IV. Big Bang Nucleosynthesis

4.1. Introduction

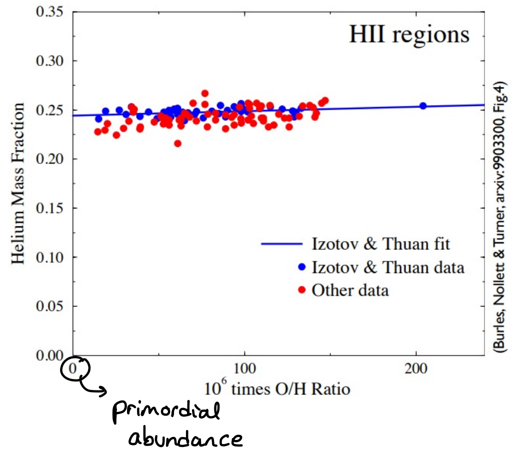
Stellar fusion

One of the questions that cosmology addresses is where do all the elements come from. We know that the elements of the periodic table can be produced by stars, either during their life (nuclear fusion to obtain energy) or after they deaths as supernovae.

Abundance discrepancies

However, stellar fusion was not enough to explain the observed ^4He abundance:

- If the Milky Way were to "shine" for 10^{10} years, it would generate 4% ^4He. Observing the spectrum of HII regions around stars, a fraction of 24% ^4He was observed, way more than expected.



There was also a problem with deuterium ($D = {}^2H = p+n$): we should not be able to observe it since fusion reactions in the sun are consuming D faster than creating it. Thus, even when stars die, the cannot enrich their environment with D. However, we do observe it in the interestellar medium.

This means that there must be other mechanisms that generate D and He asside from stellar fusion. This events must have happened during the ca. 13.7 Gyr of the Universe's existence, so we must analyse its thermal history to place them.

Big Bang nucleosynthesis

Cosmological considerations

At $z=0$ (today): $T_{CMB} = 2.73\,K$ and $\rho_b = 5\times10^{-31}\,g/cm^3$. This conditions are not similar to the ones in the centre of stars, where elements are form. However, if we go backwards in time, the Universe becomes hotter and denser ($T \propto R^{-1}$ for radiation and $\rho_b \propto R^{-3}$).

A $z \approx 10^{10}$ ($t \approx 1\,sec$) we find $T_{CM} \approx 10^{10}\,K$ and $\rho \approx 80\,g/cm^3$, sufficient for cosmological nuclear fusion events. We talk about the Big Bang Nucleosynthesis.

Developement of the theory

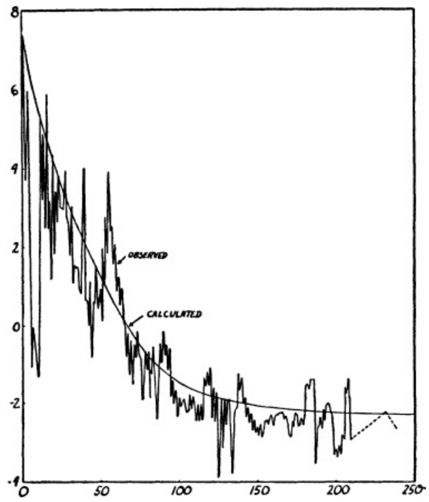
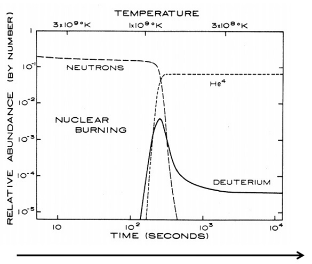


FIG. 1.
Log of relative abundance
Atomic weight

Alpher, Bethe and Gamow published they results on 1948. They studied the origin of Chemical elements solving an equation similar to the Boltzmann equation, which models the evolution of the number density of species based on its interactions

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{eq}^2).$$

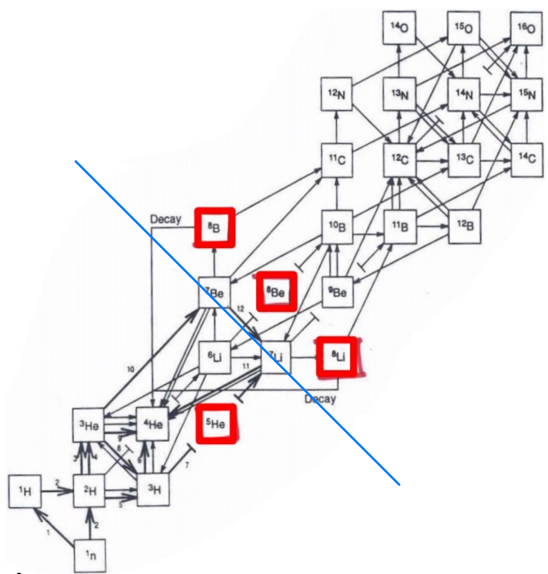
They claimed that all elements were produced in BBN, since conditions were sufficient for fusion reactions.



