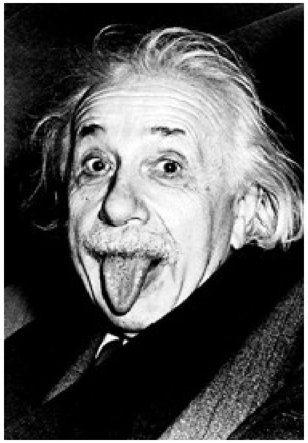
Open Problems in Cosmology II



$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

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Main points of the lecture

- Why we need Dark Energy (history+observations)
- Lemaitre-Tolman-Bondi (LTB) void models
- Scalar field and ideal fluid models
- Modified gravity and extra dimensions
- Effective fluid approach
- Conclusions

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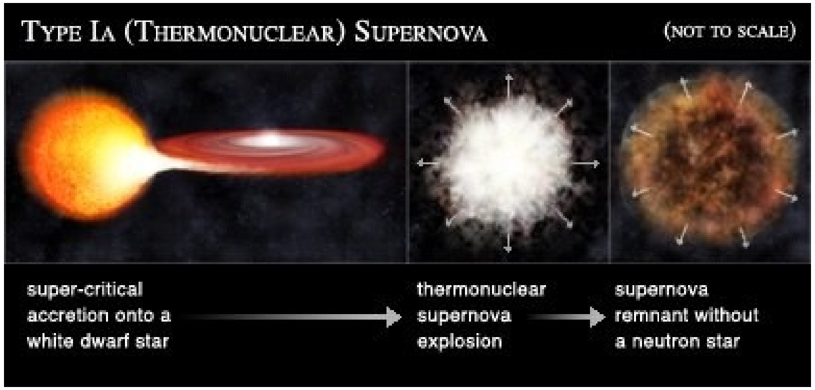
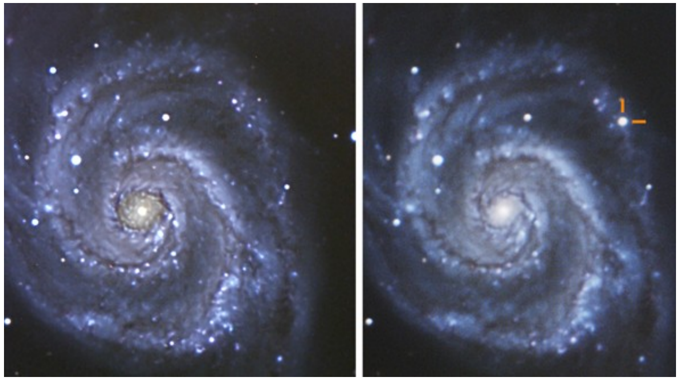
Observations of Accelerated Expansion

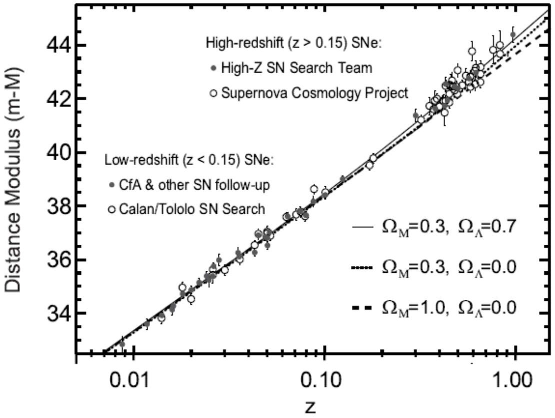
1) Type Ia supernovae

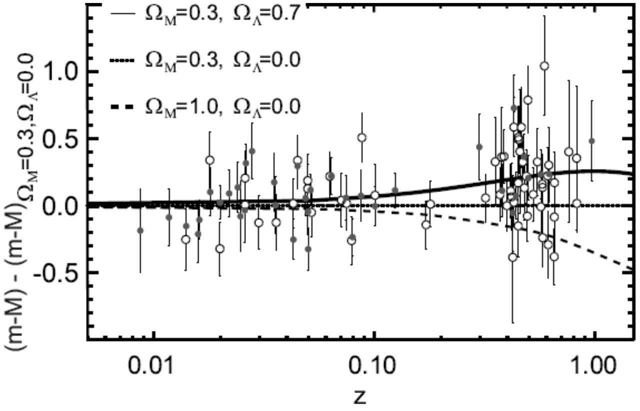




NASA

$$m - M = 5\log_{10}\left(\frac{d_L}{10\ \mathrm{pc}}\right)$$





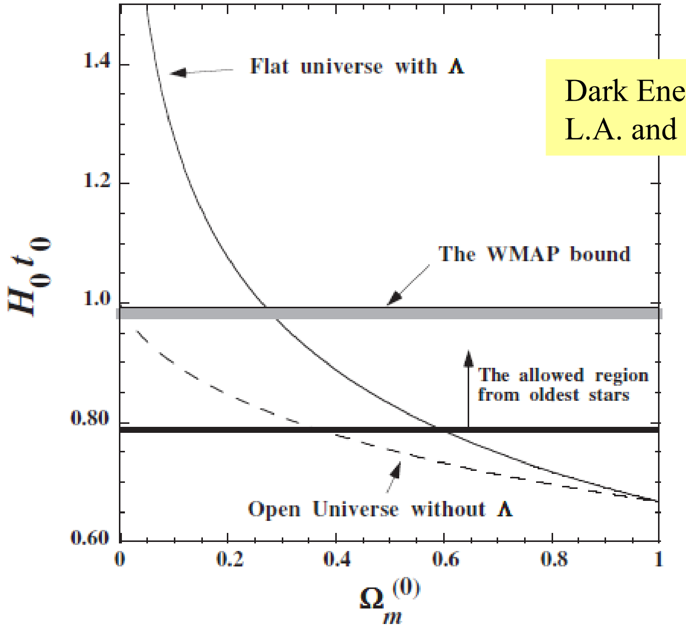
1211.2590, P. Astier

Observations of Accelerated Expansion

SnIa strongly support accelerated expansion (Riess et al 1998)

2) Age of the Universe

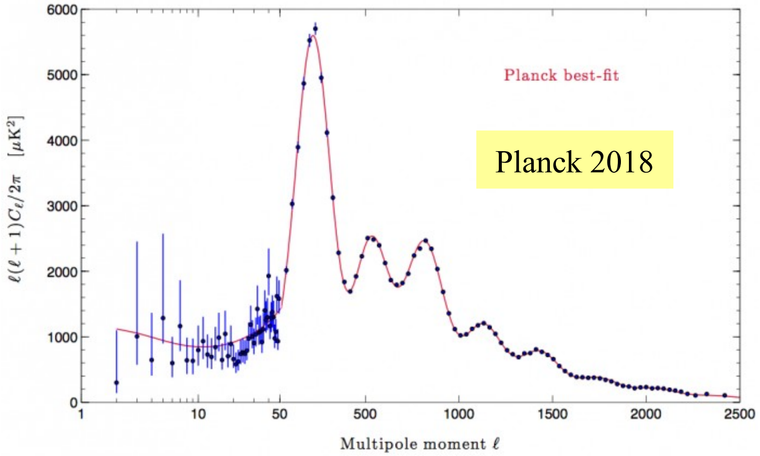
$$t_0 = H_0^{-1}\int_0^\infty \frac{dz}{E(z)\,(1+z)}$$

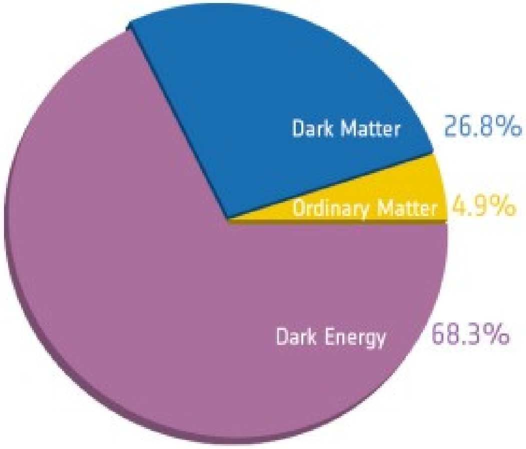


Dark Energy,
L.A. and S.T.

Observations of Accelerated Expansion

3) CMB with Planck





Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10}A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042	0.9665 ± 0.0038
$H_0\ [\mathrm{km\,s^{-1}\,Mpc^{-1}}]$	66.88 ± 0.92	68.44 ± 0.91	69.9 ± 2.7	67.27 ± 0.60	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.679 ± 0.013	0.699 ± 0.012	$0.711^{+0.033}_{-0.026}$	0.6834 ± 0.0084	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.321 ± 0.013	0.301 ± 0.012	$0.289^{+0.026}_{-0.033}$	0.3166 ± 0.0084	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1434 ± 0.0020	0.1408 ± 0.0019	$0.1404^{+0.0034}_{-0.0039}$	0.1432 ± 0.0013	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09589 ± 0.00046	0.09635 ± 0.00051	$0.0981^{+0.0016}_{-0.0018}$	0.09633 ± 0.00029	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8118 ± 0.0089	0.793 ± 0.011	0.796 ± 0.018	0.8120 ± 0.0073	0.8111 ± 0.0060	0.8102 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.840 ± 0.024	0.794 ± 0.024	$0.781^{+0.052}_{-0.060}$	0.834 ± 0.016	0.832 ± 0.013	0.825 ± 0.011
$\sigma_8\Omega_m^{0.25}$	0.611 ± 0.012	0.587 ± 0.012	0.583 ± 0.027	0.6090 ± 0.0081	0.6078 ± 0.0064	0.6051 ± 0.0058
z_{re}	7.50 ± 0.82	$7.11^{+0.91}_{-0.75}$	$7.10^{+0.87}_{-0.73}$	7.68 ± 0.79	7.67 ± 0.73	7.82 ± 0.71
$10^9 A_s$	2.092 ± 0.034	2.045 ± 0.041	2.116 ± 0.047	$2.101^{+0.031}_{-0.034}$	2.100 ± 0.030	2.105 ± 0.030
$10^9 A_s e^{-2\tau}$	1.884 ± 0.014	1.851 ± 0.018	1.904 ± 0.024	1.884 ± 0.012	1.883 ± 0.011	1.881 ± 0.010
Age [Gyr]	13.830 ± 0.037	13.761 ± 0.038	$13.64^{+0.16}_{-0.14}$	13.800 ± 0.024	13.797 ± 0.023	13.787 ± 0.020

Observations of Accelerated Expansion

4) Baryon Acoustic Oscillations and correlation function

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \langle\rho\rangle}{\langle\rho\rangle}$$

$$\xi(\vec{r}) \equiv \langle\delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle$$

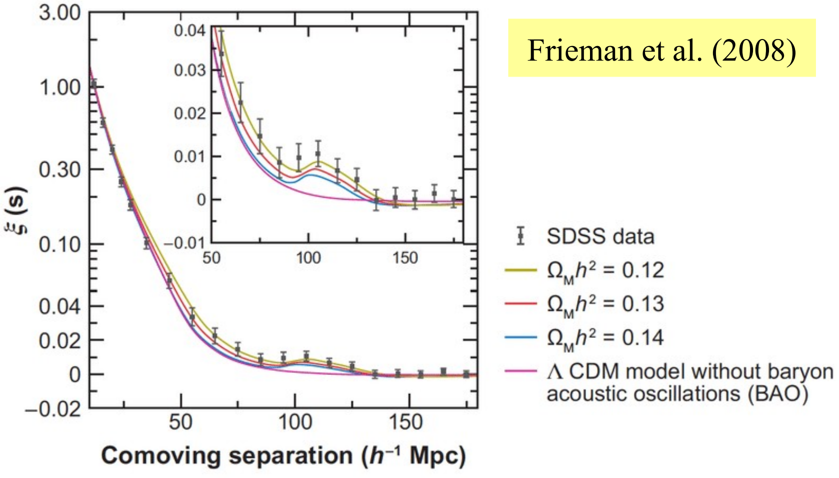
$$\xi(r) = \frac{1}{(2\pi)^3}\int P(k)\frac{\sin(kr)}{kr}4\pi k^2 dk$$

$$P(k) \equiv \langle|\delta_k|^2\rangle$$

Correlation function:

