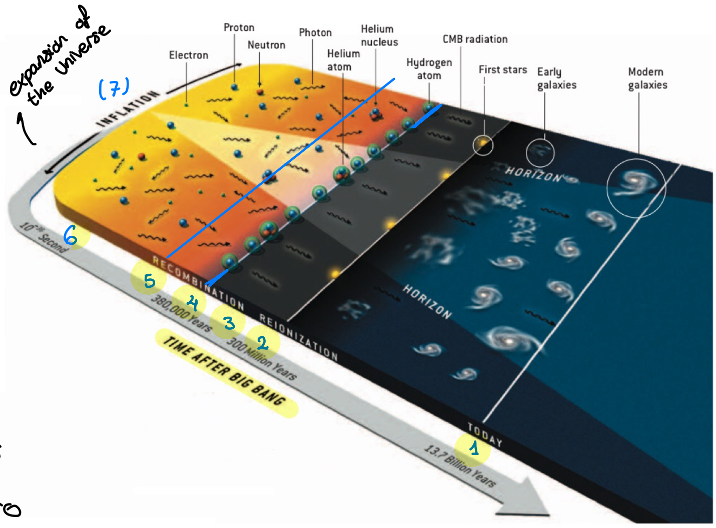
I. Cosmological principles

1.1. Introduction to Cosmology

Cosmology and the evolution of the Universe



Cosmology is the science of the origin and evolution of the Universe.

Our current picture of the Universe (1) is that galaxies form a large scale structure. If we want to understand this (as well as the morphological variety, evolution, etc.), we need a model of the formation of galaxies and structures. Thus, an important goal is to study the first galaxies and stars (2).

We know that, before this, there was an epoch where nothing was formed: the dark ages (3).

Before that, recombination ocurred (4). At recombination, photons decoupled from matter. This first decoupled photons form the cosmic microwave background (CMB), an there are observations of their temperature (energy) distribution ⟶ small anisotropies.

Going even earlier, we need to understand the Big Bang nucleosynthesis (5), since not all the elements of the periodic table can be generated in stars.

Before the formation of this elements, there was a "cosmic soup" of quarks (58%), force carriers (29%), electron-like particles (9%), neutrinos (5%) and Higgs bosons. Quarks combined in protons and neutrons during baryogenesis (6).

Preceding all that, there was an inflation (accelerated expansion) period (7). We do not have direct observations of this, but it is necessary to explain what we know about the Universe (EM observational evidences): general relativity, the SM of particle physics, galaxy formation, thermodynamics, etc. are unable to explain observations unless we include a period of inflation shortly after the Planck time.

Since our observations rely on EM radiation, there is a barrier to our knowledge: recombination (before that, photons were coupled to matter). The detection of gravitational waves open a new window to observe the Universe before recombination, since gravitational waves emitted before photon decoupling can reach us.

Time and space cosmological scales

Time scales

Event	time t
Inflation	10^{-34} s (?)
Baryogenesis	?
EW phase transition	20 ps
QCD phase transition	20 μs
Dark matter freeze-out	?
Neutrino decoupling	1 s
Electron-positron annihilation	6 s
Big Bang nucleosynthesis	3 min
Matter-radiation equality	60 kyr
Recombination	260–380 kyr
Photon decoupling	380 kyr
Reionization	100–400 Myr
Dark energy-matter equality	9 Gyr
Present	13.8 Gyr

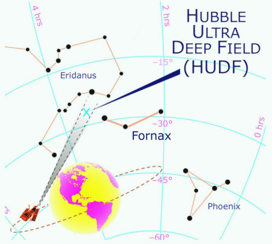
Spatial scales

- Solar system : Sun - Earth distance = 8 lightmin
- Proxima Centauri (closest star): 4 lightyears (4×10^3 Km)
- Milky Way: spiral galaxy structure ~ 100.000 lightyears
- Andromeda galaxy (closest galaxy): 2 million lightyears
- Galaxy clusters: Contain ~10.000 galaxies.

They are the largest objects that exist in the Universe. They are formed by individual galaxies and constitute the large scale structure of the Universe.