Peebles published "Primeval Helium abundance and the primeval Fireball" in 1966, where he solved Boltzmann equation numerically, obtaining a detailed calculation of the temporal evolution of element abundance. To do so, it was necessary to set up the network of nuclear reactions.

BBN Vs. Stellar fusion

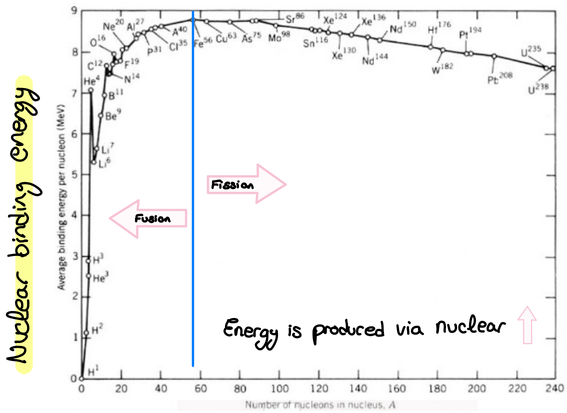
As a matter of fact, there are not stable elements with $A=5$ or $A=8$. To produce elements with higher atomic numbers, it is necessary to pass through nuclei with this atomic numbers, which decay almost immediatly, breaking the network of chemical reactions. However, we do observe elements with $A>8$ in stars (oxigen, for example).



Even if we have similar densities in the centre of the Sun ($\rho_b \approx 150\ g/cm^3$) and during BBN ($\rho_b \approx 80 g/cm^3$), we have to take into account timescales:

- During BBN, it was impossible to process anything heavier than ^7Li, ^7Be due to its very short timescale (~minutes).
- Stars like the sun are able to process heavy elements (via ^8Be triple-α process) due its extended timescale for fusion (million years) → even if one event happens every 1000 years, we would have ~1000 events of that king along the life of the star.

Summary



During BBN, the following elements were produced:
H, ^2H (D), ^3H (T), ^3He, ^4He, ^7Be, ^7Li

All the other elements were produced is stars.

The nuclear binding energy raises with Z until Fe. Elements up to this atomic number are produced in stars, which produce energy through nuclear fusion. Since fusions for $Z > Z(\mathrm{Fe})$ are endotermic, heavier elements are produced in SN explosions.

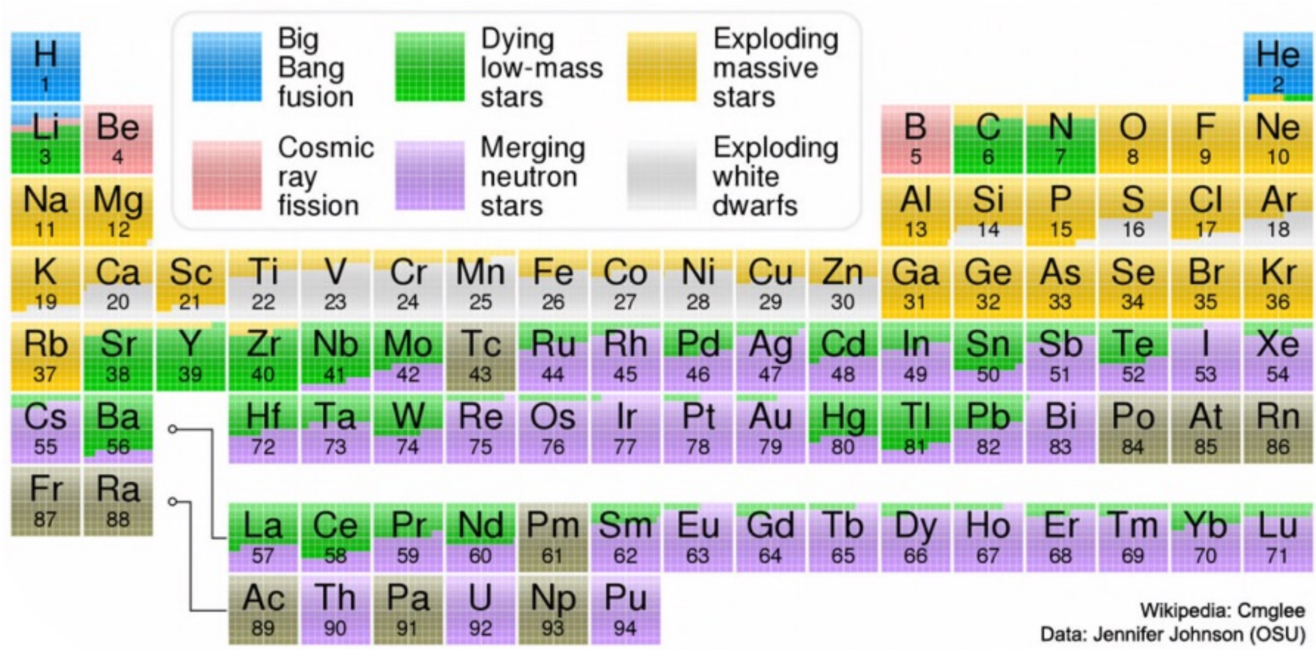
The BBN predicts the following mass fractions for helium and hydrogen:

$$Y_{He} \approx 0.24$$

$$Y_H \approx 0.73$$

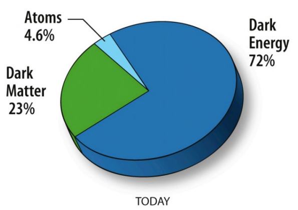
This will be calculated later, and this mass fractions have been confirmed through observations.

- Known origin of elements:

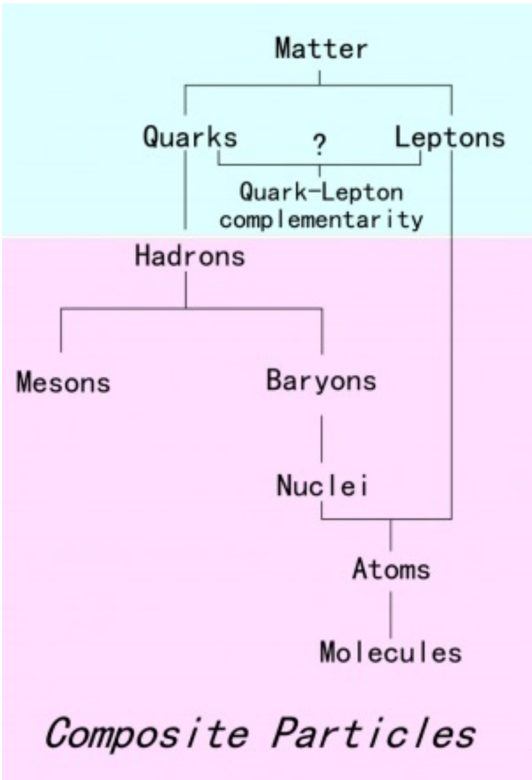


4.2. Particle physics

Ingredients for atom formation



Atoms represent a really small fraction of the components of the Universe (4.6 %), and we want to know when did they form. To do so, we study how atoms are formed.



This is the picture we have from the standard model of particle physics. Quarks and leptons were the first particles that appeared in the Universe. To form nuclei, we require neutrons and protons, which are baryons. To form atoms, we also need electrons. Recall that protons and neutrons were linked to the thermal bath via the weak interaction:

$$\nu_e + n \Leftrightarrow p + e^- \qquad e^+ + n \Leftrightarrow p + \bar{\nu}_e$$

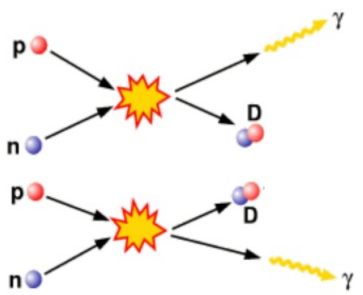
$$n \Leftrightarrow p + e^- + \bar{\nu}_e$$

Thus, we cannot form any nuclei before neutrinos decouple from the thermal bath. After neutrino freeze-out, neutrons and protons are free. To form atoms, we would have to wait until e^- are decoupled.

Nuclei formation

To form H and ^2H we just need free protons. To form heavier elements (^3H, ^3He, ^4He, ^7Be, ^7Li) we also need deuterium.

Deuterium synthesis



two-body processes only,
as probability for others too low
('bottom-up' fusion of He⁴)

Deuterium's binding energy is $E_b \approx 2\,\text{MeV}$, and the temperature at neutrino decoupling was $kT_{\nu} \approx 0.8$ MeV.

