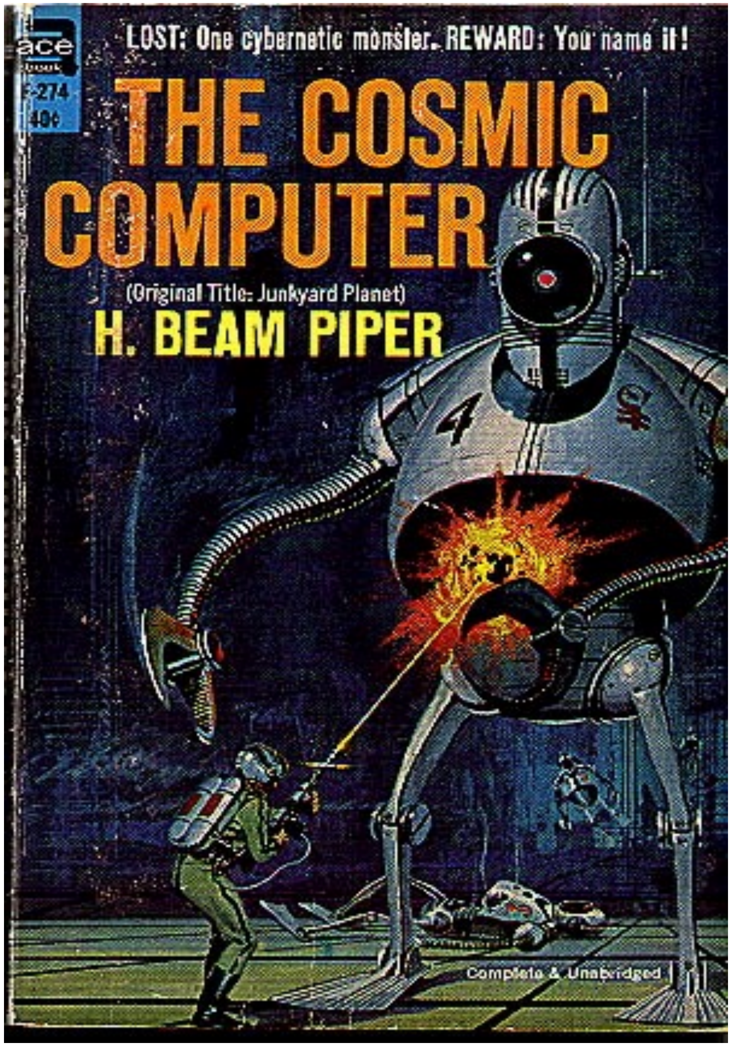
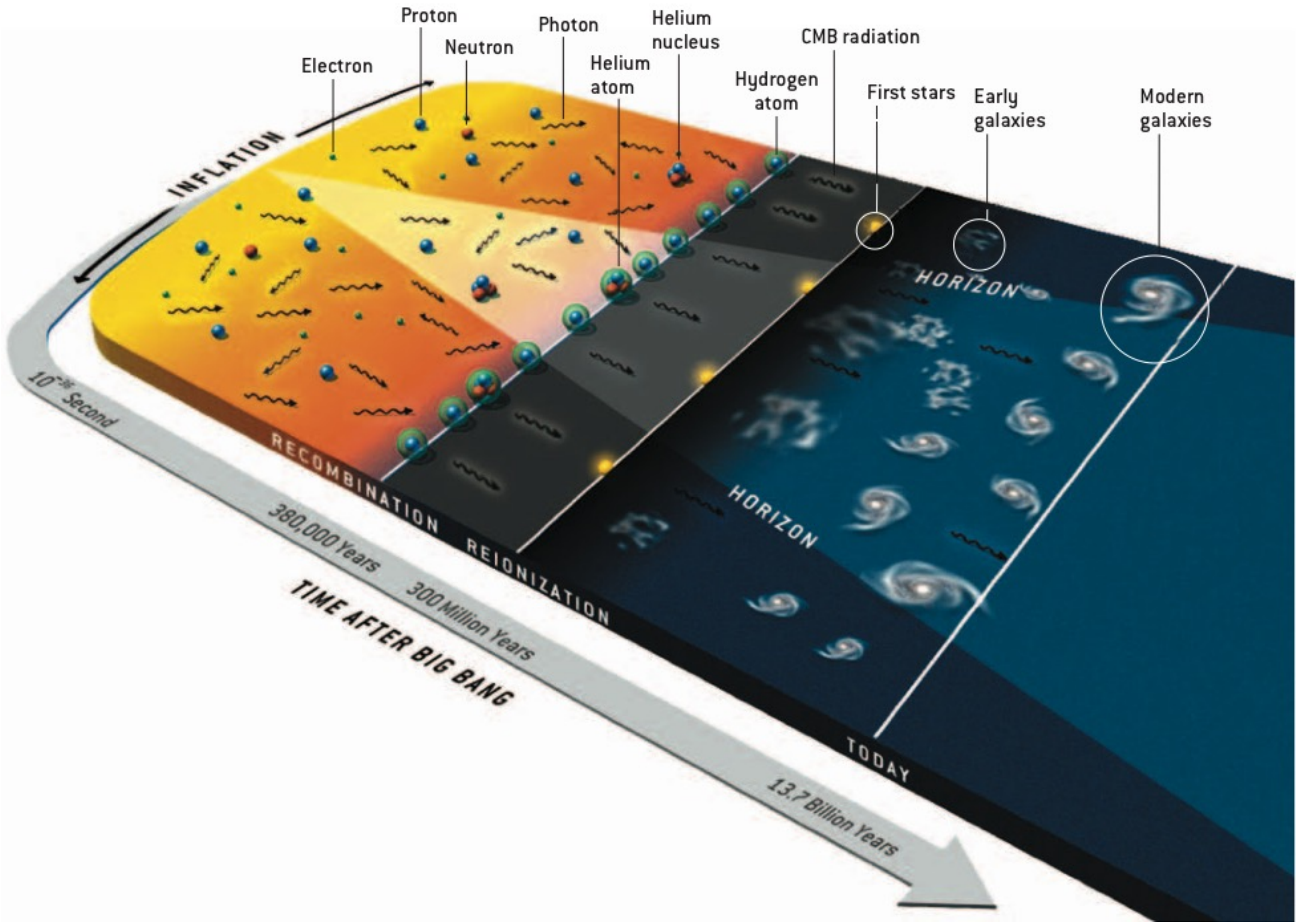
Alexander Knebe (*Universidad Autonoma de Madrid*)

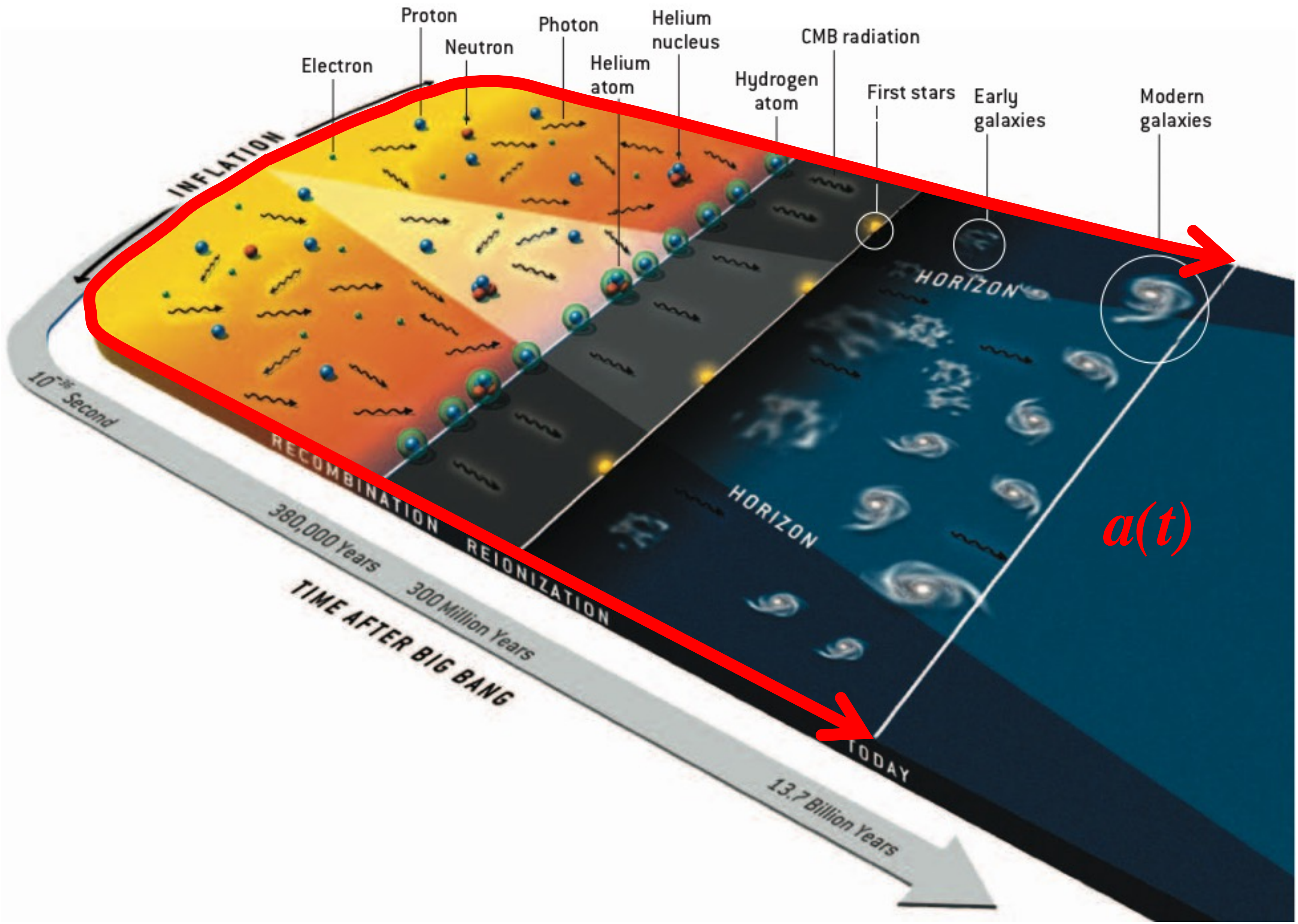


- introduction
- Boltzmann solver
- initial conditions generators
- simulation codes

- **introduction**
- Boltzmann solver
- initial conditions generators
- simulation codes



Proton
Neutron
Electron
Photon
Helium atom
Helium nucleus
Hydrogen atom
CMB radiation
First stars
Early galaxies
Modern galaxies
INFLATION
HORIZON
HORIZON
RECOMBINATION
REIONIZATION
TODAY
10^-36 Second
380,000 Years
300 Million Years
13.7 Billion Years
TIME AFTER BIG BANG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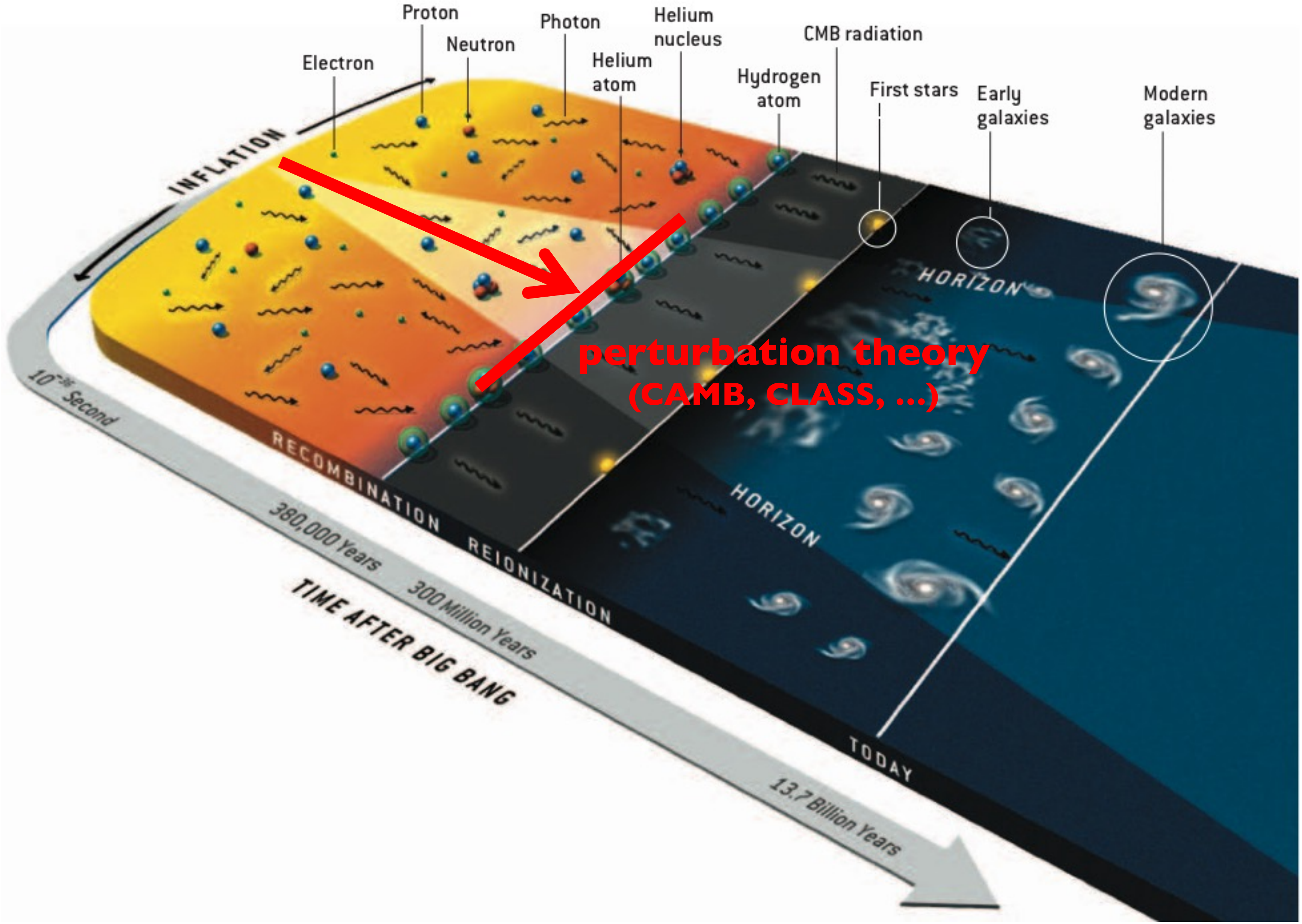
a(t)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INFLATION
HORIZON
HORIZON
perturbation theory
(CAMB, CLASS, ...)
RECOMBINATION
REIONIZATION
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10^{-36} Second
380,000 Years
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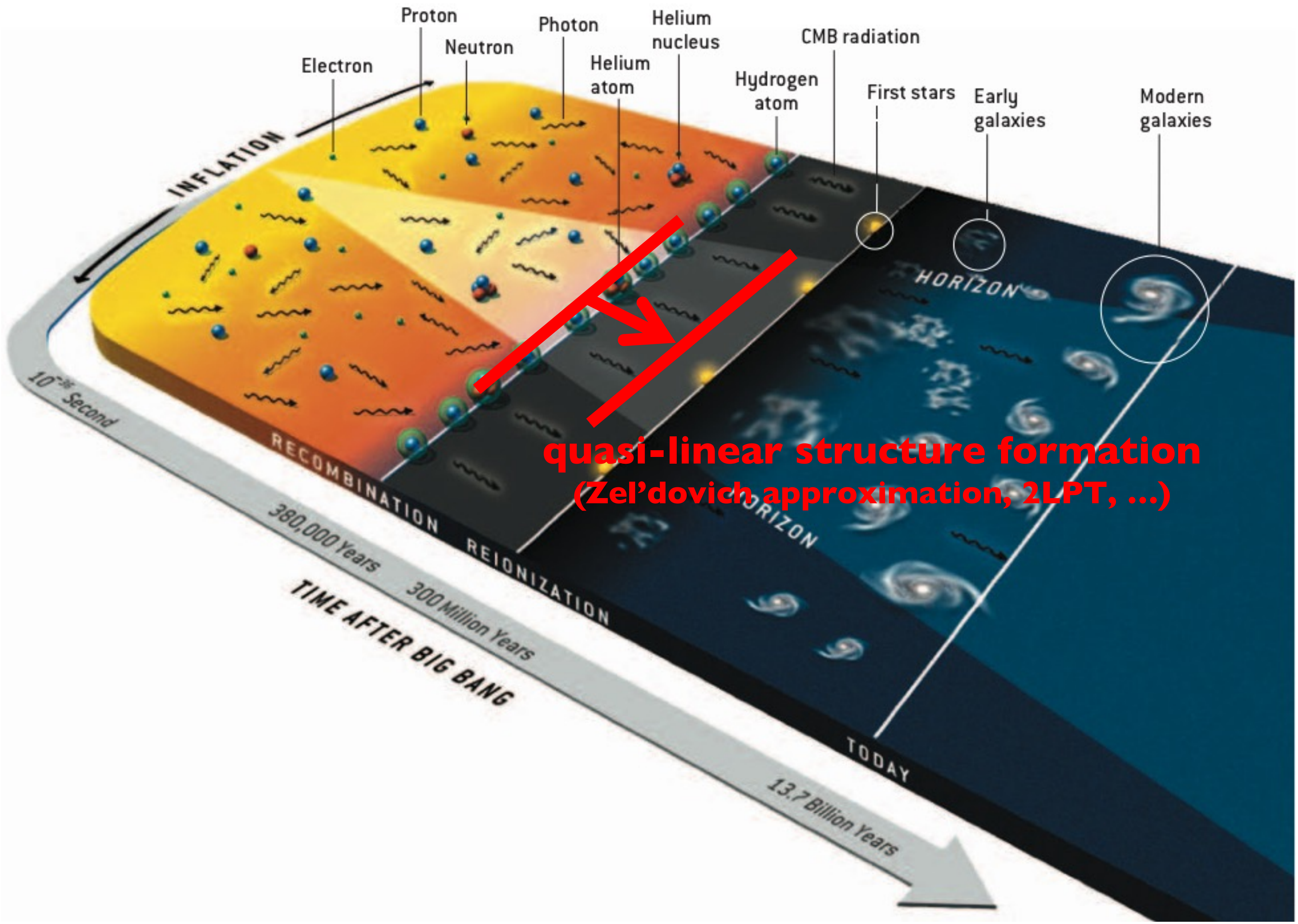
TIME AFTER BIG BANG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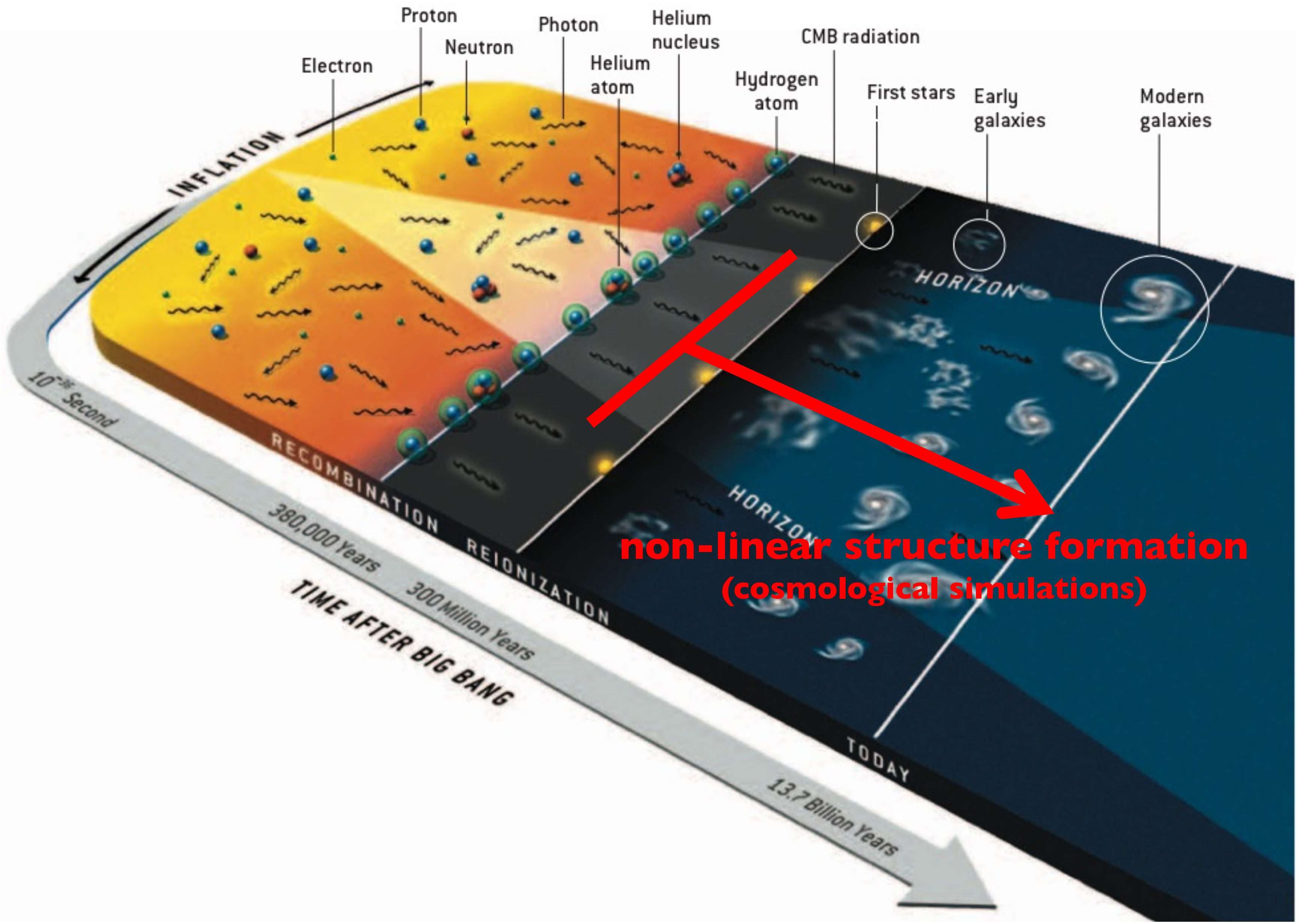
380,000 Years
300 Million Years
TIME AFTER BIG BANG
TODAY
13.7 Billion Years
quasi-linear structure formation
(Zel'dovich approximation, 2LPT, ...)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10^-36 Second
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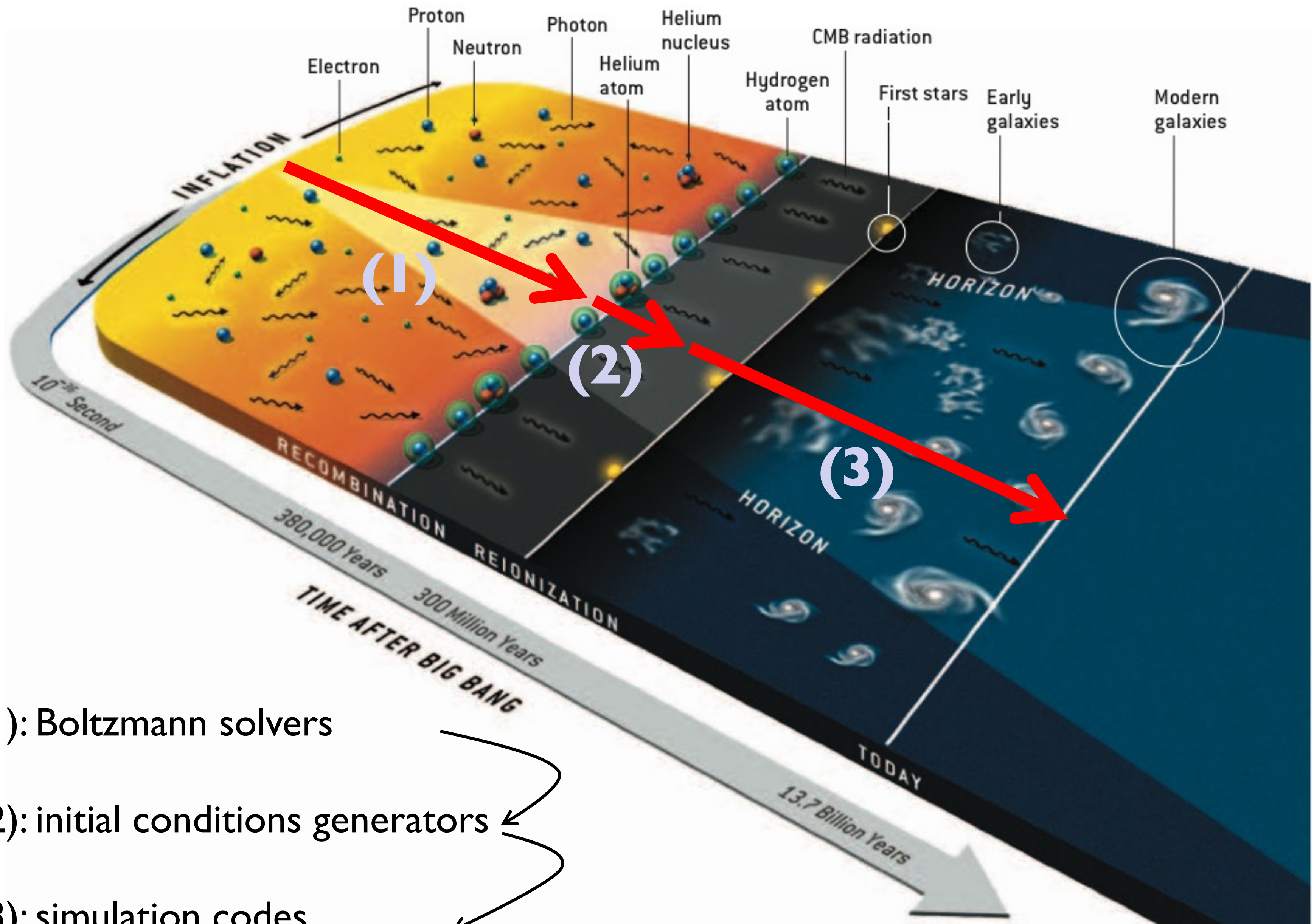
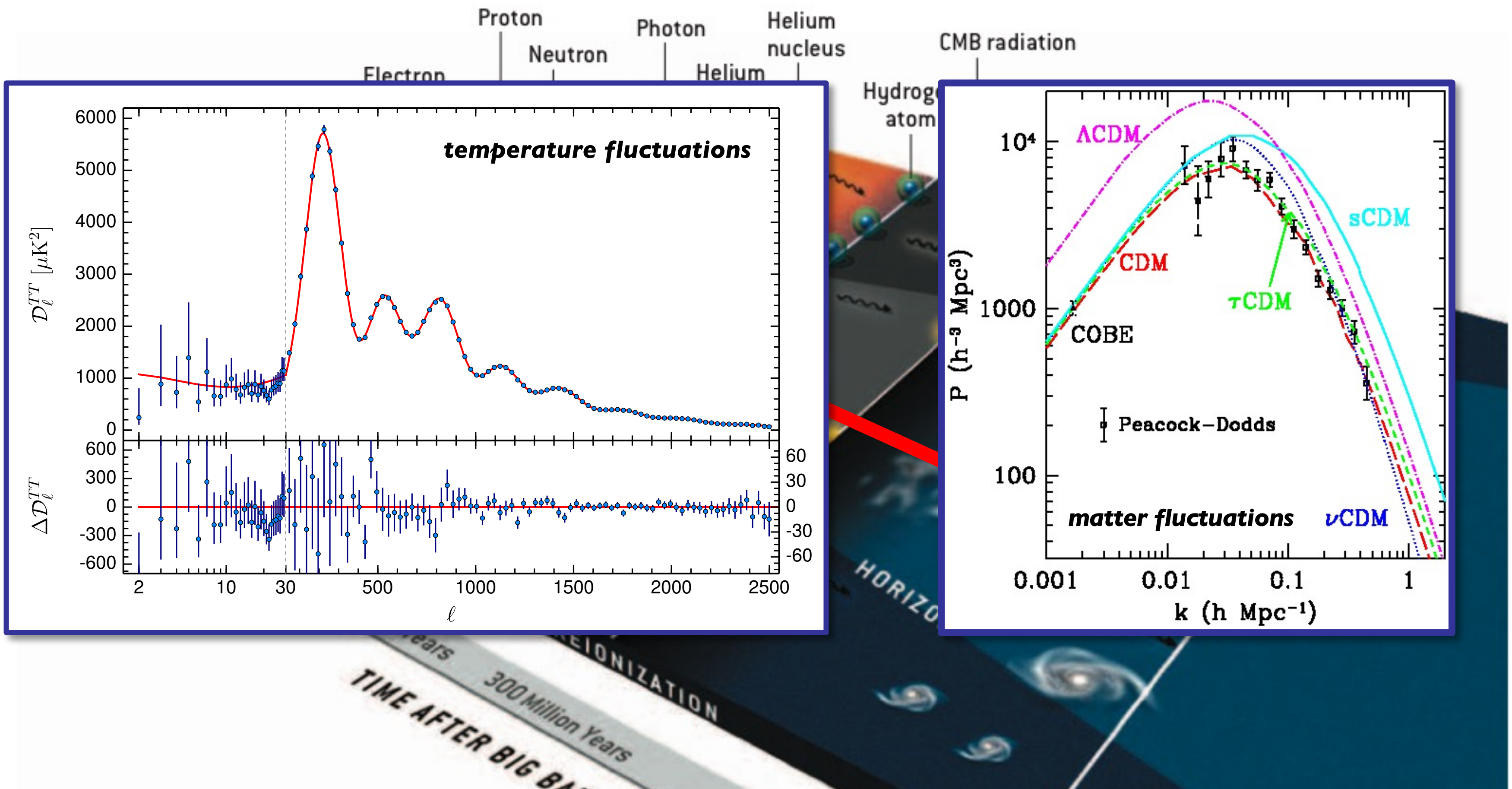
TIME AFTER BIG BANG
non-linear structure formation
(cosmological simulations)