Denotes probability to find

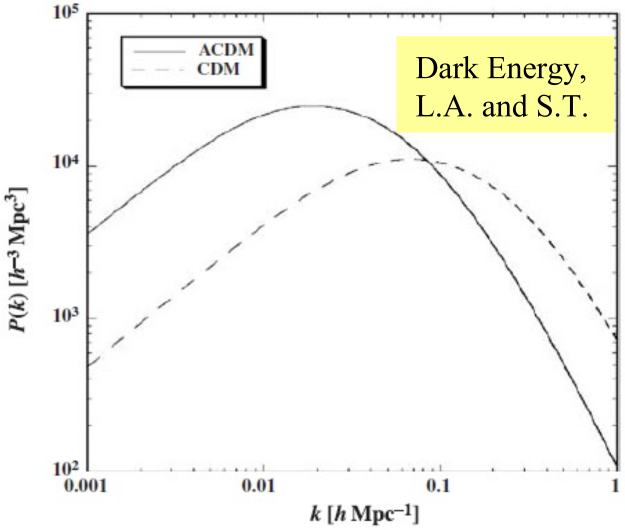
galaxy at position r



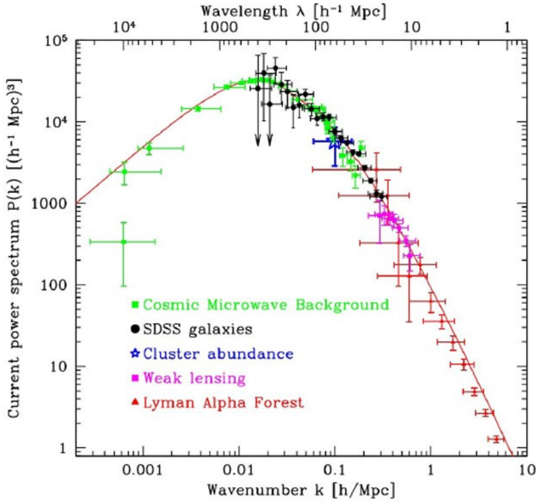
Frieman et al. (2008)

5) Large scale structure

$$P(k) \equiv \langle|\delta_k|^2\rangle$$



Dark Energy, L.A. and S.T.



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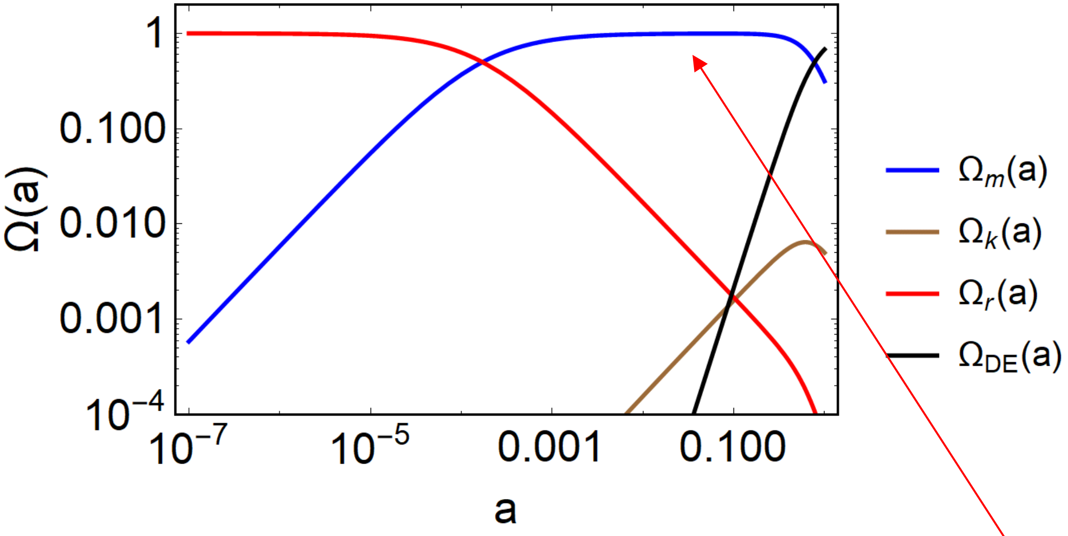
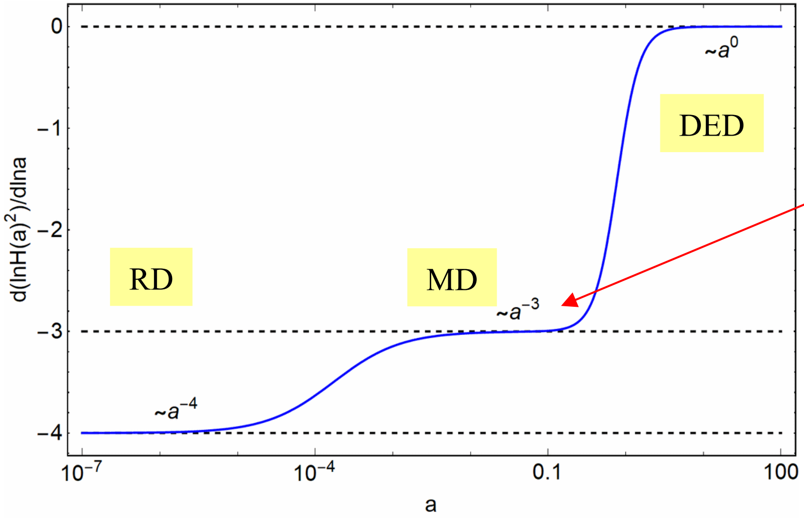
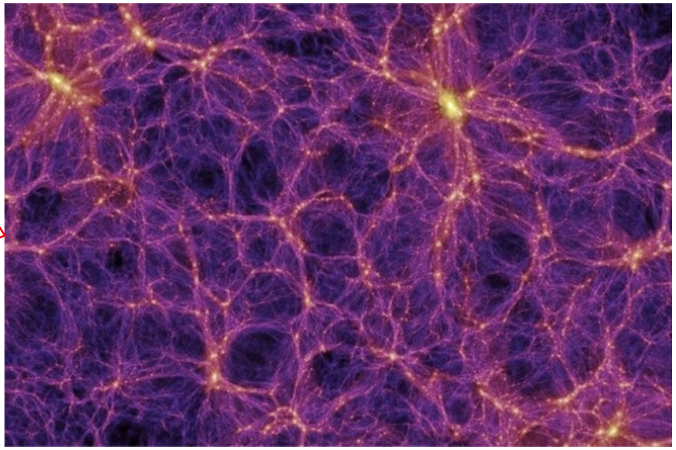
Observations of Accelerated Expansion

5) Transitions from Radiation to Matter to DE are necessary for structure formation







The Standard Cosmological model

Einstein equations
in pure GR:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

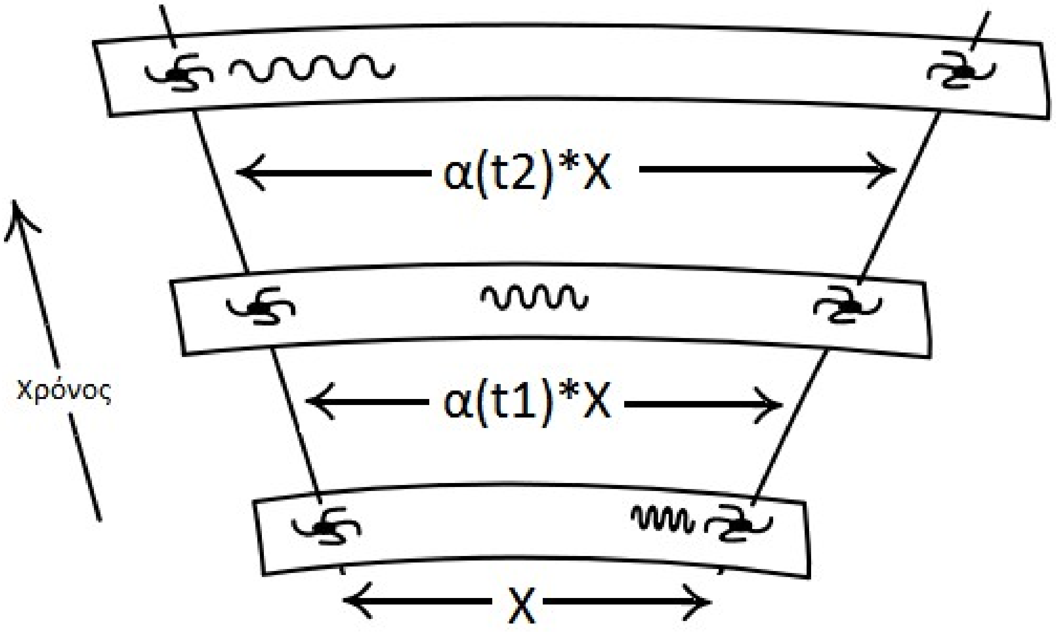
Einstein tensor

$$T^{\mu}_{\nu} = P g^{\mu}_{\nu} + (\rho + P) U^{\mu} U_{\nu}$$

Friedmann-Lemaitre-Robertson-Walker (FLRW) metric:

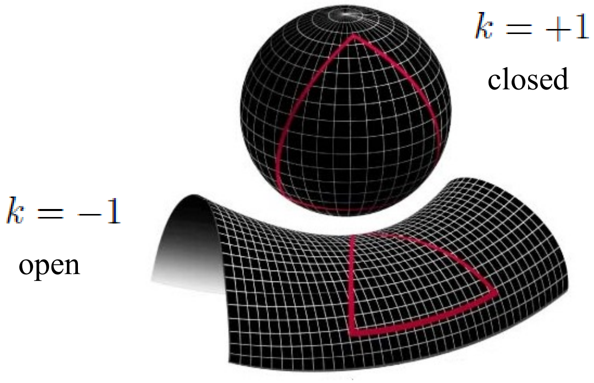
$$ds^2 = c^2 dt^2 - \alpha(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + sin(\theta)^2 d\phi^2) \right)$$

Scale factor α(t):



The Standard Cosmological model

The curvature:



Friedmann equations (1924):

$$H^2(\alpha) = \left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{8\pi G}{3}\rho(\alpha) - \frac{k}{\alpha^2}$$

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}\left(\rho(\alpha) + P(\alpha)\right)$$

Continuity equations:

(via Bianchi identities)

$$\nabla_\nu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \dot{\rho} + 3H(\rho + P) = 0$$

The Standard Cosmological model

Hubble (1929): The Universe is expanding

Redshift of distant galaxies

Riess et al. (1998): ...and it's also accelerating!

Type Ia supernovae

2nd Friedmann equation: $\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}(\rho(\alpha) + 3P(\alpha)) \Longrightarrow P < -\frac{\rho}{3}$

Equation of state $P = w\,\rho$

$w = 0$ Non-relativistic matter $P << \rho$

$w = \frac{1}{3}$ Relativistic matter (photons etc) $P = \frac{1}{3}\rho$

$$P < -\frac{\rho}{3} \Longrightarrow w < -\frac{1}{3}$$

The known forms of matter cannot explain the accelerated expansion of the Universe…

We need Dark Energy!