But D is easily photo-dissociated by γ (i.e. destroyed) until $kT_D \simeq 0.086\,\text{MeV}$ (ca. $t \approx 100\,s$, $\frac{T}{1\,\text{MeV}} \cong 1.5\, g_{*s}^{-1/4} \left(\frac{1s}{t}\right)^{1/2}$).

Thus, we have to wait to synthesize heavier elements.

This happens due to the blackbody-distribution of photons' energies.

For temperatures lower than E_b ($kT_D < E_b \approx 2\,\text{MeV}$) there will still be lots of higher energy photons in the Planck distribution, which dissociate our Deuterium. The abundance of Deuterium in the Universe must be well fixed to obtain the observed abundances:

- Too few D: important fusion agent is missing (lack of elements)
- Too much D: locks up neutrons for further synthesis.

Deuterium production (and all successive nuclei) depends on the baryon-to-photon ratio.
We define the proton to baryon ratio as:

$$\boxed{\eta = \frac{n_b}{n_\gamma} = 10^{-10}\,\eta_{10} = 10^{-10}\cdot 274\,\Omega_b h^2}$$

$H_0 = 100h$ Km/s/Mpc

NOTE that it is a really small number: There is one baryon for each 10^{10} photons.

Summary of chemical reactions

- Hydrogen: $(p + e^- \rightarrow H + \gamma)$
- Deuterium: $p + n \rightarrow D + \gamma$
- lighter elements:

$$D + n \rightarrow {}^3H + \gamma$$
$$D + D \rightarrow {}^3H + p$$
$$D + p \rightarrow {}^3He + \gamma$$
$$D + D \rightarrow {}^3He + n$$
$${}^3He + n \rightarrow {}^3H + p$$

- ^4He:

$$D + D \rightarrow {}^4He + \gamma$$
$$D + {}^3He \rightarrow {}^4He + p$$
$$D + {}^3H \rightarrow {}^4He + n$$
$${}^3He + {}^3He \rightarrow {}^4He + 2p$$
$${}^3He + n \rightarrow {}^4He + \gamma$$
$${}^3H + p \rightarrow {}^4He + \gamma$$

$E_{^4He} \approx 28.3$ MeV $\Rightarrow$ safe from dissociation

- ^7Be, ^7Li:

$${}^3He + {}^4He \rightarrow {}^7Be + \gamma$$
$${}^7Be \rightarrow {}^7Li + e^+ + \nu_e$$
$${}^3H + {}^4He \rightarrow {}^7Li + e^+ + \nu_e$$

4.3. Cosmology

Review

When matter and radiation are in thermal equilibrium, we can have reactions like: $(\gamma+\gamma \Leftrightarrow p+\bar{p})$
Reactions can happen as long as their interaction rate $(\Gamma_c \propto n\sigma v)$ is larger than the expansion rate of the Universe $(\Gamma_e \propto H)$

If $\boxed{\frac{\Gamma_c}{H} < 1}$ we talk about freeze-out

Freeze-out depends on the "temperature window", which depends on H.

radiation domination: $\left.\begin{array}{l} T \propto R^{-1} \\ H \propto R^{-2} \end{array}\right\} H \propto T^2$

BBN is sensitive to cosmology (i.e. the composition of the Universe) and its thermal history.

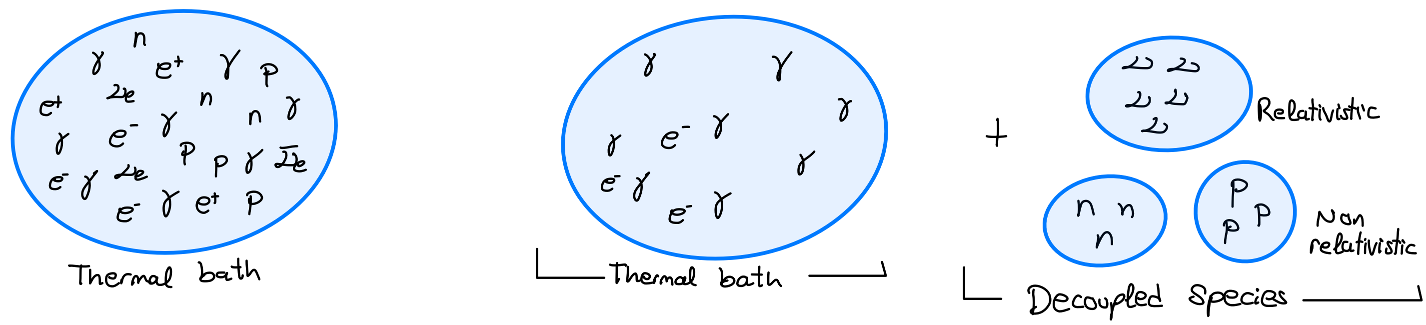
Protons and neutrons through the Thermal history of the Universe