NOTE : Galaxy surveys

- Hubble Ultra Deep Field (HUDF), 2003 - 2004

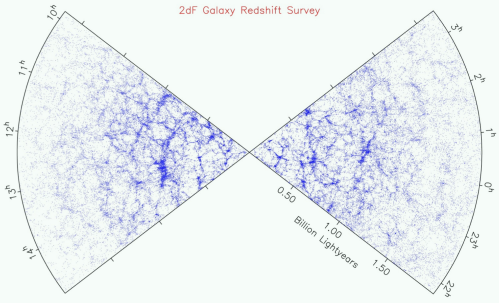


- Looked at a dark patch of sky for 4 months
- Found circa 10.000 galaxies

- AAT (Australia), 1997 - 2002

- Monitored a larger patch of sky (first systhematic survey) : billion lightyears

↳ 2dF survey: Found that galaxies are not randomly distributed → structure with fillaments and voids



2dF Galaxy Redshift Survey - 200.000 galaxies

We see less galaxies when we go to further distances

↳ Malmquist bias:

for larger distances, we only observe the brightest galaxies

↳ (That distance it is not far enough to reach z where galaxies are not formed yet).

- Latest surveys have observed ~ 50 million galaxies in the local Universe

Note that there are differences of orders of magnitudes in both scales. This needs to be explained by the different theories.

1.2. The equivalence principle and General Covariance

Weak equivalence principle

Applying Newton's second law to gravity, we can analyse the constants that appear in the equations:

$$m \frac{d^2r}{dt^2} = -m\frac{GM}{r^2}$$

- m quantifies the "resistance to force"
- m is the gravitational mass.

Gravitational and inertial masses are equivalent.

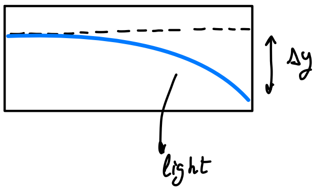
We do not have any observational evidences against this principle.

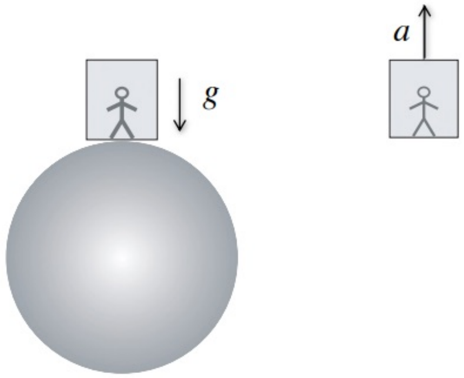
Strong equivalence principle

There is no observable distinction between the local effects of gravity and acceleration

Consecuences:

- Time dilation
- Light deflection
- Gravitational red/blueshift





1.3. The cosmological principle: Homogeneity and Isotropy.

Cosmological principle

" The Universe is homogeneous and isotropic in space".
(At sufficiently large scales)
⇒ The laws of physics are identical.

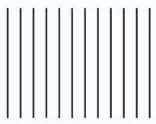
Consecuence: the Earth is not a special point of the Universe (it did not make sense otherwise).

It arose as a principle, but now there are several observational evidences supporting it.

NOTE:

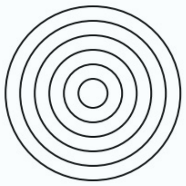
Homogeneous:

translation invariant



Isotropic:

rotation invariant



Homogeneity and isotropy observational evidences

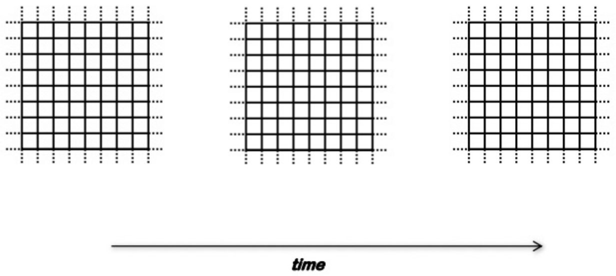
- Isotropy is proved by the observation of the CMB. Even though there are anisotropies on the level of 10^{-5}, this is nevertheless a proof that wherever we look around us the Universe is always the same. Otherwise, we would not be able to explain the cosmic microwave background radiation.

  Another proof can be obtained via the statistical analysis of galaxies sizes: we obtain similar distributions independently of the direction of observation averaging over a sufficiently large patch. This is also true for other features.
- Homogeneity: we require very large surveys. Defining a sufficiently large homogeneity scale (Wiggle Z dark energy survey: 7580 Mpc) the Universe is statiscaly homogeneous.