The cosmological constant Λ

Pure GR: $S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\;R\; + S_m \quad \Longrightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$

Modify either LHS or RHS!

GR with a cosmological constant (mod. LHS):

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,(R - 2\Lambda) + S_m \quad \Longrightarrow \quad G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Cosmological Constant

Works (see previous lectures) but has problems:

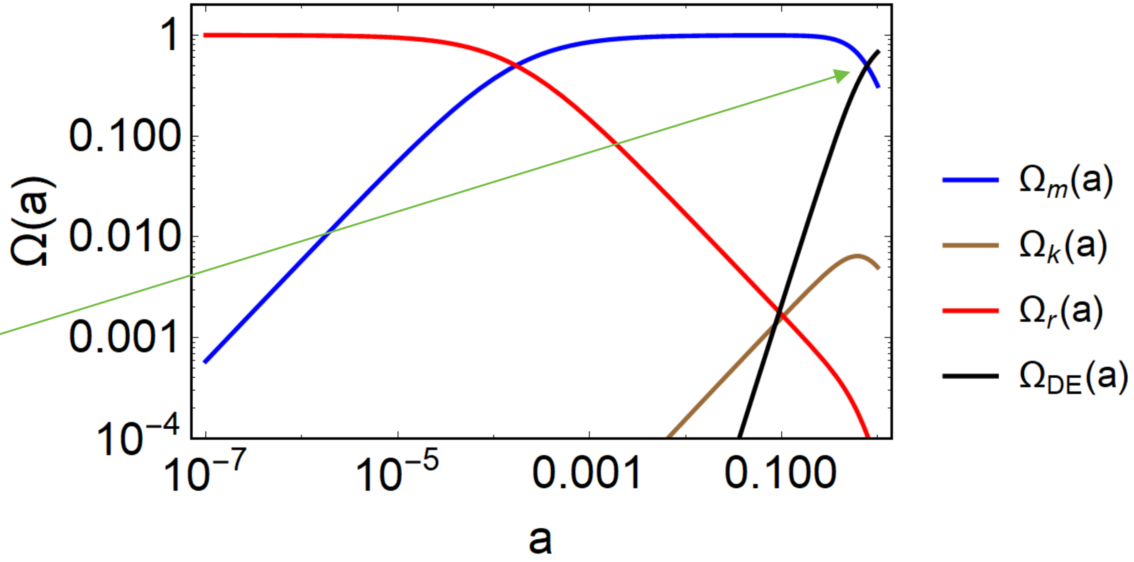
1) Fine-tuning problem:

$$\Lambda \approx H_0^2 = (2.1332h \times 10^{-42}\ \mathrm{GeV})^2 \quad \Longrightarrow \quad \rho_\Lambda \approx \frac{\Lambda m_{\mathrm{pl}}^2}{8\pi} \approx 10^{-47}\ \mathrm{GeV}^4 \approx 10^{-123} m_{\mathrm{pl}}^4$$

$$\rho_{\mathrm{vac}} = \int_0^{k_{\mathrm{max}}} \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2} = \int_0^{k_{\mathrm{max}}} \frac{4\pi k^2\mathrm{d}k}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2} \approx \frac{k_{\mathrm{max}}^4}{16\pi^2} \simeq 10^{74}\ \mathrm{GeV}^4$$

The cosmological constant Λ

2) Coincidence problem:



Why same order of magnitude today?

3) Other theoretical issues:

i) In SUSY the vacuum energy is 0.

$$Q_s|\text{boson}\rangle = |\text{fermion}\rangle \qquad \sum_{\text{all } Q_s}\{Q_s, Q_s^\dagger\} = cE \qquad E|0\rangle = 0$$
$$Q_s|\text{fermion}\rangle = |\text{boson}\rangle$$

ii) Anthropic arguments

$$-10^{-123} m_{\rm pl}^4 \lesssim \rho_\Lambda \lesssim 3 \times 10^{-121} m_{\rm pl}^4$$

iii) Other astrophysical issues (see previous lecture)

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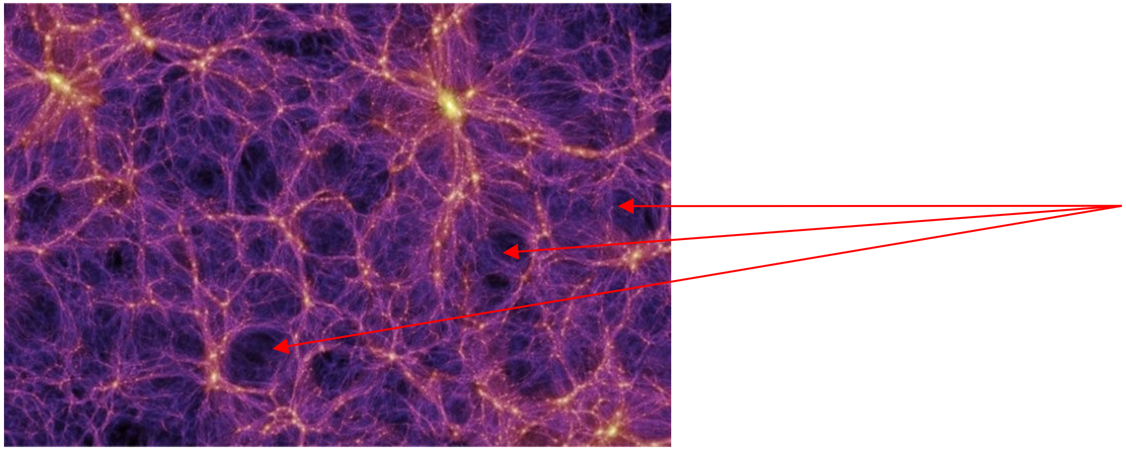
Lemaitre-Tolman-Bondi (LTB) void models

1) FRW metric is homogeneous/isotropic at large distances



Not smooth at all scales, in small scales there are voids!

2) LTB metric allows for a void:

0802.1523

$$ds^2 = -dt^2 + X^2(r,t)\,dr^2 + A^2(r,t)\,d\Omega^2$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$$

$$A(r,t) = a(t)\,r$$

$$k(r) = k\,r^2$$

$$X(r,t) \;=\; A'(r,t)/\sqrt{1-k(r)}$$

$\Longrightarrow$ FRW metric

Lemaitre-Tolman-Bondi (LTB) void models

3) Two Hubble parameters! Transverse and longitudinal:

$$H_T(r,t) \equiv \frac{\dot{A}(r,t)}{A(r,t)}$$

$$H_L(r,t) \equiv \frac{\dot{A}'(r,t)}{A'(r,t)}$$

4) Transverse Hubble parameter similar to ΛCDM:

$$H^2(r,t) = H_0^2(r)\left[\Omega_M(r)\left(\frac{A_0(r)}{A(r,t)}\right)^3 + (1-\Omega_M(r))\left(\frac{A_0(r)}{A(r,t)}\right)^2\right]$$

We require profiles for density and H0!

Lemaitre-Tolman-Bondi (LTB) void models

5) Parametric solution

$$H_0(r)t_{\rm BB}(r) = \frac{1}{\sqrt{\Omega_K(r)}}\sqrt{1+\frac{\Omega_M(r)}{\Omega_K(r)}} - \frac{\Omega_M(r)}{\sqrt{\Omega_K^3(r)}}\sinh^{-1}\sqrt{\frac{\Omega_K(r)}{\Omega_M(r)}}$$

Big Bang not happening simultaneously everywhere!

$$A(r,t) = \frac{\Omega_M(r)}{2[1-\Omega_M(r)]}[\cosh(\eta)-1]A_0(r)$$

$$H_0(r)t = \frac{\Omega_M(r)}{2[1-\Omega_M(r)]^{3/2}}[\sinh(\eta)-\eta]$$

6) Possible profiles for matter, H_0 that describe a void of size r_0:

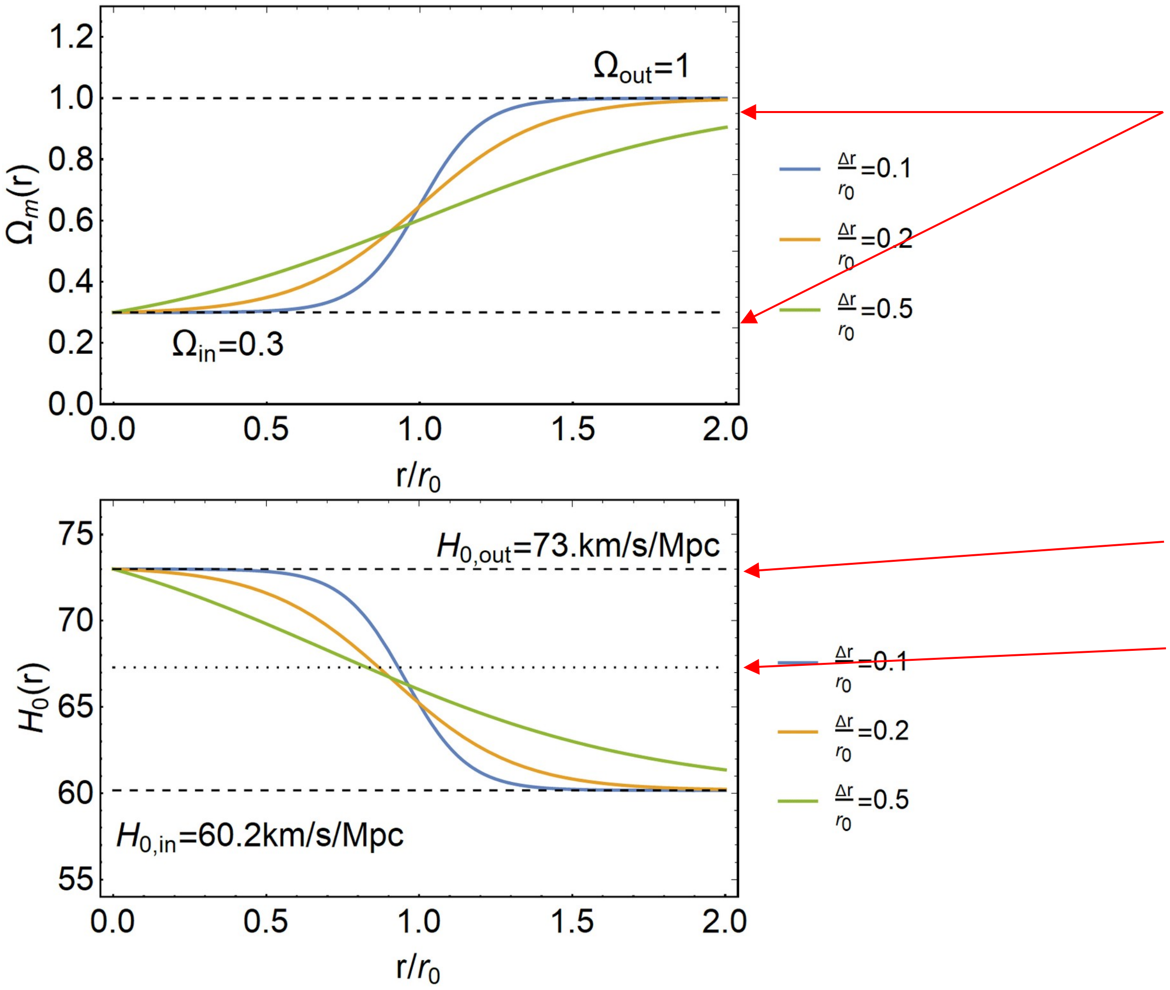
$$\Omega_M(r) = \Omega_{\rm out} + \left(\Omega_{\rm in}-\Omega_{\rm out}\right)\left(\frac{1-\tanh[(r-r_0)/2\Delta r]}{1+\tanh[r_0/2\Delta r]}\right)$$

$$H_0(r) = H_0\left[\frac{1}{\Omega_K(r)} - \frac{\Omega_M(r)}{\sqrt{\Omega_K^3(r)}}\sinh^{-1}\sqrt{\frac{\Omega_K(r)}{\Omega_M(r)}}\right] = H_0\sum_{n=0}^{\infty}\frac{2[\Omega_K(r)]^n}{(2n+1)(2n+3)}$$

Demand Big Bang happening simultaneously everywhere

Lemaitre-Tolman-Bondi (LTB) void models

7) Most profiles are problematic, require fine-tuning and/or weird primordial power spectra to fit CMB.



