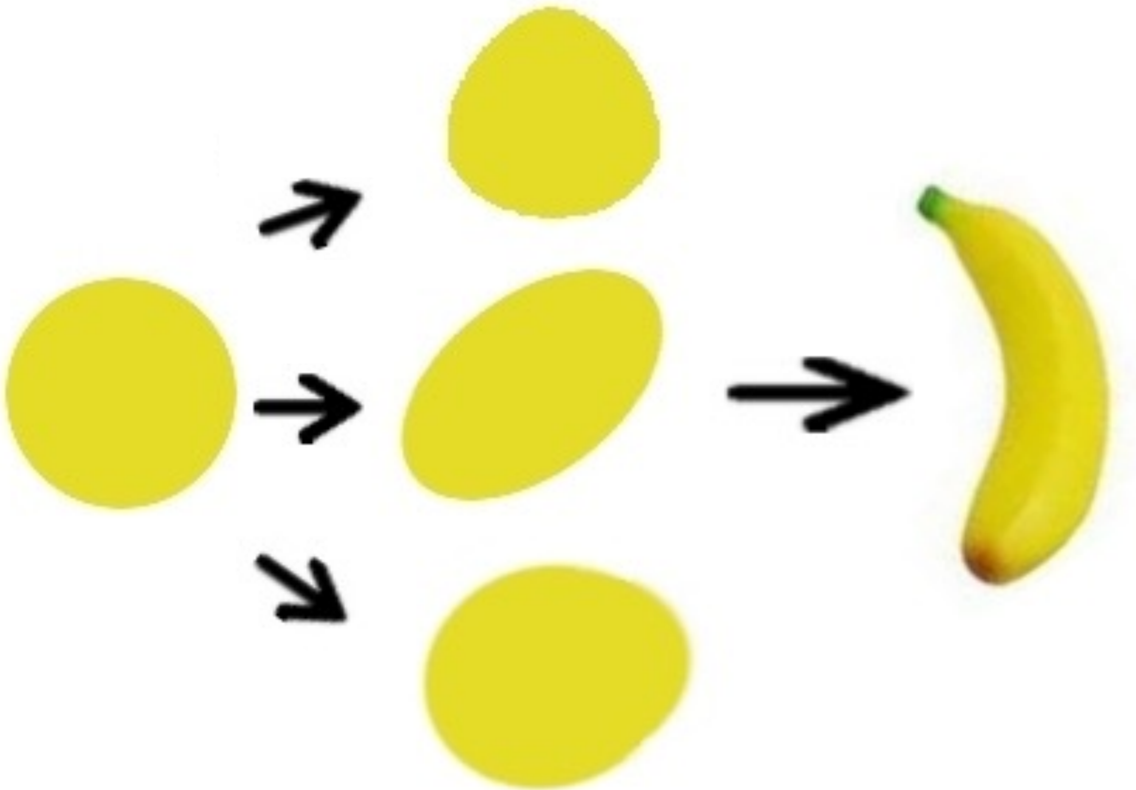
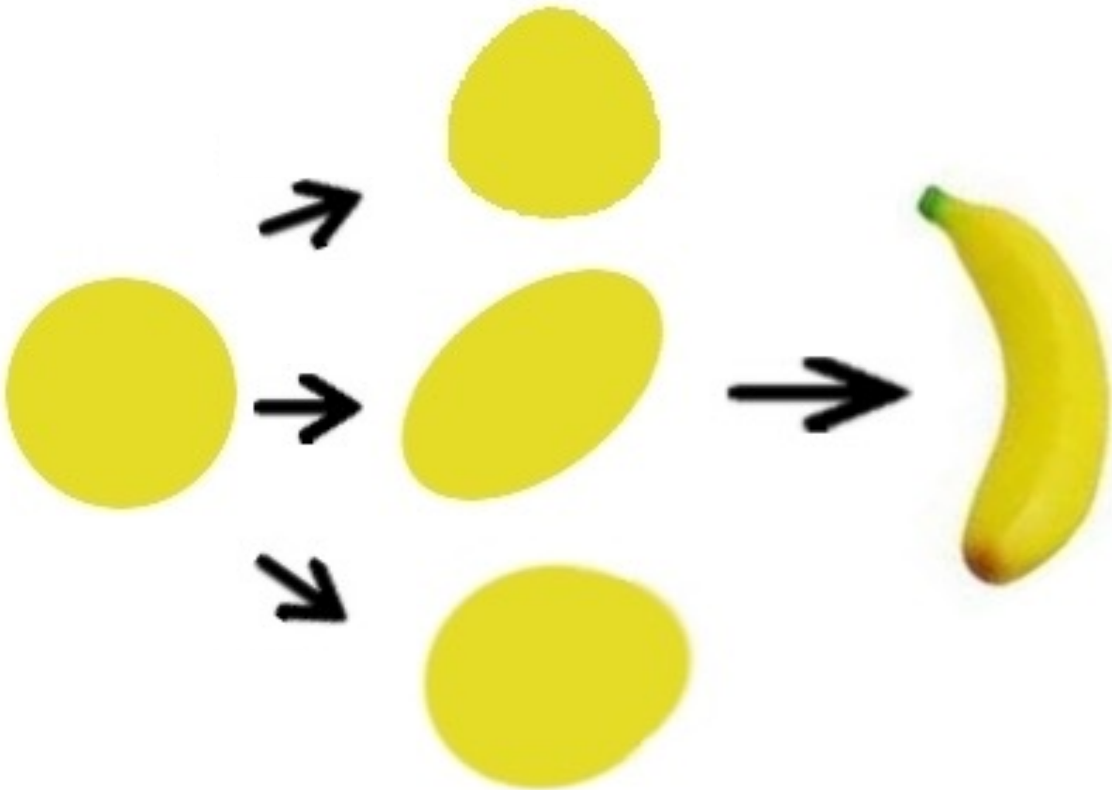
Alexander Knebe, *Universidad Autonoma de Madrid*



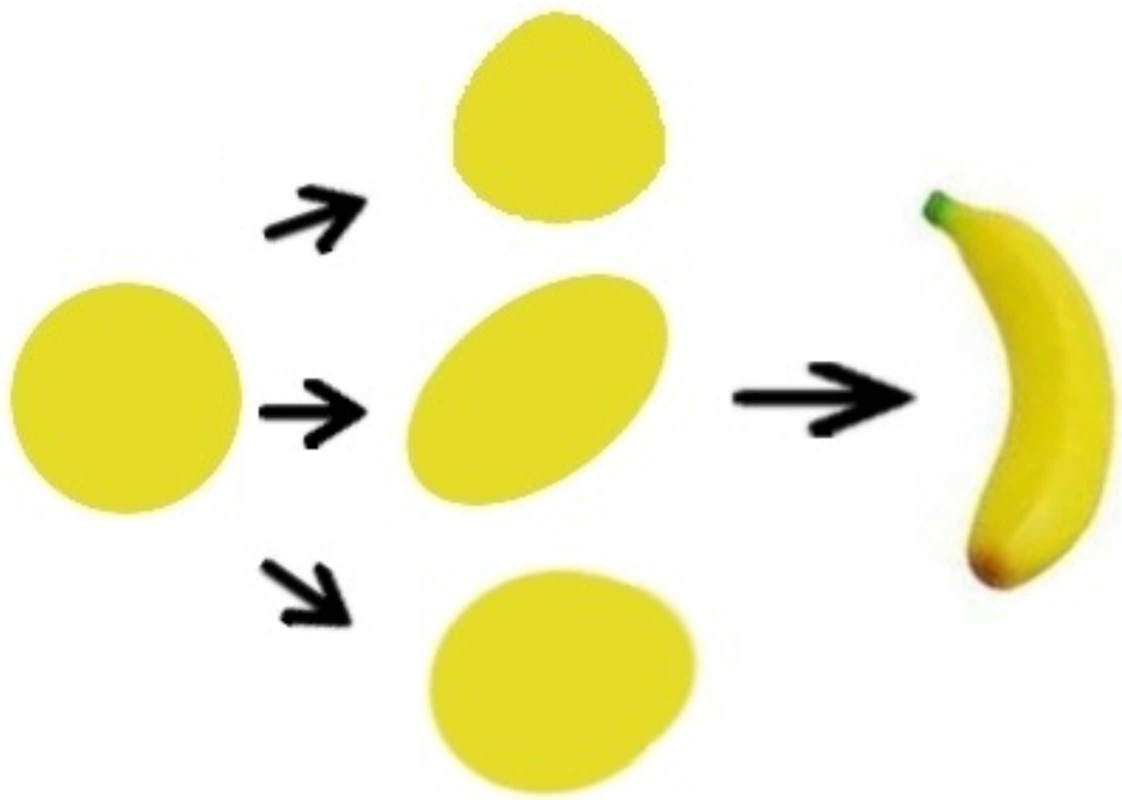
Alexander Knebe, *Universidad Autonoma de Madrid*



"You have a mass distribution about which you do not know anything

(Schneider 2006)

Alexander Knebe, *Universidad Autonoma de Madrid*

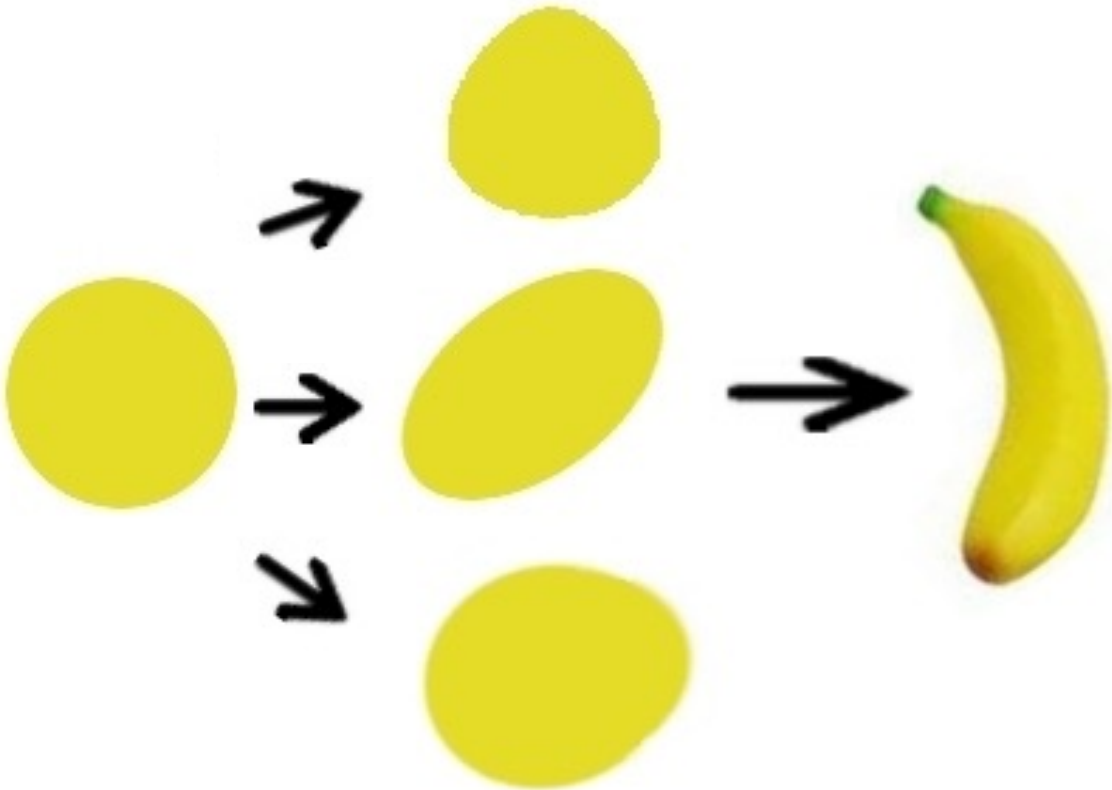


"You have a mass distribution about which you do not know anything,
and then you observe sources which you do not know either.

(Schneider 2006)

Weak Gravitational Lensing

Alexander Knebe, *Universidad Autonoma de Madrid*



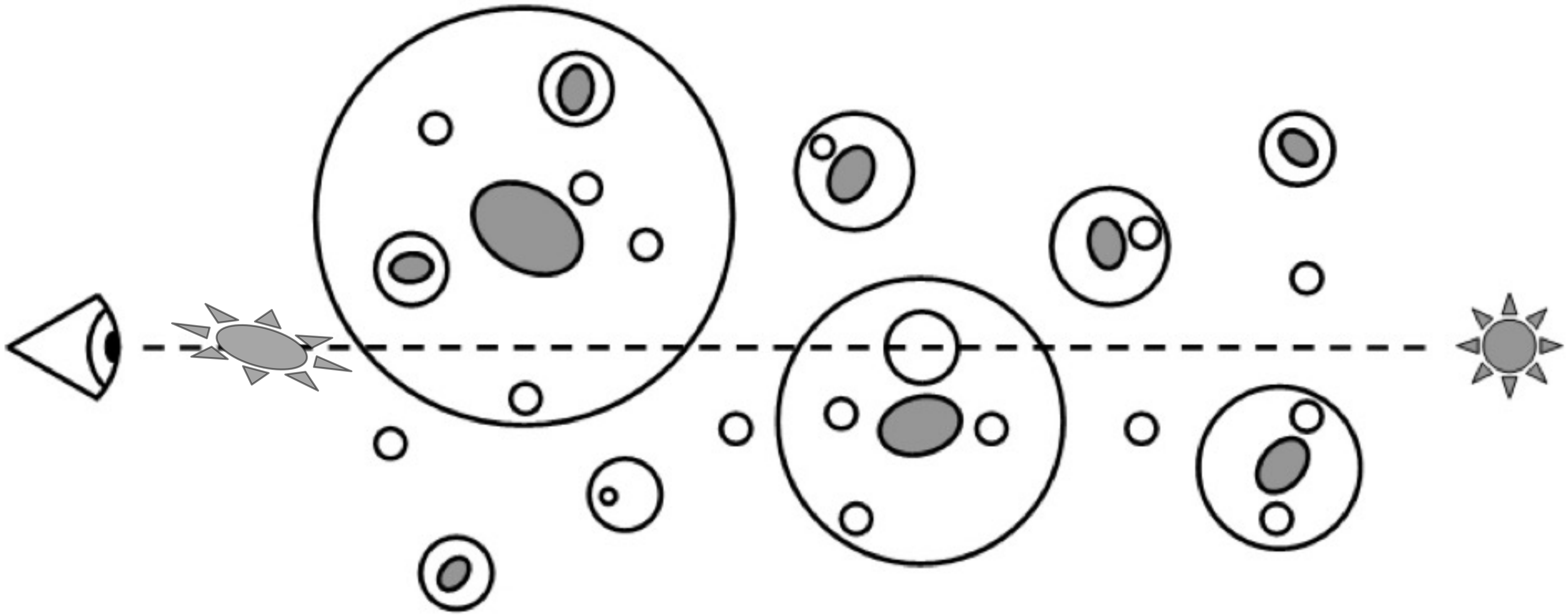
"You have a mass distribution about which you do not know anything,
and then you observe sources which you do not know either.
And then you claim to learn something about the mass distribution?"

(Schneider 2006)

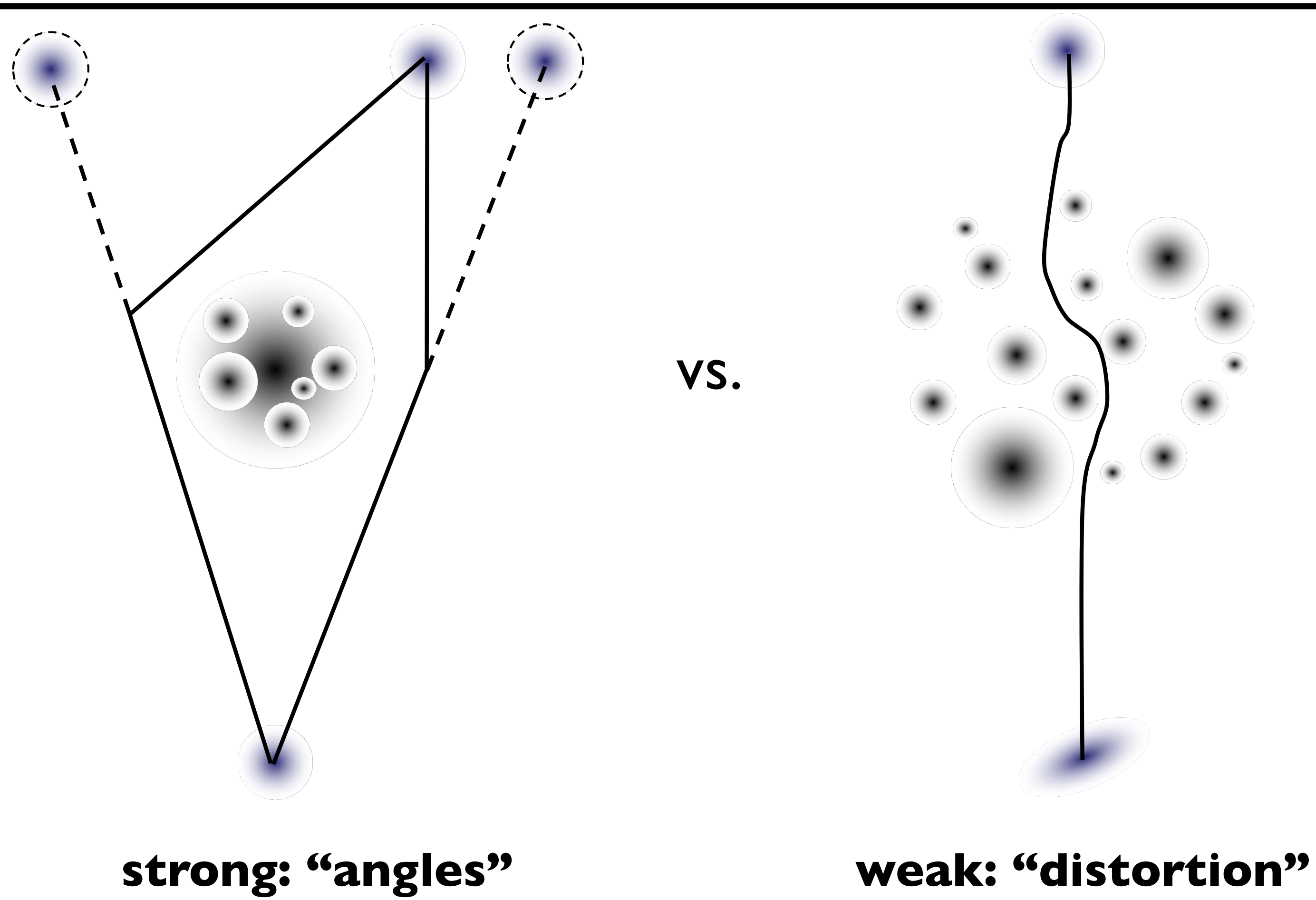
- microlensing
  - mainly referred to as lensing by objects of stellar (point) masses
    ($\rightarrow$ no distortion, mainly magnification)

- strong lensing
  - lensing of background sources by foreground galaxies, clusters, …
    ($\rightarrow$ strong distortion, magnification, and multiple images)

- weak lensing
  - lensing via large-scale structure
    ($\rightarrow$ weak distortion and magnification)



- weak lensing
  - lensing via large-scale structure
    (→ weak distortion and magnification)



- weak lensing
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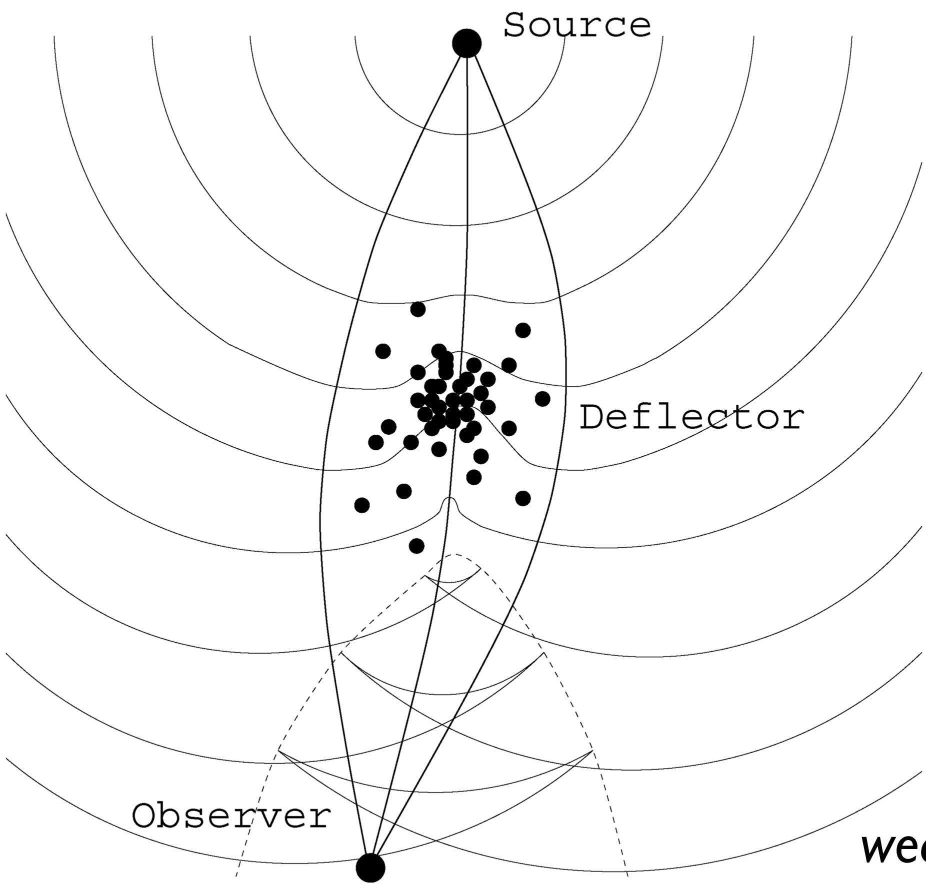


weak lensing regime

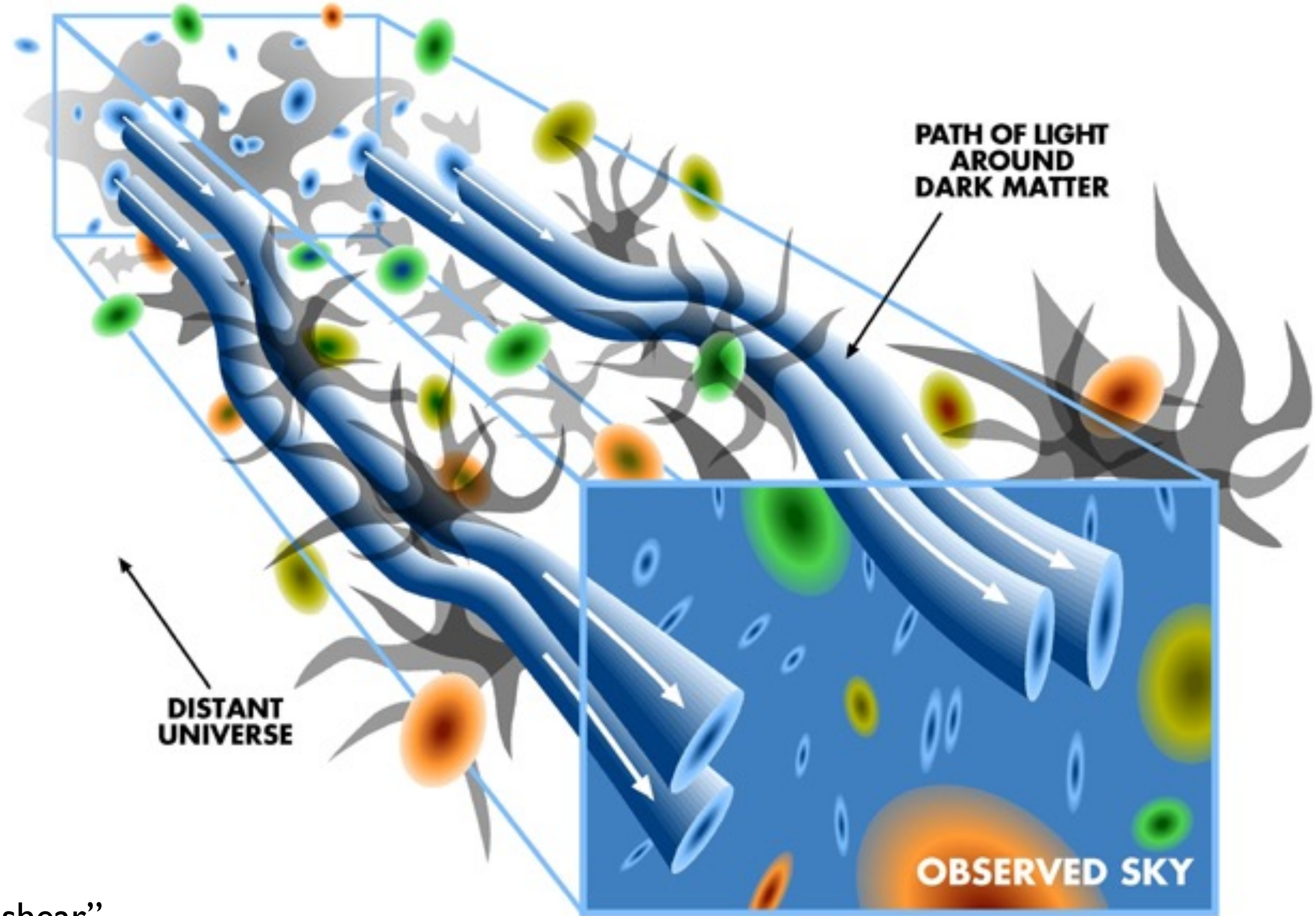
- weak lensing
  - lensing via large-scale structure
    ($\rightarrow$ weak distortion and magnification)

- concept
- theory
- application

- **concept**
- theory
- application

- image distortion* because of…

  …differential deflection of neighbouring light rays

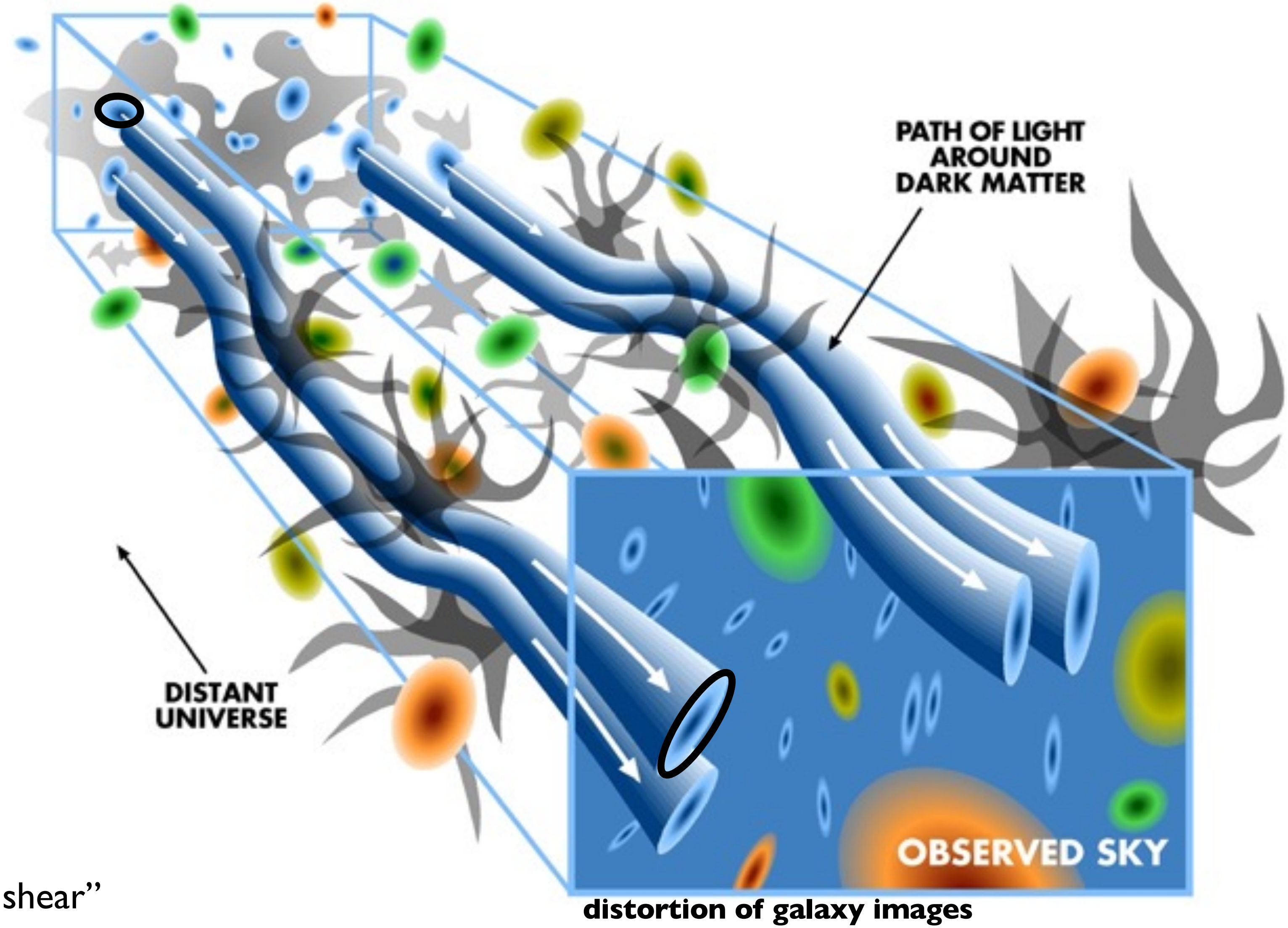


distortion of galaxy images

*aka "cosmic shear"

- image distortion* because of…

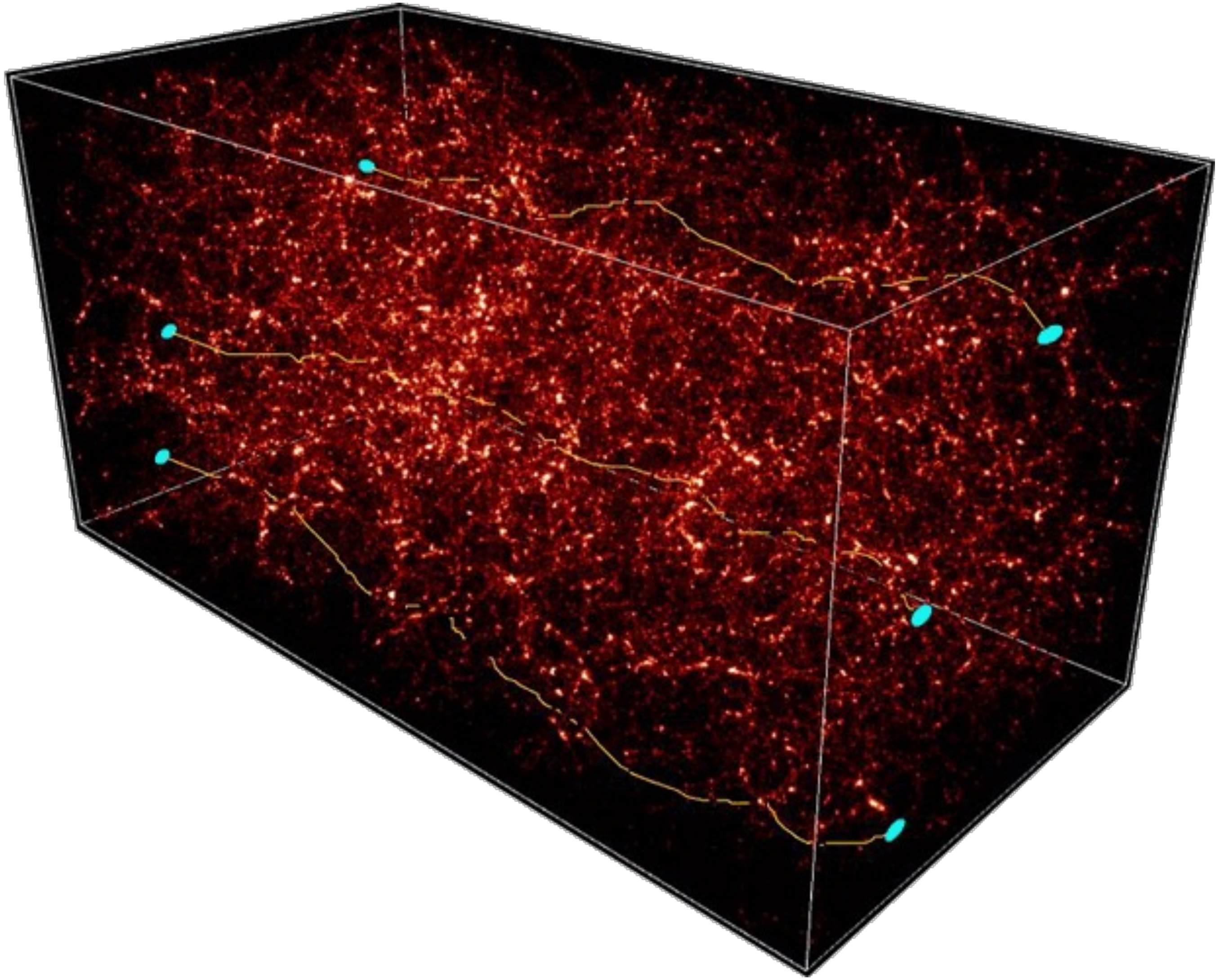
  …differential deflection of neighbouring light rays



*aka "cosmic shear"

- image distortion depends on…

  …intervening matter distribution



- image distortion depends on…

  …intervening matter distribution and hence

  - cosmic structure formation $G(a) = \frac{5}{2}\Omega_0 \frac{\dot{a}}{a}\int_0^a \frac{1}{\dot{a}^3}\,da$

  - cosmic distances $D(z) = \frac{1}{1+z}\frac{c}{H_0}\int_0^z \frac{dz}{\left[\Omega_0(1+z)^3 + \Omega_\Lambda\right]^{1/2}}$

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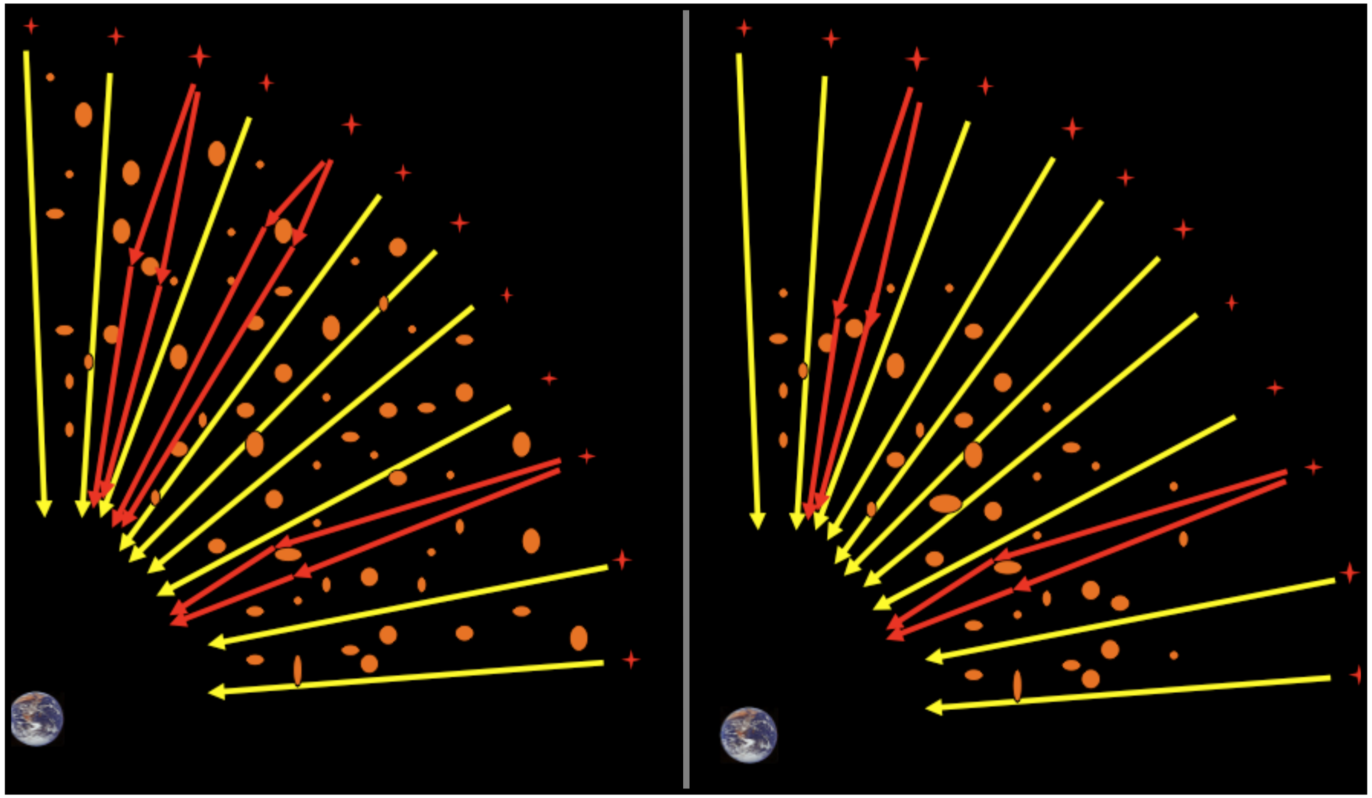

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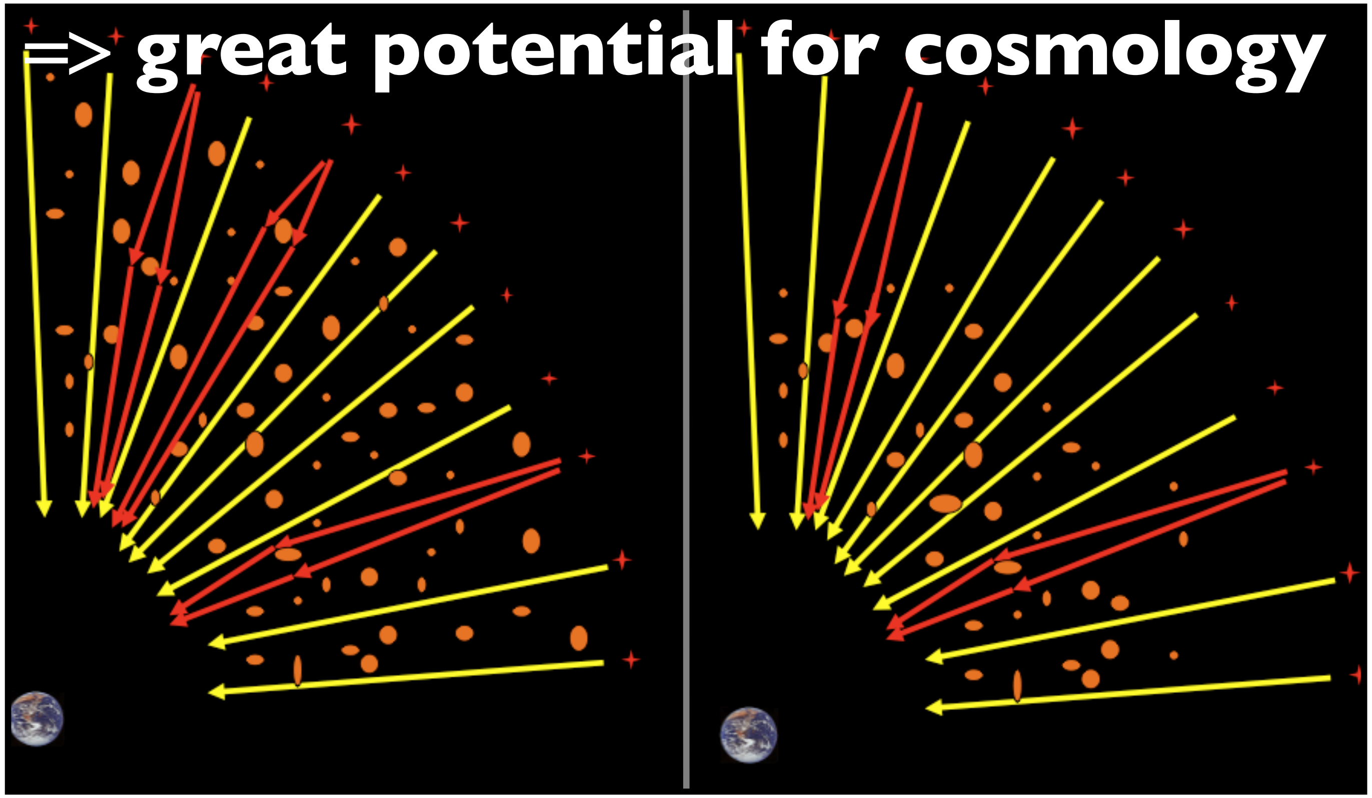
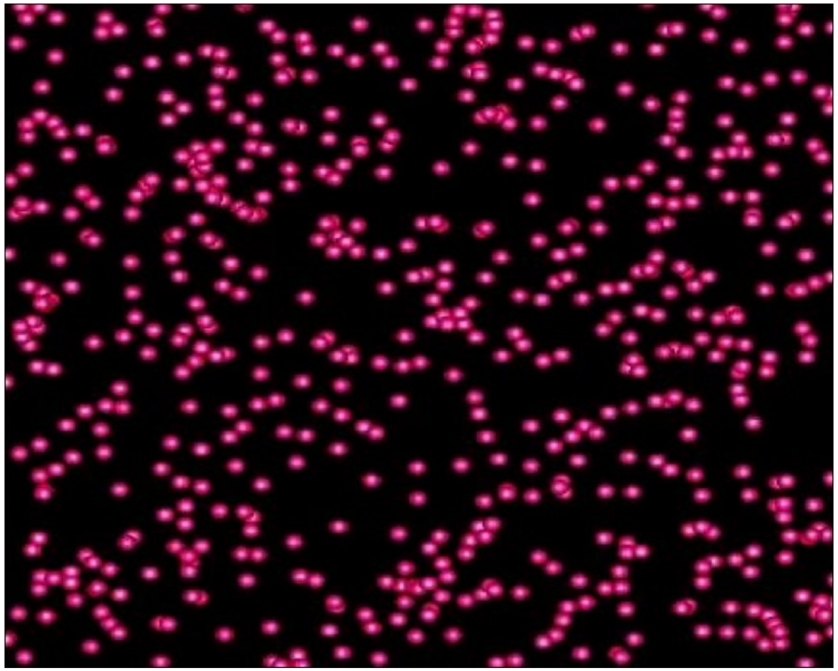