(1): Boltzmann solvers

(2): initial conditions generators

(3): simulation codes



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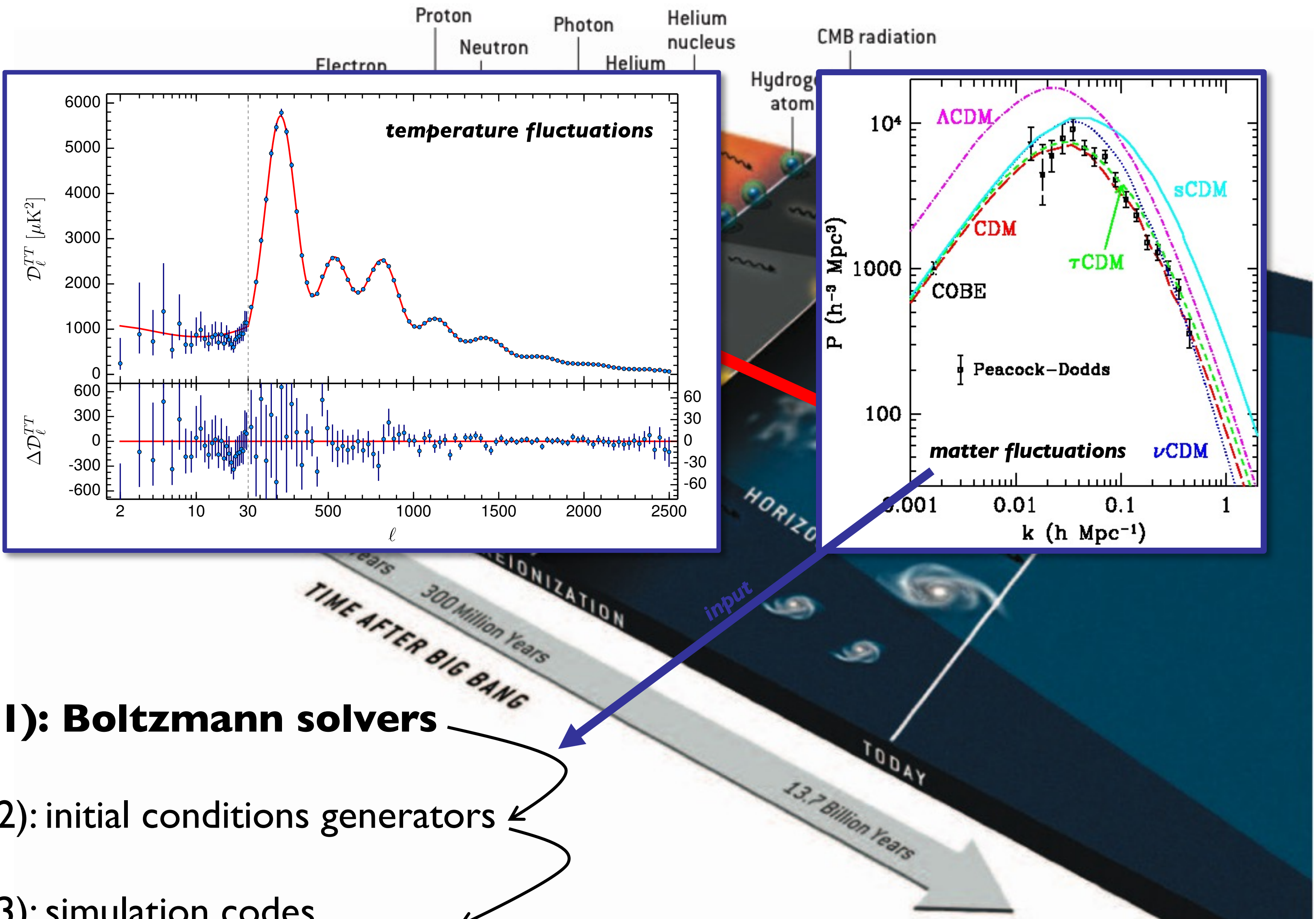

(1): Boltzmann solvers

(2): initial conditions generators

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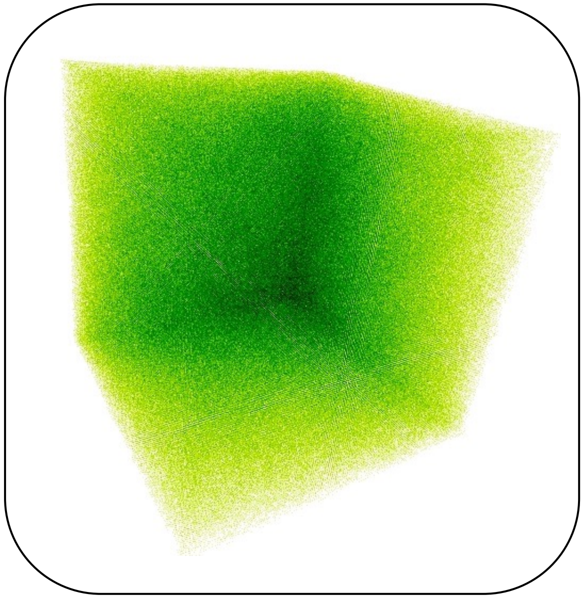
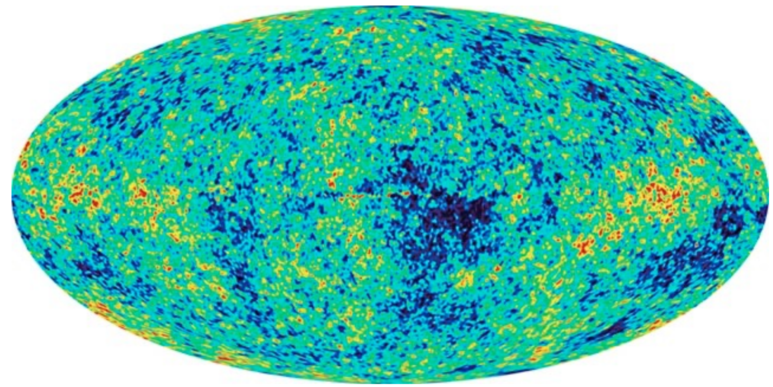
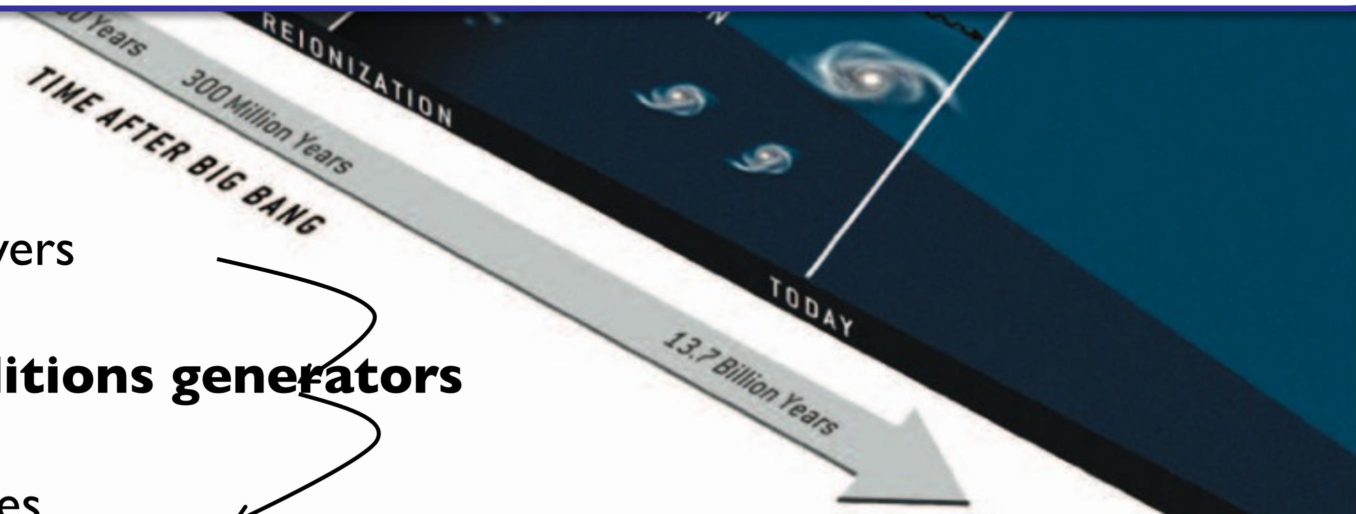


initial conditions generator



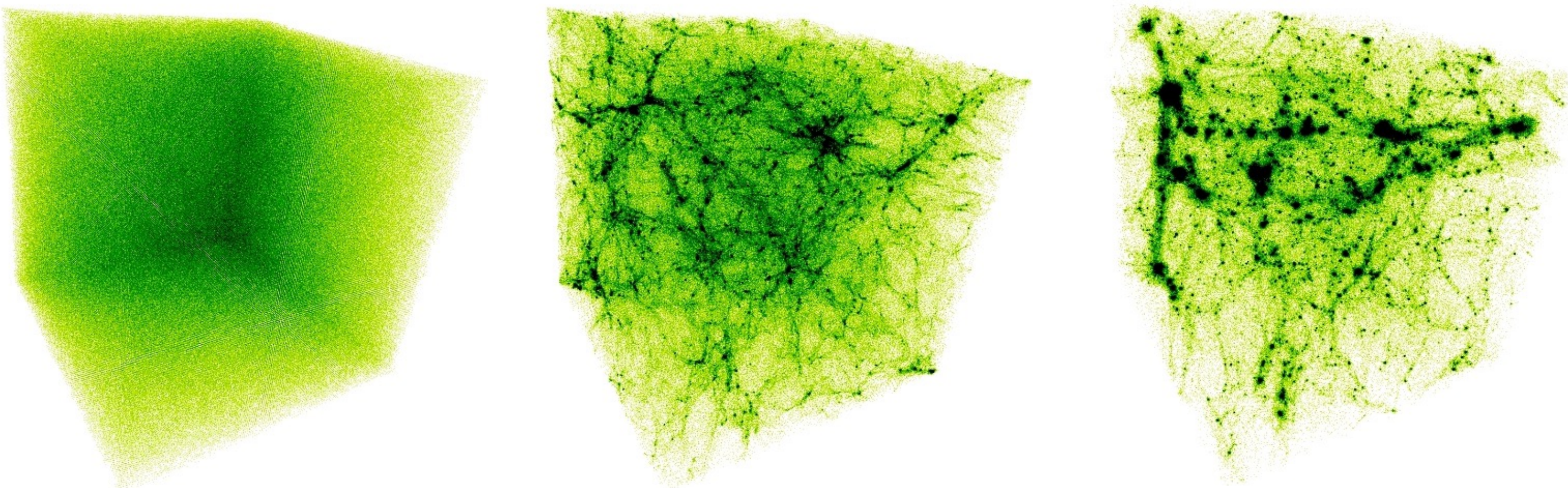
anisotropies in the
cosmic microwave background



(1): Boltzmann solvers

(2): initial conditions generators

(3): simulation codes

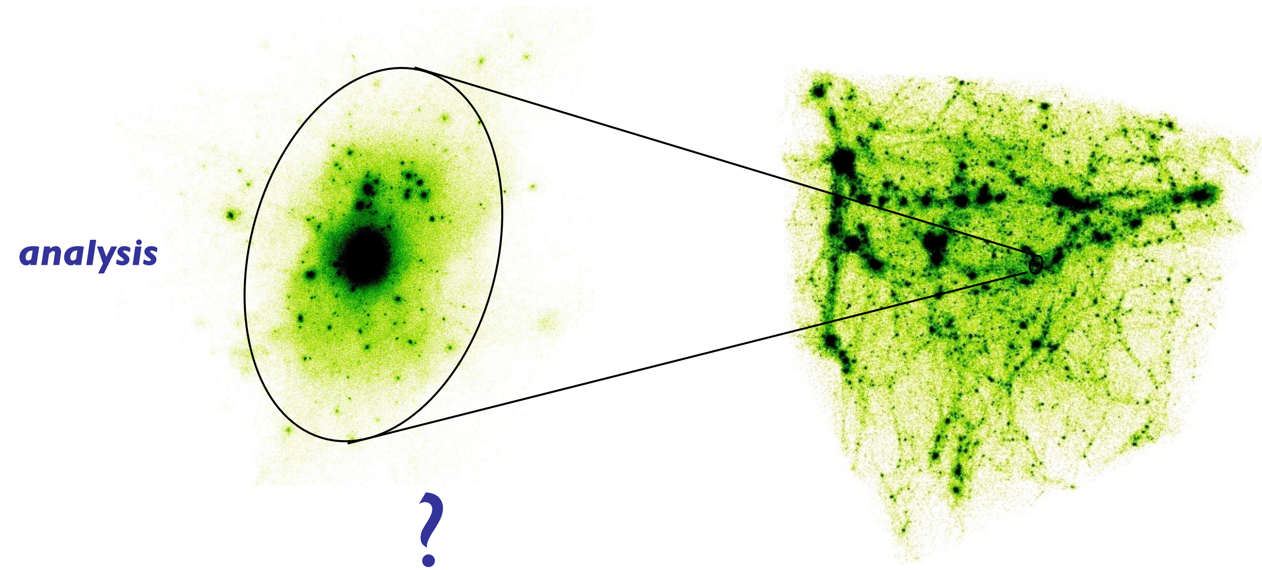


simulation code →

(1): Boltzmann solvers

(2): initial conditions generators

(3): simulation codes



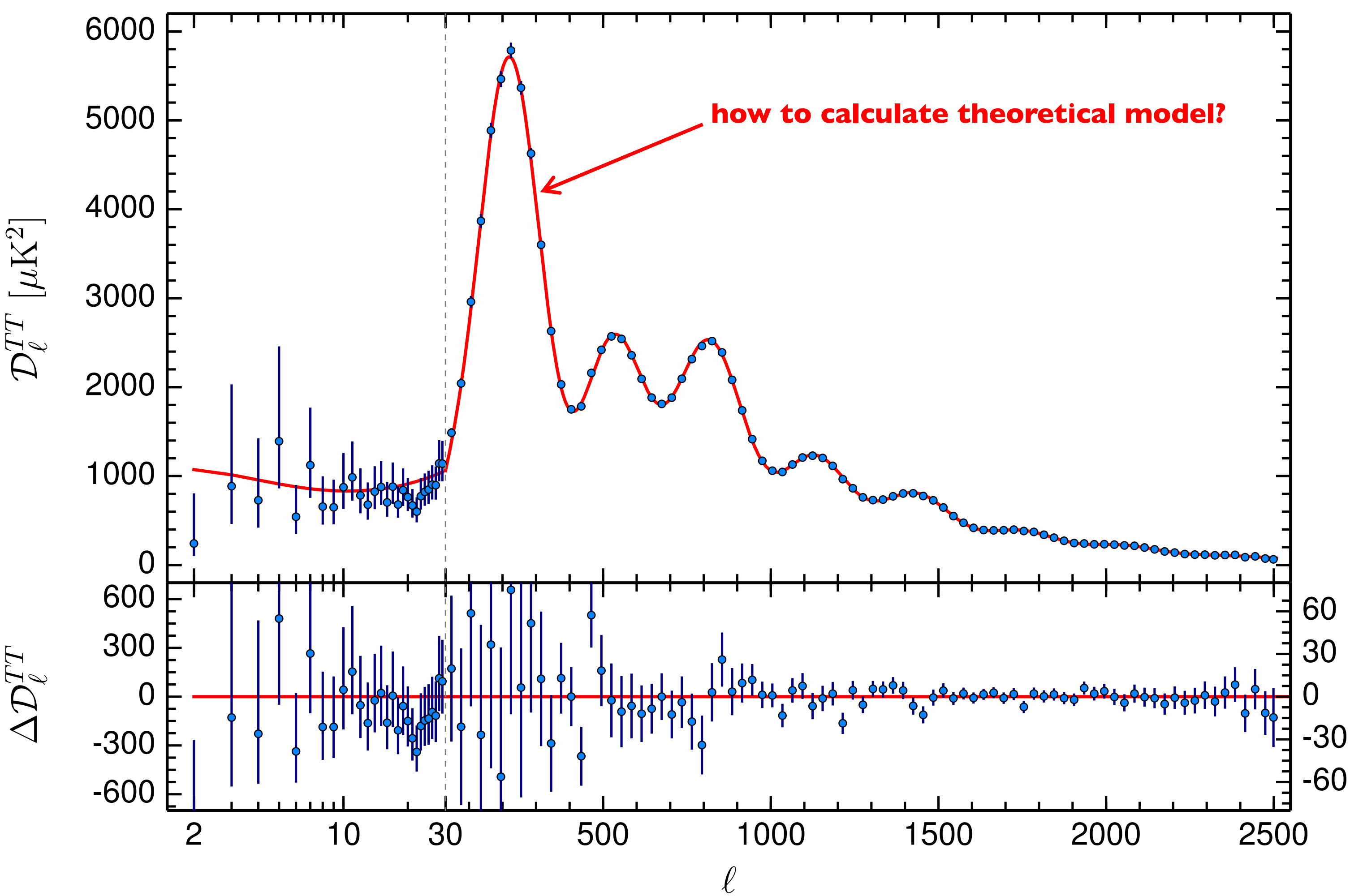
(1): Boltzmann solvers

(2): initial conditions generators

(3): simulation codes & analysis tools

- introduction
- **Boltzmann solver**
- initial conditions generators
- simulation codes

Planck 2015 data vs. Boltzmann solver results for ΛCDM



Ade et al. (arXiv:1502.01598)

- particle species...

number density

$$n = \frac{g}{(2\pi\hbar)^3}\int f(\vec{p})4\pi p^2\,dp$$

energy density

$$\rho c^2 = \frac{g}{(2\pi\hbar)^3}\int E(\vec{p})\,f(\vec{p})4\pi p^2\,dp$$

pressure

$$P = \frac{g}{(2\pi\hbar)^3}\int \frac{|\vec{p}|^2}{3E}\,f(\vec{p})4\pi p^2\,dp \qquad E^2 = |\vec{p}c|^2 + m^2c^4$$

$f(\vec{p})$: phase space distribution function

g : statistical weight

- ...in kinetic equilibrium

relativistic:

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/k_BT} \pm 1}$$

non-relativistic:

$$f(\vec{p}) \approx e^{-\left(p^2/2m-\mu\right)/k_BT}$$

(cf. Thermal History lecture)

- particle species...

number density
$$n = \frac{g}{(2\pi\hbar)^3} \int f(\vec{p}) 4\pi p^2\, dp$$

energy density
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$f(\vec{p})$: phase space distribution function

g : statistical weight

but how is *f* evolving in time?

