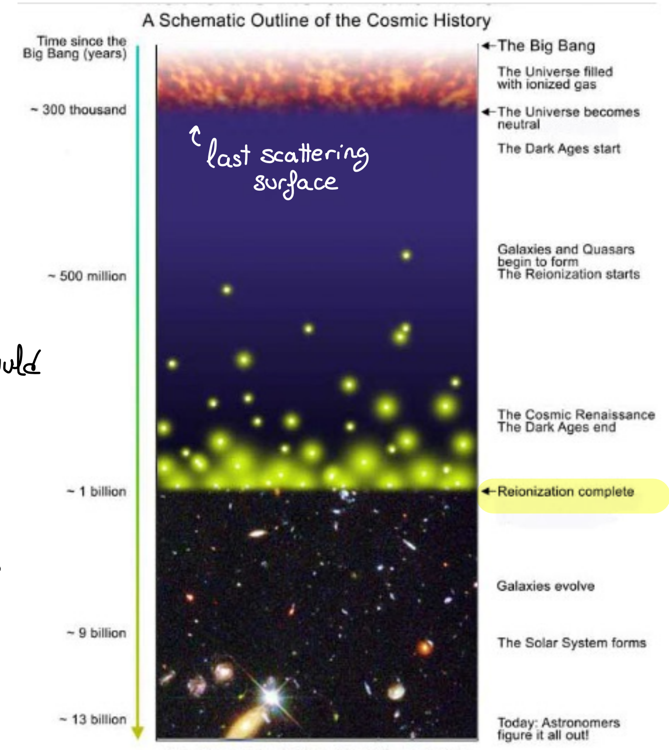
X. The first stars and galaxies

10.1. The dark ages of the Universe

After decoupling, baryons follow DM structures and fall into their potential wells. Aside from the CMB photons, we do not observe any light until the birth of the first stars, which emitted photons that ionized neutral hydrogen (almost all the free electrons were forming H, otherwise we would not have had decoupling).

During this dark ages, the Universe continued expanding and cooling, and there were no sources of energy until the birth of the first stars.



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10.2. The first stars

Summary

Star formation requires coolant for collapse (otherwise, we cannot form structures). The only coolant available when first stars were formed was H_2. The conditions for collapse were only met for $z < 100$. Numerical models suggest that first stars were very massive ($M \in [10, 500] M_\odot$), and died fast and hard: as supernovae of $M \in [8, 100] M_\odot$, which enriched the IGM with metals. Those metals facilitate subsequent star formation because they cool down the ISM.

Star formation in general

Virial theorem

We will start by deriving the virial theorem: From the expresion of the moment of inertia:

$$I = \sum_i m_i |\vec{r}_i|^2 \longrightarrow \frac{1}{2}\frac{dI}{dt} = \frac{1}{2}\frac{d}{dt}\sum_i m_i |\vec{r}_i|^2 = \frac{1}{2}\sum_i m_i \frac{d|\vec{r}_i|^2}{dt} = \sum_i m_i \frac{d\vec{r}_i}{dt}\cdot \vec{r}_i = \sum_i \vec{p}_i \cdot \vec{r}_i = G$$

$$G = \sum_i \vec{p}_i \cdot \vec{r}_i \Rightarrow \frac{dG}{dt} \overset{*}{=} \sum_i \vec{F}_i \cdot \vec{r}_i + 2E_{kin}$$

* Dem:

$$\frac{d}{dt}G = \sum_i \dot{\vec{p}}_i \cdot \vec{r}_i + \sum_i \vec{p}_i \cdot \dot{\vec{r}}_i = \sum_i \vec{F}_i \vec{r}_i + \sum_i \vec{p}_i \cdot \dot{\vec{r}}_i = \sum_i \vec{F}_i \vec{r}_i + 2\sum_i E_{kin,i} = \sum_i \vec{F}_i \cdot \vec{r}_i + 2E_{kin}$$

$$\sum_i \vec{F}_i \cdot \vec{r}_i = \sum_i (\nabla E_{pot}) \cdot \vec{r}_i = n E_{pot}$$