- image distortion

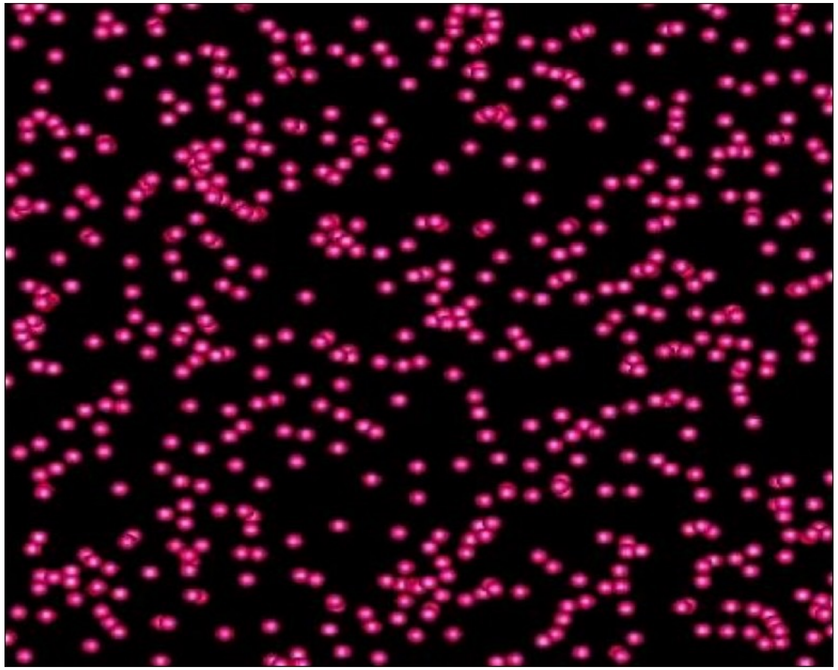
  …differential deflection of neighbouring light rays



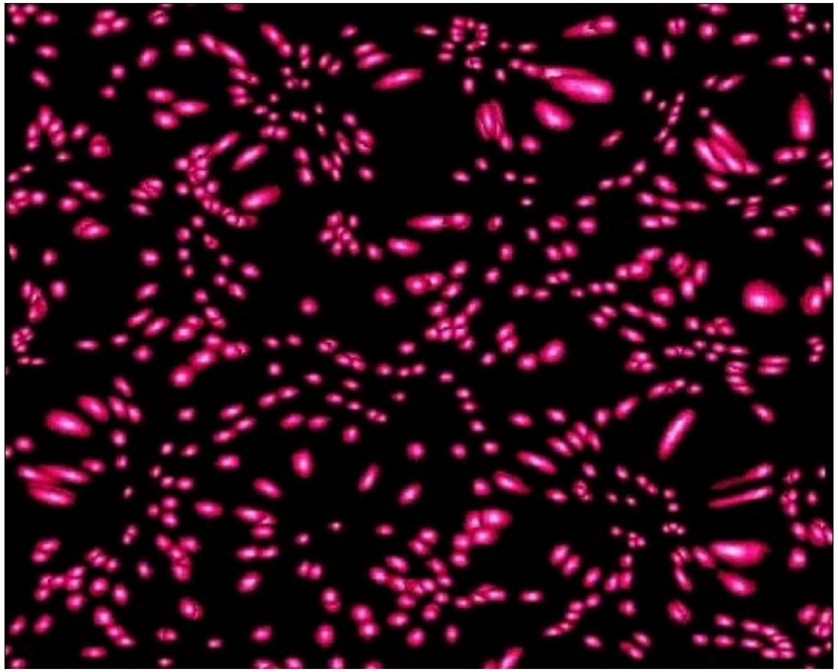
unlensed

- image distortion

  …differential deflection of neighbouring light rays

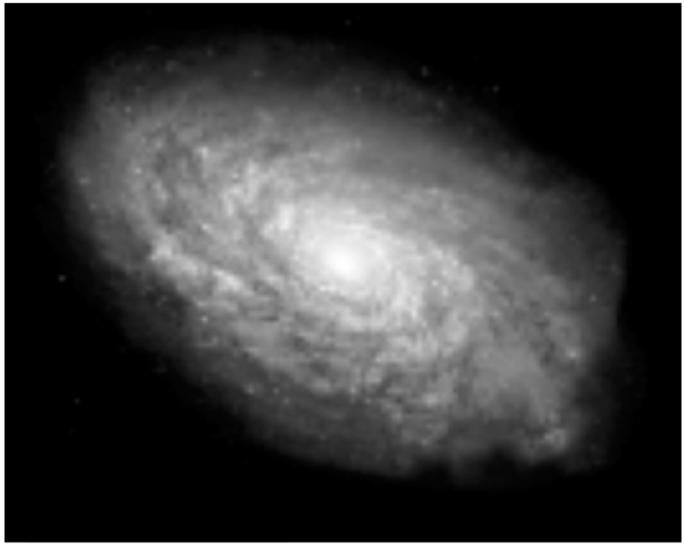


unlensed

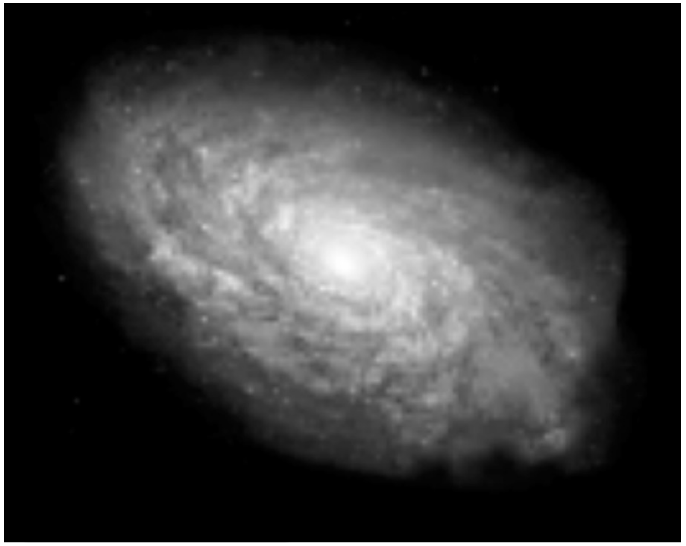


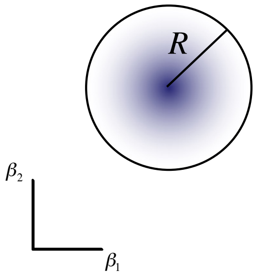
lensed

- image distortion



- image distortion



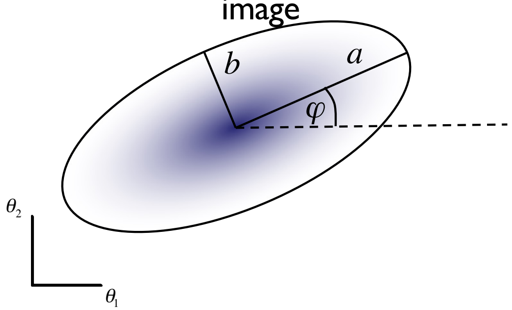


$$\gamma = |\gamma| e^{i2\varphi}$$

$$a = \frac{R}{1-\kappa-|\gamma|}$$

$$b = \frac{R}{1-\kappa+|\gamma|}$$

$$|\gamma| = \sqrt{\gamma_1^2+\gamma_2^2}$$



circular source => measuring *a* and *b* gives reduced shear $g = |\gamma|/(1-\kappa) = f(a,b)$

$$A_{ij} = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi \end{pmatrix}$$

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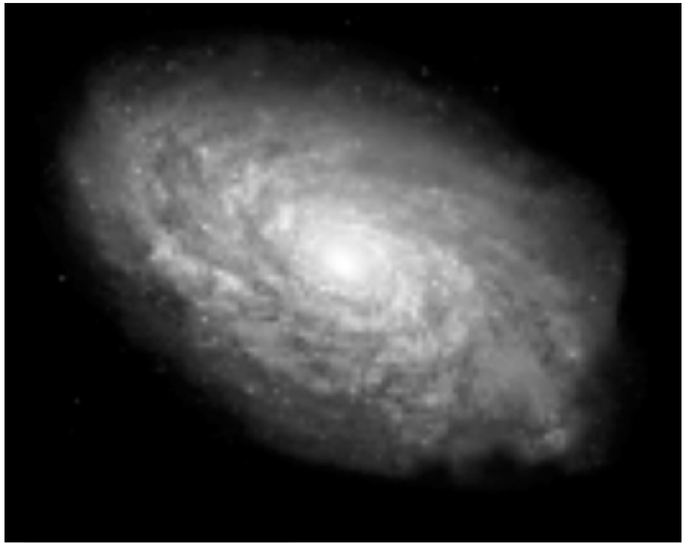
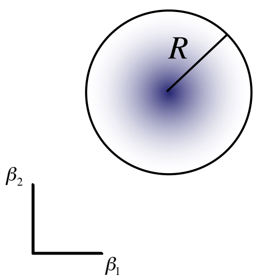


$$g \approx 0.2$$
$$\longrightarrow$$



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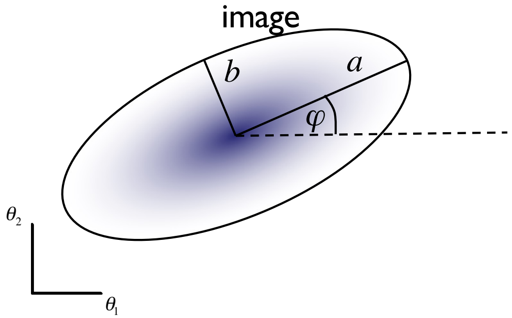

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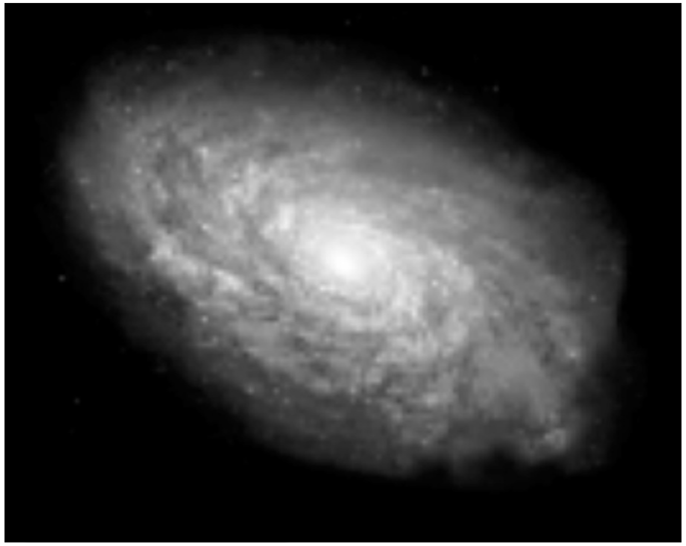

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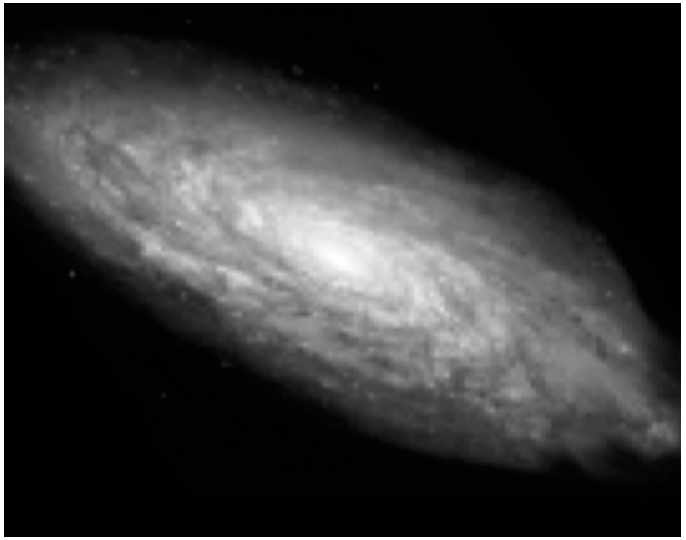

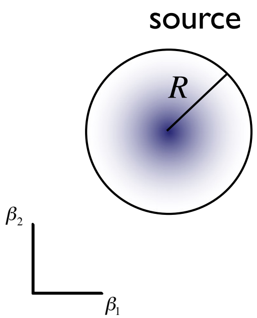

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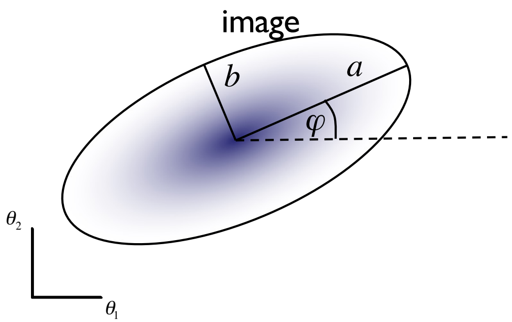

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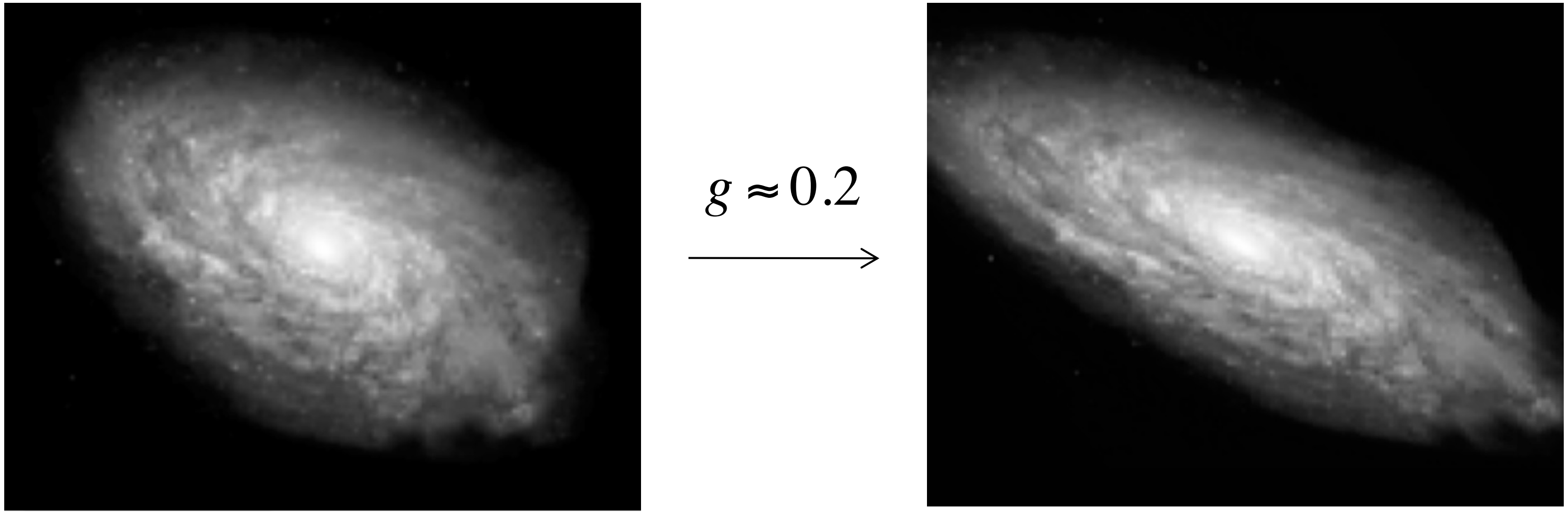
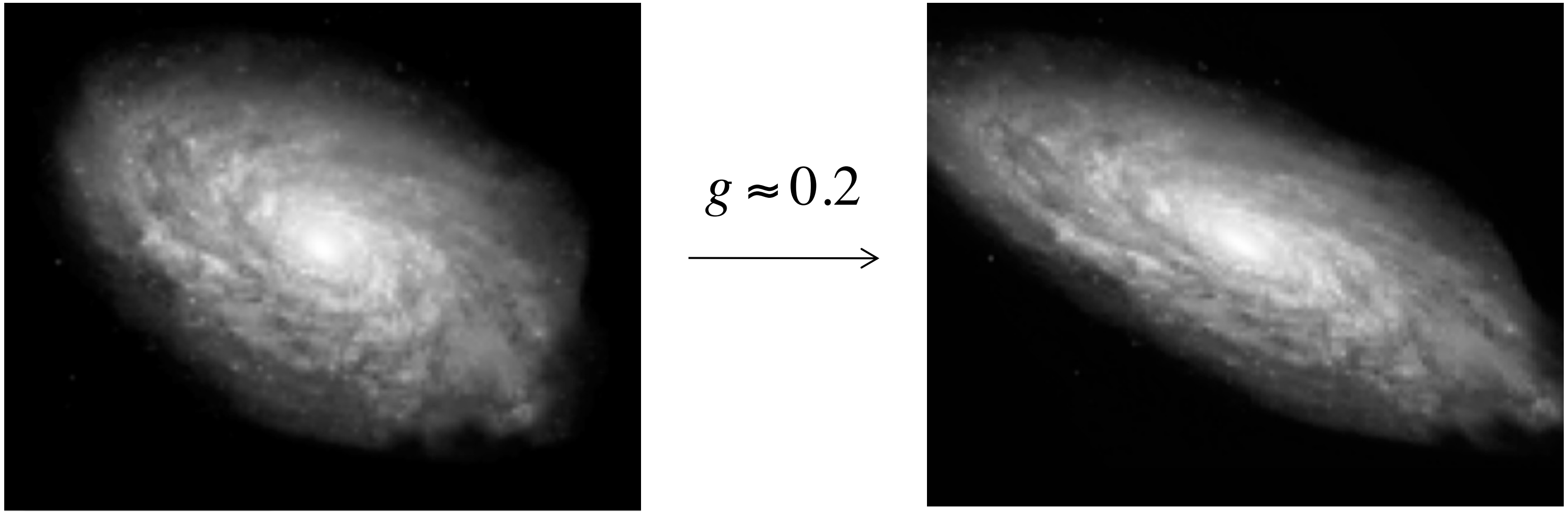


real data $g \approx 0.03$

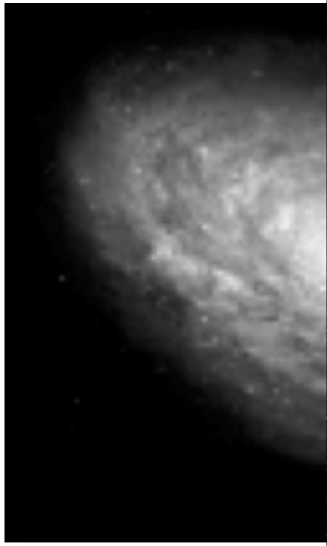
- image distortion

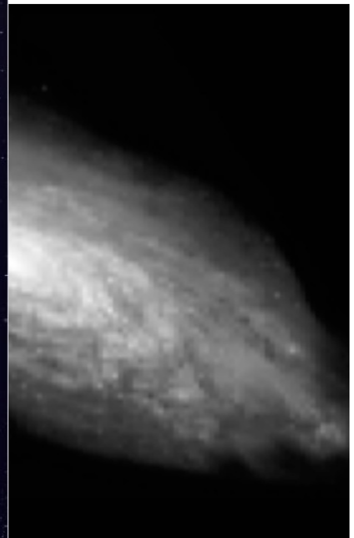


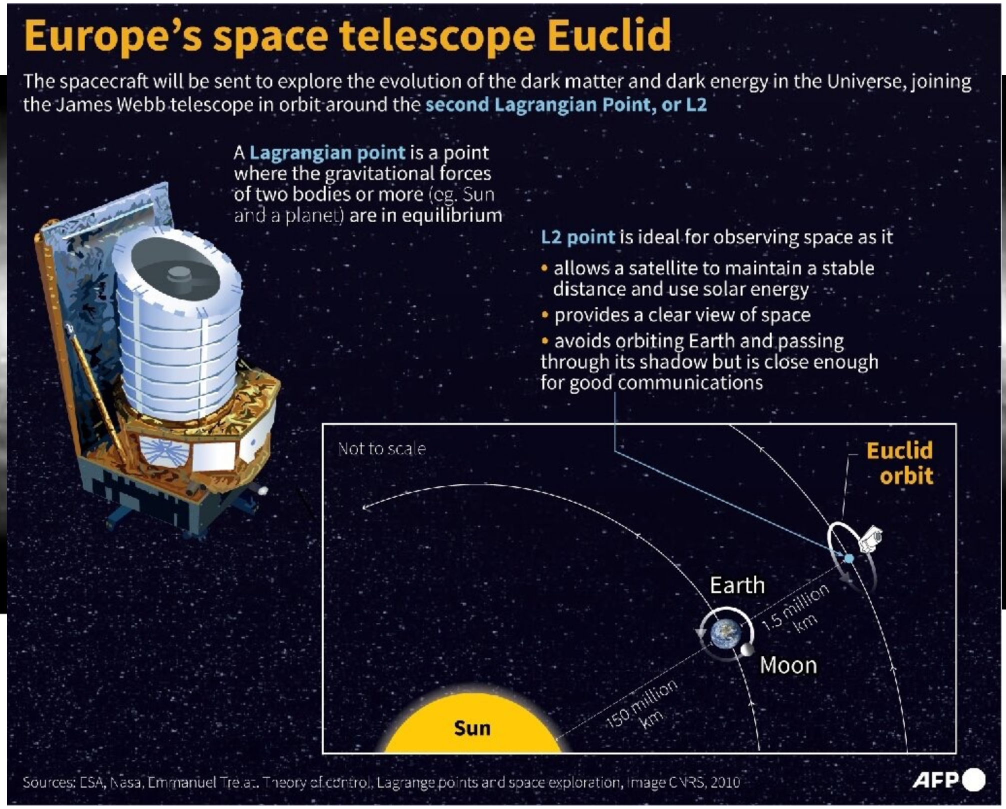
real data $g \approx 0.03$

=> necessity for *huge* surveys to obtain decent statistics!

- image distortion







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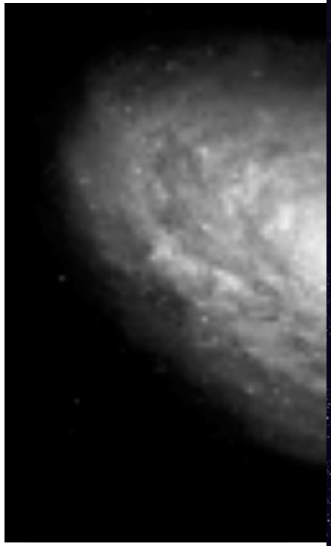
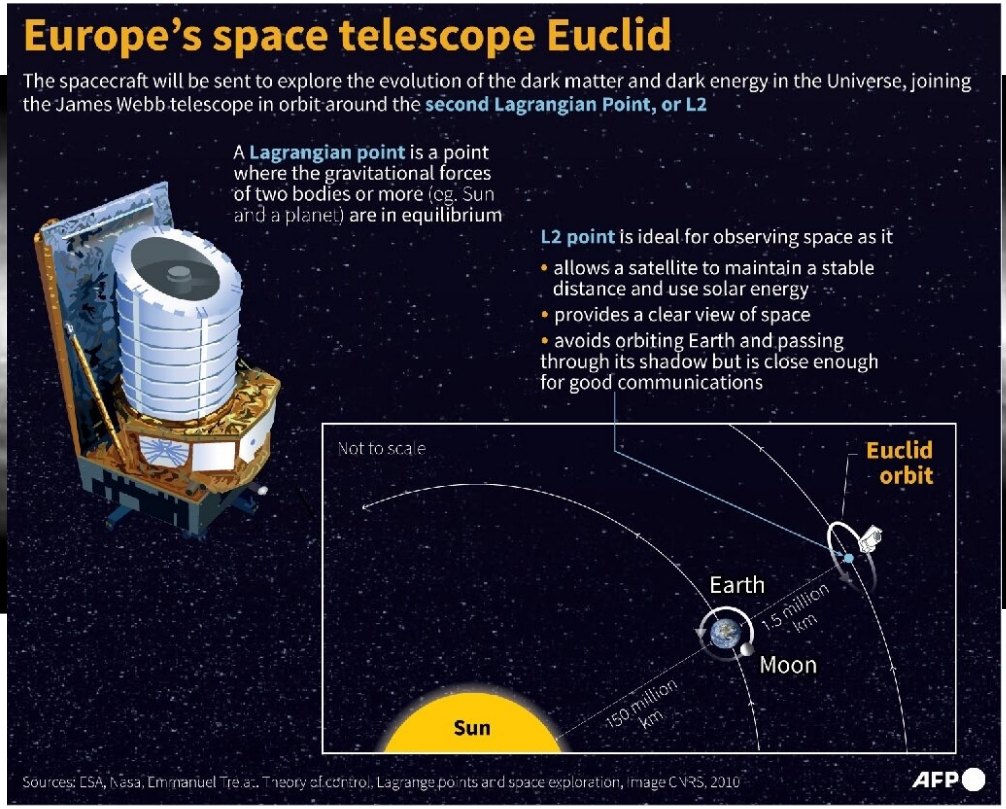
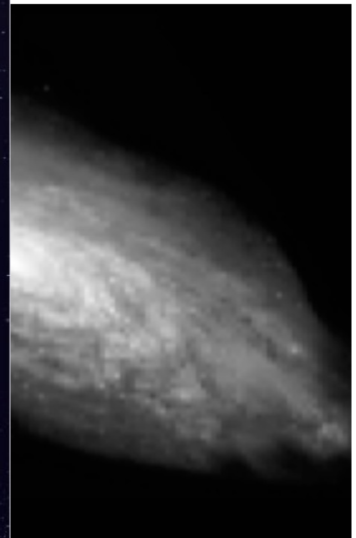






=> necessity for *huge* surveys to obtain decent statistics!

STEP (Shear Testing Programme*):

Collaborative project/forum to improve the accuracy and reliability of all weak gravitational lensing measurements in preparation for the next generation of wide-field surveys!

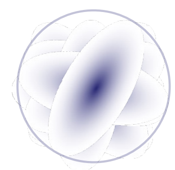
*http://www.roe.ac.uk/~heymans/step/cosmic_shear_test.html

- image distortion

small patch on sky filled with (elliptical) galaxies
(unrelated objects with different redshifts)

- image distortion

small patch on sky filled with (elliptical) galaxies
(unrelated objects with different redshifts)

=> the average shape will be circular: 

- image distortion

small patch on sky filled with (elliptical) galaxies
(unrelated objects with different redshifts)

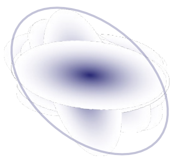
+ weak gravitational lensing!
(lightpaths become related)

- image distortion

small patch on sky filled with (elliptical) galaxies
(unrelated objects with different redshifts)

+ weak gravitational lensing!
(lightpaths become related)

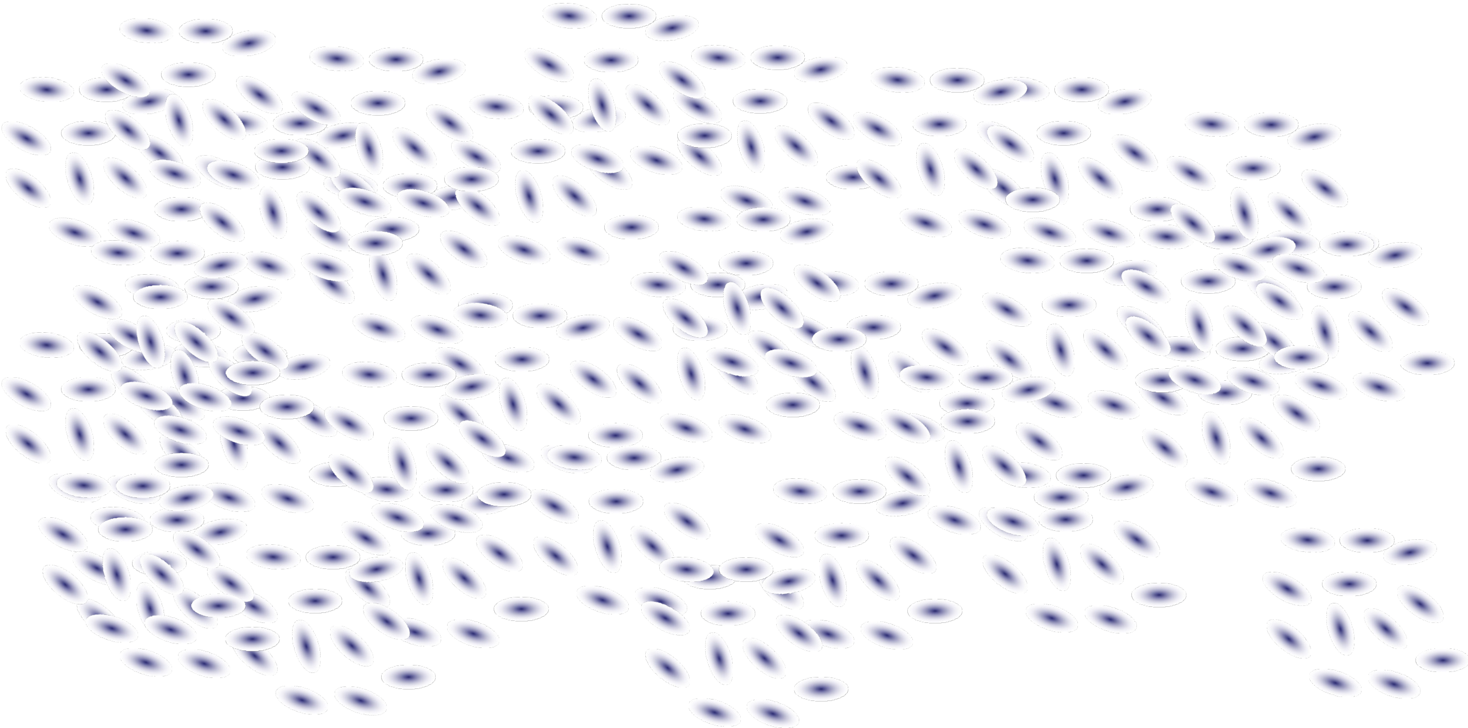
=> the average shape will be elliptical:



- image distortion – shear map on the sky?!

- image distortion – shear map on the sky!

sky filled with (elliptical) galaxies
(unrelated objects with different redshifts)



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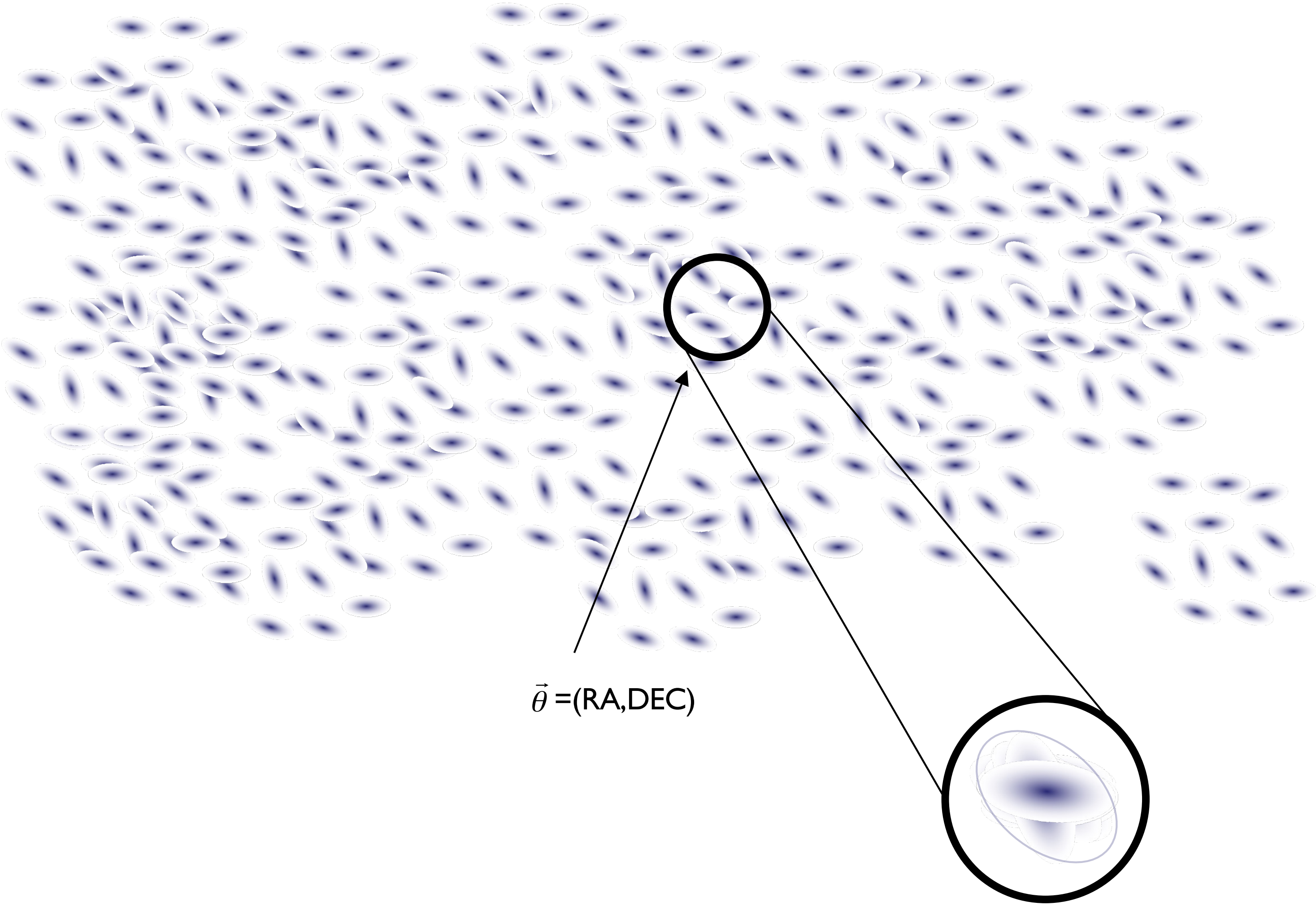

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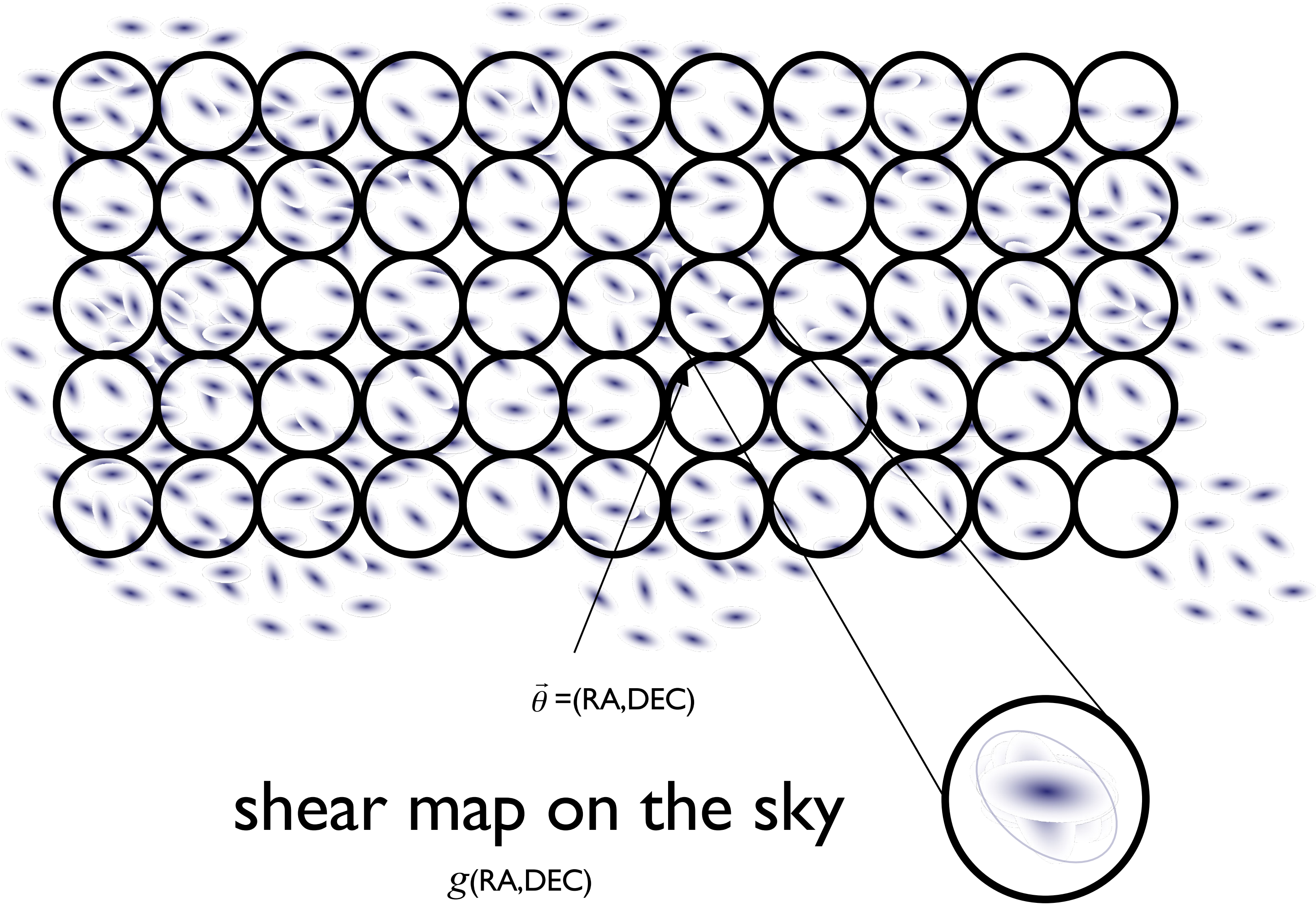


- image distortion – shear map on the sky!