- ...in kinetic equilibrium

relativistic:
$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/k_B T} \pm 1}$$

non-relativistic:
$$f(\vec{p}) \approx e^{-\left(p^2/2m-\mu\right)/k_B T}$$

(cf. Thermal History lecture)

- Boltzmann equation $\hat{L}[f_A] = \hat{C}_A[f]$

$\hat{L}$: Liouville operator

f_A : phase-space distribution function of species A

$\hat{C}_A$: collision operator for species A

f : phase-space distribution function of all species partaking in collisions

- Boltzmann equation $\hat{L}[f_A] = \hat{C}_A[f]$

$\hat{L}$: Liouville operator

- in non-relativistic limit it is just the total time derivative:

$$\hat{L}_{nr} = \frac{\partial}{\partial t} + \frac{p}{m} \cdot \nabla_x + \frac{F}{m} \cdot \nabla_p$$

- in the absence of collisions particles move on geodesics

- for homogeneous & isotropic FRW model f=$f(p)$ and hence*

$$\hat{L}[f_A] = \frac{dn_A}{dt} + 3Hn_A \quad \text{with} \quad n_A = \frac{g_A}{2\pi^2}\int p^2 f_A(p)\,dp$$

*derivation on page 33 of http://fiteoweb.unige.ch/~kunzm/lectures/cosmo_II_2005.pdf

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- **integro-differential equation for $f(p)$**
- **ordinary differential equation for $n_A(t)$**

- Boltzmann equation $\hat{L}[f_A] = \hat{C}_A[f]$

$\hat{C}_A$: collision operator for species A

  - change in number of particles A due to interactions* A+B $\leftrightarrow$ C

$$C_A = -\langle \sigma_{AB} v_{AB} \rangle n_A n_B + \beta\, n_C$$

*assuming there is only one other species B

- Boltzmann equation $\hat{L}[f_A] = \hat{C}_A[f]$

$\hat{C}_A$: collision operator for species A

- change in number of particles A due to interactions A+B ↔ C

$$C_A = \underbrace{-\langle \sigma_{AB} v_{AB} \rangle n_A n_B}_{\text{particle destruction}} + \underbrace{\beta\, n_C}_{\text{creation}}$$

- Boltzmann equation $\hat{L}[f_A] = \hat{C}_A[f]$

$\hat{C}_A$: collision operator for species A

- change in number of particles A due to interactions A+B ↔ C

$$C_A = \underbrace{-\langle \sigma_{AB} v_{AB} \rangle n_A n_B}_{\text{particle destruction}} + \underbrace{\beta\, n_C}_{\text{creation}}$$

in equilibrium: particle destruction = creation $\Rightarrow \beta = \dfrac{n_A^{(eq)} n_B^{(eq)}}{n_C^{(eq)}} \langle \sigma_{AB} v_{AB} \rangle$

- cosmological Boltzmann equation for species A interacting with species B:

$$\frac{dn_A}{dt} + 3Hn_A = -n_A^{(eq)} n_B^{(eq)} \langle \sigma_{AB} v_{AB} \rangle \left(\frac{n_A n_B}{n_A^{(eq)} n_B^{(eq)}} - \frac{n_C}{n_C^{(eq)}} \right)$$

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$$\frac{dn_A}{dt} + 3Hn_A = -n_A^{(eq)} n_B^{(eq)} \left\langle \sigma_{AB} v_{AB} \right\rangle \left(\frac{n_A n_B}{n_A^{(eq)} n_B^{(eq)}} - \frac{n_C}{n_C^{(eq)}} \right)$$

- particles in final state C are in equilibrium (i.e. $n_C = n_C^{(eq)}$):

$$\frac{dn_A}{dt} + 3Hn_A = \left\langle \sigma_{AB} v_{AB} \right\rangle \left(n_A^{(eq)} n_B^{(eq)} - n_A n_B \right)$$

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ordinary differential equation for n_A

- cosmological Boltzmann equation for species A interacting with species B:

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ordinary differential equation for n_A*

***we need to write down such an equation for each species...**

(and will eventually only consider perturbations)

- set of equations for photons and baryons:*

$$\delta_b' = -\theta_b - \frac{1}{2}h'$$

$$\delta_\gamma' = -\frac{4}{3}\theta_\gamma - \frac{2}{3}h'$$

$$\theta_b' = -\frac{1}{1+R}\left(aH\theta_b - c_s^2 k^2 \delta_b - k^2 R\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma^{TCA}\right) + R\Theta_{\gamma b}'^{TCA}\right)$$

$$\theta_\gamma' = -\frac{1}{R}\left(aH\theta_b + \theta_b' - c_s^2 k^2 \delta_b\right) + k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma^{TCA}\right)$$

δ_γ: photon perturbations (Fourier transform of $\Delta T/T$)
δ_b: baryon perturbations
$\theta_\gamma = \nabla \cdot v_\gamma$
$\theta_b = \nabla \cdot v_b$
$R = 4\rho_\gamma / 3\rho_b$
h: metric perturbations

$\Theta_{\gamma b} = \delta_\gamma - \delta_b$
$\Theta'_{\gamma b} \triangleq$ 'baryon-photon slip'
σ_γ: photon shear

*Blas, Lesgourges & Tram (arXiv:1104.2933), derivatives ' with respects to conformal time

- set of equations for photons and baryons and dark matter and massless neutrinos:

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu \mathrm{v}_b - \frac{1}{2}P_2(\mu)\Pi\right]$$

$$\dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}\left[\Theta_P + \frac{1}{2}\left(1 - P_2(\mu)\right)\Pi\right]$$

$$\dot{\delta} + ik\mathrm{v} = -3\dot{\Phi}$$

$$\dot{\mathrm{v}} + \frac{\dot{a}}{a}\mathrm{v} = -ik\Psi$$

$$\dot{\delta}_b + ik\mathrm{v}_b = -3\dot{\Phi}$$

$$\dot{\mathrm{v}}_b + \frac{\dot{a}}{a}\mathrm{v}_b = -ik\Psi + \frac{3\dot{\tau}}{4\eta}\left[\mathrm{v}_b + 3i\Theta_1\right]$$

$$\dot{\nu} + ik\mu\nu = -\dot{\Phi} - ik\mu\Psi$$

Θ:	photon perturbations (Fourier transform of $\Delta T/T$)
ν:	neutrino perturbations
δ, v:	dark matter perturbations
δ_b, v_b:	baryon perturbations
Ψ:	metric perturbations
Φ:	Newtonian perturbations
τ:	optical depth ($\dot{\tau} = -n_e \sigma_T a$)
μ:	direction of photon propagation
η:	conformal time
P():	Legendre polynomial
Π :	$\Theta_2 + \Theta_{P2} + \Theta_{P0}$

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… perturbations

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μ: direction of photon propagation

η: conformal time

P(): Legendre polynomial

Π: $\Theta_2 + \Theta_{P2} + \Theta_{P0}$

standard (Runge-Kutta) solvers break down as time-scales for interactions are much shorter than cosmological expansion! ("stiff" differential equations…)

- solving cosmological Boltzmann equations

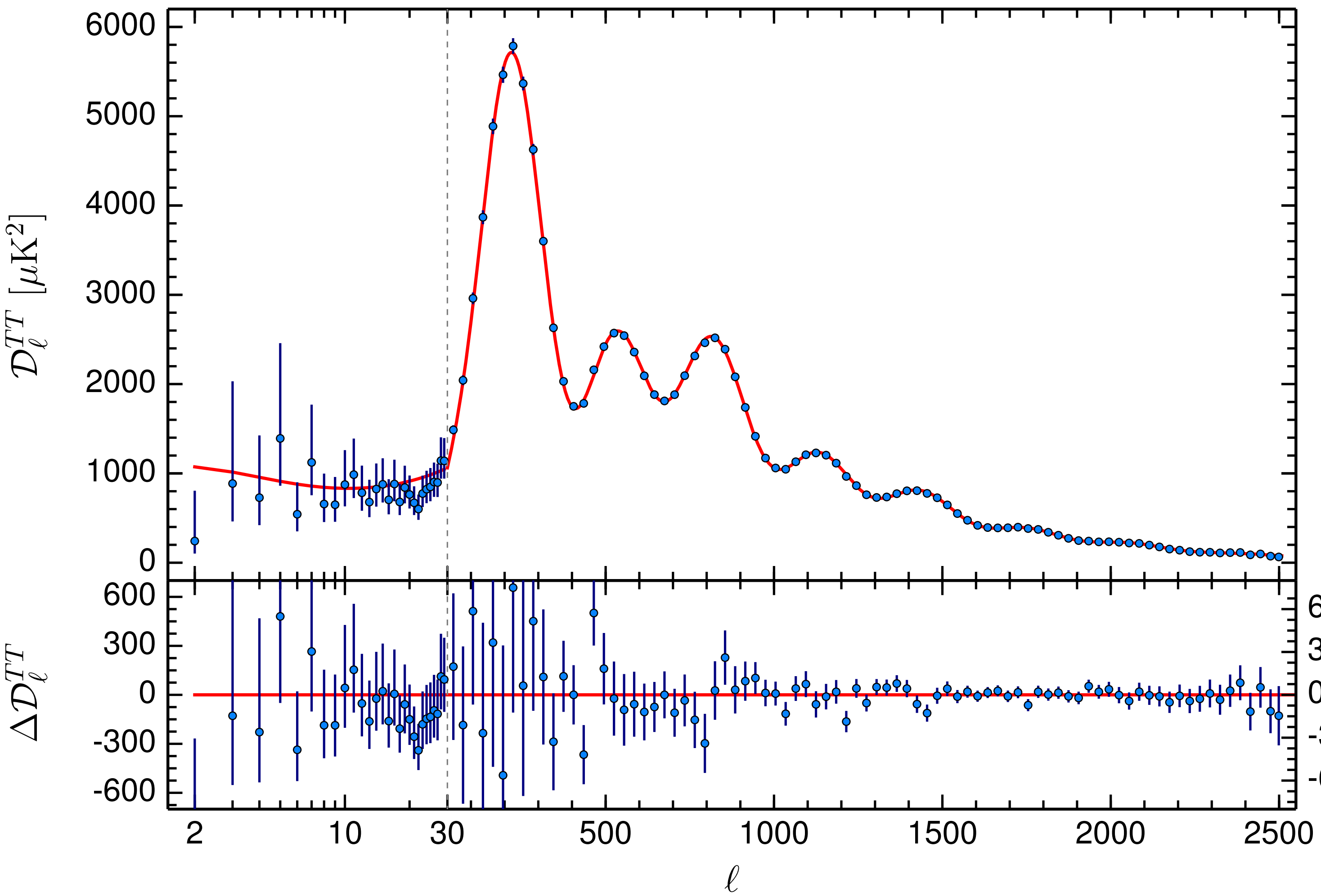
  - Peebles & Yu (1970): TCA method (Tight-Coupling-Approximation*)

    prior to recombination photons, electrons, and nuclei rapidly scattered and behaved almost like a single tightly-coupled photon-baryon plasma: $v_b = v_\gamma$

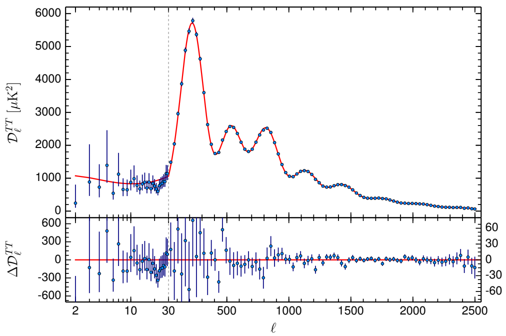
  - ...all subsequent solvers are based upon it.

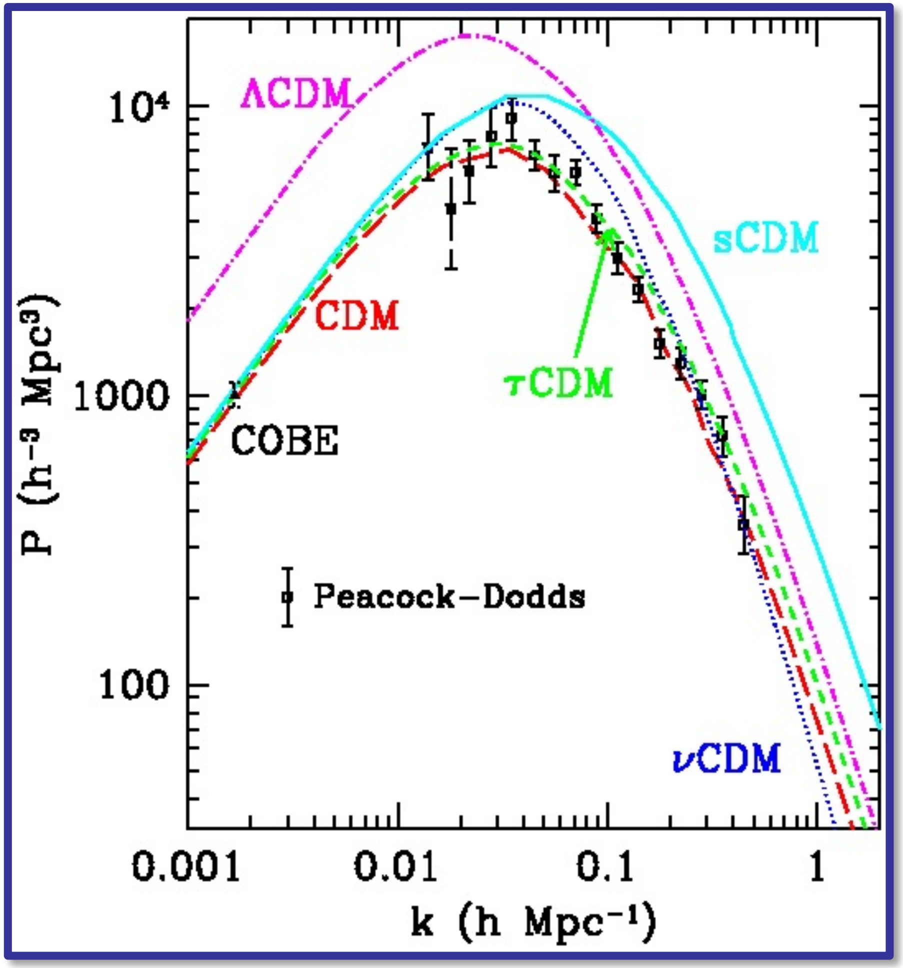
*see Cyr-Racine & Sigurdson (arXiv:1012.0569) for a validation of the TCA

- solving cosmological Boltzmann equations gives CMB fluctuations



- solving cosmological Boltzmann equations gives CMB & matter fluctuations





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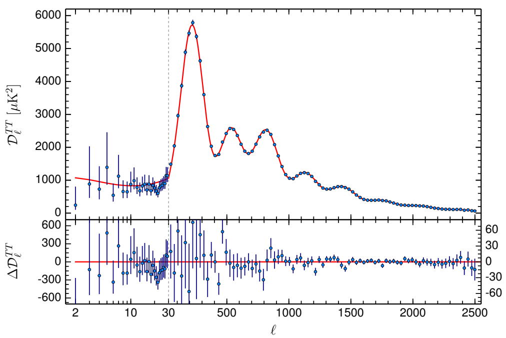
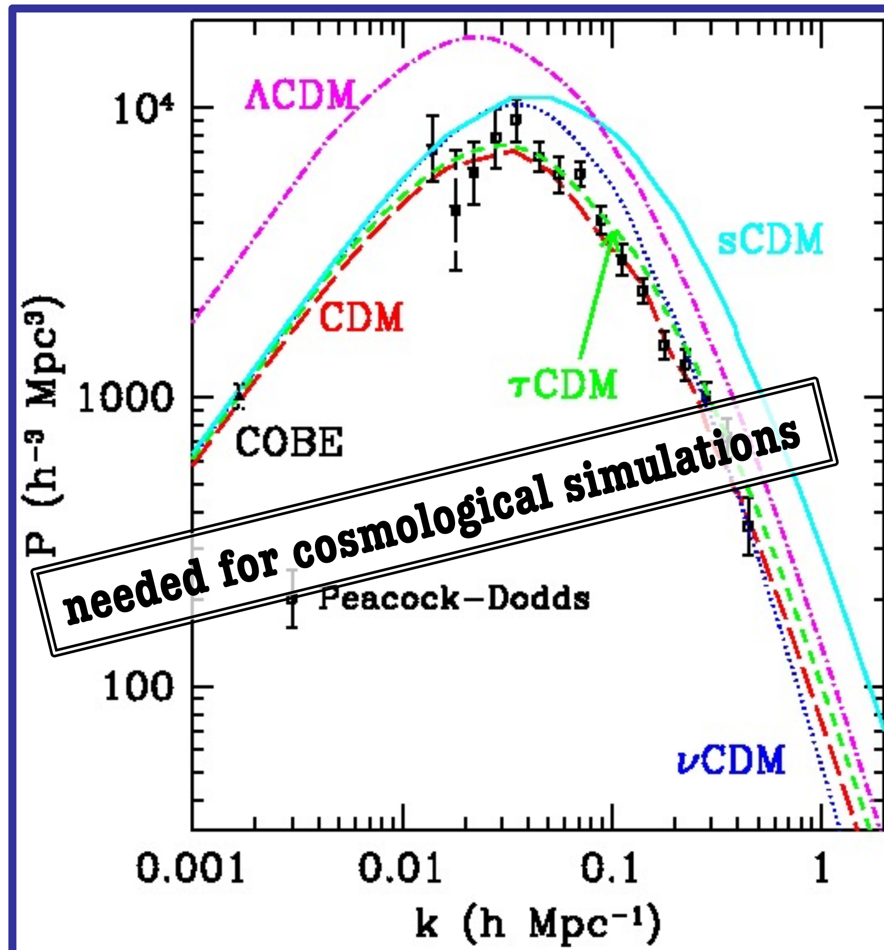




$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

$$D(a) = \frac{5}{2}\Omega_{m,0} H \int_0^a \frac{1}{\left(\Omega_{m,0}a^{-3} + (1-\Omega_{m,0}-\Omega_{\Lambda,0})a^{-2} + \Omega_{\Lambda,0}\right)} da$$

$$\vec{S} = \nabla\Psi$$

$$\Delta\Psi = \delta(\vec{x}_0)$$

$$\hat{\delta}_0\,(k) = \sqrt{\boxed{P_0(k)T^2(k)}}\,R_{\vec{k}}\,e^{i\varphi_{\vec{k}}}$$

- transfer function $T(k)$

$$P(k,a) = D^2(a)\, T^2(k)\, P_0(k)$$

- transfer function $T(k)$

$$P(k,a) = D^2(a)\, T^2(k)\, P_0(k)$$

clear separation into

- temporal evolution after decoupling
- pre-decoupling physics

- transfer function $T(k)$

$$P(k,a) = D^2(a)\, T^2(k)\, P_0(k)$$

$D(a)$: structure growth after decoupling

$T(k)$: physics of the early Universe (up to decoupling)

$P_0(k)$: primordial power spectrum as given by inflation

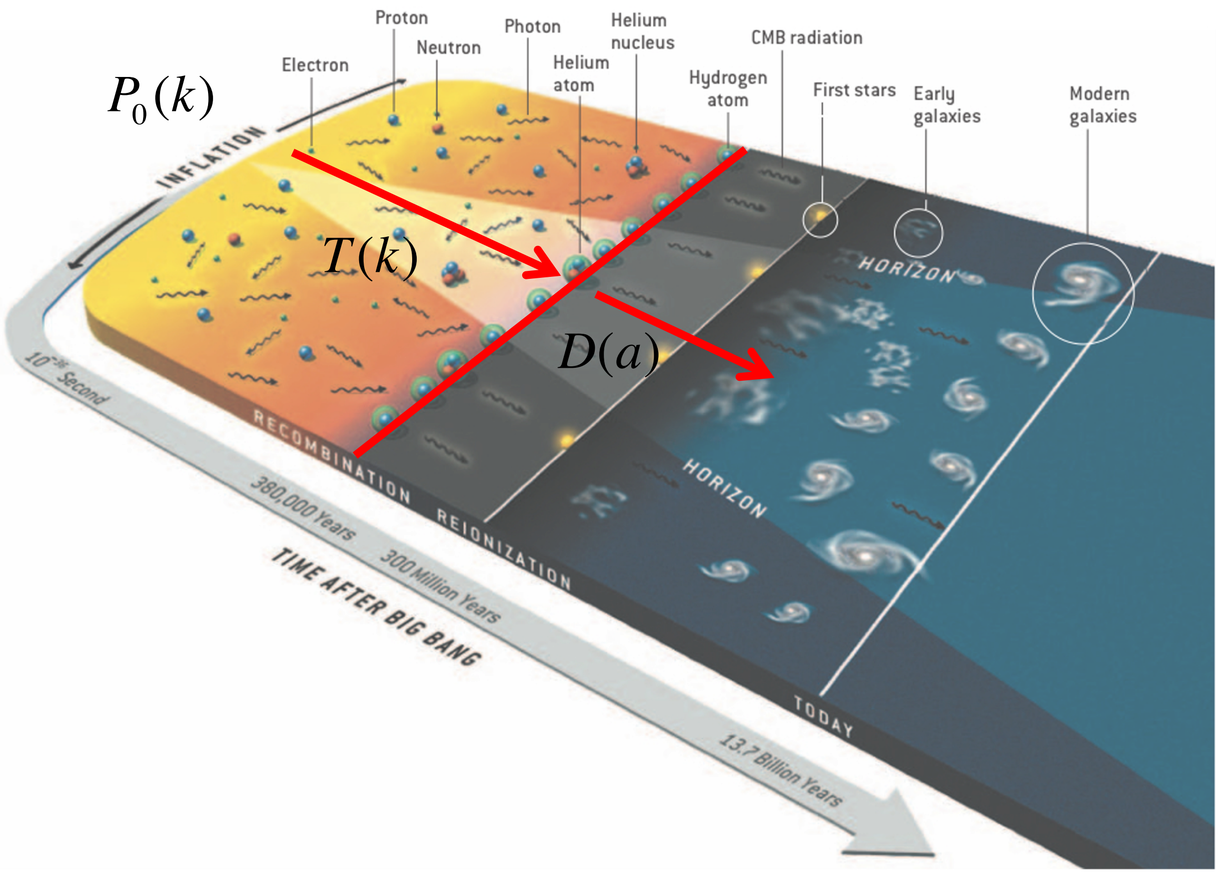
$P(k) \propto k^n$

clear separation into

- temporal evolution after decoupling
- pre-decoupling physics

- transfer function $T(k)$

$$P(k,a) = D^2(a)\,T^2(k)\,P_0(k)$$



- transfer function $T(k)$

$$P(k,a) = D^2(a)\, T^2(k)\, P_0(k)$$

- radiation domination:

$$\delta \propto a^2 \qquad k << k_{eq} \qquad \text{(outside horizon)}$$

$$\delta \propto \ln(a) \qquad k >> k_{eq} \qquad \text{(inside horizon)}$$

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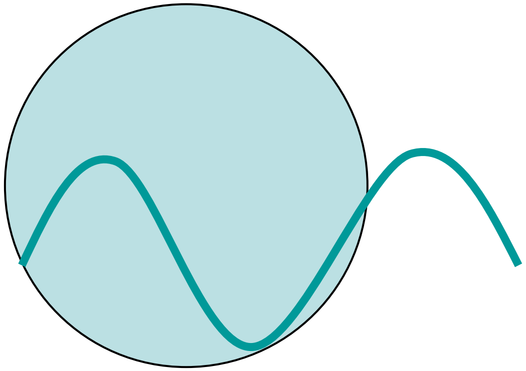