Strong cosmological principle

"The Universe is homogeneous and isotropic in space and time."

From observations, we know that the Universe is changing (in fact, expanding), thus it is not homogeneous in time.



Principle of relativity

The equations describing the laws of Physics have the same form irrespective of the coordinate system.

1.4. History of cosmology

Planck units

Cosmology is a combination of all fields in physics:

- General relativity: G (gravitational constant)
- Quantum field theory: h (Planck constant), c (speed of light), e
- Thermodynamics: k_B (Boltzmann constant)

This fields have connections with each other, and are quantified by the coupling constants written above.

Combining this constants with the proper power laws, we obtain the Planck units:

- Scale : $\sqrt{Gh/c^3} \approx 1.7 \times 10^{-33}$ cm
- Time : $\sqrt{Gh/c^5} \approx 10^{-43}$ s
- Mass : $\sqrt{hc/G} \approx 2.5 \times 10^{-5}$ g
- Energy : $\sqrt{hc^5/G} \approx 1.2 \times 10^{19}$ GeV
- Temperature : $K_B^{-1}\sqrt{\frac{hc^5}{G}} \approx 1.4 \times 10^{32}$ K

Einstein GR equations. Friedmann solution.

The foundations of cosmology were Einstein's General relativity equations (1915)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

"Matter sets up the space curvature, space tells mass how to move." The couple between them is given by a constant.

Λ was introduced to obtain a static solution, to potentially satisfy the strong cosmological principle. This happened because the first solution found by Friedmann (1922) described an expanding/collapsing Universe. Solutions also allowed singularities in spacetime (BH).

Friedmann solution:

Assuming an homogeneous and isotropic space, one gets the equation:

$$\boxed{\dot{a}^2 = \frac{8\pi G}{3} \underbrace{\rho}_{\text{matter density}} a^2 - \overbrace{k}^{\text{curvature parameter}} c^2} \qquad \text{(Friedmann equation)}$$

Where $a(t)$ is the scale factor, which is a measure of change of distances on a expanding Universe.

$\dot{a} = 0 \longrightarrow$ static universe. Only possible with $\rho = k = 0$

$\dot{a} \neq 0 \longrightarrow$ expanding or collapsing universe

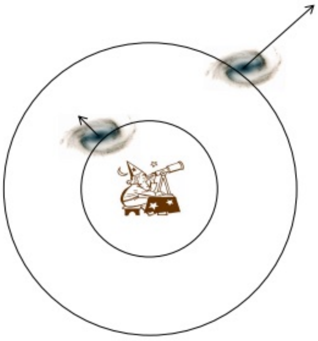
A new question arises: the universe either comes from or is heading to a singularity (since we have $\dot{a} \neq 0$, the Universe is either expanding or collapsing). This singularity was named "Big Bang" by Fred Hoyle.

Introducing the Cosmological constant Λ, Friedmann equation becomes:

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^2 + \frac{1}{3} \Lambda c^2 a^2 - kc^2$$

Which allows a solution with $\dot{a} = 0 \longrightarrow a = cte$

Hubble-Lemaitre law and the expansion of the Universe.



In 1916, Vesto Slipher observed that galaxies in our vicinity are receding from us. He also measured that the further a galaxy is from us, the faster it moves away from us.

The first systhematic study of this fact was carried out by Lemaitre in 1927, finding the relation:

$V = H_0 r$ $\qquad H_0 = 625\ \mathrm{Km/s/Mpc}$

Hubble-Lemaitre law

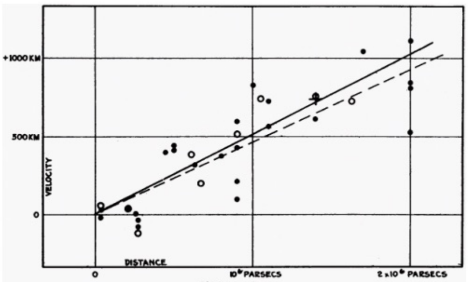
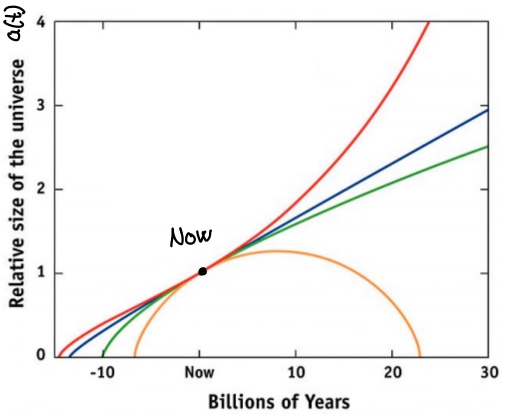


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Two years later (1929) Hubble published similar results, confirming that the Universe are expanding (based on the motion of distant galaxies

Universe evolution models. Big Bang.