In and out of the void

Cepheids

Planck

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Quintessence

1) Simplest thing we can add to GR Lagrangian (on RHS!) is a scalar field

- i) Scalar fields (bosons with spin 0) have been observed (Higgs)!
- ii) Already used in inflation (also an accelerating phase)
- iii) Dynamics well understood

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\;R + S_m \quad \Longrightarrow \quad S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R + \mathcal{L}_\phi\right] + S_M$$

$$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

2) Energy momentum tensor:

$$T^{(\phi)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_\phi)}{\delta g^{\mu\nu}} \quad = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right]$$

Quintessence

3) Effective density and pressure and equation of state w:

$$P_\phi = \frac{1}{3}T_i^{i(\phi)} = \frac{1}{2}\dot\phi^2 - V(\phi)$$

$$\rho_\phi = -T_0^{0(\phi)} = \frac{1}{2}\dot\phi^2 + V(\phi)$$

$$\Longrightarrow \quad w_\phi \equiv \frac{P_\phi}{\rho_\phi} = \frac{\dot\phi^2 - 2V(\phi)}{\dot\phi^2 + 2V(\phi)}$$

4) In quintessence, w(z) cannot cross -1! Use continuity equation:

Nesseris et al, astro-ph/0610092

$$\dot\rho_\phi + 3H(\rho_\phi + P_\phi) = 0 \qquad \dot\rho = -3\frac{\dot a}{a}(\rho + p(\rho))$$

when w→ -1 $\Longrightarrow$ $p(\rho) \rightarrow -\rho$ $\Longrightarrow$ $\dot\rho \rightarrow 0$ and $\lim\limits_{w \to -1} \frac{d^n \rho(t)}{dt^n} = 0$

So w(z) goes asymptotically to w→ -1_+!

Quintessence

5) Equations of motion:

$$H^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_M\right] ,$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\dot{\phi}^2 + \rho_M + P_M\right) ,$$

and

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$V_{,\phi} \equiv \mathrm{d}V/\mathrm{d}\phi$$

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$$

6) Example models (all high energy physics inspired):

Freezing

$$V(\phi) = M^{4+n}\phi^{-n} \quad (n > 0) ,$$
$$V(\phi) = M^{4+n}\phi^{-n}\exp(\alpha\phi^2/m_{\mathrm{pl}}^2) .$$

Thawing

$$V(\phi) = V_0 + M^{4-n}\phi^n \quad (n > 0) ,$$
$$V(\phi) = M^4\cos^2(\phi/f) .$$

Quintessence

7) Autonomous systems and critical points

arXiv:hep-th/0603057

$$\dot{x} = f(x, y, t)$$

$$\dot{y} = g(x, y, t)$$

Critical points when $(f, g)|_{(x_c, y_c)} = 0$

8) Stability when

$$x = x_c + \delta x$$

$$y = y_c + \delta y$$

$$\Longrightarrow \quad \frac{\mathrm{d}}{\mathrm{d}N}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = \mathcal{M}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix}_{(x=x_c, y=y_c)}$$

$$\delta x = C_1 e^{\mu_1 N} + C_2 e^{\mu_2 N}$$

$$\delta y = C_3 e^{\mu_1 N} + C_4 e^{\mu_2 N};$$

Quintessence

arXiv:hep-th/0603057

9) Solutions

$$\delta x = C_1 e^{\mu_1 N} + C_2 e^{\mu_2 N}$$
$$\delta y = C_3 e^{\mu_1 N} + C_4 e^{\mu_2 N}$$

⇒

(i) Stable node: $\mu_1 < 0$ and $\mu_2 < 0$.

(ii) Unstable node: $\mu_1 > 0$ and $\mu_2 > 0$.

(iii) Saddle point: $\mu_1 < 0$ and $\mu_2 > 0$ (or $\mu_1 > 0$ and $\mu_2 < 0$).

(iv) Stable spiral: The determinant of the matrix $\mathcal{M}$ is negative and the real parts of μ_1 and μ_2 are negative.

10) For quintessence

$$\mathcal{L} = \frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi)$$

$$x \equiv \frac{\kappa\dot{\phi}}{\sqrt{6}H}\,, \quad y \equiv \frac{\kappa\sqrt{V}}{\sqrt{3}H}\,,$$
$$\lambda \equiv -\frac{V_{,\phi}}{\kappa V}\,, \quad \Gamma \equiv \frac{VV_{,\phi\phi}}{V_{,\phi}^2}\,,$$

⇒

$$\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}N} &= -3x + \frac{\sqrt{6}}{2}\epsilon\lambda y^2 \\
&\quad + \frac{3}{2}x\left[(1-w_m)\epsilon x^2 + (1+w_m)(1-y^2)\right], \\
\frac{\mathrm{d}y}{\mathrm{d}N} &= -\frac{\sqrt{6}}{2}\lambda x y \\
&\quad + \frac{3}{2}y\left[(1-w_m)\epsilon x^2 + (1+w_m)(1-y^2)\right], \\
\frac{\mathrm{d}\lambda}{\mathrm{d}N} &= -\sqrt{6}\lambda^2(\Gamma-1)x\,,
\end{aligned}$$

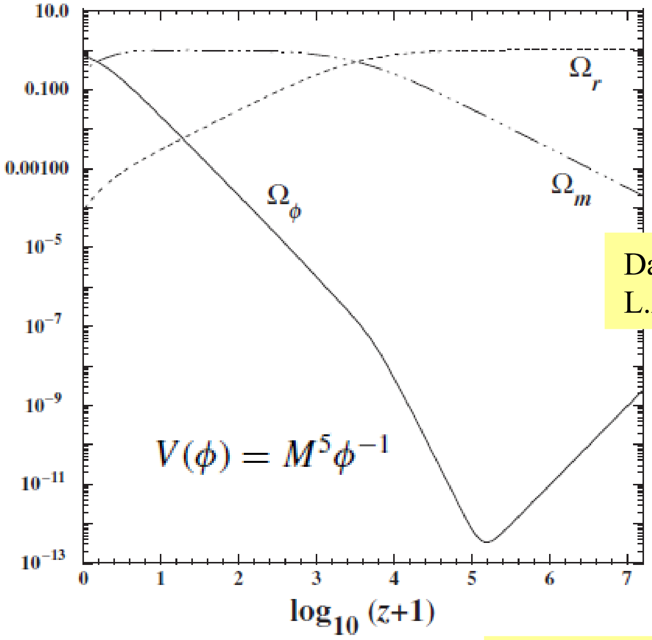
Quintessence

11) Other variables:

$$\epsilon x^2 + y^2 + \frac{\kappa^2 \rho_m}{3H^2} = 1$$

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\epsilon x^2 - y^2}{\epsilon x^2 + y^2} \,,$$

$$\Omega_\phi \equiv \frac{\kappa^2 \rho_\phi}{3H^2} = \epsilon x^2 + y^2 \,.$$



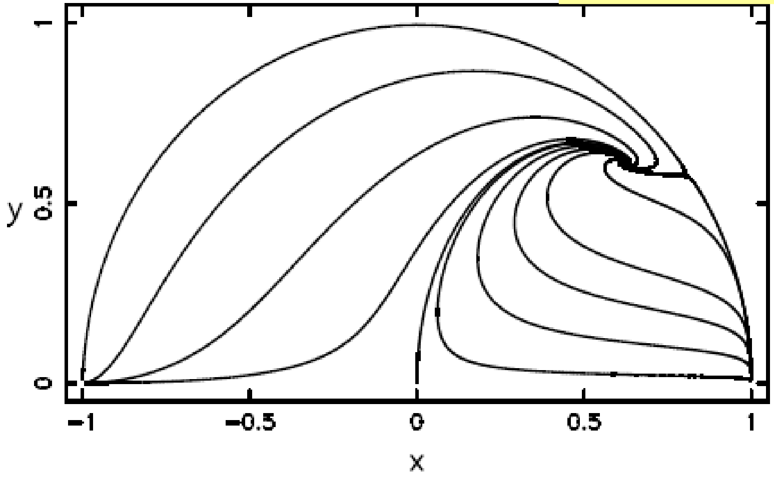
Dark Energy,
L.A. and S.T.

12) Phase space

$$x \equiv \frac{\kappa \dot{\phi}}{\sqrt{6}H} \,, \quad y \equiv \frac{\kappa \sqrt{V}}{\sqrt{3}H} \,,$$

$$\lambda \equiv -\frac{V_{,\phi}}{\kappa V} \,, \quad \Gamma \equiv \frac{V V_{,\phi\phi}}{V_{,\phi}^2} \,,$$

arXiv:hep-th/0603057



Quintessence

Dark Energy,
L.A. and S.T.

13) Potential reconstruction (E(z)=H(z)/H0):

$$H^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_M\right],$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\dot{\phi}^2 + \rho_M + P_M\right),$$

$$\Longrightarrow$$

$$\frac{\kappa^2}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}z}\right)^2 = \frac{1}{1+z}\frac{\mathrm{d}\ln E(z)}{\mathrm{d}z} - \frac{3\Omega_m^{(0)}}{2}\frac{1+z}{E^2(z)} \geq 0$$

$$\frac{\kappa^2 V}{3H_0^2} = E(z) - \frac{1+z}{6}\frac{\mathrm{d}E^2(z)}{\mathrm{d}z} - \frac{1}{2}\Omega_m^{(0)}(1+z)^3.$$

14) Condition for reconstruction

$$\frac{\mathrm{d}H^2}{\mathrm{d}z} \geq 3\Omega_m^{(0)} H_0^2 (1+z)^2 \quad \Longrightarrow \quad \rho_\phi + P_\phi \geq 0$$

(weak energy condition)

K-essence

1) K-essence (most general action for minimally coupled scalar field)

Dark Energy,
L.A. and S.T.

$$X \equiv -(1/2)(\nabla\phi)^2 \quad \Longrightarrow \quad S = \int \mathrm{d}^4x\sqrt{-g}\,p(\phi, X)$$

$$\Longrightarrow \quad S_E = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R + K(\phi)X + L(\phi)X^2 + \cdots\right],$$

2) Equation of state:

$$T^{(\phi)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,P)}{\delta g^{\mu\nu}} = P_{,X}\partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}P \quad \Longrightarrow \quad \begin{aligned} P_\phi &= P; \\ \rho_\phi &= 2XP_{,X} - P \end{aligned} \quad \Longrightarrow$$

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{P}{2XP_{,X} - P}$$

Can cross w=-1!

Chaplygin gas

1) Barotropic fluids (Chaplygin gas)

Dark Energy,
L.A. and S.T.

