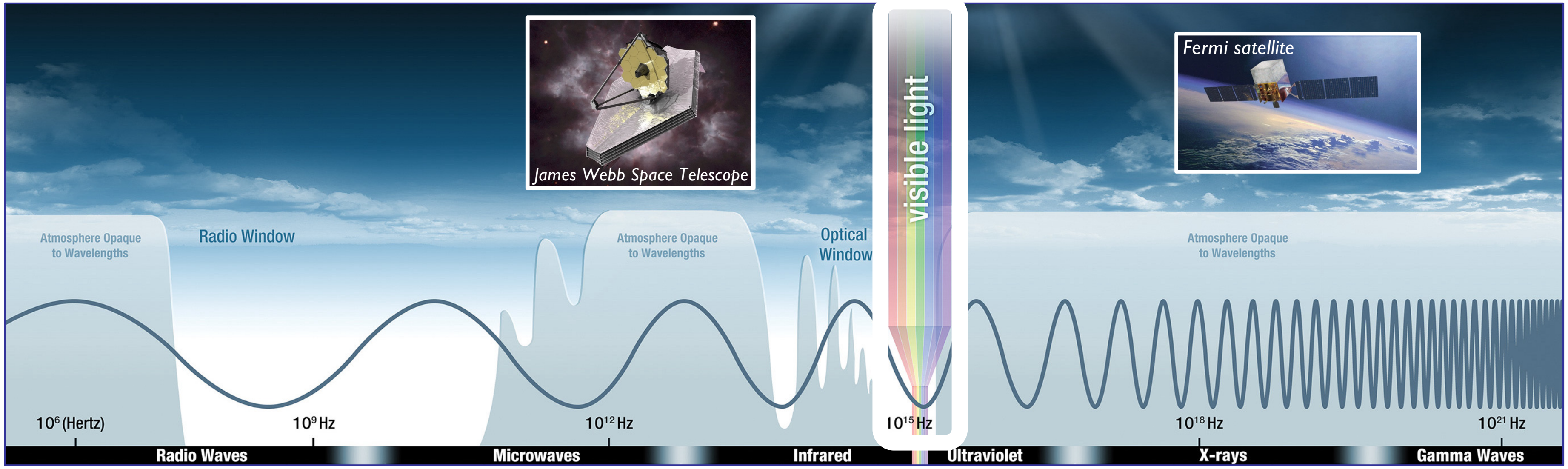


certain parts of the EM spectrum
need to be observed from space



- electromagnetic spectrum

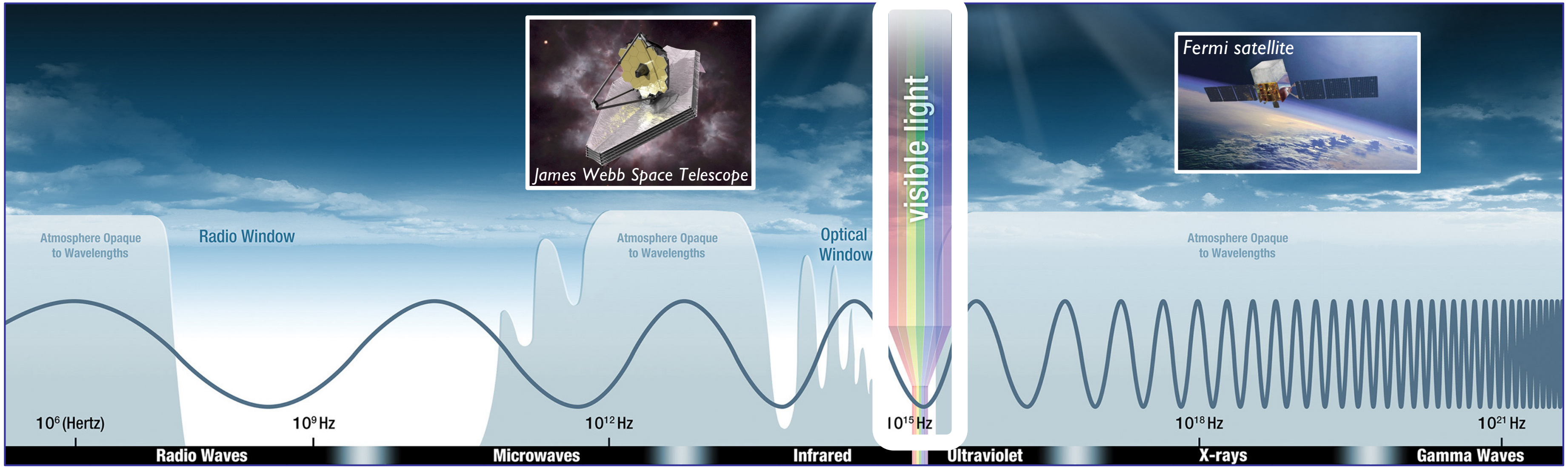
why is the sky blue and not white?



certain parts of the EM spectrum need to be observed from space

- electromagnetic spectrum

why is the sky blue and not white?

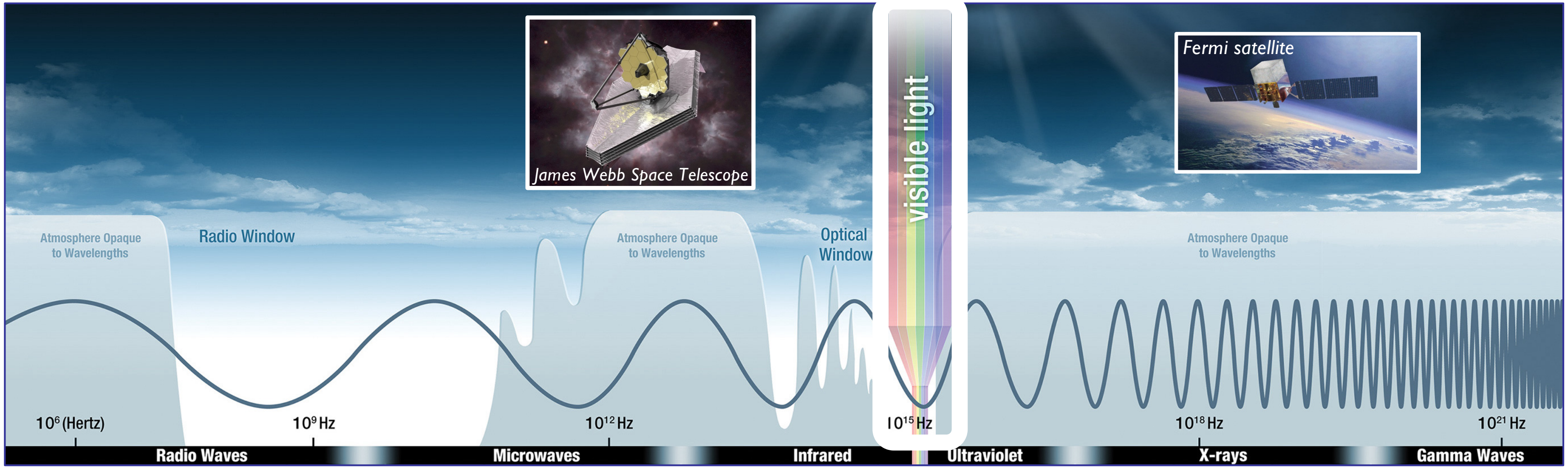


certain parts of the EM spectrum
need to be observed from space

why is the sky blue?

- electromagnetic spectrum

why is the sky blue and not white?



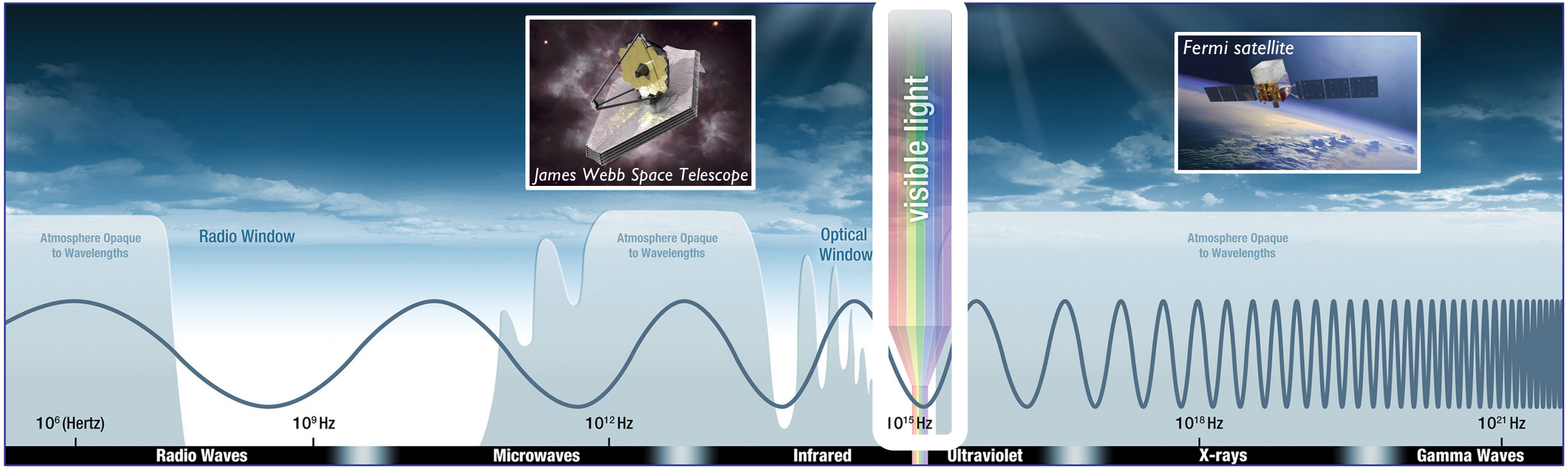
certain parts of the EM spectrum need to be observed from space

why is the sky blue?

interaction of solar photons with atmosphere...

- electromagnetic spectrum

why is the sky blue and not white?



certain parts of the EM spectrum need to be observed from space

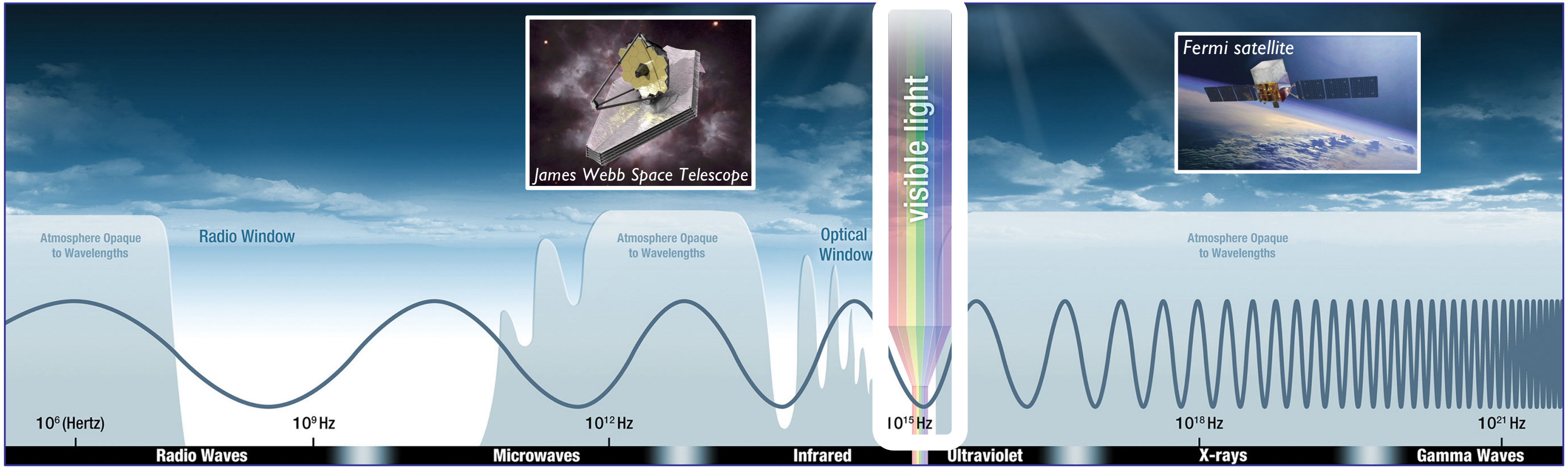
why is the sky blue?

<u>interaction of solar photons with atmosphere:</u>

Rayleigh scattering off of molecules in the sky $\sigma_s \propto {}^{1}/_{\lambda^4}$

- electromagnetic spectrum

why is the sky blue and not white?



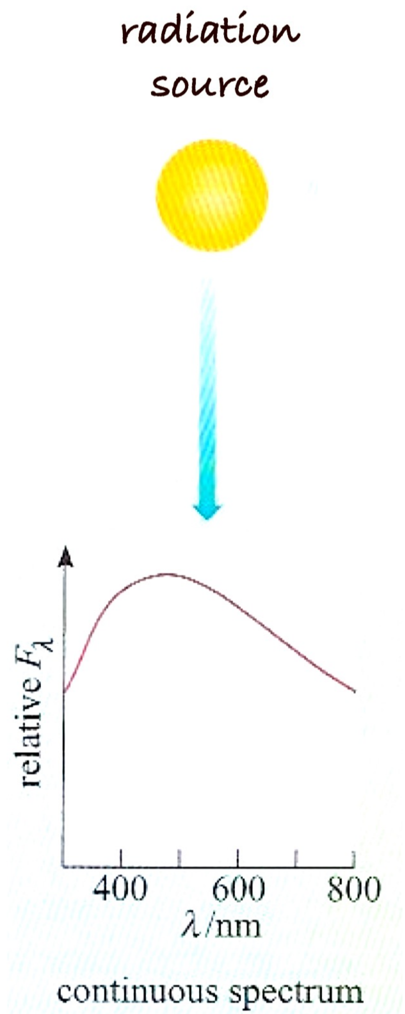
certain parts of the EM spectrum need to be observed from space

why is the sky blue?

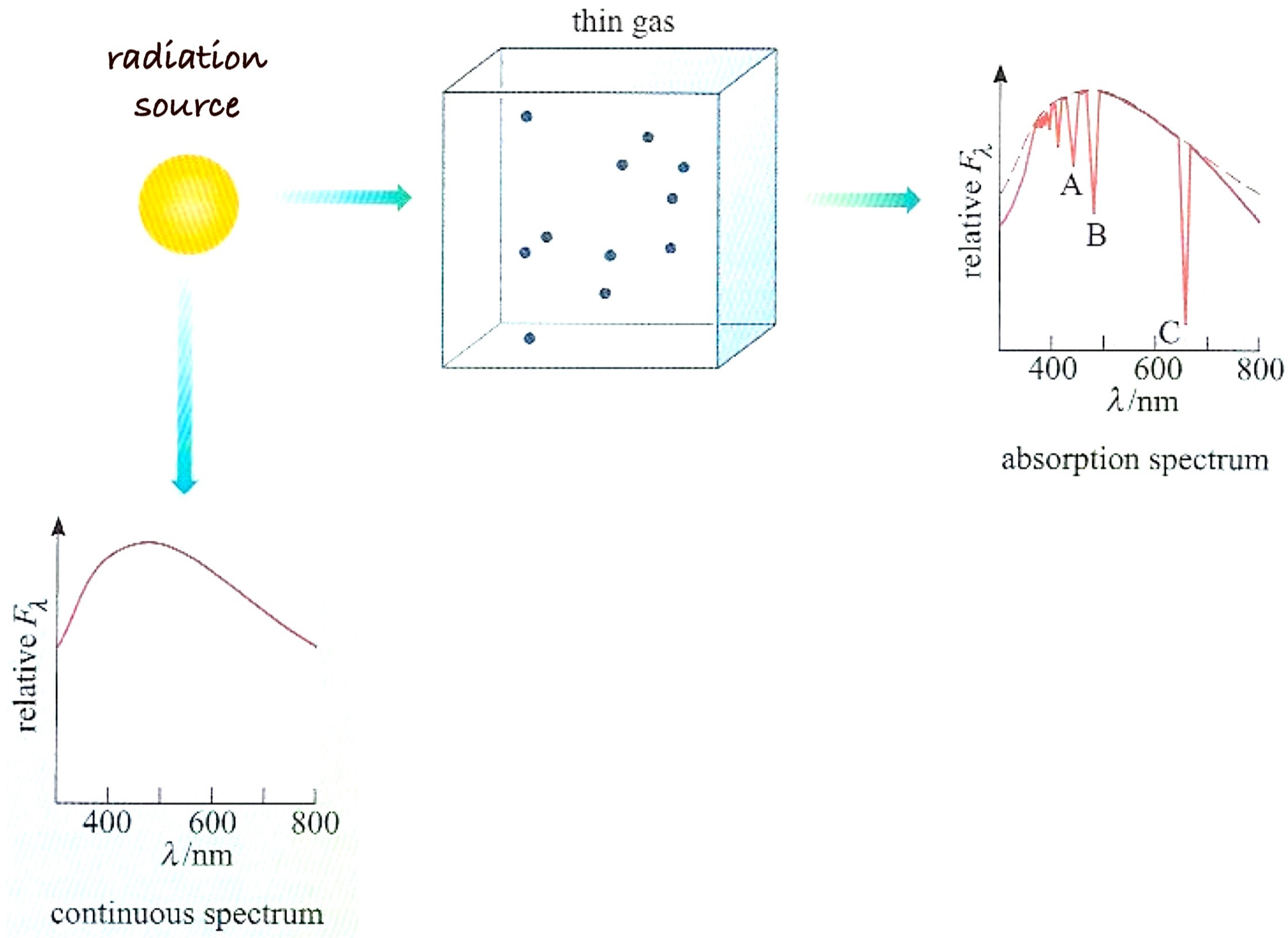
interaction of solar photons with atmosphere:

Rayleigh scattering off of molecules in the sky $\sigma_s \propto {}^1/_{\lambda^4}$ → *blue gets scattered more than red!*

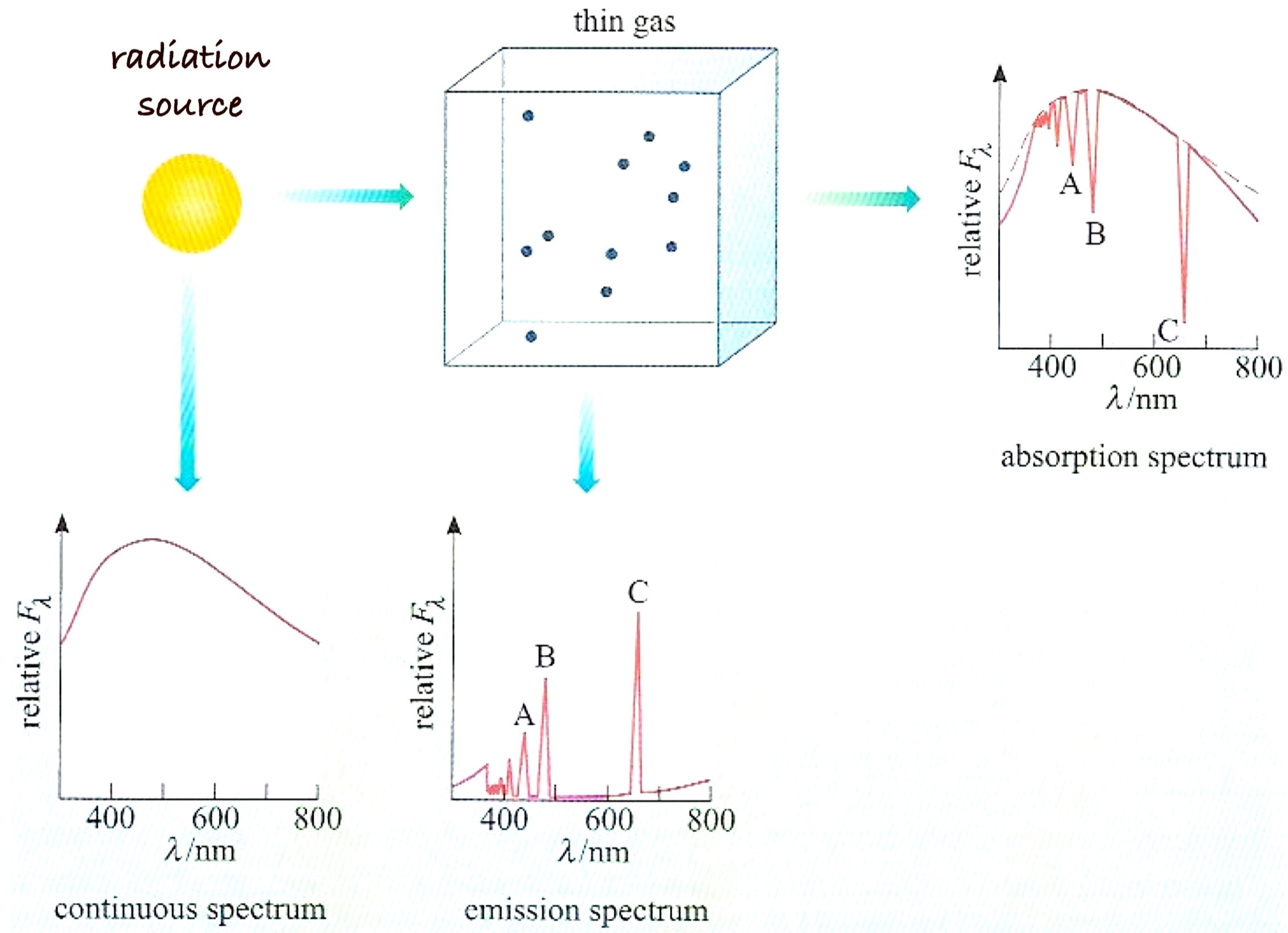
- electromagnetic spectrum – interaction w/ matter



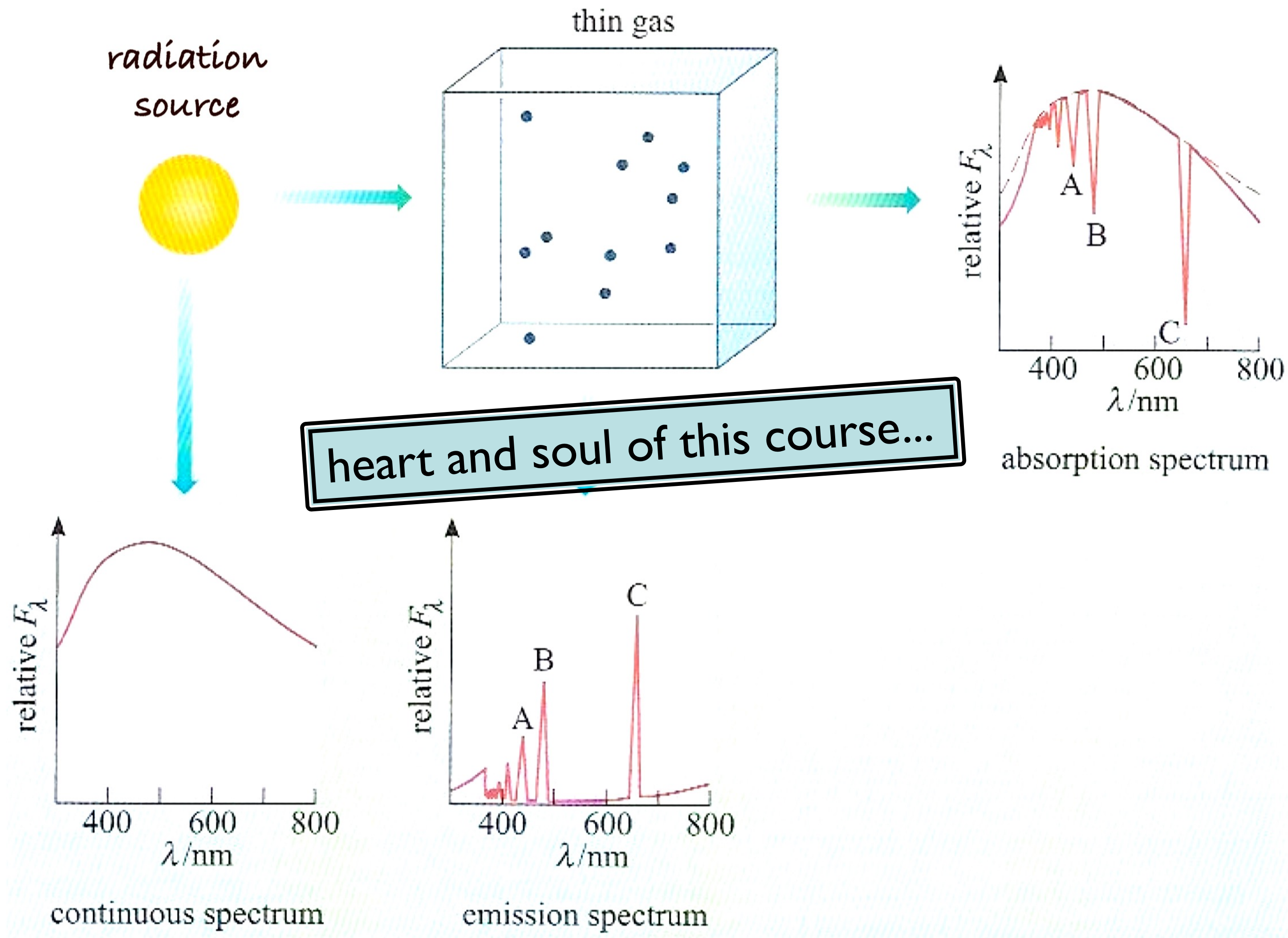
- electromagnetic spectrum – interaction w/ matter



- electromagnetic spectrum – interaction w/ matter



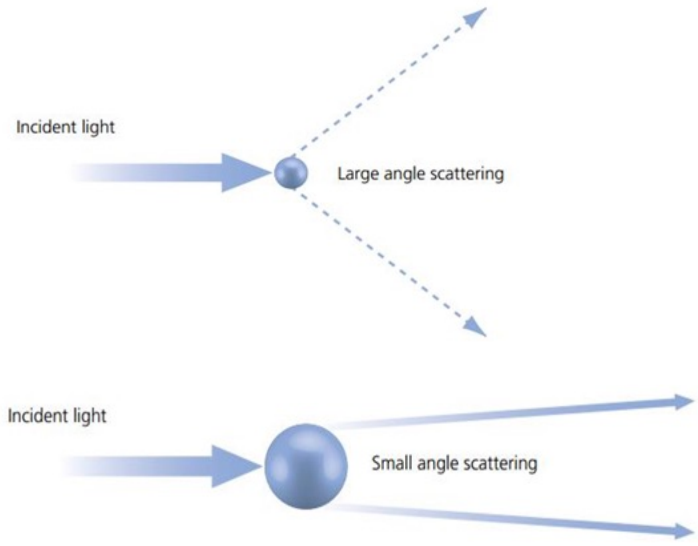
- electromagnetic spectrum – interaction w/ matter



- electromagnetic spectrum
- **description of a radiation field**
- radiative transfer equation

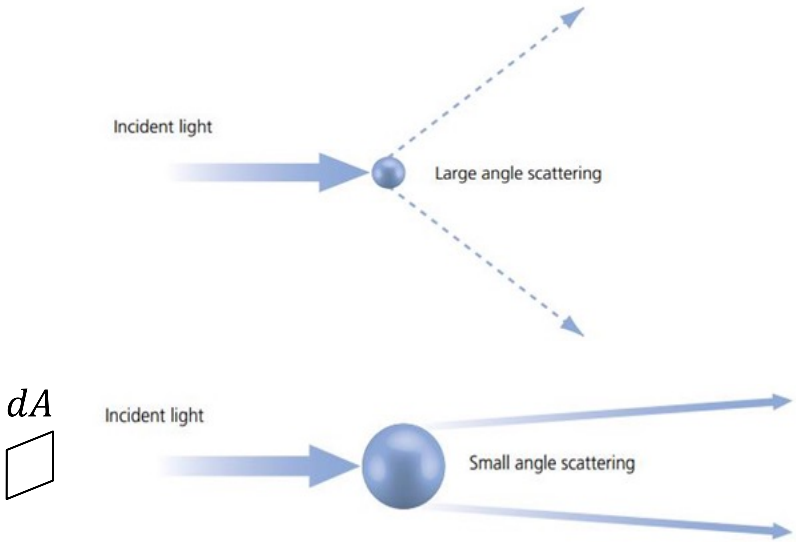
- radiation field – macroscopic description

When the scale of the system greatly exceeds the wavelength of radiation, we consider that the radiation travels in a straight line (a ray).