$\hookrightarrow \vec{F} = -\nabla E_{pot}$ $\quad \hookrightarrow E_{pot} = C r^n \Rightarrow \nabla E_{pot} = n \frac{E_{pot}}{r} \Rightarrow F \cdot r = n E_{pot}$

$$\frac{dG}{dt} = nE_{pot} + 2E_{kin}$$

bound system (coordinates and velocities have upper and lower limits)

$$\left\langle \frac{dG}{dt} \right\rangle_\tau = \frac{1}{\tau}\int_0^\tau \frac{dG}{dt}\,dt = \frac{1}{\tau}\left[G(\tau) - G(0)\right] \leq \frac{G_{max} - G_{min}}{\tau} \xrightarrow{\tau \to \infty} 0$$

$$0 = \left\langle \frac{dG}{dt} \right\rangle_\tau = 2\langle E_{kin} \rangle_\tau + n\langle E_{pot} \rangle_\tau$$

$E_{pot} \propto r^n$ ($n = -1$ for gravity)

$$0 = 2\langle E_{kin} \rangle_\tau - \langle E_{pot} \rangle_\tau$$

$$E_{pot} = Cr^{-1}$$

This can be used to derive the Jeans mass for a homogeneous sphere.

Gravitational potential of homogeneous space

For a sphere with constant density ρ:

$$M(r) = \rho \frac{4\pi}{3} r^3$$

$$dM(r) = \rho 4\pi r^2 dr \qquad R = \sqrt[3]{\frac{M}{\frac{4\pi}{3}\rho}}$$

$$dE_{pot} = -G\frac{M(r)dM(r)}{r}$$

We can calculate the potential and kinetic energy:

$$E_{pot} = -\frac{3}{5}G\frac{M^2}{R}$$

$$\left(E_{pot} = -4\pi G\rho \int_0^R \frac{M(r)}{r} r^2\,dr = -4\pi G\rho^2 \int_0^R \frac{\frac{4\pi}{3}r^3}{r} r^2\,dr = -\frac{16\pi^2}{3}G\rho^2 \int_0^R r^4\,dr = -\frac{16\pi^2}{15}G\rho^2 R^5 = -\frac{16\pi^2}{15}G\frac{M^2}{(\frac{4\pi}{3}R^3)^2}R^5 = -\frac{3}{5}G\frac{M^2}{R} \right)$$

$$E_{kin} = \frac{3}{2}Nk_BT = \frac{3}{2}\frac{M}{\mu_H m_H}k_BT \quad \text{(law of ideal gas)}$$

Using the virial theorem, we get to the following definition for the Jeans mass:

$$M_J = \sqrt{\frac{3(5k_B)^3}{4\pi(G\mu_H m_H)^3}}\sqrt{T^3}\frac{1}{\sqrt{\rho}}$$

We will only have a bound object for $M > M_J$ (otherwise, we would have too much kinetic energy compared to the potential energy. If the potential energy domains over the kinetic energy, we will have gravitational collapse.

Jeans mass

Substituting the values of the constants, we have obtained the following expression for M_J:

$$M_J = 5.46\left(\frac{k_B}{G\mu_H m_H}\right)^{3/2}\left(\frac{T^3}{\rho}\right)^{1/2}$$

If we have a relation between T and ρ, we can study its behaviour for different types of processes (nature of the gas). This relation determines the fate of collapse, since it will only start if $M > M_J$. We will consider an adiabatic and an isothermal gas.

• **Adiabatic**

$PV^\gamma = \text{const} \longrightarrow T \propto \rho^{\gamma-1} \longrightarrow T \propto \rho^{2/3}$

$P = \frac{\rho}{\mu_H m_H} k_B T$ $\qquad \gamma = \frac{5}{3}$ (monoatomic gas)

$$M_J = 5.46 \left(\frac{k_B}{G \mu_H m_H}\right)^{3/2} \left(\frac{T^3}{\rho}\right)^{1/2} \propto \sqrt{\rho}$$

When ρ increases, M_J increases : collapse stops. Even if we initially have $M > M_J$ (able to collapse), we will eventually get a value of M below Jeans mass. We will never get to ignite nuclear reactions.