As we have already discussed, to form the nuclei we need protons and neutrons. This protons and neutrons were coupled to the thermal bath until neutrinos decoupled via the weak interaction:

$$\nu_e + n \Longleftrightarrow p + e^-$$
$$e^+ + n \Longleftrightarrow p + \bar{\nu}_e$$
$$n \Longleftrightarrow p + e^- + \bar{\nu}_e$$
} weak interaction

Weak interaction freezes out at $T \approx 0.8$ MeV

BEFORE DECOUPLING $(T > 0.8\,\mathrm{MeV})$ AFTER DECOUPLING $(T < 0.8\,\mathrm{MeV})$



After decoupling ($k_B T < 0.8$ MeV) we have:

- Relativistic particles in equilibrium (e^-, e^+, γ)
- Decoupled relativistic particles (ν)
- Decoupled non-relativistic particles (n, p)

We are interested on the number densities of the latter.

REMINDER
We are building nuclei, not atoms. Electrons are not important yet.

4.4. Big Bang Nucleosynthesis

Primordial abundances: Neutron to proton ratio

The abundance of neutrons determines how many nuclei beyond $A = 1$ (hydrogen) can be formed.

We can calculate the neutron to proton ratio when they decoupled from the thermal bath using the expression for the number density of non-relativistic species:

$$n_A = g_A \left(\frac{m_A k T}{2\pi\hbar^2} \right)^{3/2} \underbrace{e^{-(m_A - \mu_A)c^2/kT}}_{\text{our distribution is exponentially suppresed w.r.t. relativistic}}$$

NOTE
We cannot ignore the chemical potential μ

Writing it for the neutron-to-proton ratio:

$$\left(\frac{n_n}{n_p} \right) = \left(\frac{m_n}{m_p} \right)^{3/2} e^{-Q/kT} e^{(\mu_n - \mu_p)/kT}$$

Where $Q = |m_n - m_p| = 1.293$ MeV is the binding energy

We need to obtain μ_n and μ_p. To do so, we can remember that the chemical potential μ is defined as the energy absorbed or released during a chemical reaction.

We know that we are under the conditions of chemical equilibrium because nuclear reactions are faster than cosmic expansion. Thus, we can write:

$$A + B \rightleftharpoons C + D \xrightarrow{\text{chemical equilibrium}} \mu_A + \mu_B = \mu_c + \mu_D$$

The reaction going on is β-decay:

$$\nu_e + n \leftrightarrow p + e^- \longrightarrow \mu_{\nu_e} + \mu_n = \mu_p + \mu_e$$

Thus, now we have to calculate μ for neutrinos and electrons. We will start with the latter.

We observe that the Universe is neutral: $n_{e^-} - n_{e^+} = n_p$ (otherwise, there would be a residual charge). We can use this to calculate μ_e as:

$$n_{e^-} - n_{e^+} = \frac{2T^3}{6\pi^2}\left[\pi^2\left(\frac{\mu_e}{T}\right) + \left(\frac{\mu_e}{T}\right)^3\right]$$

For the right hand side of the equation, we recall the baryon to photon ratio (η)

$$n_p \approx \eta n_\gamma = \eta \frac{2\zeta(3)}{\pi^2} T^3$$

Equating both expressions:

$$\frac{2T^3}{6\pi^2}\left[\pi^2\left(\frac{\mu_e}{T}\right) + \left(\frac{\mu_e}{T}\right)^3\right] = \eta \frac{2\zeta(3)}{\pi^2} T^3 \longrightarrow \frac{1}{6}\left[\pi^2\left(\frac{\mu_e}{T}\right) + \left(\frac{\mu_e}{T}\right)^3\right] = \eta \zeta(3)$$

$$\left(\frac{\mu_e}{T}\right) + \frac{1}{\pi^2}\left(\frac{\mu_e}{T}\right)^3 = \frac{6\zeta(3)}{\pi^2}\eta \Longrightarrow \boxed{\frac{\mu_e}{T} \approx \frac{6\zeta(3)}{\pi^2}\eta}$$

Note that the value of η comes from observations, and it was found that $\eta \approx 10^{-9}$.
Then, $\mu_e \ll T$, so it can be ignored.