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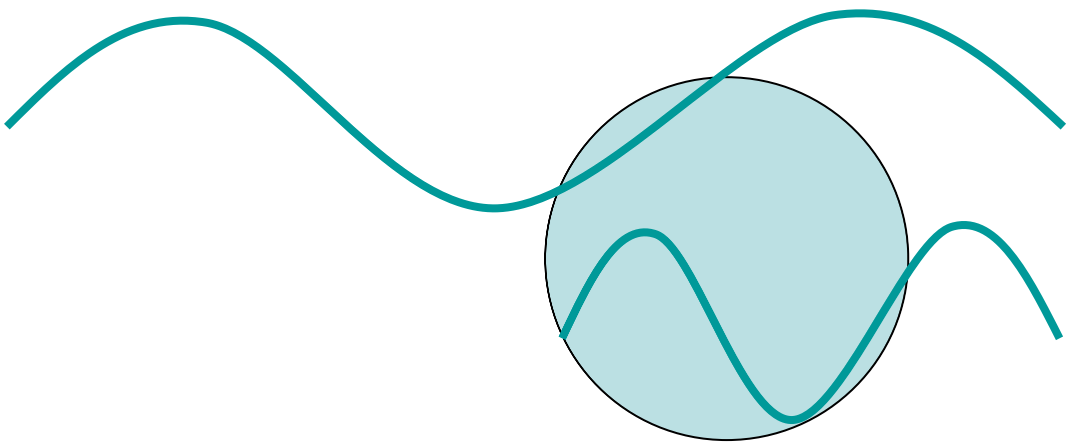


k_2 entering horizon at $a_2=a_1k_1/k_2$

k_1 entering horizon at a_1

- transfer function $T(k)$

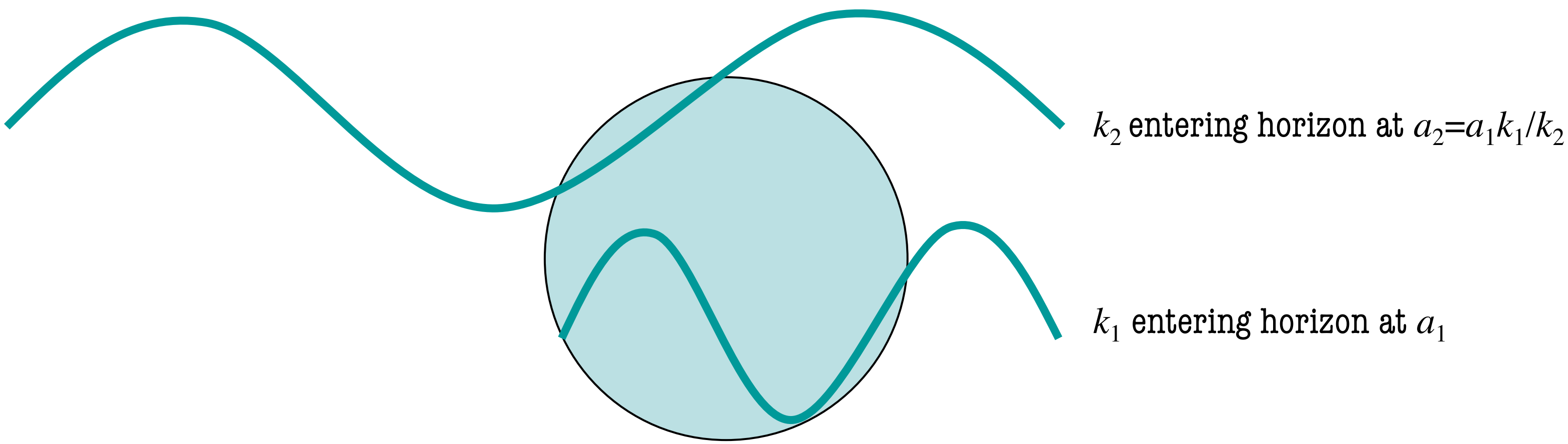
$$P(k,a) = D^2(a)\, T^2(k)\, P_0(k)$$

- radiation domination:

$$\left(\frac{a_1}{a_2}\right)^2 = \left(\frac{k_2}{k_1}\right)^2$$

$$\delta \propto a^2 \qquad k << k_{eq} \qquad \Rightarrow \qquad T(k) \propto 1 \qquad k << k_{eq}$$

$$\delta \propto \ln(a) \qquad k >> k_{eq} \qquad \qquad T(k) \propto 1/k^2 \qquad k >> k_{eq}$$



- transfer function $T(k)$

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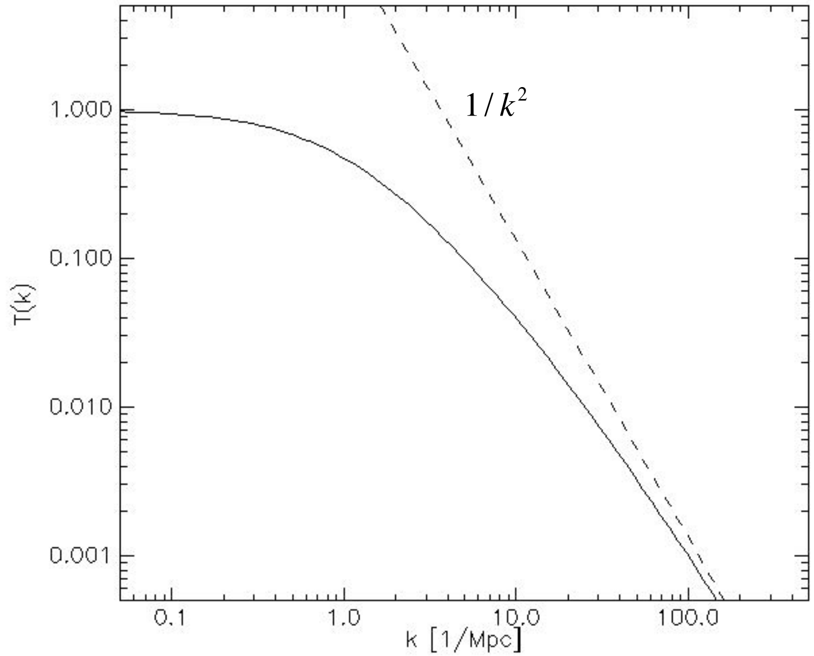


(Tom Theun's Cosmology Lecture)

- transfer function $T(k)$

$$P(k,a) = D^2(a)\,T^2(k)\,P_0(k)$$

- radiation domination:

$$T(k) \propto 1 \qquad k << k_{eq}$$

$$T(k) \propto 1/k^2 \qquad k >> k_{eq}$$

- matter domination:

$$\delta \propto a \quad => \quad T(k) \propto 1$$

- transfer function $T(k)$

$$P(k,a) = D^2(a)\, T^2(k)\, P_0(k)$$

- post-decoupling (CDM only): (Bond & Efstatiou 1984)

$$T(k) = \left(1 + \left((ak) + (bk)^{1.5} + (ck)^2\right)^{\nu}\right)^{-1/\nu}$$

$a = 6.4\ (\Omega_m h^{-2})$ Mpc

$b = 3.0\ (\Omega_m h^{-2})$ Mpc

$c = 1.7\ (\Omega_m h^{-2})$ Mpc

$\nu = 1.13$

- transfer function $T(k)$

$$P(k,a) = D^2(a)\,T^2(k)\,P_0(k)$$

- post-decoupling (CDM+baryons): (Eisenstein & Hu 1998)

$$T(k) = \frac{\Omega_b}{\Omega_m} T_b(k) + \frac{\Omega_c}{\Omega_m} T_c(k)$$

- transfer function $T(k)$

$$P(k,a) = D^2(a)\,T^2(k)\,P_0(k)$$

- post-decoupling (CDM+baryons): (Eisenstein & Hu 1998)

$$T(k) = \frac{\Omega_b}{\Omega_m} T_b(k) + \frac{\Omega_c}{\Omega_m} T_c(k)$$

$$T_b = \left[\frac{\tilde{T}_0(k;1,1)}{1+(ks/5.2)^2} + \frac{\alpha_b}{1+(\beta_b/ks)^3} e^{-(k/k_{\rm Silk})^{1.4}} \right] j_0(k\tilde{s})$$

$$T_c(k) = f\tilde{T}_0(k,1,\beta_c) + (1-f)\tilde{T}_0(k,\alpha_c,\beta_c)$$

$$k_{\rm eq} \equiv (2\Omega_0 H_0^2 z_{\rm eq})^{1/2} = 7.46\times 10^{-2}\Omega_0 h^2 \Theta_{2.7}^{-2}\ {\rm Mpc}^{-1}$$

$$k_{\rm Silk} = 1.6(\Omega_b h^2)^{0.52}(\Omega_0 h^2)^{0.73}[1+(10.4\Omega_0 h^2)^{-0.95}]$$

$$z_d = 1291\frac{(\Omega_0 h^2)^{0.251}}{1+0.659(\Omega_0 h^2)^{0.828}}[1+b_1(\Omega_b h^2)^{b_2}]$$

$$b_1 = 0.313(\Omega_0 h^2)^{-0.419}[1+0.607(\Omega_0 h^2)^{0.674}]\,,$$

$$b_2 = 0.238(\Omega_0 h^2)^{0.223}\,,$$

$$s = \frac{2}{3k_{eq}}\sqrt{\frac{6}{R_{eq}}}\ln\frac{\sqrt{1+R_d}+\sqrt{R_d+R_{eq}}}{1+\sqrt{R_{eq}}}$$

$$\alpha_b = 2.07 k_{\rm eq} s(1+R_d)^{-3/4} G\left(\frac{1+z_{\rm eq}}{1+z_d}\right)$$

$$\tilde{s}(k) = \frac{s}{[1+(\beta_{\rm node}/ks)^3]^{1/3}}$$

$$\beta_{\rm node} = 8.41(\Omega_0 h^2)^{0.435}$$

$$\Theta_{2.7} = \frac{T_{cmb}}{2.7}$$

$$\tilde{T}_0(k,\alpha_c,\beta_c) = \frac{\ln(e+1.8\beta_c q)}{\ln(e+1.8\beta_c q)+Cq^2}$$

$$C = \frac{14.2}{\alpha_c} + \frac{386}{1+69.9q^{1.08}}$$

$$q = \frac{k}{13.41 k_{eq}}$$

$$f = \frac{1}{1+(ks/5.4)^4}$$

$$\alpha_c = a_1^{-\Omega_b/\Omega_0} a_2^{-(\Omega_b/\Omega_0)^3}\,,$$

$$a_1 = (46.9\Omega_0 h^2)^{0.670}[1+(32.1\Omega_0 h^2)^{-0.532}]$$

$$a_2 = (12.0\Omega_0 h^2)^{0.424}[1+(45.0\Omega_0 h^2)^{-0.582}]$$

$$\beta_c^{-1} = 1 + b_1[(\Omega_c/\Omega_0)^{b_2} - 1]\,,$$

$$b_1 = 0.944[1+(458\Omega_0 h^2)^{-0.708}]^{-1}\,,$$

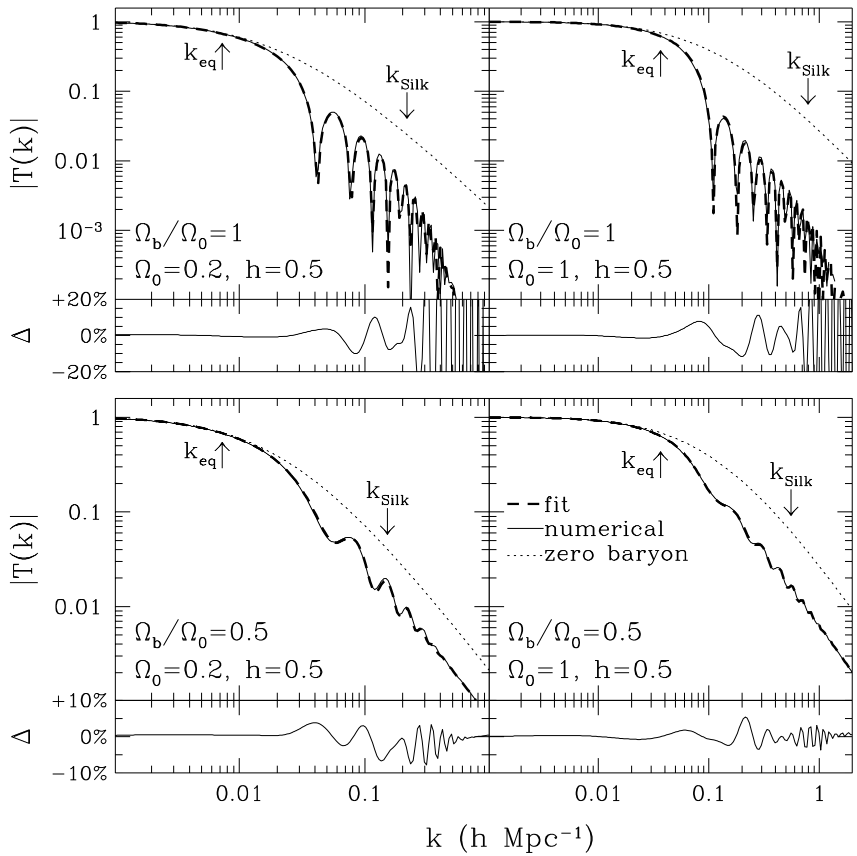
$$b_2 = (0.395\Omega_0 h^2)^{-0.0266}\,.$$

$$R \equiv 3\rho_b/4\rho_\gamma = 31.5\Omega_b h^2 \Theta_{2.7}^{-4}(z/10^3)^{-1}$$

- transfer function $T(k)$

$$P(k,a) = D^2(a)\,T^2(k)\,P_0(k)$$

- post-decoupling (CDM+baryons): (Eisenstein & Hu 1998)



- 1995: `COSMICS` package
- 1996: `CMBFAST`
- 1999: `RECFAST`

  `CAMB`
- 2003: `CMBEASY`
- 2010: `CLASS`

  `CosmoRec`

- 1995: **COSMICS** package (Bertschinger)
  - first ever public release of Boltzmann solver
  - bundled with package to generate initial conditions for simulations
- 1996: CMBFAST
- 1999: RECFAST

  CAMB
- 2003: CMBEASY
- 2010: CLASS

  CosmoRec

- 1995: `COSMICS` package
- 1996: **`CMBFAST`** (Seljak & Zaldarriaga)
  - adding functions to `COSMICS` for computing the sources
  - convolution with Bessel functions
  - *much* faster than `COSMICS`
- 1999: `RECFAST`

  `CAMB`
- 2003: `CMBEASY`
- 2010: `CLASS`

  `CosmoRec`

- 1995: `COSMICS` package
- 1996: `CMBFAST`
- 1999: **`RECFAST`** (Seager, Sasselov & Scott)
    - solves recombination of H and He simultaneously giving
    - ionised fractions as a function of redshift

  `CAMB`
- 2003: `CMBEASY`
- 2010: `CLASS`

  `CosmoRec`

- 1995: COSMICS package
- 1996: CMBFAST
- 1999: RECFAST

  CAMB (Lewis et al.)

  - taking CMBFAST apart and re-writing it in f90
  - improved expressions for sources, lensing, etc.
- 2003: CMBEASY
- 2010: CLASS

  CosmoRec

- 1995: COSMICS package
- 1996: CMBFAST
- 1999: RECFAST

  CAMB
- 2003: **CMBEASY** (Doran)
  - re-write of CMBFAST in C++
- 2010: CLASS

  CosmoRec

- 1995: `COSMICS` package
- 1996: `CMBFAST`
- 1999: `RECFAST`

  `CAMB`
- 2003: `CMBEASY`
- 2010: **`CLASS`** (Lesgourges et al. http://www.class-code.net)
  - highly modular code
  - easy to install and use
  - exactly following Bertschinger's notation to avoid confusion

  `CosmoRec`

- 1995: COSMICS package
- 1996: CMBFAST
- 1999: RECFAST

  CAMB
- 2003: CMBEASY
- 2010: CLASS

  CosmoRec (Chluba)
  - highly improved C++ version of RECFAST

- 1995: `COSMICS` package
- 1996: `CMBFAST`
- 1999: `RECFAST`

  `CAMB`
- 2003: `CMBEASY`
- 2010: **`CLASS`**

  `CosmoRec`

(still) maintained

- 1995: `COSMICS` package
- 1996: `CMBFAST`
- 1999: `RECFAST`

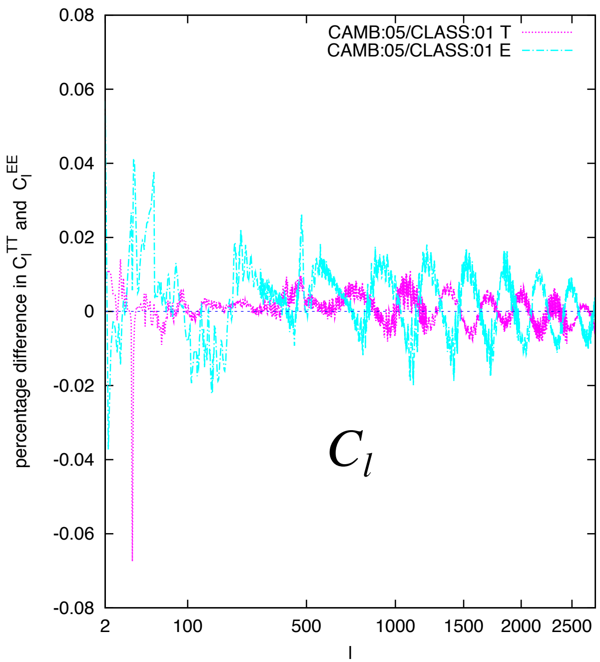
  `CAMB`
- 2003: `CMBEASY`
- 2010: **`CLASS`** $\longrightarrow$ hands-on training in "Advanced Cosmology" course

  `CosmoRec`

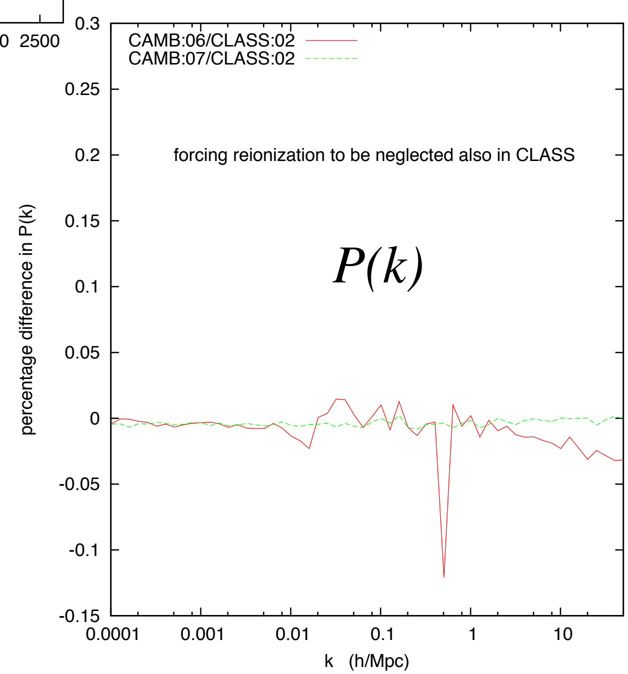
- 1995: COSMICS package
- 1996: CMBFAST
- 1999: RECFAST

  CAMB →
- 2003: CMBEASY
- 2010: **CLASS** →

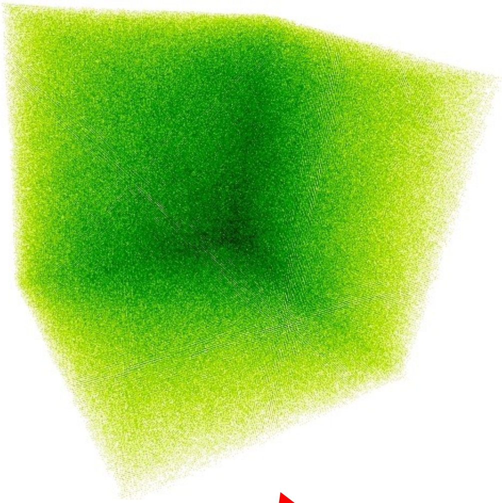
  CosmoRec

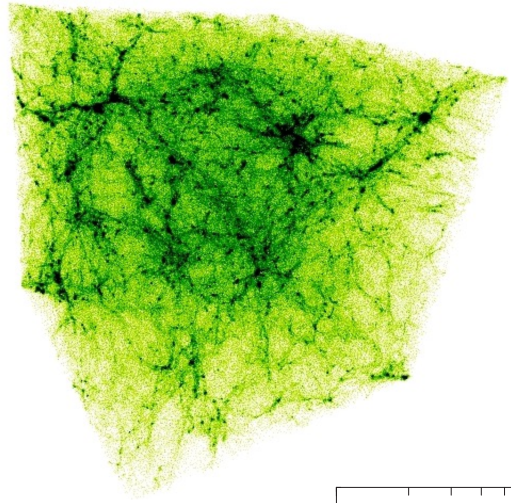


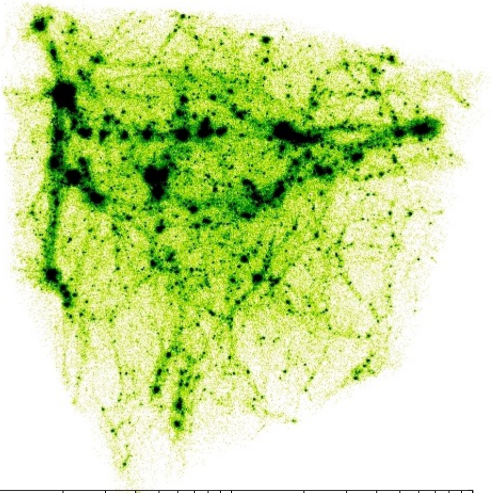
difference between CAMB & CLASS
(Lesgourges, arxiv:1104.2934)



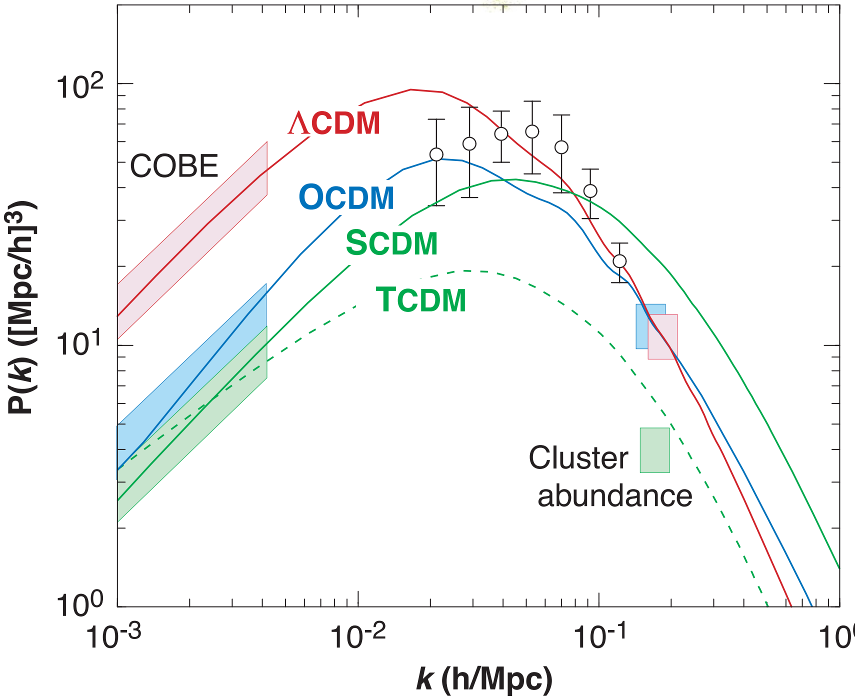
- introduction
- Boltzmann solver
- **initial conditions generators**
- simulation codes







how to translate into initial conditions?