• **Isothermal**:

$T = \text{const} \longrightarrow M_J = 5.46 \left(\frac{k_B}{G \mu_H m_H}\right)^{3/2} \left(\frac{T^3}{\rho}\right)^{1/2} \propto \frac{1}{\sqrt{\rho}}$

For an initial mass $M > M_J$: if ρ increases, M_J decreases : "runaway" collapse

Collapse converges to isothermal sphere.

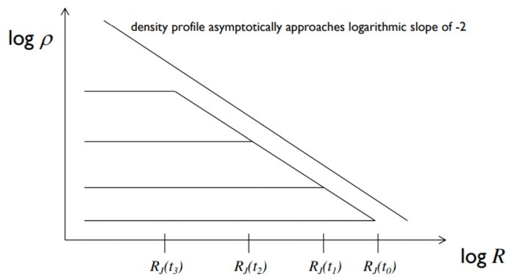
Isothermal gravitational collapse

We can calculate the density profile:

$$\rho \propto \frac{M_J}{R_J^3} \propto \frac{1/\sqrt{\rho}}{R_J^3}$$

$$\rho^{3/2} \propto 1/R_J^3 \longrightarrow \rho \propto \frac{1}{R_J^2}$$

$$\boxed{\log \rho \propto -2 \log R}$$



Cooling

Isothermal collapse requires cooling. Dust grains / metals can absorb and re-emit energy, but there are no such things at cosmic dawn. This is where and why the first star formation differ's from today : in the primeval Universe there was no dust/ metal acting as coolant.

The dominant coolant is (molecular) hydrogen H_2, which acts via rotational/vibrational channels:

- excitation through collision (R/V)
- de-excitation via radiation (→ cooling) or collision

The formation of H_2 requires free electrons and protons (remnants of the epoch of recombination plus ionized H by energy of initial collapse) and high densities $n_H > 10^8\ cm^{-3}$

- $H + e^- \rightarrow H^- + \gamma$
- $H + H^- \rightarrow H_2 + e^-$
- $H + p \rightarrow H_2^+ + \gamma$
- $H + H_2^+ \rightarrow H_2 + H^+$

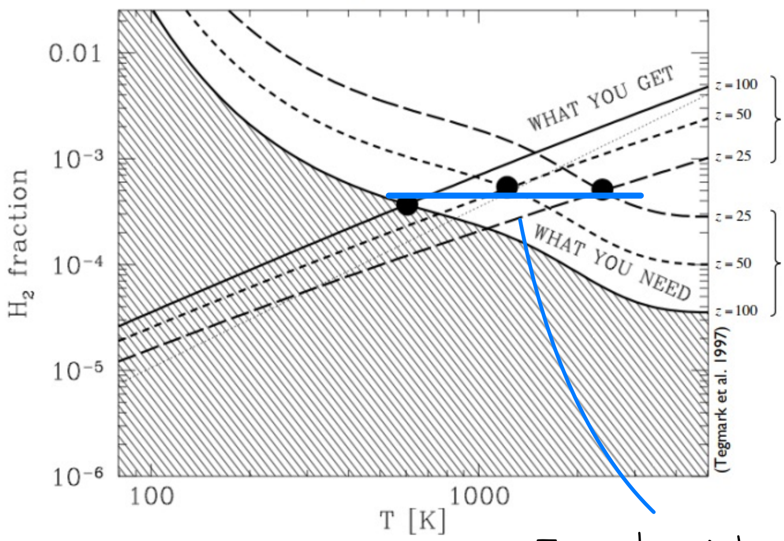
} presence of free electrons and protons

- $H + H + H \rightarrow H_2 + H$
- $H + H + H_2 \rightarrow H_2 + H_2$

} high densities, $n_H > 10^8 cm^{-3}$

The H_2 fraction X_{H_2} must exceed 10^{-4} for cooling to be effective.