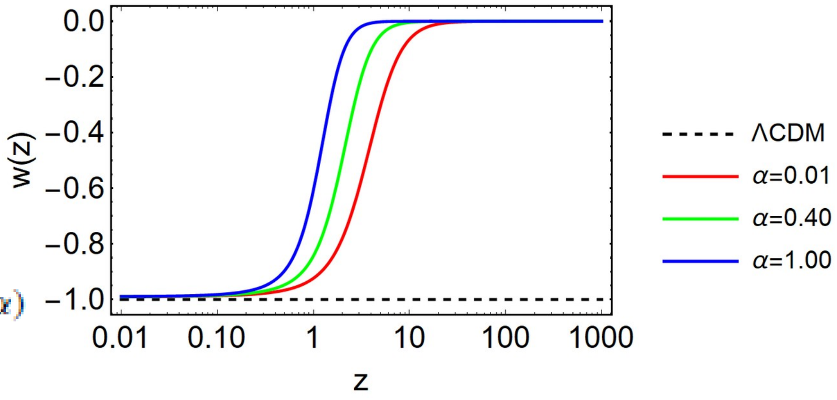
$$P = -A\rho^{-\alpha}$$

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0 \quad \Longrightarrow \quad \rho(t) = \left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{1/(1+\alpha)}$$

2) Equation of state w(z)

$$\Omega_m^* \equiv \frac{B}{A+B}\,, \quad \Longrightarrow$$

$$\rho(z) = \rho_*\left[1 - \Omega_m^* + \Omega_m^*(1+z)^{3(1+\alpha)}\right]^{1/(1+\alpha)}$$



$$w(z) = -\left[1 + \frac{\Omega_m^*}{1-\Omega_m^*}(1+z)^{3(1+\alpha)}\right]^{-1}$$

w(z=0)→ -1 mimics DE at late times!
w(z>>0)→ 0 mimics DM at early times!

Main points of the lecture

- Why we need Dark Energy (history+observations)
- Lemaitre-Tolman-Bondi (LTB) void models
- Scalar field and ideal fluid models
- Modified gravity and extra dimensions
- Effective fluid approach
- Conclusions

Modified gravity

1) Simplest thing we can add to GR Lagrangian (on LHS!) is R→ f(R)

- i) Just a scalar degree of freedom
- ii) Has been used in inflation (Starobinsky model!)
- iii) Dynamics well understood, but rich phenomenology
- iv) High energy physics inspired

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\;R\;+S_m \quad\Longrightarrow\quad S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\;f\left(R\right)+S_m$$

2) Simplest example of f(R) is ΛCDM!

$$f(R) \simeq f(R_0) + f'(R_0)R + \cdots \quad\Longrightarrow$$

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\;f\left(R\right)+S_m \quad\Longrightarrow\quad S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R-2\Lambda\right)+S_m$$

GR is just a special case, not unique theory!

Modified gravity

3) High energy physics inspired. New terms appear when trying to renormalize GR at one-loop order:

$$R \Rightarrow R + \alpha \left[\frac{1}{180} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - \frac{1}{180} R_{\mu\nu} R^{\mu\nu} - \frac{1}{6} \left(\frac{1}{5} - \xi \right) \Box R + \frac{1}{2} \left(\frac{1}{6} - \xi \right)^2 R^2 + \cdots \right]$$

Birrell & Davis 1986, Sec 6.2, pg 159

=?

f(R)!

4) Most general (pure) modified gravity theory is of the form:

$$R \Rightarrow f(R, P, Q, \Box^n, G)$$

$$\begin{aligned} R &= g_{\mu\nu} R^{\mu\nu} \\ P &= R_{\mu\nu} R^{\mu\nu} \\ Q &= R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \\ \Box &= g^{\mu\nu} \nabla_\mu \nabla_\nu \\ G &= Q - 4P + R^2 \end{aligned}$$

D'Alambertian in curved space

Gauss-Bonnet term
(topological invariant in 4D)

f(R) models

1) f(R) equations of motion (vary action with respect to metric):

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\, f(R) + S_m$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \delta g_{\mu\nu} \leftrightarrow \tilde{g}^{\mu\nu} = g^{\mu\nu} - \delta g^{\mu\nu}$$

$$\begin{aligned}
\delta\Gamma^{\lambda}_{\mu\nu} &= \frac{1}{2} g^{\lambda a}\left(\nabla_\mu \delta g_{a\nu} + \nabla_\nu \delta g_{a\mu} - \nabla_a \delta g_{\mu\nu}\right)\\
\delta R^{\nu}_{k\lambda a} &= \nabla_\lambda \delta\Gamma^{\nu}_{ka} - \nabla_a \delta\Gamma^{\nu}_{k\lambda}\\
\delta R_{\mu\nu} &= \frac{1}{2}\left(-\Box\delta g_{\mu\nu} + \nabla_a\nabla_\mu \delta g^a_\nu + \nabla_a\nabla_\nu\delta g^a_\mu - \nabla_\mu\nabla_\nu\delta g^a_a\right)\\
\delta\sqrt{-g} &= -\frac{1}{2}\sqrt{-g}g_{ab}\delta g^{ab}\\
\delta R &= \delta(g^{\mu\nu}R_{\mu\nu}) = \delta g^{\mu\nu}R_{\mu\nu} + g_{\mu\nu}\Box\delta g^{\mu\nu} - \nabla_\mu\nabla_\nu\delta g^{\mu\nu}
\end{aligned}$$

$$\Longrightarrow \quad \boxed{F G_{\mu\nu} - \frac{1}{2}(f(R) - R\ F)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\,F = \kappa\, T^{(m)}_{\mu\nu}}$$

2) Conservation equation:

$$\begin{aligned}
S &= \int d^4x\sqrt{-g}\mathcal{L} \Rightarrow\\
\delta S &= \int d^4x\sqrt{-g}\left[\frac{\sqrt{-g}\mathcal{L}}{\delta g^{\mu\nu}}\frac{1}{\sqrt{-g}}\right]\delta g^{\mu\nu}\\
&= \int d^4x\sqrt{-g}S_{\mu\nu}\delta g^{\mu\nu}
\end{aligned}$$

$$\Longrightarrow$$

$$\begin{aligned}
\delta g_{\mu\nu} &= \pounds_V g_{\mu\nu} = V^\sigma\nabla_\sigma g_{\mu\nu} + (\nabla_\mu V^\lambda)g_{\lambda\nu} + (\nabla_\nu V^\lambda)g_{\mu\lambda}\\
&= \nabla_\mu V_\nu + \nabla_\nu V_\mu
\end{aligned}$$

$$\begin{aligned}
\delta S &= \int d^4x\sqrt{-g}\Big[S_{\mu\nu}(\nabla^\mu V^\nu + \nabla^\nu V^\mu)\Big]\\
&= -\int d^4x\sqrt{-g}V^\nu\Big[\nabla^\mu S_{\mu\nu} + \nabla^\mu S_{\nu\mu}\Big]\\
&= -2\int d^4x\sqrt{-g}\,V^\nu\ \nabla^\mu S_{\mu\nu}\\
&= 0 \qquad\qquad \Rightarrow
\end{aligned}$$

$$\boxed{\nabla_\mu S^{\mu\nu} \ = \ 0}$$

f(R) models

3) f(R) Friedman equations for FRW and acceleration!

$$ds^2 = c^2dt^2 - \alpha(t)^2\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin(\theta)^2d\phi^2)\right) \quad \Longrightarrow$$

$$\begin{aligned} 3FH^2 &= \rho_{\rm m} + \rho_{\rm rad} + \frac{1}{2}(FR - f) - 3H\dot{F} \\ -2F\dot{H} &= \rho_{\rm m} + \frac{4}{3}\rho_{\rm rad} + \ddot{F} - H\dot{F} \end{aligned}$$

Properly chosen,
can give acceleration!

4) Autonomous systems

$$\begin{aligned} x_1 &= -\frac{F'}{F}\,, \\ x_2 &= -\frac{f}{6FH^2}\,, \\ x_3 &= \frac{R}{6H^2} = \frac{H'}{H} + 2\,, \\ x_4 &= \frac{\rho_{\rm rad}}{3FH^2} = \Omega_r\,. \end{aligned}$$

$$' = \frac{d}{d\ln\alpha} \equiv \frac{d}{dN} = \frac{1}{H}\frac{d}{dt}$$

$$\Longrightarrow$$

$$x_1' = -1 - x_3 - 3x_2 + x_1^2 + x_4$$

$$x_2' = \frac{x_1x_3}{m} - x_2(2x_3 - x_1 - 4)$$

$$x_3' = -\frac{x_1x_3}{m} - 2x_3(x_3 - 2)$$

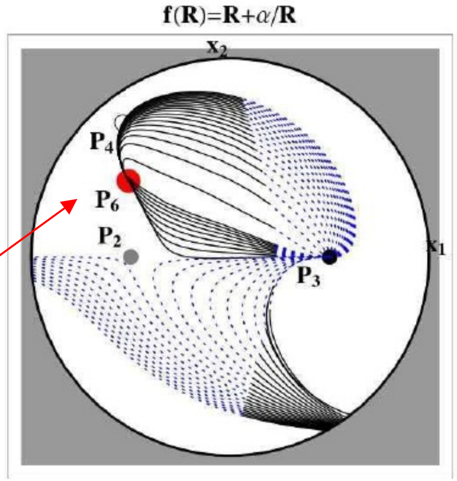
$$x_4' = -2x_3x_4 + x_1x_4$$

$$m \equiv \frac{F'R}{f'} = \frac{f_{,RR}\,R}{f_{,R}}$$

f(R) models

5) Critical points

$$P_1 : (x_1, x_2, x_3) = (0, -1, 2), \quad \Omega_{\rm m} = 0, \quad w_{\rm eff} = -1\,,$$
$$P_2 : (x_1, x_2, x_3) = (-1, 0, 0), \quad \Omega_{\rm m} = 2, \quad w_{\rm eff} = 1/3\,,$$
$$P_3 : (x_1, x_2, x_3) = (1, 0, 0), \quad \Omega_{\rm m} = 0, \quad w_{\rm eff} = 1/3\,,$$
$$P_4 : (x_1, x_2, x_3) = (-4, 5, 0), \quad \Omega_{\rm m} = 0, \quad w_{\rm eff} = 1/3\,,$$
$$P_5 : (x_1, x_2, x_3) = \left(\frac{3m}{1+m}, -\frac{1+4m}{2(1+m)^2}, \frac{1+4m}{2(1+m)}\right),$$
$$\Omega_{\rm m} = 1 - \frac{m(7+10m)}{2(1+m)^2}, \quad w_{\rm eff} = -\frac{m}{1+m}\,,$$
$$P_6 : (x_1, x_2, x_3) = \left(\frac{2(1-m)}{1+2m}, \frac{1-4m}{m(1+2m)}, -\frac{(1-4m)(1+m)}{m(1+2m)}\right),$$
$$\Omega_{\rm m} = 0, \quad w_{\rm eff} = \frac{2-5m-6m^2}{3m(1+2m)}$$



Attractor!