- Zeldovich approximation: (cf. LSS lecture)

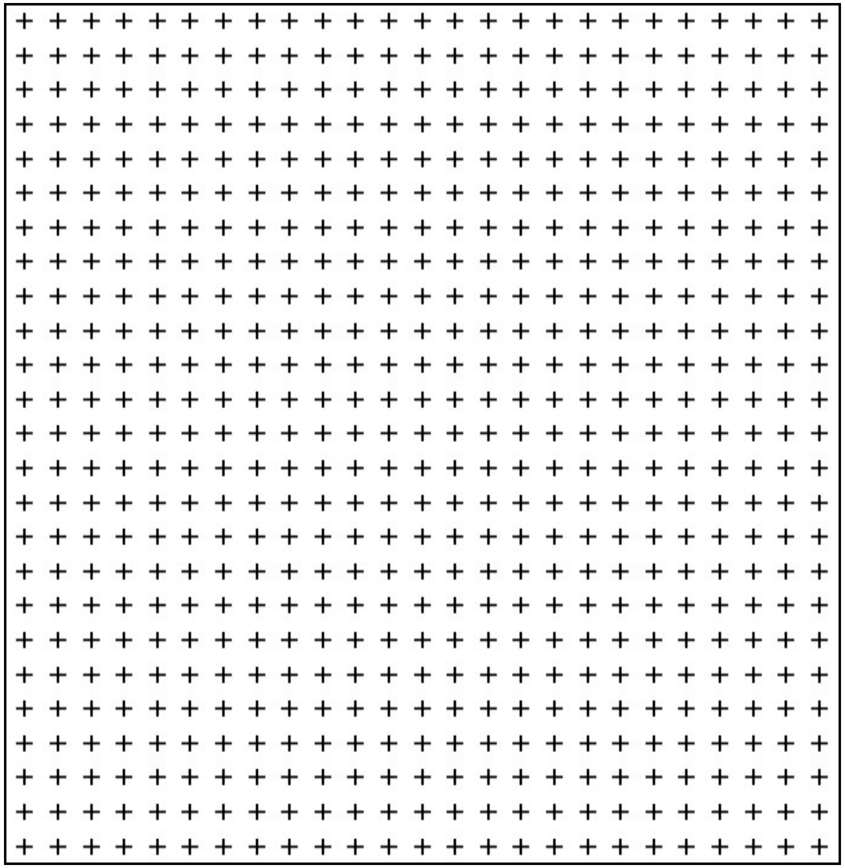
$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

$$D(a) = \frac{5}{2}\Omega_{m,0} H \int_0^a \frac{1}{\left(\Omega_{m,0} a^{-3} + (1 - \Omega_{m,0} - \Omega_{\Lambda,0}) a^{-2} + \Omega_{\Lambda,0}\right)} da$$

$$\vec{S} = \nabla \Psi$$

$$\Delta\Psi = \delta(\vec{x}_0)$$

$$\hat{\delta}_0\,(k) = \sqrt{P_0(k) T^2(k)} R_{\vec{k}}\, e^{i\varphi_{\vec{k}}}$$



- **Zeldovich approximation:**

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

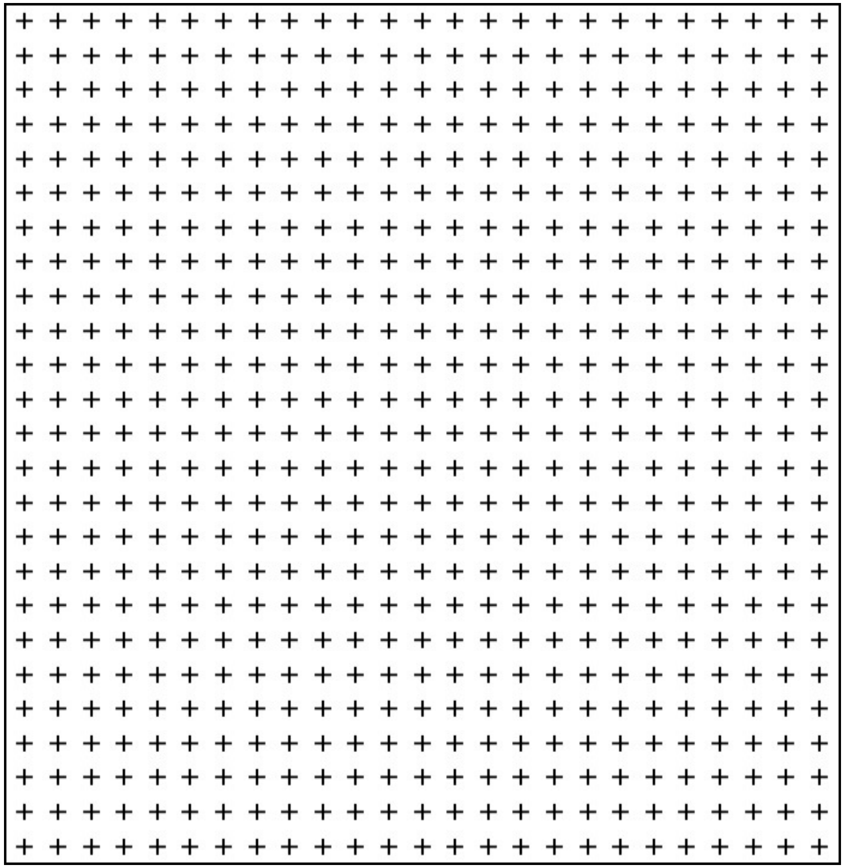
$$D(a) = \frac{5}{2}\Omega_{m,0} H \int_0^a \frac{1}{\left(\Omega_{m,0} a^{-3} + (1 - \Omega_{m,0} - \Omega_{\Lambda,0}) a^{-2} + \Omega_{\Lambda,0}\right)} da$$

$$\vec{S} = \nabla\Psi$$

$$\Delta\Psi = \delta(\vec{x}_0)$$

$$\hat{\delta}_0(k) = \sqrt{P_0(k) T^2(k) R_{\vec{k}}}\, e^{i\varphi_{\vec{k}}}$$

we require the post-decoupling power spectrum of density perturbations
(result of Boltzmann solver or use fitting formula...)



- ## Zeldovich approximation:

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

$$D(a) = \frac{5}{2}\Omega_{m,0} H \int_0^a \frac{1}{\left(\Omega_{m,0} a^{-3} + (1-\Omega_{m,0}-\Omega_{\Lambda,0})a^{-2} + \Omega_{\Lambda,0}\right)} da$$

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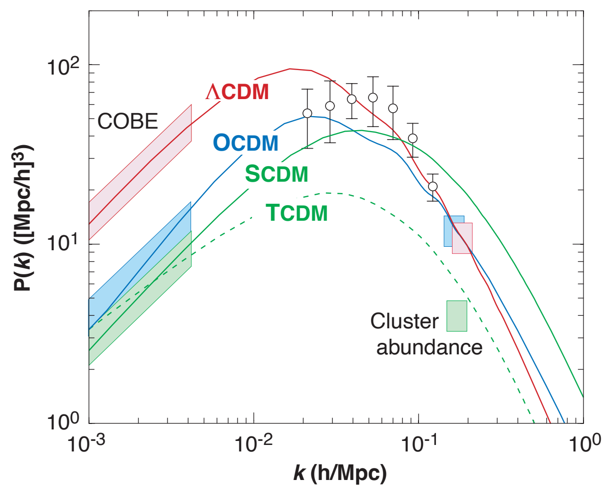

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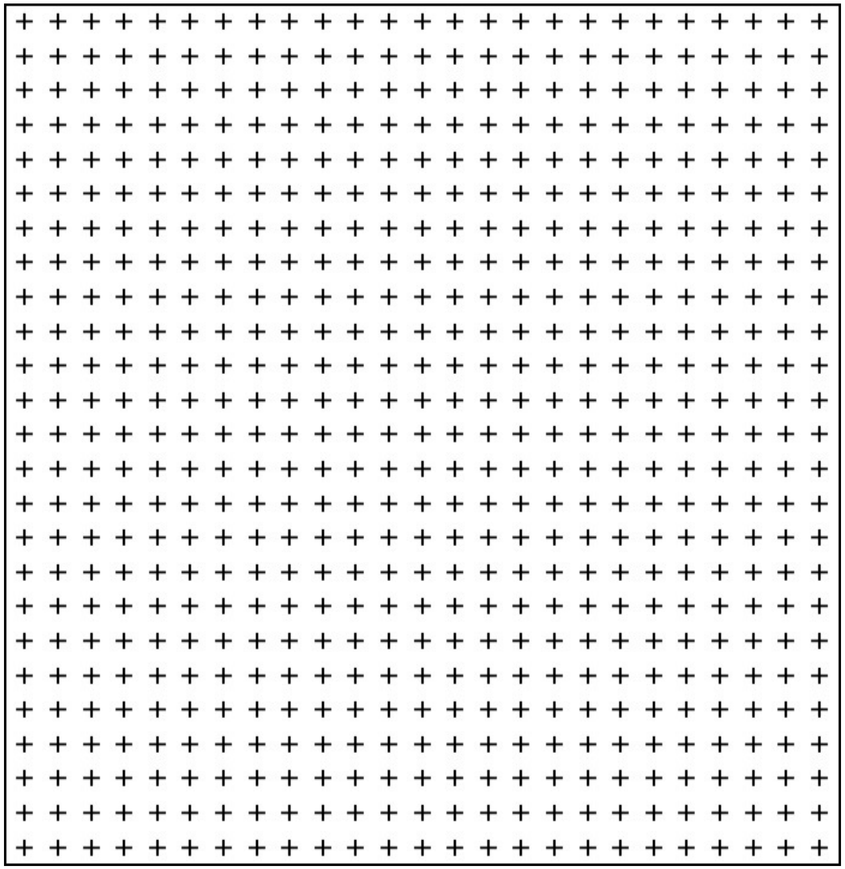

- <u>Zeldovich approximation:</u>

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

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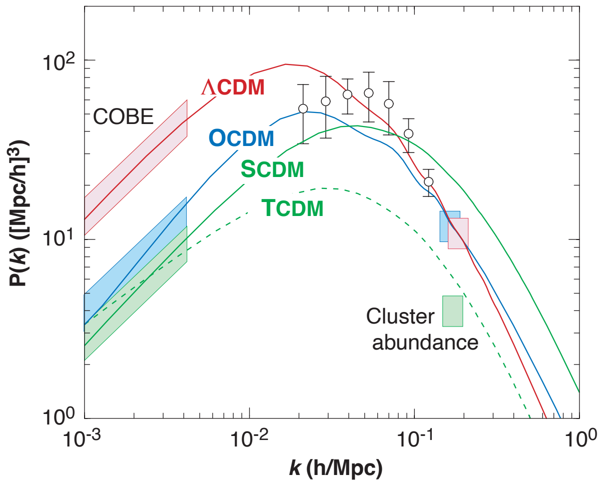

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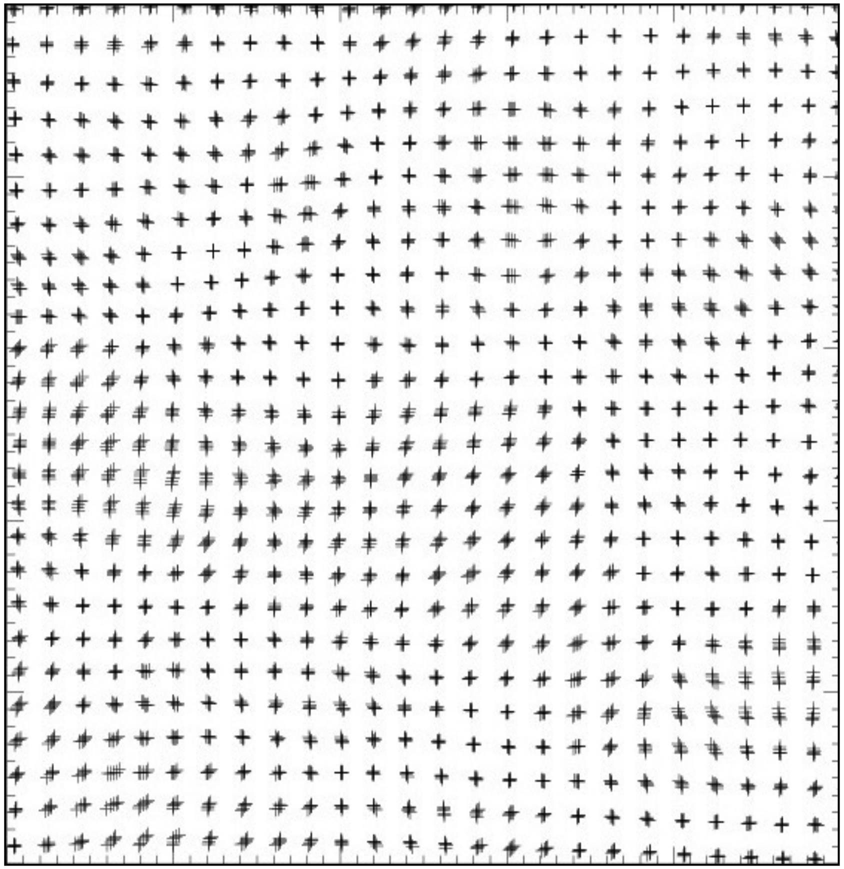


- Zeldovich approximation:

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

$$D(a) = \frac{5}{2}\Omega_{m,0} H \int_0^a \frac{1}{\left(\Omega_{m,0} a^{-3} + (1-\Omega_{m,0}-\Omega_{\Lambda,0}) a^{-2} + \Omega_{\Lambda,0}\right)} da$$

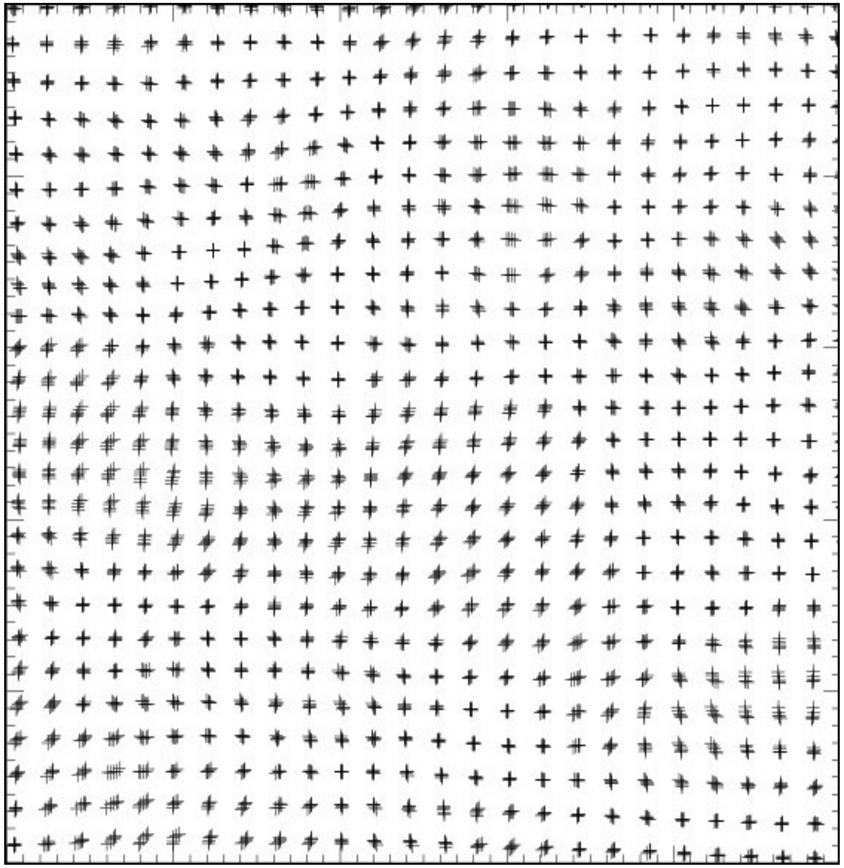
$$\vec{S} = \nabla \Psi$$

$$\Delta \Psi = \delta(\vec{x}_0)$$

$$\hat{\delta}_0(k) = \sqrt{P_0(k) T^2(k)}\, R_{\vec{k}}\, e^{i\varphi_{\vec{k}}}$$?

R : Gaussian random number

φ : random phase



- <u>Zeldovich approximation:</u>

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

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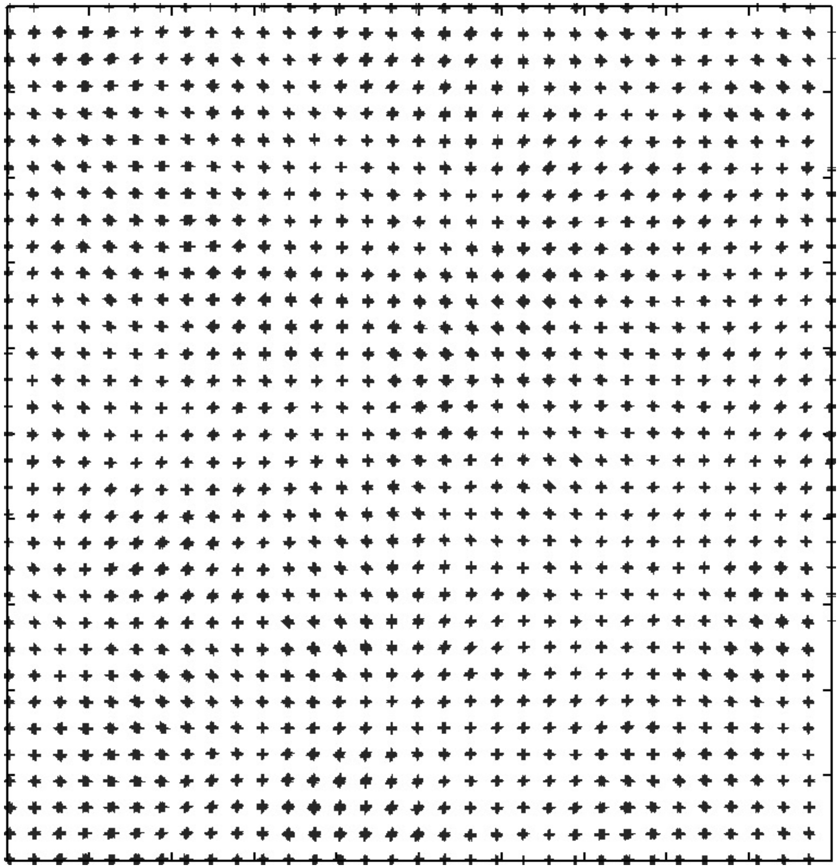
R : Gaussian random number

φ : random phase



- <u>Zeldovich approximation:</u>

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$$\vec{S} = \nabla\Psi$$

$$\Delta\Psi = \delta(\vec{x}_0)$$

$$\hat{\delta}_0(k) = \sqrt{P_0(k)T^2(k)}\, R_{\vec{k}}\, e^{i\varphi_{\vec{k}}} \;\; ?$$

R : Gaussian random number
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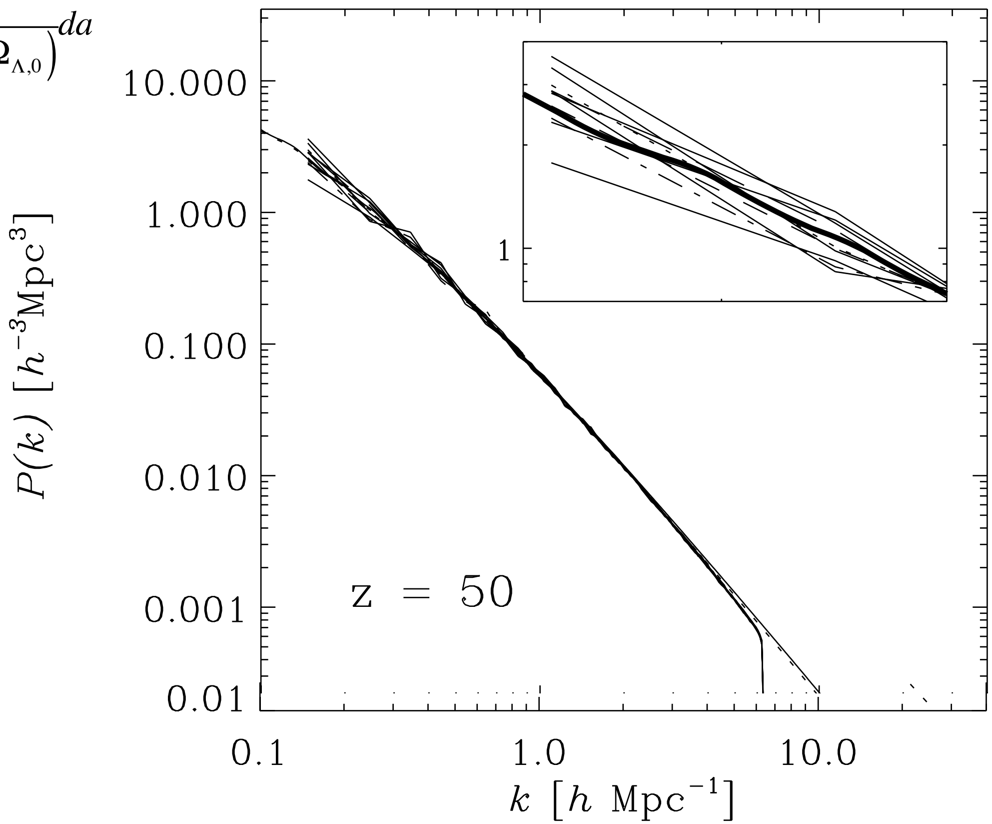
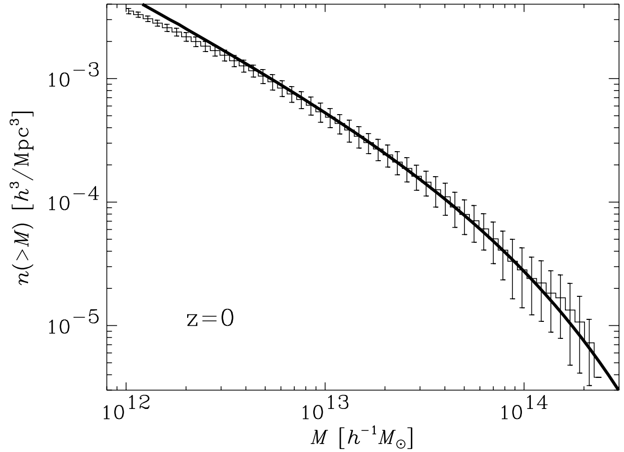