H_2 fraction

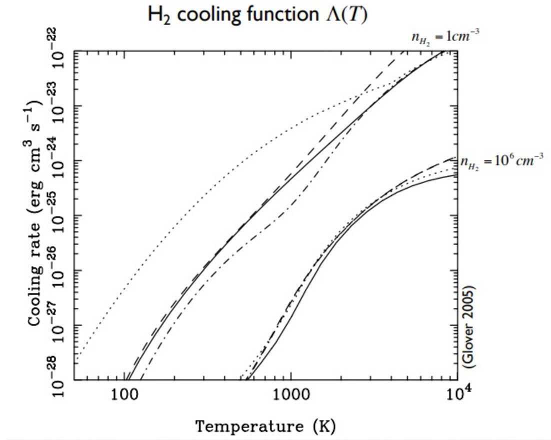


H_2 fraction produced in Hubble time $t_{Hubble} \propto H^{-1}(z)$

H_2 fraction able to cool in Hubble time $t_{cool} = \frac{3kT}{2n\Lambda(T)}$ $\Lambda(T)$ cooling function

~ Temperature independent

H_2 cooling function $\Lambda(T)$



We require of order $X_{H_2} \gtrsim 5 \times 10^{-3}$

To figure out if the amout of hydrogen that we are producing is able to cool enough to enable star formation, we need to look at the cooling function (which depens on the temperature).

The efficiency of the cooling increases with T

$\Rightarrow$ Even if the initial conditions are not enough to have isotermal collapse, an increase of the temperature will increase the efficiency of the cooling.

Collapse phases

Due to the conditions involved, the collapse will happen as:

1. Adiabatic collapse due to the lack of sufficient H_2. The increasing density leads to more H_2, and the increasing temperature leads to more efficient cooling.
2. Collapse becomes isothermal

First stars formation

Masses of the first stars

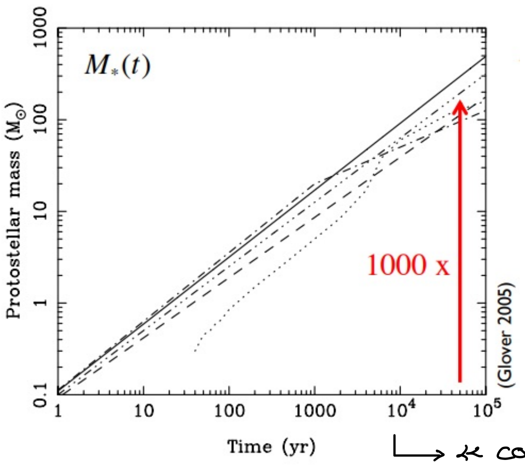
The mass growth of a proto-stellar gas cloud is given by:

$$M_*(t) = M_{pr} + \int_0^t \dot{M}(\tau)\, d\tau, \quad \text{where} \quad \dot{M} = \frac{dM}{dt}$$

$\int_0^t \dot{M}(\tau)\, d\tau$ → infall rate

M_{pr} → Jeans mass that triggers collapse

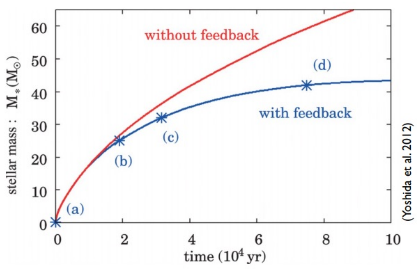
Gravity pulls in some of the material of the vecinity into the collapsing region. Numerical models for the mass accretion rate $\dot{M}$ lead to:



→ ≪ cosmic time

In very short times, the proto-star becomes extremely massive. However, feedback can substancially reduce the accretion rates (and hence M_*)

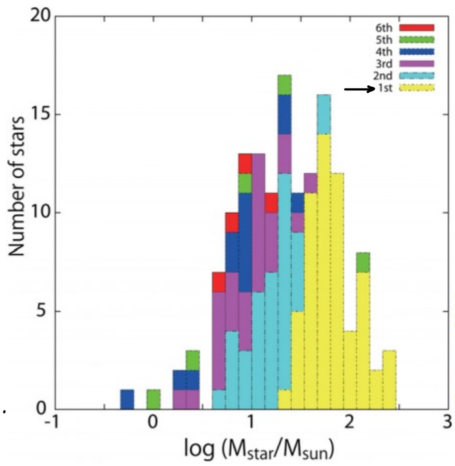
[Feedback effects due to nuclear fusion inside the proto-star]

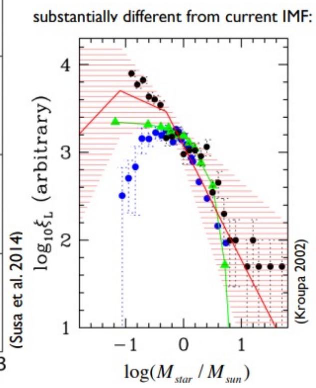


The primeval initial mass function is determined via simulations. The range of masses for the first stars is

$M \in [10, 500] M_\odot$

The next generations have different mass ranges and distribution, mostly because of the changes in the cooling mechanism.