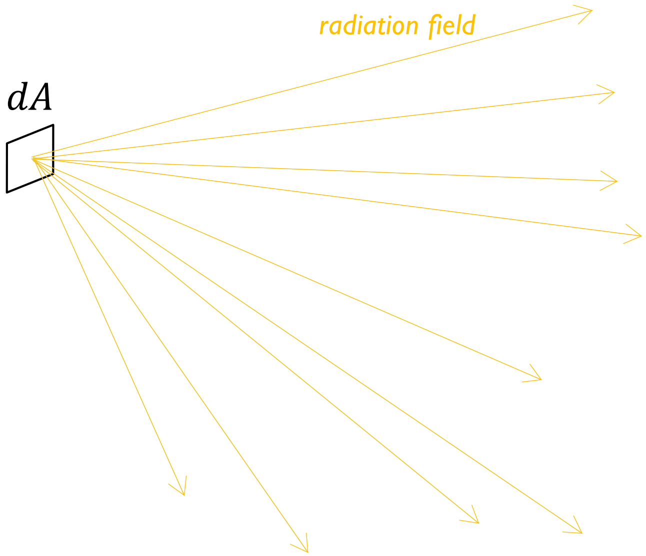


- radiation field – macroscopic description

When the scale of the system greatly exceeds the wavelength of radiation, we consider that the radiation travels in a straight line (a ray).

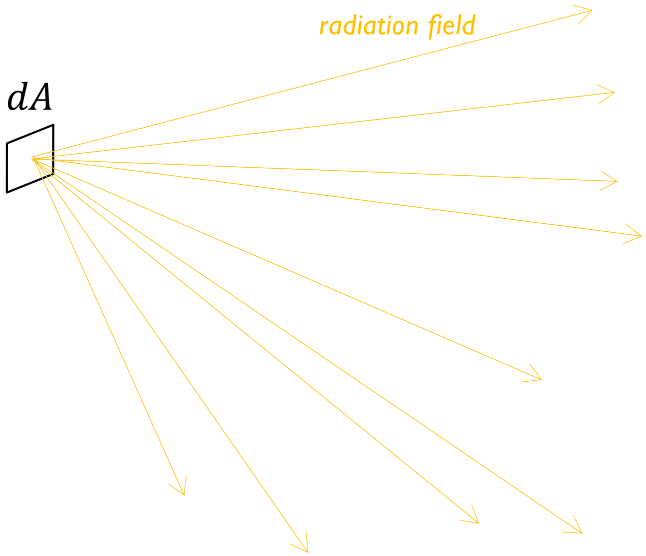


- radiation field – macroscopic description



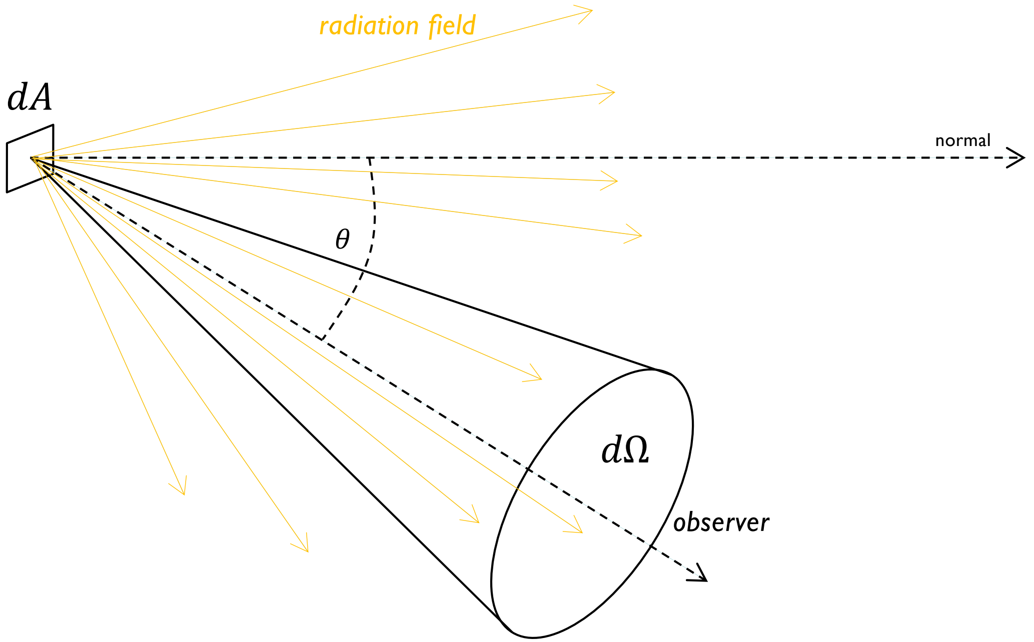
▪ radiation field – macroscopic description

- we seek a description that…
  - ✓ describes the intrinsic radiation field



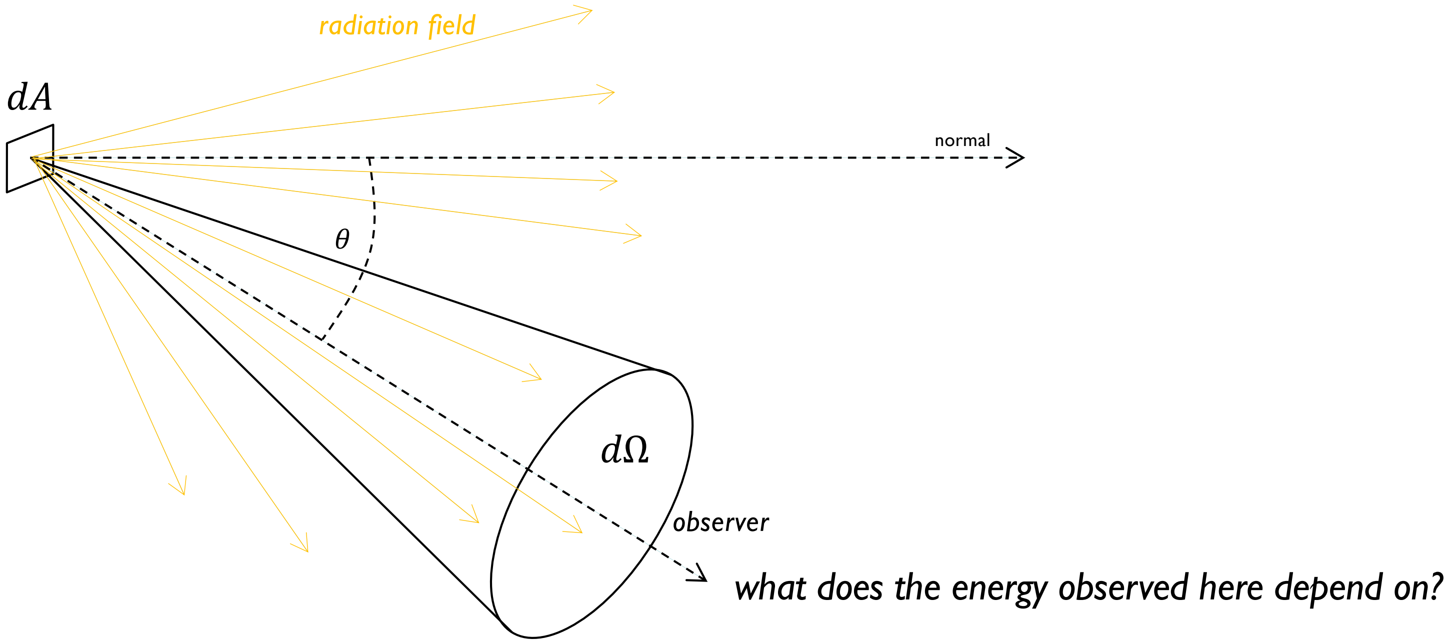
▪ radiation field – macroscopic description

- we seek a description that...
  - ✓ describes the intrinsic radiation field, and
  - ✓ does not depend on the observer



▪ radiation field – macroscopic description

- we seek a description that...
  - ✓ describes the intrinsic radiation field, and
  - ✓ does not depend on the observer

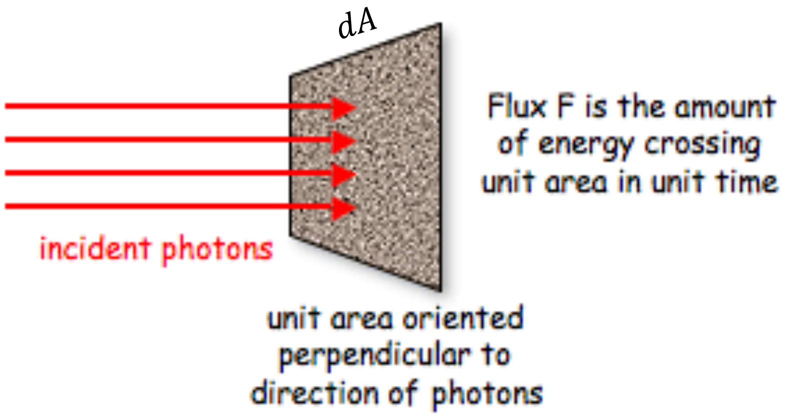


- radiation field – macroscopic description
  - flux
  - intensity
  - luminosity
  - momentum
  - energy density
  - radiation pressure

- radiation field – macroscopic description
  - **flux**
  - intensity
  - luminosity
  - momentum
  - energy density
  - radiation pressure

- radiation field – flux

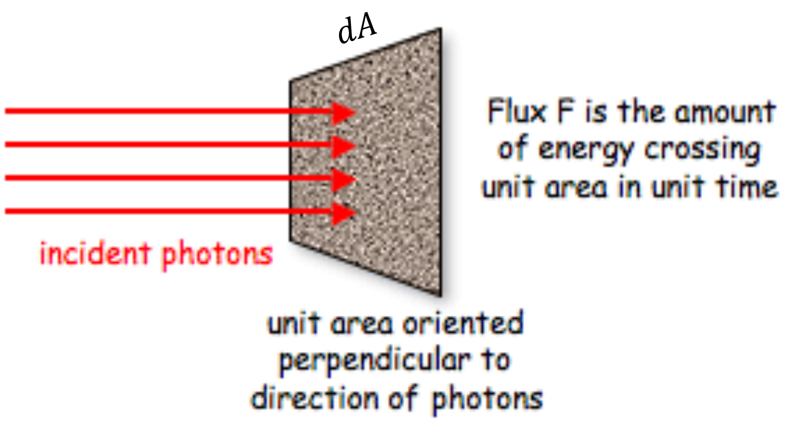
radiative flux is the total amount of energy that crosses a unit area per unit time



$$dE = F\, dA\, dt$$

- radiation field – flux

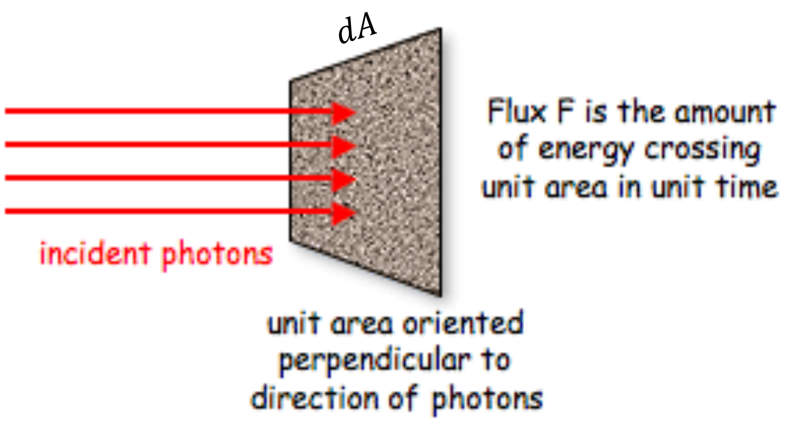
radiative flux is the total amount of energy that crosses a unit area per unit time



$dE = F\, dA\, dt$ → flux is a measure of the energy carried by all rays

▪ radiation field – flux

radiative flux is the total amount of energy that crosses a unit area per unit time



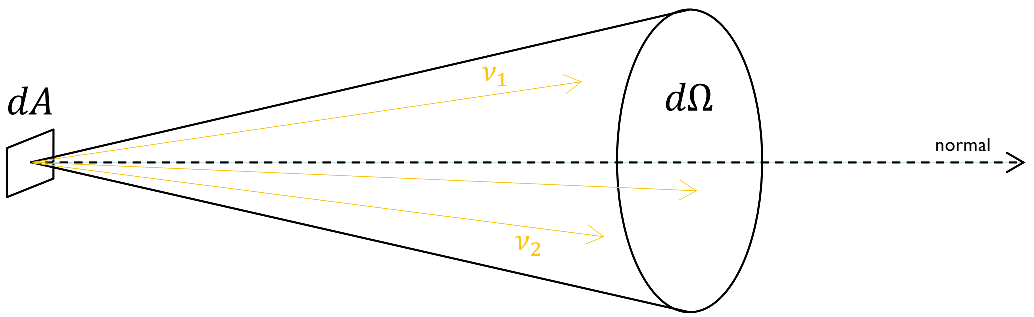
$$dE = F\,dA\,dt$$ → flux is a measure of the energy carried by all rays

When we observe a radiation source, we actually measure the energy E collected by a detector over a period of time, which obviously represents the integrated energy flux over the size of the detector and time observed.

- radiation field – flux
  - flux is a measure of the energy carried by *all* rays $dE = F\, dA\, dt$

▪ radiation field – flux

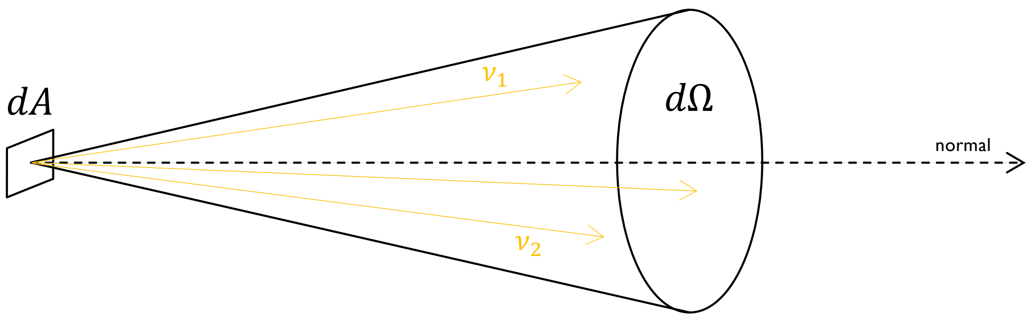
- flux is a measure of the energy carried by *all* rays $dE = F\, dA\, dt$



but the radiation is not necessarily isotropic nor equal for all wavelengths

▪ radiation field – flux

- flux is a measure of the energy carried by *all* rays $dE = F\, dA\, dt$

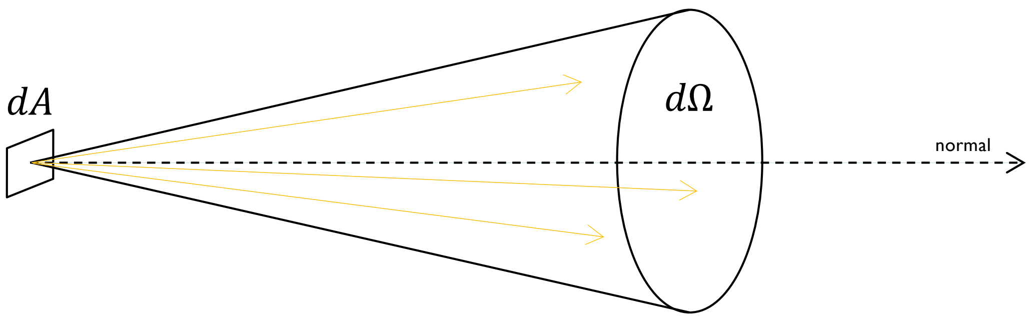


but the radiation is not necessarily isotropic nor equal for all wavelengths

→ *intensity* = flux normalized by solid angle ($d\Omega$) and wavelength interval ($d\nu$)

- radiation field – macroscopic description
  - flux
  - **intensity**
  - luminosity
  - momentum
  - energy density
  - radiation pressure

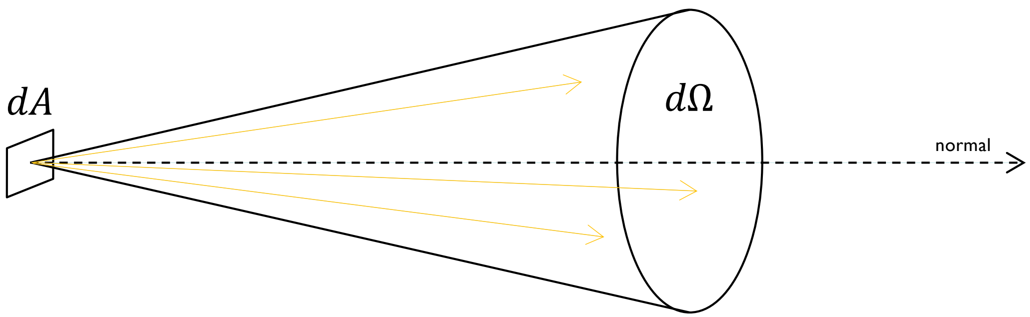
- radiation field – intensity
  - intensity is a measure of the energy carried by *individual* rays



▪ radiation field – intensity

- intensity is a measure of the energy carried by *individual* rays

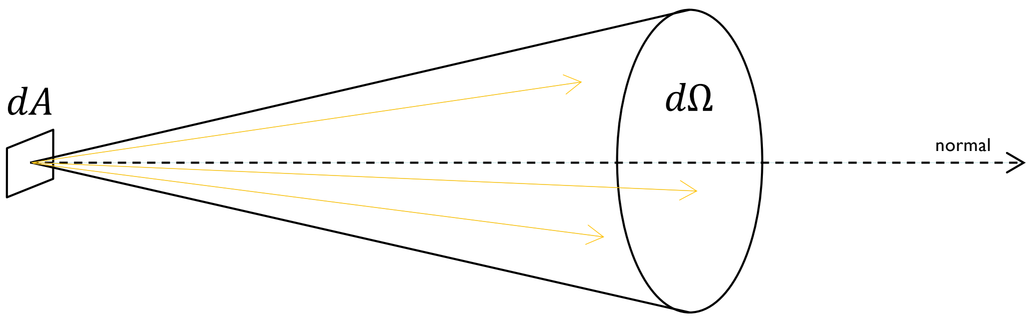
$$dE = I_\nu(\Omega)\, d\Omega\, d\nu\, dA\, dt$$



■ radiation field – intensity

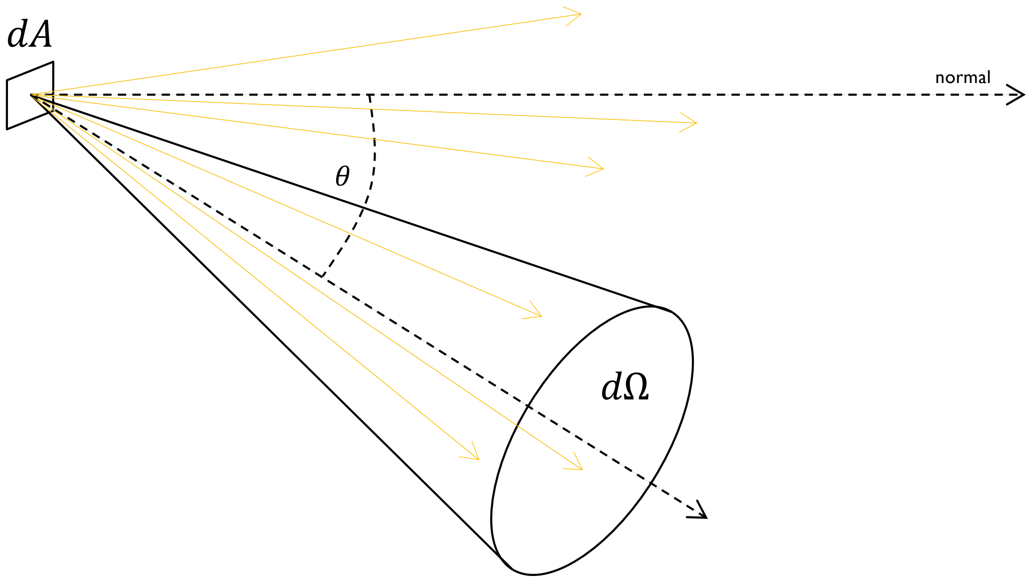
- intensity is a measure of the energy carried by *individual* rays

$$dE = \overbrace{I_\nu(\Omega)\, d\Omega\, d\nu}^{F_\nu(\Omega)}\, dA\, dt$$



- radiation field – intensity
  - intensity is a measure of the energy carried by *individual* rays

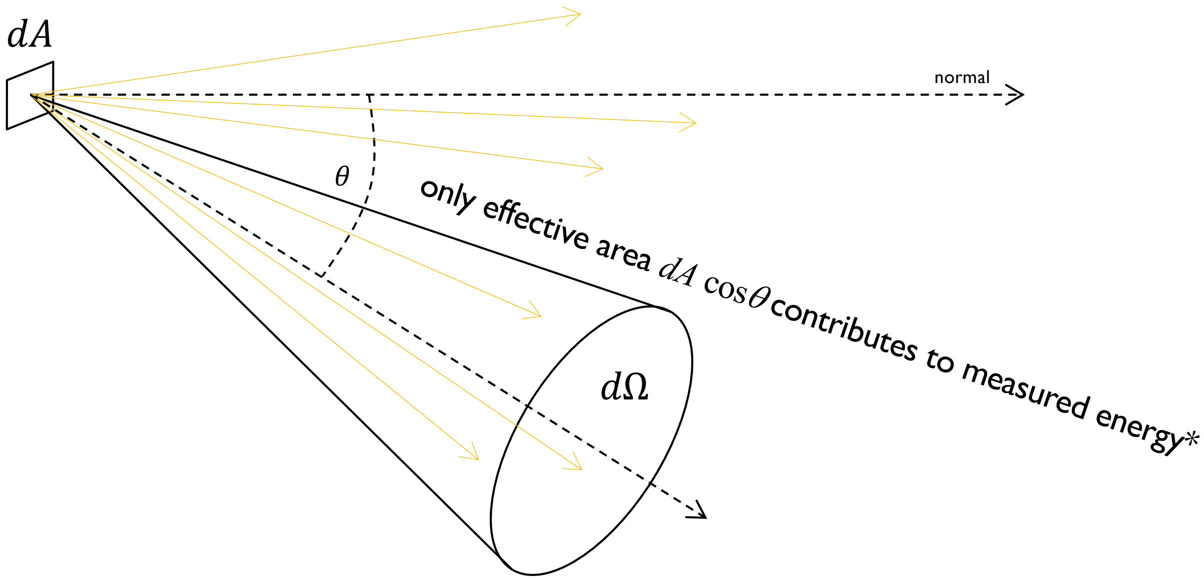
$$dE = I_\nu(\Omega)\, d\Omega\, d\nu\, dA_{eff}\, dt$$