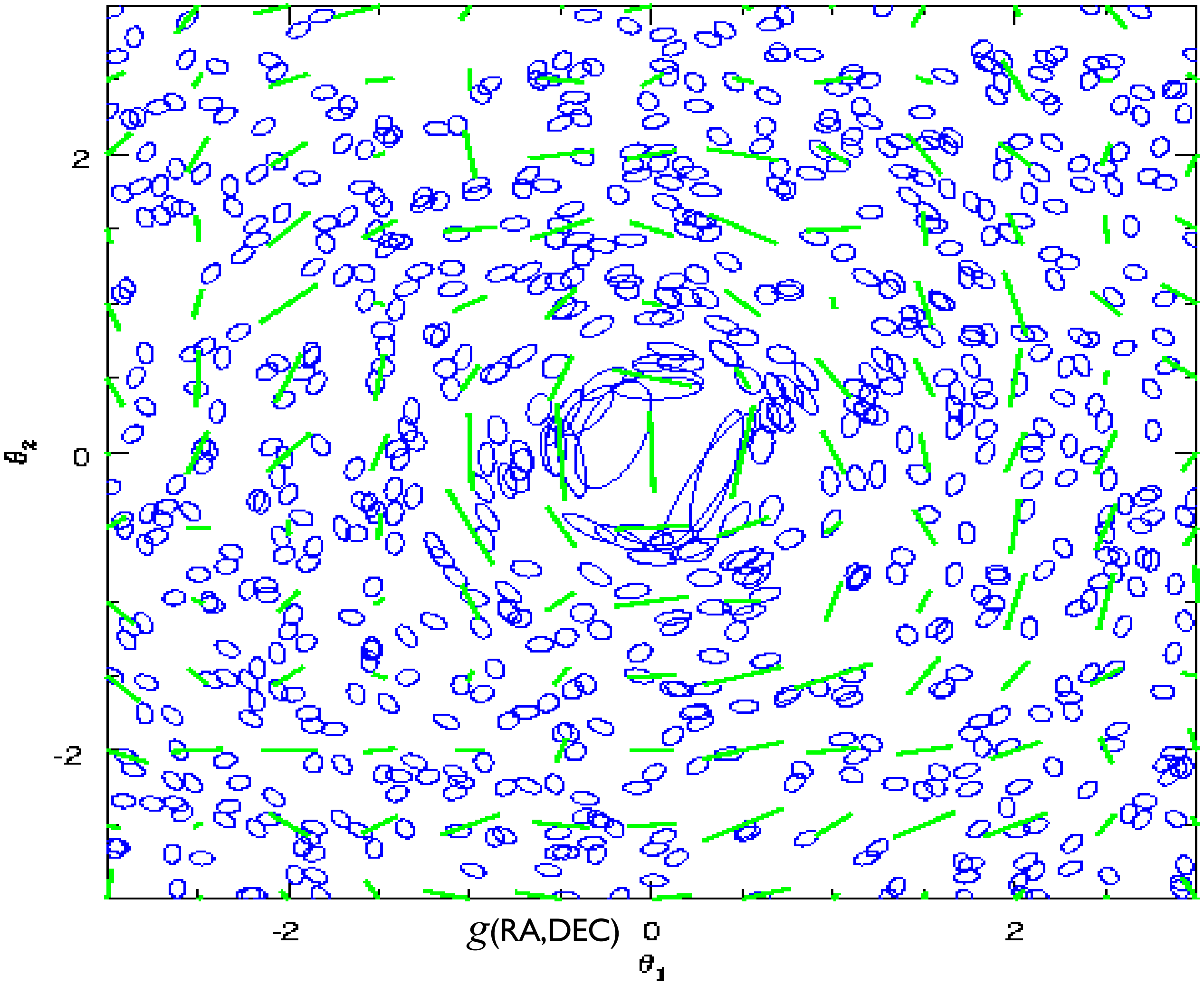
sky filled with (elliptical) galaxies
(unrelated objects with different redshifts)

g(RA,DEC)

- image distortion – shear map on the sky!

sky filled with (elliptical) galaxies

(unrelated objects with different redshifts)



g(RA,DEC)

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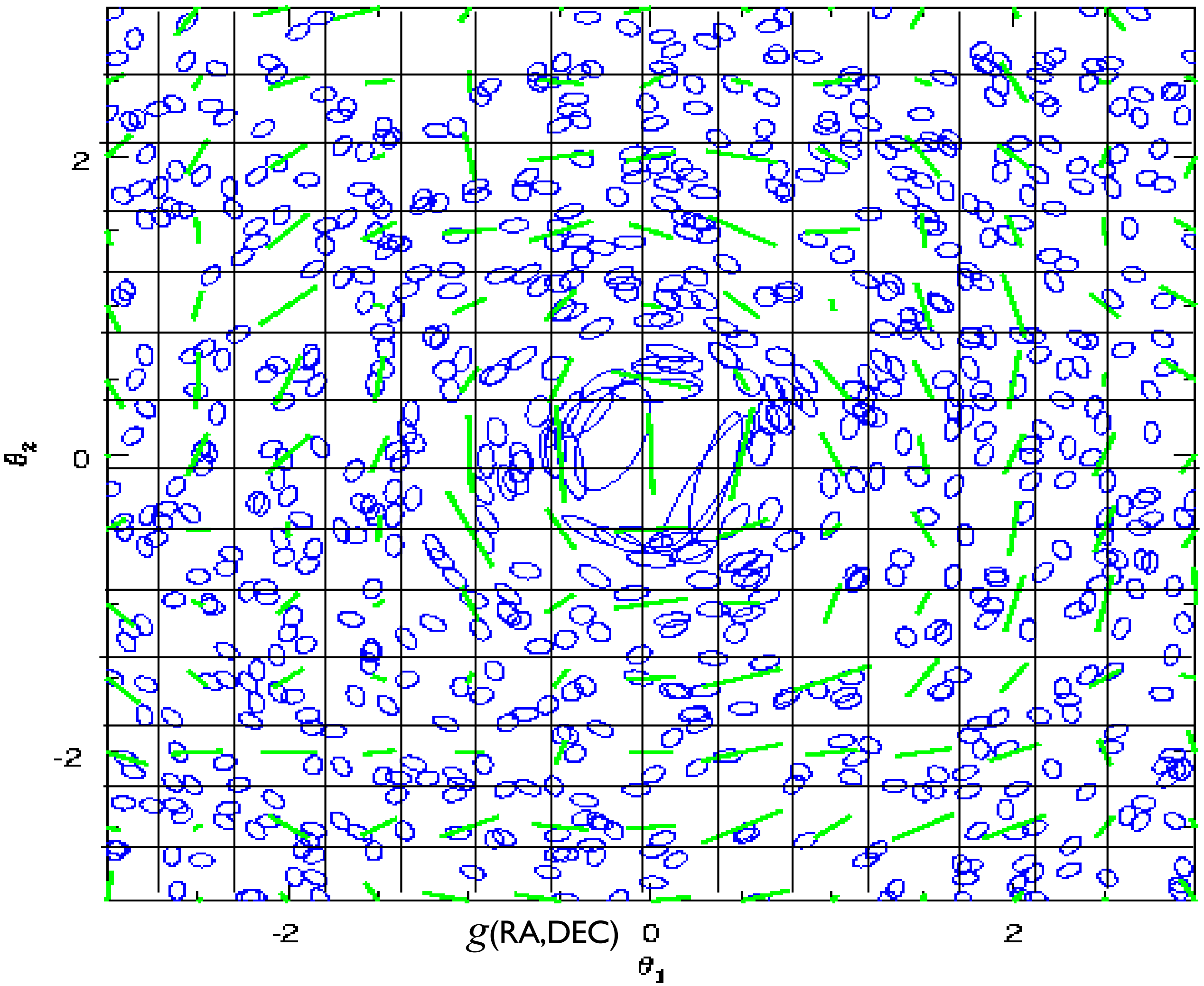

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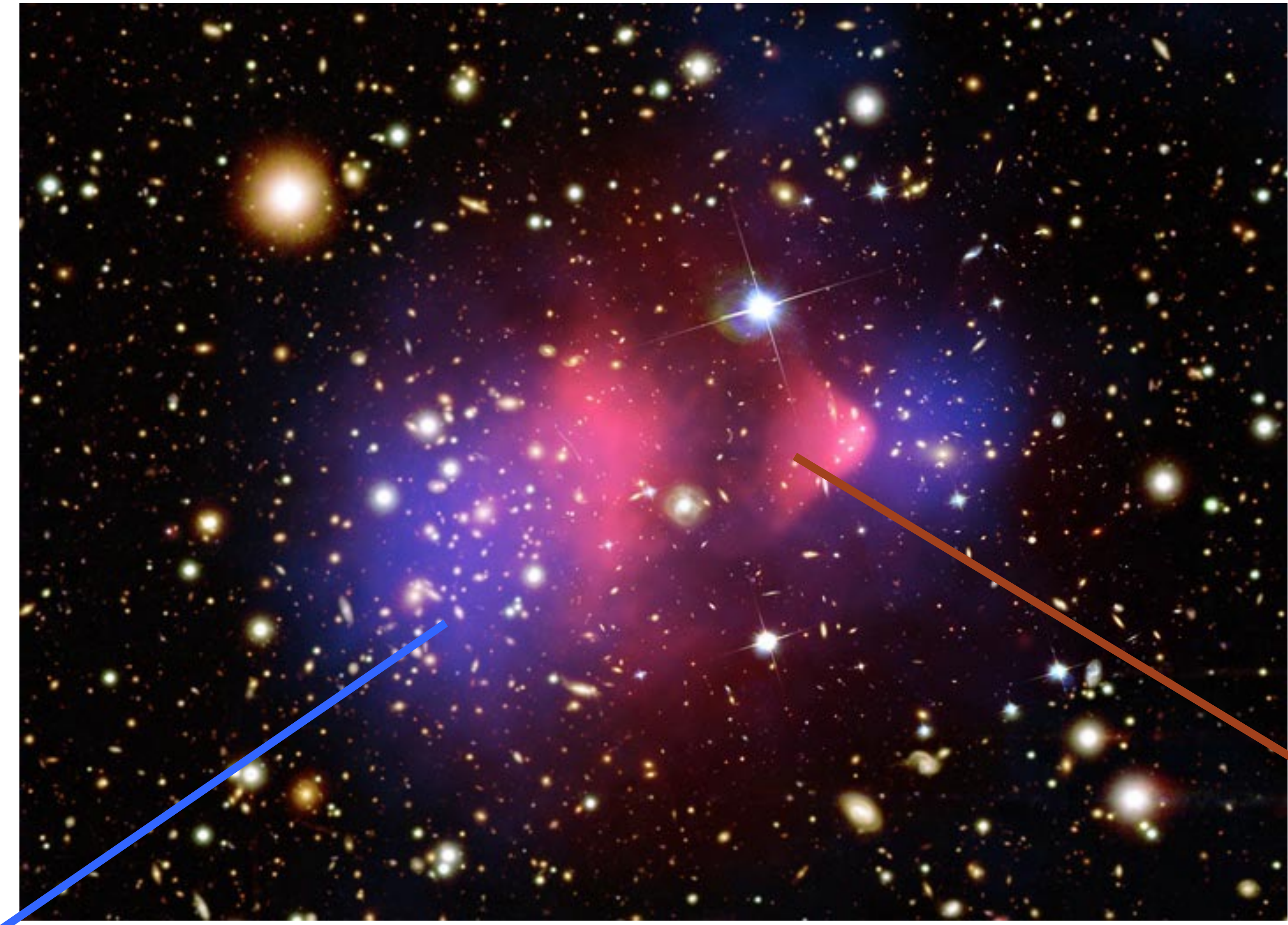

g(RA,DEC)

X-ray gas

gravitating matter

- image distortion – shear map on the sky!

sky filled with (elliptical) galaxies
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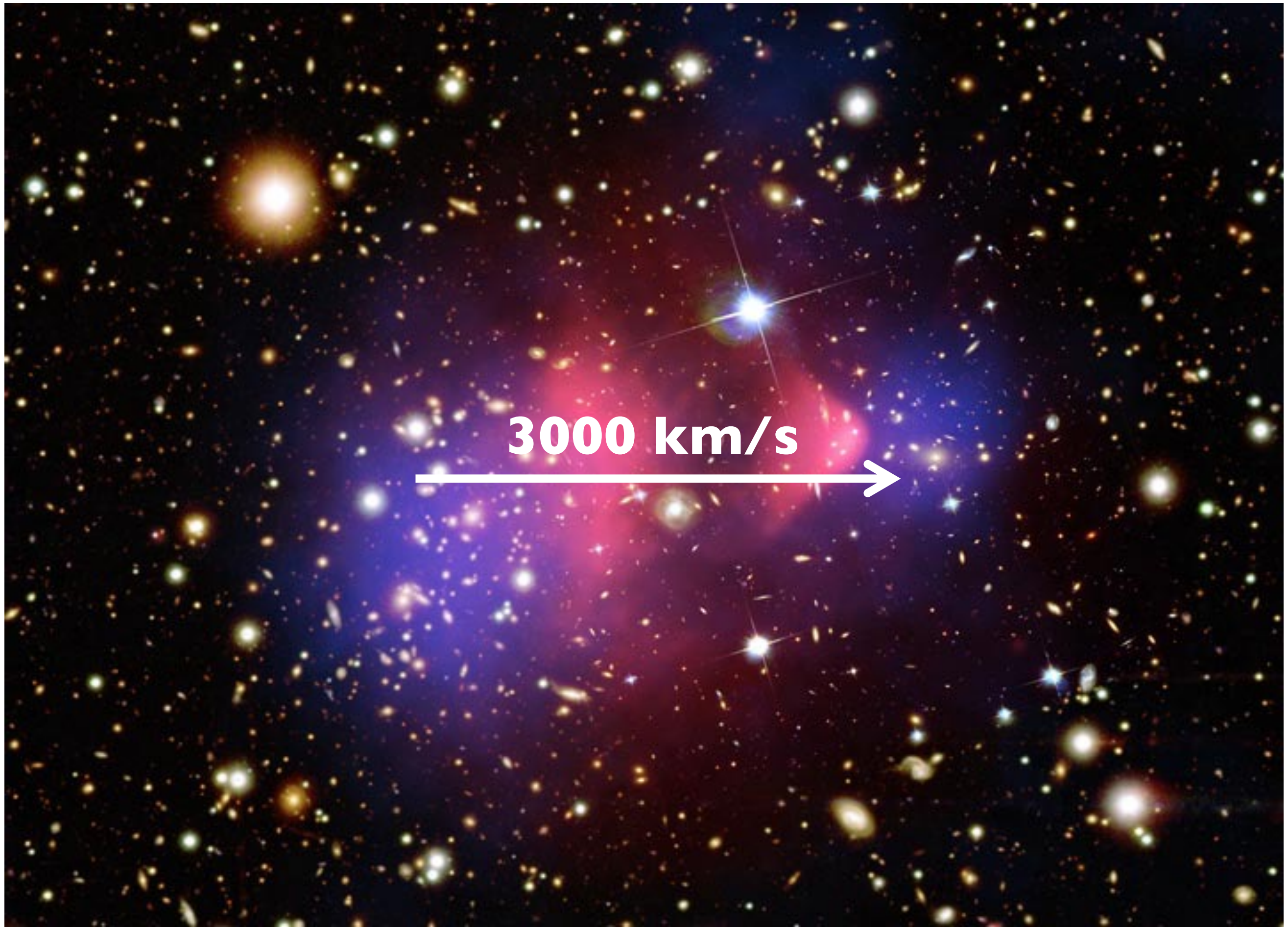

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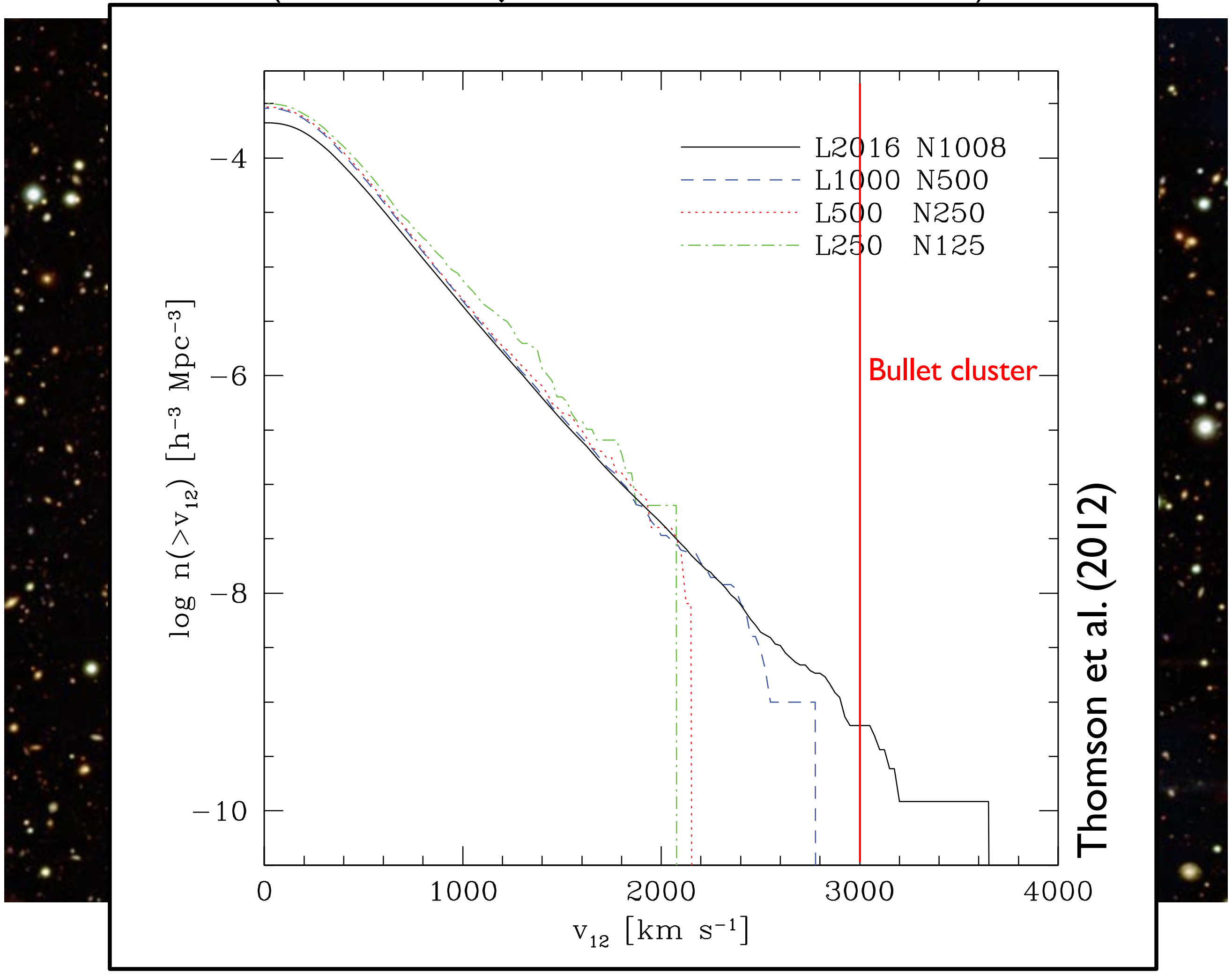
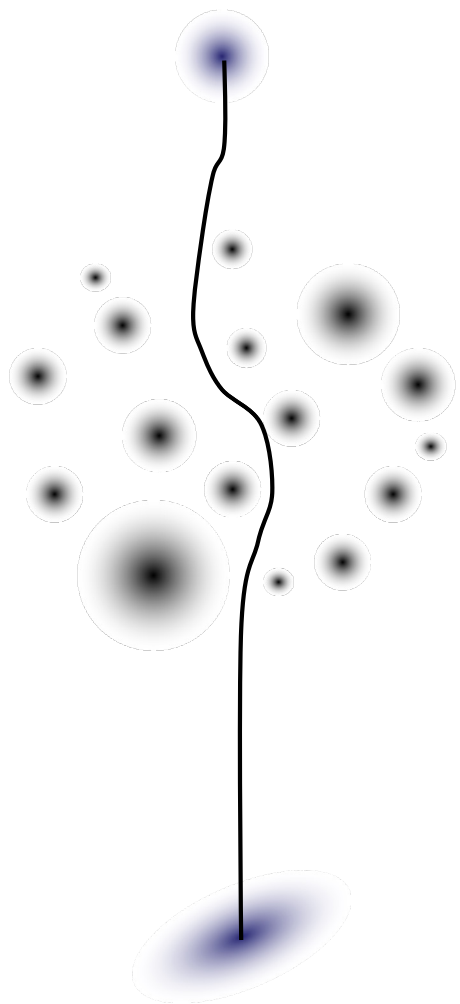
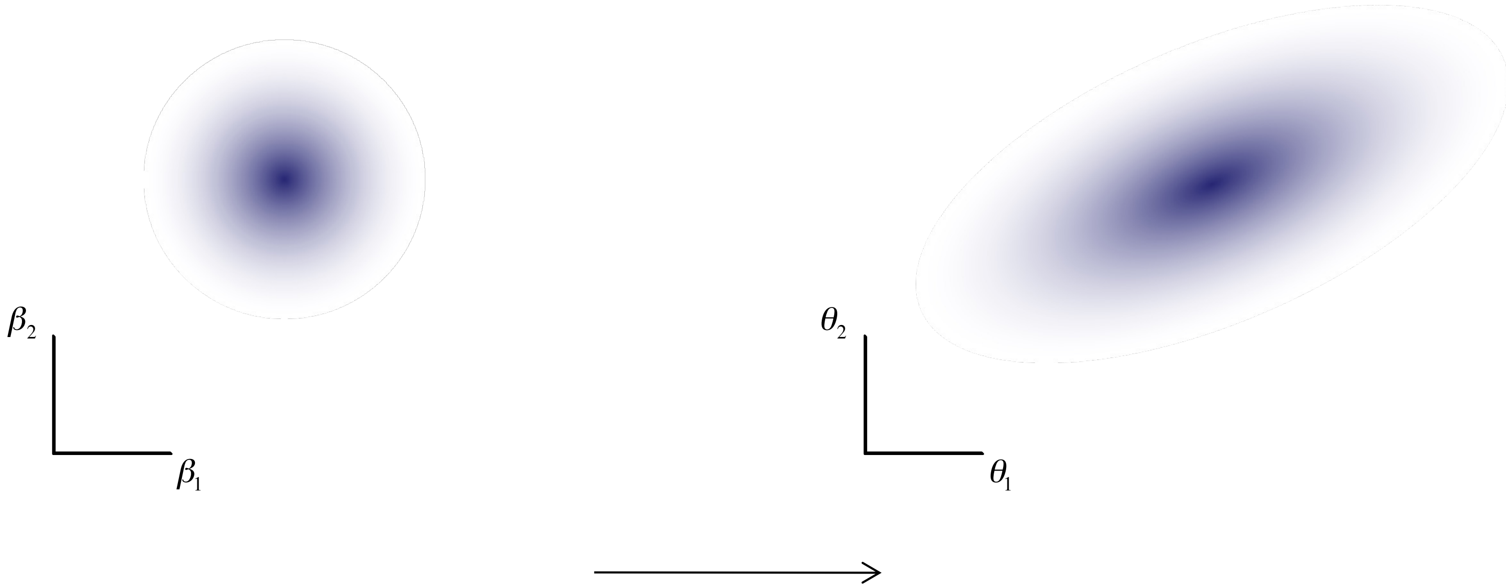


- concept
- **theory**
- application

- the distortion matrix



- the distortion matrix

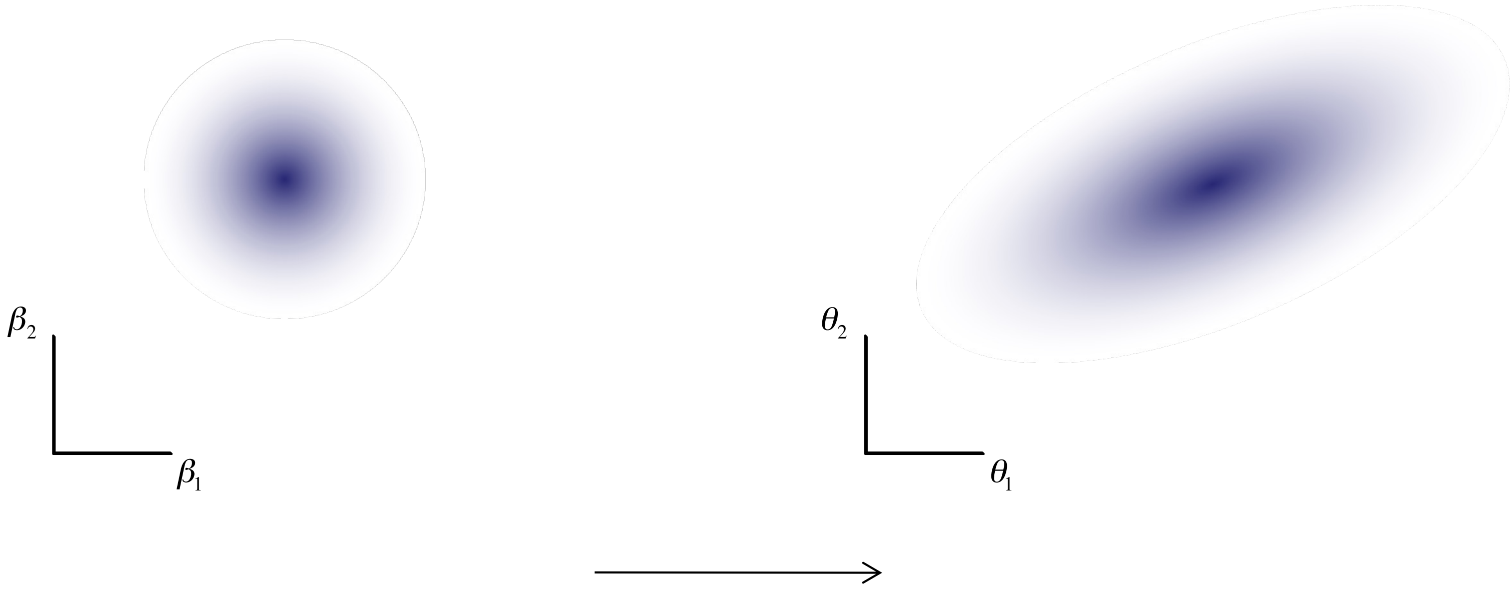


coordinate transformation from $\boldsymbol{\beta}$ to $\boldsymbol{\theta}$:

$$\beta = \theta - \alpha(\theta)$$

(the lens equation)

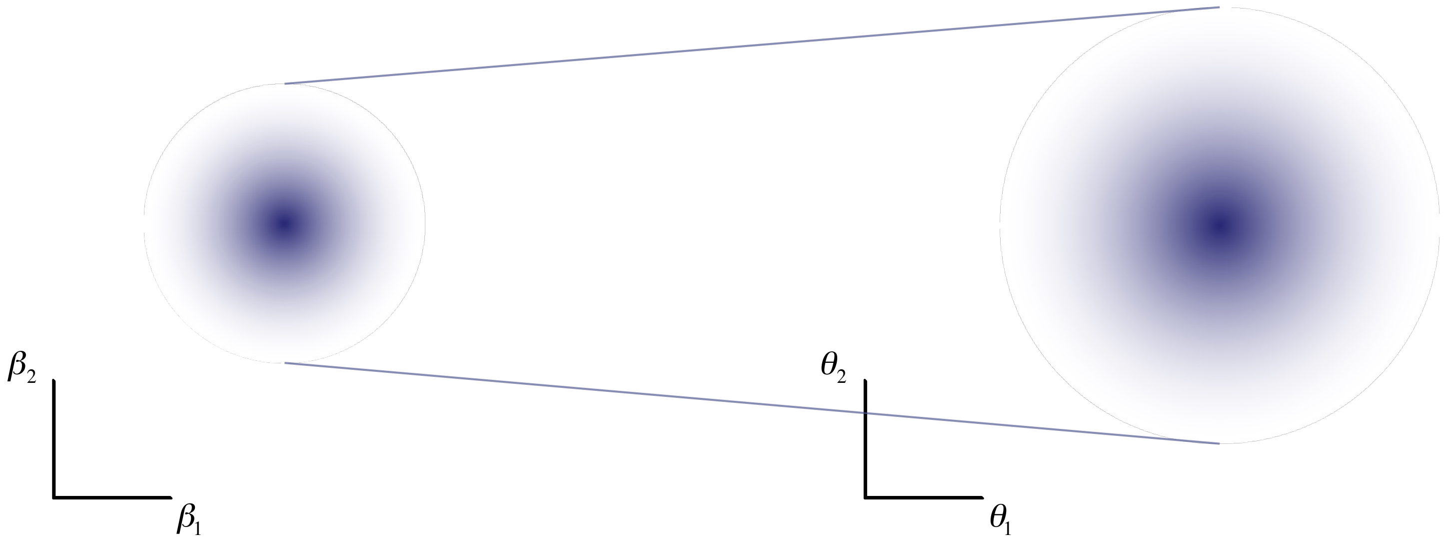
- the distortion matrix



coordinate transformation from $\boldsymbol{\beta}$ to $\boldsymbol{\theta}$:

$$A_{ij} = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

- the distortion matrix



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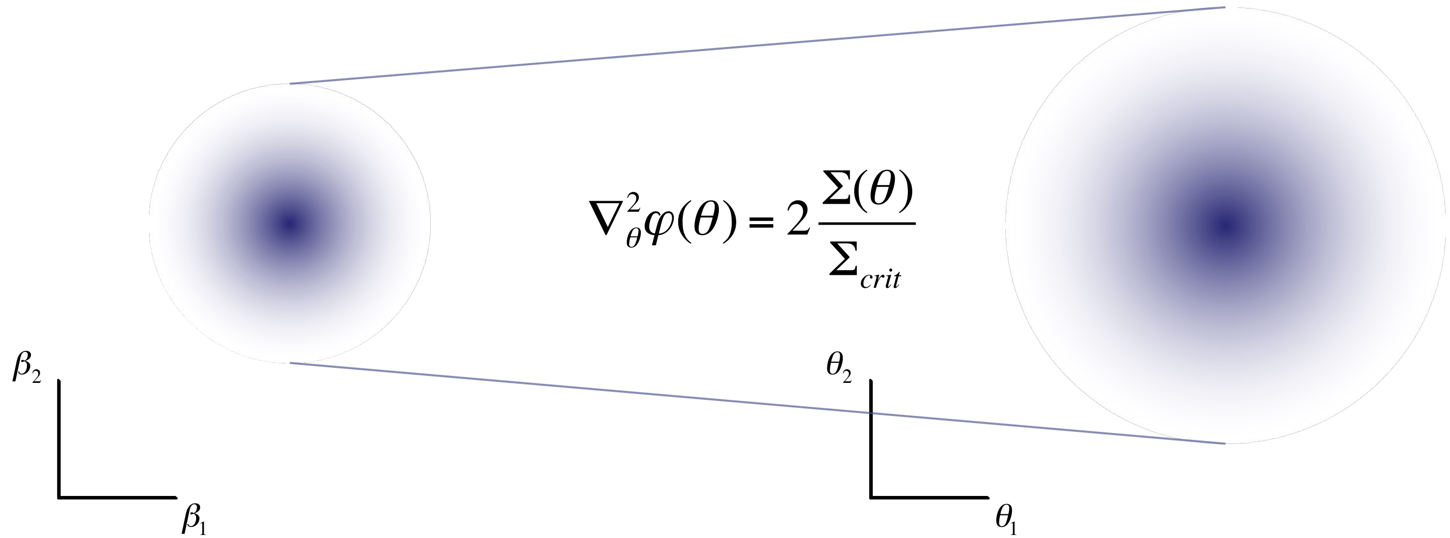

$$\nabla_\theta^2 \varphi(\theta) = 2\frac{\Sigma(\theta)}{\Sigma_{crit}}$$

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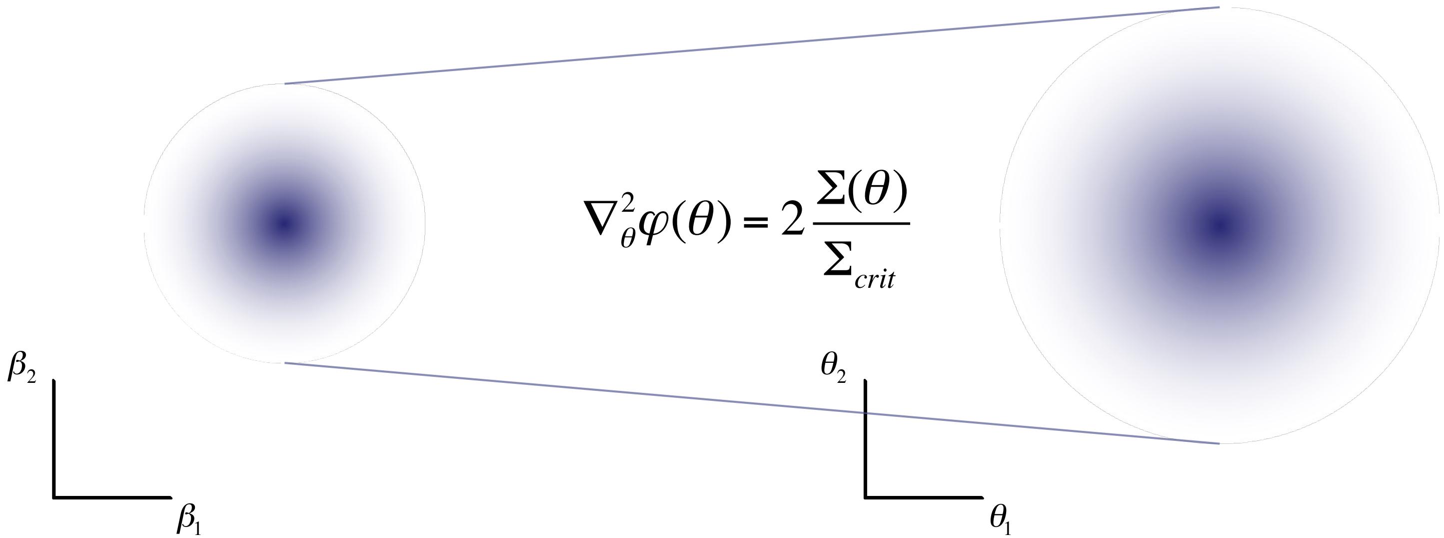

$$\nabla_\theta^2 \varphi(\theta) = 2\frac{\Sigma(\theta)}{\Sigma_{crit}}$$

(exercise) $\kappa = \frac{1}{2}\left(\partial_{11}\varphi + \partial_{22}\varphi\right)$

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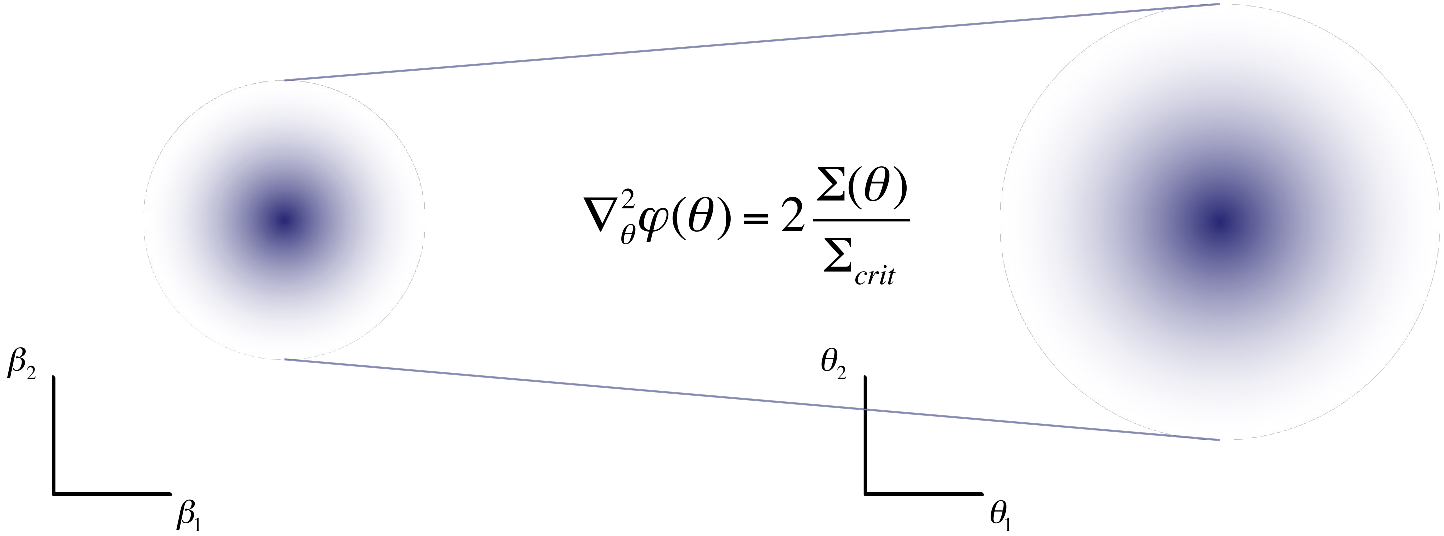
- the distortion matrix



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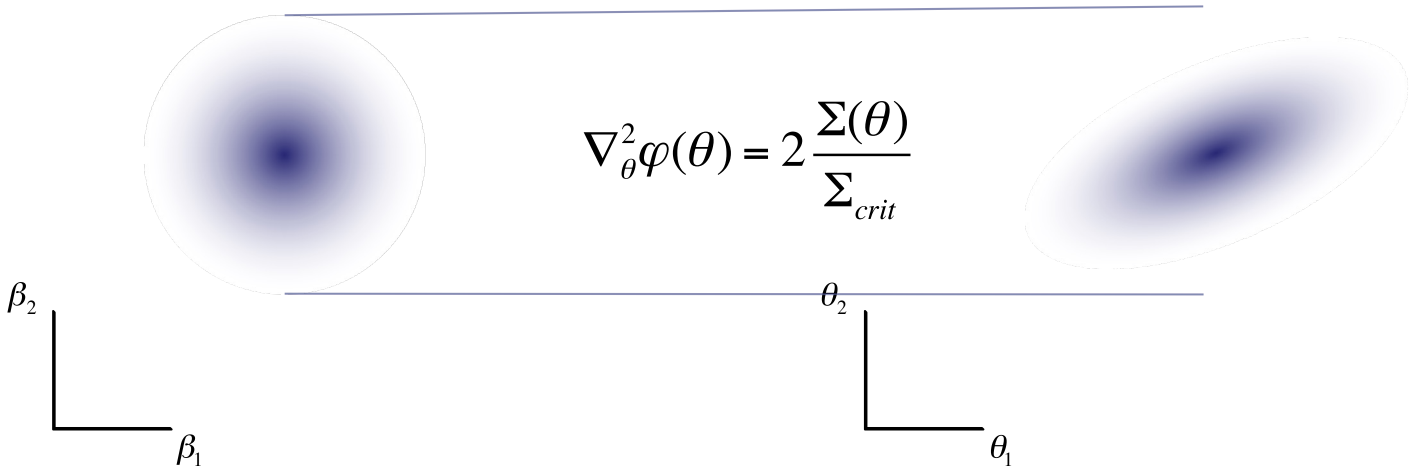


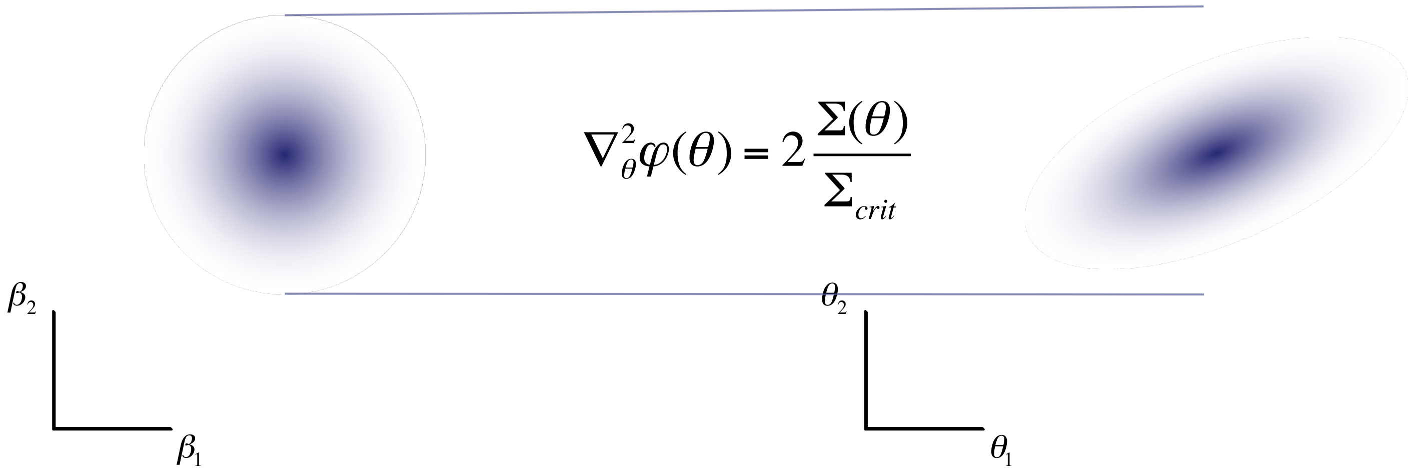
$$\nabla_\theta^2 \varphi(\theta) = 2\frac{\Sigma(\theta)}{\Sigma_{crit}}$$

$$\kappa = \frac{1}{2}\left(\partial_{11}\varphi + \partial_{22}\varphi\right)$$

$$\kappa = \frac{\Sigma(\theta)}{\Sigma_{crit}}$$

$$A_{ij} = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

- the distortion matrix



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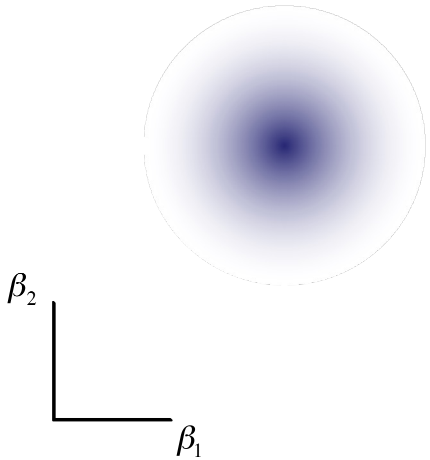
$$\kappa = \frac{\Sigma(\theta)}{\Sigma_{crit}}$$

$$\gamma_1 = \frac{1}{2}\left(\partial_{11}\varphi - \partial_{22}\varphi\right)$$

