Cosmology

Part A (problems to be handed in)

1. Plot the Hubble law, travel time, comoving, luminosity, and angular diameter distances (in Gpc) as a function of redshift for a flat ΛCDM model with $\Omega_{m,0}$=0.3 and h=0.7. Is there anything peculiar for any of the distances? Explain its implications and also submit your code used for the calculation and plotting. (**6 points**)

2. The surface brightness of an astronomical object is defined as its observed flux divided by its observed angular scale, i.e. $\Sigma \sim F_{obs}/(\theta_{obs})^2$. What is the redshift evolution of $\Sigma(z)$ for a class of objects that are both standard candle and ruler? (**6 points**)

3. Plot the redshift evolution of the mass of various "νσ halos", i.e. $M_{\nu\sigma}(z)$, where ν is defined via

$$\nu = \frac{\delta_c}{D(z)\,\sigma(M_{\nu\sigma})}$$

Here $D(a=1/(1+z))$ is the growth factor that can be approximated as

$$D(a) = \frac{5a}{2}\Omega_m(a)\left[\Omega_m^{4/7}(a) - \Omega_\Lambda(a) + \left(1+\frac{\Omega_m(a)}{2}\right)\left(1+\frac{\Omega_\Lambda(a)}{70}\right)\right]^{-1}$$

and

$$\sigma^2(M) = \frac{1}{2\pi^2}\int_0^\infty P_0(k)\,\widehat{W}^2(kR)k^2dk$$

where M and R are related in a way as given in the lecture notes. Please use a scale-free cosmology characterized by $P_0(k)=A\,k^n$, $\Omega_{m,0}=1.0$, and $\Omega_{\Lambda,0}=0$. Vary both ν and n in a reasonable range.

hints: During the calculation it makes sense to define a 'typical collapsing mass' $M_* = \sigma_0^{\frac{6}{n+3}}\frac{4\pi\langle\rho\rangle}{3}$ which you can safely assume to be $M_*=10^{13}M_\odot$, irrespective of n.
Note that σ_0 captures everything that is constant, even though still depends on n (but see previous hint!)
As explained in class, for each redshift z you need to find $M_{\nu\sigma}(z)$ for which it helps to first find an analytical expression for $\sigma^2(M) = \frac{1}{2\pi^2}\int_0^\infty P_0(k)\,\widehat{W}^2(kR)k^2dk$ (**6 points**)

4. You are given some preliminary data from a GW observation:
i) The spectral noise density of the detector is approximately Sn(f)~ 0.00938426*(f/1Hz)^{-20} sec,
ii) The maximum frequency of observations is f_{max}~200Hz,
iii) The signal-to-noise ratio of the observation was S/N~8,
iv) The frequencies of the GW for the last 0.1secs of the observation before the merger were (time in secs, freq. in Hz):
{{0.1, 81.8089}, {0.09, 85.1059}, {0.08, 88.9492}, {0.07, 93.5166}, {0.06, 99.0818}, {0.05, 106.093}, {0.04, 115.353}, {0.03, 128.493}, {0.02, 149.594}, {0.01, 193.999}}

Then:
1. Assuming circular orbits (averaged over the inclination), find the chirp mass and distance (in Mpc) of the system.
2. Finally, assuming the inclination angle was π/3, plot the reconstructed strains as a function of time for the last 0.1secs before the merger. (**7 points**)

Part B (problems to be discussed in class)

1) Explain the Saha equation
2) Derive the relation between (adiabatic) matter and temperature perturbations
3) Derive the scaling relations:
 radiation domination: $\delta_m \sim \ln(a)$
 matter domination: $\delta_m \sim a$
 Λ domination: $\delta_m \sim 1/a^2$
4) Explain the (idea of) the Press-Schechter mass function
5) Explain the dependence of the CMB peaks on the parameters Ω_m, Ω_b, Ω_k
6) Calculate the sound horizon at recombination for Ω_m =0.3 and Ω_m =1. Discuss what you found.
7) Explain the behavior of the matter power spectrum $P(k)$