These are analytical solution to the Friedmann equation depending on the components of the Universe.

In any case, one reaches size zero at some point of the past: The Big Bang.

If the Universe really started with a BB, it is not naive to think that we should see the photons coming from it: some sort of fireball.

This is not possible because photons could not travel freely before recombination (because they were coupled to matter. However, we can observe the first decoupled photons: the CMB.

The Cosmic Microwave Background (CMB).

Discovery

As it was mentioned above, scientist were looking for the photons emited at the Big Bang, which were emitted when the Universe was very hot. In the 1930's, people realised that, when the Universe was expanding, something could be happening to the energy of those photons: maybe they were not remaining as hot as they were because they had cooled down by some mechanism. This photons were discovered by accident by Penzias and Wilson.

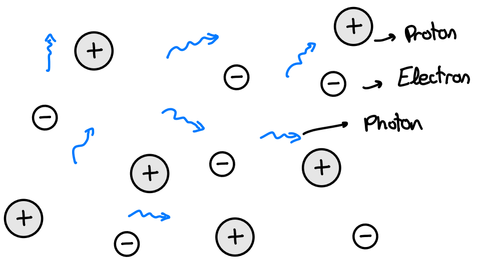
Penzias and Wilson worked on 1965 for Bell labs. Investigating radio emission from the Milky way they found some strange residual background noise. This noise was isotropic, so it was not being emitted by any neighbour source. An explanation was given by Dicke, Peebles and Wilkinson: it could be the residual photons coming from the Big Bang, that had been cooled down due to the expansion of the Universe. This was the final proof of The Big Bang model, since it was a clear prediction coming out of GR with the cosmological principle (since it was isotropic). The energy loss was explained through the principle of redshift: The emitted photon would have an energy given by its wavelength: $E = hc/\lambda$. This wavelength would get streched when space expanded.

CMB and recombination

A new question arises: It is really the fireball of the Big Bang or are we seeing some other epoch?

What is being observed is certainly a background of photons coming from all directions, having the same energy ($T \sim 2.76$ K) up to the 5th decimal. This is the most perfect black body radiation that has been observed in the whole Universe.

To answer the question, we have to look back in the history of the Universe.



There was an epoch where the Universe was filled with electrons, protons and photons, so there were no periodic elements. There were no atoms, everything was ionised, and in that epoch photons were not able to travel freely: they were coupled to electrons via Compton scattering.

Only when the electrons and photons started to combine and form hydrogen atoms, and there were no (or few) electrons left, could photons start travelling through the Universe. This means that we can only see the photons after the so-called recombination. The cosmic microwave background is the earliest thing that it can observe, and it was emitted when the Universe had cooled down to approximately 3000 K, far from the Planck temperature ($\sim 10^{32}$ K).

We are seeing the photons coming from the fireball of the Big Bang, but a lot has happened between those times. This will be discussed when we take a look to the Thermal history of the Universe.

Nobel prizes related to Cosmology

Penzias and Wilson were awarded with the Nobel Prize in 1978. This was the first Nobel Prize awarded to Cosmology, which was followed by:

(2002) Giacconi - Discovery of cosmic X-ray sources

(2006) Smoot & Mather - Discovery of anisotropies in the CMB

(2011) Perlmutter, Schmidt & Riess - Discovery of accelerated expansion of the Universe

(2017) Weiss, Barish & Thorne - Discovery of gravitational waves

(2019) Peebles and Mayor & Queloz - Discovery of gravitational waves

(2020) Penrose and Genzel & Ghez - Relativity and Black holes

1.5. Measuring the components of the Universe.

Recap: The Friedmann equation. Introduction to Density parameters.

All started with GR an Einstein's equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Which under the assumption of isotropy an homogeneity can be reduced to the Friedmann equation:

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^2 + \frac{1}{3} \Lambda c^2 a^2 - Kc^2$$

It describes the background expansion of the Universe through the scale factor $a(t)$.

