

## Cosmology

## Part A (problems to be handed in)

1. What is the present density parameter of massive neutrinos (assuming direct hierarchy, and consider both relativistic and non-relativistic neutrinos)?

(7 points)

2. Show that due to the relative motion of an observer with rest-frame K and the CMB with rest frame K' (moving with velocity  $\beta=v/c$  at an angle  $\theta$  with respect to K), the observer measures the rest-frame blackbody spectrum temperature  $T'$ , as a blackbody spectrum with temperature  $T_{\text{obs}}$ :

$$T_{\text{obs}} = \frac{T'}{\gamma(1 - \beta \cos \theta)}$$

(7 points)

3. Show that the maximum possible value of the primordial helium fraction is

$$Y_{\text{max}} = \frac{2f}{(1 + f)}$$

where  $f=n_n/n_p \leq 1$ .

(6 points)

**4. Decoupling of relativistic species. Neutrino temperature today.**

a) Show that when relativistic species decouple from all interactions, their particle distribution remains self-similar to an equilibrium distribution but with a temperature that decreases inversely proportional to the scale factor.

(2 points)

b) Using the conservation of entropy, compute the effective temperature of the relic background of relativistic neutrinos today.

(3 points)

## Part B (problems to be discussed in class)

- 1) Derive redshift of matter-radiation equality.
- 2) Calculate the slow roll parameters for  $V(\Phi) = \lambda \Phi^n$ .
- 3) Discuss the different cosmological horizons.
- 4) Discuss the magnetic monopole problem.
- 5) Discuss alternatives to single scalar field inflation.
- 6) Discuss the Planck 2018 constraints on inflation.