▪ radiation field – intensity

- intensity is a measure of the energy carried by *individual* rays

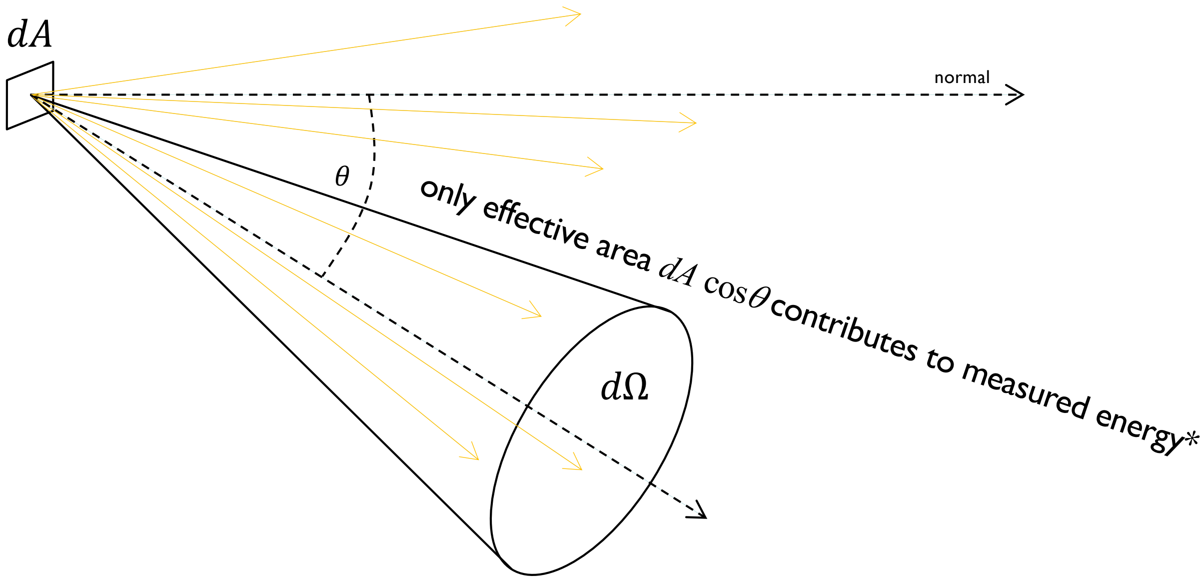
$$dE = I_\nu(\Omega)\, d\Omega\, d\nu\, dA_{eff}\, dt$$



*but we want to know the actual intensity I_ν

- radiation field – intensity
  - intensity is a measure of the energy carried by *individual* rays

$$dE = I_\nu(\Omega)\, d\Omega\, d\nu\, dA\, cos\theta\, dt$$



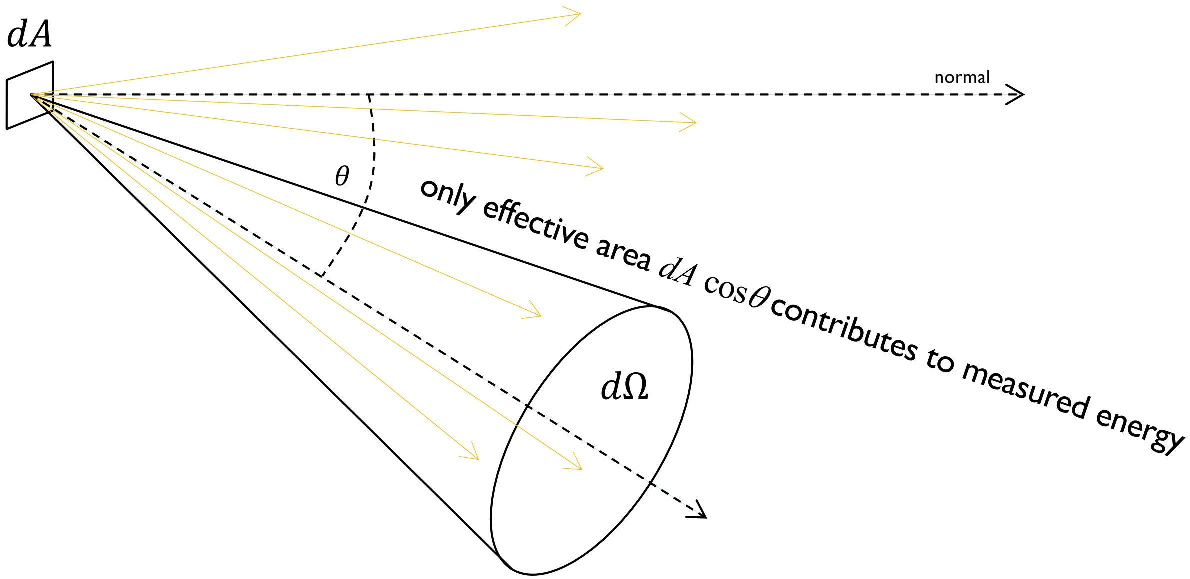
*but we want to know the actual intensity I_ν

▪ radiation field – intensity

- intensity is a measure of the energy carried by *individual* rays

$$\frac{dI_\nu(\Omega)}{ds} = 0$$ intensity is conserved* along a ray s (**exercise**)

$$dE = I_\nu(\Omega)\, d\Omega\, d\nu\, dA\, cos\theta\, dt$$



*if there are no interactions, of course

- radiation field – flux vs. intensity
  - flux $dE = F\ dA\ dt$
  - intensity $dE = I_\nu(\Omega)\ dA\ cos\theta\ dt\ d\Omega\ d\nu$

▪ radiation field – flux vs. intensity

- flux $dE = F\ dA\ dt$ → *all rays*
- intensity $dE = I_\nu(\Omega)\ dA\ cos\theta\ dt\ d\Omega\ d\nu$ → *individual ray*

▪ radiation field – flux vs. intensity

- flux $dE = F \, dA \, dt$ → *all rays*
- intensity $dE = I_\nu(\Omega) \, dA \, cos\theta \, dt \, d\Omega \, d\nu$ → *individual ray*

▪ radiation field – flux vs. intensity

- flux $dE = F\ dA\ dt$ $dF_\nu = I_\nu(\Omega)\ cos\theta\ d\Omega$, $dF = F_\nu\ d\nu$

- intensity $dE = I_\nu(\Omega)\ dA\ cos\theta\ dt\ d\Omega\ d\nu$

radiation field – flux vs. intensity

- flux $dE = F\, dA\, dt$ $F_\nu = \int I_\nu(\Omega)\, cos\theta\, d\Omega$, $F = \int F_\nu\, d\nu$
- intensity $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu$

▪ radiation field – flux vs. intensity

- flux $dE = F\, dA\, dt$ $F_\nu = \int I_\nu(\Omega)\, cos\theta\, d\Omega\ ,\ \ F = \int F_\nu\, d\nu$
- intensity $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu$

✓ *intensity defines how the source radiates*

✓ *flux depends on…*

…the intensity, and

…the apparent size of the source on the observer's sky.

- radiation field – macroscopic description
  - flux
  - intensity
  - **luminosity**
  - momentum
  - energy density
  - radiation pressure

▪ radiation field – luminosity

- flux $dE = F\ dA\ dt$
- intensity $dE = I_\nu(\Omega)\ dA\ cos\theta\ dt\ d\Omega\ d\nu$
- luminosity

▪ radiation field – luminosity

- flux $dE = F\, dA\, dt$
- intensity $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu$
- luminosity $dE = L\, dt$ *luminosity is the total amount of energy per unit time*

▪ radiation field – flux vs. luminosity

- flux $dE = F\, dA\, dt$
- intensity $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu$
- luminosity $dE = L\, dt$ *luminosity is the total amount of energy per unit time*

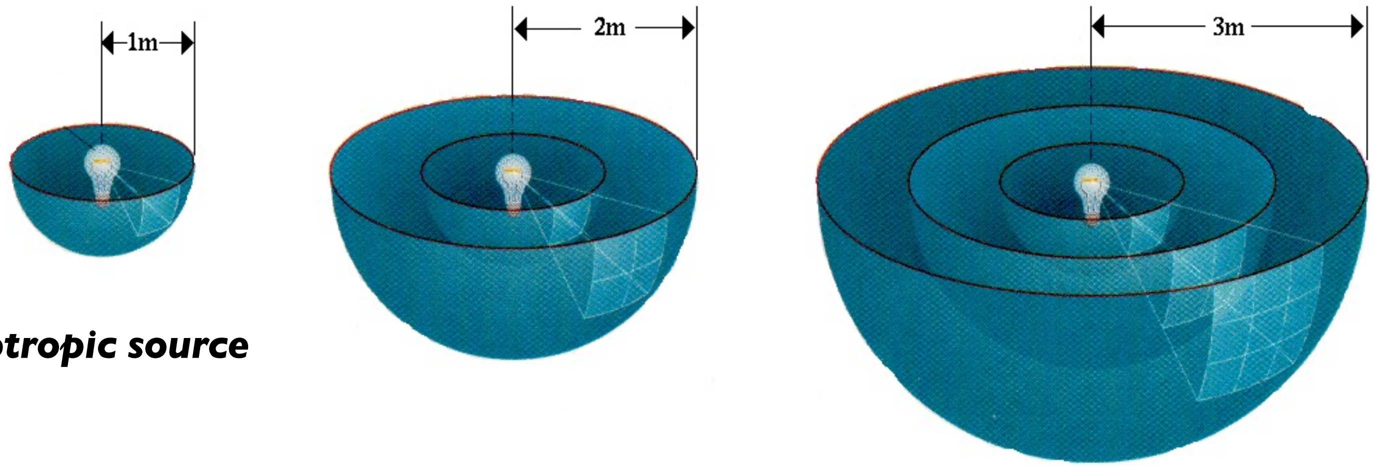
$dL = F\, dA$

- radiation field – flux vs. luminosity

  - flux $dE = F\, dA\, dt$
  - intensity $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu$
  - luminosity $dE = L\, dt$ *luminosity is the total amount of energy per unit time*

$dL = F\, dA$

$L_1 = F_1\, 4\pi\, r_1^2$ $L_2 = F_2\, 4\pi\, r_2^2$ $L_3 = F_3\, 4\pi\, r_3^2$



isotropic source

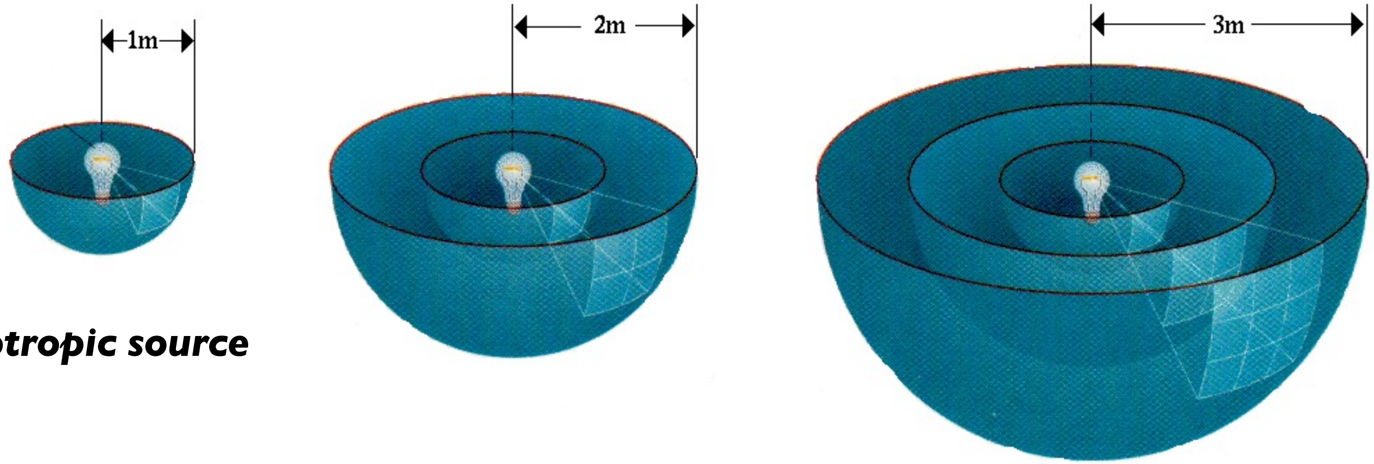
▪ radiation field – flux vs. luminosity

- flux $dE = F\, dA\, dt$
- intensity $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu$
- luminosity $dE = L\, dt$ *luminosity is the total amount of energy per unit time*

$dL = F\, dA$

energy conservation: $L_1 = L_2 = L_3$

$L_1 = F_1\, 4\pi\, r_1^2$ $L_2 = F_2\, 4\pi\, r_2^2$ $L_3 = F_3\, 4\pi\, r_3^2$



isotropic source

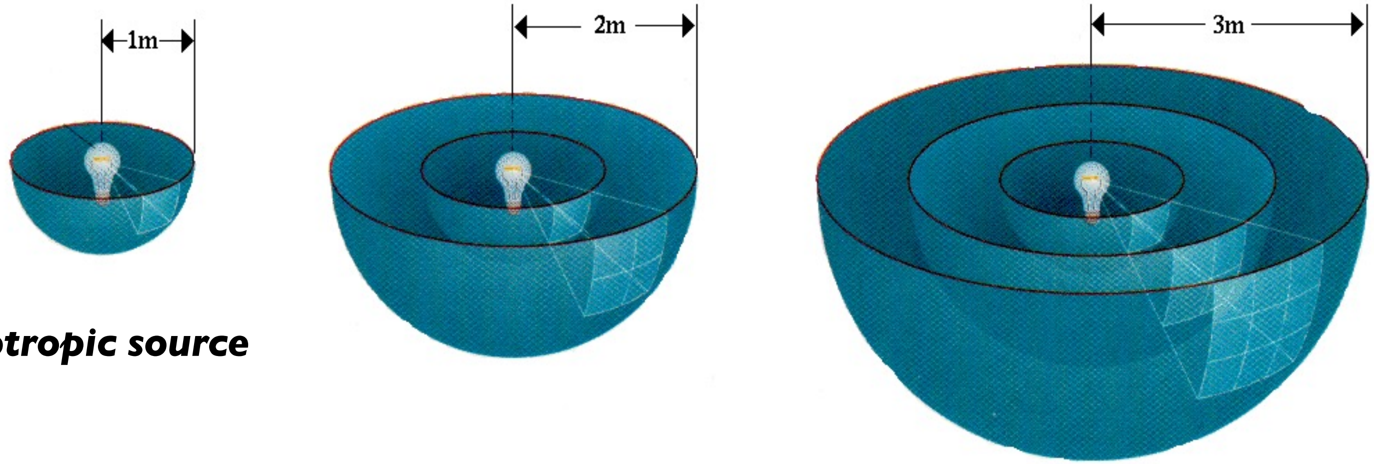
▪ radiation field – flux vs. luminosity

- flux $dE = F\ dA\ dt$
- intensity $dE = I_\nu(\Omega)\ dA\ cos\theta\ dt\ d\Omega\ d\nu$
- luminosity $dE = L\ dt$ *luminosity is the total amount of energy per unit time*

$dL = F\ dA$

energy conservation: $L_1 = L_2 = L_3$ ⟶ $\boxed{F(r) = \frac{L}{4\pi r^2}}$ *(isotropic source)*

$L_1 = F_1\ 4\pi\ r_1^2$ $L_2 = F_2\ 4\pi\ r_2^2$ $L_3 = F_3\ 4\pi\ r_3^2$

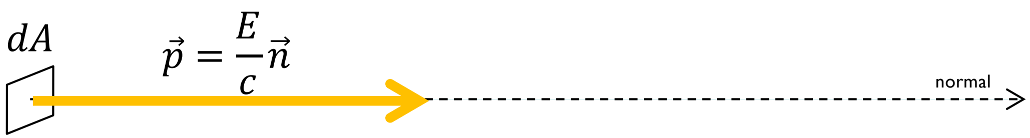


isotropic source

- radiation field – macroscopic description
  - flux
  - intensity
  - luminosity
  - **momentum**
  - energy density
  - radiation pressure

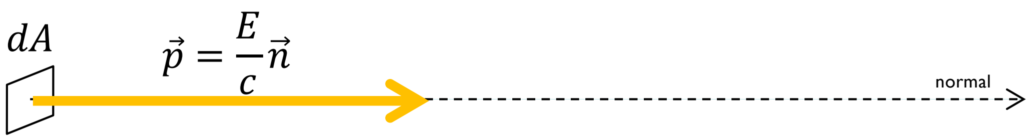
▪ radiation field – momentum

- photons also carry a momentum $\vec{p} = \frac{E}{c}\vec{n}$



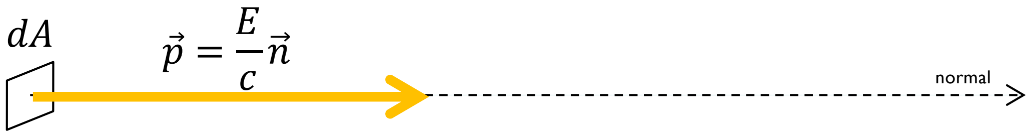
▪ radiation field – momentum

- photons also carry a momentum $\vec{p} = \frac{E}{c}\vec{n}$
- energy $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu$



▪ radiation field – momentum

- photons also carry a momentum $\vec{p} = \frac{E}{c}\vec{n}$
- energy $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu \quad \times \frac{\vec{n}}{c}$

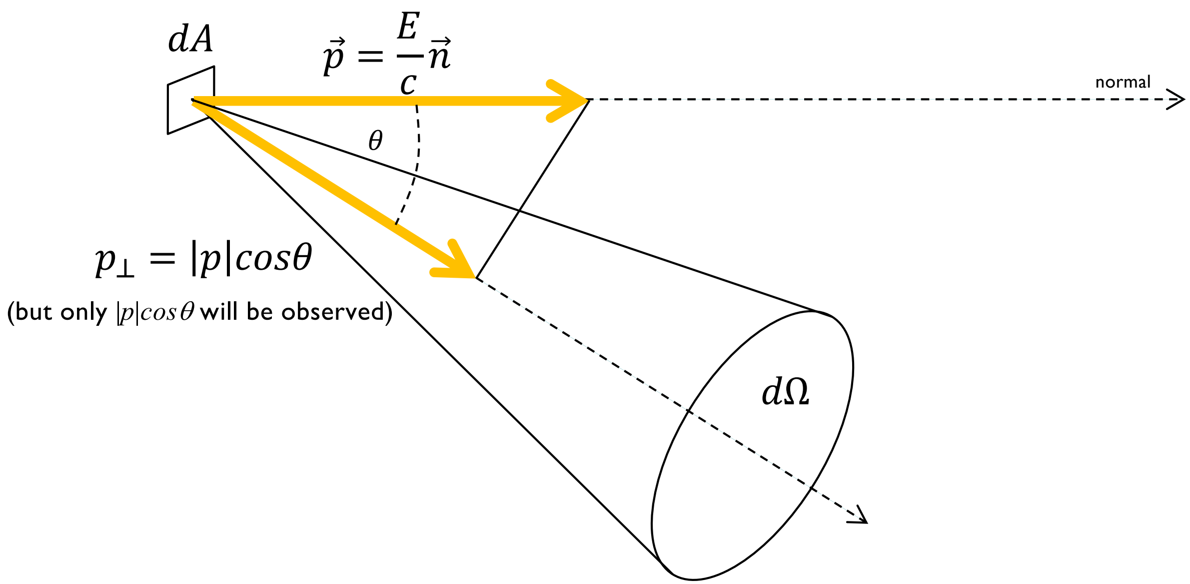


▪ radiation field – momentum

- photons also carry a momentum $\vec{p} = \frac{E}{c}\vec{n}$
- energy $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu$
- momentum $d\vec{p}_\nu = \frac{I_\nu(\Omega)}{c}\vec{n}\, dA\, cos\theta\, dt\, d\Omega\, d\nu$

▪ radiation field – momentum

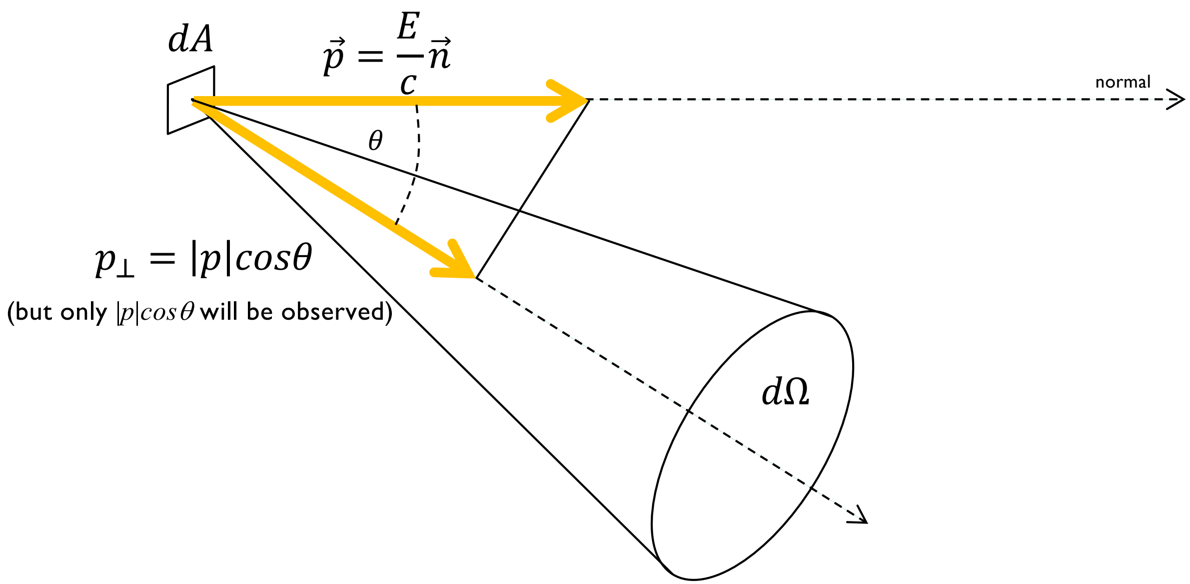
- photons also carry a momentum $\vec{p} = \frac{E}{c}\vec{n}$
- energy $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu$
- momentum $dp_\nu = \frac{I_\nu(\Omega)}{c}\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$



▪ radiation field – momentum

- photons also carry a momentum $\vec{p} = \frac{E}{c}\vec{n}$
- energy $dE = I_\nu(\Omega)\, dA\, cos\theta\, dt\, d\Omega\, d\nu$
- momentum $dp_\nu = \frac{I_\nu(\Omega)}{c} dA\, cos^2\theta\, dt\, d\Omega\, d\nu$

energy density u_ν...



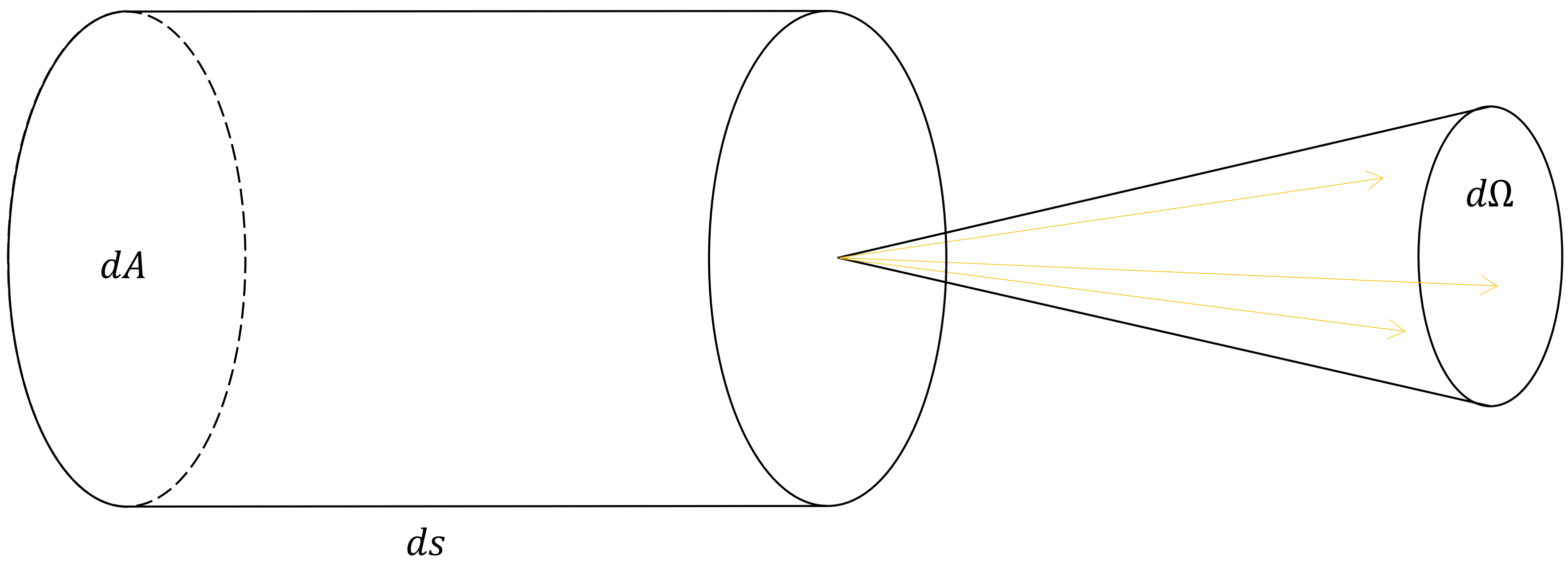
- radiation field – macroscopic description
  - flux
  - intensity
  - luminosity
  - momentum
  - **energy density**
  - radiation pressure

- radiation field – energy density

- radiation field – energy ***density***
  - how much energy escapes $dV = dA\, ds$ after time dt?