(exercise) $\gamma_2 = \partial_{12}\varphi - \partial_{21}\varphi$

- the distortion matrix



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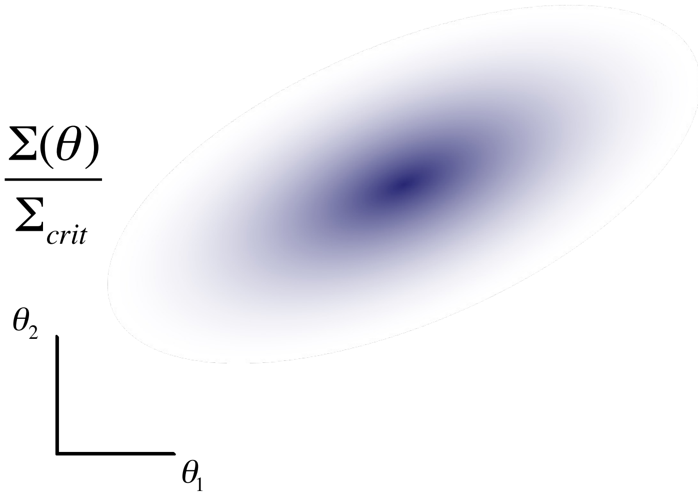

$$\kappa = \frac{1}{2}\left(\partial_{11}\varphi + \partial_{22}\varphi\right)$$

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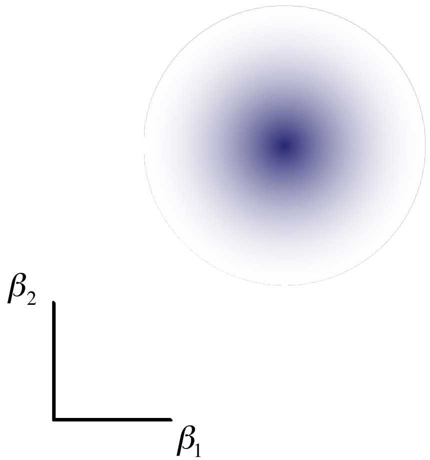
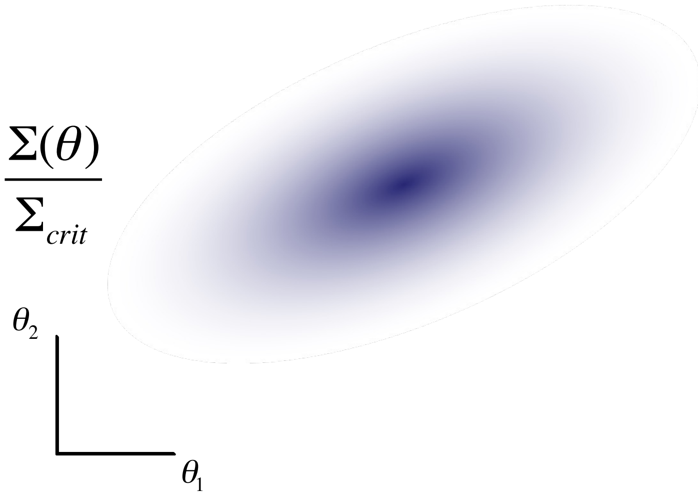
- the distortion matrix

relation to observables!?



$$\nabla_\theta^2 \varphi(\theta) = 2\,\frac{\Sigma(\theta)}{\Sigma_{crit}}$$



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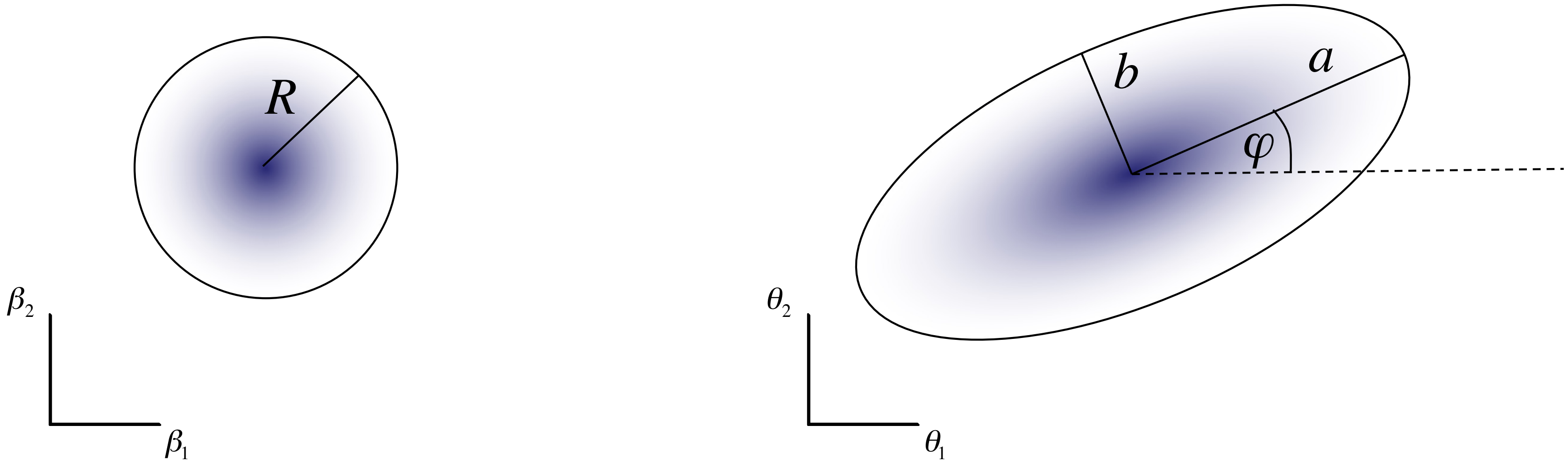

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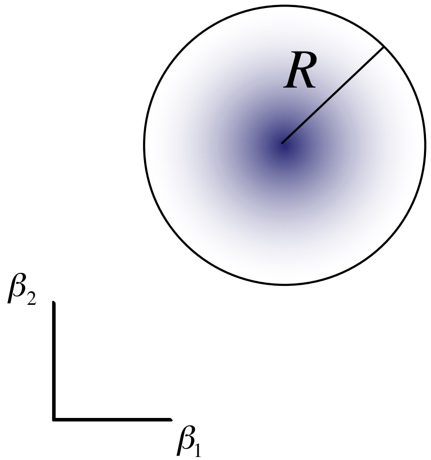
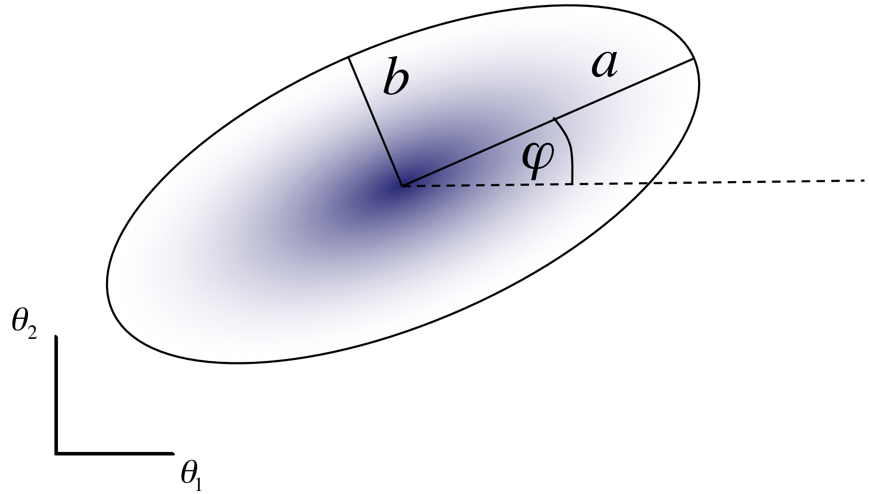
- the distortion matrix



$$\gamma = |\gamma| \, e^{i2\varphi}$$

$$a = \frac{R}{1-\kappa-|\gamma|}$$

$$b = \frac{R}{1-\kappa+|\gamma|}$$



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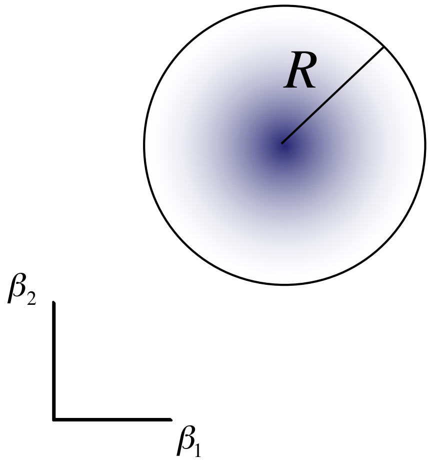

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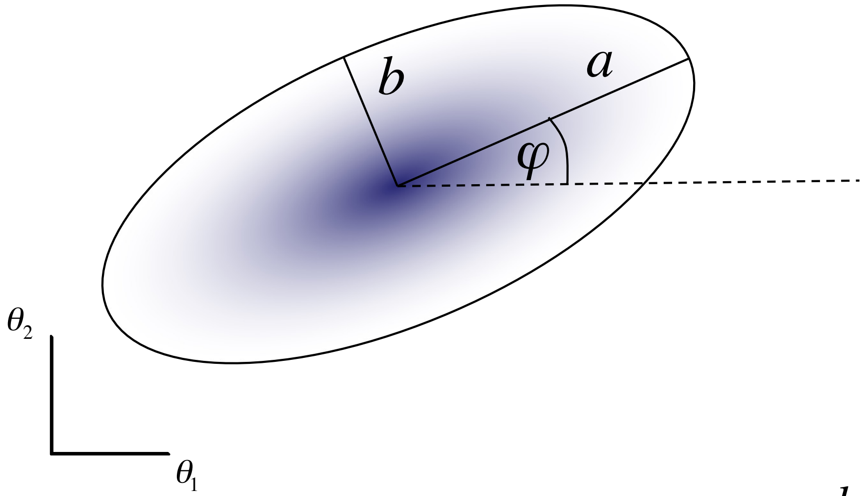

circular source => measuring a, b (and φ) gives reduced shear $g = \dfrac{|\gamma|}{1-\kappa} = \dfrac{1-\frac{b}{a}}{1+\frac{b}{a}}$

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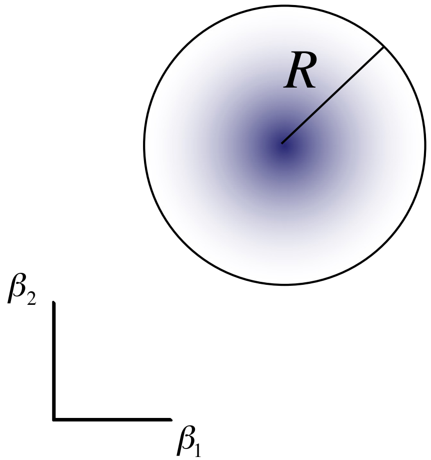
- the distortion matrix

how to actually measure the ellipticity?



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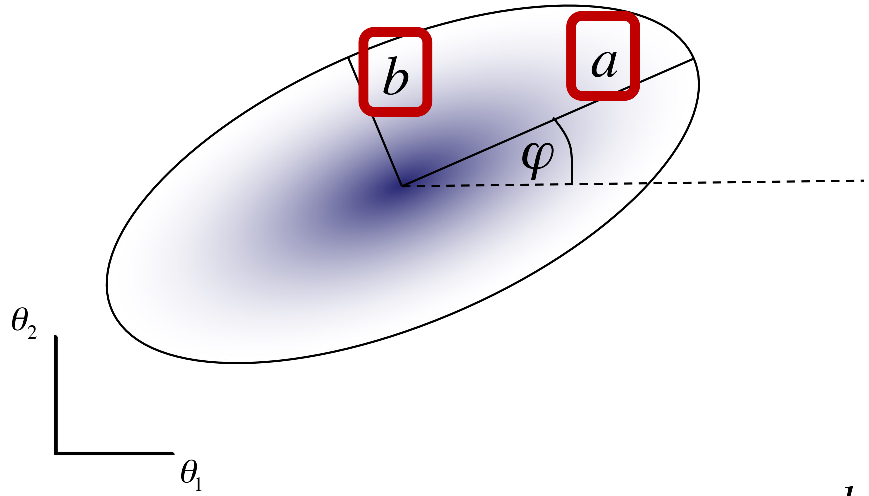

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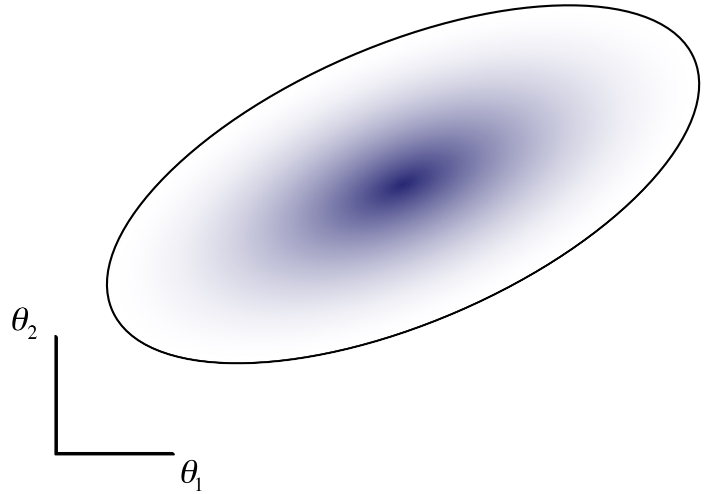
- galaxy shapes

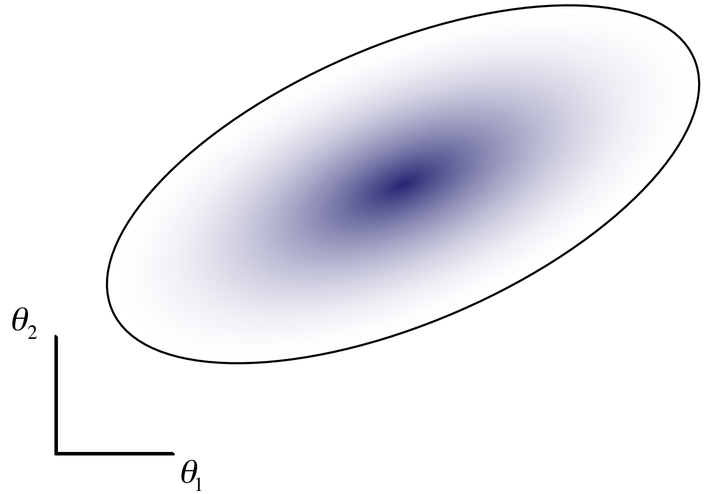


how to assign a measure for ellipticity to a galaxy image?

- galaxy shapes

1. find the image centre

$I(\boldsymbol{\theta})$ surface brightness

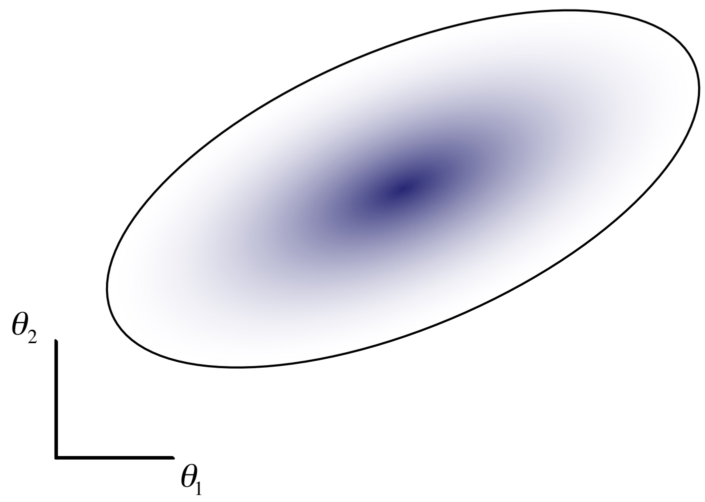


- galaxy shapes

1. find the image centre

$$\bar{\boldsymbol{\theta}} = \frac{\int \boldsymbol{\theta}\, I(\boldsymbol{\theta})\, q_I\big(I(\boldsymbol{\theta})\big)\, d^2\theta}{\int I(\boldsymbol{\theta})\, q_I\big(I(\boldsymbol{\theta})\big)\, d^2\theta}$$ (= image centre)

$I(\boldsymbol{\theta})$ surface brightness



▪ galaxy shapes

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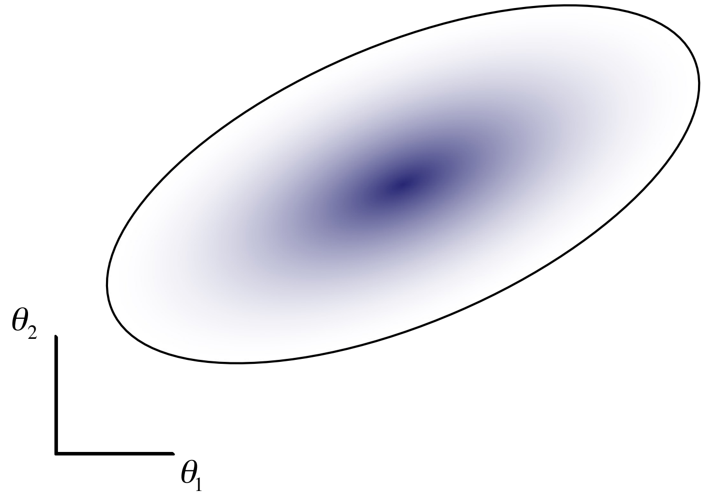

$q_I\big(I(\boldsymbol{\theta})\big)$: suitably chosen weight function (to compensate for telescope flaws: ?)

▪ galaxy shapes

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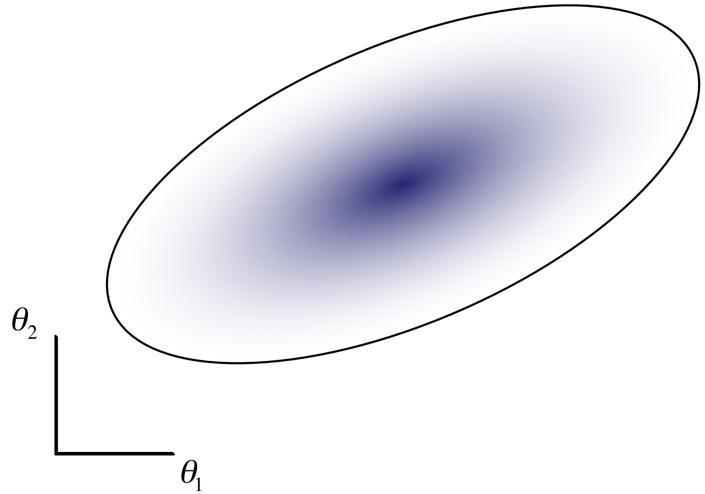
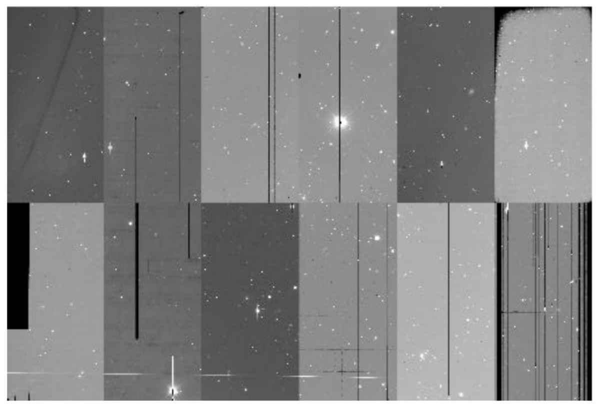


$q_I\big(I(\boldsymbol{\theta})\big)$: suitably chosen weight function (to compensate for telescope flaws: )

▪ galaxy shapes

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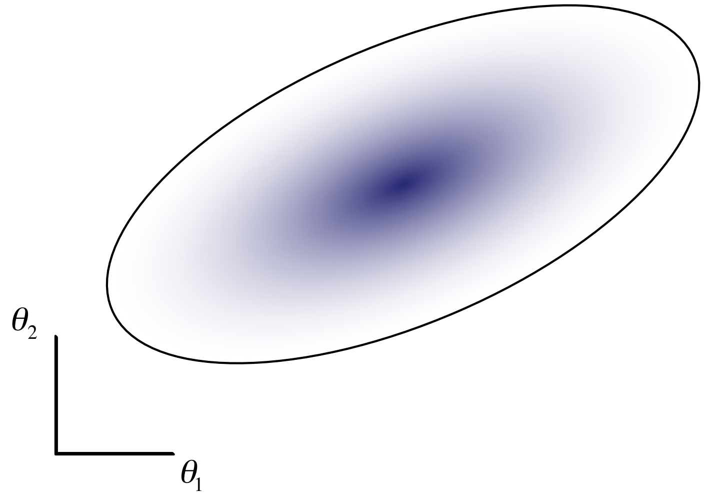

2. calculate its 2nd order moments on the sky

$$Q_{ij} = \frac{\int \Big[(\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)\Big]\, I(\boldsymbol{\theta})\, q_I\big(I(\boldsymbol{\theta})\big)\, d^2\theta}{\int I(\boldsymbol{\theta})\, q_I\big(I(\boldsymbol{\theta})\big)\, d^2\theta}$$

$q_I\big(I(\boldsymbol{\theta})\big)$: suitably chosen weight function

▪ galaxy shapes

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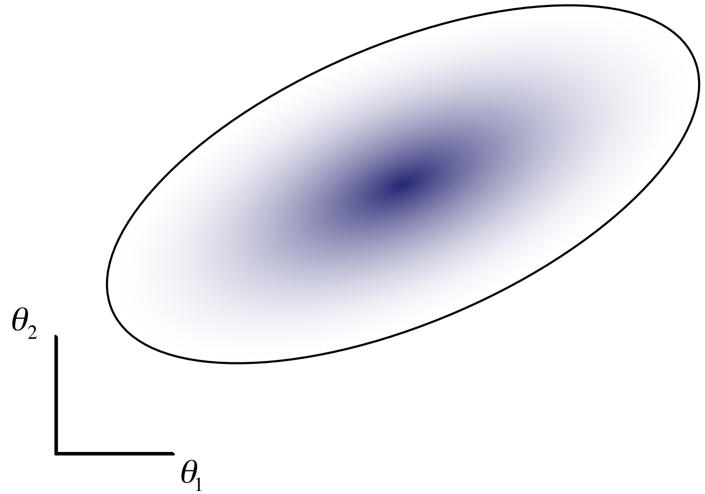
$I(\boldsymbol{\theta})$ surface brightness



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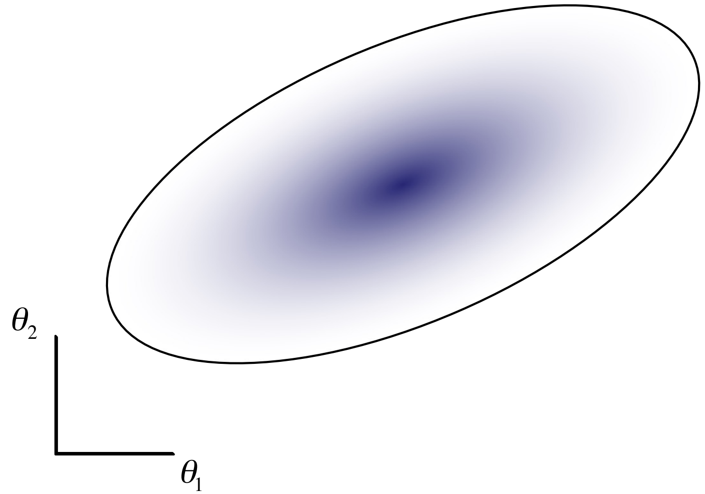
3. define its ellipticity from the moments

$$\varepsilon \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2\,(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

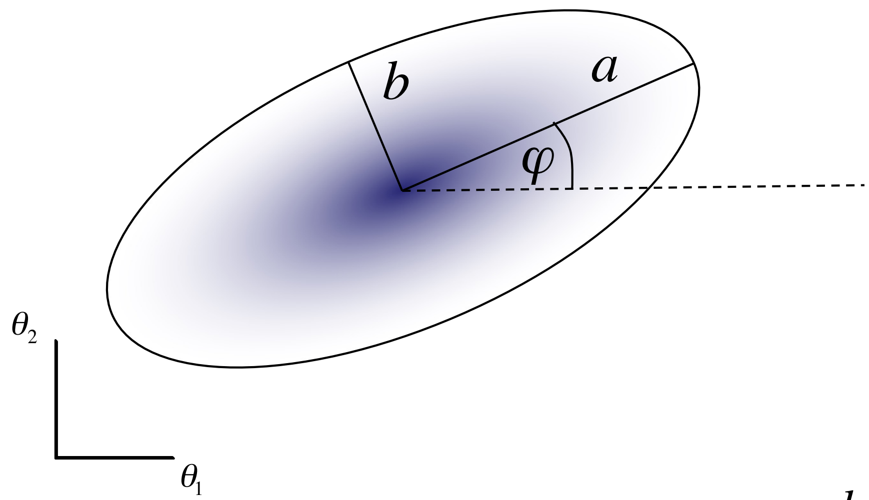
$q_I\big(I(\boldsymbol{\theta})\big)$: suitably chosen weight function

- galaxy shapes



• how to actually measure ellipticity: $\varepsilon \equiv \dfrac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2\,(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$

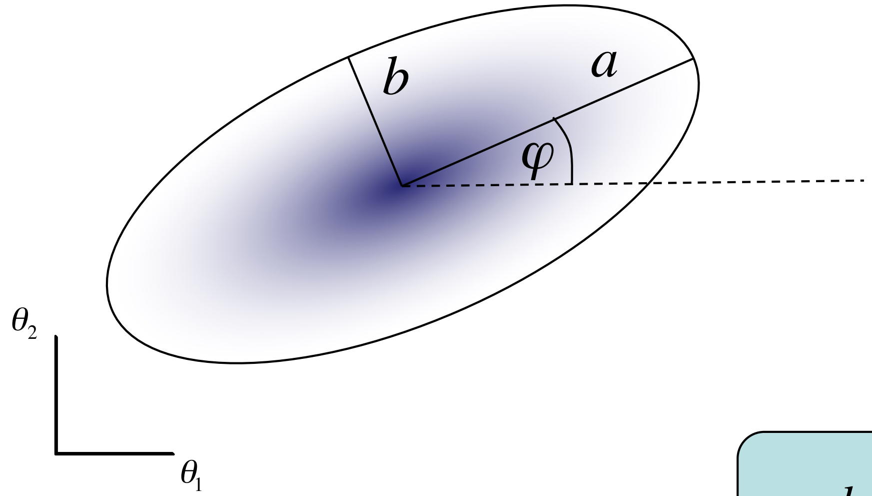
- galaxy shapes



measuring a, b (and φ) gives reduced shear $g = \frac{|\gamma|}{1-\kappa} = \frac{1-\frac{b}{a}}{1+\frac{b}{a}}$

measure ellipticity: $\varepsilon \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2\,(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$

- galaxy shapes



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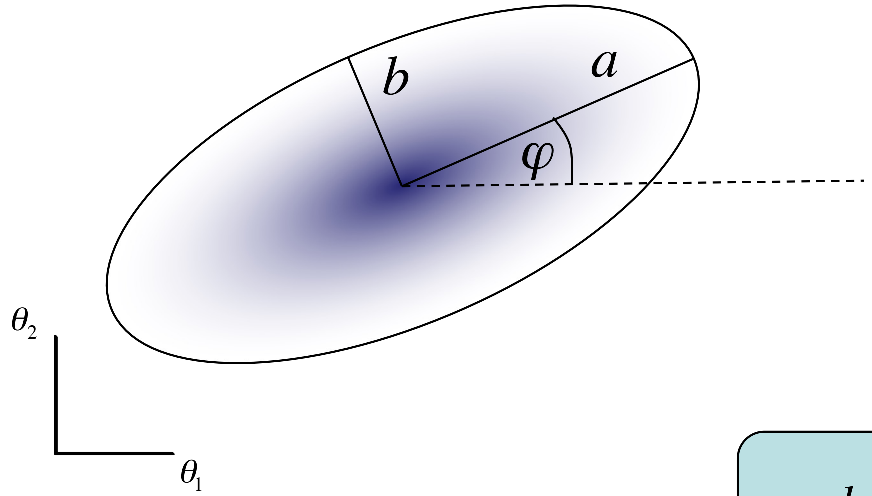
how is ε related to a and b?

- galaxy shapes



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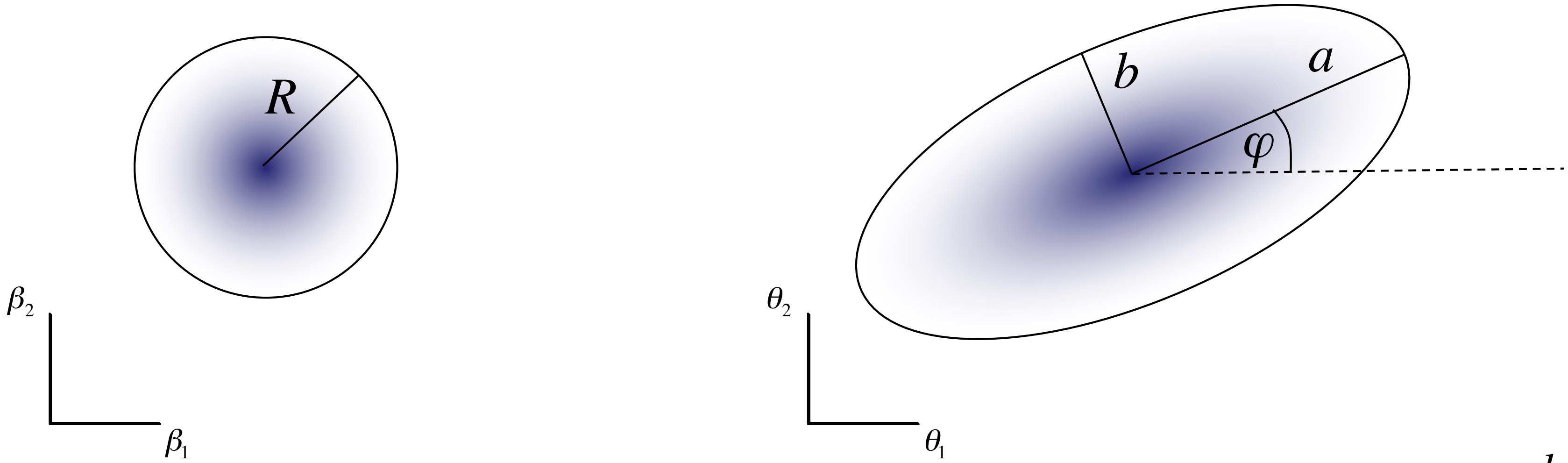
how is ε related to a and b? ...we'll see later that this is not needed!

- galaxy shapes



circular source => measuring a, b (and φ) gives reduced shear $g = \frac{|\gamma|}{1-\kappa} = \frac{1-\frac{b}{a}}{1+\frac{b}{a}}$

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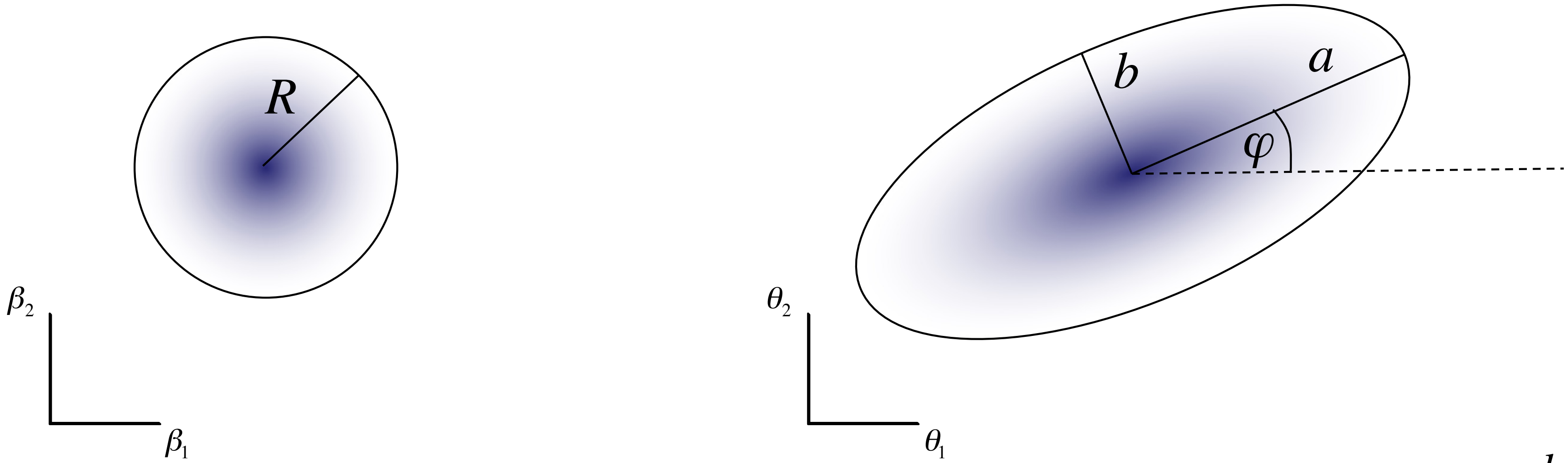
- the distortion matrix



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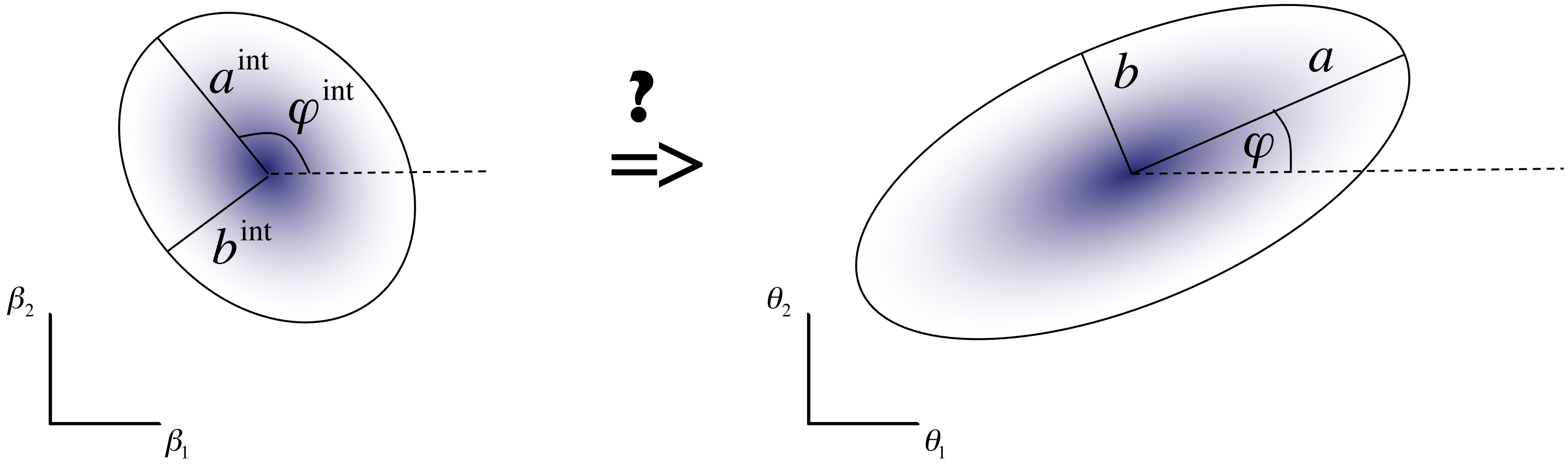
- galaxy shapes



elliptical source => measuring a, b (and φ) does not give reduced shear

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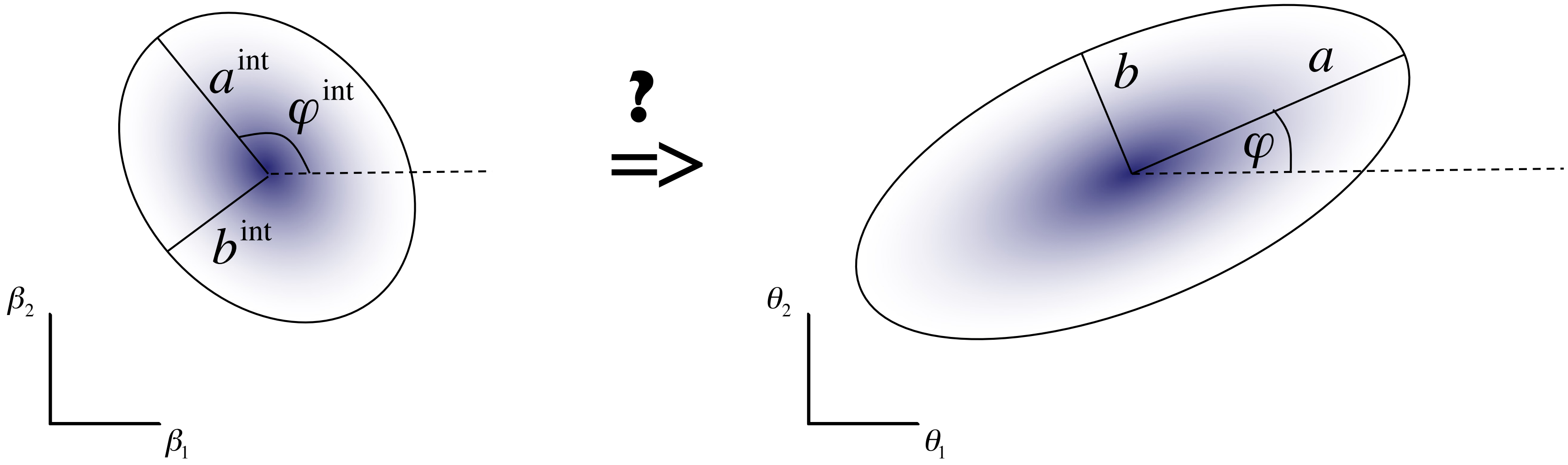
- galaxy shapes



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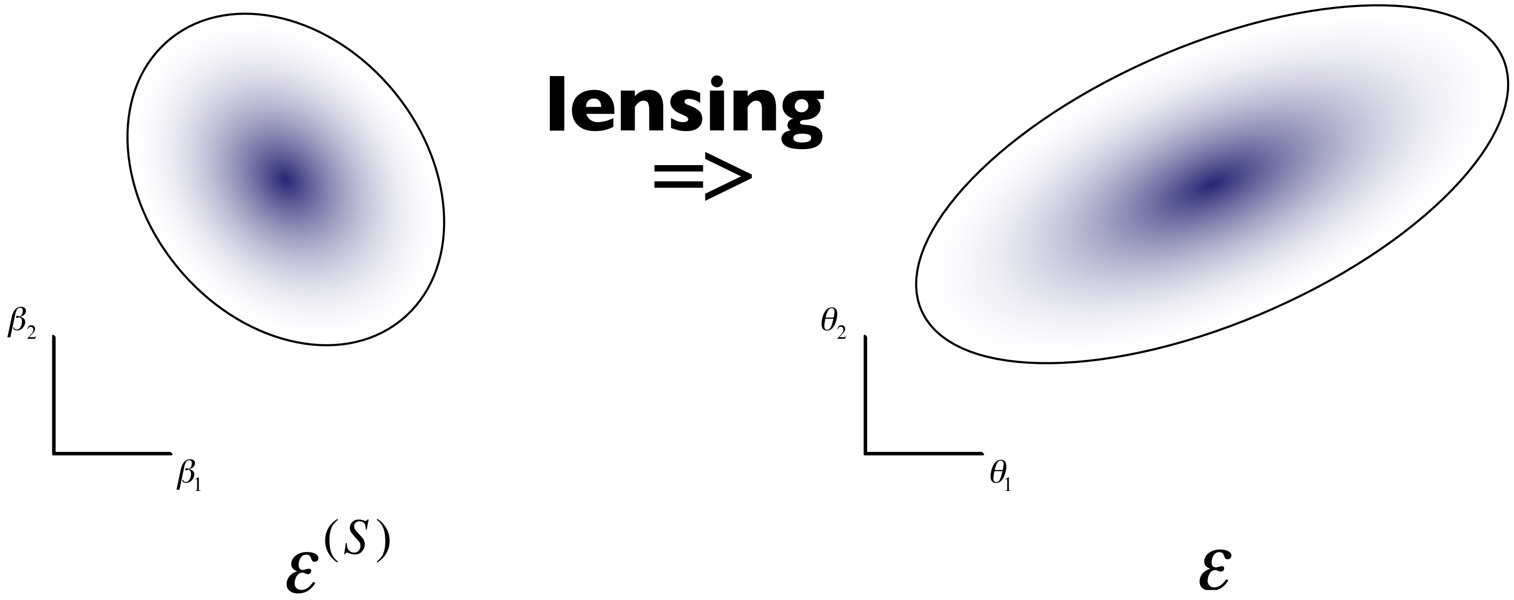
how to derive (reduced) shear from ε?