6) Viable models (Hu+Sawicki, Starobinski) are perturbations around ΛCDM!

$$
\begin{aligned}
f(R) &= R - m^2 \frac{c_1 (R/m^2)^n}{1 + c_2 (R/m^2)^n}, \\
&= R - \frac{m^2 c_1}{c_2} + \frac{m^2 c_1/c_2}{1 + c_2 (R/m^2)^n} \\
&= R - 2\Lambda \left(1 - \frac{1}{1 + (R/(b\,\Lambda))^n}\right) \\
&= R - \frac{2\Lambda}{1 + \left(\frac{b\Lambda}{R}\right)^n},
\end{aligned}
$$

$$f(R) = R - 2\Lambda\left(1 - \frac{1}{\left(1 + \left(\frac{R}{b\Lambda}\right)^2\right)^n}\right),$$

$$
\begin{aligned}
\lim_{b\to 0} f(R) &= R - 2\Lambda, \\
\lim_{b\to\infty} f(R) &= R.
\end{aligned}
$$

f(R) models

7) Effective fluids: take modifications from LHS to RHS → Dark Energy fluid!

$$F G_{\mu\nu} - \frac{1}{2}(f(R) - R\ F) g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) F = \kappa T^{(m)}_{\mu\nu} \Longrightarrow G_{\mu\nu} = \kappa \left(T^{(m)}_{\mu\nu} + T^{(DE)}_{\mu\nu} \right)$$

$$\boxed{\kappa T^{(DE)}_{\mu\nu} = (1 - F) G_{\mu\nu} + \frac{1}{2}(f(R) - R\ F) g_{\mu\nu} - (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) F}$$

$$\nabla^\mu T^{(DE)}_{\mu\nu} = 0$$

8) Perturbations and Geff: f(R) modifies Newton's constant!

$$f(R) \simeq f(R_0) + f'(R_0) R + \cdots \Longrightarrow$$

$$S = \frac{1}{8\pi G_N} \int d^4x \sqrt{-g} f(R) \simeq \frac{1}{8\pi G_N} \int d^4x \sqrt{-g} \left[f(R_0) + f'(R_0) R \right] \simeq \frac{1}{8\pi G_{eff}} \int d^4x \sqrt{-g} \left[R - 2\Lambda \right]$$

$$G_{eff} \sim G_N / f'(R_0)$$

More properly: perturb FRW, find Poisson equation:

$$ds^2 = a(\tau)^2 \left[-(1 + 2\Psi(\vec{x},\tau)) d\tau^2 + (1 - 2\Phi(\vec{x},\tau)) d\vec{x}^2 \right] \Longrightarrow$$

$$\begin{aligned} \Psi &= -4\pi G_N \frac{a^2}{k^2} \frac{G_{eff}}{G_N} \bar{\rho}_m \delta_m, \\ G_{eff}/G_N &= \frac{1}{F} \frac{1 + 4\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1 + 3\frac{k^2}{a^2}\frac{F_{,R}}{F}}, \end{aligned}$$

f(R) models

9) Conformal transformation (Jordan→Einstein frame): f(R) is just a scalar field!

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\,, \qquad \Rightarrow \qquad R = \Omega^2(\tilde{R} + 6\tilde{\Box}\omega - 6\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega) \qquad \Rightarrow$$

$$\omega \equiv \ln\,\Omega$$

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}F\Omega^{-2}(\tilde{R} + 6\tilde{\Box}\omega - 6\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega) - \Omega^{-4}U\right] + \int \mathrm{d}^4x\mathcal{L}_M(\Omega^{-2}\,\tilde{g}_{\mu\nu}, \Psi_M)$$

$$\Omega^2 = F \qquad U = \frac{FR - f}{2\kappa^2}$$

Redefine “field”:

$$\kappa\phi \equiv \sqrt{3/2}\;\ln\;F \qquad \Rightarrow$$

$$\boxed{S_E = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + \int \mathrm{d}^4x\mathcal{L}_M(F^{-1}(\phi)\tilde{g}_{\mu\nu}, \Psi_M)}$$

Potential

Non-minimal coupling

Quintessence!!!

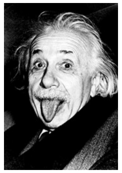
$$V(\phi) = \frac{U}{F^2} = \frac{FR - f}{2\kappa^2 F^2}$$

Ostrogradsky’s theorem on higher derivatives

1506.02210

Ostrogradsky’s theorem and higher order derivatives:

- i) GR has 2nd order derivatives.
- ii) Modified gravity theories in general have >2nd order!
- iii) Theories with more than 2nd order derivs are unstable (exceptions apply...)
- iv) These theories may also suffer from ghosts!
- v) Modifying GR is tough :(



Example:

$$L = L(x,\dot{x}) \Longrightarrow \frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = 0 \Longrightarrow \begin{aligned} H(X,P) &\equiv P\dot{x} - L\ , \\ &= PV(X,P) - L\Big(X, V(X,P)\Big) \end{aligned}$$

$$\ddot{x} = \mathcal{F}(x,\dot{x})$$

Quadratic in P→ bounded from below.

$$L(x,\dot{x},\ddot{x}) \Longrightarrow \frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{x}} = 0 \Longrightarrow H(X_1,X_2,P_1,P_2) \equiv \sum_{i=1}^{2} P_i x^{(i)} - L\ ,$$

$$\dddot{x} = \mathcal{F}(x,\dot{x},\ddot{x},\dddot{x})$$

$$X_1 \equiv x \quad P_1 \equiv \frac{\partial L}{\partial \dot{x}} - \frac{d}{dt}\frac{\partial L}{\partial \ddot{x}}$$

$$X_2 \equiv \dot{x} \quad P_2 \equiv \frac{\partial L}{\partial \ddot{x}}\ .$$

$$= P_1 X_2 + P_2 A(X_1,X_2,P_2) - L\Big(X_1, X_2, A(X_1,X_2,P_2)\Big)$$

Linear in P1→ unbounded from below!!!!!

Modified gravity and ghosts

Ghosts+propagators in MoG:

ArXiv:0911.3094

$$S = \int d^4x\sqrt{-g}f(R,P,Q)$$

$$P \equiv R_{ab}R^{ab}$$
$$Q \equiv R_{abcd}R^{abcd}$$

$$\Longrightarrow$$

$$F \equiv \frac{\partial f}{\partial R}, \quad f_P \equiv \frac{\partial f}{\partial P}, \quad f_Q \equiv \frac{\partial f}{\partial Q}$$

$$\begin{aligned} FG_{\mu\nu} \;=\; & \frac{1}{2}g_{\mu\nu}\left(f - R\ F\right) - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F \\ & -2\left(f_P R^a_\mu R_{a\nu} + f_Q\ R_{abc\mu}R^{abc}{}_\nu\right) \\ & -g_{\mu\nu}\nabla_a\nabla_b(f_P R^{ab}) - \Box(f_P R_{\mu\nu}) \\ & +2\nabla_a\nabla_b\left(f_P\ R^a{}_{(\mu}\delta^b{}_{\nu)} + 2f_Q\ R^a{}_{(\mu\nu)}{}^b\right) \end{aligned}$$

Fourth order derivatives… Problem!!!

Linearize and find propagator G(k):

$$g_{\mu\nu} \;=\; \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{2}\ \eta_{\mu\nu} + \eta_{\mu\nu}h_f$$

$$\Longrightarrow$$

$$\left(k^2 + \frac{k^4}{m^2_{spin2}}\right)\bar{h}_{\mu\nu} = 0$$

$$\Box h_f = m_s^2 h_f$$

$$m^2_{spin2} \equiv -\frac{F_0}{f_{P0}+4f_{Q0}}$$

$$m_s^2 \equiv \frac{1}{3}\frac{F_0}{F_{,R0}+\frac{2}{3}(f_{P0}+f_{Q0})}$$

$$\Longrightarrow$$

$$\boxed{G(k) \propto \frac{1}{k^2} - \frac{1}{k^2 + m^2_{spin2}}}$$

Negative sign…

Other MoG models

1) f(R,G) gravity, G is Gauss-Bonnet term:

arXiv:
0911.1811,
1309.1055

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

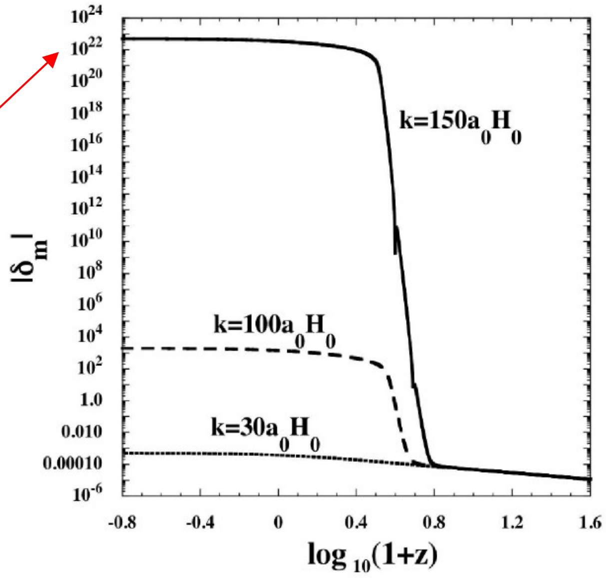
$$\begin{aligned}\mathcal{G} &= 24H^2\frac{\ddot{a}}{a}\\ &= 24H^2(H^2+\dot{H})\end{aligned}$$

$$S = \frac{1}{8\pi G_N}\int d^4x\sqrt{-g}\,(R/2 + f(\mathcal{G})) + S_m \quad\Longrightarrow\quad 3H^2 = \mathcal{G}f_{\mathcal{G}} - f - 24H^3\dot{f}_{\mathcal{G}} + 8\pi G_N\rho_m$$

Perturbations have catastrophic instabilities

$$\ddot{\delta}_m + C_1(k,a)\dot{\delta}_m + C_2(k,a)\delta_m \approx 0,$$

Blows up at late times
on reasonable scales
k~150H0~0.05/Mpc



Other MoG models

1) f(T) gravity, T is the torsion scalar (tetrad formalism):

1803.09278

$$T^{\lambda}_{\ \mu\nu} = \overset{w\lambda}{\Gamma}_{\nu\mu} - \overset{w\lambda}{\Gamma}_{\mu\nu} = e^{\lambda}_{A}\left(\partial_\mu e^{A}_{\nu} - \partial_\nu e^{A}_{\mu}\right),$$

$$g_{\mu\nu}(x) = \eta_{AB}\, e^{A}_{\mu}(x)\, e^{B}_{\nu}(x),$$

$$\Rightarrow \quad T \equiv \frac{1}{4}T^{\rho\mu\nu}T_{\rho\mu\nu} + \frac{1}{2}T^{\rho\mu\nu}T_{\nu\mu\rho} - T_{\rho\mu}{}^{\rho}T^{\nu\mu}{}_{\nu},$$

2) Lagrangian ~T is identical to GR! Use f(T):

$$I = \frac{1}{16\pi G}\int d^4x e\left[T + f(T) + L_m + L_r\right] \quad \Rightarrow$$

Energy momentum tensor of matter

$$e^{-1}\partial_\mu(e e^{\rho}_{A} S_{\rho}{}^{\mu\nu})[1+f_T] + e^{\rho}_{A} S_{\rho}{}^{\mu\nu}\partial_\mu(T) f_{TT} - [1+f_T] e^{\lambda}_{A} T^{\rho}{}_{\mu\lambda} S_{\rho}{}^{\nu\mu} + \frac{1}{4} e^{\nu}_{A}[T+f(T)] = 4\pi G e^{\rho}_{A} \overset{m}{T}_{\rho}{}^{\nu}$$

$$T = -6H^2 \quad \Rightarrow$$

$$\boxed{\begin{aligned} H^2 &= \frac{8\pi G}{3}(\rho_m + \rho_r) - \frac{f}{6} + \frac{T f_T}{3} \\ \dot{H} &= -\frac{4\pi G(\rho_m + P_m + \rho_r + P_r)}{1 + f_T + 2T f_{TT}}, \end{aligned}}$$

Can give acceleration!