This equation can be rewriten as:

$$\boxed{\frac{\dot{a}^2}{a} = H_0^2 \left(\frac{\Omega_m}{a^3} + \frac{\Omega_K}{a^2} + \Omega_\Lambda \right)}$$

$\longrightarrow$ Present expansion rate: $H_0^2 = \frac{\dot{a}_0^2}{a_0^2}$

Where:

a = measure for radius of the Universe

$\dot{a}$ = measure for expansion rate

$$\left.\begin{array}{l} \Omega_m = \text{mass content} \\ \Omega_\Lambda = \text{cosmological} \\ \Omega_K = \text{curvature} \end{array}\right\} \text{normalised densities}$$

Taking $a = 1$ (today), we find that only two of the three density parameters are independent:

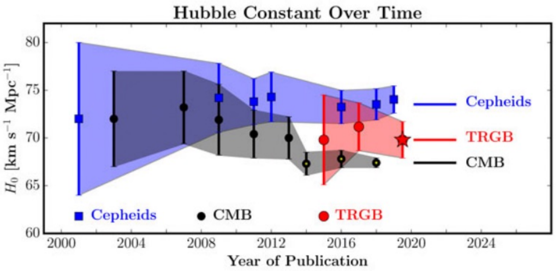
$$\boxed{\Omega_m + \Omega_\Lambda + \Omega_K = 1}$$

Knowing two of them, the third one can be calculated. We also need to measure H_0.

Measuring density parameters (and the Hubble constant H_0).

Hubble-Lemaitre parameter H_0

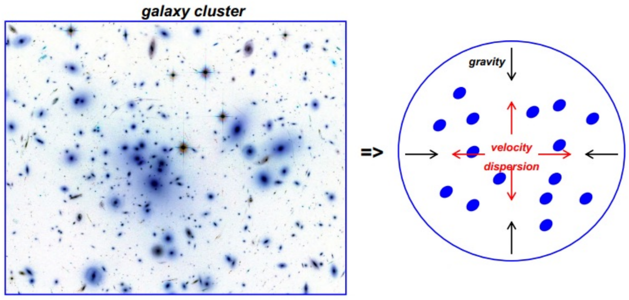
As it was mentioned before, H_0 is the proportionality constant in the Hubble-Lemaitre law ($v = H_0 r$). Thus, it can be obtained from the measurement of the distance and recession velocity of galaxies. These measurements depend heavily on the method used to determine the distance: a mistake in the distance estimation gives a wrong H_0 value.



In the last few years there has been a bifurcation on the results found for H_0 using different techniques.
Using cepheids, one is using observations in the local Universe, while the radiation from the CMB comes from very far away. People use de mean between the results of both methods: $H_0 = 70$ (km/s)/Mpc.
Some people argue that this discrepancy is because the local measurements are still in a regime where homogeneity has not been reached. However, this is not yet settled.

Ω_m - Matter density

Captures all the matter in the Universe (whatever type of matter). An approach to calculate it is estimating the mass in galaxy clusters and add a bit more to account for galaxies that do not live in Galaxy clusters.
The first attempt of measuring the mass of a galaxy cluster was carried out by Fritz Zwicky in 1933 using the hydrostatic equilibrium. He took all the galaxies in the cluster and calculated the velocity dispersion.



Assuming isotropy, it does not matter having only the line-of-sight velocity because the distribution is the same in any direction. This velocity dispersion is a measure of the kinetic energy:
$T \simeq \frac{1}{2} M \sigma_v^2$
There is also some gravitational energy $U = -G \, {M^2}/{R}$
Since we have an object in equilibrium, there is a relation between this potential and kinetic energy, which we take to be the Virial equilibrium:
$2T + U = 0$

Measuring σ_v and the radius, one can find the mass M. He found that the mass M needed to support the system was larger than the sum of all the observable mass (in galaxies). This was one of the evidences of the existence of dark matter.

Another evidence is the rotation curves of galaxies measured using stars and the 21cm-line of hydrogen.