On the life and dead of high mass stars:

The mass-luminosity relation for main sequence stars is given by:

$$L \propto \frac{dE}{dt} \propto M^{3.5}$$

*approximate derivation:

perfect black-body radiator: $L = 4\pi R^2 \sigma T^4$

hydrostatic equilibrium: $\frac{dP}{dr} = -\frac{GM\rho}{r^2} \Rightarrow \langle P \rangle = -\frac{1}{3}\frac{E_{pot}}{V} \Rightarrow \langle P \rangle V = -\frac{1}{3}E_{pot} = \frac{1}{5}\frac{GM^2}{R} = NkT = \frac{M}{m_H}kT = \frac{M}{m_H}k\frac{L^{1/4}}{4\pi R^{1/2}}$

$$M^{3.33} \propto L \Leftarrow M^4 \propto L\, M^{2/3} \overset{R \propto M^{1/3}}{\Leftarrow} M^4 \propto L R^2 \Leftarrow M \propto L^{1/4} R^{1/2} \Leftarrow \frac{M^2}{R} \propto M \frac{L^{1/4}}{R^{1/2}}$$

This radiated energy comes from nuclear reactions in the centre of the start, which is transported to its surface (either by radiation or convection). There is radiated away as photons.

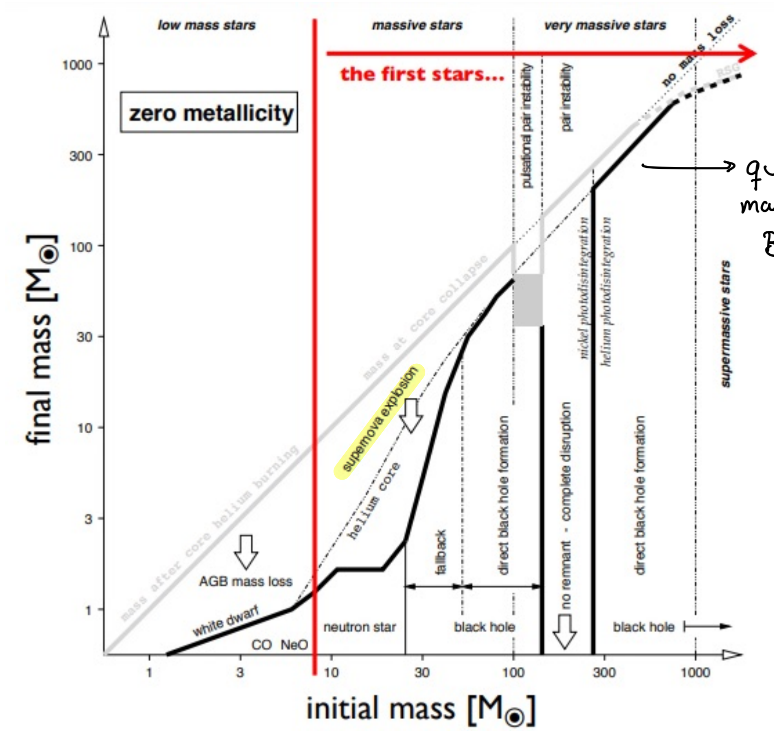
We can calculate the maximum lifetime a star of a given mass can have if it radiates away all its energy. The energy reservoir is proportional to mass:

$E \propto M$

And so:

$L \propto dE/dt \longrightarrow \tau = E/L \propto M^{-2.5}$ (typical time on main sequence)

High mass stars die hard (with spectacular end stages) and fast (after a few Myrs only). Metal-free high mass stars try to use gravity as an alternative energy reservoir. Due to this, they rapidly collapse. They either form a black hole or completely disrupt ("pair instability supernova").