For the chemical potential of neutrinos, there is another set of observations that gives us an upper limit on its value. These are measurements of the CMB radiation, which give

$$\frac{\mu_{\nu e}}{T} < 0.2 \quad \text{(today)}$$

To figure out this ratio at the time of BBN, we can use the scaling of T, obtaining:

$$\frac{\mu_{\nu e}}{T} < 10^{-10}$$

Thus, we obtain:

$$\boxed{\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-Q/kT}}$$

Neutron to proton ratio

It is important to remember that we neglected the chemical potentials using observations, not just ignoring them.

Relative abundance

The resulting equation only contains the binding energy of deuterium ($Q = 1.293$ MeV).
We observe that:

- $KT > 1.3$ MeV $\Rightarrow n_n \approx n_p$
- $KT < 1.3$ MeV $\Rightarrow n_n < n_p$

But there is a gap between this temperature and $KT \sim 0.8$ MeV, when the weak interaction freezes-out and BBN starts. Thus, we will start with fewer neutrons than protons.

For $KT \approx 0.72$ MeV (since the weak interaction will not instantaneously freeze at 0.8 MeV):

$$\frac{n_n}{n_p} \approx \frac{1}{6}$$

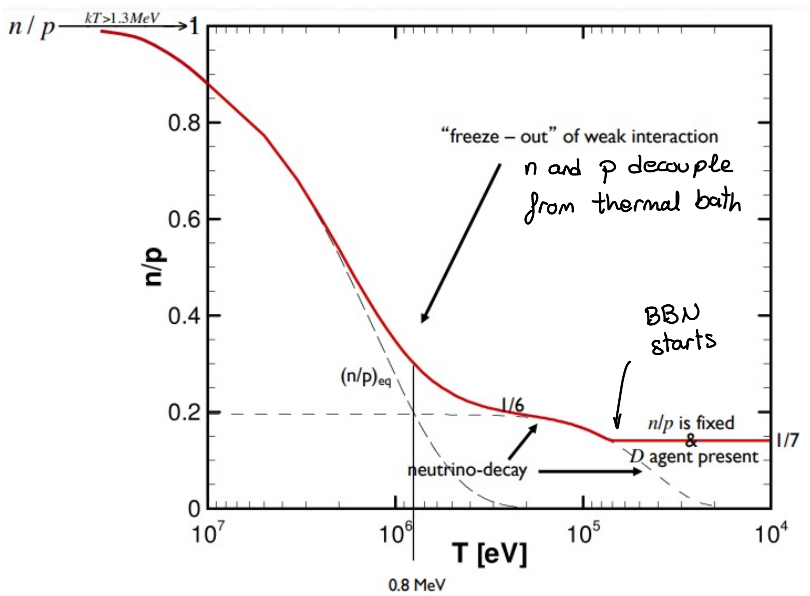
But this is not the ratio that we observe: we have to remember the Deuterium bottleneck: D is easily photo-dissociated by γ until $K_B T_D \simeq 0.086$ MeV. Also, neutrons are ustable: they have a limited lifetime $\tau_n \approx 887$ s.

From $KT \approx 0.8$ MeV until $KT_D \approx 0.086$ MeV there is a race between free neutrons decaying away and remaining free neutrons being incorporated into nuclei (e.g. D).

The neutrons that do not decay and can form heavier nuclei will be:

$$\boxed{\frac{n_n}{n_p} = \underbrace{\frac{1}{6}}_{\text{at decoupling}} \underbrace{e^{-t/\tau_n}}_{\text{decay with } \tau_n = 887\text{ s}} \approx \frac{1}{7}}$$

$t \simeq 140$ s ($E_{b,D} = 2.22$ MeV $\rightarrow T_D = 0.086$ MeV)



Primordial He abundance

Helium abundance is defined as:

REMINDER
$$\frac{n_n}{n_p} = \frac{1}{7}$$

$$\boxed{X_{He} = Y = \frac{m_{He}}{m_{He} + m_H}}$$

Since $m_p \approx m_n$:

$$X_{He} = Y = \frac{4n_{He}}{4n_{He} + n_H}$$

(comparing the masses of He and H without taking into account the binding energy)

Taking into account that $n_{He} = \frac{n_n}{2}$ (because we need to neutrons to form Helium) we can write:

$$X_{He} = Y = \frac{4n_{He}}{4n_{He} + n_H} = \frac{2n_n}{2n_n + \underbrace{(n_p - n_n)}_{\text{protons not in He}}} = \frac{2n_n}{n_n + n_p} = \frac{2(n_n/n_p)}{1 + (n_n/n_p)} = \frac{2 \times 1/7}{1 + 1/7} = 0.25$$

$Y = 0.25$ is a clear prediction of BBN that fits observations (recall the plot at the beginning.