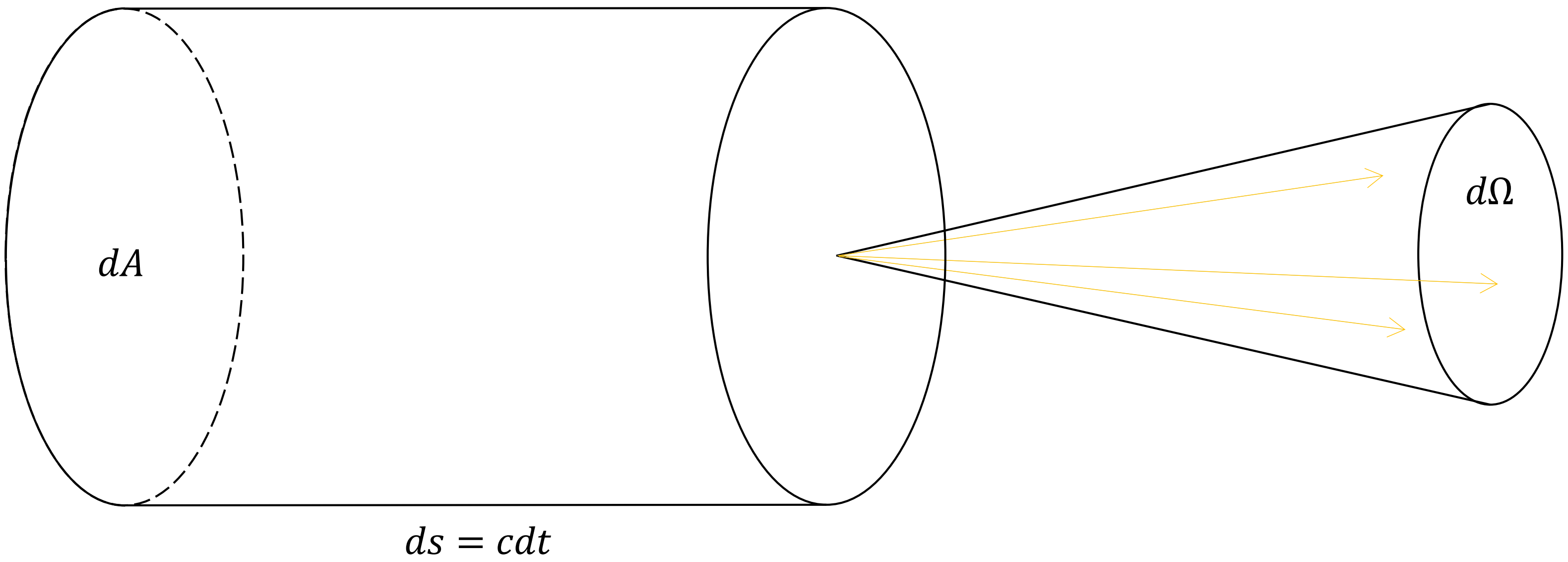
▪ radiation field – energy density

- how much energy escapes $dV = dA\ ds$ after time dt?



- radiation field – energy density

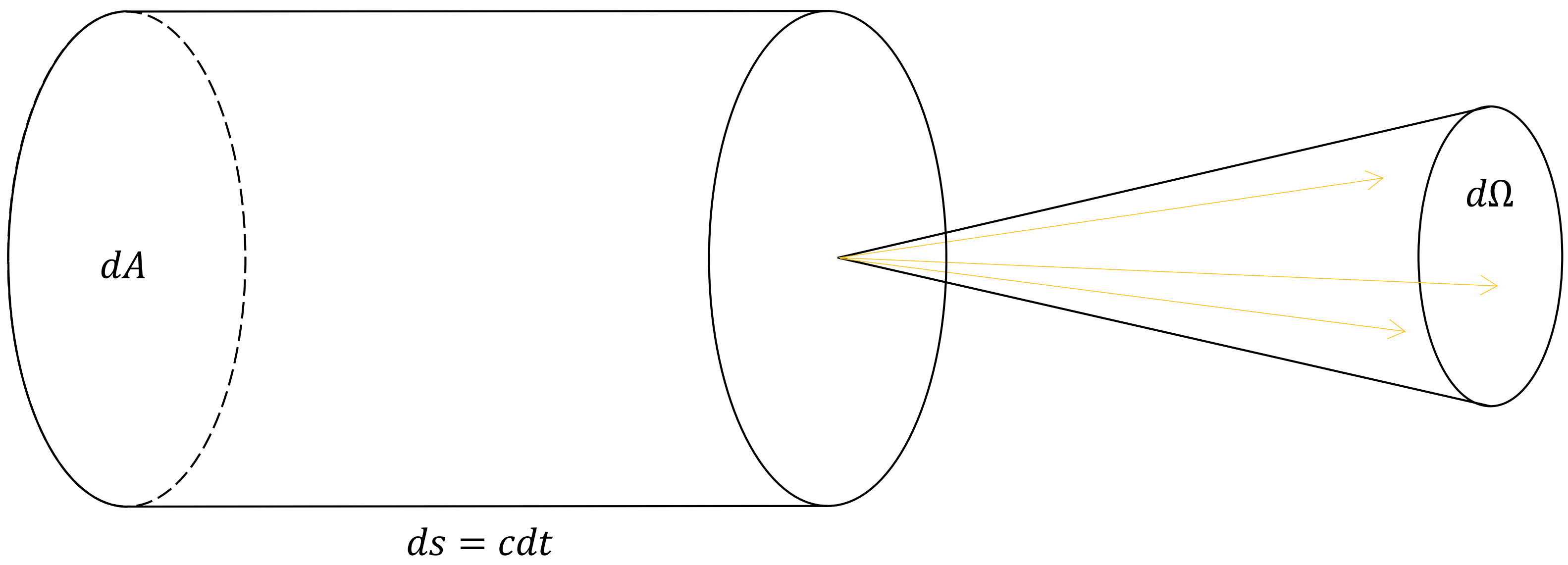
  - how much energy escapes $dV = dA\, ds = dA\, cdt$ after time dt?



▪ radiation field – energy density

- how much energy escapes $dV = dA\, ds = dA\, cdt$ after time dt?

$$dE = u_\nu(\Omega)\; dA\; cdt\; d\Omega\; d\nu \qquad , u_\nu(\Omega)\text{: energy density}$$

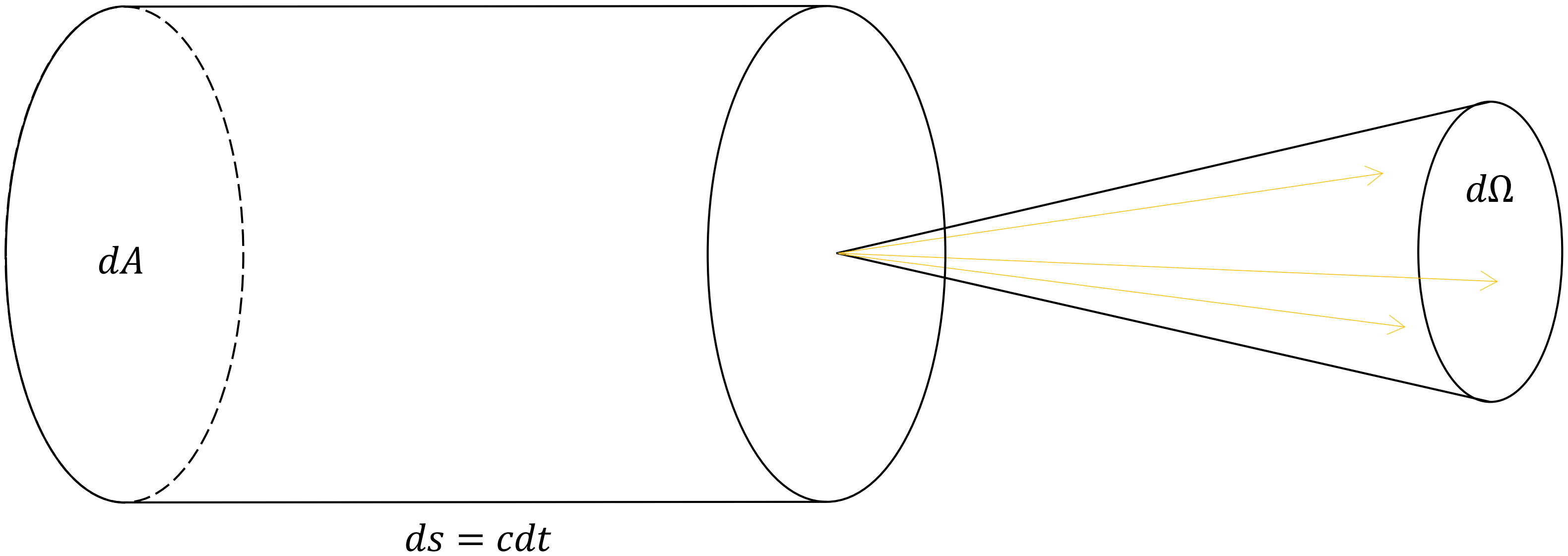


▪ radiation field – energy density

- how much energy escapes $dV = dA\, ds = dA\, cdt$ after time dt?

$$dE = u_\nu(\Omega)\, dA\, cdt\, d\Omega\, d\nu \qquad , u_\nu(\Omega)\text{: energy density}$$

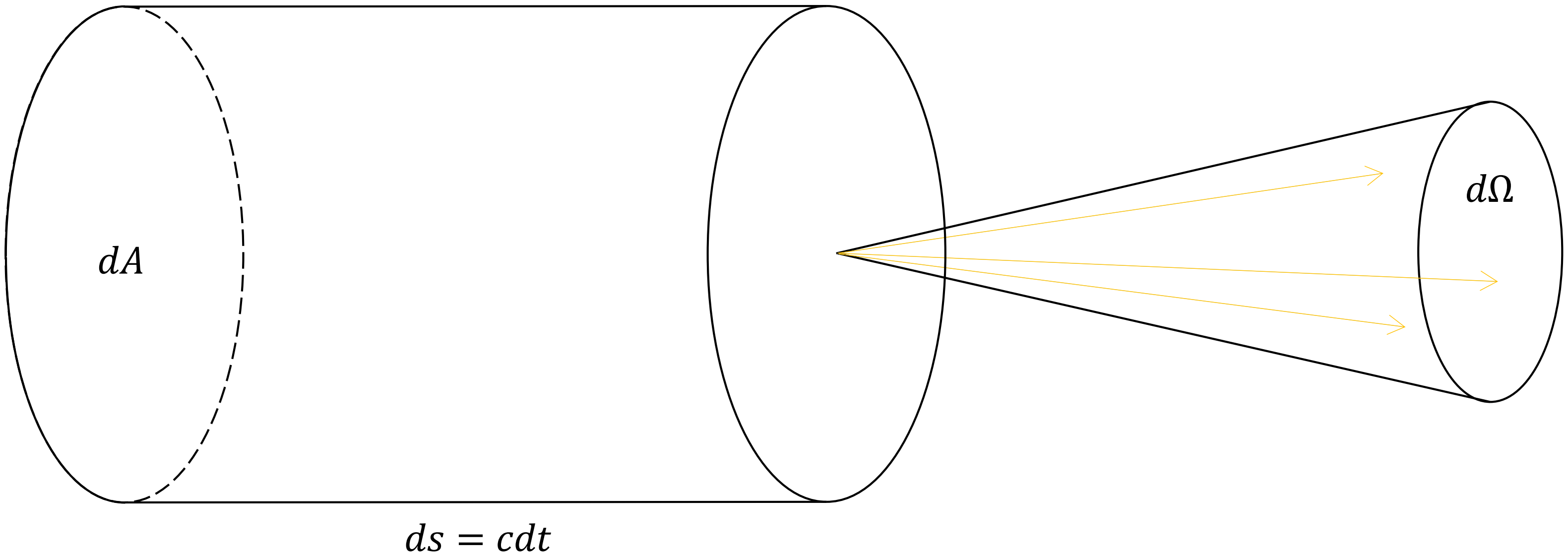
$$dE = I_\nu(\Omega)\, dA\, dt\, d\Omega\, d\nu$$



▪ radiation field – energy density

- how much energy escapes $dV = dA\,ds = dA\,cdt$ after time dt?

$$u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c} \quad \begin{cases} dE = u_\nu(\Omega)\, dA\, cdt\, d\Omega\, d\nu & , u_\nu(\Omega)\text{: energy density} \\ dE = I_\nu(\Omega)\, dA\, dt\, d\Omega\, d\nu \end{cases}$$



▪ radiation field – energy density

- energy density $u_\nu(\Omega) = \dfrac{I_\nu(\Omega)}{c}$

▪ radiation field – energy density

- energy density $$u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$$

$$u_\nu = \int \frac{I_\nu(\Omega)}{c} d\Omega = \frac{4\pi}{c} J_\nu$$, J_ν: mean intensity

▪ radiation field – energy density

- energy density $$u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$$

$$u_\nu = \int \frac{I_\nu(\Omega)}{c} d\Omega = \frac{4\pi}{c} J_\nu$$, J_ν: mean intensity

- isotropic radiation $I_\nu = J_\nu$ $$u_\nu = \frac{4\pi}{c} I_\nu$$

- radiation field – macroscopic description
  - flux
  - intensity
  - luminosity
  - momentum flux
  - energy density
  - **radiation pressure**

- radiation field – radiation pressure

relation between radiation pressure and energy density?

▪ radiation field – radiation pressure

- energy density $u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$
- momentum $dp_\nu = \frac{I_\nu(\Omega)}{c}\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$

- radiation field – radiation pressure

  - energy density $$u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$$

  - momentum $$dp_\nu = \frac{I_\nu(\Omega)}{c}\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu(\Omega)\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

- radiation field – radiation pressure for isotropic radiation
  - energy density $u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$
  - momentum $dp_\nu = \frac{I_\nu(\Omega)}{c}\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$

$$= u_\nu(\Omega)\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

- radiation field – radiation pressure for isotropic radiation

  - energy density $$u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$$
  - momentum $$dp_\nu = \frac{I_\nu(\Omega)}{c}\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu(\Omega)\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$p = \int u_\nu\, d\nu \int cos^2\theta\, d\Omega\;\, dA\, dt$$

▪ radiation field – radiation pressure for isotropic radiation

- energy density $$u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$$
- momentum $$dp_\nu = \frac{I_\nu(\Omega)}{c}\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu(\Omega)\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$p = \int u_\nu\, d\nu \int cos^2\theta\, d\Omega\ dA\, dt$$

$$= \quad u \quad \int cos^2\theta\, d\Omega\ dA\, dt$$

▪ radiation field – radiation pressure for isotropic radiation

- energy density $$u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$$
- momentum $$dp_\nu = \frac{I_\nu(\Omega)}{c}\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu(\Omega)\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$p = \int u_\nu\, d\nu \int cos^2\theta\, d\Omega\;\, dA\, dt$$

$$= \quad u \quad \int cos^2\theta\, d\Omega\;\, dA\, dt$$

$$= \quad u \quad \int_0^{2\pi} d\varphi \int_0^{\pi/2} cos^2\theta\, sin\theta\, d\theta\, dA\, dt$$

- radiation field – radiation pressure for isotropic radiation

  - energy density $$u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$$

  - momentum $$dp_\nu = \frac{I_\nu(\Omega)}{c}\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu(\Omega)\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$p = \int u_\nu\, d\nu \int cos^2\theta\, d\Omega\;\, dA\, dt$$

$$= \quad u \quad \int cos^2\theta\, d\Omega\;\, dA\, dt$$

$$= \quad u \quad \int_0^{2\pi} d\varphi \int_0^{\pi/2} cos^2\theta\, sin\theta\, d\theta\, dA\, dt$$

$$= \quad u \quad \frac{1}{3} \quad dA\, dt$$

▪ radiation field – radiation pressure for isotropic radiation

- energy density $u_\nu(\Omega) = \dfrac{I_\nu(\Omega)}{c}$
- momentum $dp_\nu = \dfrac{I_\nu(\Omega)}{c}\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$

$$= u_\nu(\Omega)\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$= u_\nu\, dA\, cos^2\theta\, dt\, d\Omega\, d\nu$$

$$p = \int u_\nu\, d\nu \int cos^2\theta\, d\Omega\;\; dA\, dt$$

$$= \quad u \quad \int cos^2\theta\, d\Omega\;\; dA\, dt$$

$$= \quad u \quad \int_0^{2\pi} d\varphi \int_0^{\pi/2} cos^2\theta\, sin\theta\, d\theta\, dA\, dt$$

$$= \quad u \quad \frac{1}{3} \quad dA\, dt$$

- radiation pressure $P = \dfrac{p}{dAdt} = \dfrac{1}{3}\, u$

▪ radiation field – macroscopic description

- flux $dE = F\,dA\,dt$ $dF_\nu = I_\nu(\Omega)\,cos\theta\,d\Omega$
- intensity $dE = I_\nu(\Omega)\,dA\,cos\theta\,dt\,d\Omega\,d\nu$
- luminosity $dE = L\,dt$
- energy density $u_\nu(\Omega) = \dfrac{I_\nu(\Omega)}{c}$
- radiation pressure $P = \dfrac{1}{3}u_\nu$

- radiation field – macroscopic description

  - flux $dE = F\ dA\ dt$ $\quad dF_\nu = I_\nu(\Omega)\ cos\theta\ d\Omega$
  - intensity $dE = I_\nu(\Omega)\ dA\ cos\theta\ dt\ d\Omega\ d\nu$
  - luminosity $dE = L\ dt$
  - energy density $u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$
  - radiation pressure $P = \frac{1}{3} u_\nu$

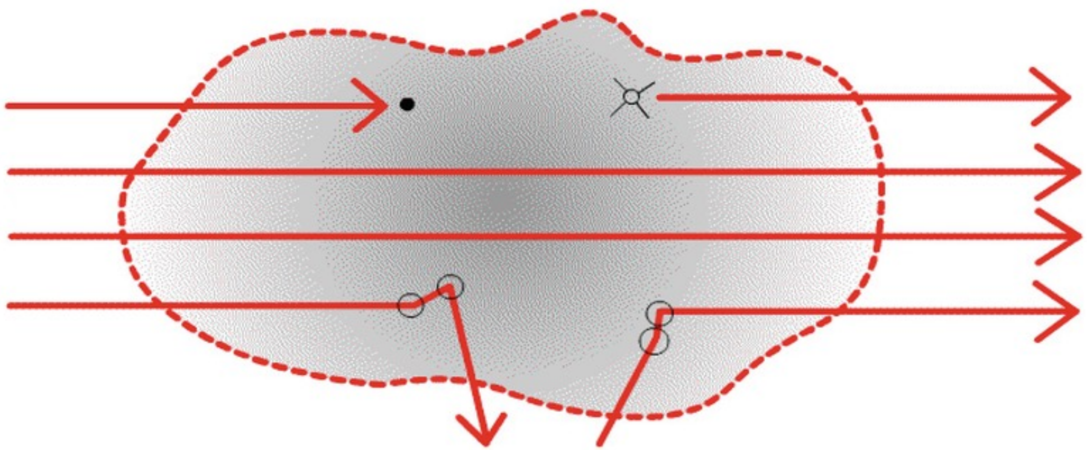
how do these quantities change along the ray?

- electromagnetic spectrum
- description of a radiation field
- **radiative transfer equation**

- vaccum: $$\frac{dI_\nu}{ds} = 0$$ intensity is conserved along a ray s

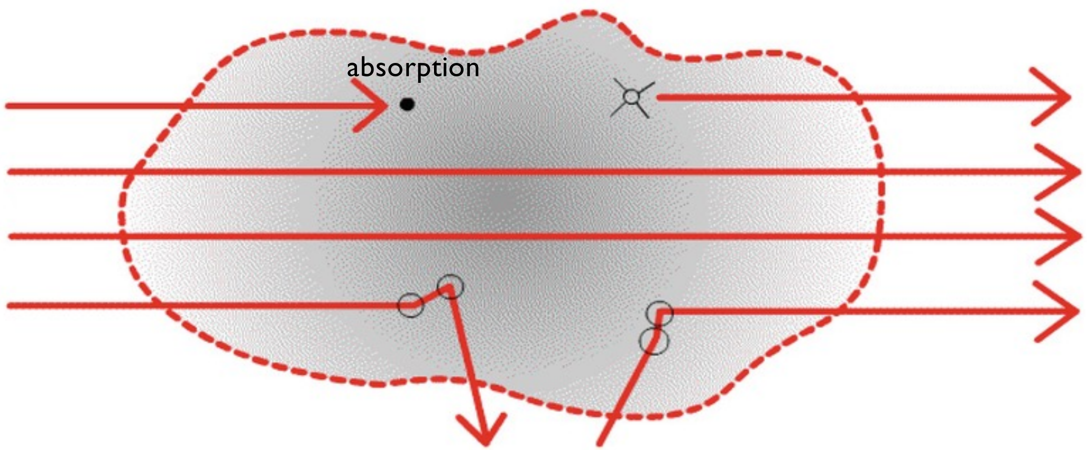
- vaccum: $\frac{dI_\nu}{ds} = 0$ intensity is conserved along a ray s

- in matter:



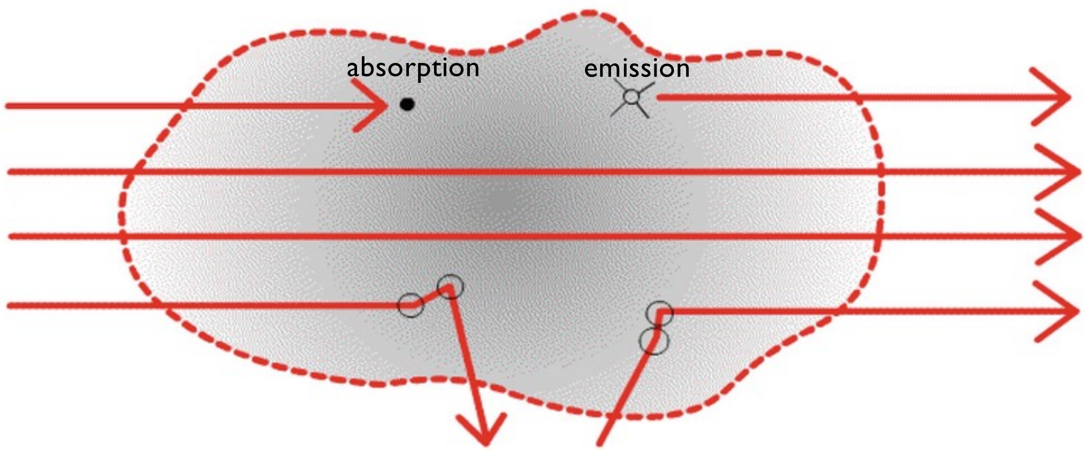
- vaccum: $\frac{dI_\nu}{ds} = 0$ intensity is conserved along a ray s

- in matter:



- vaccum: $$\frac{dI_\nu}{ds} = 0$$ intensity is conserved along a ray s

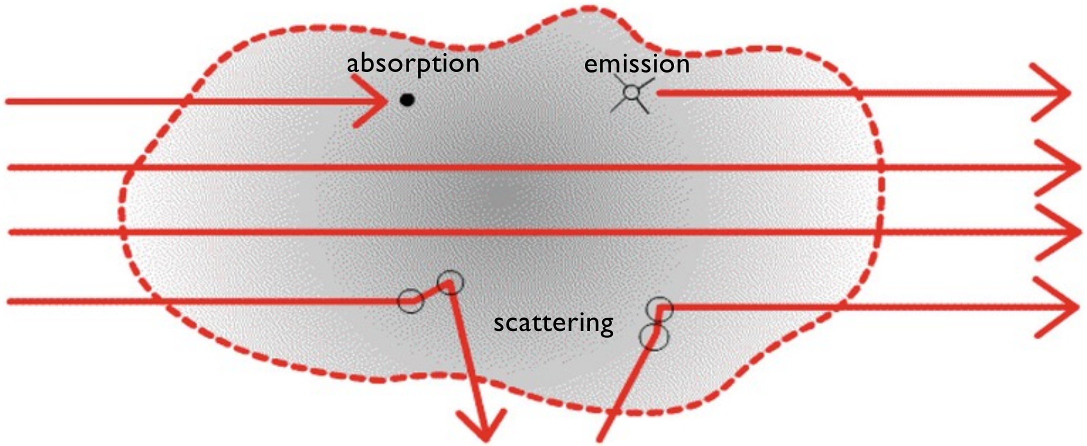
- in matter:



- vaccum: $\frac{dI_\nu}{ds} = 0$ intensity is conserved along a ray s

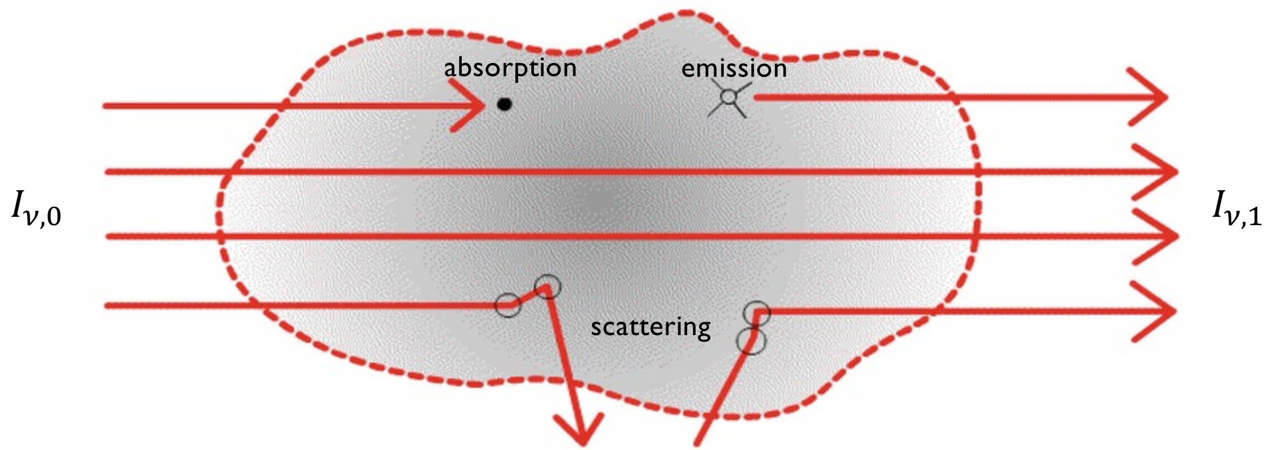
$I_{\nu,0}$  $I_{\nu,0}$

- in matter:



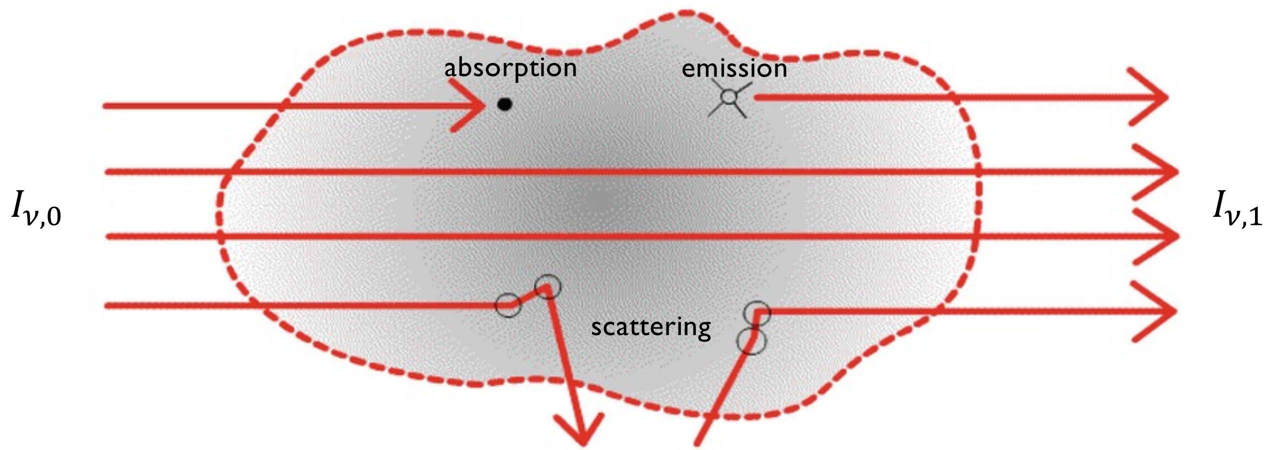
- vaccum: $\frac{dI_\nu}{ds} = 0$ intensity is conserved along a ray s

- in matter:



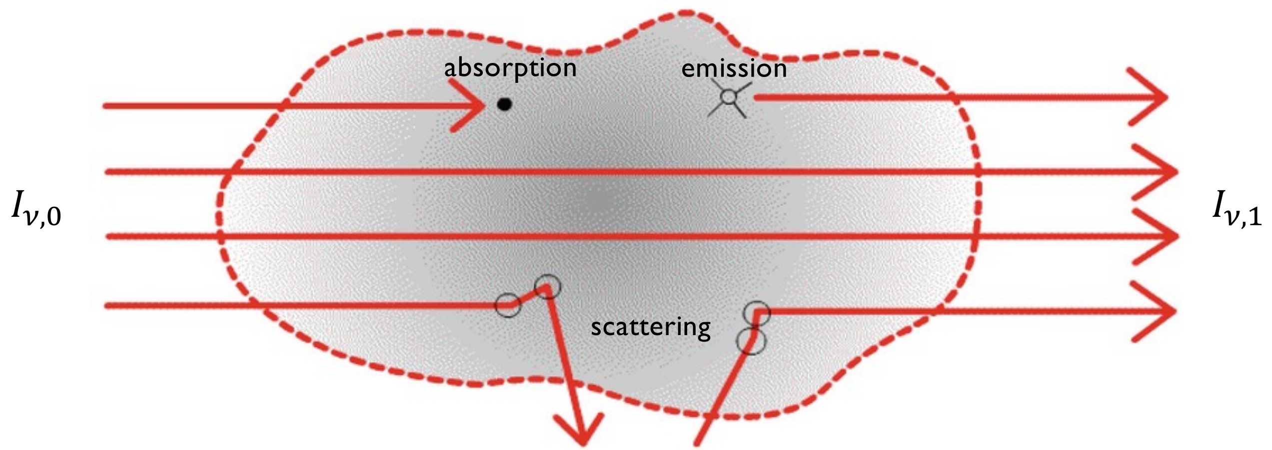
- vaccum: $\frac{dI_\nu}{ds} = 0$ intensity is conserved along a ray s

- in matter: $\frac{dI_\nu}{ds} \neq 0$



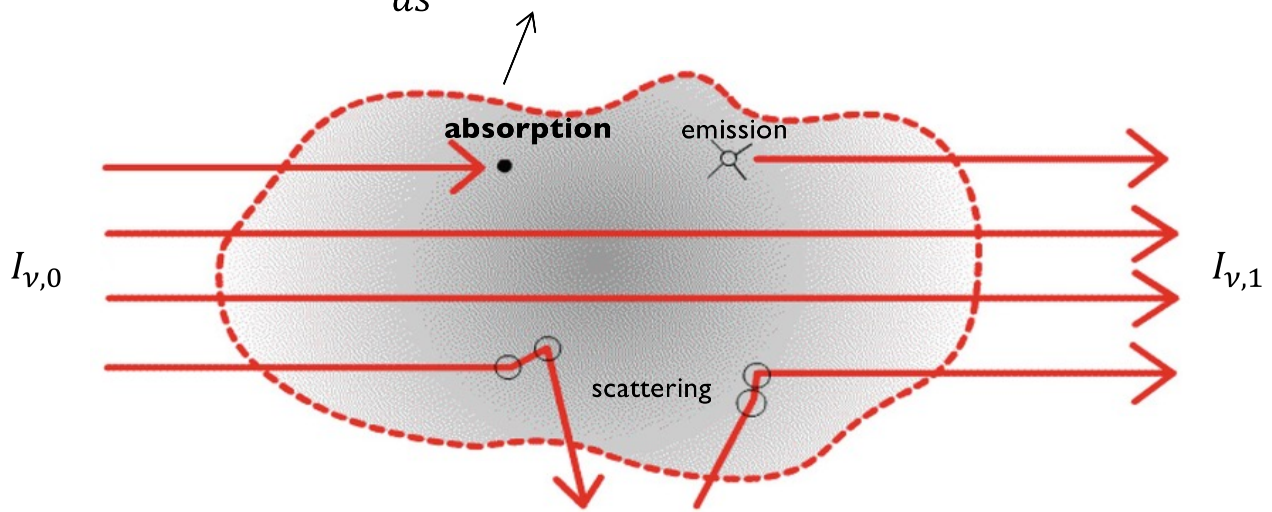
- vaccum: $\frac{dI_\nu}{ds} = 0$ intensity is conserved along a ray s

- in matter: $\frac{dI_\nu}{ds} = ?$



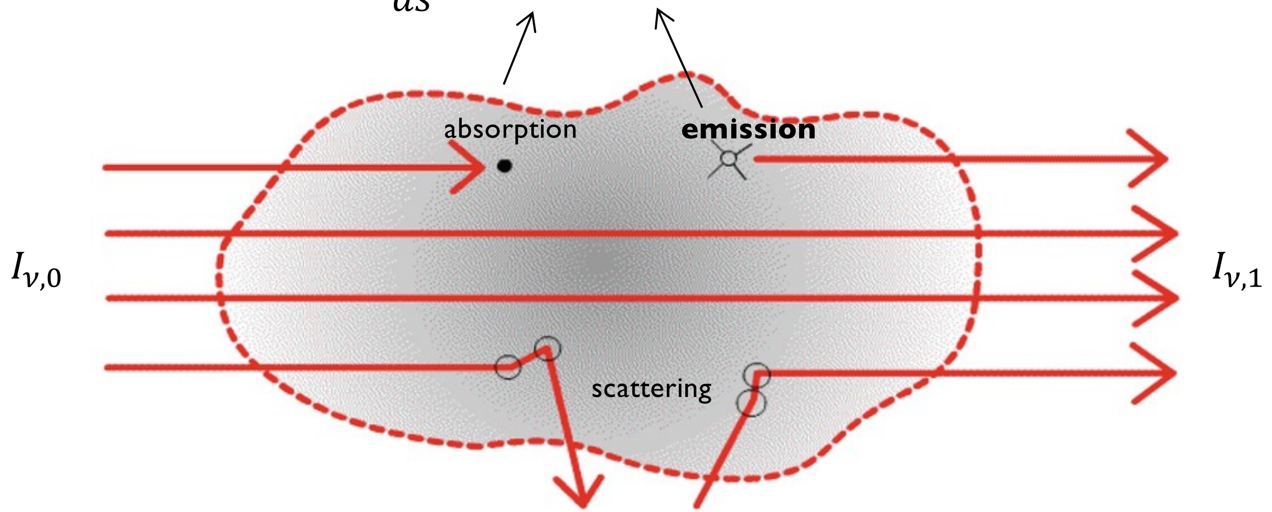
- vaccum: $\frac{dI_\nu}{ds} = 0$ intensity is conserved along a ray s

- in matter: $\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$



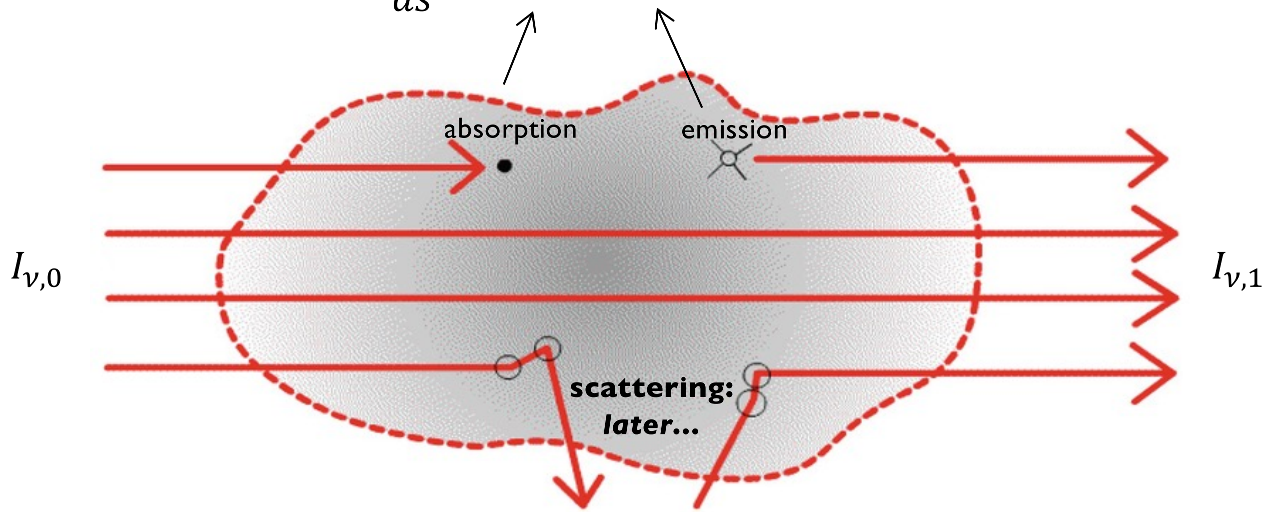
- vaccum: $\frac{dI_\nu}{ds} = 0$ intensity is conserved along a ray s

- in matter: $\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$



- vaccum: $\frac{dI_\nu}{ds} = 0$ intensity is conserved along a ray s

- in matter: $\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$



- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- is assumed that the radiation propagates like particles,
  i.e. the wave nature of radiation is neglected:

  - no refraction,
  - no diffraction,
  - no interference

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- is assumed that the radiation propagates like particles, i.e. the wave nature of radiation is neglected:
  - no refraction,
  - no diffraction,
  - no interference

→ *in astronomy, especially at radio wavelengths, this assumption is not appropriate.*

- equation of radiative transfer

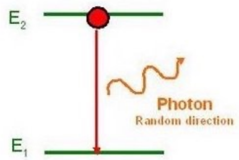
$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \boxed{+ j_\nu}$$

- emission
  - matter (atoms, molecules, etc)...
    - ...converts thermal motion into photons,
    - ...emits photons

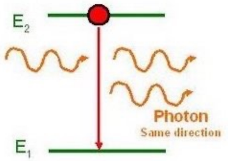
- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \boxed{+\, j_\nu}$$

- emission
  - matter (atoms, molecules, etc)...
    - ...converts thermal motion into photons,
    - ...emits photons:
      - spontaneous emission



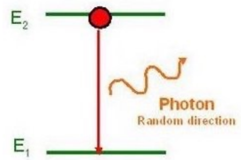
      - induced emission



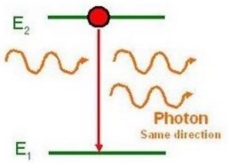
- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \boxed{+\, j_\nu}$$

- emission
  - matter (atoms, molecules, etc)...
    - ...converts thermal motion into photons,
    - ...emits photons:



- spontaneous emission = independent of radiation field

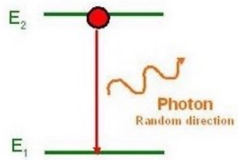


- induced emission = dependent on radiation field

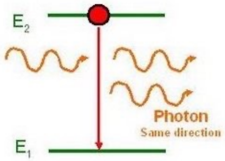
- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- emission
  - matter (atoms, molecules, etc)...
    - ...converts thermal motion into photons,
    - ...emits photons:



- *spontaneous emission* = *independent of radiation field*



- induced emission = dependent on radiation field

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \boxed{+\, j_\nu}$$

- emission – spontaneous

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \quad \xrightarrow{\text{no absorption}} \quad dI_\nu = j_\nu ds$$

- emission – spontaneous

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \boxed{+ j_\nu} \xrightarrow{\text{no absorption}} dI_\nu = j_\nu ds$$

- emission – spontaneous

radiation intensity: $dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

no absorption

$$dI_\nu = j_\nu ds$$

- emission – spontaneous

radiation intensity: $dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$

$$= j_\nu \, ds \, dA \, dt \, d\Omega \, d\nu$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

no absorption

$$dI_\nu = j_\nu ds$$

- emission – spontaneous

radiation intensity:

$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$
$$= j_\nu \, ds \, dA \, dt \, d\Omega \, d\nu$$
$$= j_\nu \, dV \, dt \, d\Omega \, d\nu$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \boxed{+ j_\nu}$$

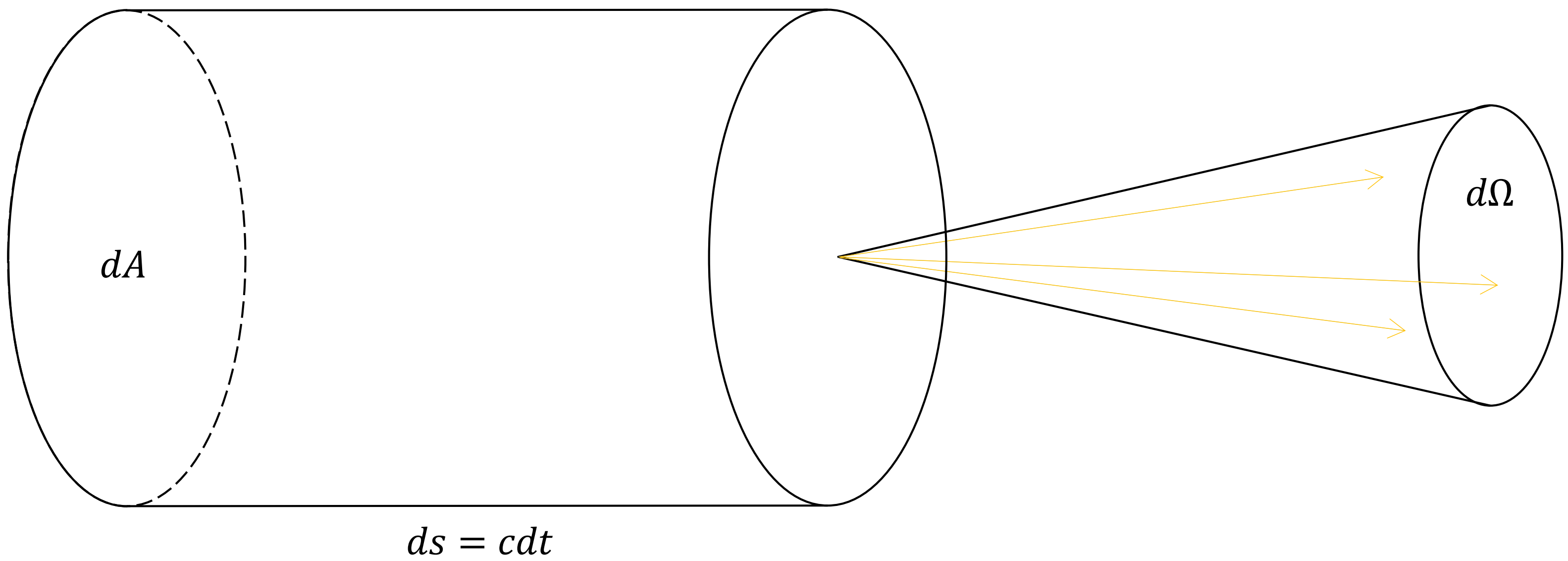
no absorption

$$dI_\nu = j_\nu ds$$

- emission – spontaneous

radiation intensity:

$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$
$$= j_\nu \, ds \, dA \, dt \, d\Omega \, d\nu$$
$$= j_\nu \, dV \, dt \, d\Omega \, d\nu$$



- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

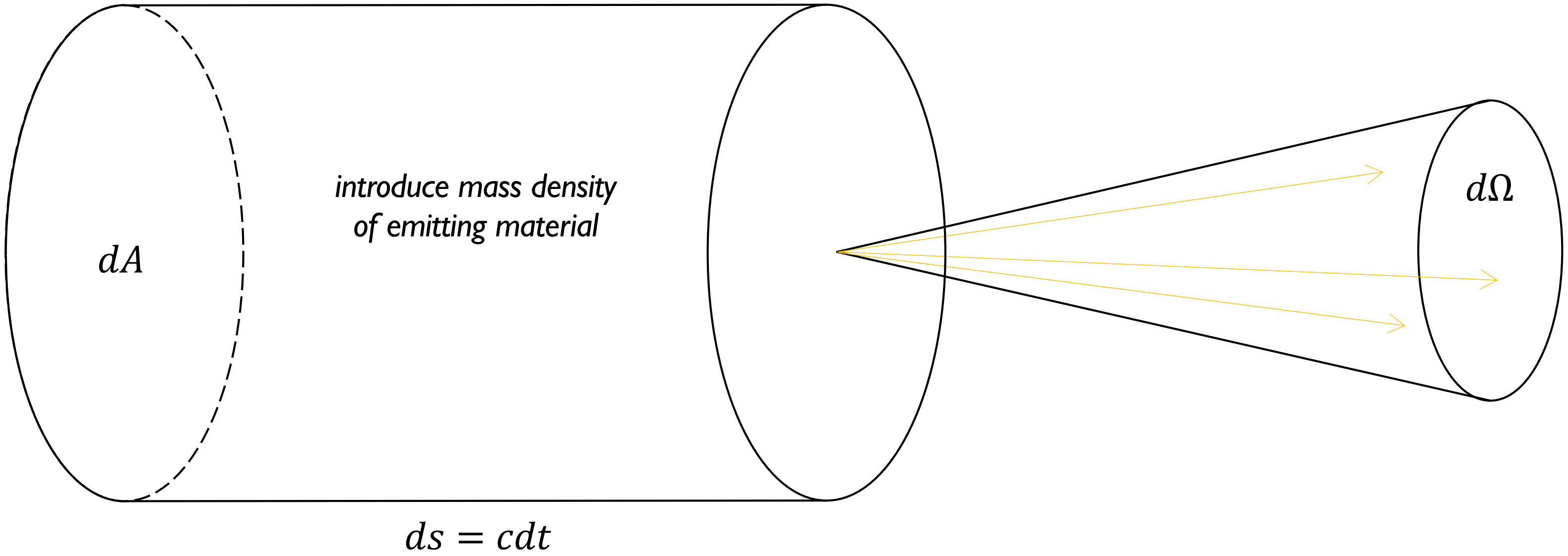
no absorption

$$dI_\nu = j_\nu ds$$

- emission – spontaneous

radiation intensity:

$$dE = I_\nu\, dA\, dt\, d\Omega\, d\nu$$
$$= j_\nu\, ds\, dA\, dt\, d\Omega\, d\nu$$
$$= j_\nu\, dV\, dt\, d\Omega\, d\nu$$



- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

no absorption

$$dI_\nu = j_\nu ds$$

- emission – spontaneous

radiation intensity:

$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$
$$= j_\nu \, ds \, dA \, dt \, d\Omega \, d\nu$$
$$= j_\nu \, dV \, dt \, d\Omega \, d\nu$$

$$dE = \epsilon_\nu \, \rho \, dV \, dt \, d\nu \, d\Omega$$

ϵ_ν: emissivity

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

no absorption → $dI_\nu = j_\nu ds$

- emission – spontaneous

radiation intensity: $dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$

$$= j_\nu \, ds \, dA \, dt \, d\Omega \, d\nu$$

$$= j_\nu \, dV \, dt \, d\Omega \, d\nu$$

$$dE = \epsilon_\nu \, \rho \, dV \, dt \, d\nu \, d\Omega$$

ϵ_ν: emissivity

ρ: mass density of emitting material

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

no absorption

$$dI_\nu = j_\nu ds$$

- emission – spontaneous

radiation intensity:

$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$
$$= j_\nu \, ds \, dA \, dt \, d\Omega \, d\nu$$
$$= j_\nu \, dV \, dt \, d\Omega \, d\nu$$

$$dE = \epsilon_\nu \, \rho \, dV \, dt \, d\nu \, d\Omega$$

ϵ_ν: emissivity

$$j_\nu = \epsilon_\nu \, \frac{\rho}{4\pi}$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

no absorption → $dI_\nu = j_\nu ds$

- emission – spontaneous

radiation intens

Material	Emissivity
Polished silver	0.02
Polished copper	0.03
Polished gold	0.03
Aluminum foil	0.07
Wood	0.85
Asphalt pavement	0.9
White paint	0.9
Vegetation	0.94
White paper	0.94
Water	0.95
Black paint	0.98

ϵ_ν: emissivity

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \boxed{+ j_\nu}$$

- emission
  - matter (atoms, molecules, etc)...
    - ...converts thermal motion into photons,
    - ...emits photons:
      - spontaneous emission = independent of radiation field
      - *induced emission* = *dependent on radiation field*

include in absorption!

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

- absorption
  - matter (atoms, molecules, etc) absorbs photons

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

- absorption
  - matter (atoms, molecules, etc) absorbs photons

relation of α_ν to physical properties!?