Other MoG models

3) DE equation of state and specific models:

$$w \equiv \frac{P_{DE}}{\rho_{DE}} = -\frac{f/T - f_T + 2T f_{TT}}{[1 + f_T + 2T f_{TT}][f/T - 2f_T]}$$

Can give crossing of w=-1!

$$f(T) = \alpha(-T)^b$$

$$f(T) = \alpha T_0(1 - e^{-p\sqrt{T/T_0}})$$

$$f(T) = \alpha T_0(1 - e^{-pT/T_0})$$

4) Perturbations:

$$ds^2 = a(\tau)^2\left[-(1+2\Psi(\vec{x},\tau))d\tau^2 + (1-2\Phi(\vec{x},\tau))d\vec{x}^2\right]$$

$$\Rightarrow \boxed{\frac{G_{\rm eff}(a)}{G_N} = \frac{1}{1+f_T}}$$

Very close to 1 and no dependence on k!

Other MoG models

1) f(R,φ,X)

0705.1032

i) Generalization of non-minimally coupled scalar field

ii) Contains f(R), scalar-tensor, quintessence, K-essence

iii) Still viable after GWs

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{1}{2} f(R, \phi, X) + \mathcal{L}_m \right]$$

$$X = -\phi^{;c}\phi_{,c}/2$$

$\Rightarrow$

$$FG_{\mu\nu} = \frac{1}{2}(f - RF)g_{\mu\nu} + F_{,\mu;\nu} - \Box F g_{\mu\nu} + \frac{1}{2} f_{,X}\phi_{,\mu}\phi_{,\nu} + T^{(m)}_{\mu\nu} \,,$$

$$(f_{,X}\phi^{;c})_{;c} + f_{,\phi} = 0 \,,$$

2) Background equations give rich phenomenology

$$w_{\rm DE} = -1 + \frac{2f_{,X}X + 2\ddot{F} - 4H\dot{F} - 4\dot{H}(F_0 - F)}{2f_{,X}X + FR - f - 6H\dot{F} + 6H^2(F_0 - F)}$$

3) Perturbations:

Anisotropic stress, could be detected by Weak Lensing

$$G_{\rm eff} \simeq \frac{1}{8\pi F} \frac{f_{,X} + 4\left(f_{,X}\frac{k^2}{a^2}\frac{F_{,R}}{F} + \frac{F_{,\phi}^2}{F}\right)}{f_{,X} + 3\left(f_{,X}\frac{k^2}{a^2}\frac{F_{,R}}{F} + \frac{F_{,\phi}^2}{F}\right)}$$

$$\eta \equiv \frac{\Phi - \Psi}{\Psi} = \frac{2f_{,X}\frac{k^2}{a^2}\frac{F_{,R}}{F} + \frac{2F_{,\phi}^2}{F}}{f_{,X}\left(1 + \frac{2k^2}{a^2}\frac{F_{,R}}{F}\right) + \frac{2F_{,\phi}^2}{F}}$$

Other MoG models

1) Horndesky theory

i) Most general case of non-minimally coupled scalar field
ii) Has shift symmetry φ→ φ+c
iii) Contains f(R,φ,X) plus more!
iv) 2nd order equations only by construction → no instabilities!
v) Terms beyond f(R,φ,X) excluded after GWs (see GW lecture)
vi) Not motivated from High Energy Physics… :-(

$$S[g_{\mu\nu},\phi] = \int d^4x\sqrt{-g}\left[\sum_{i=2}^{4}\frac{1}{8\pi G_N}\mathcal{L}_i\left[g_{\mu\nu},\phi\right] + \mathcal{L}_m\left[g_{\mu\nu},\psi_M\right]\right]$$

These terms are put by hand to cancel the higher order derivatives

$$\begin{aligned}
\mathcal{L}_2 &= G_2\left(\phi,X\right) \\
\mathcal{L}_3 &= -G_3\left(\phi,X\right)\Box\phi \\
\mathcal{L}_4 &= G_4\left(\phi,X\right)R + G_{4X}\left(\phi,X\right)\left[\left(\Box\phi\right)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right] \\
\mathcal{L}_5 &= G_5\left(\phi,X\right)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}\left(\phi,X\right)\left[\left(\Box\phi\right)^3 + 2\phi_{;\mu}^{\nu}\phi_{;\nu}^{\alpha}\phi_{;\alpha}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]
\end{aligned}$$

Other MoG models

2) Horndesky sub-cases:

i) f(R): $G_2 = -\frac{M_{pl}^2}{2}\left(Rf_{,R} - f\right), \quad G_3 = G_5 = 0, \quad G_4 = \frac{1}{2}M_{pl}\phi, \quad \phi = M_{pl} f_{,R}$

ii) Brans-Dicke: $G_2 = \frac{M_{pl}\omega_{BD}X}{\phi} - V(\phi), \quad G_3 = G_5 = 0, \quad G_4 = \frac{1}{2}M_{pl}\phi$

iii) Covariant Galileon: $G_2 = -c_2 X, \quad G_3 = \frac{c_3}{M^3}X, \quad G_4 = \frac{1}{2}M_{pl}^2 - \frac{c_4}{M^6}X^2, \quad G_5 = \frac{3c_5}{M^9}X^2$

iv) Kinetic Braiding: $G_2 = G_2(X), \quad G_3 = G_3(X), \quad G_4 = \frac{1}{2}M_{pl}^2, \quad G_5 = 0$

Other MoG models

1) Models with extra dimensions: Kaluza-Klein

i) Assume extra dimension y, which is compactified with cylindrical boundary conditions. Then 5D metric g_{MN} satisfies

$$f(x,y) = f(x, y + 2\pi r) \Longrightarrow \frac{\partial g_{MN}}{\partial y} = 0$$

Similar to U(1) symmetry!

ii) Expand 5D metric in Fourier modes:

$$g_{MN}(x,y) = \sum_n g_{MN}^{(n)}(x) e^{iny/r} \Longrightarrow g_{MN}^{(0)} = \phi^{-1/3} \begin{pmatrix} g_{\mu\nu} + \phi A_\mu A\nu & \phi A_\mu \\ \phi A_\nu & \phi \end{pmatrix}$$

Very general decomposition)

Other MoG models

iii) GR in 5D:

4D GR+Maxwell+scalar field!

$$S = \frac{1}{16\pi G_N^5}\int d^4x dy \sqrt{-g^{(5)}} R^{(5)}$$

$$= \frac{1}{16\pi G_N^4}\int d^4x \sqrt{-g^{(4)}} \left(R + \frac{1}{4}\phi F_{\mu\nu}F^{\mu\nu} + \frac{1}{6\phi^2}\partial^\mu\phi\partial_\mu\phi \right)$$

$$G_N^{(4)} = \frac{G_N^{(5)}}{2\pi r}$$

iv) Add extra scalar field:

$$S_\Phi = \int d^4x dy \sqrt{-g^{(5)}} \left(g_{MN}^{(0)} \partial_M \Phi \partial_N \Phi \right)$$

$$= (2\pi r) \sum_n \int d^4x \sqrt{-g^{(4)}} \left[g^{\mu\nu} \left(\partial_\mu + \frac{in}{r} A_\mu \right) \Phi_n \left(\partial_\nu + \frac{in}{r} A_\nu \right) \Phi_n - \frac{n^2}{\phi r^2} \Phi_n^2 \right]$$

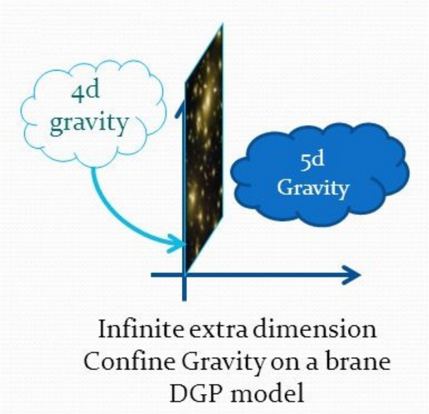
⇨ $Q_n = \frac{8\pi G_N^{(4)} n}{r}\sqrt{\frac{2}{\phi}}$ $\qquad M_n = \frac{|n|}{r\sqrt{\phi}}$

Qn~Mn… Problem!!!

Other MoG models

2) Models with extra dimensions: DGP

hep-th/0005016



$$\Rightarrow \quad S = M^3 \int d^5X \sqrt{G}\, \mathcal{R}_{(5)} + M_P^2 \int d^4x \sqrt{|g|}\, R$$

$$g_{\mu\nu}(x) \equiv G_{\mu\nu}(x,\ y=0)$$

Brane part and 4D part

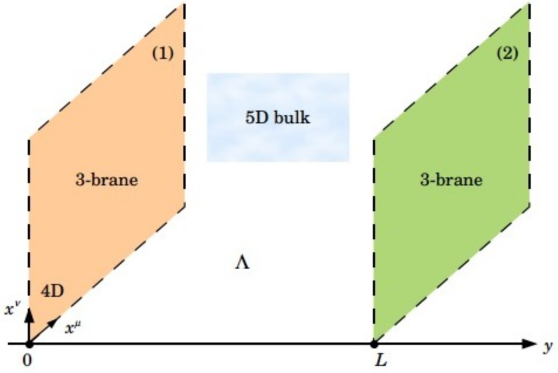
$$\Rightarrow \quad \boxed{V(r) \simeq -\frac{1}{8\pi^2 M_P^2}\frac{1}{r}\left\{\frac{r_0}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)\right\}}$$

Gravity is weaker at r>>r0

Other MoG models

3) Models with extra dimensions: Randal-Sundrum (2 branes!)

hep-ph/9905221



$$ds^2 = e^{-2kr_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2$$

$$0 \leq \phi \leq \pi$$

$$g^{vis}_{\mu\nu}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \phi = \pi)$$

$$g^{hid}_{\mu\nu}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \phi = 0)$$

1) Can solve hierarchy problem
2) Affects dynamics at large distances

$$\begin{aligned} S &= S_{gravity} + S_{vis} + S_{hid} \\ S_{gravity} &= \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-G}\{-\Lambda + 2M^3 R\} \\ S_{vis} &= \int d^4x \sqrt{-g_{vis}}\{\mathcal{L}_{vis} - V_{vis}\} \\ S_{hid} &= \int d^4x \sqrt{-g_{hid}}\{\mathcal{L}_{hid} - V_{hid}\} \end{aligned}$$

Visible (us) and invisible branes

Main points of the lecture

- Why we need Dark Energy (history+observations)
- Lemaitre-Tolman-Bondi (LTB) void models
- Scalar field and ideal fluid models
- Modified gravity and extra dimensions
- Effective fluid approach
- Conclusions

Effective fluid approach

1811.02469

1) Re-write MoG theory as GR and an effective DE fluid. Eg for f(R):

$$F G_{\mu\nu} - \frac{1}{2}(f(R) - R\,F) g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu)\,F = \kappa T^{(m)}_{\mu\nu} \quad \Longrightarrow$$

$$G_{\mu\nu} = \kappa \left(T^{(m)}_{\mu\nu} + T^{(DE)}_{\mu\nu} \right) \quad \Longrightarrow \quad \begin{aligned} \kappa T^{(DE)}_{\mu\nu} &= (1-F) G_{\mu\nu} + \frac{1}{2}(f(R) - R\,F) g_{\mu\nu} \\ &- (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu)\,F. \end{aligned}$$

1) Makes it easier to include in codes
2) Gives connection with lab physics

$$\nabla^\mu T^{(DE)}_{\mu\nu} = 0.$$

2) Effective pressure/density and Friedman equations

Effective pressure and density!