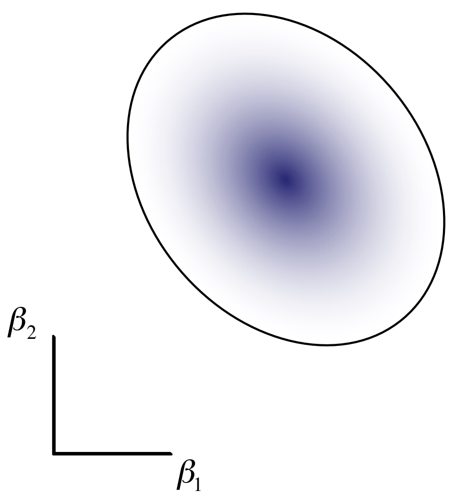
- galaxy shapes

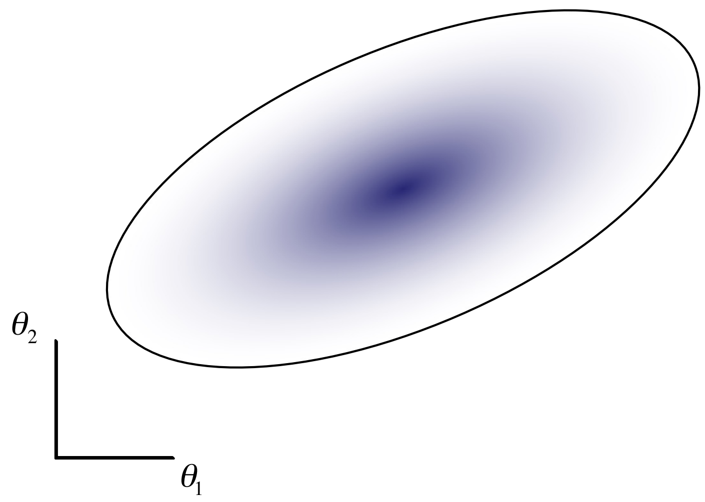


ε: measured ellipticity

$\varepsilon^{(S)}$: source ellipticity

- galaxy shapes





$\varepsilon^{(S)}$ ε

lens equation* => $$\varepsilon = \frac{\varepsilon^{(S)} + g}{1 - g^{*}\varepsilon^{(S)}}$$ with $g = \frac{\gamma}{1-\kappa}$ (reduced shear)

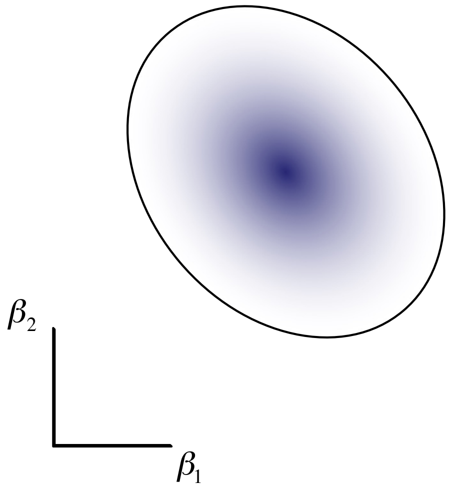
ε: measured ellipticity

$\varepsilon^{(S)}$: source ellipticity

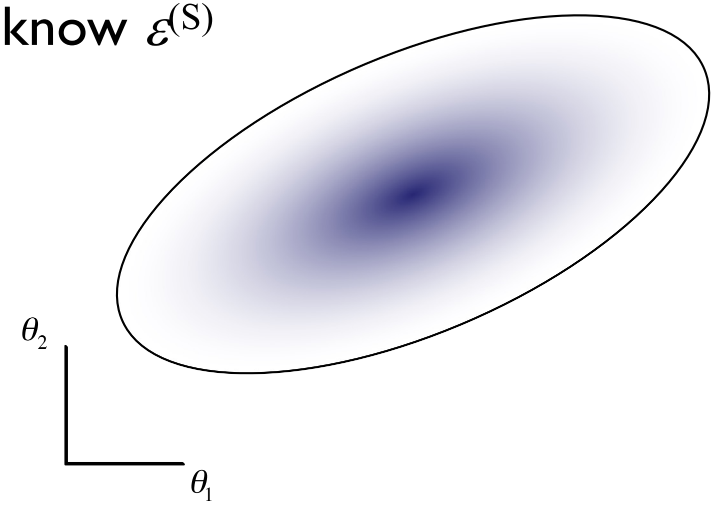
*Seitz & Schneider (1996)

- galaxy shapes

while we can measure ε
we still do not know $\varepsilon^{(S)}$



$\varepsilon^{(S)}$



ε

lens equation
=>

$$\varepsilon = \frac{\varepsilon^{(S)} + g}{1 - g^{*}\varepsilon^{(S)}}$$

with $g = \frac{\gamma}{1-\kappa}$ (reduced shear)

ε: measured ellipticity
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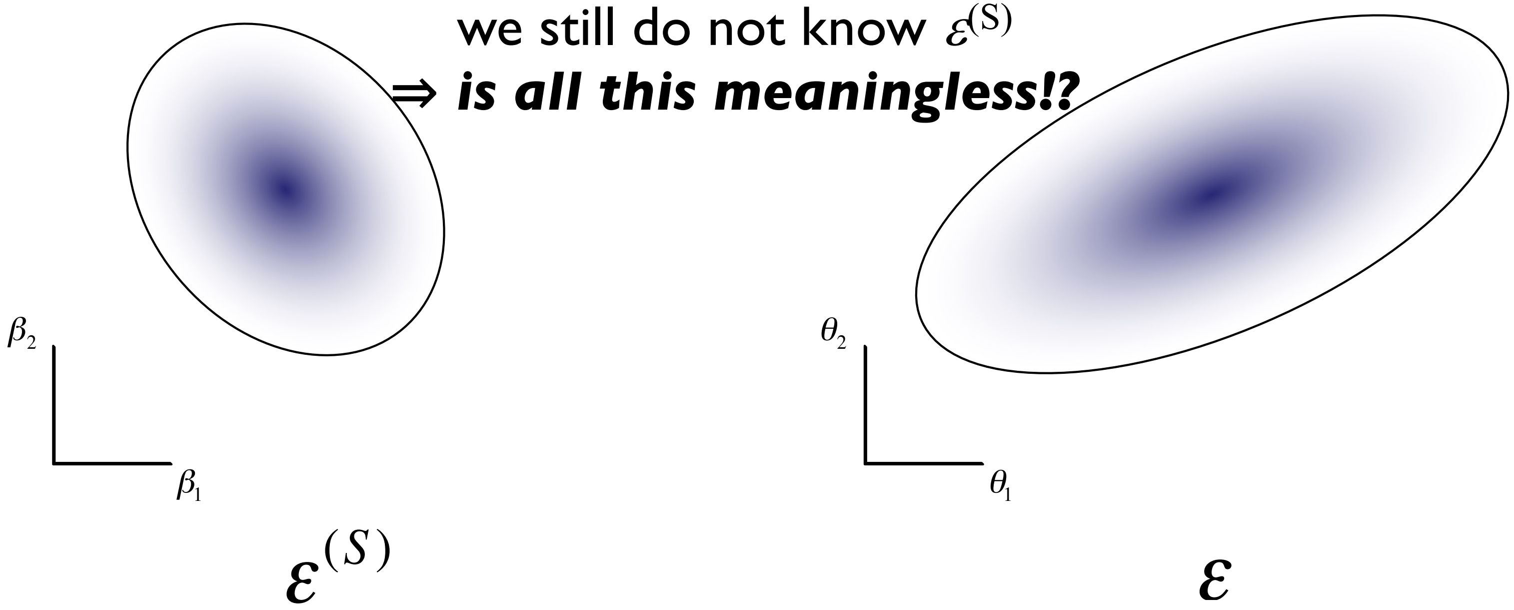
- galaxy shapes

while we can measure ε
we still do not know $\varepsilon^{(S)}$
⇒ ***is all this meaningless!?***



lens equation
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$$\varepsilon = \frac{\varepsilon^{(S)} + g}{1 - g^{*}\varepsilon^{(S)}}$$

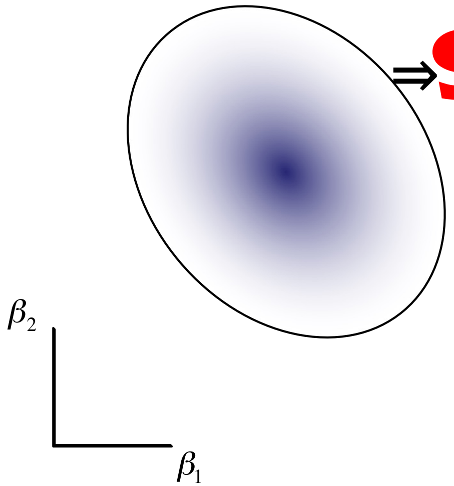
with $g = \frac{\gamma}{1-\kappa}$ (reduced shear)

ε: measured ellipticity
$\varepsilon^{(S)}$: source ellipticity

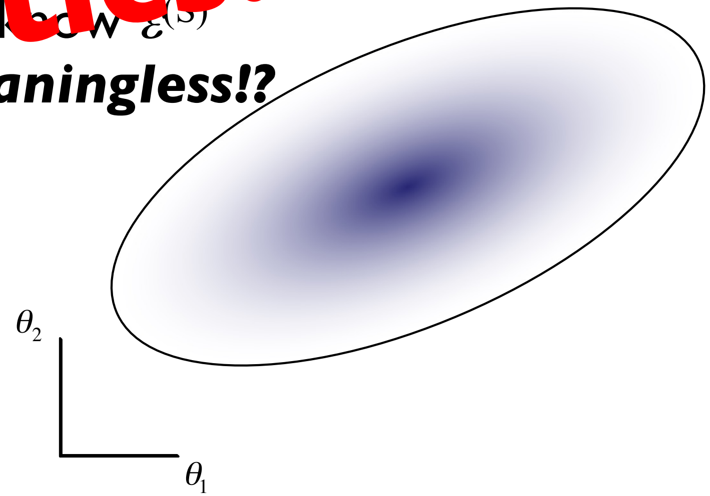
- galaxy shapes

while we can measure ,
we still do not know $\varepsilon^{(S)}$
⇒ ***is all this meaningless!?***

statistics!



$\varepsilon^{(S)}$



ε

lens equation
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with $g = \frac{\gamma}{1-\kappa}$ (reduced shear)

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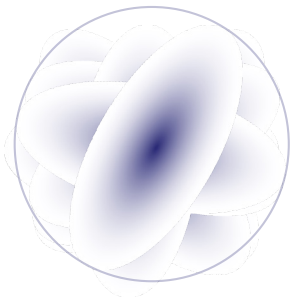
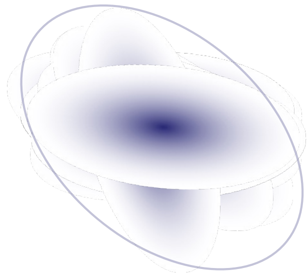
- galaxy shapes



assumption: $\left\langle \varepsilon^{(S)} \right\rangle = 0$



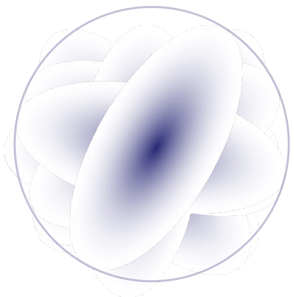
$\langle \varepsilon \rangle \neq 0$

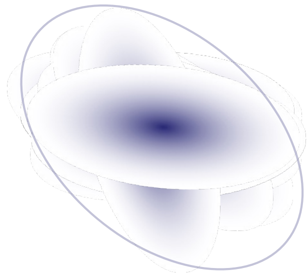
$$\varepsilon = \frac{\varepsilon^{(S)} + g}{1 - g^{*}\varepsilon^{(S)}}$$ with $g = \frac{\gamma}{1-\kappa}$ (reduced shear)

ε: measured ellipticity

$\varepsilon^{(S)}$: source ellipticity

- galaxy shapes





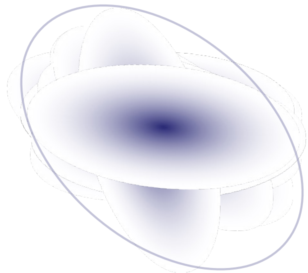
$$\left\langle \varepsilon^{(S)} \right\rangle = 0 \qquad \longrightarrow \qquad \left\langle \varepsilon \right\rangle = g$$

$$\left\langle \varepsilon \right\rangle = \frac{\left\langle \varepsilon^{(S)} \right\rangle + g}{1 - g^{*} \left\langle \varepsilon^{(S)} \right\rangle}$$ with $g = \frac{\gamma}{1-\kappa}$ (reduced shear)

ε: measured ellipticity

$\varepsilon^{(S)}$: source ellipticity

- galaxy shapes

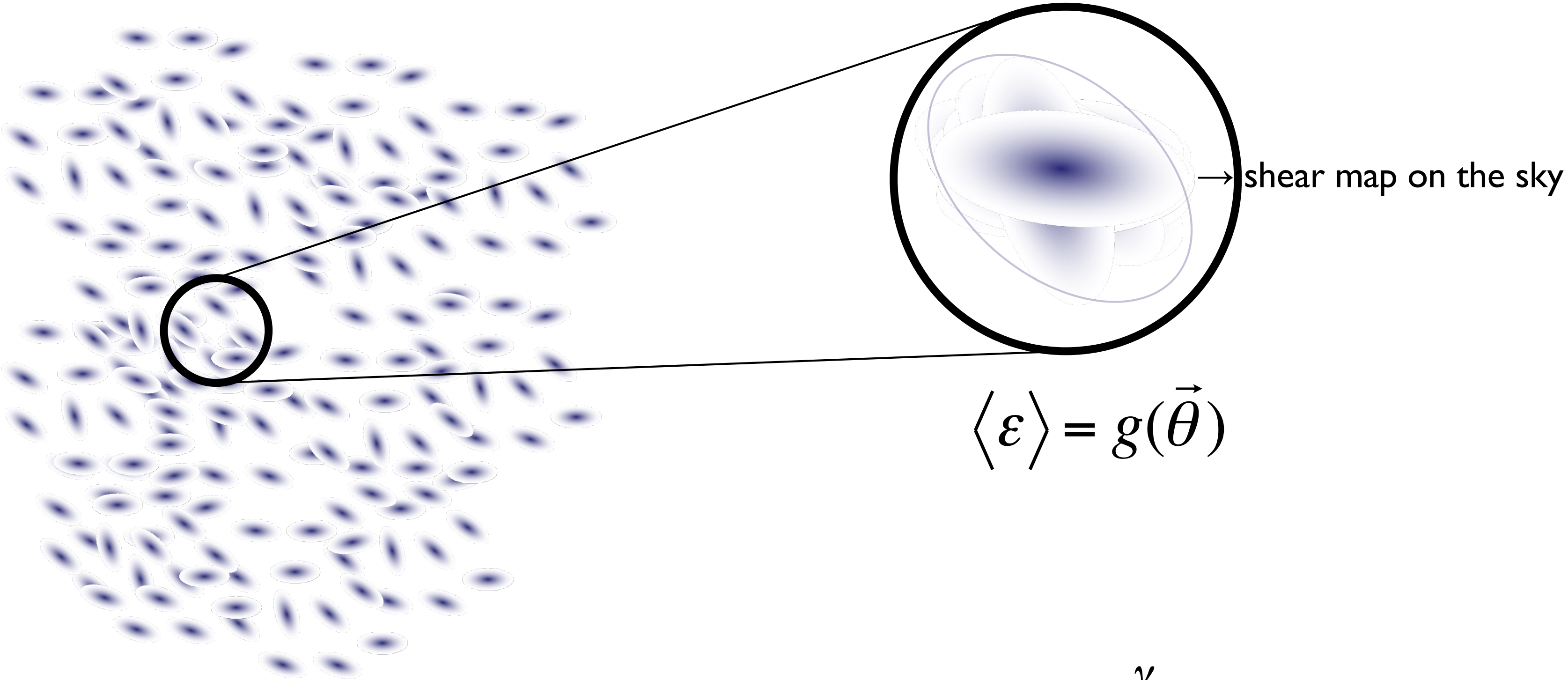


→ shear map on the sky

$$\langle \varepsilon \rangle = g$$

with $g = \dfrac{\gamma}{1-\kappa}$ (reduced shear)

- galaxy shapes



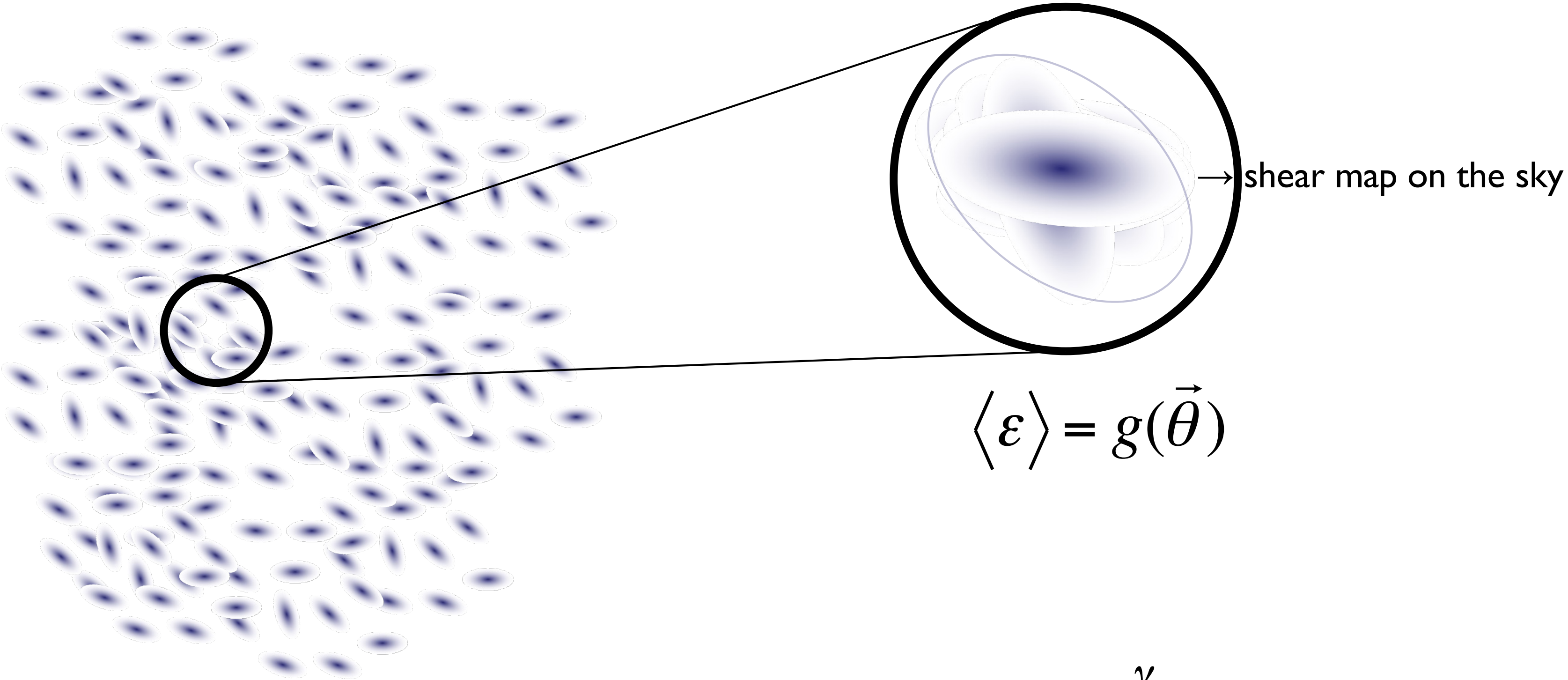
$$\langle \varepsilon \rangle = g(\vec{\theta})$$

with $g = \frac{\gamma}{1-\kappa}$ (reduced shear)

- galaxy shapes

Note: to get $g(\theta)$ we need to average over (enough!) galaxies at the same position θ

(As a rough guide, on a 3-hour exposure with a 4-meter class telescope, about 30 galaxies per arcmin² can be used for a shape measurement)



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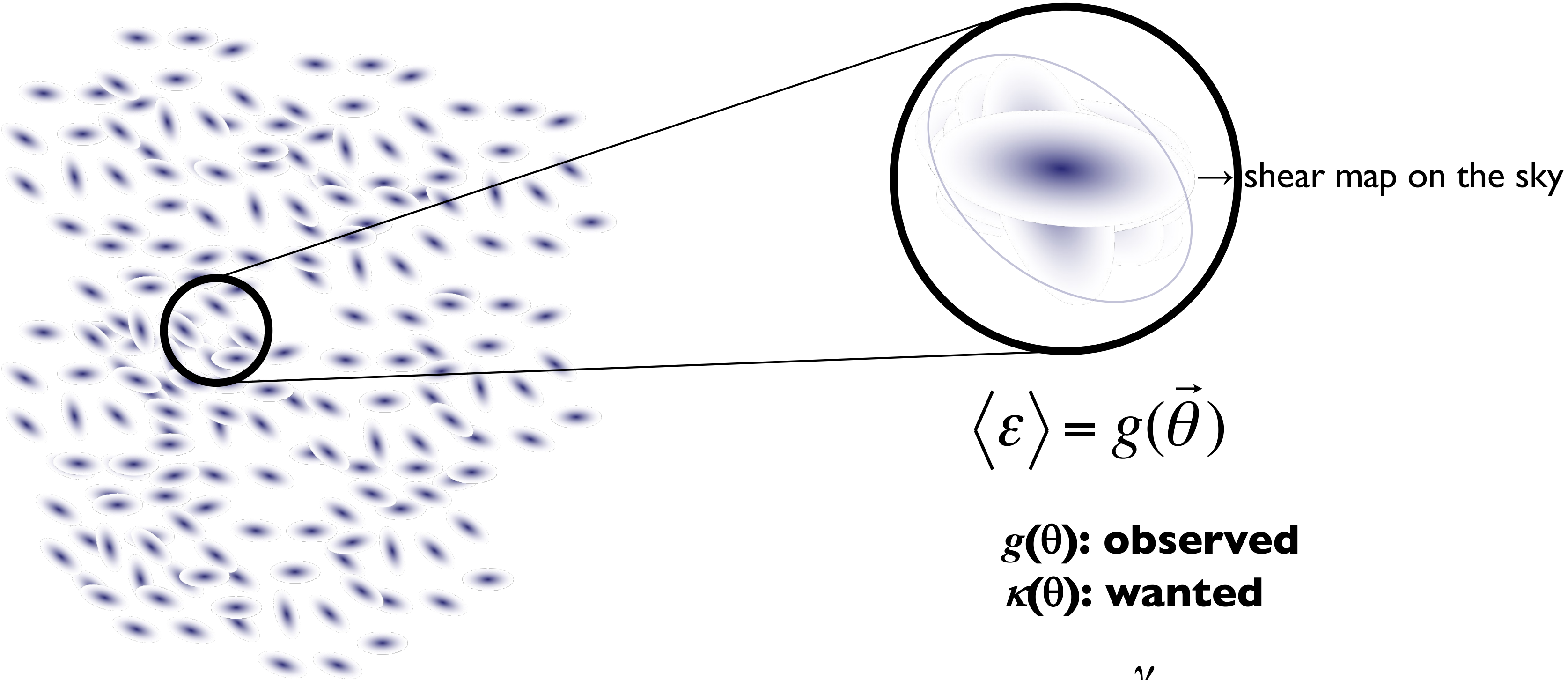
- galaxy shapes – ***mass reconstruction!?***



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$g(\theta)$: observed
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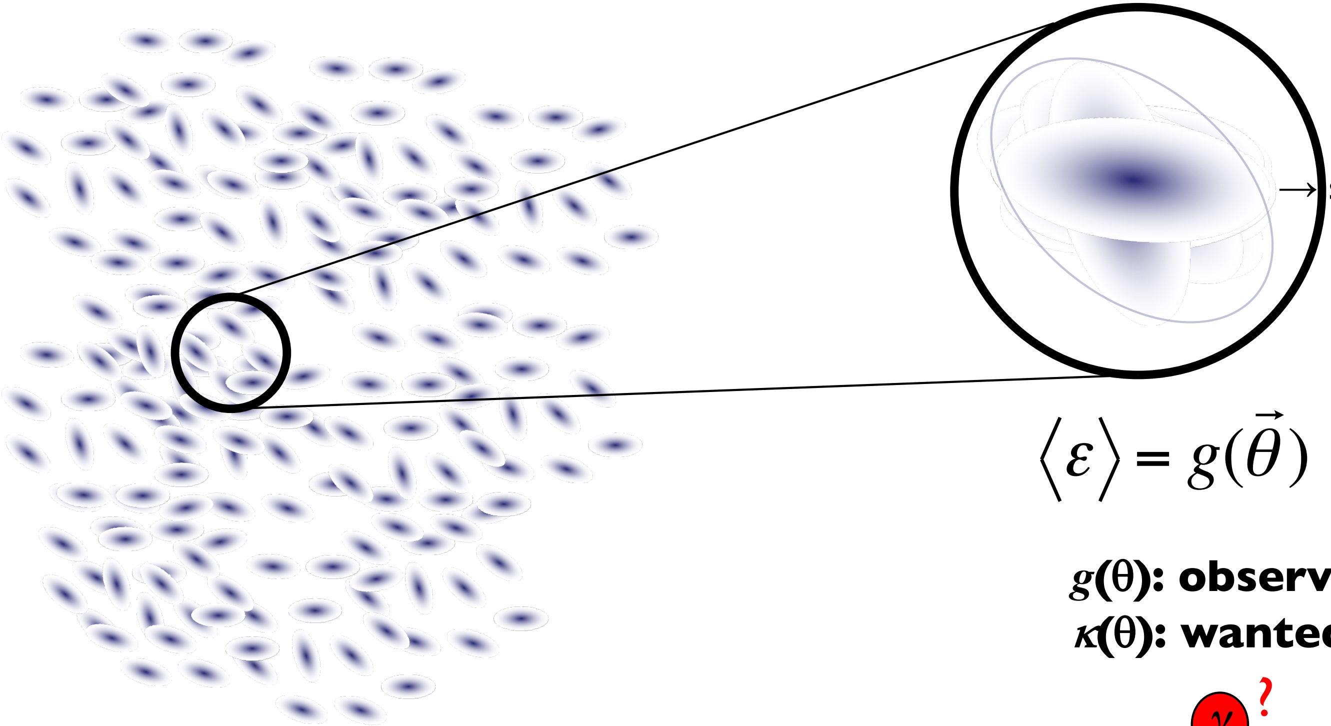

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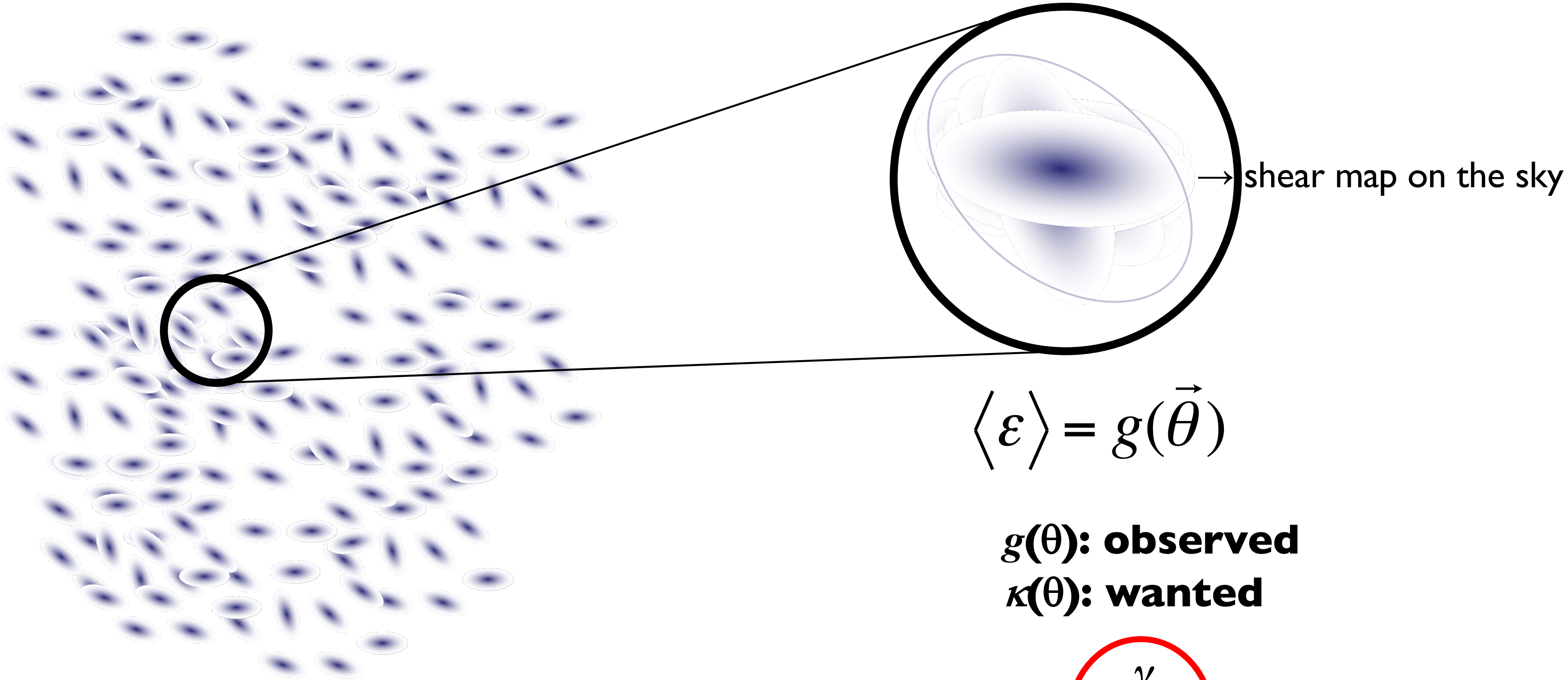

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relation between γ and κ?!

▪ galaxy shapes – mass reconstruction

- relation between κ and γ ✭

$$\kappa(\boldsymbol{\theta}) - \kappa_0 = \frac{1}{\pi}\int \mathrm{Re}\left[D^*(\boldsymbol{\theta} - \boldsymbol{\theta}')\gamma(\boldsymbol{\theta}')\right] d^2\theta'$$

with $D(\boldsymbol{\theta} - \boldsymbol{\theta}') = \dfrac{\left((\theta_1 - \theta_1') + i(\theta_2 - \theta_2')\right)^2}{\left|\boldsymbol{\theta} - \boldsymbol{\theta}'\right|^4}$

✭Seitz & Schneider (1996)

- galaxy shapes – mass reconstruction
  - relation between κ and γ

$$\kappa(\boldsymbol{\theta}) - \kappa_0 = \frac{1}{\pi} \int \mathrm{Re}\left[D^*(\boldsymbol{\theta} - \boldsymbol{\theta}')\gamma(\boldsymbol{\theta}') \right] d^2\theta'$$

a constant surface mass density does not cause any shear!

with $D(\boldsymbol{\theta} - \boldsymbol{\theta}') = \dfrac{\left((\theta_1 - \theta_1') + i(\theta_2 - \theta_2')\right)^2}{|\boldsymbol{\theta} - \boldsymbol{\theta}'|^4}$

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relate to observable quantity!

with $D(\boldsymbol{\theta} - \boldsymbol{\theta}') = \dfrac{\left((\theta_1 - \theta_1') + i(\theta_2 - \theta_2')\right)^2}{\left|\boldsymbol{\theta} - \boldsymbol{\theta}'\right|^4}$

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$$g = \frac{\gamma}{1-\kappa} = \langle\varepsilon(\theta)\rangle$$

is observable!

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$$g = \frac{\gamma}{1-\kappa} = \langle \varepsilon(\theta) \rangle$$

is observable!

$$\kappa(\boldsymbol{\theta}) - \kappa_0 = \frac{1}{\pi} \int (1 - \kappa(\theta'))\, \mathrm{Re}\left[D^*(\boldsymbol{\theta} - \boldsymbol{\theta}')g(\boldsymbol{\theta}')\right] d^2\theta'$$

- galaxy shapes – mass reconstruction
  - relation between κ and γ

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galaxy shapes – mass reconstruction

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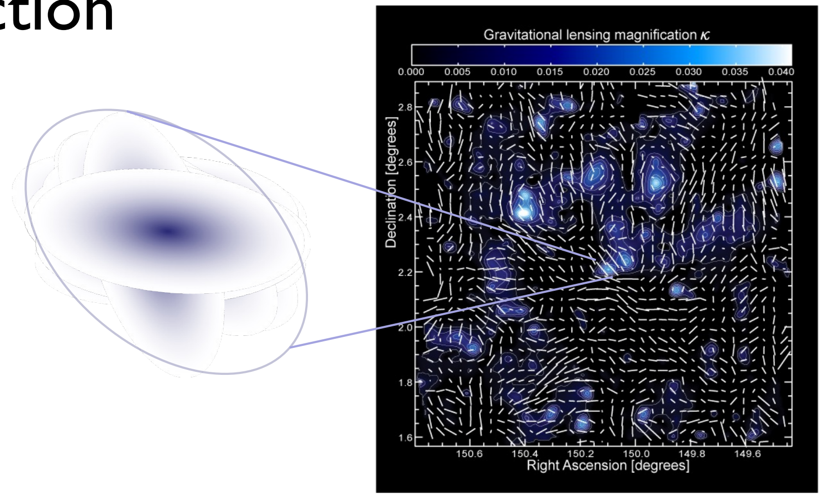
remember: $\kappa(\theta) = 2\dfrac{\Sigma(\theta)}{\Sigma_{crit}}$

- galaxy shapes – mass reconstruction

$$g(\theta) = \frac{\gamma}{1-\kappa} = \langle \varepsilon(\theta) \rangle$$



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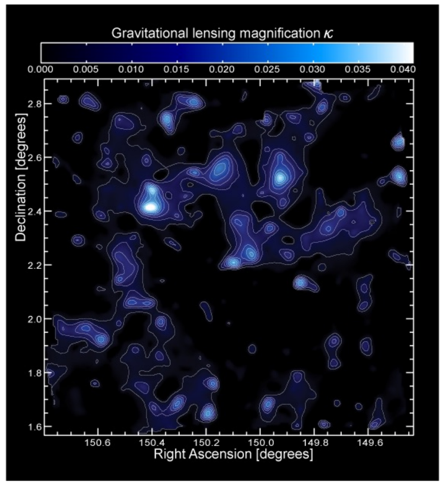
$$\frac{\Sigma(\theta)}{\Sigma_{crit}} = \kappa(\theta)$$



- galaxy shapes – mass reconstruction w/ lensing potential
  - lensing equation:

$$\beta = \theta - \nabla_\theta \varphi(\theta)$$

  - distortion matrix:

$$A_{ij} = \frac{\partial \beta_i}{\partial \theta_j} = 1 - \frac{\partial^2 \varphi}{\partial \theta_i \partial \theta_j} = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

$$\kappa = \frac{1}{2}(\partial_{11}\varphi + \partial_{22}\varphi) = \frac{\Sigma(\theta)}{\Sigma_{crit}}$$

=>

$$\gamma_1 = \frac{1}{2}(\partial_{11}\varphi - \partial_{22}\varphi)$$

$$\gamma_2 = \partial_{12}\varphi = \partial_{21}\varphi$$

- obstacles

  - point-spread-function of telescope
  - read-out errors of CCD's
  - signal-to-noise (S/N) ratio
  - intrinsic shape variations of galaxies

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  - **point-spread-function of telescope**
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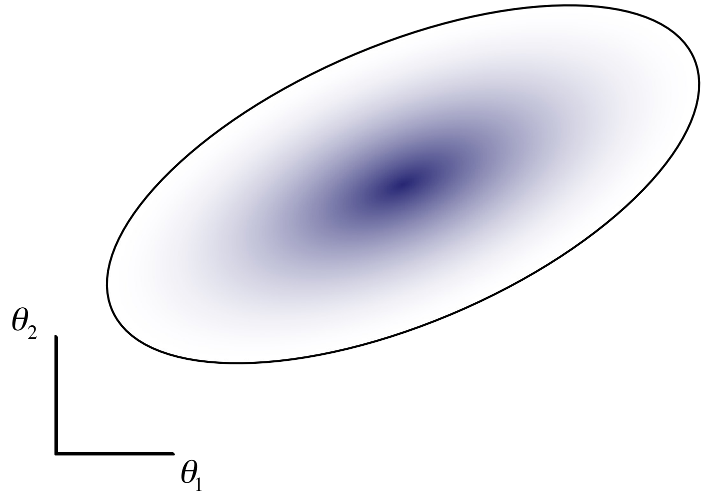
▪ telescope limitations

1. find the image centre

$$\bar{\boldsymbol{\theta}} = \frac{\int \boldsymbol{\theta}\, I(\boldsymbol{\theta})\, q_I\big(I(\boldsymbol{\theta})\big)\, d^2\theta}{\int I(\boldsymbol{\theta})\, q_I\big(I(\boldsymbol{\theta})\big)\, d^2\theta}$$ (= image centre)

$I(\boldsymbol{\theta})$ surface brightness



2. calculate its 2nd order moments on the sky

$$Q_{ij} = \frac{\int \left[(\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)\right] I(\boldsymbol{\theta})\, q_I\big(I(\boldsymbol{\theta})\big)\, d^2\theta}{\int I(\boldsymbol{\theta})\, q_I\big(I(\boldsymbol{\theta})\big)\, d^2\theta}$$

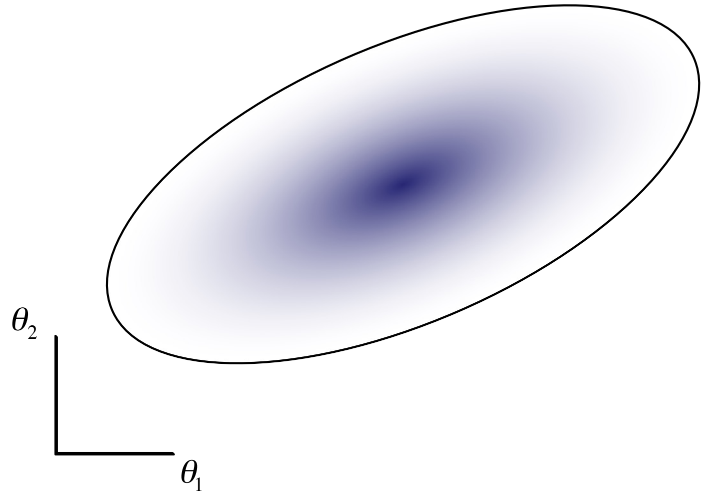
3. define its ellipticity from the moments

$$\varepsilon \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2\,(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