(Knebe et al. 2003)

▪ Zeldovich approximation:

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

resulting differences in today's halo mass function



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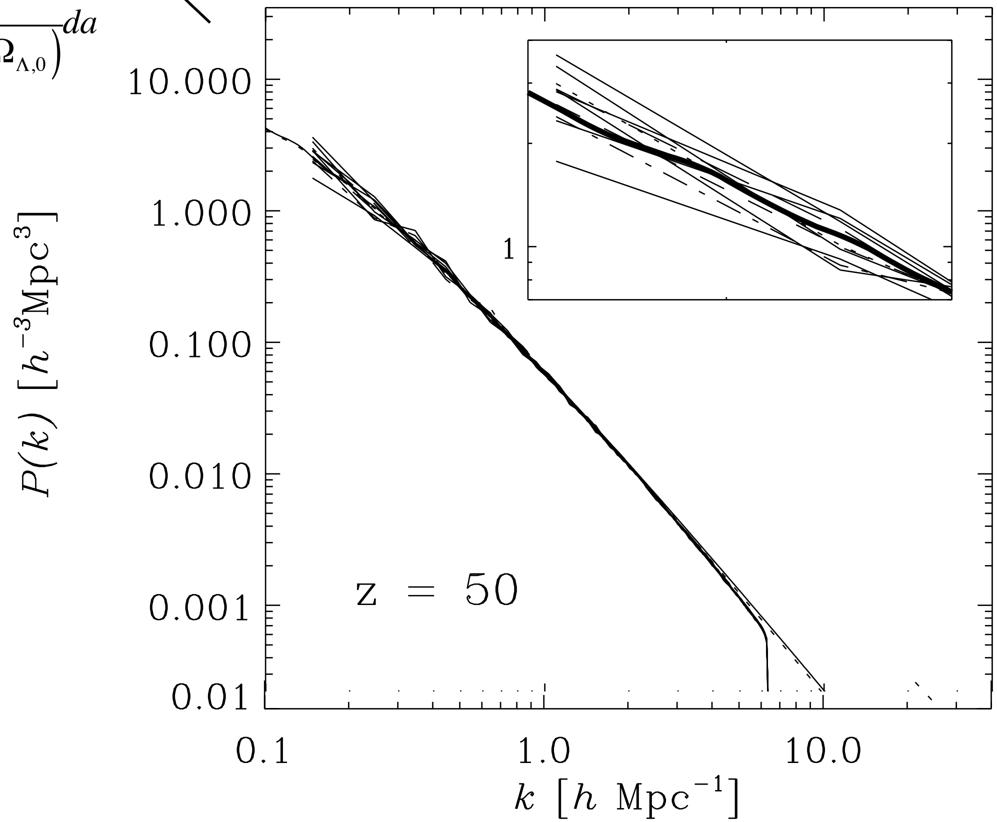
$$\vec{S} = \nabla\Psi$$

$$\Delta\Psi = \delta(\vec{x}_0)$$

$$\hat{\delta}_0(k) = \sqrt{P_0(k)T^2(k)}\, R_{\vec{k}}\, e^{i\varphi_{\vec{k}}}$$ **?**

R : Gaussian random number

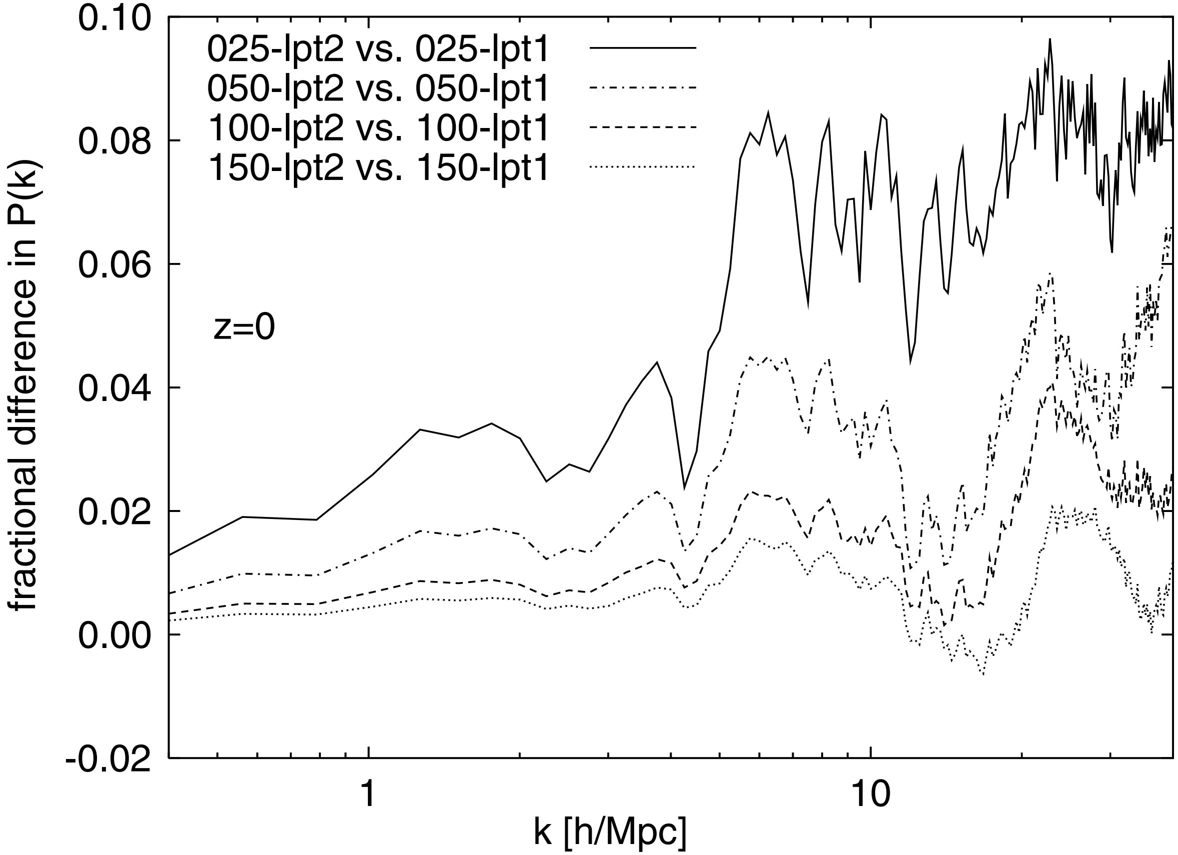
φ : random phase



(Knebe et al. 2003)

- Lagrangian perturbation theory

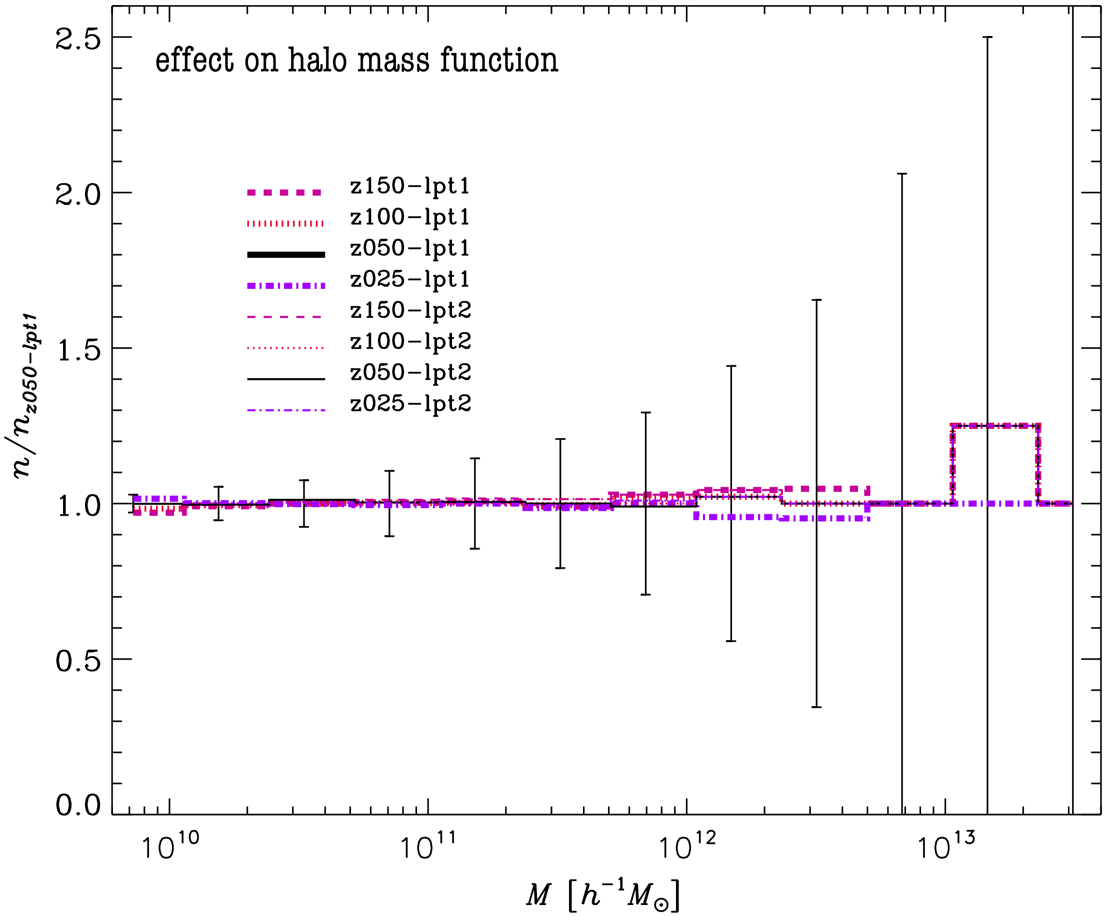
$$\vec{x}(t) = \vec{q} + D(a)\nabla\Psi - D^{(2)}\nabla\Psi^{(2)}$$



(Knebe et al. 2009)

- Lagrangian perturbation theory

$$\vec{x}(t) = \vec{q} + D(a)\nabla\Psi - D^{(2)}\nabla\Psi^{(2)}$$



(Knebe et al. 2009)

- available codes

1990:

- COSMICS: http://web.mit.edu/edbert
- GRAFIC-2: http://web.mit.edu/edbert
- PMstartM http://astro.nmsu.edu/~aklypin/PM/pmcode

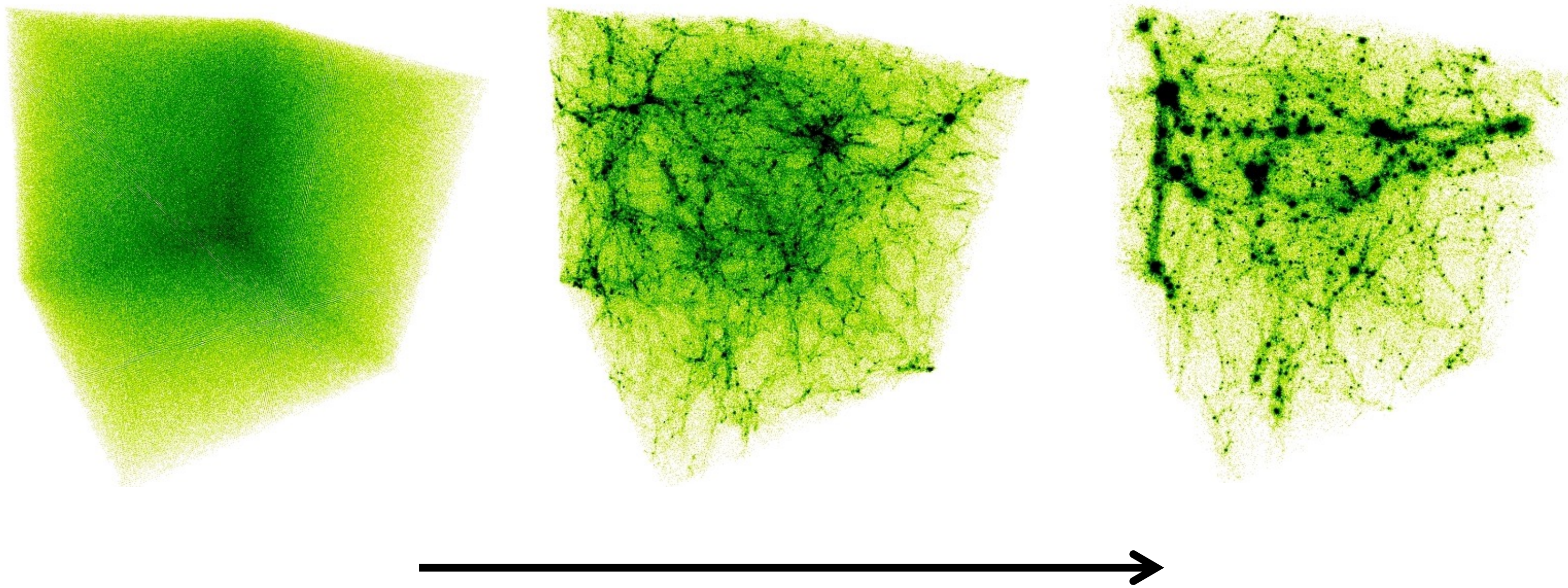
2000:

- N-genIC: http://www.h-its.org/tap-software-e/ngenic-code
- 2LPTic: http://cosmo.nyu.edu/roman/2LPT

2010:

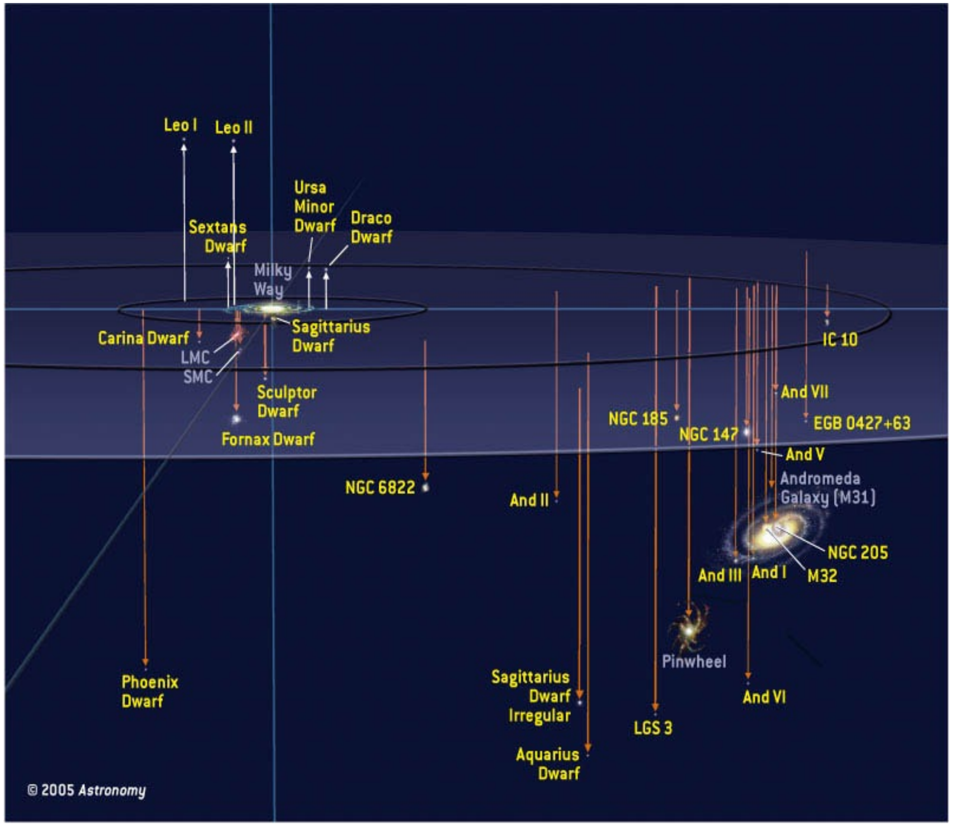
- CICsASS: http://faculty.washington.edu/mcquinn/Init_Cond_Code.html
- Panphasia http://icc.dur.ac.uk/Panphasia.php
- MUSIC: https://www-n.oca.eu/ohahn/MUSIC
- ginnungagap: https://github.com/ginnungagapgroup/ginnungagap

- introduction
- Boltzmann solver
- initial conditions generators
- **simulation codes**

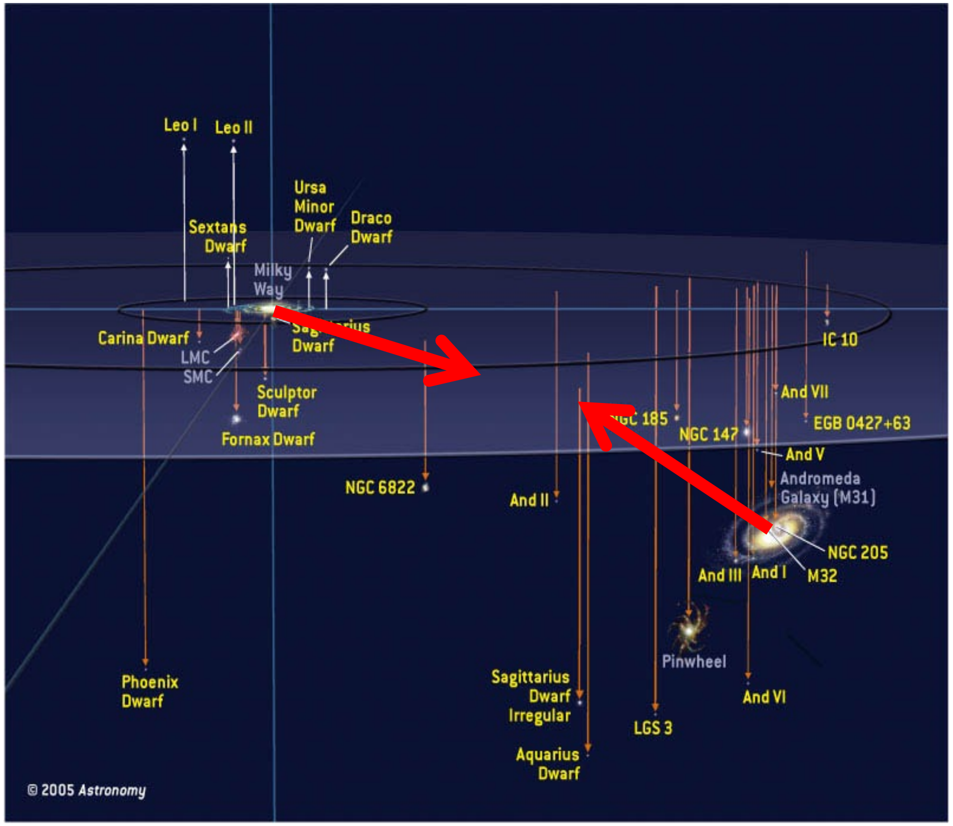


?

The collision of our Milky Way with the Andromeda Galaxy!



The collision of our Milky Way with the Andromeda Galaxy!



The collision of our Milky Way with the Andromeda Galaxy!

(courtesy Arman Khalatyan)

The collision of our Milky Way with the Andromeda Galaxy!

do we really need supercomputers for this?

(courtesy Arman Khalatyan)

1941

Erik Holmberg

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME 94 NOVEMBER 1941 NUMBER 3

ON THE CLUSTERING TENDENCIES AMONG THE NEBULAE

II. A STUDY OF ENCOUNTERS BETWEEN LABORATORY MODELS OF STELLAR SYSTEMS BY A NEW INTEGRATION PROCEDURE

ERIK HOLMBERG

ABSTRACT

In a previous paper[1] the writer discussed the possibility of explaining the observed clustering effects among extragalactic nebulae as a result of captures. The present investigation deals with the important problem of whether the loss of energy resulting from the tidal disturbances at a close encounter between two nebulae is large enough to effect a capture. The tidal deformations of two models of stellar systems, passing each other at a small distance, are studied by reconstructing, piece by piece, the orbits described by the individual mass elements. The difficulty of integrating the total gravitational force acting upon a certain element at a certain point of time is solved by replacing gravitation by light. The mass elements are represented by light-bulbs, the candle power being proportional to mass, and the total light is measured by a photocell (Fig. 1). The nebulae are assumed to have a flattened shape, and each is represented by 37 light-bulbs. It is found that the tidal deformations cause an increase in the attraction between the two objects, the increase reaching its maximum value when the nebulae are separating, i.e., after the passage. The resulting loss of energy (Fig. 6) is comparatively large and may, in favorable cases, effect a capture. The spiral arms developing during the encounter (Figs. 4) represent an interesting by-product of the investigation. The direction of the arms depends on the direction of rotation of the nebulae with respect to the direction of their space motions.

I. THE EXPERIMENTAL ARRANGEMENTS

The present paper is a study of the tidal disturbances appearing in stellar systems which pass one another at small distances. These tidal disturbances are of some importance since they are accompanied by a loss of energy which may result in a capture between the two objects. In a previous paper[1] the writer discussed the clustering tendencies among extragalactic nebulae. A theory was put forth that the observed clustering effects are the result of captures between individual nebulae. The capture theory seems to be able to account not only for double and multiple nebulae but also for the large extragalactic clusters. The present investigation tries to give an answer to the important question of whether the loss of energy accompanying a close encounter between two nebulae is large enough to effect a capture.

A study of tidal disturbances is greatly facilitated if it can be restricted to only two dimensions, i.e., to nebulae of a flattened shape, the principal planes of which coincide with the plane of their hyperbolic orbits. In order to reconstruct the orbit described by

[1] *Mt. W. Contr.*, No. 633; *Ap. J.*, **92**, 200, 1940.

