Cosmology

Part A (problems to be handed in)

1. Consider a Universe with pressureless matter, a cosmological constant, and spatial curvature. Show that the 1st Friedmann equation can be written as the equation of motion of a particle moving in one dimension with total energy zero and potential $V(a)$ as follows (where $a=R/R_0$, with $R_0=1$)

$$V(a) = -\frac{4\pi G}{3}\frac{\rho_{m,0}}{a} + \frac{k}{2} - \frac{\Lambda}{6}a^2$$

(**7 points**)

2. Show that the age of the Universe in a spatially flat model $\Omega_{\Lambda,0}+\Omega_{m,0}=1$ is given by

$$t(z) = \frac{2}{3H_0\sqrt{\Omega_{\Lambda,0}}}\ln\left(\sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\left(\frac{1}{1+z}\right)^3} + \sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\left(\frac{1}{1+z}\right)^3 + 1}\right)$$

(**6 points**)

3. Show that ($x = \Omega_m(a)$, $y = \Omega_\Lambda(a)$) satisfy the homogeneous system of equations:

$$x' = -x(1 - x + 2y)$$
$$y' = \ \ y(x + 2 - 2y)$$

where ()'=d/d log a. Compute the critical points and draw the cosmic flow/phase space diagram in the plane (x, y).

(**6 points**)

4. Show that our Universe has *not* suffered a bounce since decoupling because the maximum redshift $z_{\max}$ would satisfy $z^2_{\max}(z_{\max}+3) < 2/\Omega_{m,0}$ but we have certainly observed galaxies at $z > 8$.

hint: A bounce corresponds to a Universe that has suffered first contraction and then expansion.

(**6 points**)

Part B (problems to be discussed in class)

1) Derive the expression a=1/(1+z)
2) Show that $\rho\, R^{3(1+w)}$=const. for barotropic fluids under adiabatic expansion
3) Derive the evolution of the density parameter for a fluid with $w(z)$
4) Derive the evolution of the scale factor $a(t)$ with time during matter and radiation domination, ie $a(t) \sim t^n$, n=1/2 or 2/3
5) Same as before, but with conformal time η defined as dη =d$t/a(t)$

general remarks:

- 'show' means that you should derive the equations rather than verifying their correctness; this also means that you have to give every single step of calculation and not just say 'as we can see'.
- if you refer to some calculations or results from somewhere else you must provide the proper reference.
- be clear about what you do and write
- for all the exercise you should ignore any contribution coming from radiation, i.e. set Ω_r=0.