▪ derive the lensing equation

$$\beta = \theta - \frac{D_{LS}}{D_S}\hat{\alpha}(\theta)$$

▪ derive the formula for the general relativistic deflection angle of a point mass

$$\hat{\alpha} = \frac{4GM}{c^2}\frac{1}{b}$$

▪ derive Poisson's equation for the lensing potential

$$\nabla_\theta{}^2\varphi = 2\frac{\Sigma(\theta)}{\Sigma_{crit}}$$

▪ calculate the eigenvalues of the distortion matrix

$$A_{ij} = (1-\kappa)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}\gamma_1 & \gamma_2\\ \gamma_2 & -\gamma_1\end{pmatrix}$$

▪ derive the formula for the reduced shear and its relation to the ellipse axes a and b

$$g = \frac{|\gamma|}{1-\kappa} = f(a,b)$$

▪ derive the following relations between convergence, shear and lensing potential

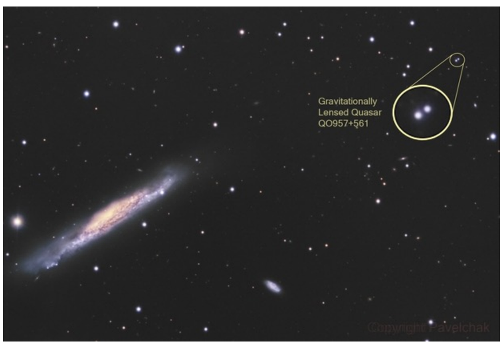
$$\kappa = \frac{1}{2}(\partial_{11}\varphi + \partial_{22}\varphi) = \frac{\Sigma(\theta)}{\Sigma_{crit}}$$

$$\gamma_1 = \frac{1}{2}(\partial_{11}\varphi - \partial_{22}\varphi)$$

$$\gamma_2 = \partial_{12}\varphi = \partial_{21}\varphi$$

▪ Consider the gravitationally lensed quasar QO957+561. The two images are located at θ+ = 5.35" and θ− = −0.80". The redshift of the quasar and the lens are z_S = 1.41 and z_L = 0.36. If Ω_m = 0.3, Ω_Λ = 0.7, H_0 = 72 km s^{-1} Mpc^{-1} these redshifts translate into angular diameter distances of D_S = 1693 Mpc, D_L = 1011 Mpc, and D_{LS} = 1123 Mpc.

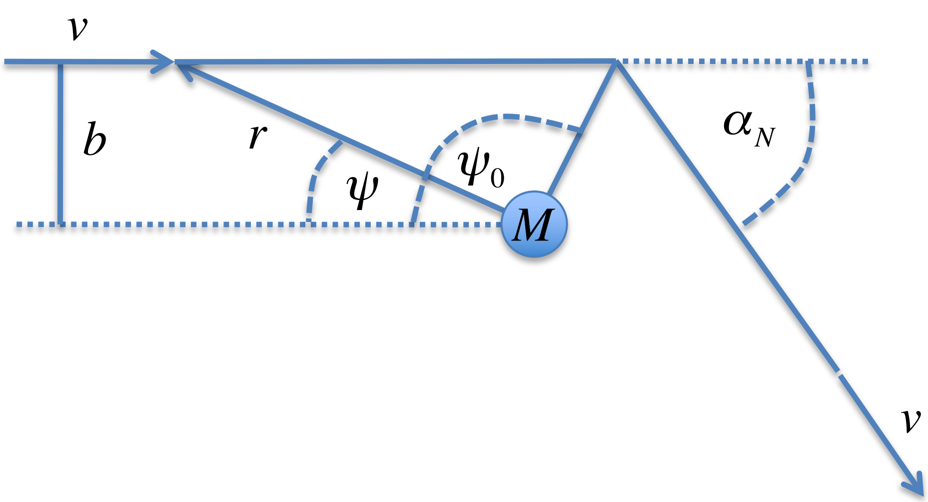
Estimate the mass M of the lens.



…more on the flipside →

▪ derive the formula for the Newtonian deflection angle

$$\alpha_N = \frac{2GM}{c^2}\frac{1}{b}$$



▪ hints:

• use the solution for a Keplerian orbit: $r(\psi) = \dfrac{l^2/GM}{1+e\cos(\psi-\psi_0)}$ with $e = C\dfrac{l^2}{GM}$

…where $l = bv = r^2\dot{\psi}$ is the specific angular momentum
and C and ψ_0 are given by the initial conditions

• use the fact that $r(0) = \infty$ and $\dot{r}(0) = -v$ to actually determine C and eventually ψ_0

• sometimes it's helpful to consider $1/r$ instead of r