1941

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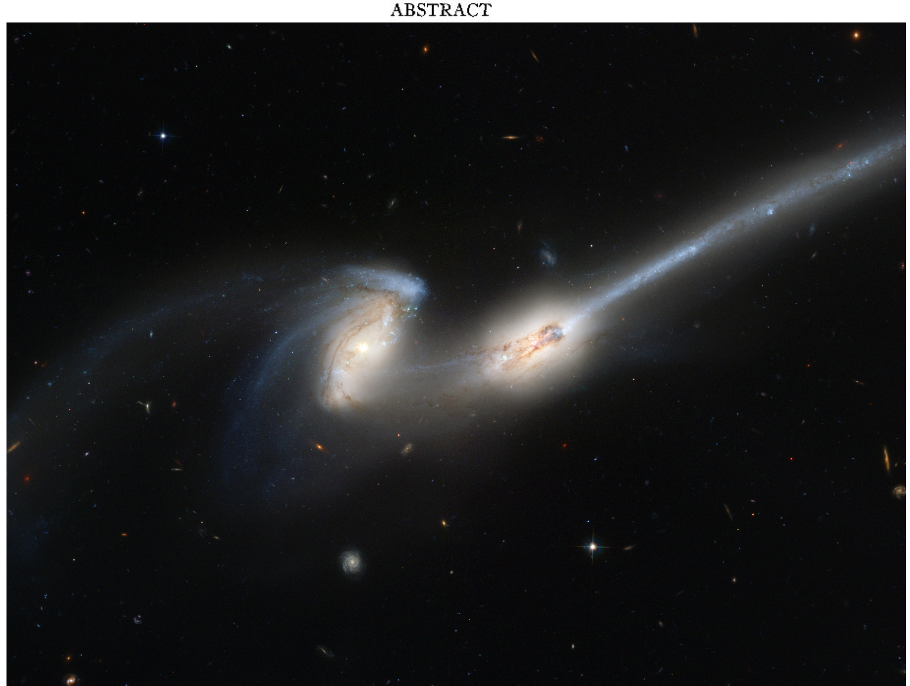
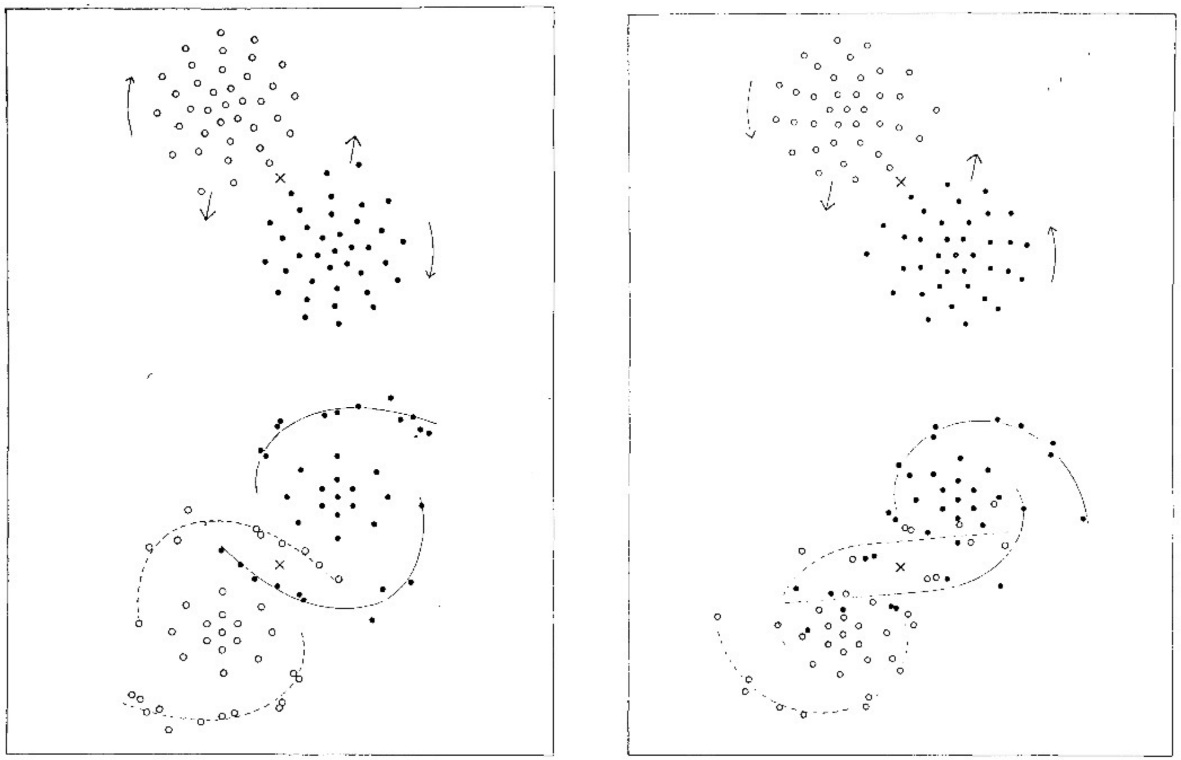


(Mice Galaxies)

1941

Erik Holmberg

- replacing gravity by light (same $1/r^2$ law)
- formation of tidal features



4m

3m

$N = 2\text{x } 37$

- gravity of N bodies

$$\boxed{m_i \ddot{\vec{r}}_i = \vec{F}(\vec{r}_i) \quad \forall i \in N}$$

- the "brute force approach" scales like N^2:

$$\vec{F}(\vec{r}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

the summation over (N-1) particles has to be done for all N particles:

$\Rightarrow$ number of floating point operations $\propto N(N\text{-}1) \propto N^2$

- gravity of N bodies

$$m_i \ddot{\vec{r}}_i = \vec{F}(\vec{r}_i) \quad \forall i \in N$$

- the "brute force approach" scales like N^2:

$$\vec{F}(\vec{r}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{(\vec{r}_i - \vec{r}_j)^3} (\vec{r}_i - \vec{r}_j)$$

even nowadays not a feasible approach!

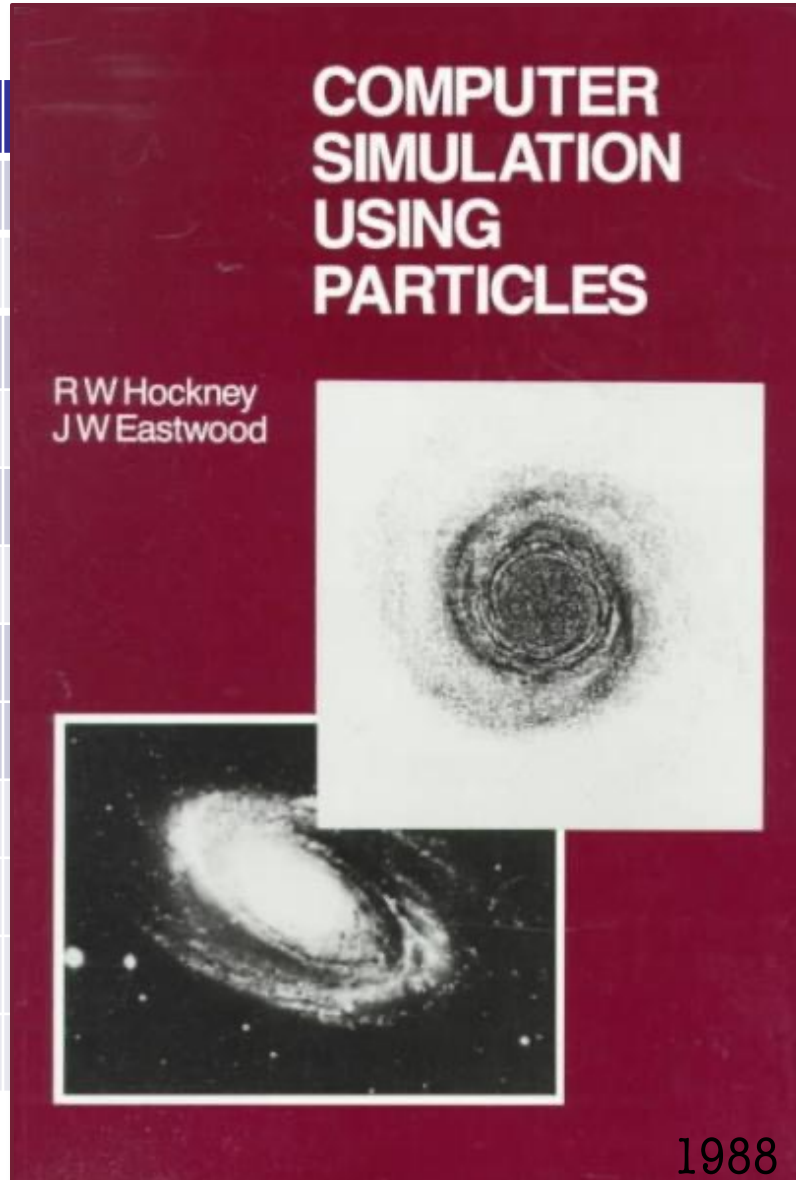
➔ sophisticated techniques are required...

the summation over (N-1) particles has to be done for all N particles:

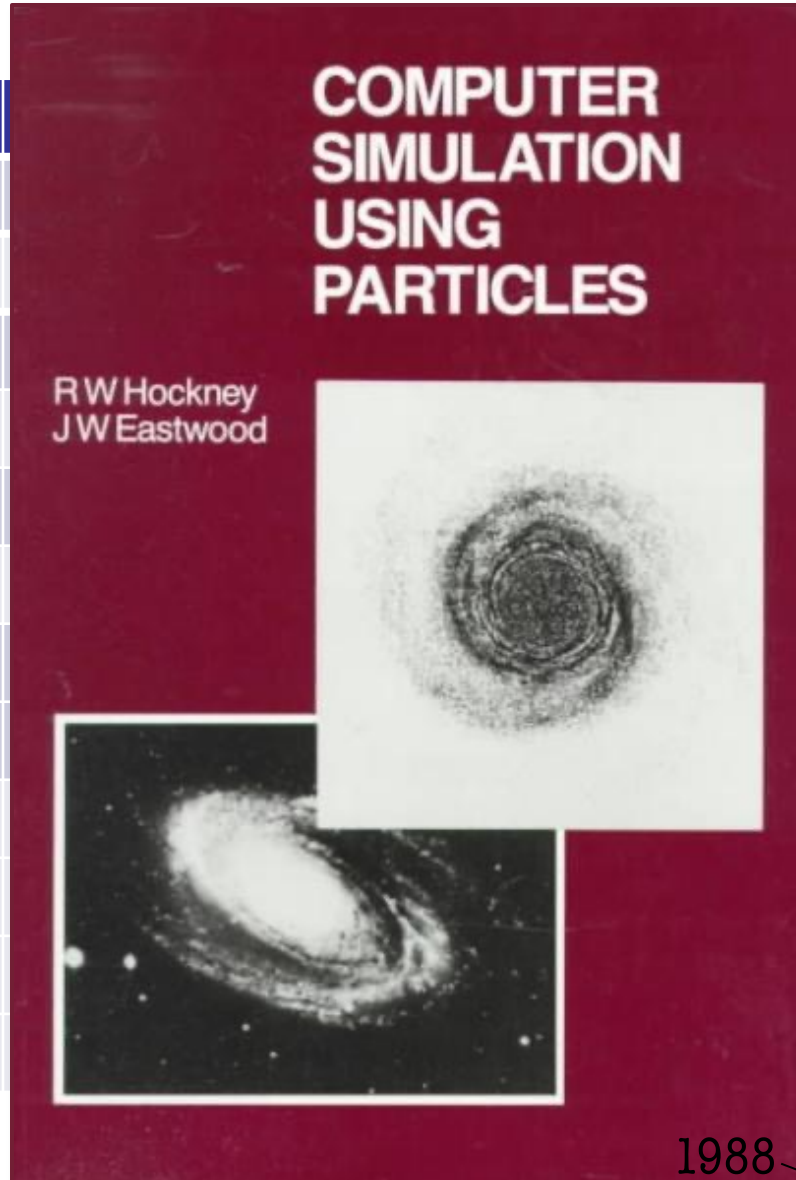
$\Rightarrow$ number of floating point operations $\propto N(N-1) \propto N^2$

year	who	what
1941	Erik Holmberg	light bulbs
1963	Svere Aarseth	NBODY
1981	George Efstathiou	P^3M
1983	Anatoly Klypin	PM
1986	Barnes & Hut	tree
1991	Hugh Couchman	AP^3M
1995	Suisalu & Saar	AMR (**A**daptive **M**esh **R**efinement)
1997	Kravtsov	ART
…		
2000++	Springel	GADGET
	Springel	Arepo
	Hopkins	GIZMO

year	
1941	
1963	
1981	
1983	
1986	
1991	
1995	esh **R**efinement)
1997	
…	
2000++	

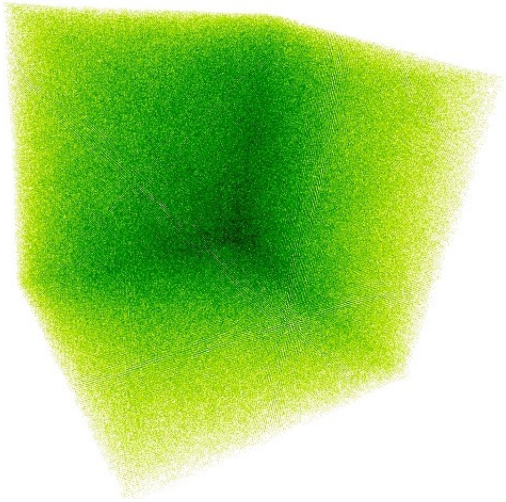


year	
1941	
1963	
1981	
1983	
1986	
1991	
1995	esh **R**efinement)
1997	
…	
2000++	



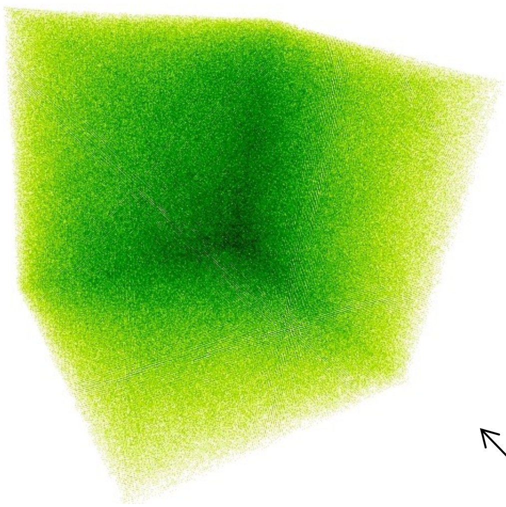
1988 → ***still the one-and-only reference!***

- generating initial conditions



1. primordial matter density field

- generating initial conditions



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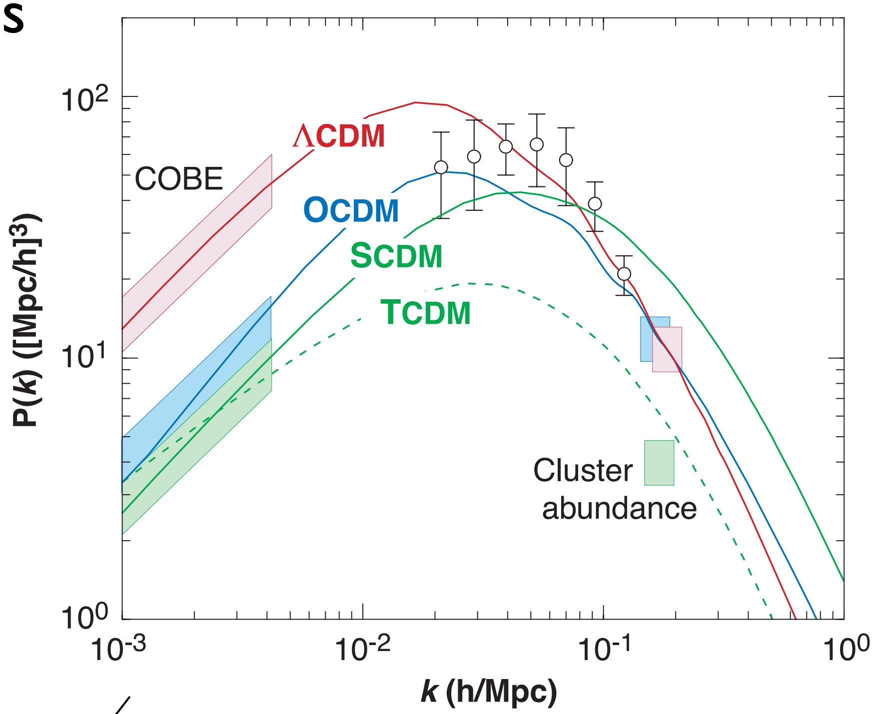
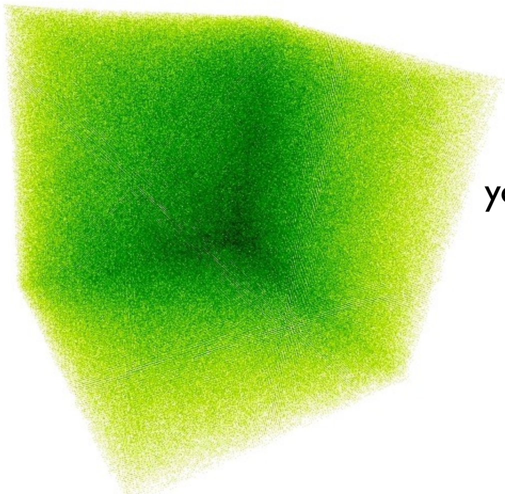
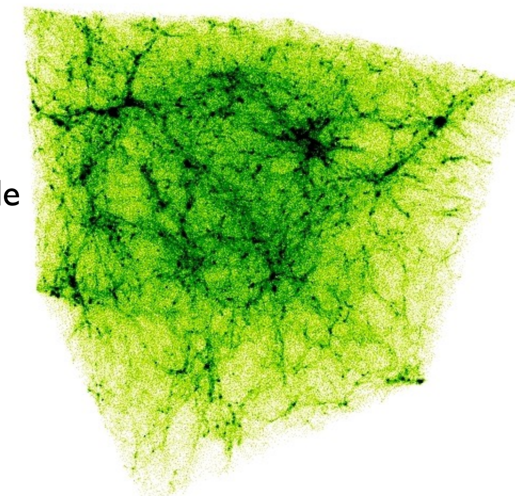


anisotropies in the matter field
(as calculated by Boltzmann solvers)

- running the simulation



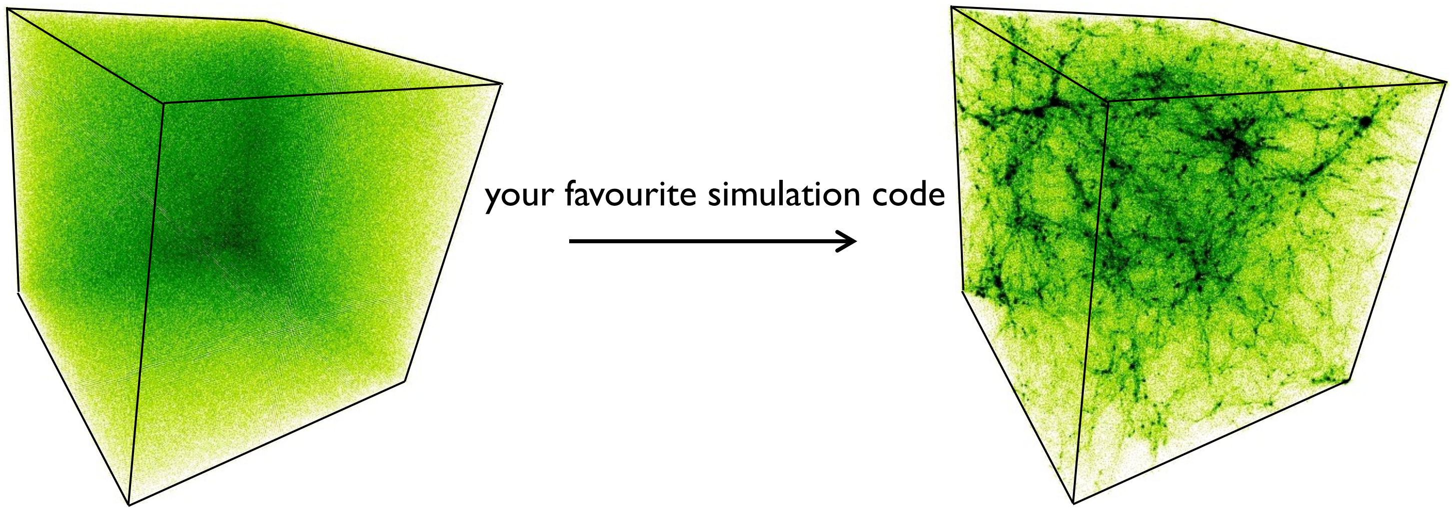
your favourite simulation code →



1. primordial matter density field

2. today's matter density field

- running the simulation

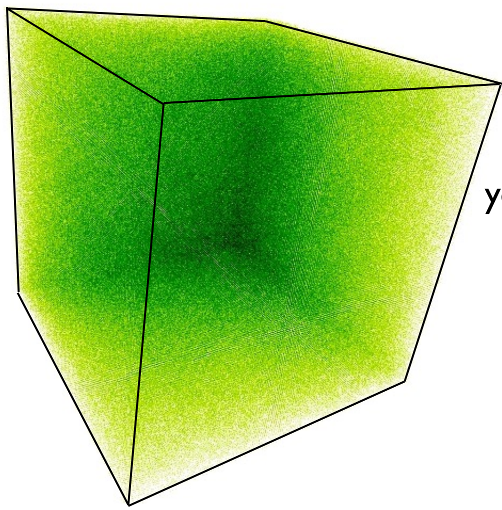


1. primordial matter density field

2. today's matter density field

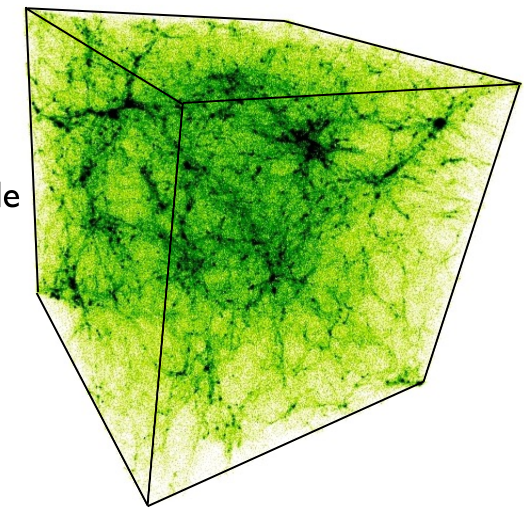
cubical universe?

- running the simulation



1. primordial matter density field

your favourite simulation code →



2. today's matter density field

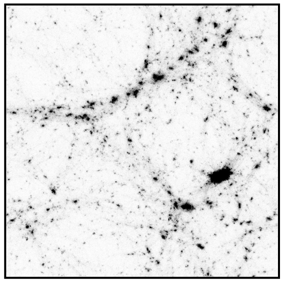
cubical universe: infinity via periodic boundaries!

