Cosmology

Part A (problems to be handed in)

1. George Gamov predicted the existence of the CMBR already back in 1948. He knew that nucleosynthesis must have taken place at a temperature $T_{\rm nuc}\approx 10^9$K, and that the age of the Universe is $t_0\approx$10Gyr. Assume that the Universe is radiation dominated:

a) What was the energy density at the time of nucleosynthesis?

Compare your result to the mass density at the sun's centre.

b) What was the Hubble parameter at the time of nucleosynthesis in units of km/s/Mpc?

Compare your result to today's value for H_0.

c) What time did nucleosynthesis take place?

d) what is the current temperature of the radiation?

compare against the observed T_{CMB} and explain the difference.

e) Explain the assumptions $T_{\rm nuc}\approx 10^9$K and $t_0\approx$10Gyr. (**7 points**)

2. The Saha Equation. The purpose of this problem is to determine how the uncertainty in the value of the baryon-to-photon ratio η affects the recombination temperature in the early Universe. Plot the fractional ionization X as a function of temperature, in the range 2500 K < T < 5500 K. Use $\eta = 10^{-10}$ and $\eta = 10^{-9}$. How much does such a change in η affect the computed value of the recombination temperature $T_{\rm rec}$, if we define it as the temperature at which $X = 0.1$.

(**6 points**)

3. a) Use the Euler equations to demonstrate that the vorticity of the peculiar velocity field in a theory of structure formation via gravitational collapse of small inhomogeneities decays away like $1/a$. Further, confirm that in the absence of external forces the velocity field itself decays like $1/a$, too. (**3 points**)

b) Then show that the peculiar velocity is proportional to the acceleration induced by the density field,

$$\vec{u} = \frac{2f(a)}{3H\Omega_m}\vec{g}$$

where $f(a) = {\rm d}\ln\delta(a)/{\rm d}\ln a$ is the growth function and $\vec{g} = -\frac{1}{a}\nabla\Phi$ the peculiar grav. acceleration. (**3 points**)

4. Show that the growth factor and the luminosity distance in a flat ΛCDM model are given by the following hypergeometric functions:

$$\delta(a) = a\,{}_2F_1\left(\frac{1}{3}, 1, \frac{11}{6}; a^3\left(1-\Omega_{m,0}^{-1}\right)\right)$$

$$d_L(a) = \frac{c}{H_0}\frac{2\Omega_{m,0}^{-1/2}}{a}\left[{}_2F_1\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}; \left(1-\Omega_{m,0}^{-1}\right)\right) - \sqrt{a}\,{}_2F_1\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}; a^3\left(1-\Omega_{m,0}^{-1}\right)\right)\right]$$

(**6 points**)

Part B (problems to be discussed in class)

1) Explain the Deuterium bottleneck.
2) Show that the neutron-to-proton ratio at the start of BBNS is n/p ≈ 1/7.
3) Show that the primordial Helium abundance can be estimated to be $X_{He}\approx 0.25$.
4) Explain the Sakharov conditions.
5) Explain the problem of the baryon asymmetry.
6) Show that the baryon asymmetry is zero if there is no baryon number violation.

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