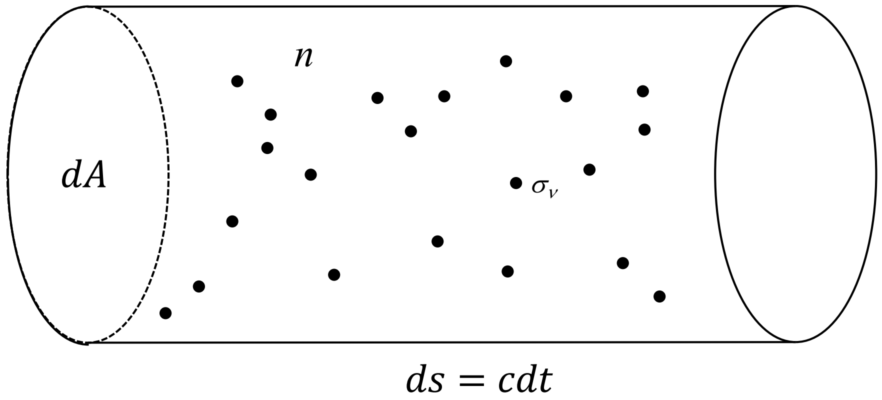
- equation of radiative transfer

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

- absorption
  - matter (atoms, molecules, etc) absorbs photons

$n,\ \rho$: number/mass density of matter
σ_ν : cross-section of individual particles

randomly distributed in tube $dA\ ds$



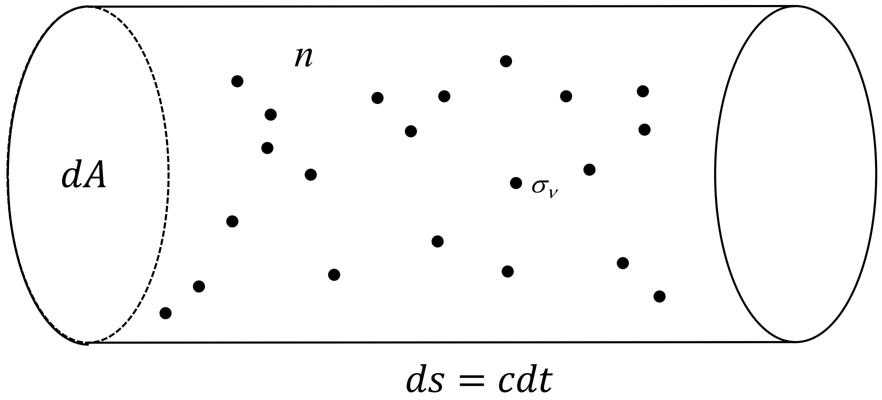
- equation of radiative transfer

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

- absorption
  - matter (atoms, molecules, etc) absorbs photons

$n,\ \rho$: number/mass density of matter
σ_ν : cross-section of individual particles

randomly distributed in tube $dA\ ds$



change in intensity: $dI_\nu = -I_\nu\, n\, \sigma_\nu\, ds$

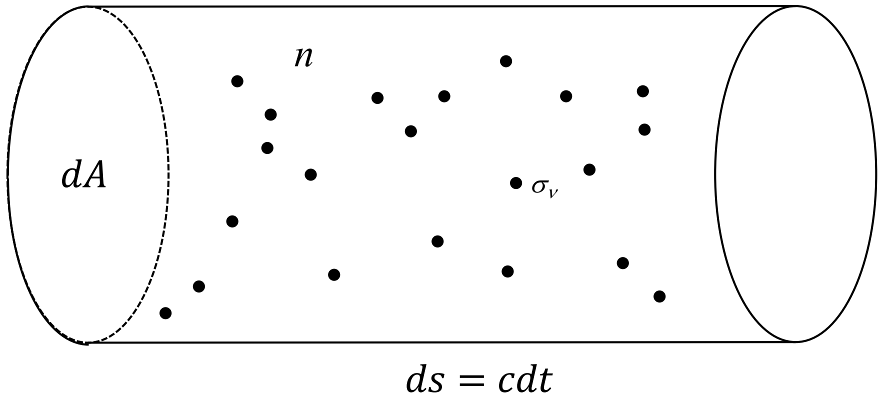
- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- absorption
  - matter (atoms, molecules, etc) absorbs photons

n, ρ : number/mass density of matter
σ_ν : cross-section of individual particles

randomly distributed in tube $dA\, ds$



no emission: $\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$

change in intensity: $dI_\nu = -I_\nu\, \underbrace{n\, \sigma_\nu}_{\alpha_\nu}\, ds$

- equation of radiative transfer

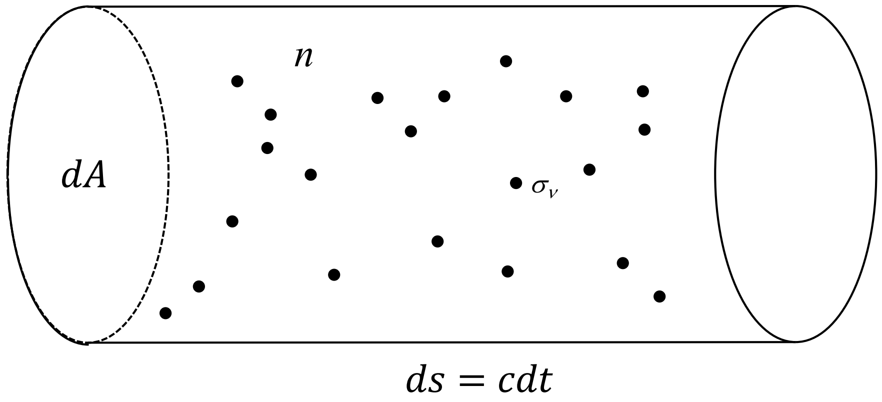
$\alpha_\nu = n\,\sigma_\nu$ (mean free path[-1])

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

- absorption
  - matter (atoms, molecules, etc) absorbs photons

n, ρ : number/mass density of matter
σ_ν : cross-section of individual particles

randomly distributed in tube $dA\ ds$



change in intensity: $dI_\nu = -I_\nu\, n\, \sigma_\nu\, ds$

- equation of radiative transfer

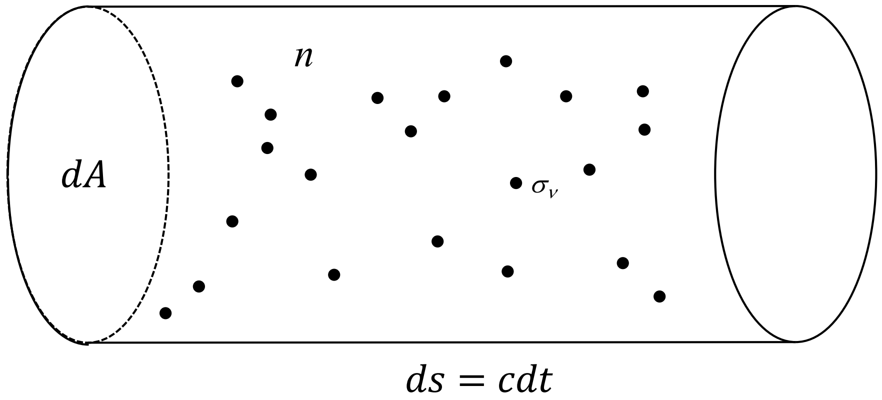
$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

$\alpha_\nu = n\,\sigma_\nu$ (mean free path^{-1}) ?

- absorption
  - matter (atoms, molecules, etc) absorbs photons

$n,\ \rho$: number/mass density of matter
σ_ν : cross-section of individual particles

randomly distributed in tube $dA\ ds$



change in intensity: $dI_\nu = -I_\nu\, n\, \sigma_\nu\, ds$

- equation of radiative transfer

$\alpha_\nu = n\,\sigma_\nu$ (mean free path^{-1})

?

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

- mean free path

$$l = \frac{\text{distance travelled}}{\text{number of collisions}}$$

- equation of radiative transfer

$$\alpha_\nu = n\,\sigma_\nu \quad \text{(mean free path}^{-1}\text{)}$$

?

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

- mean free path

$$l = \frac{\text{distance travelled}}{\text{number of collisions}} = \frac{\text{distance travelled}}{\text{number of particles per volume * volume of interaction}}$$

- equation of radiative transfer

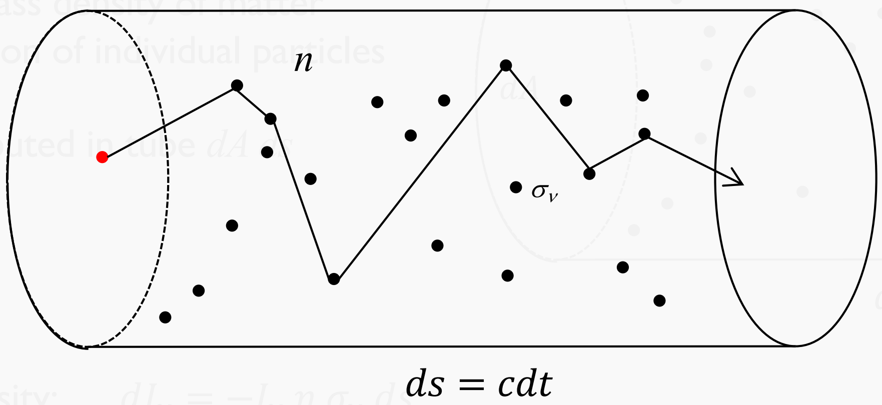
$$\alpha_\nu = n\,\sigma_\nu \quad \text{(mean free path}^{-1}\text{)}$$

?

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

- mean free path

$$l = \frac{\text{distance travelled}}{\text{number of collisions}} = \frac{\text{distance travelled}}{\text{number of particles per volume * volume of interaction}}$$



distance travelled = ds
number of particles per volume = n
volume of interaction = $\sigma\, ds$

$$l = \frac{1}{n\,\sigma}$$

- equation of radiative transfer

$$\alpha_\nu = n\,\sigma_\nu \quad \text{(mean free path}^{-1}\text{)}$$

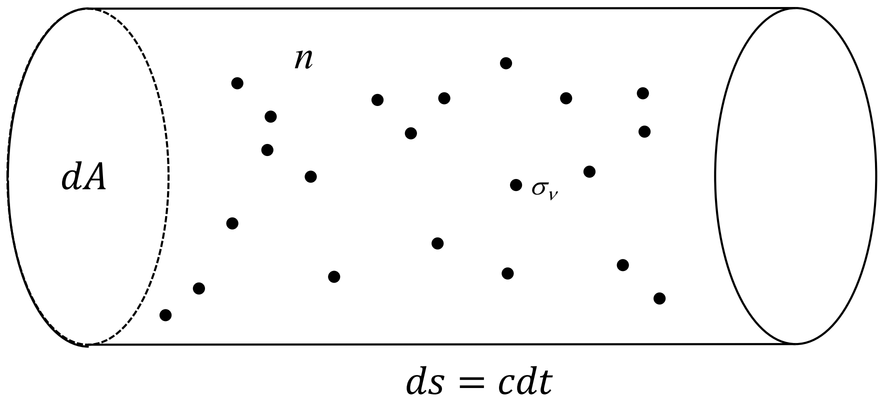
$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

- absorption

  - matter (atoms, molecules, etc) absorbs photons

$n,\ \rho$: number/mass density of matter
σ_ν : cross-section of individual particles

randomly distributed in tube $dA\ ds$



change in intensity: $dI_\nu = -I_\nu\, n\, \sigma_\nu\, ds$

$$\frac{dI_\nu}{ds} = -I_\nu\, n\, \sigma_\nu = -\alpha_\nu I_\nu$$

- equation of radiative transfer

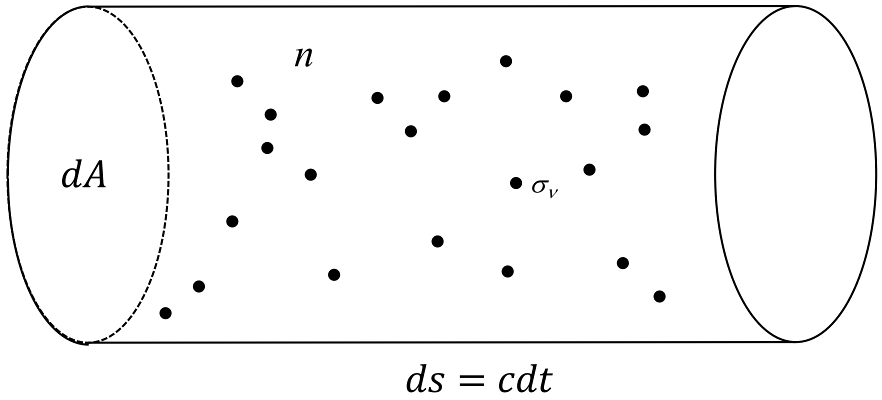
$\alpha_\nu = n\,\sigma_\nu$ (mean free path^{-1})

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- absorption
  - matter (atoms, molecules, etc) absorbs photons

n, ρ : number/**mass density** of matter
σ_ν : cross-section of individual particles

randomly distributed in tube $dA\;ds$



change in intensity: $dI_\nu = -I_\nu\, n\, \sigma_\nu\, ds$

$$\frac{dI_\nu}{ds} = -I_\nu\, n\, \sigma_\nu = -\alpha_\nu I_\nu$$

- equation of radiative transfer

$$\alpha_\nu = n\,\sigma_\nu \quad \text{(mean free path}^{-1}\text{)}$$

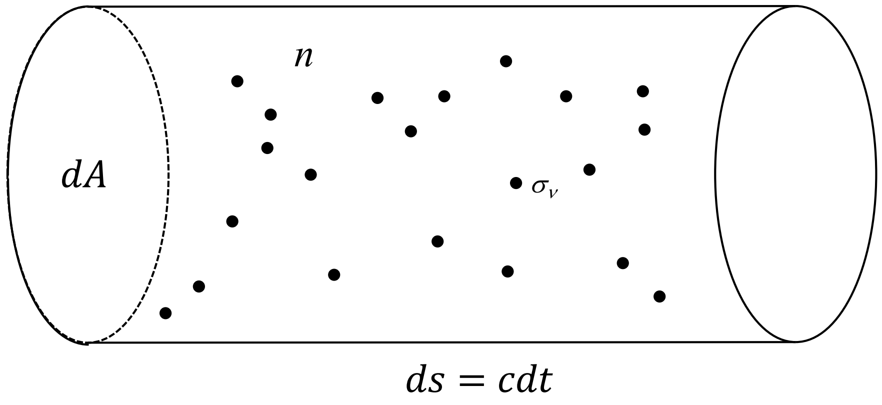
$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

- absorption

  - matter (atoms, molecules, etc) absorbs photons

n, ρ : number/**mass density** of matter
σ_ν : cross-section of individual particles

randomly distributed in tube $dA\,ds$



change in intensity: $dI_\nu = -I_\nu\, n\, \sigma_\nu\, ds$

$$\frac{dI_\nu}{ds} = -I_\nu\, n\, \sigma_\nu = -\alpha_\nu I_\nu = -I_\nu\, \rho\, \kappa_\nu$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

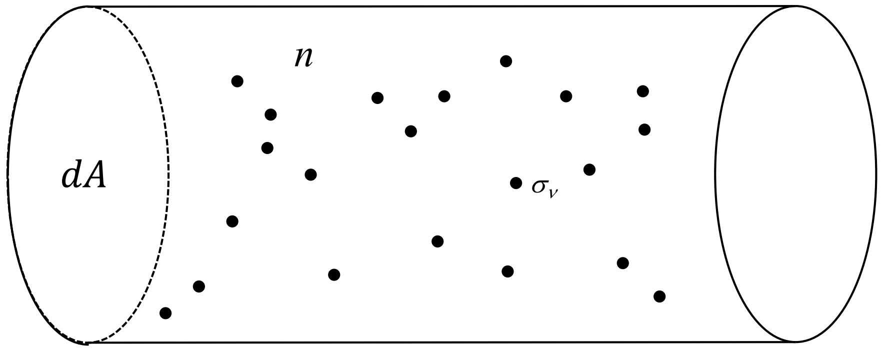
$\alpha_\nu = n\,\sigma_\nu$ (mean free path^{-1})

$\alpha_\nu = \rho\,\kappa_\nu$ κ_ν : mass absorption coefficient

- absorption
  - matter (atoms, molecules, etc) absorbs photons

$n,\ \rho$: number/mass density of matter
σ_ν : cross-section of individual particles

randomly distributed in tube $dA\, ds$



change in intensity: $dI_\nu = -I_\nu\, n\, \sigma_\nu\, ds$

$$\frac{dI_\nu}{ds} = -I_\nu\, n\, \sigma_\nu = -\alpha_\nu I_\nu = -I_\nu\, \underline{\underline{\rho\, \kappa_\nu}}$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

$\alpha_\nu = n\,\sigma_\nu$ (mean free path[-1])

$\alpha_\nu = \rho\,\kappa_\nu$ κ_ν : mass absorption coefficient

- absorption

  - matter (atoms, m

$n,\ \rho$: number/mass den

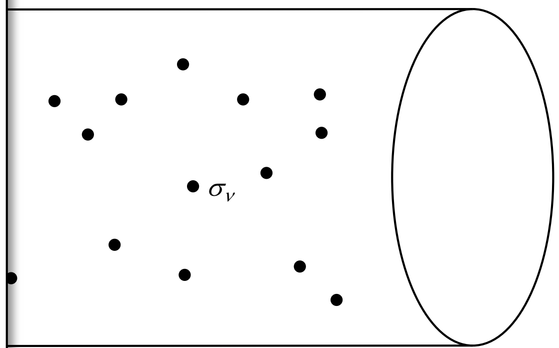
σ_ν : cross-section of i

randomly distributed i

change in intensity:

Mass absorption coefficients for x-rays of wavelength λ = 0.56, 0.71 and 1.54 Å

	Mass Absorption Coefficient (μ_m), cm²/g				Mass Absorption Coefficient (μ_m), cm²/g		
Absorber	Ag K_α 0.56 Å	Mo K_α λ = 0.71 Å	Cu K_α λ = 1.54 Å	Absorber	Ag K_α 0.56 Å	Mo K_α λ = 0.71 Å	Cu K_α λ = 1.54 Å
H	0.371	0.3727	0.435	Zr	58.5	16.10	143
Li	0.187	0.1968	0.716	Nb	61.7	16.96	153
Be	0.229	0.2451	1.50	Mo	64.8	18.44	162
B	0.279	0.3451	2.39	Pd	12.3	24.42	206
C	0.400	0.5348	5.50	Ag	13.1	26.38	218
N	0.544	0.7898	7.52	Cd	14.0	27.73	231
O	0.740	1.147	12.7	In	14.9	29.13	243
F	0.976	1.584	16.4	Sn	15.9	31.18	256
Na	1.67	2.939	30.1	Sb	16.9	33.01	270
Mg	2.12	3.979	38.6	Te	17.9	33.92	282
Al	2.65	5.043	48.6	I	19.0	36.33	294
Si	3.28	6.533	60.6	Cs	21.3	40.44	318
P	4.01	7.870	74.1	Ba	22.5	42.37	358.9
S	4.84	9.625	89.1	La	23.7	45.34	341
Cl	5.77	11.64	106	Ce	25.0	48.56	352
K	8.00	16.20	143	Pr	26.3	50.78	363
Ca	9.28	19.00	162	Nd	27.7	53.28	374
Sc	10.7	21.04	184	Sm	30.6	57.96	397
Ti	12.3	23.25	208	Gd	33.8	62.79	437
V	14.0	25.24	233	Tb	35.5	66.77	273
Cl	15.8	29.25	260	Dy	37.2	68.89	286
Mn	17.7	31.86	285	Er	40.8	75.61	134
Fe	19.7	37.74	308	Yb	44.8	80.23	146
Co	21.8	41.02	313	Hf	48.8	86.33	159
Ni	24.1	47.24	45.7	Ta	50.9	89.51	166
Cu	26.4	49.34	52.9	W	53.0	95.76	172
Zn	28.8	55.46	60.3	Re	55.2	98.74	178
Ga	31.4	56.90	67.9	Os	57.3	100.2	186
Ge	34.1	60.47	75.6	Ir	59.4	103.4	193
As	36.9	65.97	83.4	Pt	61.4	108.6	200
Se	39.8	68.82	91.4	Au	63.1	111.3	208
Rb	48.9	83	117	Hg	64.7	114.7	216
Sr	52.1	88.04	125	Pb	67.7	122.8	232
Y	55.3	97.56	134	Bi	69.1	125.9	240



$ds = c\,dt$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

$\alpha_\nu = n\,\sigma_\nu$ (mean free path^{-1})

$\alpha_\nu = \rho\,\kappa_\nu$ κ_ν : mass absorption coefficient

- absorption
  - matter (atoms, molecules, etc) absorbs photons

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

$\alpha_\nu = n\,\sigma_\nu$ (mean free path^{-1})

$\alpha_\nu = \rho\,\kappa_\nu$ κ_ν : mass absorption coefficient

- absorption
  - matter (atoms, molecules, etc) absorbs photons
    - ○ cross section must be smaller than inter-particle distance

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$\alpha_\nu = n\,\sigma_\nu$ (mean free path^{-1})

$\alpha_\nu = \rho\,\kappa_\nu$ κ_ν : mass absorption coefficient

- absorption
  - matter (atoms, molecules, etc) absorbs photons
    - cross section must be smaller than inter-particle distance
    - absorbers need to be independent and randomly distributed

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = \boxed{-\alpha_\nu I_\nu} + j_\nu$$

$\alpha_\nu = n\,\sigma_\nu$ (mean free path^{-1})

$\alpha_\nu = \rho\,\kappa_\nu$ κ_ν : mass absorption coefficient

- absorption
  - matter (atoms, molecules, etc) absorbs photons
    - ○ cross section must be smaller than inter-particle distance
    - ○ absorbers need to be independent and randomly distributed
    - ○ α_ν can include induced emission that is also proportional to I_ν !

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

  - macroscopic formalism to solve for intensity
  - requires knowledge of α_ν and j_ν

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

  - macroscopic formalism to solve for intensity
  - requires knowledge of α_ν and j_ν
  - or equivalently mass absorption coefficient κ_ν and emissivity ϵ_ν

$$\alpha_\nu = \rho\,\kappa_\nu \qquad j_\nu = \frac{1}{4\pi}\,\rho\,\epsilon_\nu$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

  - macroscopic formalism to solve for intensity
  - requires knowledge of α_ν and j_ν
  - or equivalently mass absorption coefficient κ_ν and emissivity ϵ_ν

$$\alpha_\nu = \rho\,\kappa_\nu \qquad j_\nu = \frac{1}{4\pi}\,\rho\,\epsilon_\nu$$

  - solution to simple limiting cases:

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

  - macroscopic formalism to solve for intensity
  - requires knowledge of α_ν and j_ν
  - or equivalently mass absorption coefficient κ_ν and emissivity ϵ_ν

$$\alpha_\nu = \rho\,\kappa_\nu \qquad j_\nu = \frac{1}{4\pi}\,\rho\,\epsilon_\nu$$

  - solution to simple limiting cases:
    - emission-only $\quad \frac{dI_\nu}{ds} = j_\nu$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

  - macroscopic formalism to solve for intensity
  - requires knowledge of α_ν and j_ν
  - or equivalently mass absorption coefficient κ_ν and emissivity ϵ_ν

$$\alpha_\nu = \rho\,\kappa_\nu \qquad j_\nu = \frac{1}{4\pi}\,\rho\,\epsilon_\nu$$

  - solution to simple limiting cases:
    - emission-only $\quad \frac{dI_\nu}{ds} = j_\nu \qquad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

  - macroscopic formalism to solve for intensity
  - requires knowledge of α_ν and j_ν
  - or equivalently mass absorption coefficient κ_ν and emissivity ϵ_ν

$$\alpha_\nu = \rho\, \kappa_\nu \qquad j_\nu = \frac{1}{4\pi}\, \rho\, \epsilon_\nu$$

  - solution to simple limiting cases:
    - emission-only $\quad \frac{dI_\nu}{ds} = j_\nu \qquad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$
    - absorption-only $\quad \frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

  - macroscopic formalism to solve for intensity
  - requires knowledge of α_ν and j_ν
  - or equivalently mass absorption coefficient κ_ν and emissivity ϵ_ν

$$\alpha_\nu = \rho\,\kappa_\nu \qquad j_\nu = \frac{1}{4\pi}\,\rho\,\epsilon_\nu$$

  - solution to simple limiting cases:
    - emission-only $\quad \frac{dI_\nu}{ds} = j_\nu \quad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$
    - absorption-only $\quad \frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \quad I_\nu(s) = I_\nu(s_0)\,e^{-\int_{s_0}^{s}\alpha_\nu(s')ds'}$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

  - macroscopic formalism to solve for intensity
  - requires knowledge of α_ν and j_ν
  - or equivalently mass absorption coefficient κ_ν and emissivity ϵ_ν

$$\alpha_\nu = \rho\,\kappa_\nu \qquad j_\nu = \frac{1}{4\pi}\,\rho\,\epsilon_\nu$$

  - solution to simple limiting cases:
    - emission-only $\quad \frac{dI_\nu}{ds} = j_\nu \qquad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$
    - absorption-only $\quad \frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \qquad I_\nu(s) = I_\nu(s_0)\,e^{-\boxed{\int_{s_0}^{s}\alpha_\nu(s')ds'}}$

optical depth:

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu\, ds$$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu \, ds$$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu \, ds$$

$\rightarrow$ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

$e^{-\tau_\nu}$

probability of a photons traveling at least one optical depth before being absorbed/scattered

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu \, ds$$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\langle \tau_\nu \rangle = \int \tau_\nu \, e^{-\tau_\nu} d\tau_\nu$$

mean optical depth travelled before being absorbed/scattered

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu \, ds$$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\langle \tau_\nu \rangle = \int \tau_\nu \, e^{-\tau_\nu} d\tau_\nu = 1$$

mean optical depth travelled before being absorbed/scattered

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu\, ds$$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\langle\tau_\nu\rangle = \int \tau_\nu\, e^{-\tau_\nu} d\tau_\nu = 1$$