$q_I\big(I(\boldsymbol{\theta})\big)$: suitably chosen weight function

▪ telescope limitations

$I(\boldsymbol{\theta})$ surface brightness



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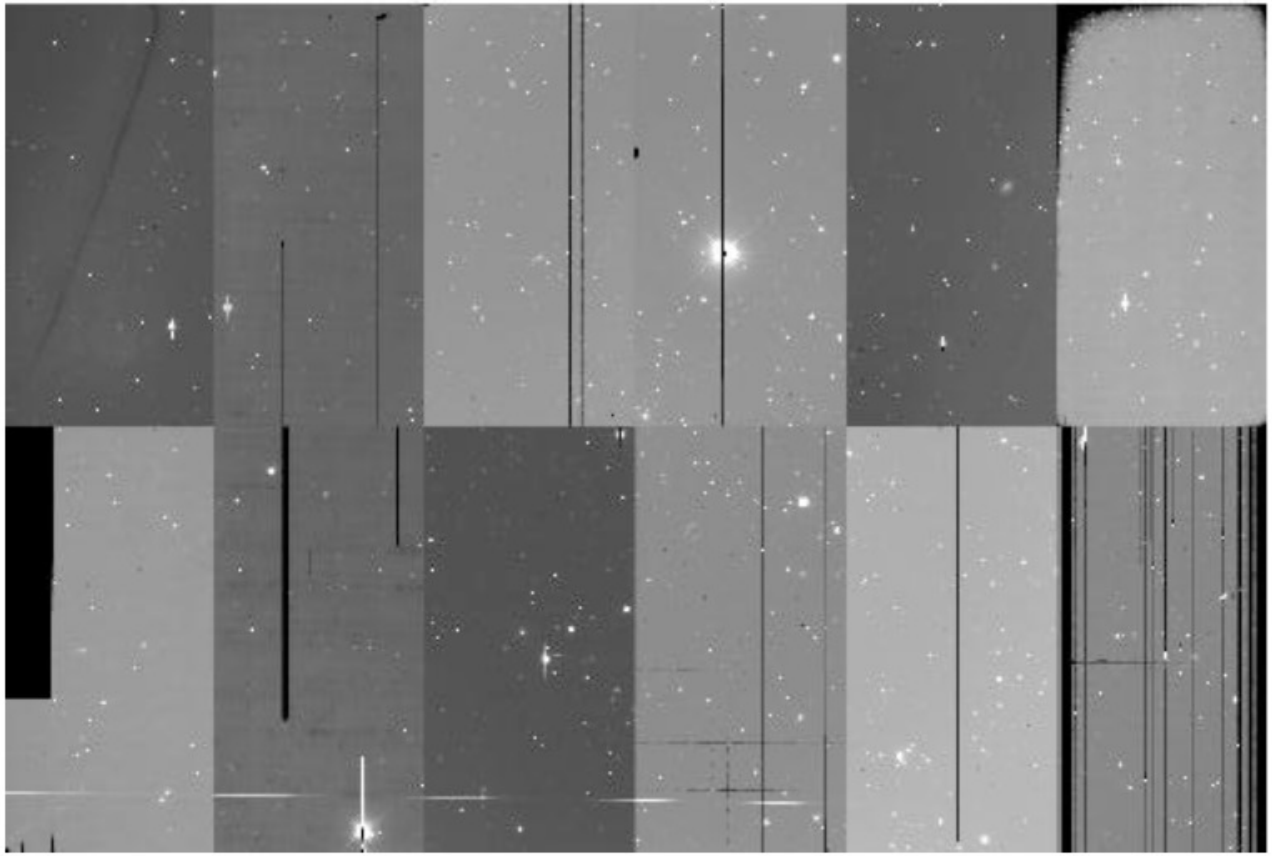
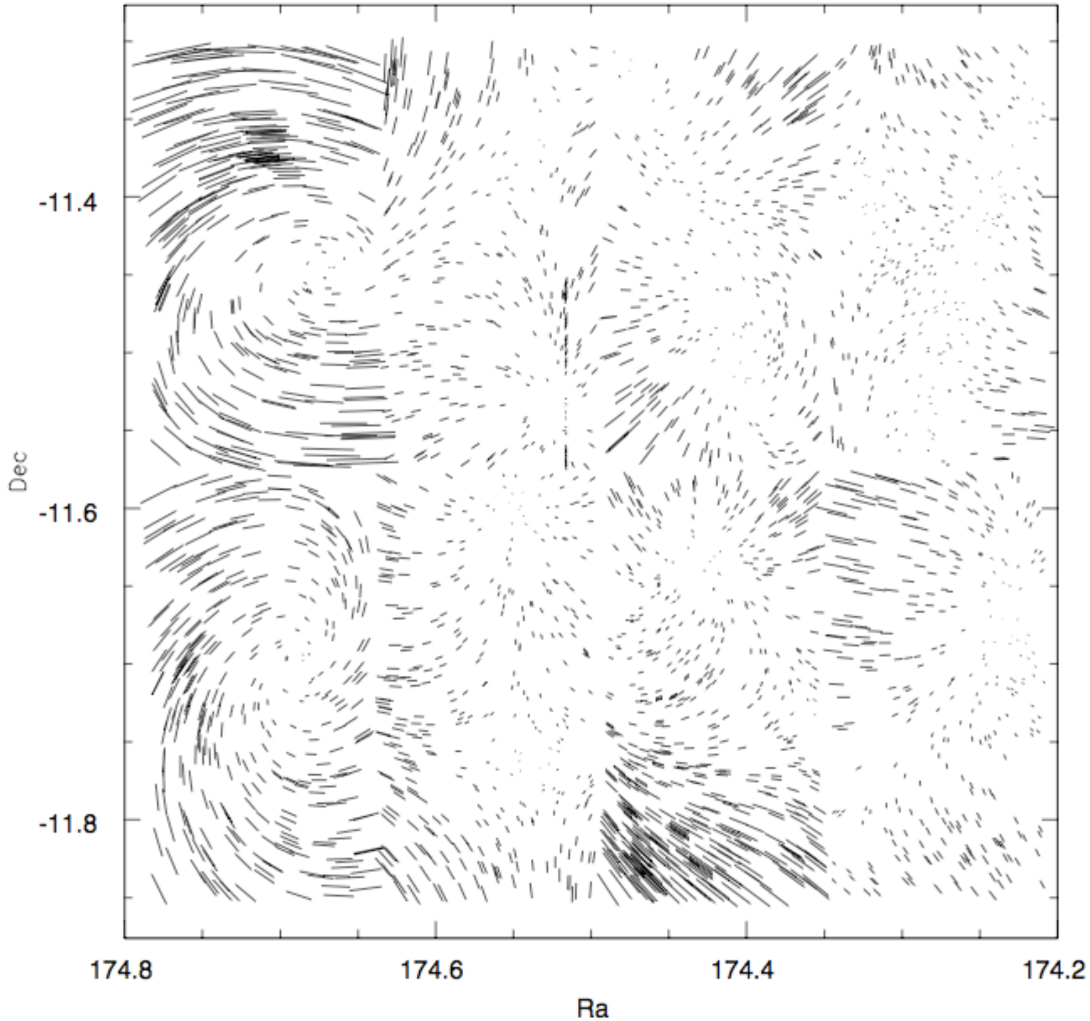


raw image from the CFH12K camera

Schneider (2001, p.18)

- telescope limitations



geometric distortion of the Wide Field Imager at the ESO/MPG 2.2m telecsope at La Silla

Schneider (2001, p.18)

- obstacles

  - point-spread-function of telescope
  - read-out errors of CCD's
  - **signal-to-noise (S/N) ratio**
  - intrinsic shape variations of galaxies

- detection of weak lensing

$$\frac{S}{N} = 12.7\left(\frac{n_S}{30\ \mathrm{arcmin}^{-2}}\right)^{1/2}\left(\frac{\sigma_\varepsilon}{0.2}\right)^{-1}\left(\frac{\sigma_v}{600\ \mathrm{km\ s}^{-1}}\right)^{2}\left(\frac{\ln(\theta_{out}/\theta_{in})}{\ln 10}\right)^{1/2}\left\langle\frac{D_{LS}}{D_S}\right\rangle$$

Bartelmann & Schneider (2001, p.351)

- detection of weak lensing

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number distribution of sources

Bartelmann & Schneider (2001, p.351)

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dispersion of ellipticities

Bartelmann & Schneider (2001, p.351)

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velocity dispersion of lenses
(remember singular isothermal sphere example...)

Bartelmann & Schneider (2001, p.351)

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geometry of averaging

Bartelmann & Schneider (2001, p.351)

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cosmology

Bartelmann & Schneider (2001, p.351)

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- σ_v ~ 600 km/sec (galaxy clusters) => detectable
- σ_v ~ 200 km/sec (galaxies) => undetectable
  (superposition necessary!)

Bartelmann & Schneider (2001, p.351)

- obstacles

  - at least 100 galaxies required to increase S/N ratio
  - point-spread-function of telescope
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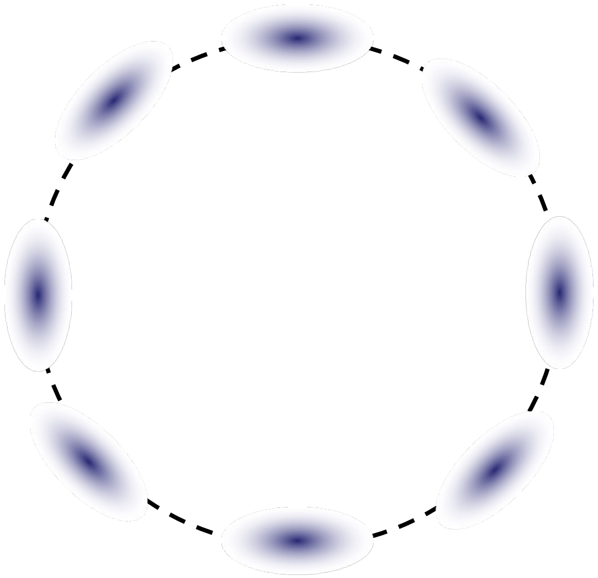
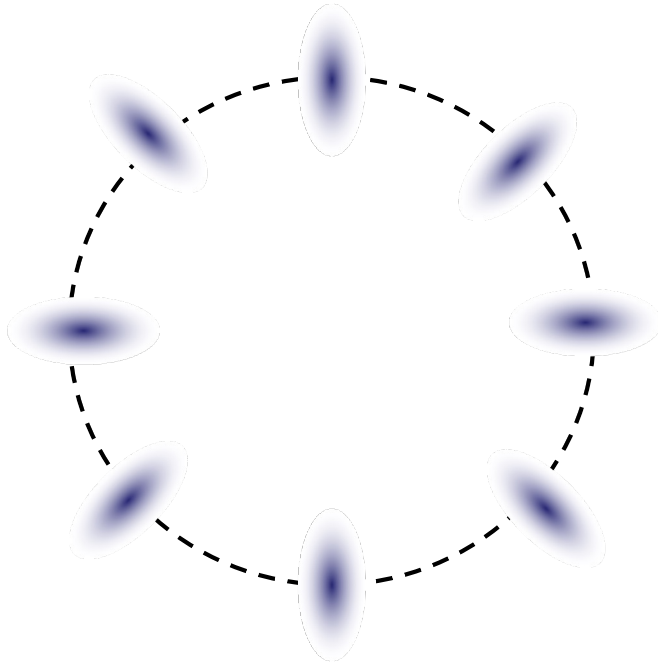
→ **diagnostics of shear map!?**

- diagnostics of lensing signal (image distortion)
  - weak lensing produces curl-free E-modes…



mass overdensity
(cf. strong lensing arcs)



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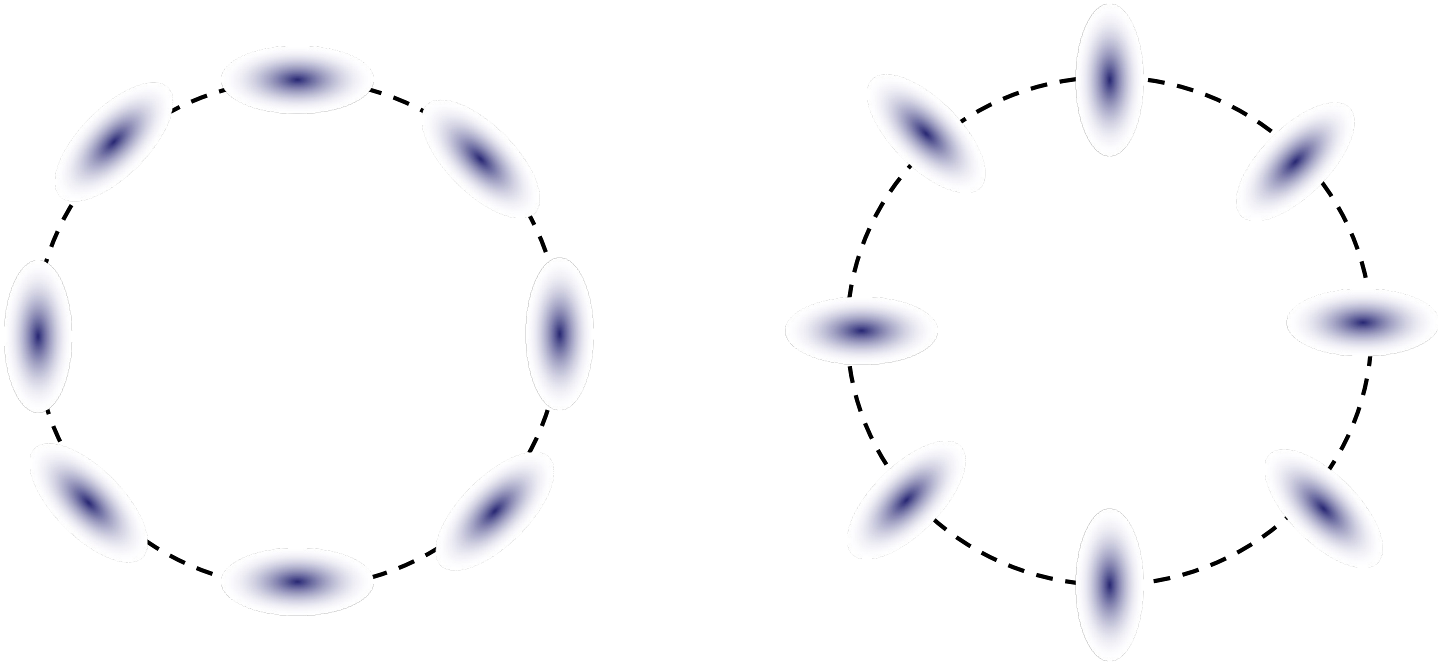
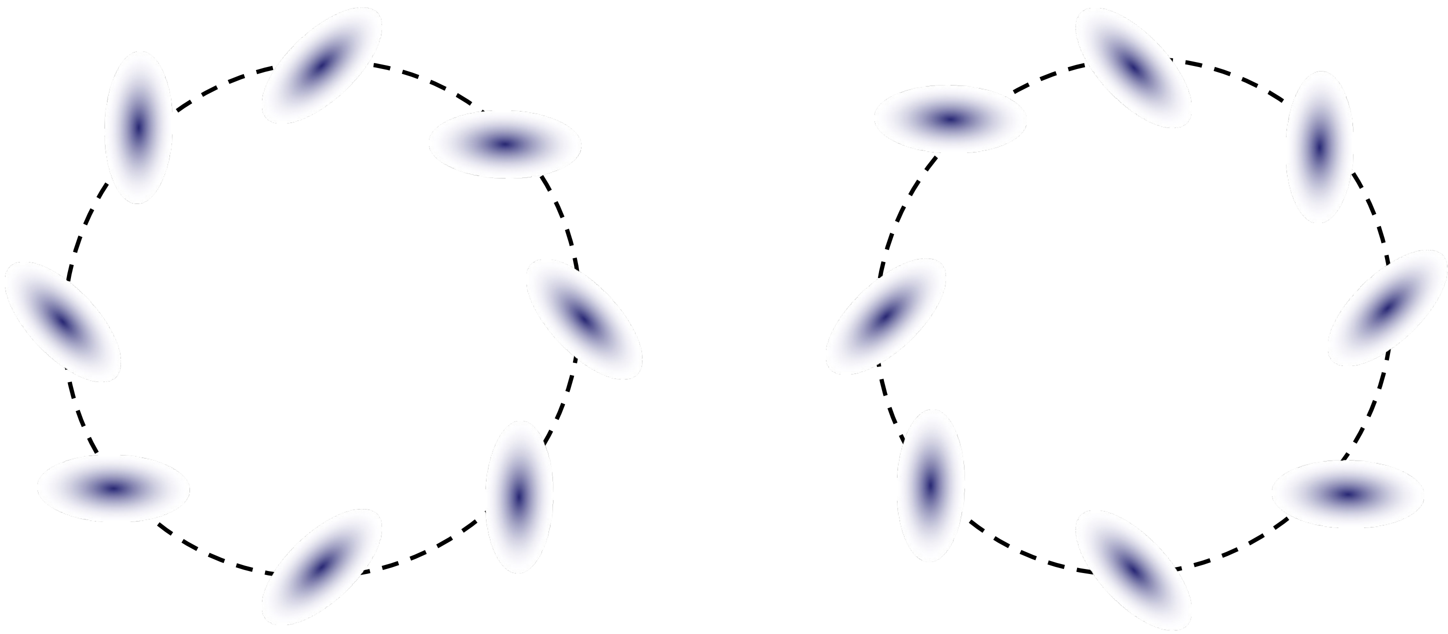


mass overdensity
(cf. strong lensing arcs)

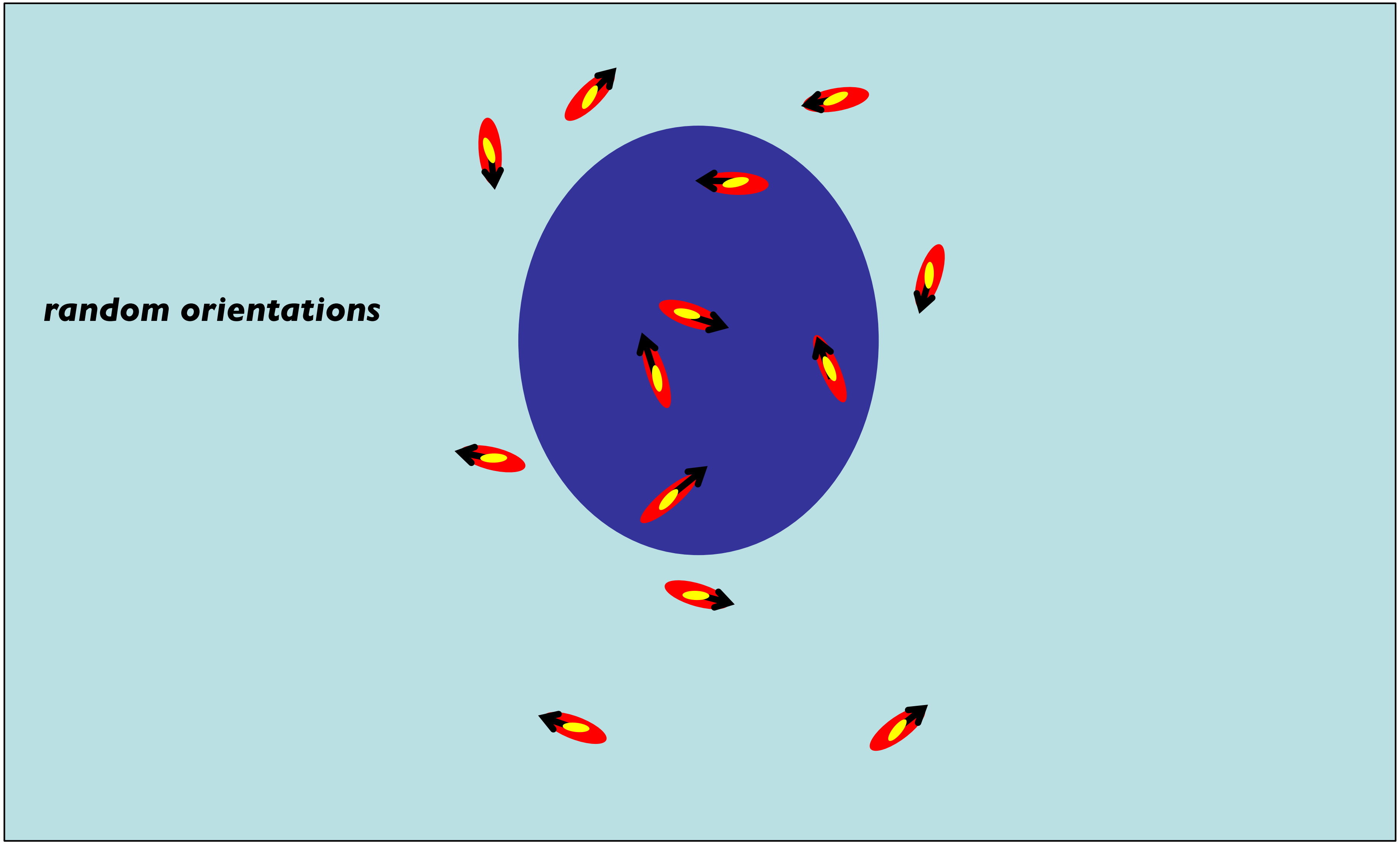
mass underdensity

…because the shear stems from a scalar (lensing) potential φ

- diagnostics of lensing signal (image distortion)
  - “noise” produces divergence-free B-modes



(Note that intrinsic alignment is also “noise”…)



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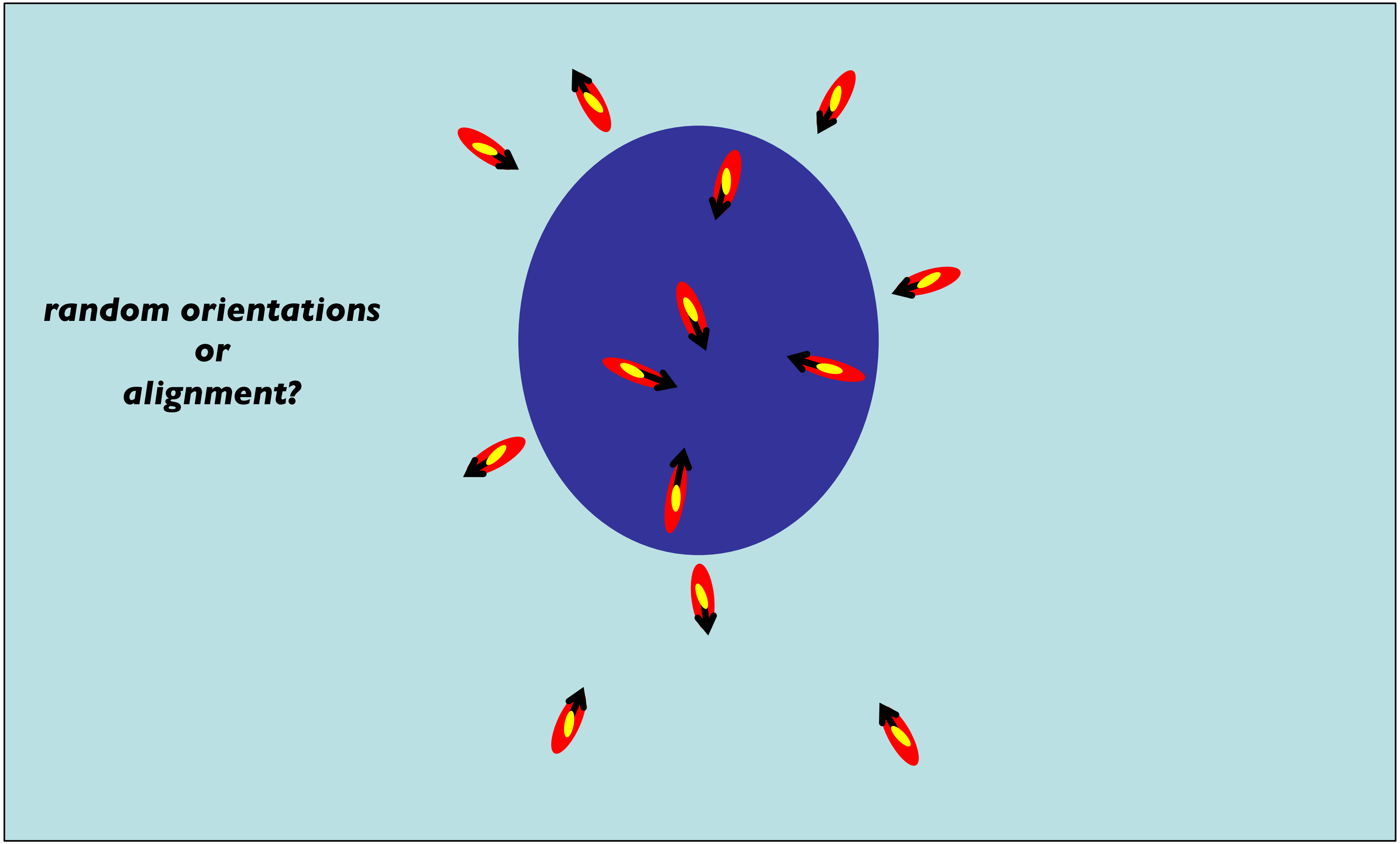
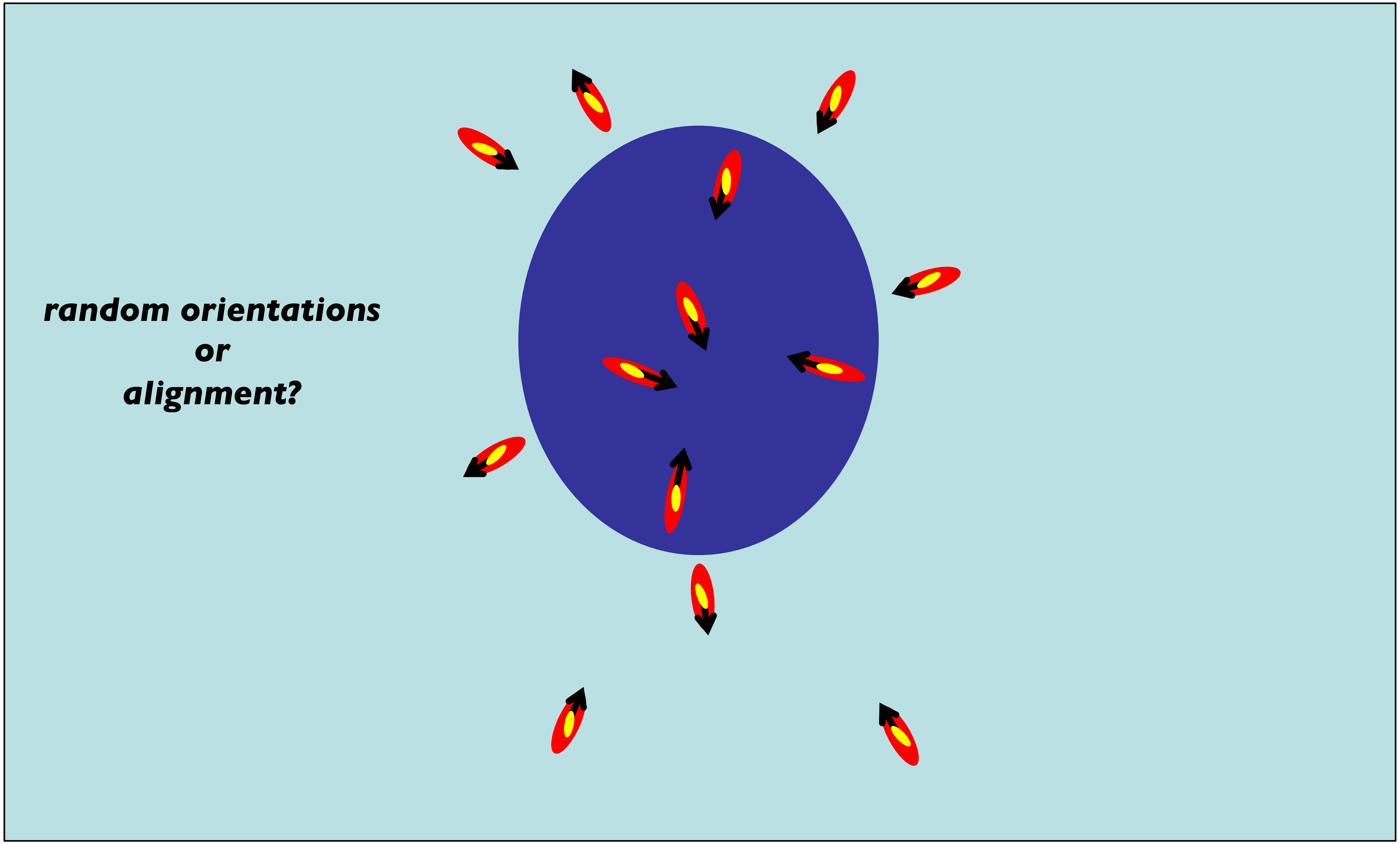


(Note that **intrinsic alignment** is also "noise"…)

excellent review with focus on lensing: Kiessling et al. (arXiv:1504.05546)



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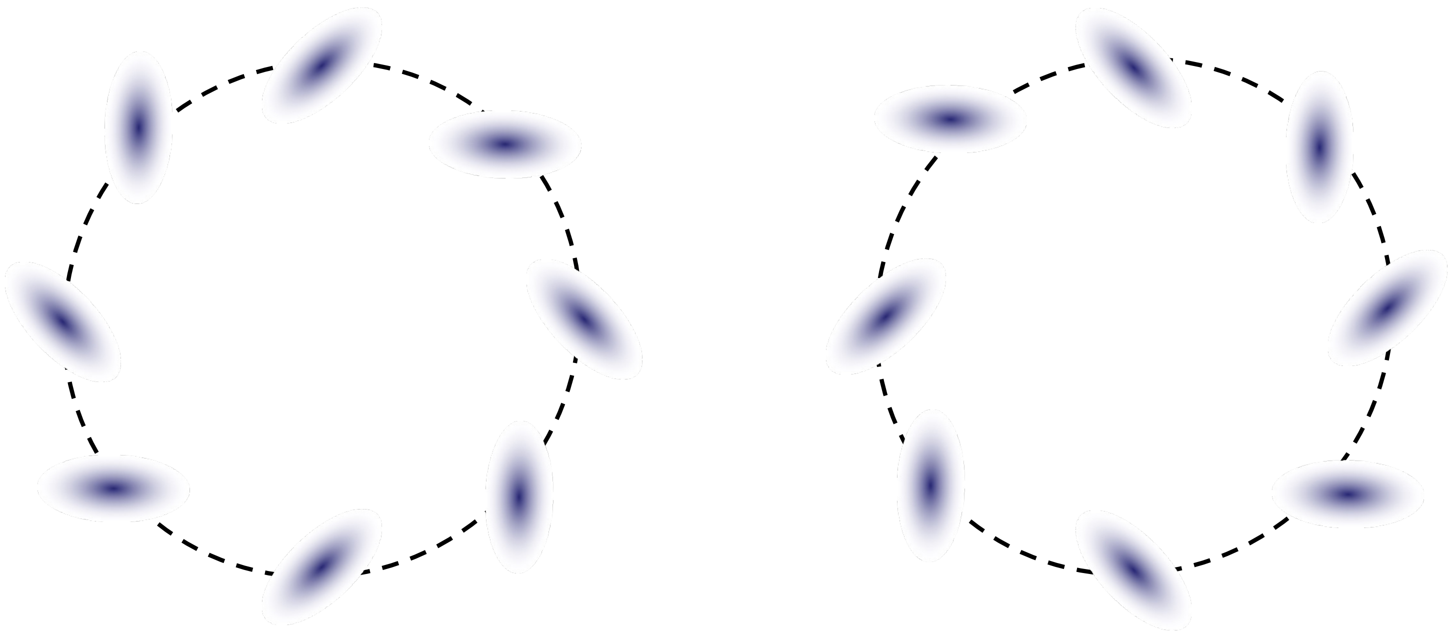

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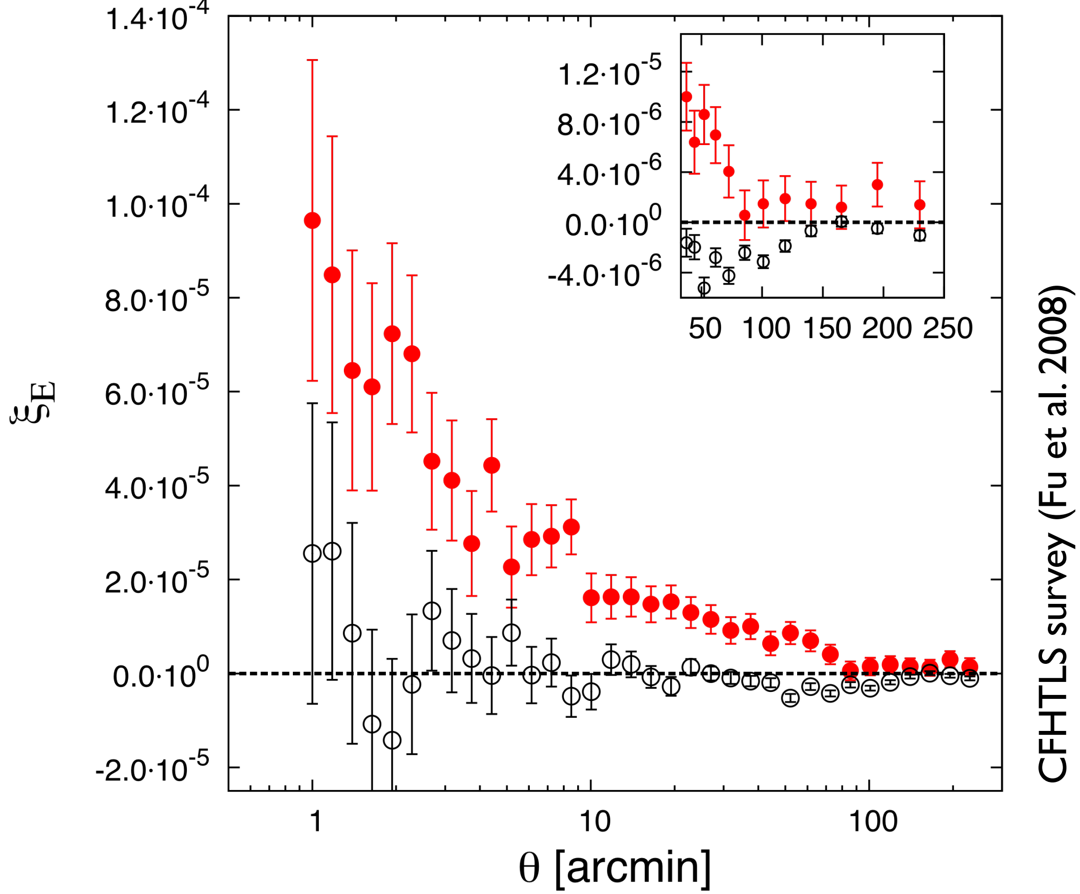
- diagnostics of lensing signal (image distortion)
  - weak lensing produces curl-free E-modes: ●
  - “noise” produces divergence-free B-modes: **O**

shear correlation function



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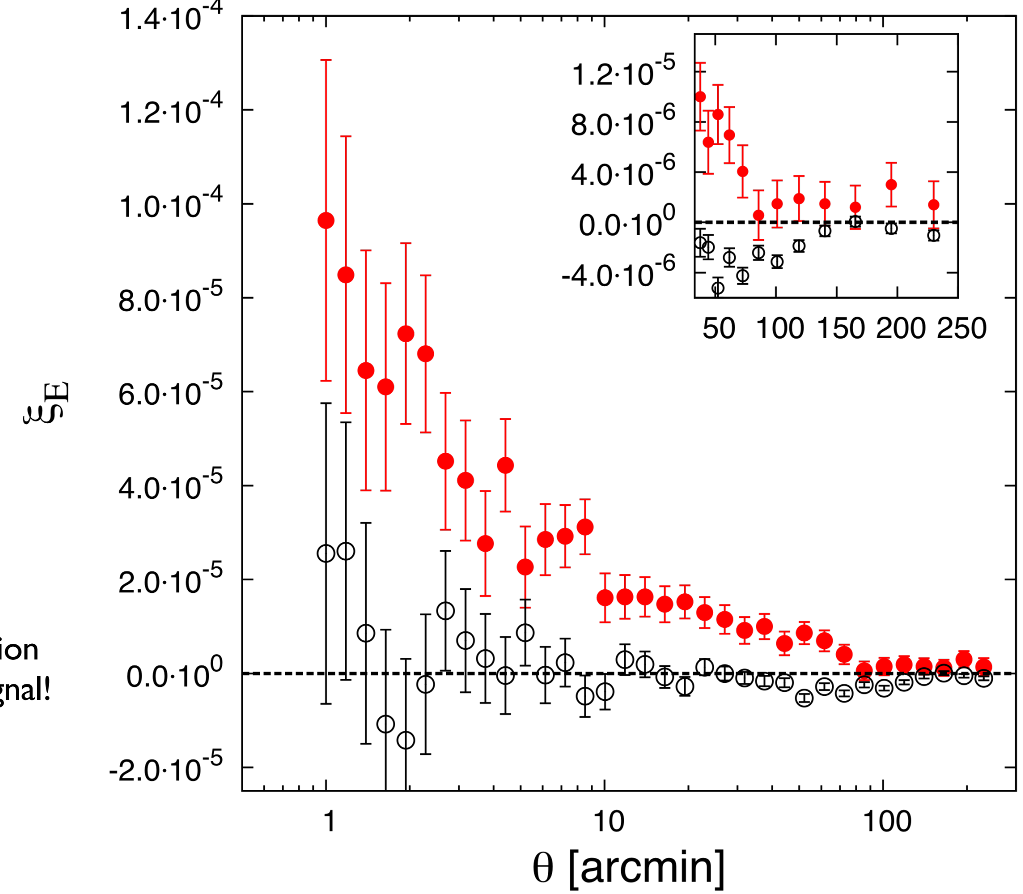
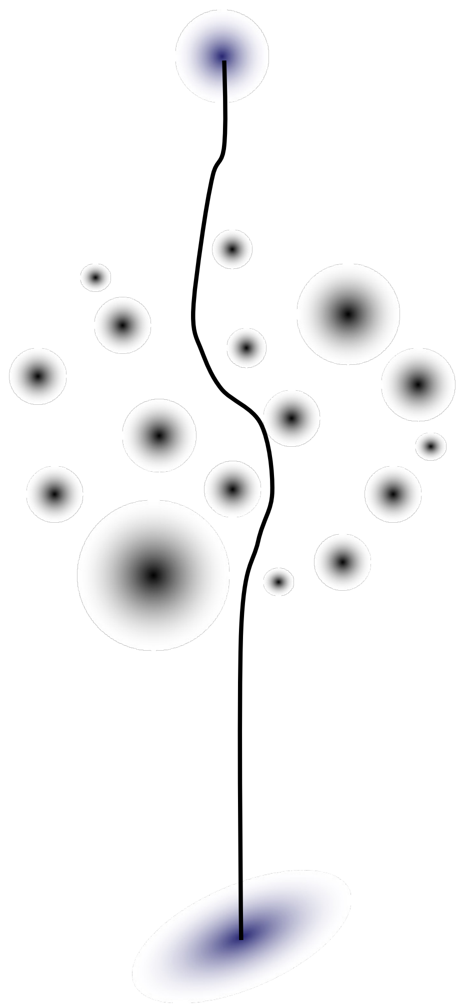


CFHTLS survey (Fu et al. 2008)

low B-mode correlation indicates low-noise signal!

- cosmological considerations



- cosmological considerations

$$\kappa = \frac{1}{2}(\partial_{11}\varphi + \partial_{22}\varphi) = \frac{1}{2}\nabla \cdot \boldsymbol{\alpha}(\boldsymbol{\theta})$$

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$$\alpha = \frac{2}{c^2}\int \frac{D_{LS}}{D_S}\nabla_{\xi}\Phi(\xi, z) \; dz$$

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re-write using 'cosmological quantities' (i.e. density contrast δ)

- cosmological considerations

$$\kappa = \frac{1}{2}(\partial_{11}\varphi + \partial_{22}\varphi) = \frac{1}{2}\nabla \cdot \boldsymbol{\alpha}(\boldsymbol{\theta})$$

$$\alpha = \frac{2}{c^2}\int \frac{D_{LS}}{D_S} \nabla_\xi \Phi(\xi, z) \; dz$$

$$=> \quad \kappa(\theta) = \frac{1}{c^2}\int \frac{D_{LS}}{D_S} \nabla_\theta \cdot \nabla_\xi \Phi(\xi, z) \; dz$$

$\xi = D_L \theta$

$$= \frac{1}{c^2}\int \frac{D_{LS} D_L}{D_S} \nabla_\xi \cdot \nabla_\xi \Phi(\xi, z) \; dz$$

$\Delta\Phi = \frac{3H_0^2\Omega_0}{a}\delta$

(Note that $\langle \partial\Phi / \partial z \rangle = 0$)

$$= \frac{3H_0^2\Omega_0}{2c^2}\int \frac{D_{LS} D_L}{D_S} \frac{\delta(\theta, z)}{a(z)} \; dz$$

- cosmological considerations

$$\kappa(\theta) = \frac{3H_0^2\Omega_0}{2c^2}\int \frac{D_{LS}D_L}{D_S}\frac{\delta(\theta,z)}{a(z)}\, dz$$

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cosmological setup

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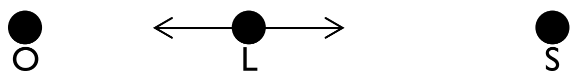
varying lens position D_L

what will happen to $D_{LS}D_L/D_S$?