Doing this, one finds that 30% of the Universe consist of matter, but only 17% of this matter is visible.

$$\boxed{\Omega_m = 0.3} \longrightarrow \Omega_K + \Omega_\Lambda = 0.7$$

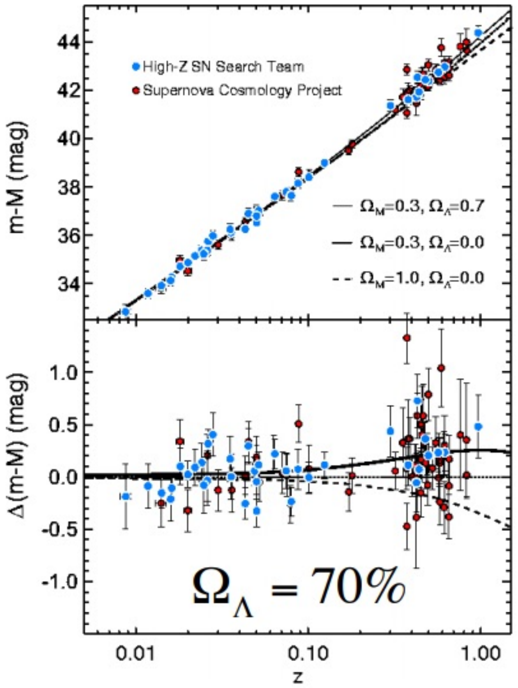
Ω_Λ - Cosmological constant

Ω_Λ is the renormalization of the cosmological constant.

It was measured using standard candles, i.e. objects that always emit the same number of photons per time. One known (a really bright) standard candle are Type Ia Supernovae. From Earth, we will see a brighter SN if it is close to us ($\text{Flux} \propto 1/d^2$). This proportionality is only true in Euclidean space, but space might be curved. We need to include the curvature of space in the relation between flux and distance using GR and the Cosmological principle. This expression ($m(z) = \dots$) will be derived in another chapter, and involves Ω_m and Ω_Λ. ($\Omega_K = 1 - \Omega_\Lambda - \Omega_m$).

From the observables we can deduce Ω_Λ, finding:

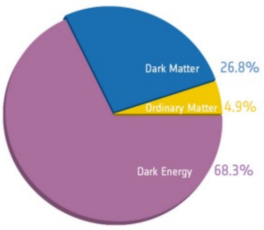
$$\boxed{\Omega_\Lambda = 0.7}$$



Lines are theoretical models for the dependence $m(z)$. The parameter set which provides the best fit is:

$\Omega_m = 0.3, \quad \Omega_\Lambda = 0.7$

This leaves us with the following composition of the Universe:



Only ordinary matter comes out from the standard model of particle physics.

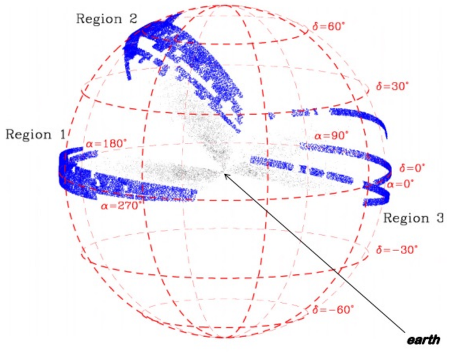
Ω_K - Curvature

Renormalization of the curvature term in the Friedmann equation. Determined from the values of Ω_m and Ω_Λ.
Since $\Omega_K \sim 0$, our Universe is almost flat.

1.6. Cosmological structure formation

It is necessary to study structure formation within an expanding background. This structure formation is highly non-linear. This is done creating models that have to fit the observed distribution of galaxies and the structures they form (fillaments and voids).

Homogeneity, isotropy and structure formation.



There are lots of telescopes and satellites mapping the distribution of galaxies in the Universe.
If we had an homogeneous and isotropic Universe, those structures (or any galaxy at all) would not be forming. This is something that people realised shortly after the discovery of the CMB, and started to look for anisotropies in it.
This anisotropies were found by Smoot & Mather (who received a Nobel Prize because of it) up to an order of 10^{-4}. Only having this anisotropies is possible to explain structure formation.
Temperature fluctuations can be translated into matter fluctuations, which are the seeds of galaxies and everything that we see in the Universe.

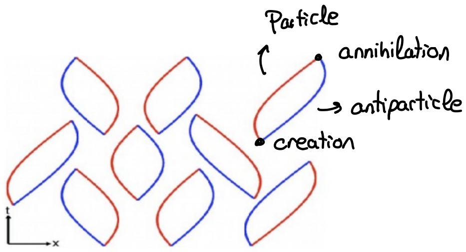
Matter perturbations and structure formation. Overdensity.