$\tau_\nu > 1$: optically thick medium (opaque)

$\tau_\nu < 1$: optically thin medium (transparent)

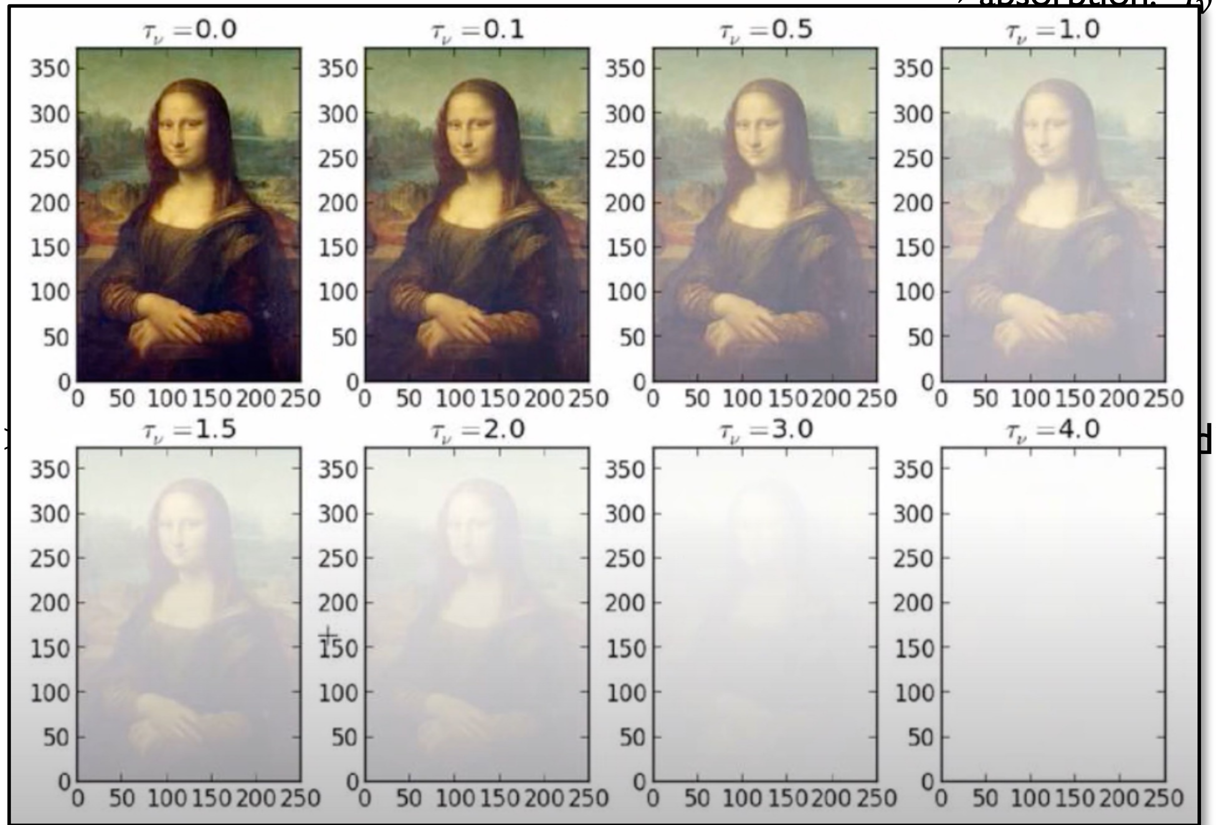
- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν $\quad d\tau_\nu = \alpha_\nu\, ds$

$\rightarrow$ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

τ_ν … dium



https://www.youtube.com/watch?v=hGqKOwBRKwA

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu \, ds$$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s') ds'$$

→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\langle \tau_\nu \rangle = \int \tau_\nu \, e^{-\tau_\nu} d\tau_\nu = 1$$

$\tau_\nu > 1$: optically thick medium (opaque)

$\tau_\nu < 1$: optically thin medium (transparent)

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu \, ds$$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

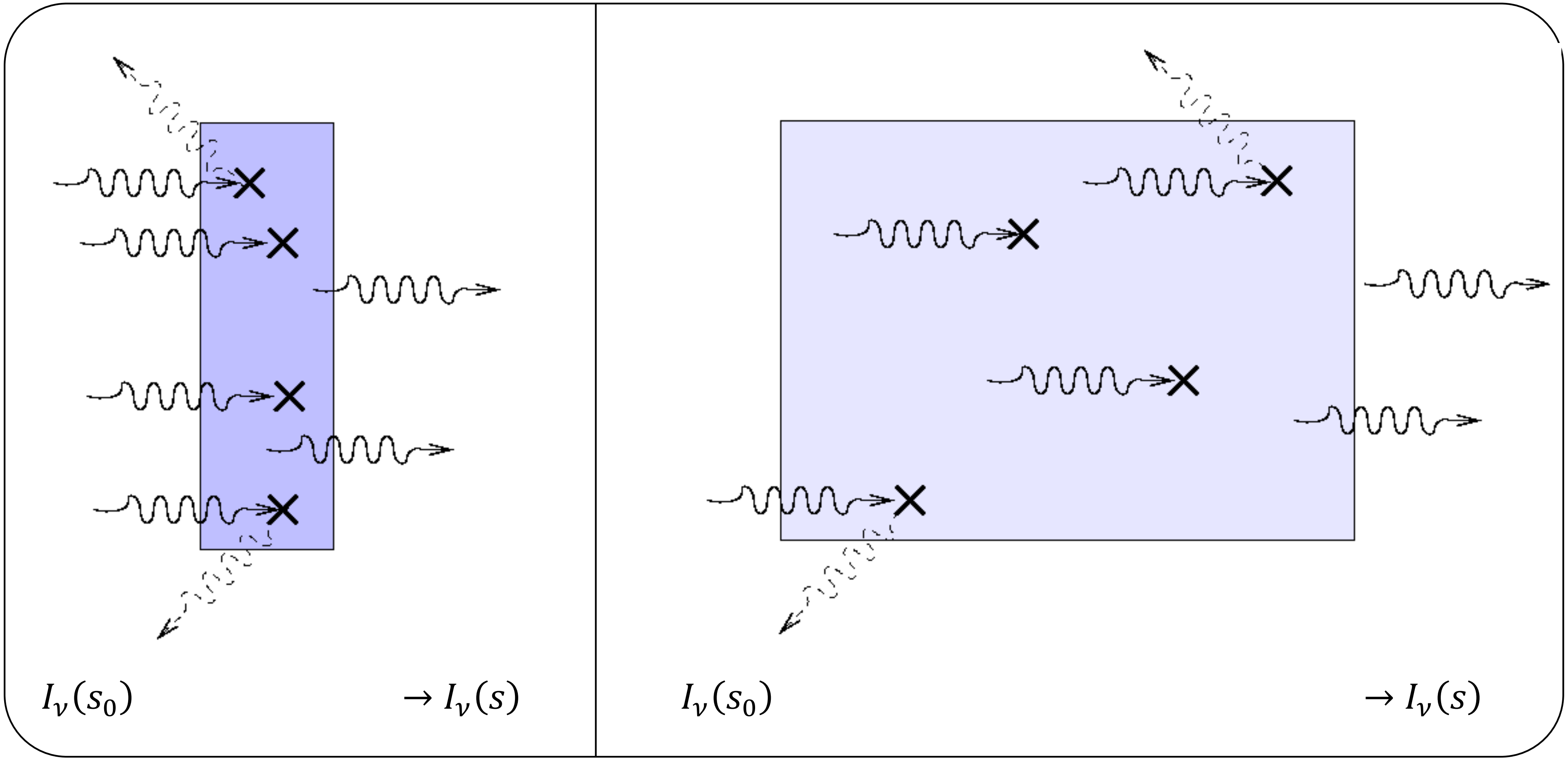
→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\langle \tau_\nu \rangle = \int \tau_\nu \, e^{-\tau_\nu} d\tau_\nu = 1$$

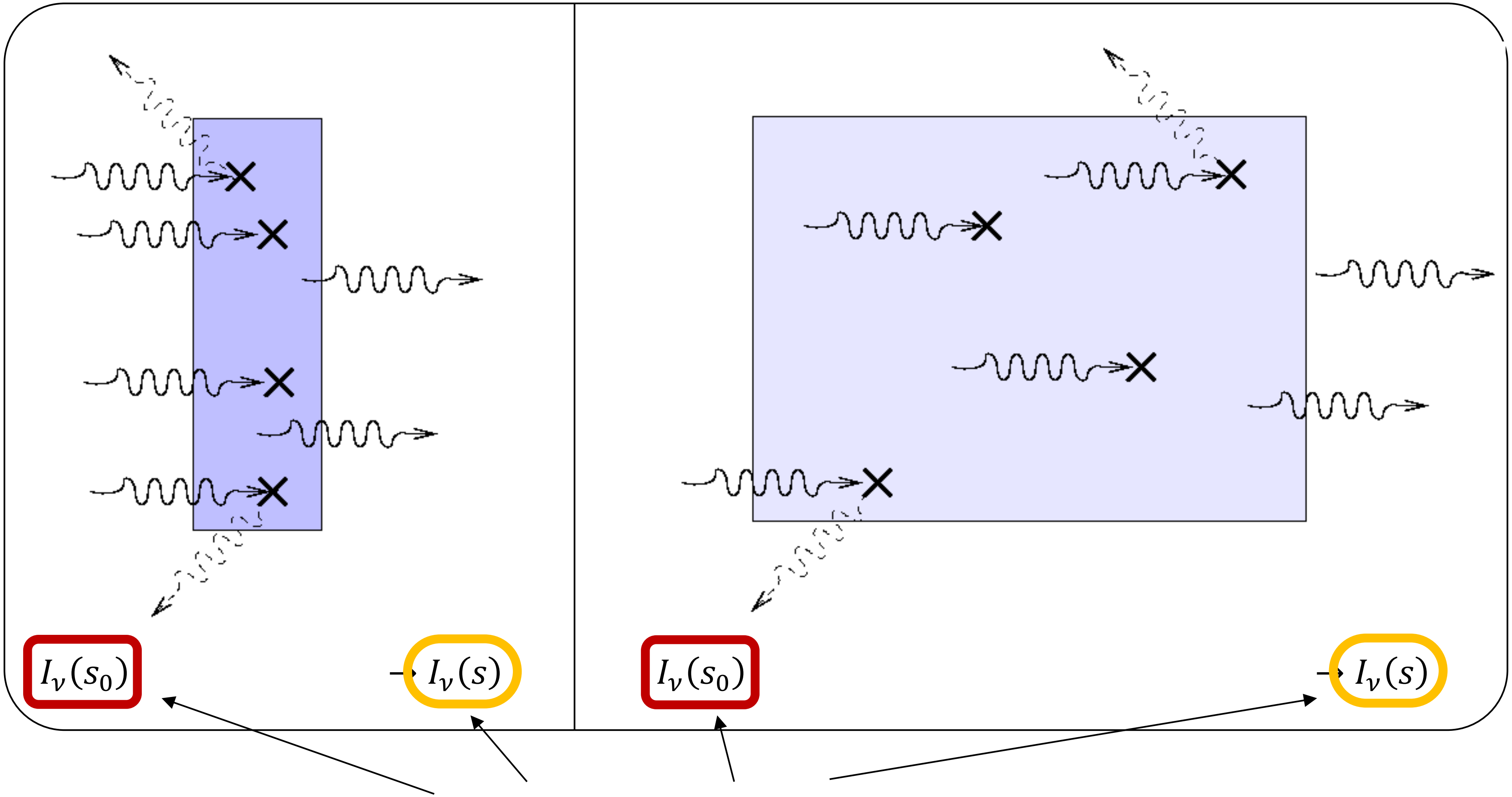
$\tau_\nu > 1$: optically thick medium (opaque)

$\tau_\nu < 1$: optically thin medium (transparent)

why do we prefer to use optical depth?

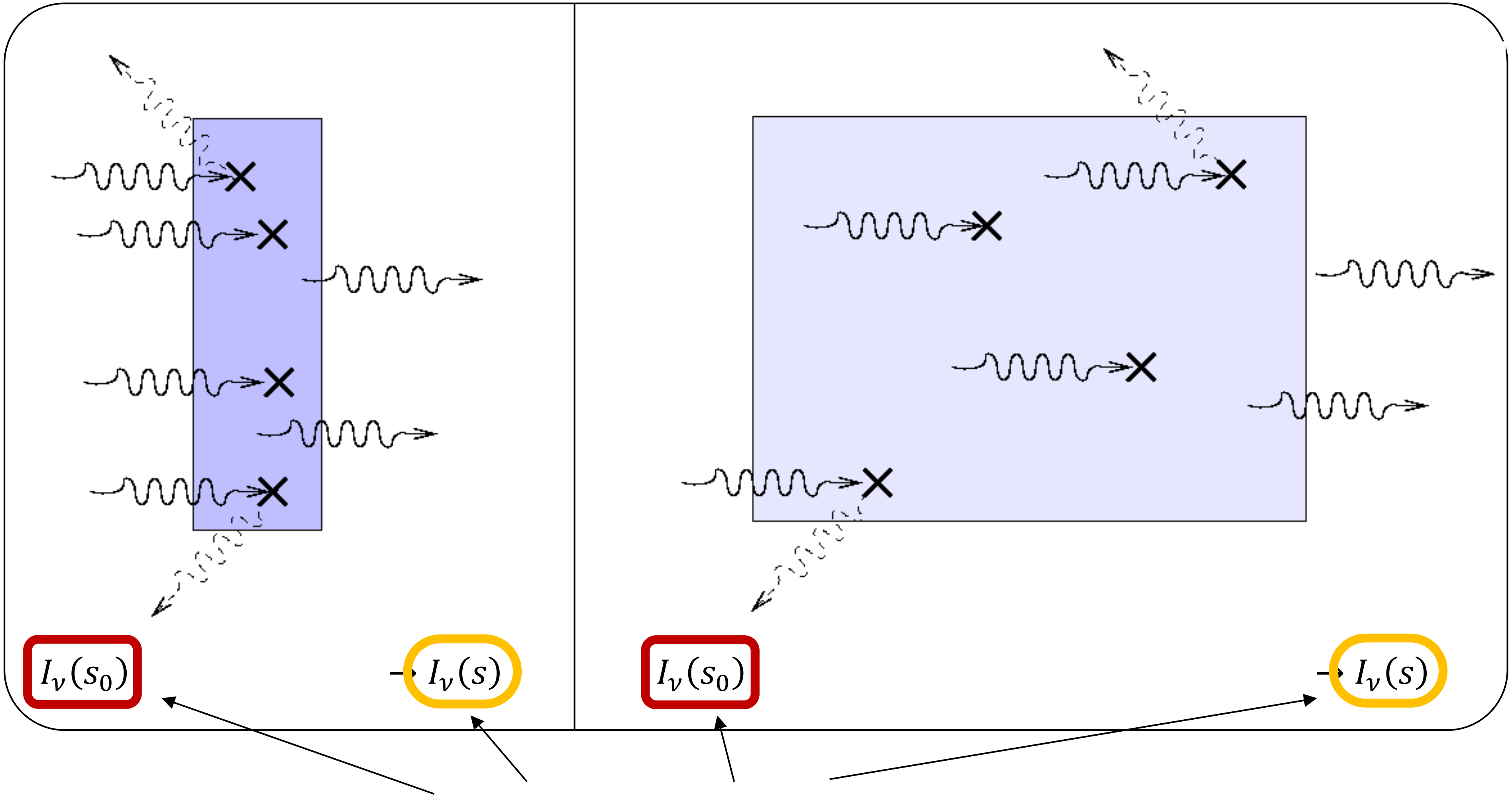


why do we prefer to use optical depth?



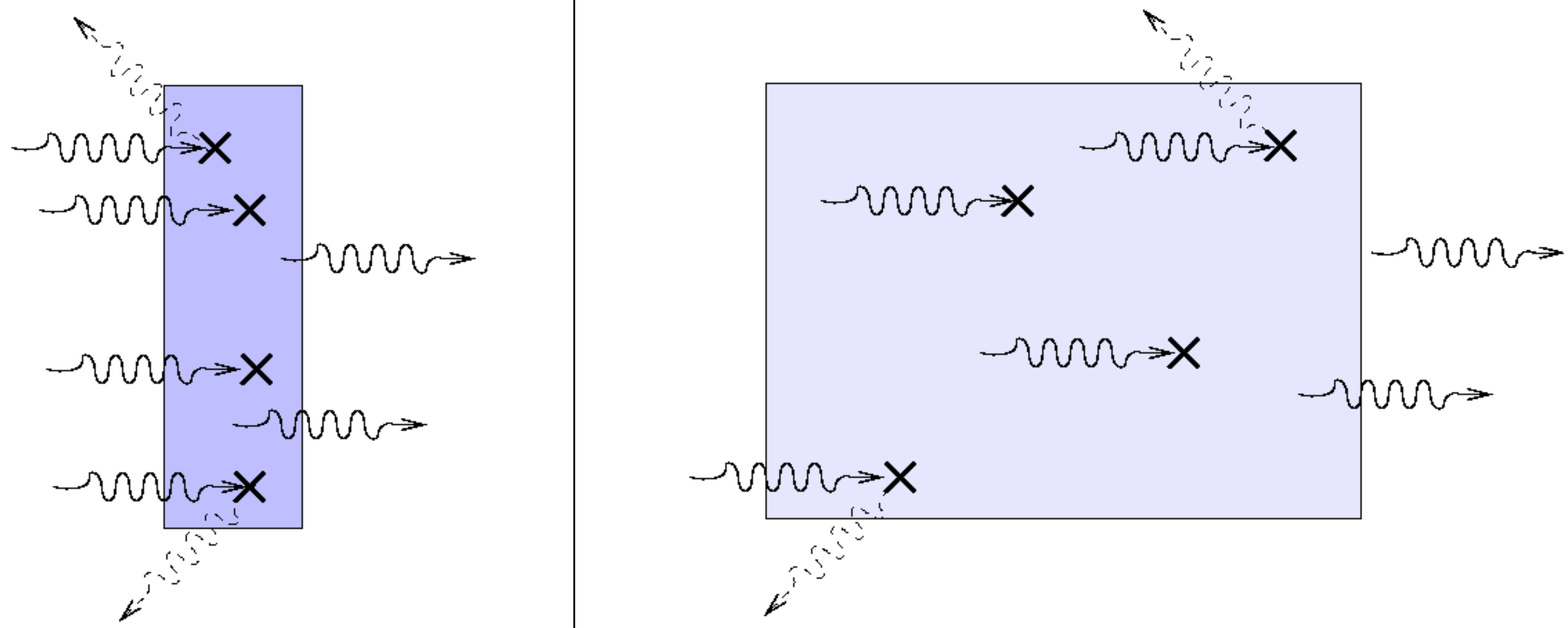
assume same $I_\nu(s_0)$ and $I_\nu(s)$ for both situations

why do we prefer to use optical depth?



assume same $I_\nu(s_0)$ and $I_\nu(s)$ for both situations, and write down absorption solution...

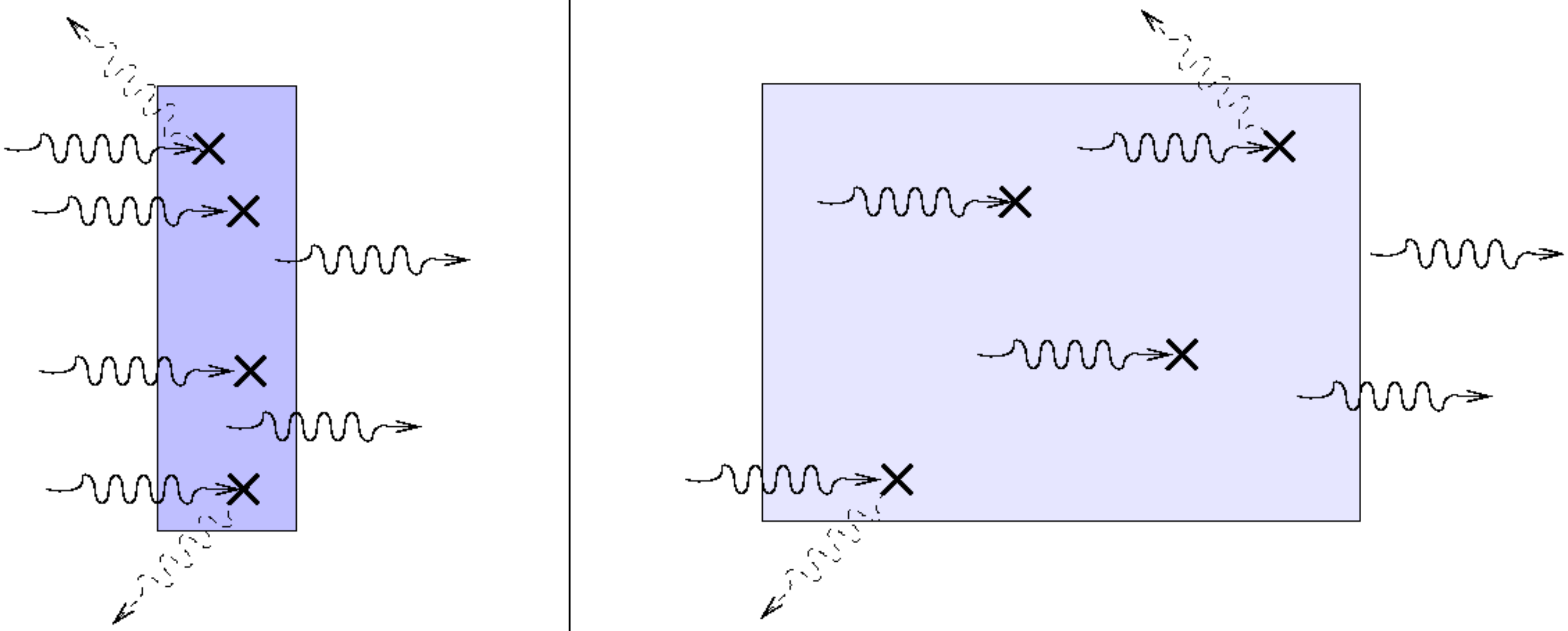
why do we prefer to use optical depth?



$$I_\nu(s_0)\; e^{-\int_{s_0}^{s_A} \alpha_\nu^A(s')ds'} = I_\nu(s)$$

$$I_\nu(s_0)\; e^{-\int_{s_0}^{s_B} \alpha_\nu^B(s')ds'} = I_\nu(s)$$

why do we prefer to use optical depth?