A proper calculation leads to:

$$X_{He} = Y \approx 0.2454 + 0.0198\,(N_\nu - 3)$$

N_ν → neutrino species

Other primordial abundances

For non-relativistic nucleus, $A = N_n + Z$ = # neutrons + # protons

The number density is given by:

$$n_A = g_A \left(\frac{m_A kT}{2\pi\hbar^2}\right)^{3/2} e^{-(m_A - \mu_A)c^2/kT}$$

To find μ_A, we look at the chemical reaction $A \leftrightharpoons p + n$ (at the end of the day, the reaction will imply protons and neutrons.

In chemical equilibrium: (since nuclear reactions are faster than cosmic expansion)

$$\mu_A = Z\mu_p + N_n \mu_n = Z\mu_p + (A - Z)\mu_n$$

Thus, the number density can be written as:

$$n_A = g_A \left(\frac{m_A kT}{2\pi\hbar^2}\right)^{3/2} e^{-(m_A - (Z\mu_p + (A-Z)\mu_n))c^2/kT}$$

We can eliminate μ_n and μ_p introducing n_p and n_n:

$$n_p = 2\left(\frac{m_p kT}{2\pi\hbar^2}\right)^{3/2} e^{-(m_p - \mu_p)c^2/kT} \qquad\qquad e^{-\mu_p c^2/kT} = \frac{2}{n_p}\left(\frac{m_p kT}{2\pi\hbar^2}\right)^{3/2} e^{-m_p c^2/kT}$$

$$\Longrightarrow$$

$$n_n = 2\left(\frac{m_n kT}{2\pi\hbar^2}\right)^{3/2} e^{-(m_n - \mu_n)c^2/kT} \qquad\qquad e^{-\mu_n c^2/kT} = \frac{2}{n_n}\left(\frac{m_n kT}{2\pi\hbar^2}\right)^{3/2} e^{-m_n c^2/kT}$$

$$\longrightarrow \boxed{\begin{aligned} n_A &= g_A \frac{A^{3/2}}{2^A} \left(\frac{m_A}{A} \frac{KT}{2\pi\hbar^2} \right)^{3(1-A)/2} n_p^{\,z} \, n_A^{\,(A-z)} \, e^{B_A/kT} \\ B_A &= \left(z m_p + (A-z) m_n - m_A \right) c^2 \end{aligned}}$$

$B_A \equiv$ Binding energy of the nucleus

All the binding energies are above the temperature of the universe at these times:

Nucleus	^2H	^3H	^3He	^4He
B_A	2.2. MeV	8.48 MeV	7.72 MeV	28.3 MeV

Mass fraction

$X_A = \dfrac{A n_A}{n_b}$, where $A n_A$ is the mass fraction of that particular species and n_b is the total barionic mass: $n_b = \sum A_i n_i$

Now we can replace n_b using the baryon to proton ratio: $\eta = \dfrac{n_b}{n_\gamma}$

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} \left(\frac{K_B}{\hbar c} \right)^3 T^3$$

$$\Rightarrow \boxed{X_A \propto g_A A^{5/2} \left(\frac{KT}{m_A} \right)^{3(A-1)/2} X_p^{\,z} X_n^{\,(A-z)} \eta^{A-1} e^{B_A/kT}}$$

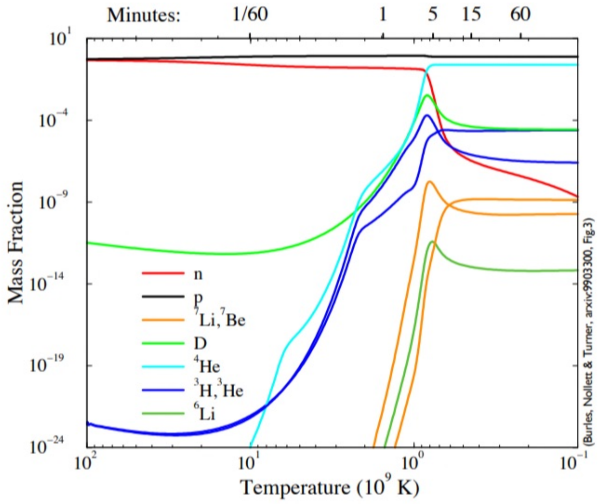
Thus, the mass fraction can be obtained using the proton mass fraction X_p, the neutron mass fraction and the baryon-to-photon ratio η.