Origin of matter perturbations. Inflation.

On the Big Bang, only space-time itself and energy were created. But Heisenberg uncertainty principle tells us that there is no empty vacuum: there are always particles (and their associanted antiparticles) going in and out of existence.

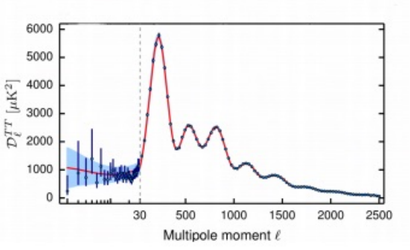
NOTE

This is a theory, since we do not have observations of events before recombination.

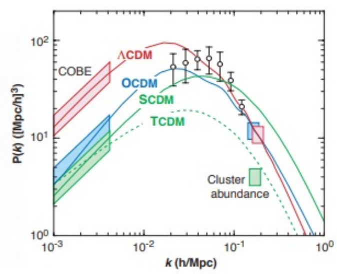


This gives some structure, but at a scale where the Universe is still homogeneous and isotropic, unless all of a sudden space expands really quickly (so that particles and antiparticles are too far to annihilate each other). This expansion is known as inflation.

During inflation the Universe does not follow Friedmann equations. Inflation solves multiple problems (flatness problem, horizon problem, monopole problem, ...) and explains the existence of fluctuations: quantum fluctuations magnified to cosmic size during inflation, becoming the seeds for structures in the Universe.

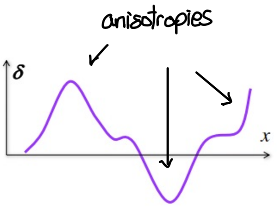


temperature fluctuations



Temperature fluctuations can be translated into matter perturbations, which will be the seeds for galaxies, stars, Galaxy clusters and everything that we see in the Universe.

Growth of matter perturbations



To quantify perturbations, we can define the density contrast as:

$$\boxed{\delta(\vec{x},t) = \frac{\rho(\vec{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)}}$$

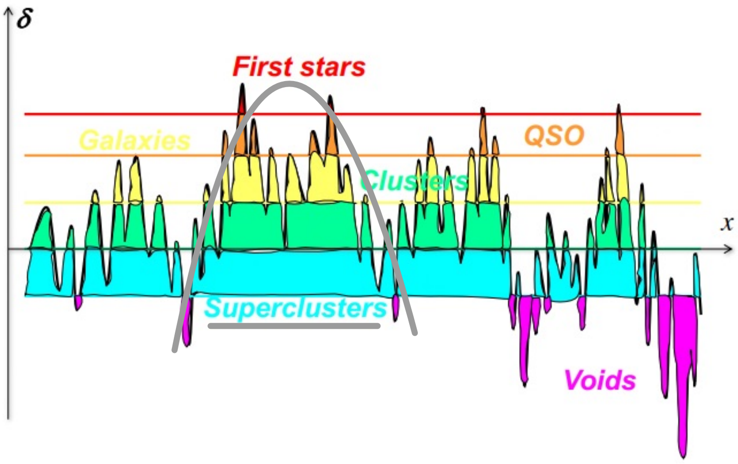
$\bar{\rho}(t)$ ⟶ background density

$\rho(\vec{x},t)$ ⟶ local density perturbations

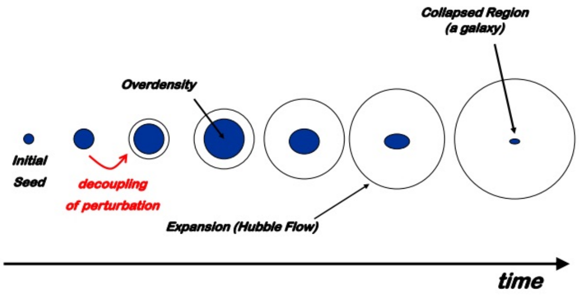
We can also define the power spectrum of density perturbations, which is the average of the density contrast in k-space.

$$\boxed{P(k) = \langle|\hat{\delta}(\vec{k})|^2\rangle_{|\vec{k}|=k}}$$

We assume that, when δ is over some threshold, structures are formed.



- For one of the peaks:



Decouple: self gravity allows matter to recolapse