$$\begin{aligned} T^0_0 &= -(\bar\rho + \delta\rho), \\ T^0_i &= (\bar\rho + \bar P) u_i, \\ T^i_j &= (\bar P + \delta P)\delta^i_j + \Sigma^i_j \end{aligned}$$

$$\begin{aligned} \kappa \bar P_{DE} &= \frac{f}{2} - \mathcal{H}^2/a^2 - 2F\mathcal{H}^2/a^2 + \mathcal{H}\dot F/a^2 \\ &- 2\dot{\mathcal{H}}/a^2 - F\dot{\mathcal{H}}/a^2 + \ddot F/a^2, \\ \kappa \bar\rho_{DE} &= -\frac{f}{2} + 3\mathcal{H}^2/a^2 - 3\mathcal{H}\dot F/a^2 + 3F\dot{\mathcal{H}}/a^2 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} \mathcal{H}^2 &= \frac{\kappa}{3} a^2 \left(\bar\rho_m + \bar\rho_{DE}\right), \\ \dot{\mathcal{H}} &= -\frac{\kappa}{6} a^2 \left((\bar\rho_m + 3\bar P_m) + (\bar\rho_{DE} + 3\bar P_{DE}) \right) \end{aligned}$$

Effective fluid approach

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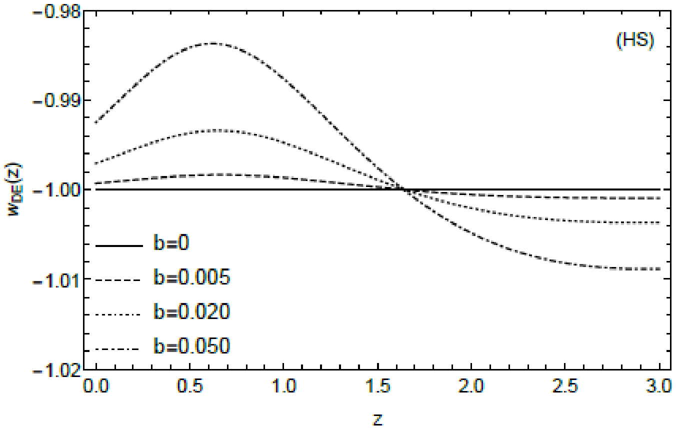
3) The equation of state w(z)

$$w_{DE} = \frac{-a^2 f + 2\left((1+2F)\mathcal{H}^2 - \mathcal{H}\dot{F} + (2+F)\dot{\mathcal{H}} - \ddot{F}\right)}{a^2 f - 6(\mathcal{H}^2 - \mathcal{H}\dot{F} + F\dot{\mathcal{H}})}$$



4) The effective fluid perturbations

$$\begin{aligned}\delta' &= 3(1+w)\Phi' - \frac{V}{a^2H} - \frac{3}{a}\left(\frac{\delta P}{\bar{\rho}} - w\delta\right),\\ V' &= -(1-3w)\frac{V}{a} + \frac{k^2}{a^2H}\frac{\delta P}{\bar{\rho}} + (1+w)\frac{k^2}{a^2H}\Psi\\ &- \frac{2}{3}\frac{k^2}{a^2H}\pi,\end{aligned}$$

$$c_s^2 \equiv \frac{\delta P}{\delta\rho}$$

$$\begin{aligned}\ddot{\delta} &+ (\cdots)\dot{\delta} + (\cdots)\delta =\\ &- k^2\left((1+w)\Psi + c_s^2\delta - (1+w)\sigma\right) + \cdots\\ &= -k^2\left((1+w)\Psi + c_s^2\delta - \frac{2}{3}\pi\right) + \cdots,\end{aligned}$$

See Advanced Cosmo class next semester!

Has to be positive!

$$c_{s,eff}^2 = c_s^2 - \frac{2}{3}\pi/\delta$$

Effective fluid approach

1811.02469

5) The effective pressure, density and velocity perturbations

$$\frac{\delta P_{DE}}{\bar{\rho}_{DE}} \simeq \frac{1}{3F}\frac{2\frac{k^2}{a^2}\frac{F_{,R}}{F}+3(1+5\frac{k^2}{a^2}\frac{F_{,R}}{F})\ddot{F}k^{-2}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$\delta_{DE} \simeq \frac{1}{F}\frac{1-F+\frac{k^2}{a^2}(2-3F)\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

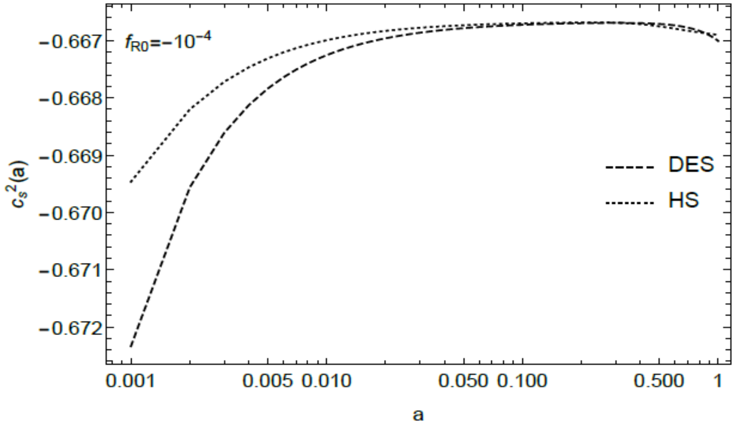
$$\begin{aligned} V_{DE} &\equiv (1+w_{DE})\theta_{DE} \\ &\simeq \frac{\dot{F}}{2F}\frac{1+6\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m \end{aligned}$$

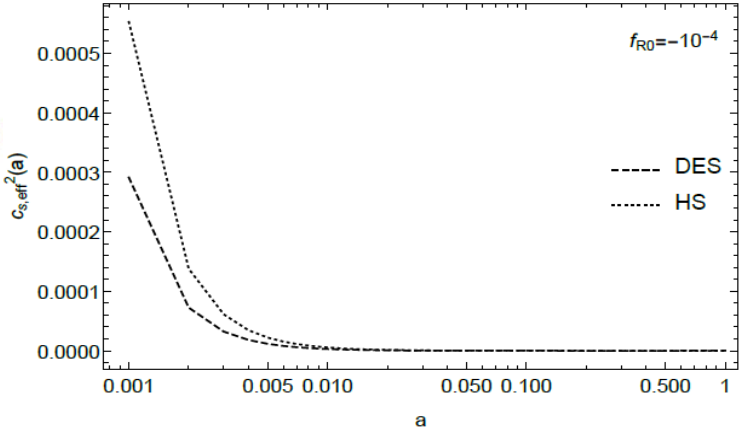
$$\begin{aligned} \pi_{DE} &= \frac{\frac{k^2}{a^2}(\Phi-\Psi)}{\kappa\,\bar{\rho}_{DE}} \\ &\simeq \frac{1}{F}\frac{\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m \end{aligned}$$

$$c_s^2 \equiv \frac{\delta P}{\delta \rho}$$

$$c^2_{s,eff} = c_s^2 - \frac{2}{3}\pi/\delta$$

$$\begin{aligned} T^0_0 &= -(\bar{\rho}+\delta\rho), \\ T^0_i &= (\bar{\rho}+\bar{P})u_i, \\ T^i_j &= (\bar{P}+\delta P)\delta^i_j+\Sigma^i_j \end{aligned}$$





Effective fluid approach

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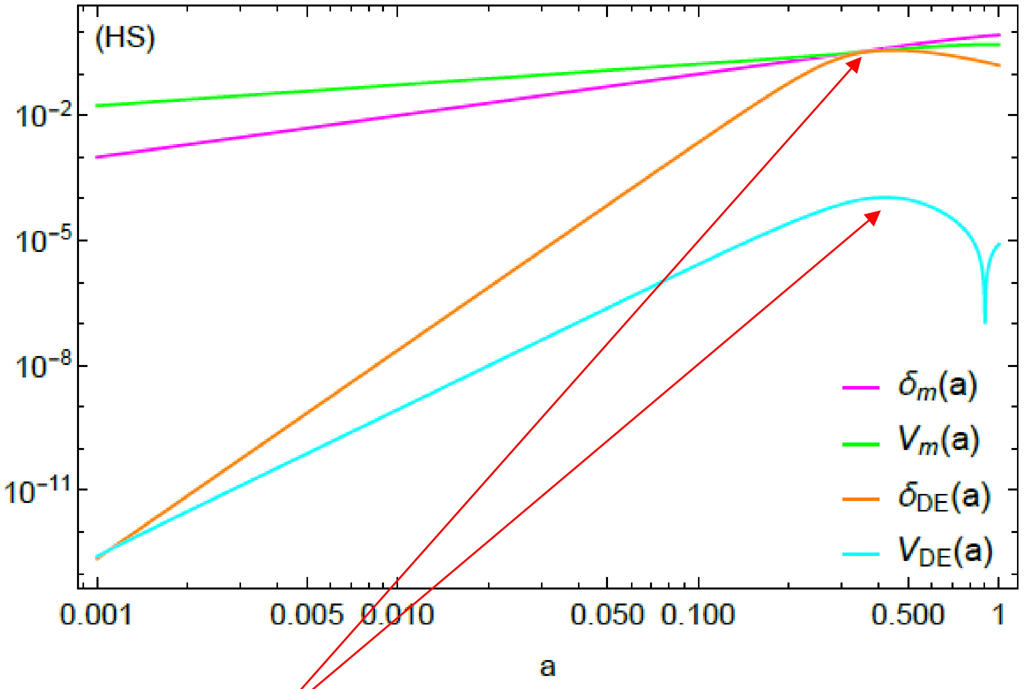
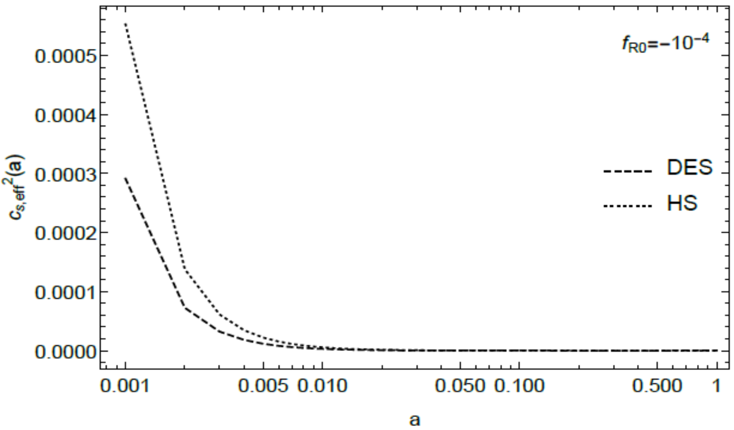
6) Solution of the perturbation equations is now very simple!

$$\delta' = 3(1+w)\Phi' - \frac{V}{a^2 H} - \frac{3}{a}\left(\frac{\delta P}{\bar{\rho}} - w\delta\right),$$

$$V' = -(1-3w)\frac{V}{a} + \frac{k^2}{a^2 H}\frac{\delta P}{\bar{\rho}} + (1+w)\frac{k^2}{a^2 H}\Psi - \frac{2}{3}\frac{k^2}{a^2 H}\pi,$$





DE perturbations grow and then reach a plateau
In agreement with ceff

Summary

1) Dark energy is needed to explain accelerated expansion of the Universe

2) DE model zoo: Scalar fields, Modified Gravity, Extra dimensions

3) f(R) is simplest modification of GR that works (sort of)...

4) Effective fluid approach simplifies things a lot: write MoG as GR+DE!

5) Conclusions: Lots of work to do, many models to study!