$$\eta = \frac{n_b}{n_\gamma} = 10^{-10} \eta_{10} = 10^{-10} \cdot 274 \, \Omega_b h^2$$

To know how X changes as a function of time, we need to solve the Boltzmann equation:

$$\boxed{\frac{dX_A}{dt} = -\Gamma_A \left(X_A - (1 - X_A) e^{-B_A/kT} \right)}$$

Numerical calculations of cosmic evolution of mass fractions



This is an update of Peebles calculations. Mass fractions are calculated using the Boltzmann equation, and the predicted abundances match observations.

Dependence on baryon density

- Helium (pink): $\Omega_b \nearrow \rightarrow Y \nearrow$

  (as BBN starts earlier)

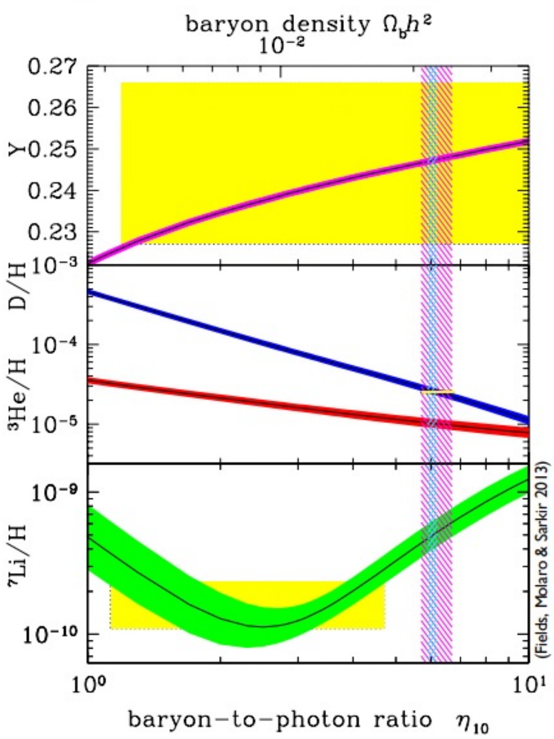
- Deuterium and tritium: they are primarily consumed to form other elements

- Lithium and Berilium: $\Omega_b \nearrow \rightarrow {}^7Be \nearrow$, $\Omega_b \nearrow \rightarrow {}^7Li \searrow$

  Destruction: $p + {}^7Li \rightarrow {}^4He + {}^4He$

  Formation: ${}^3H + {}^4He \rightarrow {}^7Li + e^+ + \nu_e$

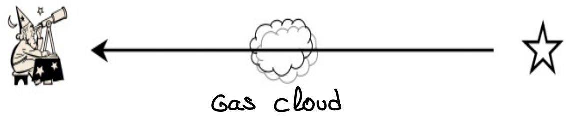
  Dominates for high η: $\begin{cases} {}^3He + {}^4He \rightarrow {}^7Be + \gamma \\ {}^7Be \rightarrow {}^7Li + e^+ + \nu_e \end{cases}$



Note that BBN predictions cover 9 orders of magnitude for abundances. The strong dependence on η makes possible to determine it from observations through the Deuterium fraction (i.e. deuterium is a good baryometer). This happens because D is very sensitive to η and is only produced during BBN.

BBN observations

Ly-α clouds (not polluted by stars)



The line strength in QSQ absorption spectra provide abundance measurements, being specially significant the abundance of deuterium. IGM observations: $\begin{cases} \eta \simeq 2.5 \times 10^{-5} \\ \eta \simeq \frac{n_b}{n_\gamma} = 10^{-10} \eta_{10} = 10^{-10}\, 274\, \Omega_b h^2 \end{cases}$

Nearby dwarf galaxies

These galaxies have a high gas/star ratio, and their low metal/H in gas suggest that their inter stellar medium is close to primordial. ISM observations: $\eta \simeq 1.6 \times 10^{-5}$

As above, $\Omega_b h^2 \approx 0.0214 \rightarrow$ even BBN claims for the existence of non-baryonic matter.

Galactic halo

Contains very old stellar population. ${}^7Li$ was observed in spectra of cool low-mass stars in Galactic halo.

H II regions

Low-density cloud of partially ionized gas in which star formation took/takes place. ${}^4He$ proved via emission from optical recombination lines in HII regions.