→ quite massive BH

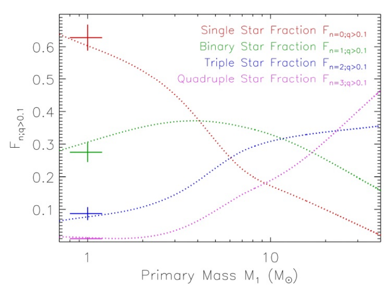
The figures shows the possible initial and final states for stars with different masses.

Supernova exposions enrich the IGM, which cool their environment more efficiently than H_2, which facilitates subsequent star formation.

First stars open questions

- Do the first stars come for binaries?

Nowadays, almost all the stars form binaries, but we do not Know if this was also true for the first stars.



- How did Pop III star formation come to an end? (old stars)

They might be still forming somewhere in the Universe.

- What is the influence of magnetic fields?
- How exactly works turbulence/fragmentation?
- What about dark matter?

10.3. The first galaxies

Overview

Dark matter is able to form structures (haloes) befor baryons. Assuming a biased formation scenario (White & Rees, 1974), protogalaxies (gravitationally bound gas clouds) are formed within dark matter haloes by baryons that fall into dark matter potential wells (after decoupling). Thus, it is impossible to find a galaxy without a halo, but it is possible to have haloes without galaxies.

Since we need haloes to form galaxies, we will look at the condition to form them first (Press-Schechter function). First, we will characterize DM overdensity peaks by their height ν, which is defined as:

$$\nu = \frac{\delta}{D(a)\,\sigma_0(M)} = \frac{\delta}{\sigma_M(z)} \quad , \quad \sigma_0^2(M) = \frac{1}{2\pi^2}\int_0^\infty P_0(k)\,\hat{W}_M^2(k)\,k^2\,dk \qquad \ddot{D} + 2H\dot{D} - \frac{3}{2}\Omega_m H^2 D = 0$$

If we compare dark matter $M_{200}(a)$ to its Jeans mass $M_J(a) \propto a^{-3/2}$, it is possible to find the time at which each halo can be formed. $3-\sigma$ DM haloes can be formed at $z \approx 30$. These dark matter haloes virialize due to relaxation processes.

Characterization of DM peaks: peak height

The number density of dark matter haloes (according to the Press-Schechter formalism) is given by:

$$\frac{dn}{dM}\,dM = \sqrt{\frac{2}{\pi}}\,\frac{\bar{\rho}}{M}\,\frac{\delta_c}{\sigma_M}\left|\frac{d\ln\sigma_M}{d\ln M}\right| \exp\left(\frac{-\delta_c^2}{2\sigma_M^2}\right)\frac{dM}{M}$$

where

$$\begin{cases} \sigma_0^2(M) = \dfrac{1}{2\pi^2}\displaystyle\int_0^\infty P_0(k)\,\hat{W}_M^2(k)\,k^2\,dk \\ \hat{W}(x) = \dfrac{3}{x^3}\left(\sin(x) - x\cos x\right) \\ P(k) = \left(\dfrac{D(a)}{D(a_0)}\right)^2 P_0(k) \end{cases}$$

we can combine these expressions and introduce the peak height as: $\nu = \dfrac{\delta_c}{D(z)\,\sigma_0(M)}$

Temporal evolution →

$$\frac{dn}{dM}\,dM = \sqrt{\frac{2}{\pi}}\,\frac{\langle\rho\rangle}{M}\left|\frac{d\nu}{dM}\right|\exp\left(-\frac{\nu^2}{2}\right)dM$$

$$\nu = \frac{\delta_c}{D(z)\,\sigma_0(M)}$$

Jeans mass and collapse

Dark matter will collapse and form a halo if its mass is above Jeans mass. It is important to note that before we defined Jeans mass for objects with pressure support, but DM is pressureless. In this case, pressure does not counterbalance self gravity, but velocity dispersion does.