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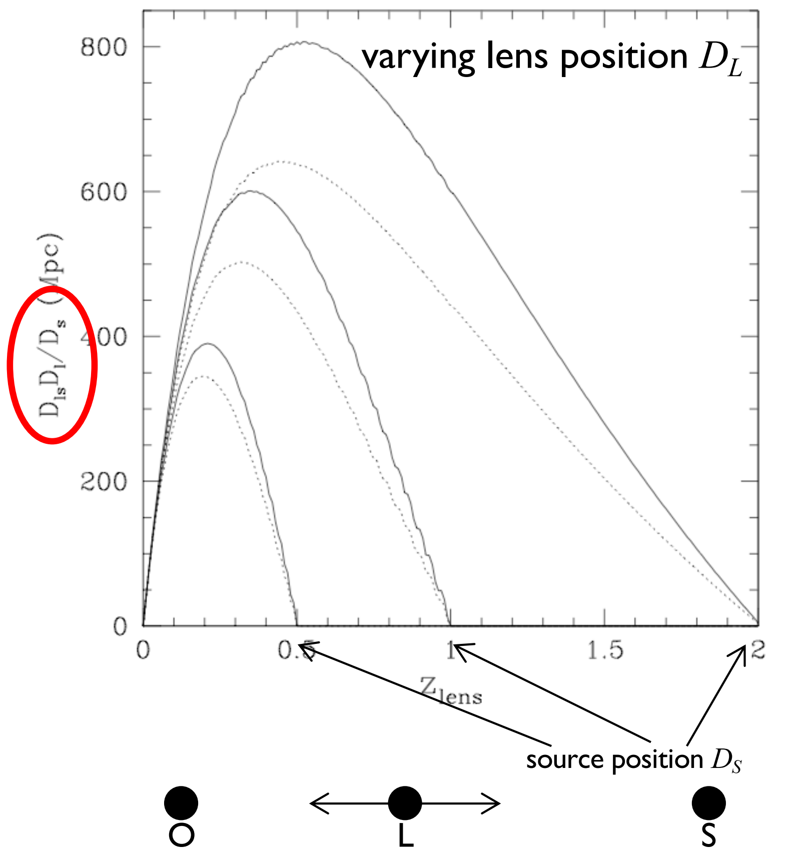
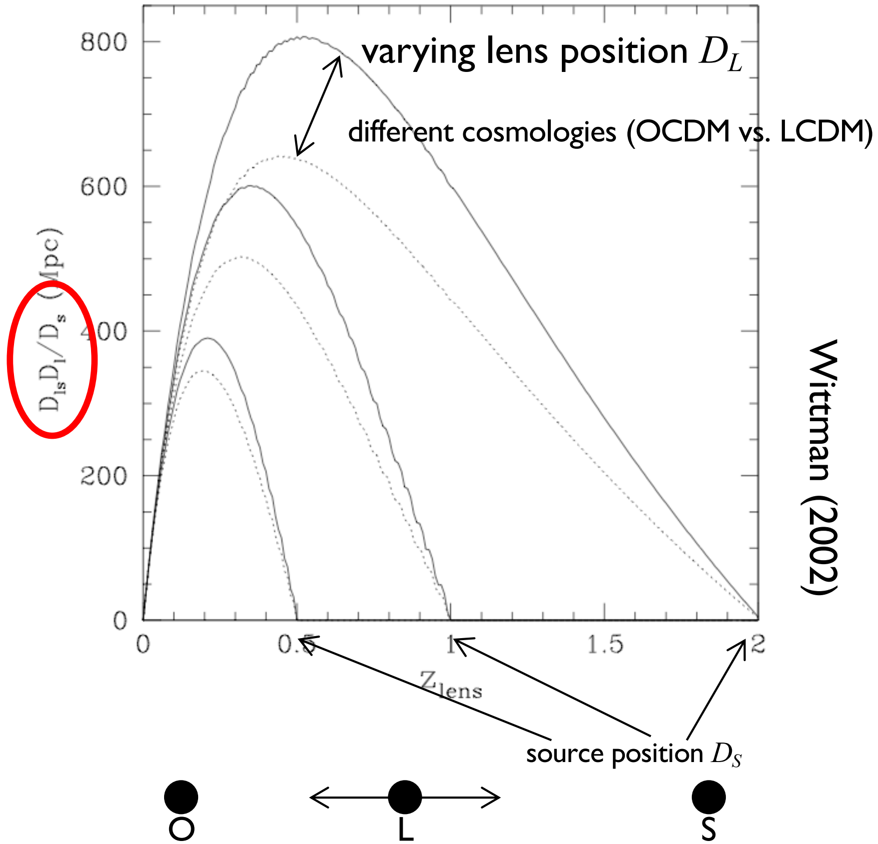


Wittman (2002)

- cosmological considerations

cosmological setup

$$\kappa(\theta) = \frac{3H_0^2\Omega_0}{2c^2}\int \frac{D_{LS}D_L}{D_S}\frac{\delta(\theta,z)}{a(z)}\,dz$$



Wittman (2002)

- cosmological considerations

cosmological setup

$$\kappa(\theta) = \frac{3H_0^2\Omega_0}{2c^2}\int \frac{D_{LS}D_L}{D_S}\frac{\delta(\theta,z)}{a(z)}\, dz$$

varying distance to sources D_S

what will happen to $D_{LS}D_L/D_S$?

- cosmological considerations

cosmological setup

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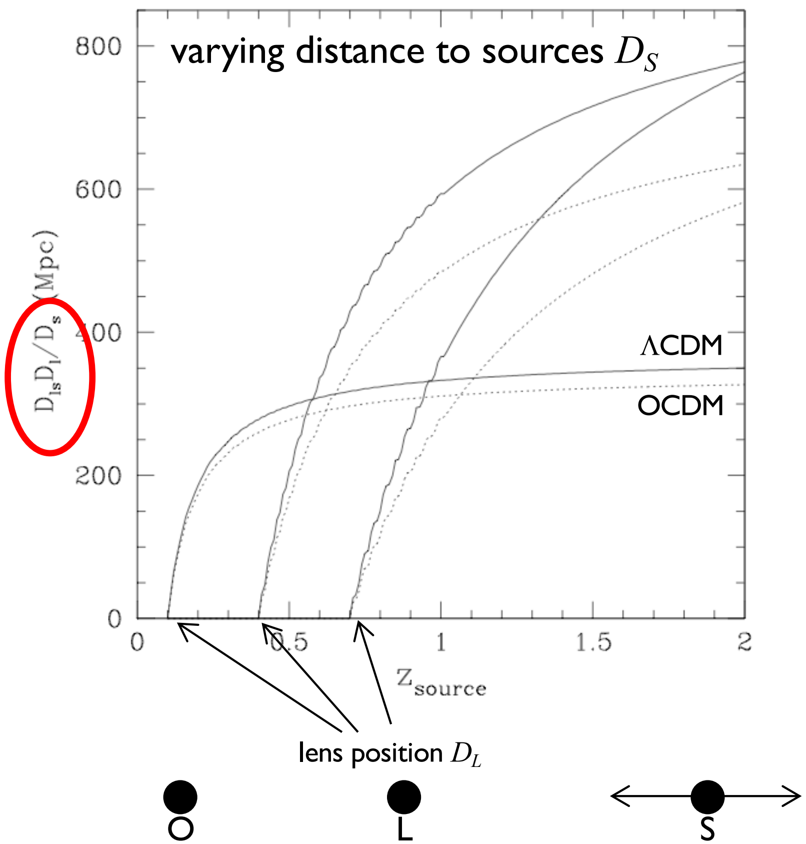

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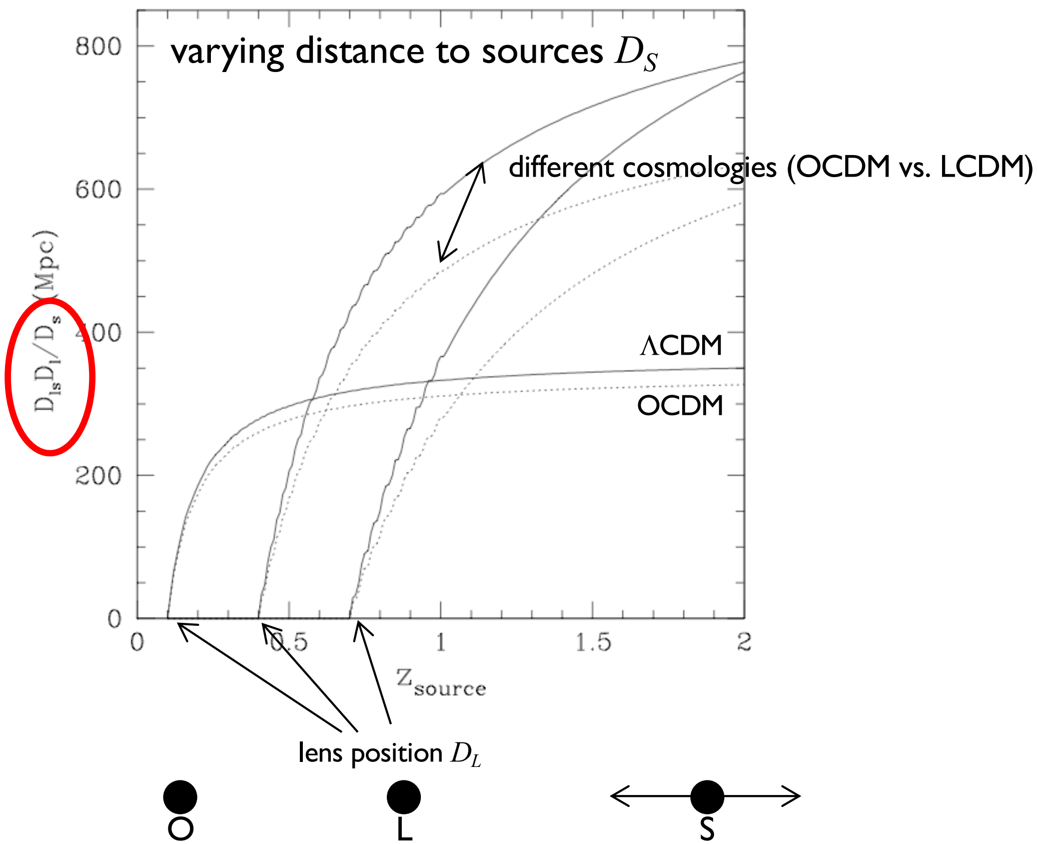

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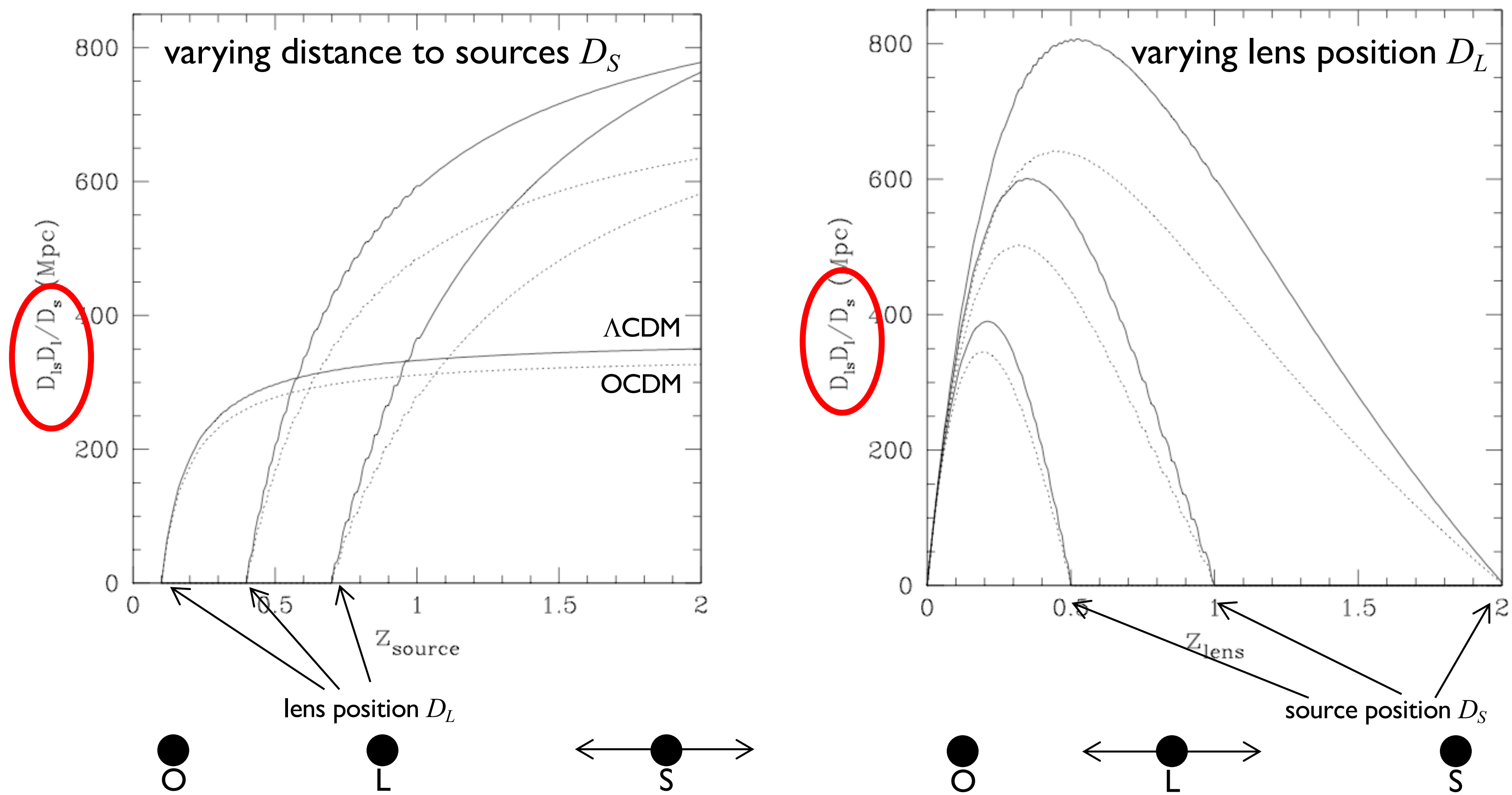
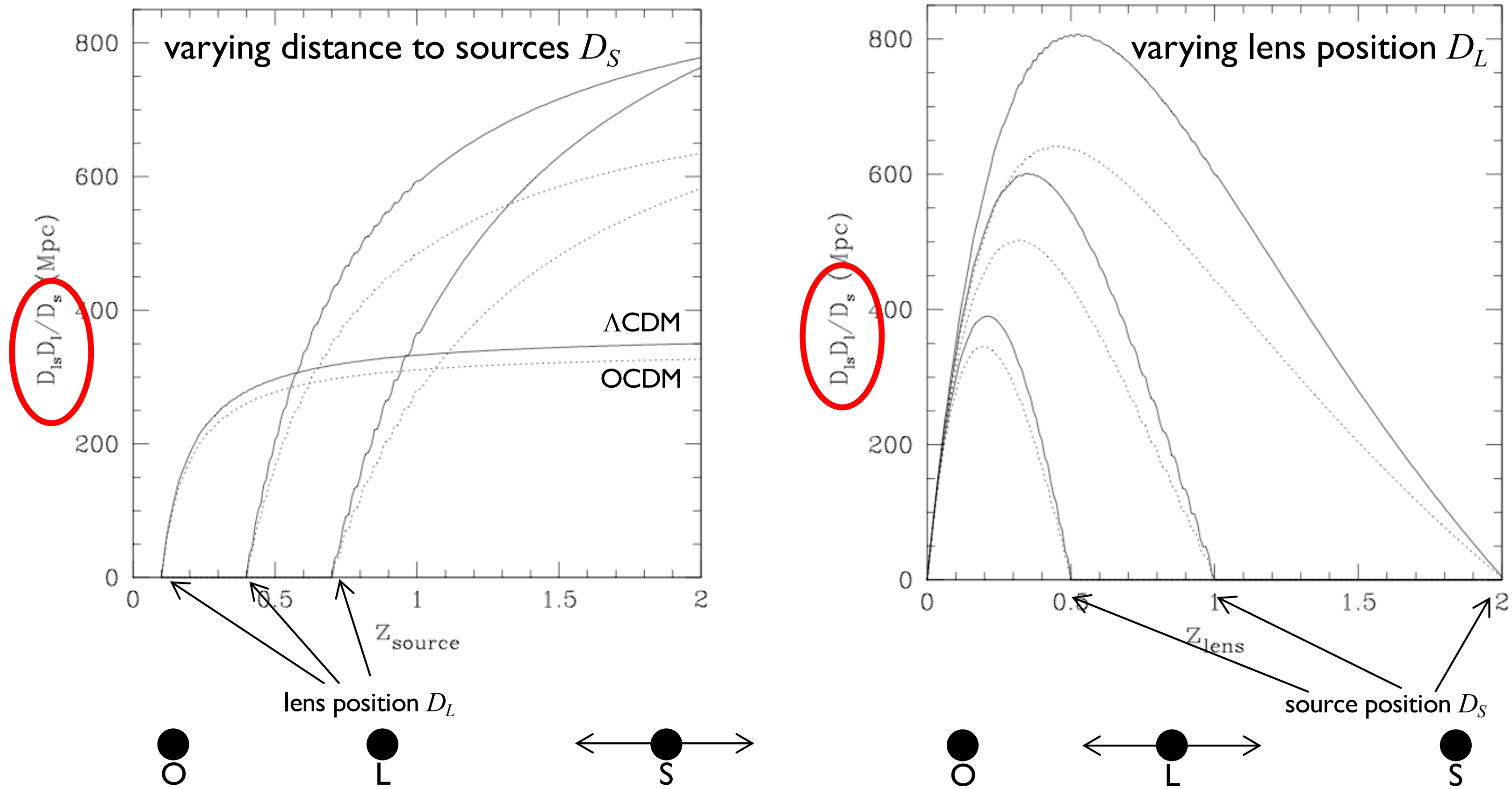


Wittman (2002)

- cosmological considerations

lensing kernel/efficiency

$$\kappa(\theta) = \frac{3H_0^2\Omega_0}{2c^2}\int \frac{D_{LS}D_L}{D_S}\frac{\delta(\theta,z)}{a(z)}\;dz$$



Wittman (2002)

- concept
- theory
- **application**

- past, present & future projects

2006	KIDS	Kilo Degree Survey
2007	COSMOS	Cosmic Evolution Survey
2008	Pan-STARRS	Panoramic Survey Telescope & Rapid Response System
2008	STAGES	Space Telescope A901/2 Galaxy Evolution Survey
2009	CFHTLS	Canada-France-Hawaii Telescope Legacy Survey
2012	DES	Dark Energy Survey
2010	HETDEX	Hobby-Eberly Telescope Dark Energy Experiment
2015	SNAP	Supernova Acceleration Probe
2015	ADEPT	Advanced Dark Energy Physics Telescope
2015	DESTINY	Dark Energy Space Telescope
2020	Vera Rubin	Legacy Survey of Space and Time
2022?	Euclid	Dark Universe Explorer

Note: this list does not claim to be complete!

- past, present & future projects

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Note: this list does not claim to be complete!

- some examples…

…mass maps of (colliding) galaxy clusters and the Universe, respectively

…discovery of (previously unknown) mass concentrations

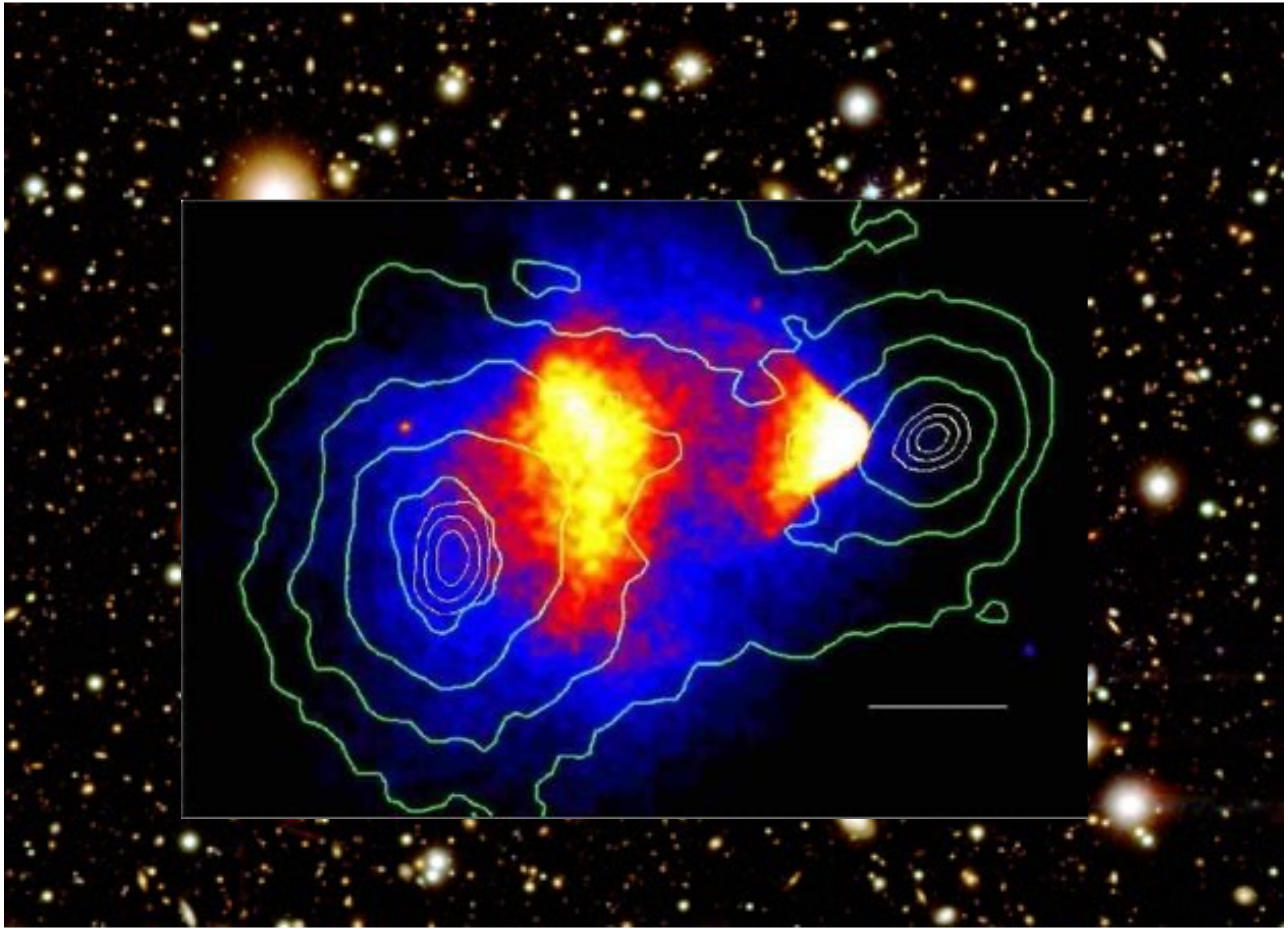
…mapping the large-scale structure of the Universe

…lensing of the CMB photons

- mass map - “Bullet” cluster

(Clowe et al. 2004)

- mass map - "Bullet" cluster



(Clowe et al. 2004)

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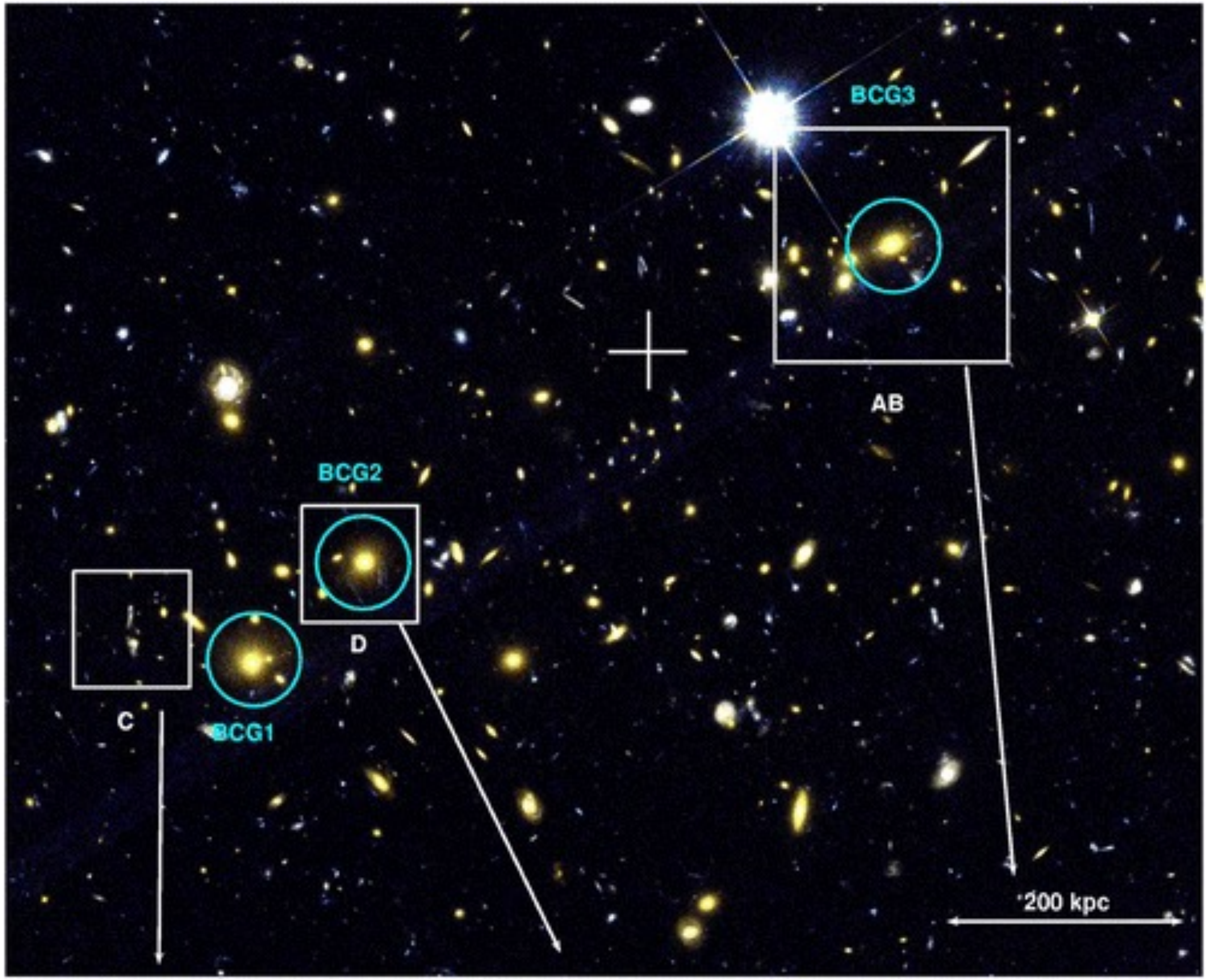
- mass map - MACS J0025.4-1222



(Bradac et al. 2008b)

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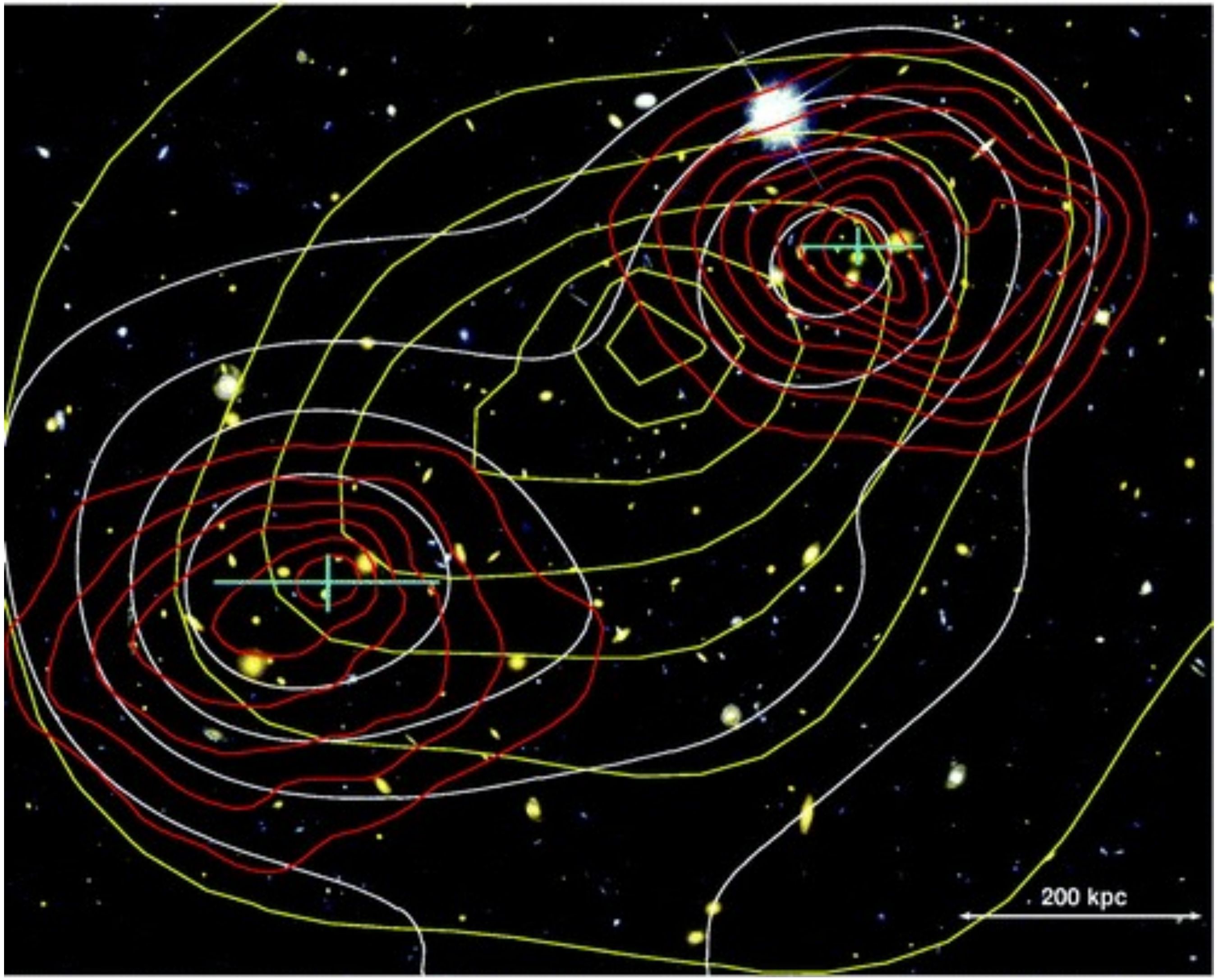

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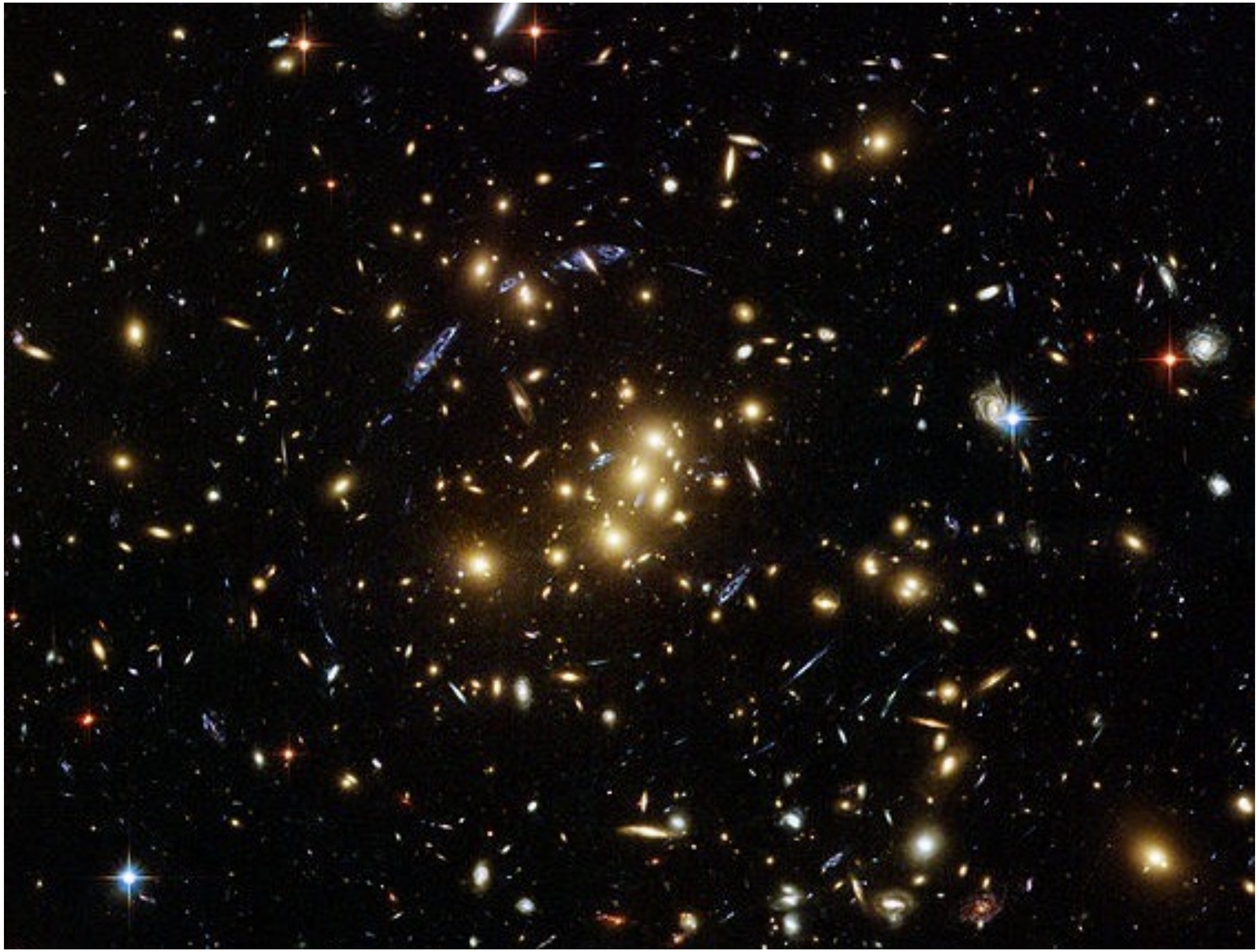
- mass map - "Dark Matter Ring" in Cl0024+17



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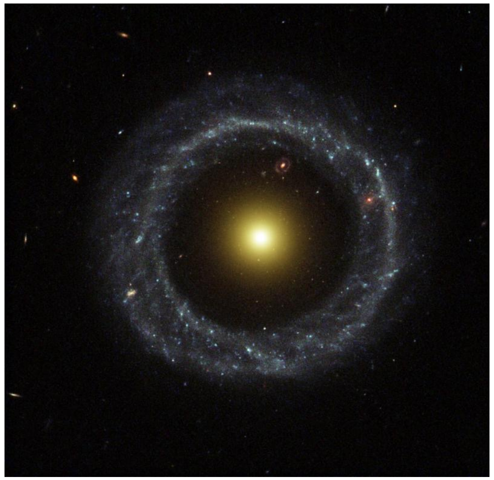
Cartwheel galaxy



Hoag's object

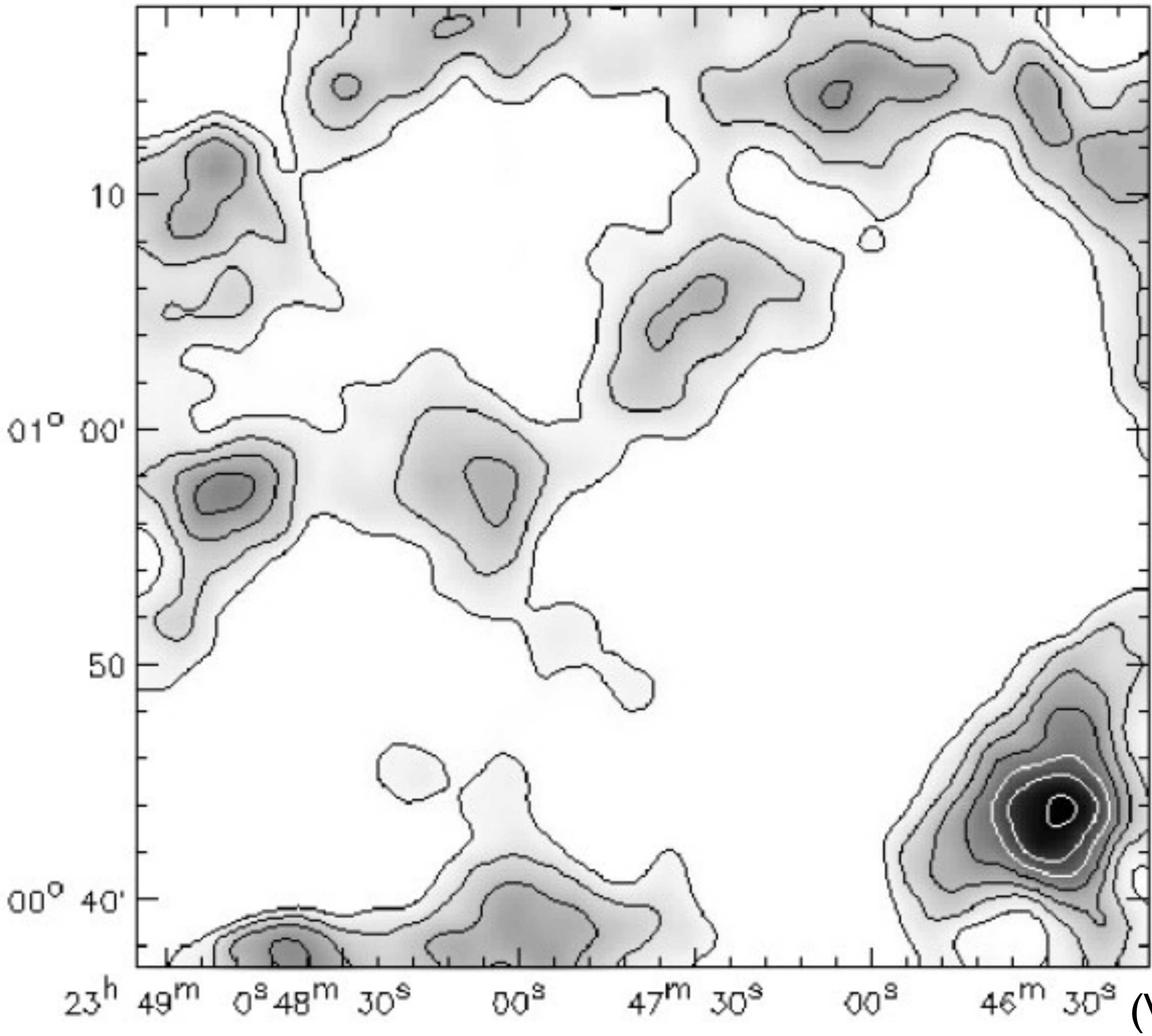
?