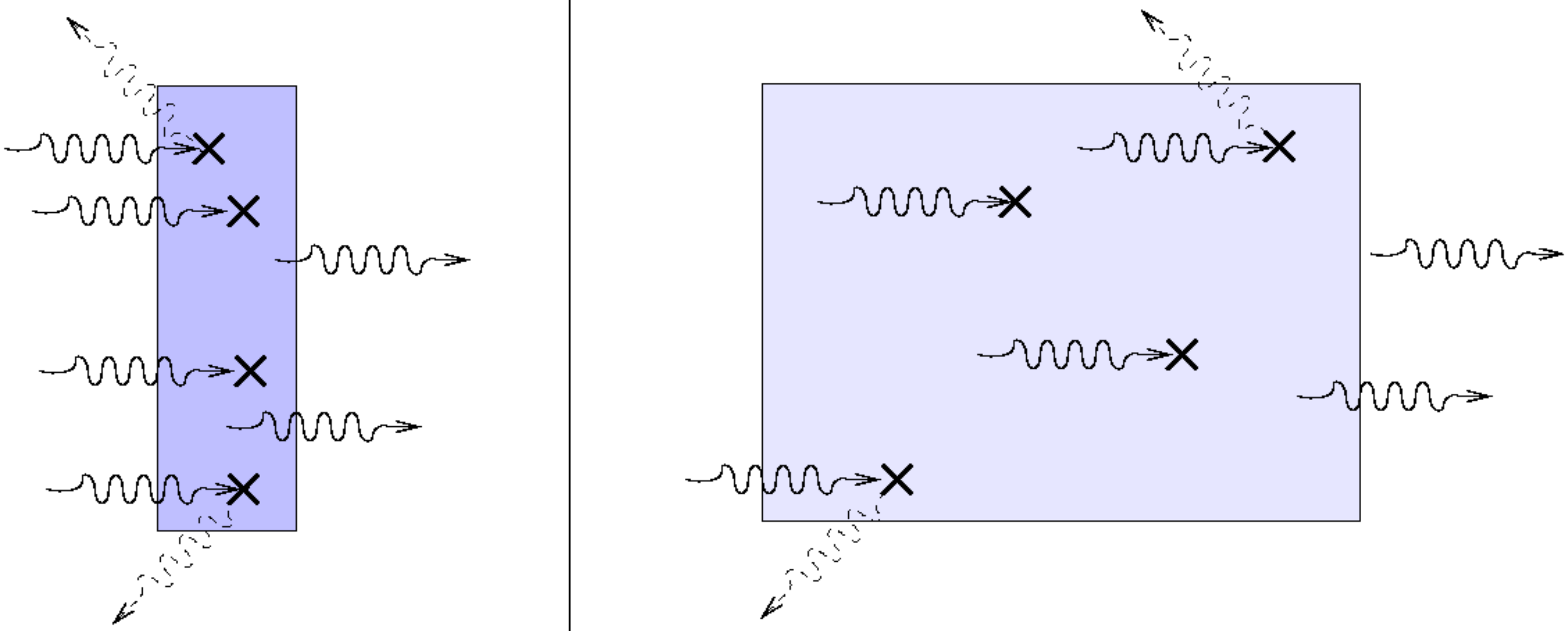
$$I_\nu(s_0)\ e^{-\int_{s_0}^{s_A} \alpha_\nu^A(s')ds'} = I_\nu(s)$$

$$I_\nu(s_0)\ e^{-\int_{s_0}^{s_B} \alpha_\nu^B(s')ds'} = I_\nu(s)$$

$$\tau_A = \int_{s_0}^{s_A} \alpha_\nu^A(s')ds'$$

$$\tau_B = \int_{s_0}^{s_B} \alpha_\nu^B(s')ds'$$

why do we prefer to use optical depth?



$$I_\nu(s_0)\ e^{-\int_{s_0}^{s_A} \alpha_\nu^A(s')ds'} = I_\nu(s)$$

$$I_\nu(s_0)\ e^{-\int_{s_0}^{s_B} \alpha_\nu^B(s')ds'} = I_\nu(s)$$

$$\tau_A = \int_{s_0}^{s_A} \alpha_\nu^A(s')ds'$$

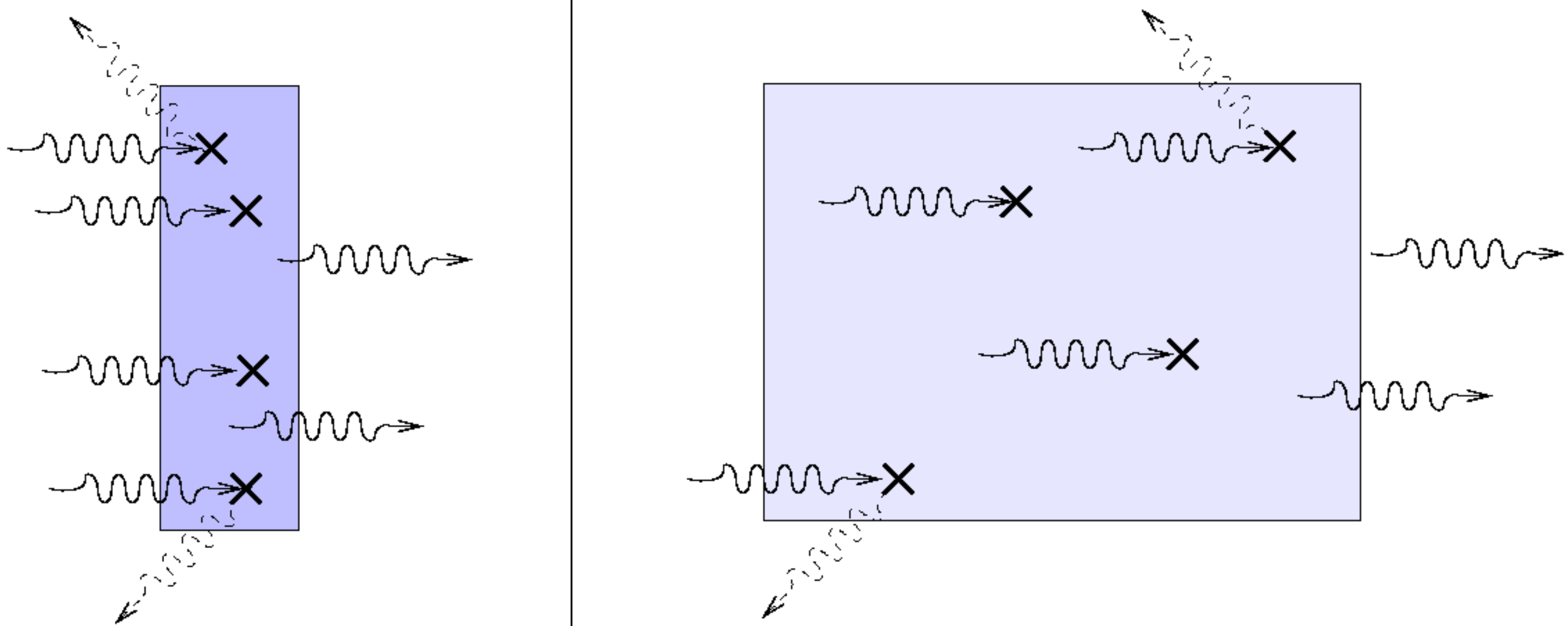
$$\tau_B = \int_{s_0}^{s_B} \alpha_\nu^B(s')ds'$$

$$\tau_A = \tau_B = \tau_\nu$$

why do we prefer to use optical depth?

Fundamentals

radiative transfer equation



$$I_\nu(s_0)\; e^{-\int_{s_0}^{s_A} \alpha_\nu^A(s')ds'} = I_\nu(s)$$

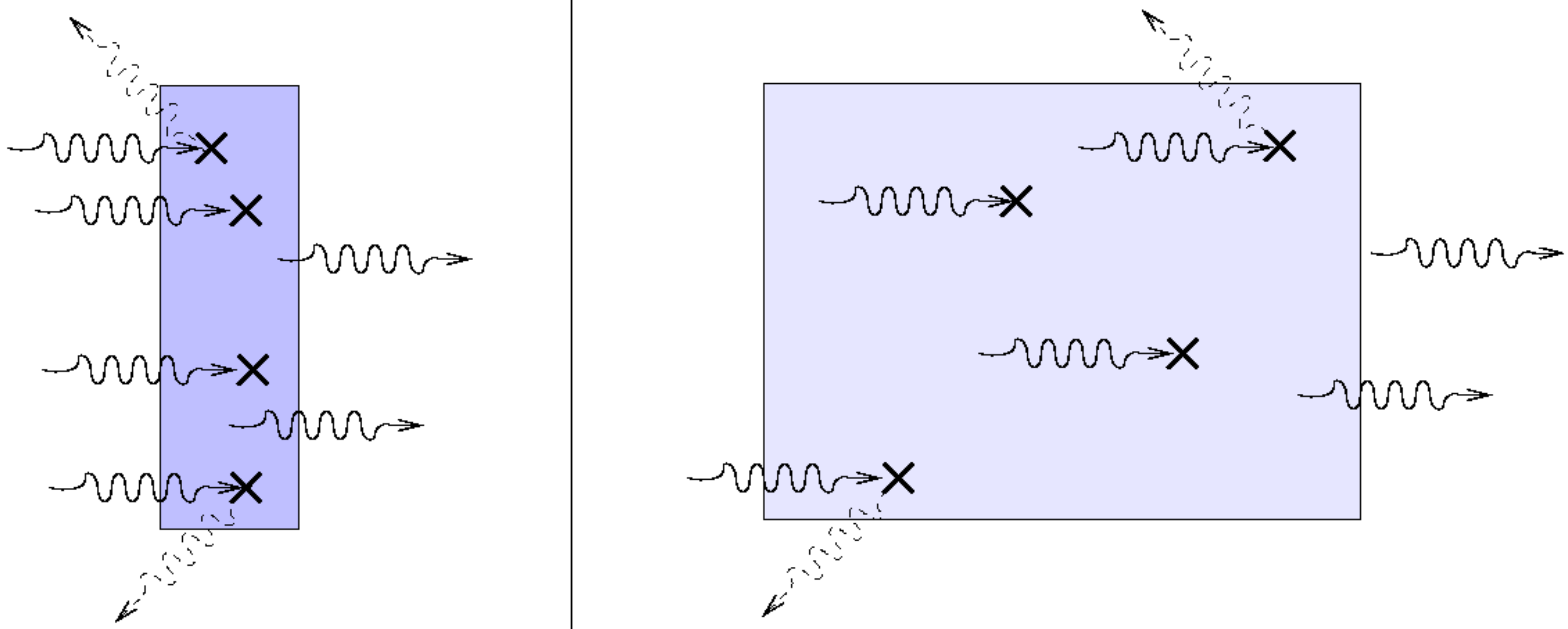
$$I_\nu(s_0)\; e^{-\int_{s_0}^{s_B} \alpha_\nu^B(s')ds'} = I_\nu(s)$$

$$\tau_A = \int_{s_0}^{s_A} \alpha_\nu^A(s')ds'$$

$$\tau_B = \int_{s_0}^{s_B} \alpha_\nu^B(s')ds'$$

$$\xrightarrow{\tau_A = \tau_B = \tau_\nu} \quad I_\nu(s) = I_\nu(0)\, e^{-\tau_\nu}$$

why do we prefer to use optical depth?



$$I_\nu(s_0)\ e^{-\int_{s_0}^{s_A} \alpha_\nu^A(s')ds'} = I_\nu(s)$$

$$I_\nu(s_0)\ e^{-\int_{s_0}^{s_B} \alpha_\nu^B(s')ds'} = I_\nu(s)$$

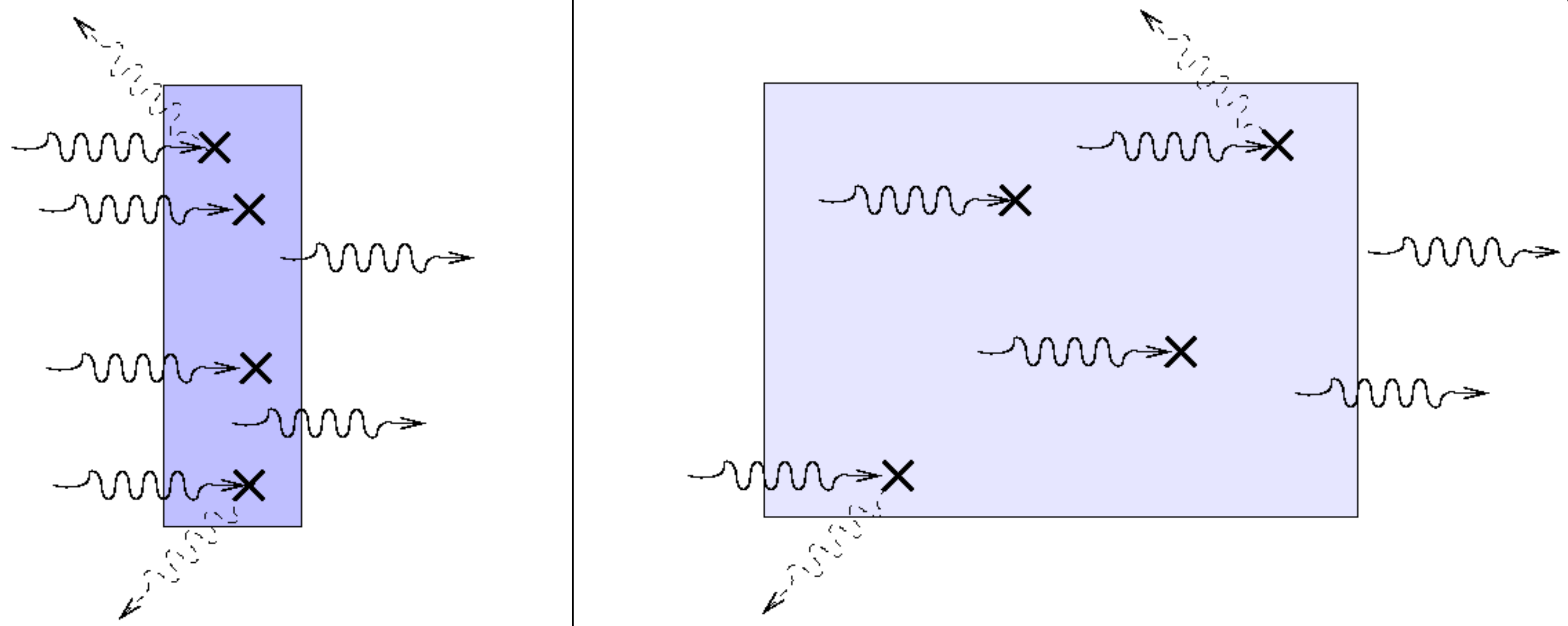
$$\tau_A = \int_{s_0}^{s_A} \alpha_\nu^A(s')ds'$$

$$\tau_B = \int_{s_0}^{s_B} \alpha_\nu^B(s')ds'$$

$\tau_A = \tau_B = \tau_\nu$ → $I_\nu(s) = I_\nu(0)\ e^{-\tau_\nu}$

same net decrease of intensity, perfectly described by τ_ν!

why do we prefer to use optical depth?



$$I_\nu(s_0)\ e^{-\int_{s_0}^{s_A} \alpha_\nu^A(s')ds'} = I_\nu(s)$$

$$I_\nu(s_0)\ e^{-\int_{s_0}^{s_B} \alpha_\nu^B(s')ds'} = I_\nu(s)$$

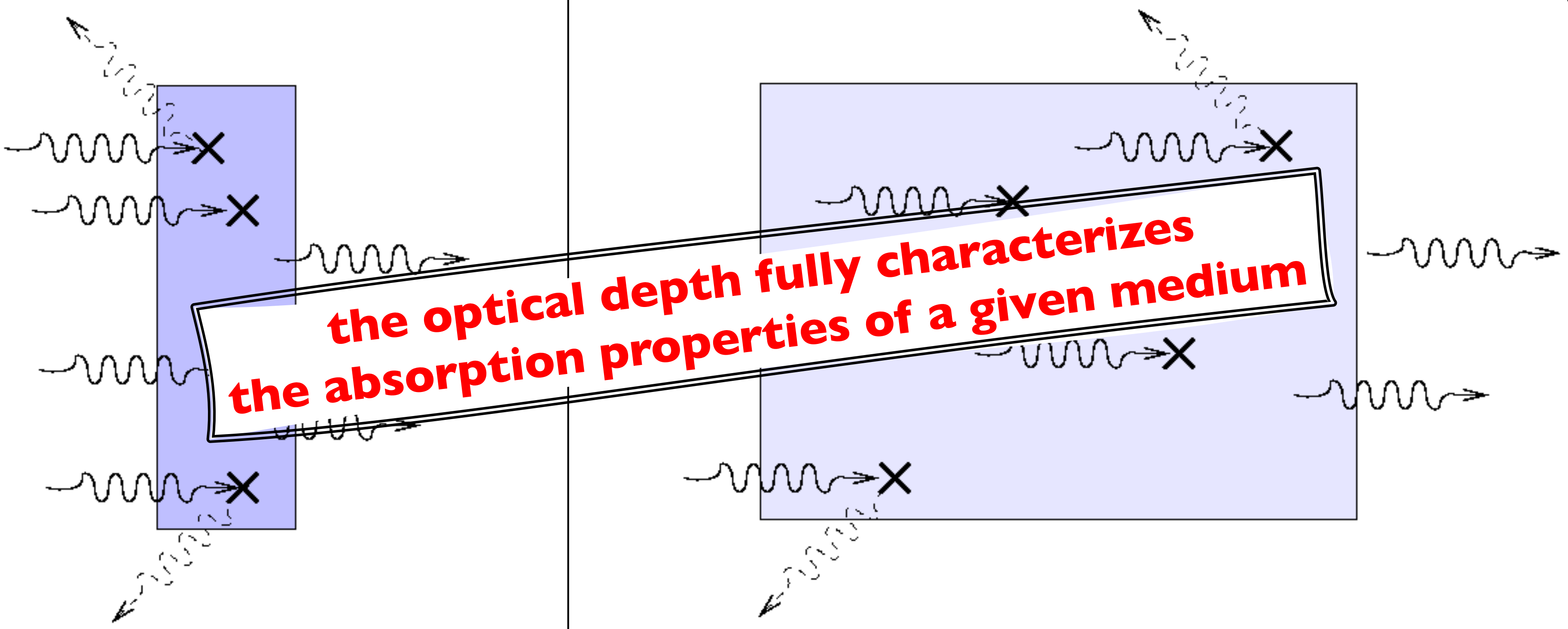
$$\tau_A = \int_{s_0}^{s_A} \alpha_\nu^A(s')ds'$$

$$\tau_B = \int_{s_0}^{s_B} \alpha_\nu^B(s')ds'$$

$\tau_A = \tau_B = \tau_\nu$ → $I_\nu(s) = I_\nu(0)\ e^{-\tau_\nu}$

same net decrease of intensity, perfectly described by τ_ν!

we prefer to use optical depth, because we only care about the fraction of light that is absorbed!



$$I_\nu(s_0)\, e^{-\int_{s_0}^{s_A} \alpha_\nu^A(s')ds'} = I_\nu(s)$$

$$I_\nu(s_0)\, e^{-\int_{s_0}^{s_B} \alpha_\nu^B(s')ds'} = I_\nu(s)$$

$$\tau_A = \int_{s_0}^{s_A} \alpha_\nu^A(s')ds'$$

$$\tau_B = \int_{s_0}^{s_B} \alpha_\nu^B(s')ds'$$

$\xrightarrow{\tau_A = \tau_B = \tau_\nu}$ $I_\nu(s) = I_\nu(0)\, e^{-\tau_\nu}$

same net decrease of intensity,
perfectly described by τ_ν!

we prefer to use optical depth, because
we only care about the fraction of light that is absorbed!

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu\, ds$$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\langle\tau_\nu\rangle = \int \tau_\nu\, e^{-\tau_\nu} d\tau_\nu = 1$$

$\tau_\nu > 1$: optically thick medium (opaque)

$\tau_\nu < 1$: optically thin medium (transparent)

we prefer to use optical depth!

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu\, ds$$

→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

change of variables:
$s \rightarrow \tau$

$$\langle\tau_\nu\rangle = \int \tau_\nu\, e^{-\tau_\nu} d\tau_\nu = 1$$

$\tau_\nu > 1$: optically thick medium (opaque)

$\tau_\nu < 1$: optically thin medium (transparent)

we prefer to use optical depth!

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu \, ds$$

→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

change of variables:
$s \rightarrow \tau$

$$\langle\tau_\nu\rangle = \int \tau_\nu \, e^{-\tau_\nu} d\tau_\nu = 1$$

$\tau_\nu > 1$: optically thick medium (opaque)

$\tau_\nu < 1$: optically thin medium (transparent)

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu \, ds$$

→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

change of variables:
$s \rightarrow \tau$

$$\langle \tau_\nu \rangle = \int \tau_\nu \, e^{-\tau_\nu} d\tau_\nu = 1$$

$\tau_\nu > 1$: optically thick medium (opaque)

$\tau_\nu < 1$: optically thin medium (transparent)

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \boxed{\frac{j_\nu}{\alpha_\nu}}$$

yet another definition...

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu \, ds$$

→ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

change of variables:
$s \rightarrow \tau$

$$\langle \tau_\nu \rangle = \int \tau_\nu \, e^{-\tau_\nu} d\tau_\nu = 1$$

$\tau_\nu > 1$: optically thick medium (opaque)

$\tau_\nu < 1$: optically thin medium (transparent)

- source function

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$S_\nu = \dfrac{j_\nu}{\alpha_\nu}$ source function

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- optical depth τ_ν

$$d\tau_\nu = \alpha_\nu\, ds$$

$\rightarrow$ absorption: $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

change of variables:
$s \rightarrow \tau$

$$\langle\tau_\nu\rangle = \int \tau_\nu\, e^{-\tau_\nu} d\tau_\nu = 1$$

$\tau_\nu > 1$: optically thick medium (opaque)

$\tau_\nu < 1$: optically thin medium (transparent)

- source function

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$S_\nu = \dfrac{j_\nu}{\alpha_\nu}$ source function

the source function describes the ratio between newly created and absorped photons

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \quad \text{source function}$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \quad \text{source function}$$

- solution in general

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

(**exercise**)

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \quad \text{source function}$$

- solution in general

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} \, S_\nu(\tau'_\nu) \, d\tau'_\nu$$

absorption of incident radiation $I_\nu(0)$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \quad \text{source function}$$

- solution in general

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

absorption of incident radiation $I_\nu(0)$

integral over newly created photons as they propagate to optical depth τ_ν

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

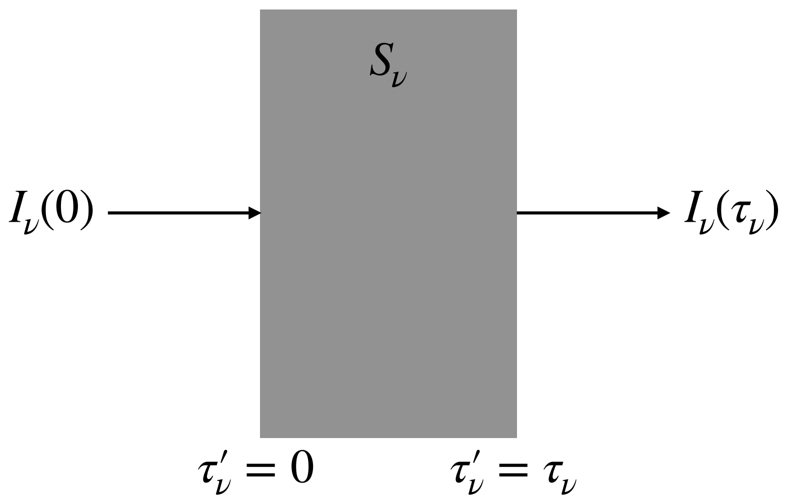
$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \quad \text{source function}$$

- solution in general

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

absorption of incident radiation $I_\nu(0)$

integral over newly created photons as they propagate to optical depth τ_ν



- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \quad \text{source function}$$

- solution in general

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

- solution – special cases
  - emission-only $\quad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$
  - absorption-only $\quad I_\nu(s) = I_\nu(s_0)\, e^{-\int_{s_0}^{s} \alpha_\nu(s')ds'}$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \quad \text{source function}$$

- solution in general

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

- solution – special cases
  - emission-only $\quad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$
  - absorption-only $\quad I_\nu(s) = I_\nu(s_0)\, e^{-\int_{s_0}^{s} \alpha_\nu(s')ds'}$
  - S_ν= const. $\quad I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu\,(1 - e^{-\tau_\nu})$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \quad \text{source function}$$

- solution in general

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

- solution – special cases
  - emission-only $\quad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$
  - absorption-only $\quad I_\nu(s) = I_\nu(s_0)\, e^{-\int_{s_0}^{s} \alpha_\nu(s')ds'}$
  - S_ν= const. $\quad I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$
    - ○ $\tau_\nu \gg 1$: $\quad I_\nu(\tau_\nu) = S_\nu$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu}$$ source function

- solution in general

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

- solution – special cases
  - emission-only $\quad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$
  - absorption-only $\quad I_\nu(s) = I_\nu(s_0)\, e^{-\int_{s_0}^{s} \alpha_\nu(s')ds'}$
  - S_ν= const. $\quad I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$
    - $\tau_\nu \gg 1$: $\quad I_\nu(\tau_\nu) = S_\nu$

$I_\nu(0) > S_\nu$ → photons will be absorbed from the beam until $I_\nu(\tau_\nu) = S_\nu$

$I_\nu(0) < S_\nu$ → photons will be added to the beam until $I_\nu(\tau_\nu) = S_\nu$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \quad \text{source function}$$

- solution in general

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

- solution – special cases
  - emission-only $\quad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$
  - absorption-only $\quad I_\nu(s) = I_\nu(s_0)\, e^{-\int_{s_0}^{s} \alpha_\nu(s')ds'}$
  - S_ν= const. $\quad I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu\,(1 - e^{-\tau_\nu})$
    - ○ $\tau_\nu \gg 1$: $\quad I_\nu(\tau_\nu) = S_\nu$
    - ○ $\tau_\nu \ll 1$: $\quad I_\nu(\tau_\nu) = I_\nu(0)\,(1 - \tau_\nu) + \tau_\nu\, S_\nu$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \qquad S_\nu = \frac{j_\nu}{\alpha_\nu} \quad \text{source function}$$

- solution in general

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

- solution – special cases
  - emission-only $\quad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$
  - absorption-only $\quad I_\nu(s) = I_\nu(s_0)\, e^{-\int_{s_0}^{s} \alpha_\nu(s')ds'}$
  - S_ν= const. $\quad I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu\,(1 - e^{-\tau_\nu})$
    - $\tau_\nu \gg 1$: $\quad I_\nu(\tau_\nu) = S_\nu$
    - $\tau_\nu \ll 1$: $\quad I_\nu(\tau_\nu) = I_\nu(0)\,(1 - \tau_\nu) + \tau_\nu\, S_\nu \quad \rightarrow \quad I_\nu(0)$

- electromagnetic spectrum
- description of a radiation field
- radiative transfer equation

- electromagnetic spectrum
- description of a radiation field
- radiative transfer equation

summary

▪ radiation field – macroscopic description

- flux $dE = F\,dA\,dt$ $dF_\nu = I_\nu(\Omega)\,cos\theta\,d\Omega$
- intensity $dE = I_\nu(\Omega)\,dA\,cos\theta\,dt\,d\Omega\,d\nu$
- luminosity $dE = L\,dt$
- energy density $u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c}$
- radiation pressure $p_\nu = \frac{1}{3}u_\nu$
- optical depth $d\tau_\nu = \alpha_\nu\,ds$, absorption coefficient α_ν
- mean free path $l_\nu = \frac{1}{\alpha_\nu}$

emission coefficient j_ν $dE = j_\nu\,dV\,dt\,d\Omega\,d\nu$

▪ equation of radiative transfer

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$ source function $$S_\nu = \frac{j_\nu}{\alpha_\nu}$$