$$M_J \propto \left(\frac{T^3}{\rho}\right)^{1/2} \longrightarrow M_J \propto \left(\frac{\sigma_v^6}{\rho}\right)^{1/2}$$

$$E_{kin} = \frac{3}{2} N k_B T = \frac{1}{2} m \sigma_v^2$$

Velocity dispersion and density scale as:

$\sigma_v \propto a^{-1}$ (because velocities scale like $1/a$)

$\rho \propto a^{-3}$

Combining both, we obtain the dependence with z of M_J:

$$M_J \propto a^{-3/2}$$

$\longrightarrow$ When a increases, M_J decreases. Formation becomes easier

> **NOTE**
> This Jeans mass refers to the mass of a dark matter halo, but determines whether its baryonic component is able to collapse or will be prevented from it. If the DM is not bound, galaxies will not be able to form.

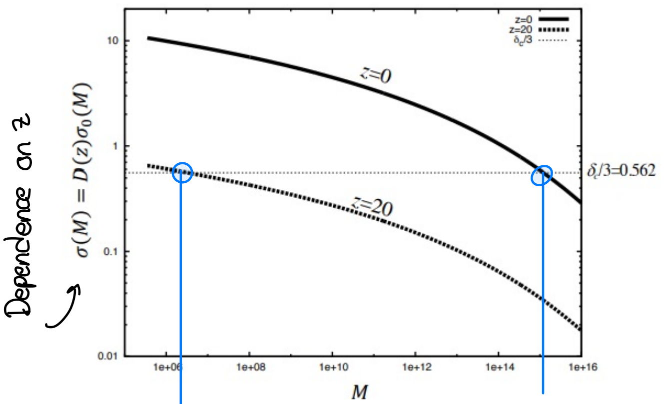
The first bound objects

We can fix a height to compare the mass of the haloes and M_J. Taking 3-σ haloes:

$$\nu = \frac{\delta_c}{D(z)\,\sigma_0(M_{3\sigma})} = 3 \,, \qquad D(z) \text{ linear growth factor}$$

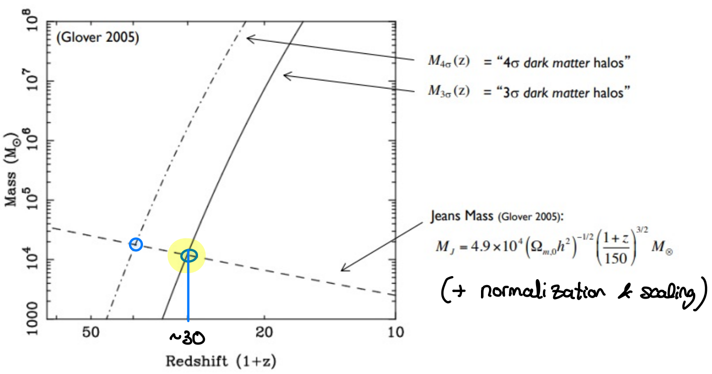
For a given mass, we can plot the variance of the power spectrum (σ_M) when smoothed for that particular mass. This is calculated for different redshifts and compared with $\nu = 3$ $\left(\sigma_M = \frac{\delta_c}{\nu} = \frac{\delta_c}{3}\right)$.

We can obtain the mass of 3σ peaks as a function of redshift (○)



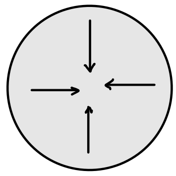
$M_{3\sigma}(z=20) \approx 10^{6.7} M_\odot$ $\quad$ $M_{3\sigma}(z=0) \approx 10^{15} M_\odot$ $\longrightarrow M_{3\sigma}(z)$

We can compare this with the evolution of M_J to check where do they cross. That tells at what redshift can a certain peak collapse.



4σ peak can form earlier, but are less frequent. Considering that 3σ peaks can host galaxies, the formation of the first proto-galaxies ocurred at $z \approx 30$.