Cl0024+17 (Jee et al. 2007)

- unexpected discovery of galaxy cluster

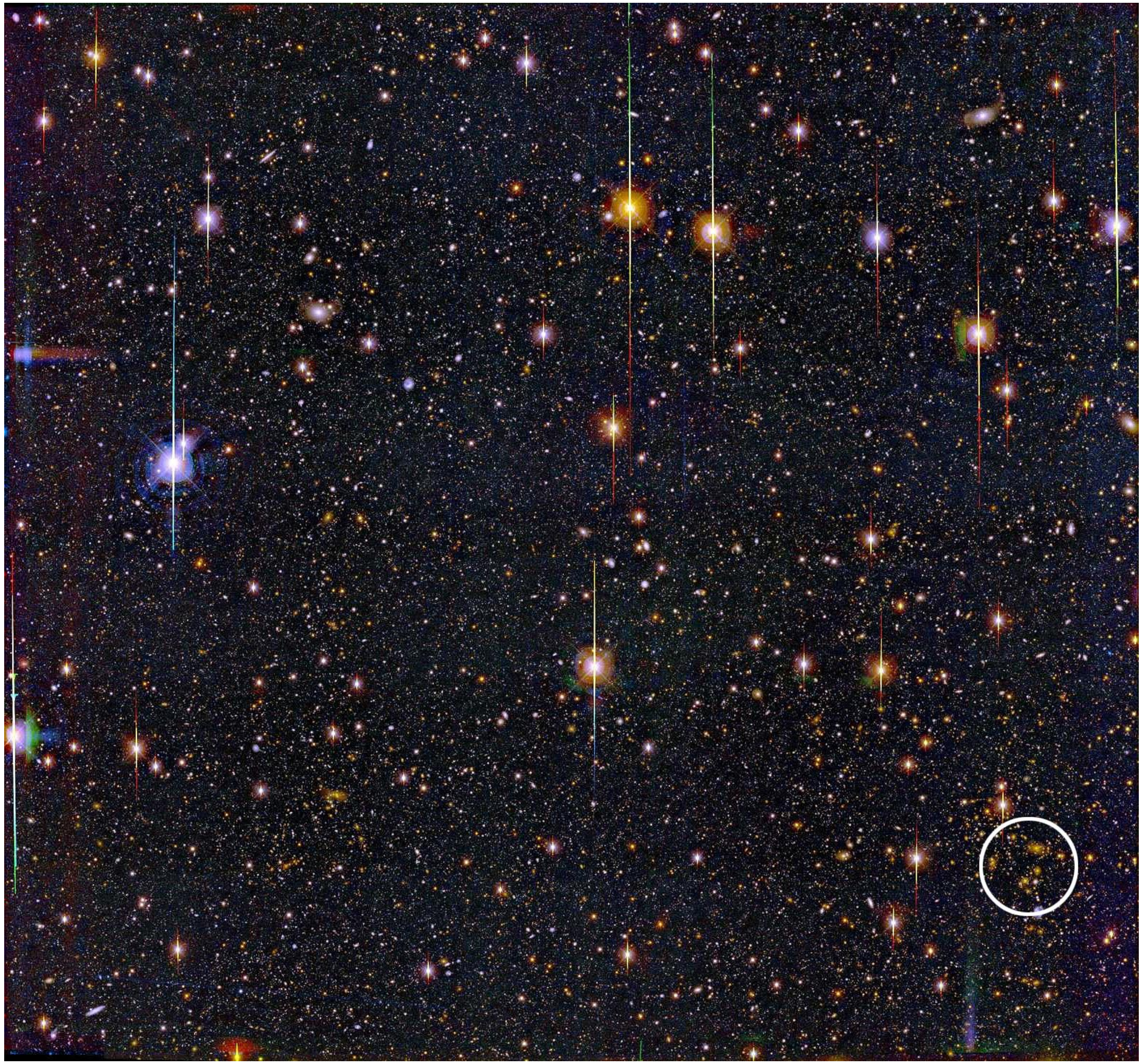
(Wittman et al. 2001)

- unexpected discovery of galaxy cluster



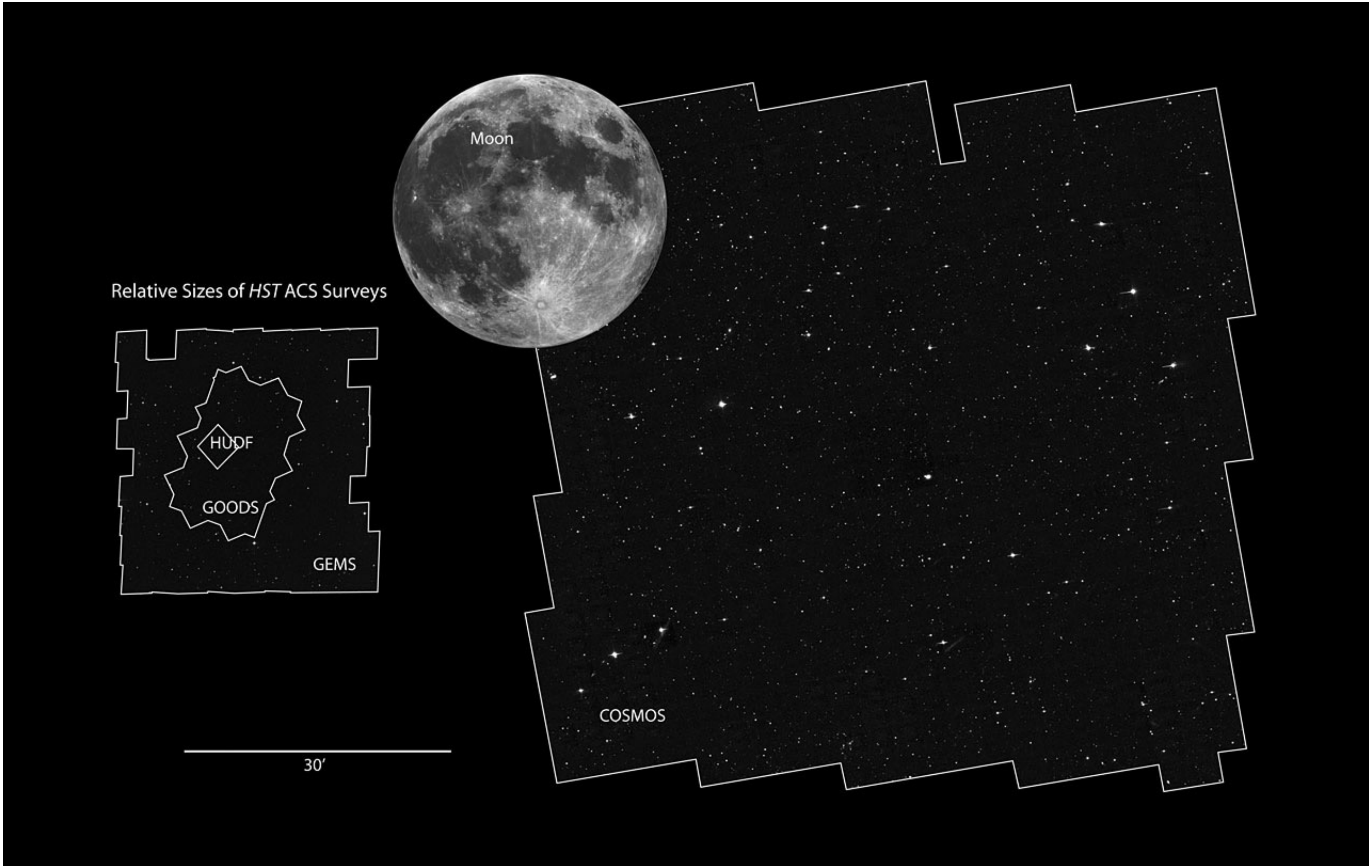
(Wittman et al. 2001)

- unexpected discovery of galaxy cluster



(Wittman et al. 2001)

- mass map of the Universe: COSMOS survey



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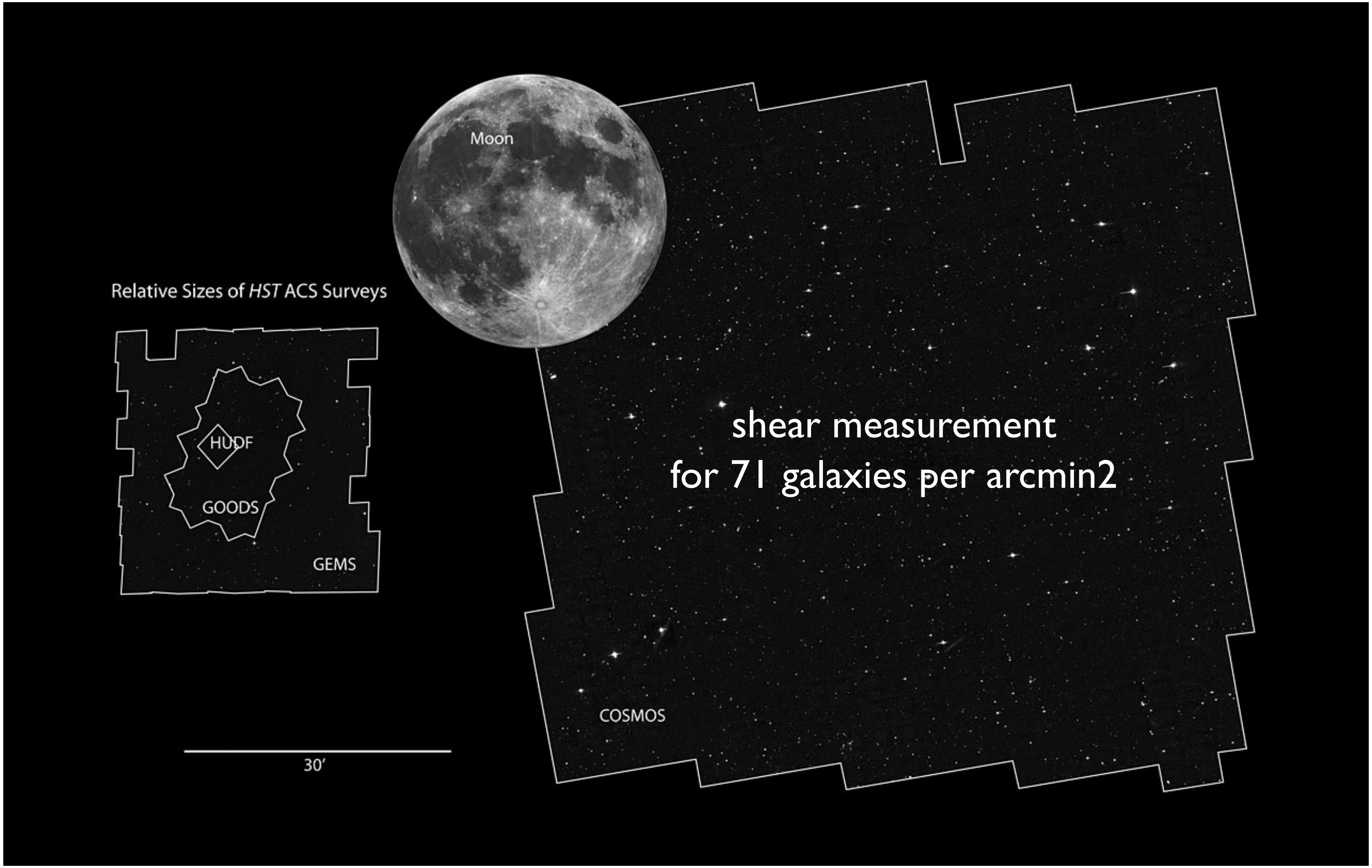

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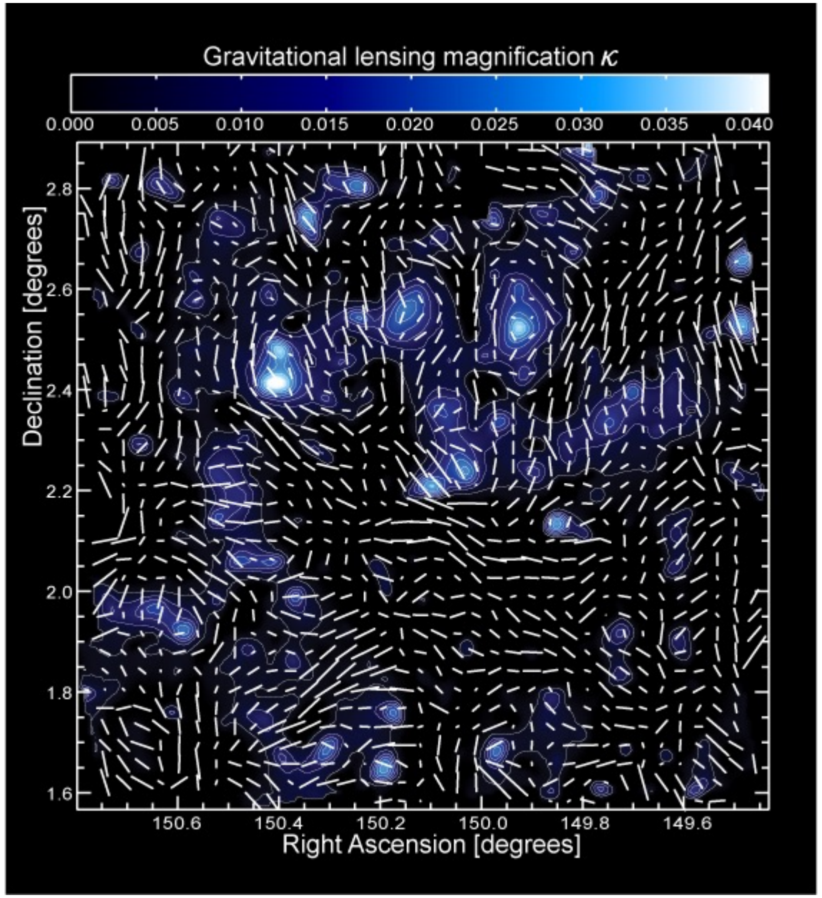

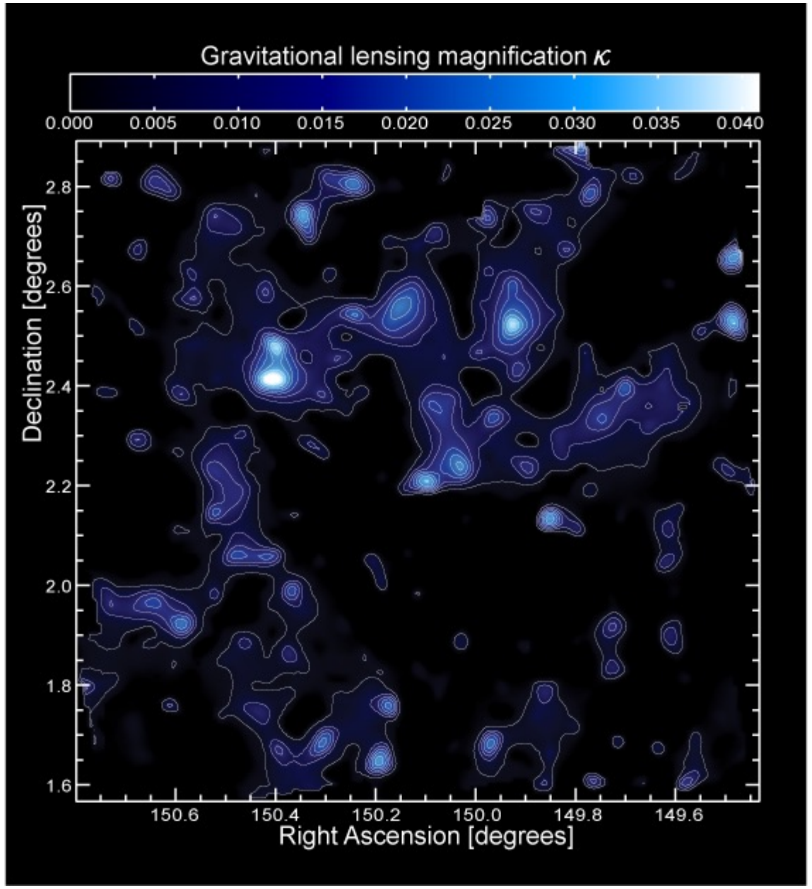

(Massey et al. 2007)

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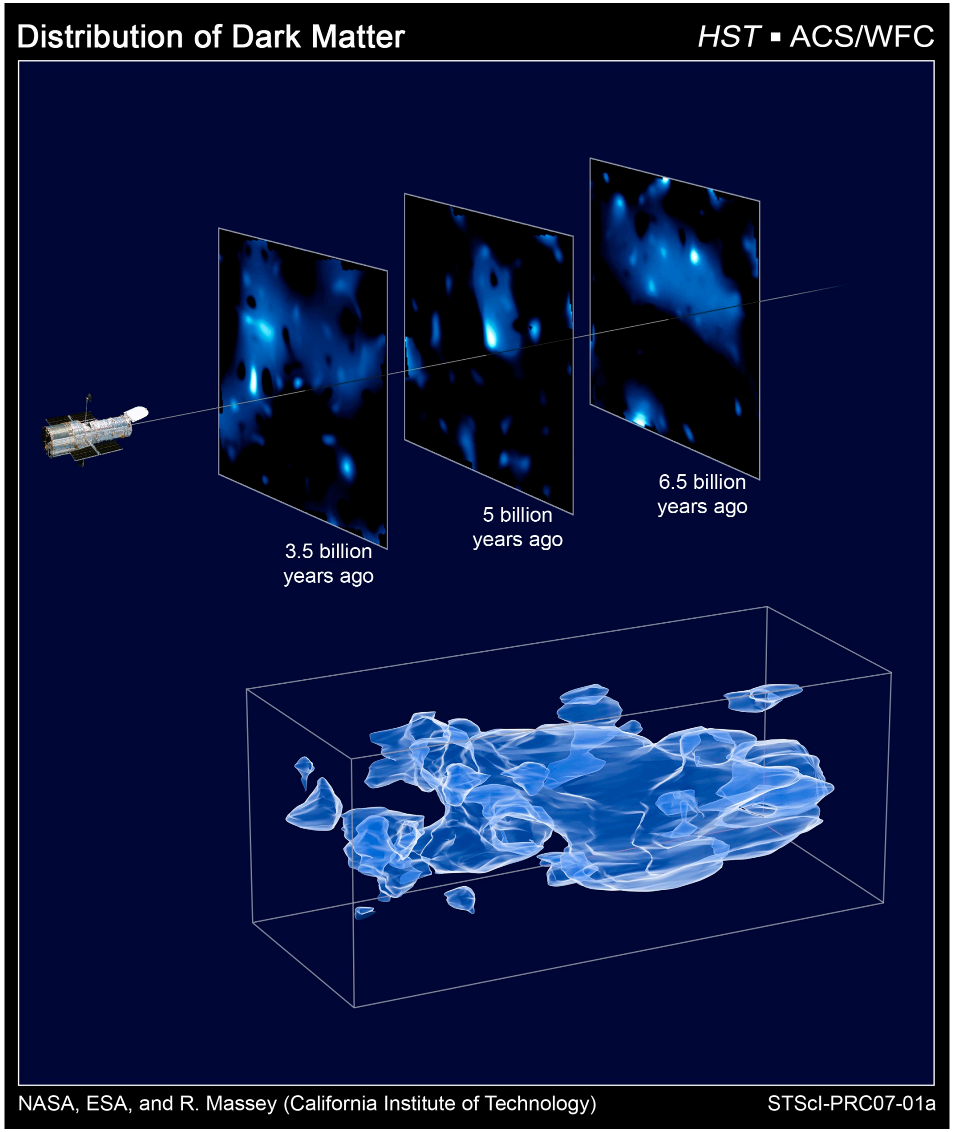

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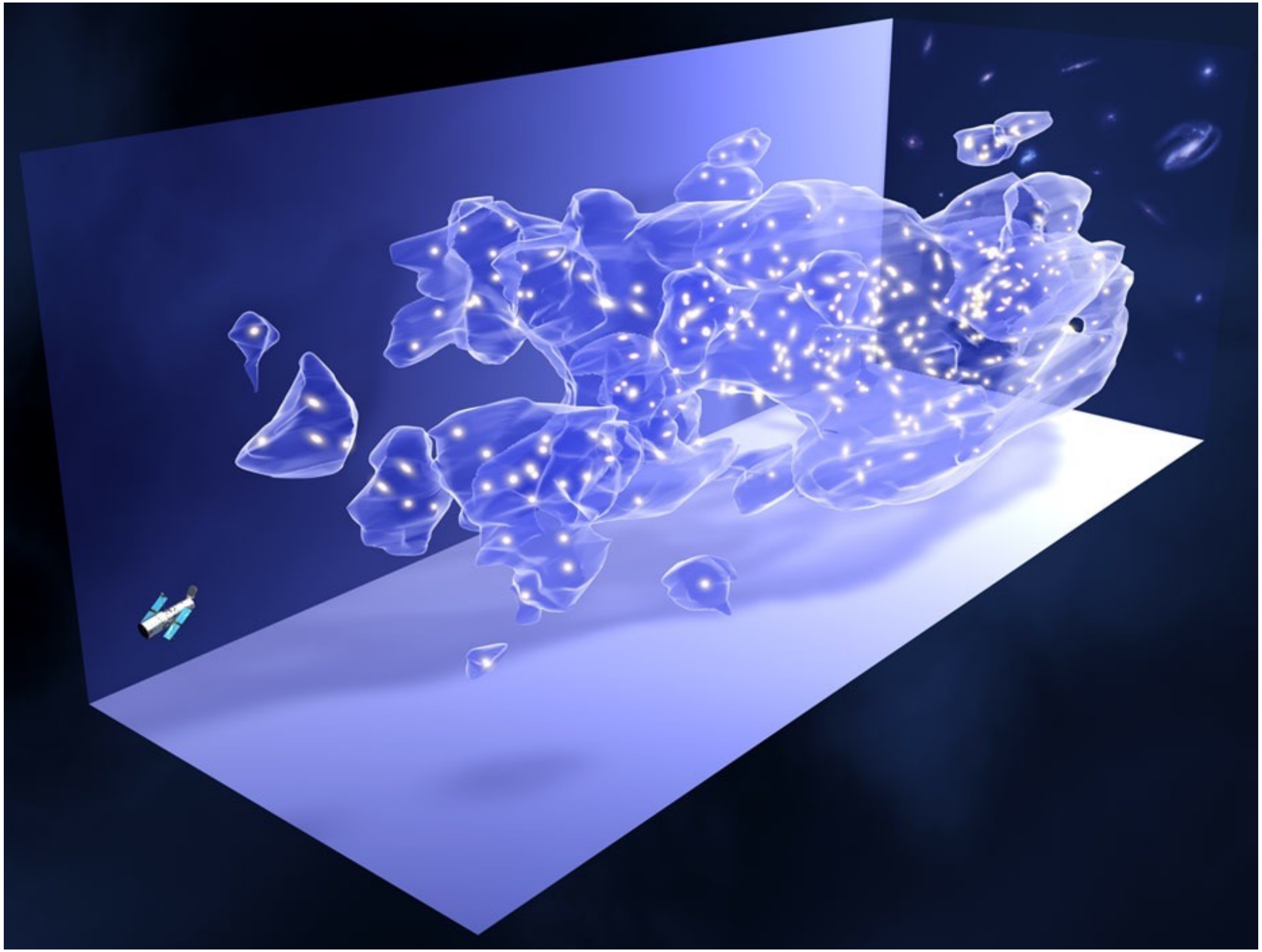

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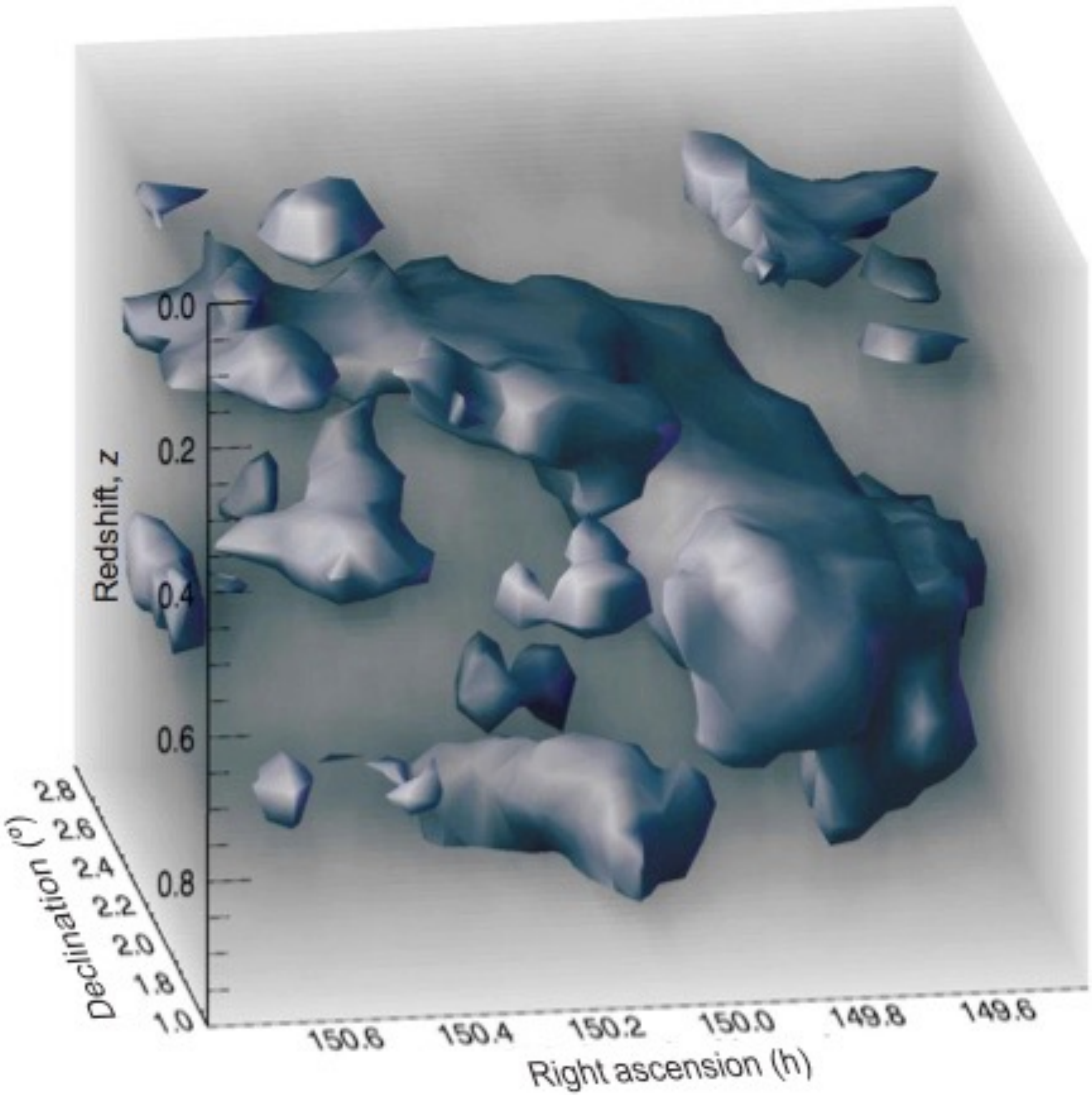
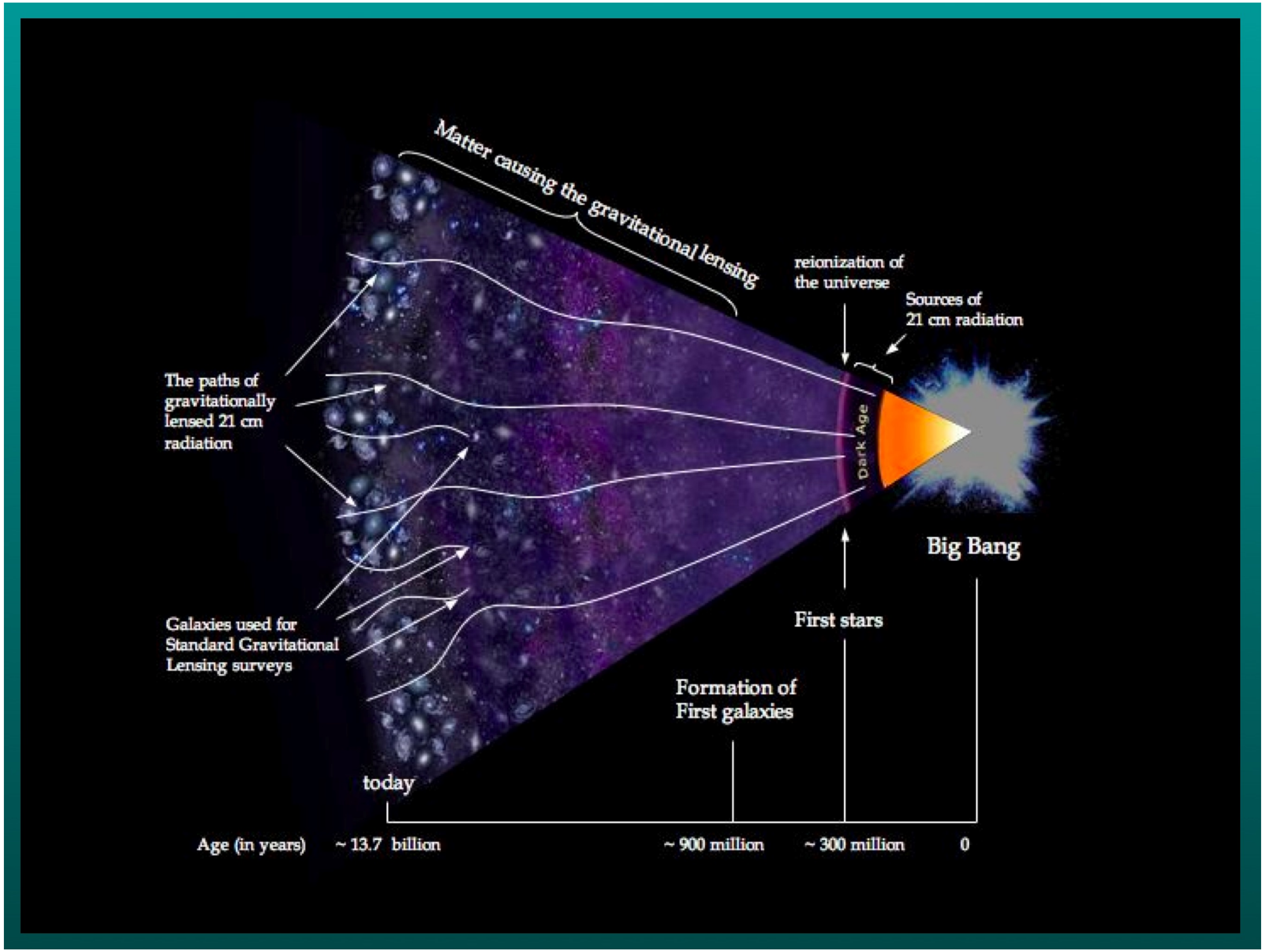


(Massey et al. 2007)

- lensing of the CMB

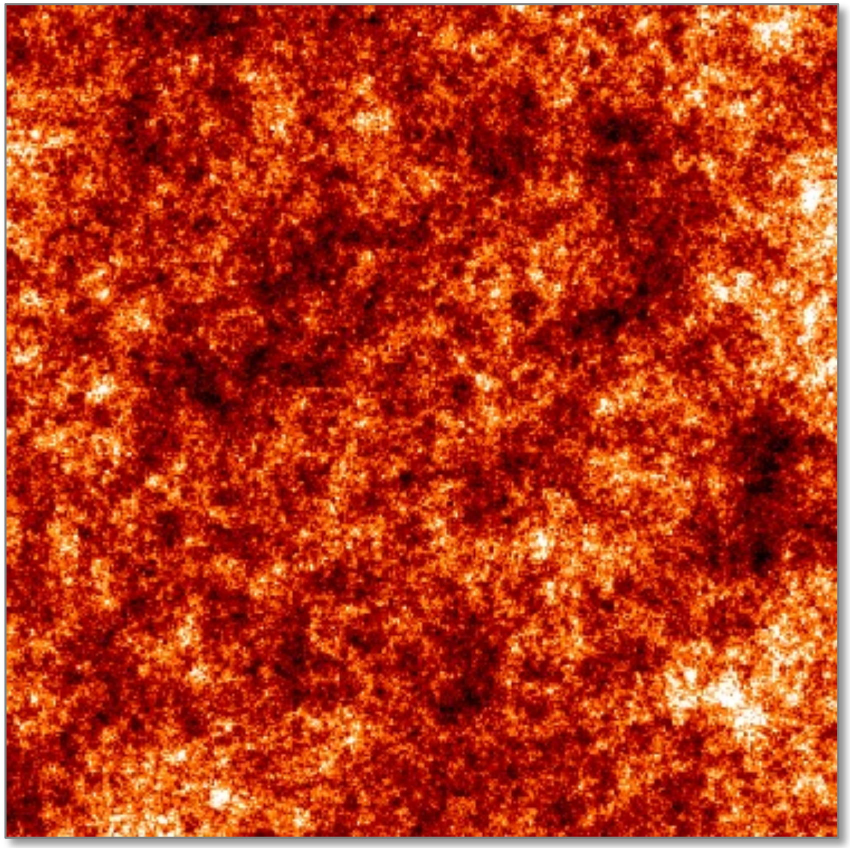
!?

- lensing of the CMB



- lensing of the CMB

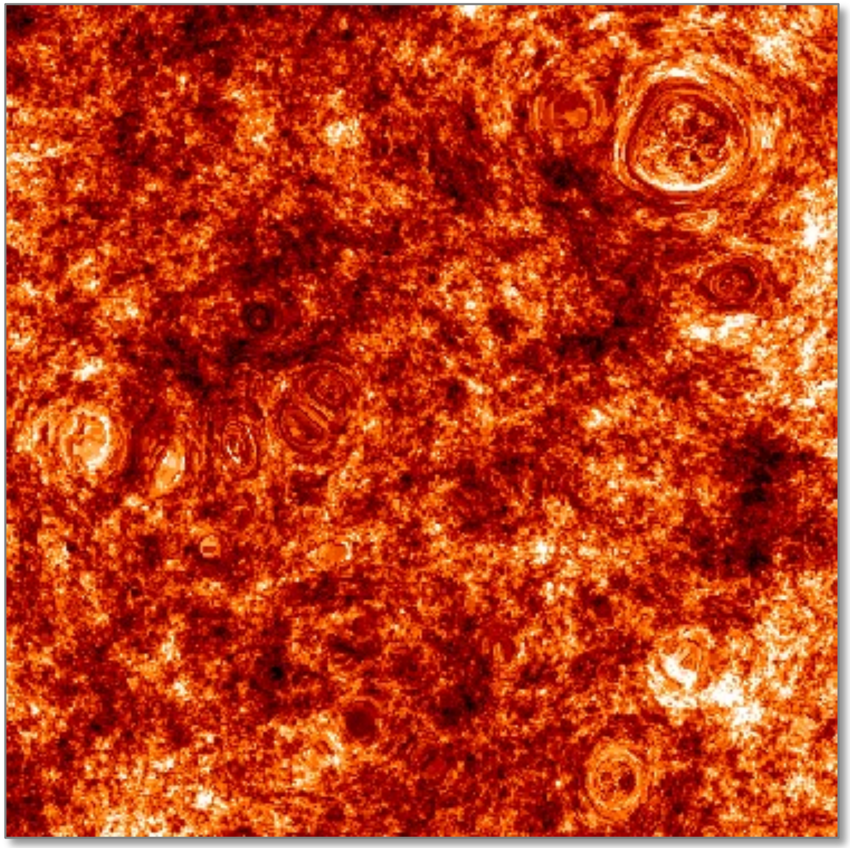
unlensed



← 6 arcmin →

- lensing of the CMB

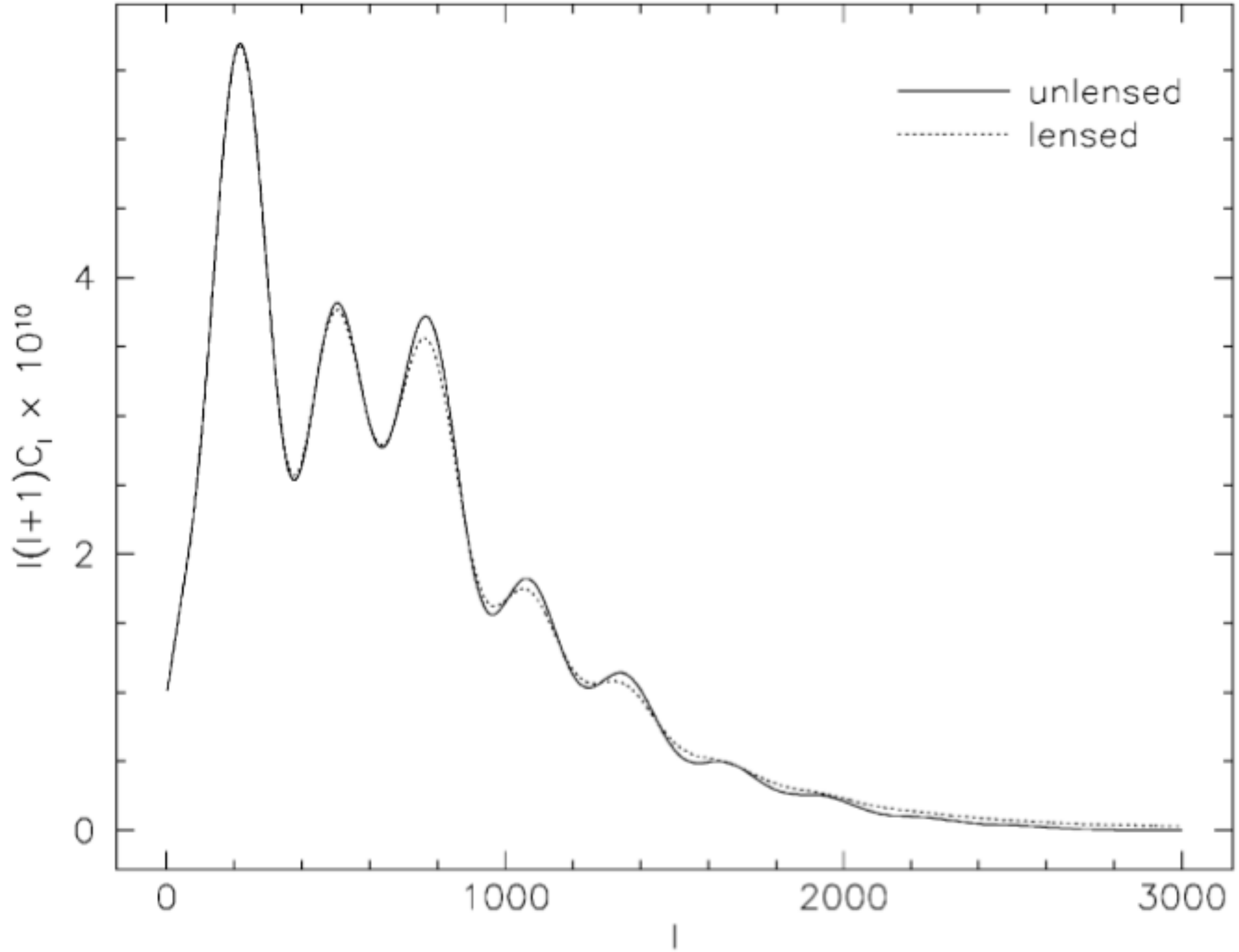
lensed by LSS



← 6 arcmin →

- lensing of the CMB

effect on CMB anisotropy measurements



Bartelmann & Schneider (2001, p.457)

- lensing of the CMB

  – the CMB polarisation field can be decomposed into E and B modes

- lensing of the CMB

  – the CMB polarisation field can be decomposed into E and B modes

$\Rightarrow$ lensing mixes these modes*:

- polarisation picks a favourite direction
- lensing distorts those lightrays

*Note that this is distinct from generating B mode shear!

- **weak lensing** – some results

  – the "Bullet cluster"

  direct evidence for the existence of dark matter!?

  (Clowe et al. 2004)

  – 3D map of the dark matter in the Universe

  confirmation of "cosmic scaffolding"

  (Massey et al. 2007)

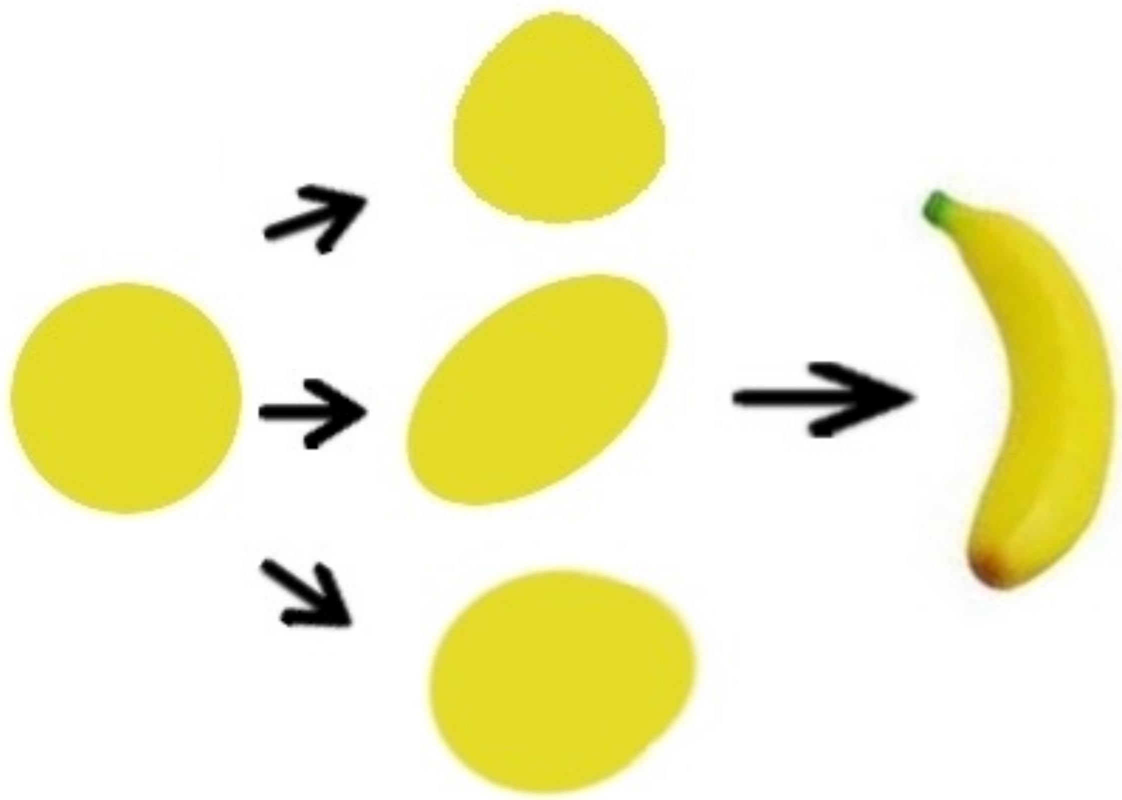
  – cosmic shear analysis*

  Ω_0=0.30 $\Omega_{\Lambda,0}$=0.70 σ_8=0.80

  (Hetterscheidt et al. 2007)

*Note that the values depend on the actual data set analysed :-(

Alexander Knebe, *Universidad Autonoma de Madrid*



"You have a mass distribution about which you do not know anything,
and then you observe sources which you do not know either.
And then you claim to learn something about the mass distribution?"

(Schneider 2006)