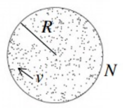
Spherical top-hat collapse



We know that haloes collapse gravitationally and virialize, getting a final δ of: $1+\delta_{TH}(t_{vir}) = 18\pi^2 \approx 178$

It is left to know how do they reach that final state, since there is no pressure to stop the collapse. It is necessary to develope a velocity dispersion through a mechanism that transforms potential energy into kinetic energy. This is done via relaxation processes:

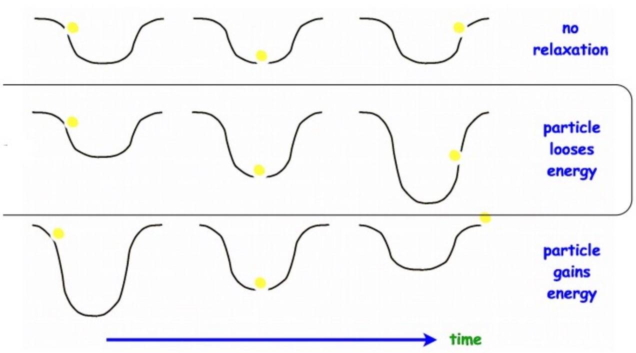
- Two body relaxation: two body interactions



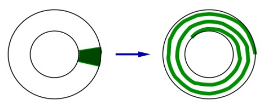
$$t_{relax} \approx \frac{N}{10 \ln N} t_{cross} \qquad t_{cross} \approx \frac{R}{v}$$

$t_{relax} \gg t_{Hubble}$ (for all cosmological objects that interest to us).

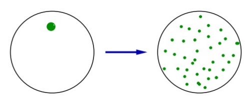
- Violent relaxation: change in energy due to change in overall potential



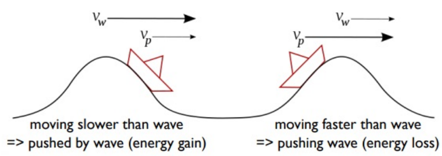
- Phase-mixing: spreading of phase-space due to different frequencies of orbits



- Chaotic mixing: spread of phase-space due to chaotic nature of orbits



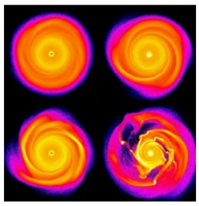
- Landau damping: damping and decay of perturbations due to interaction of particles with (density) waves

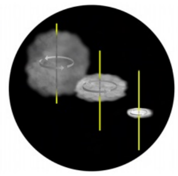


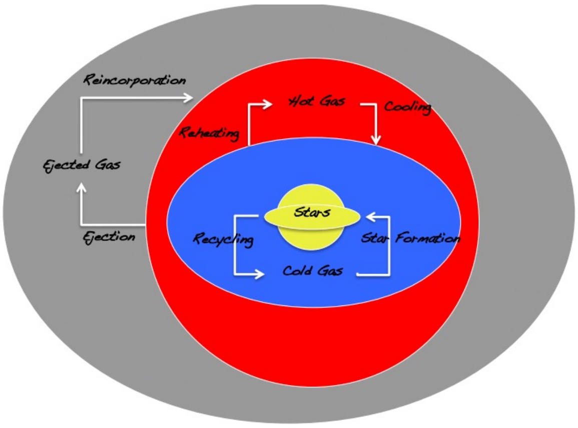
After these processes, we finally reach a virialized object with $2T = -U$

Protogalaxies

The presence of a DM halo appears inevitable, but the potential well of DM halo needs to be sufficiently deep to retain gas heated to high temperatures ($>10^4$ K) by first stars. In order to collapse the gas to form a proto-galaxy, it is necessary to cool the gas. The gas collapses to disc-like structures because of angular momentum conservation (from tidal torques). We also need fragmentation, which is thougt to be obtained via turbulence (because we need to form stars from the gas).







10.4. Implications for subsequent structure formation

General effects

We already know that the birth and death of the first stars produced the enrichment of the Universe with heavy elements and its re-ionisation. Thus, the first objects affect everything that comes afterwards. Any model of galaxy / star formation that targets low redshift formation needs to model this effects properly.

Reionising the Universe

The energy released by the first objects ionizes neutral hydrogen. This is detected via QSO spectra: neutral hydrogen along line-of-sight absorbs photons, but this was not detected through in spectra for QSO's with $z < 6$.

This can also be detected analysing the Thomson scattering of CMB photons: erasing small scale anisotropies, polarization of the CMB, ... It was found with Planck 2013 data that reionisation started at $z = 11$.