- running the simulation

1. primordial matter density field

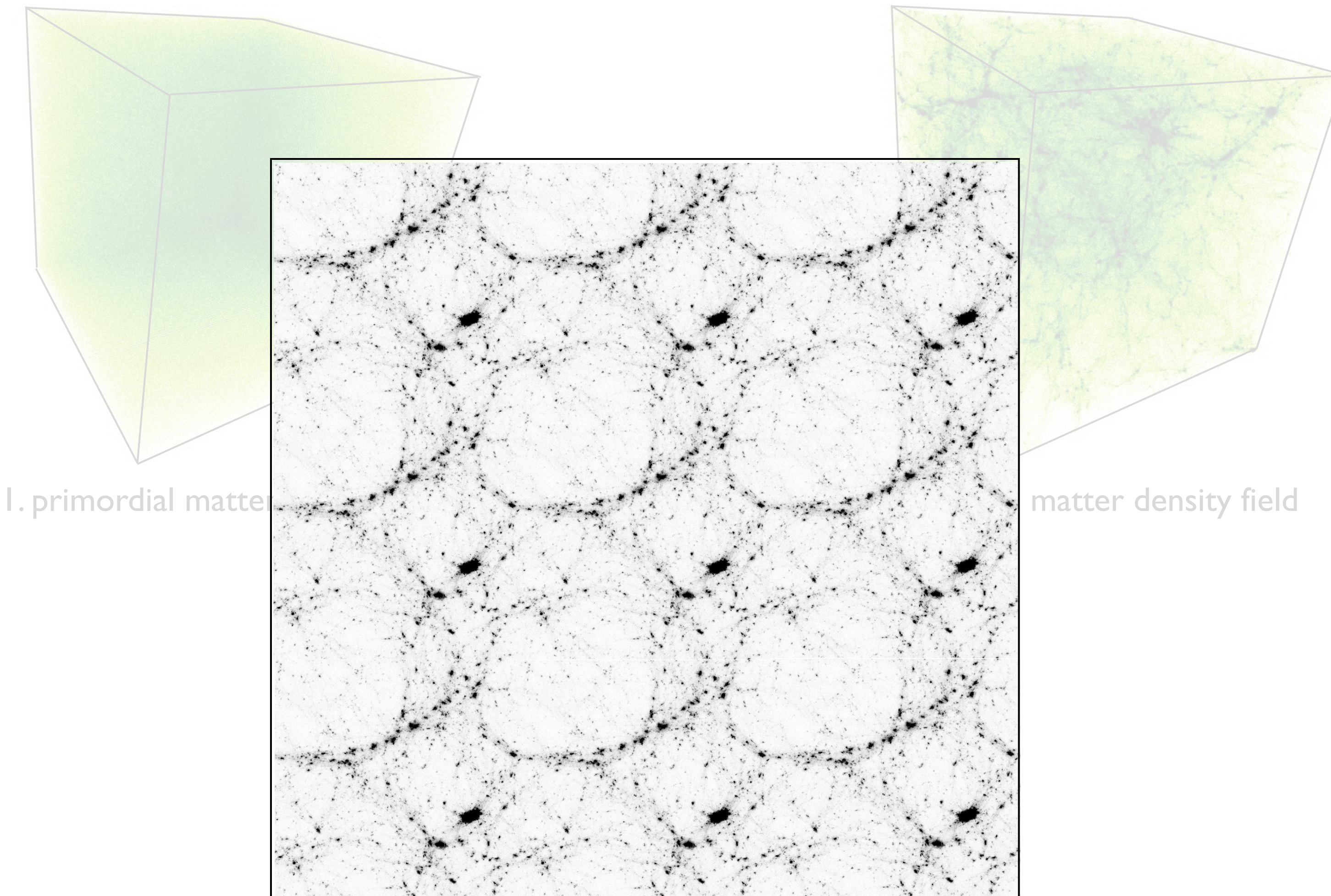
your favourite simulation code →

2. today's matter density field



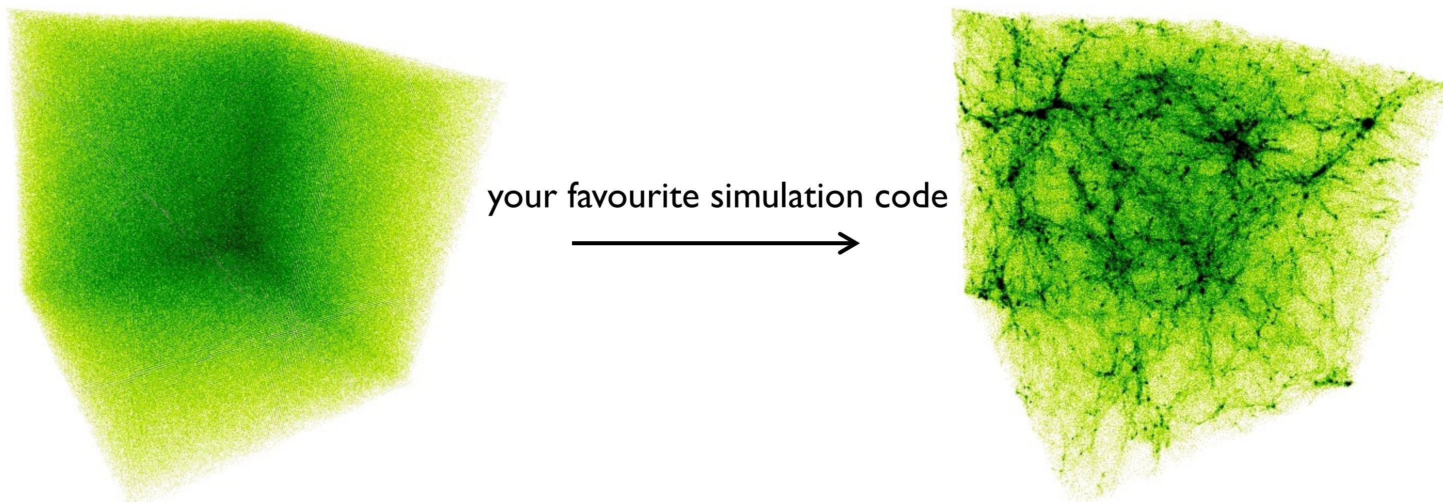
cubical universe: infinity via periodic boundaries!

- running the simulation



cubical universe: infinity via periodic boundaries!

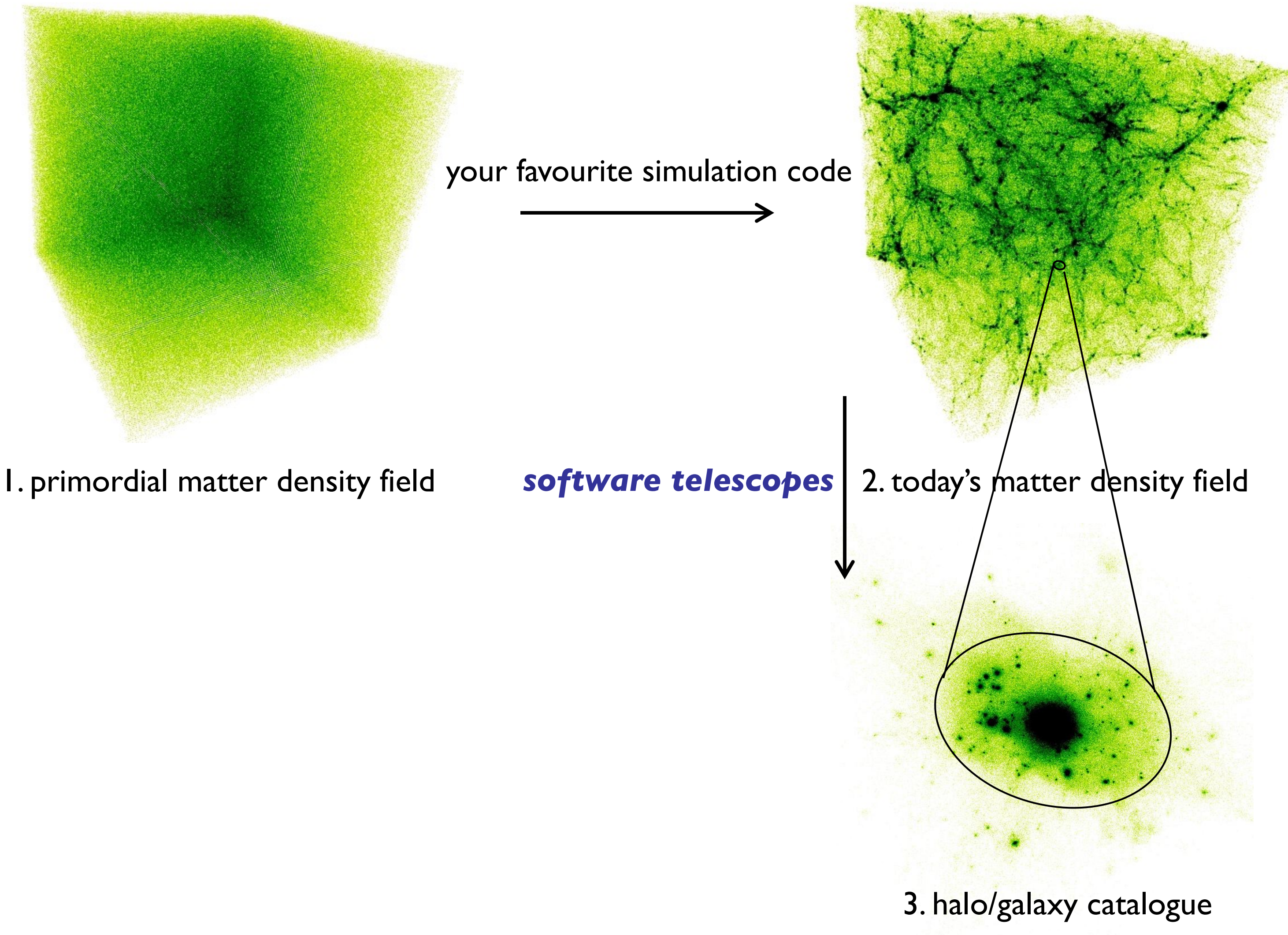
- running the simulation



1. primordial matter density field

2. today's matter density field

- analysing the outputs



ourite si

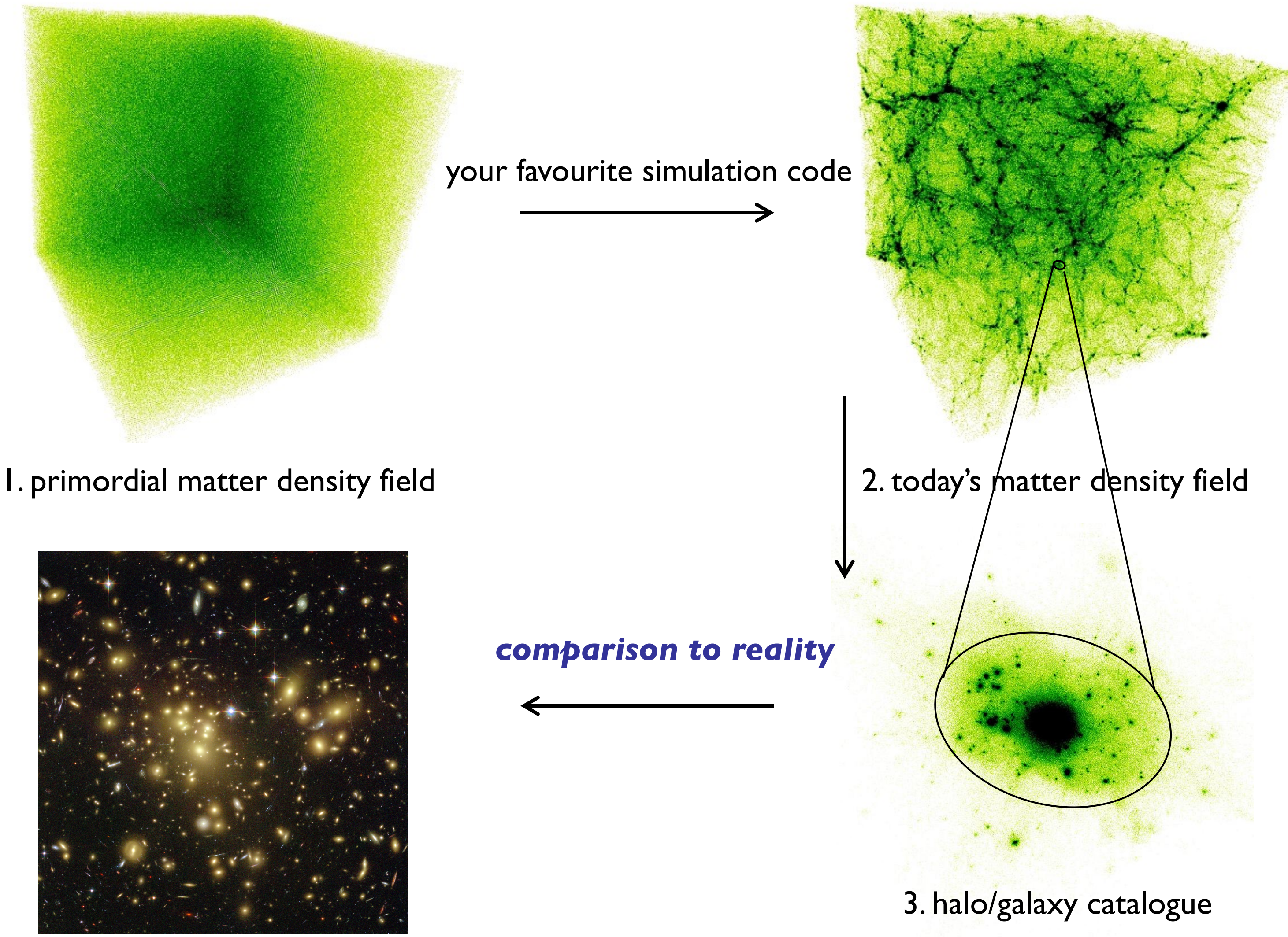
d

software telescopes

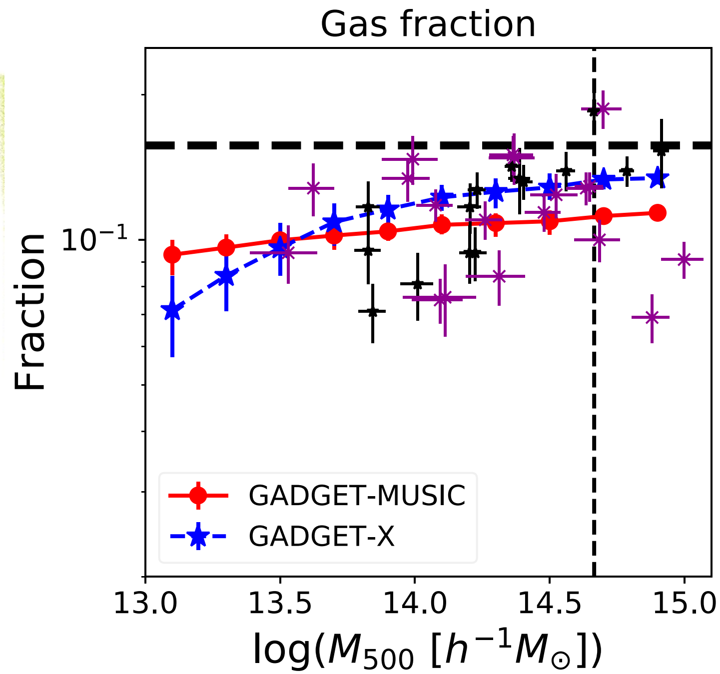
2. today's matter density field

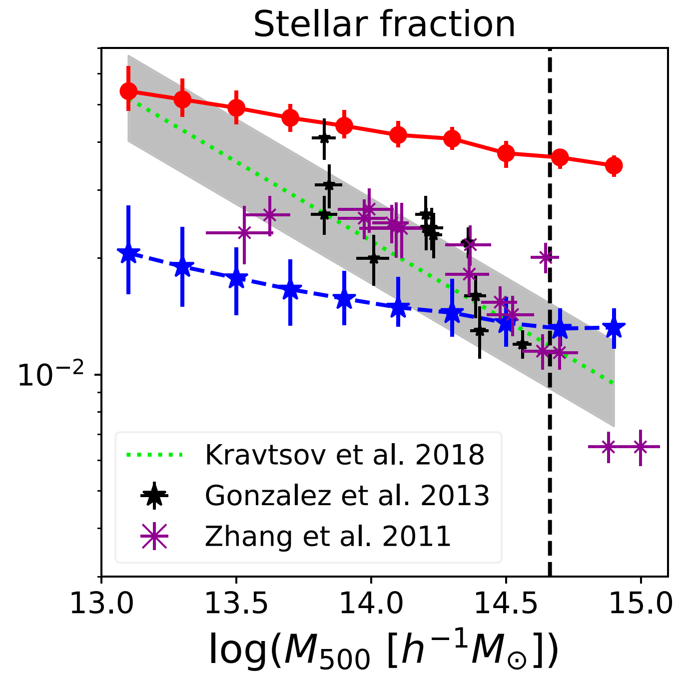
3. halo/galaxy catalogue

• analysing the outputs: comparison



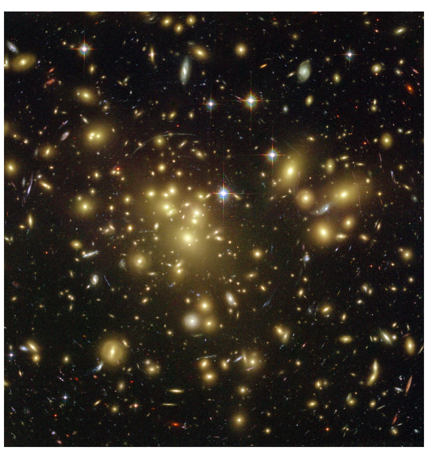
- analysing the outputs: comparison



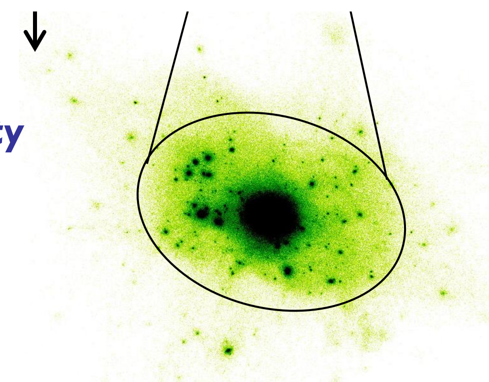


(Cui et al. 2018)

1.

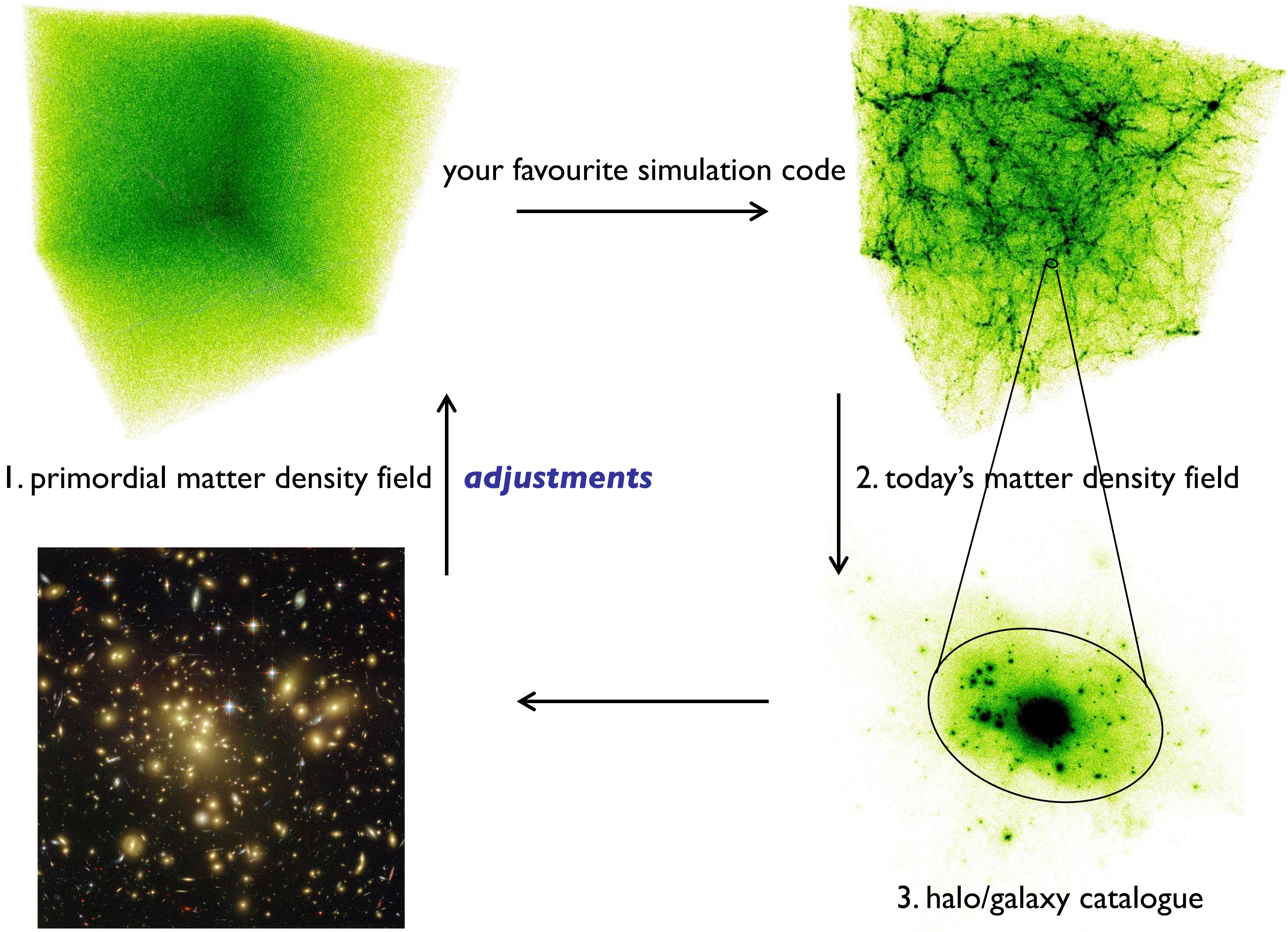


comparison to reality

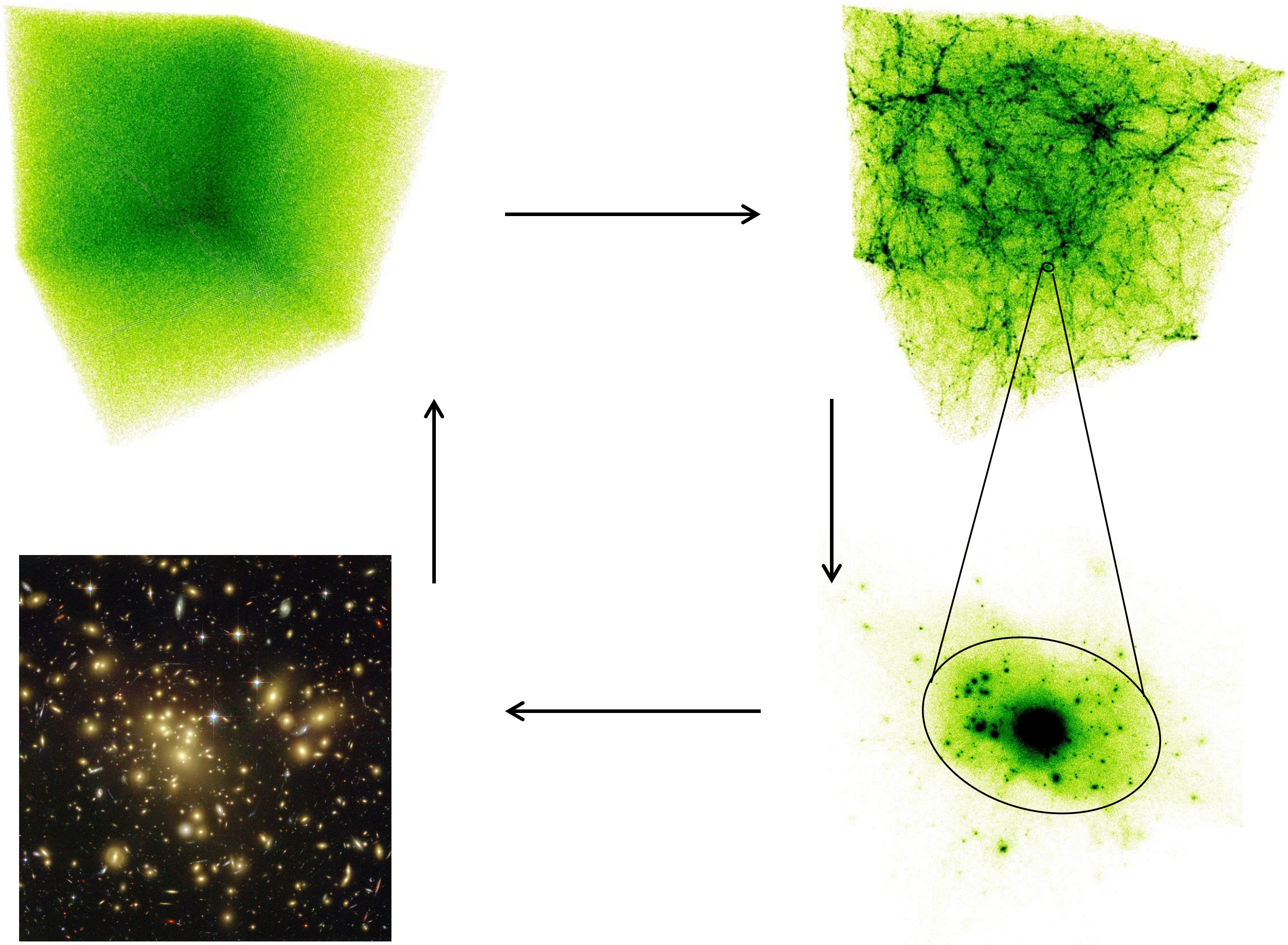


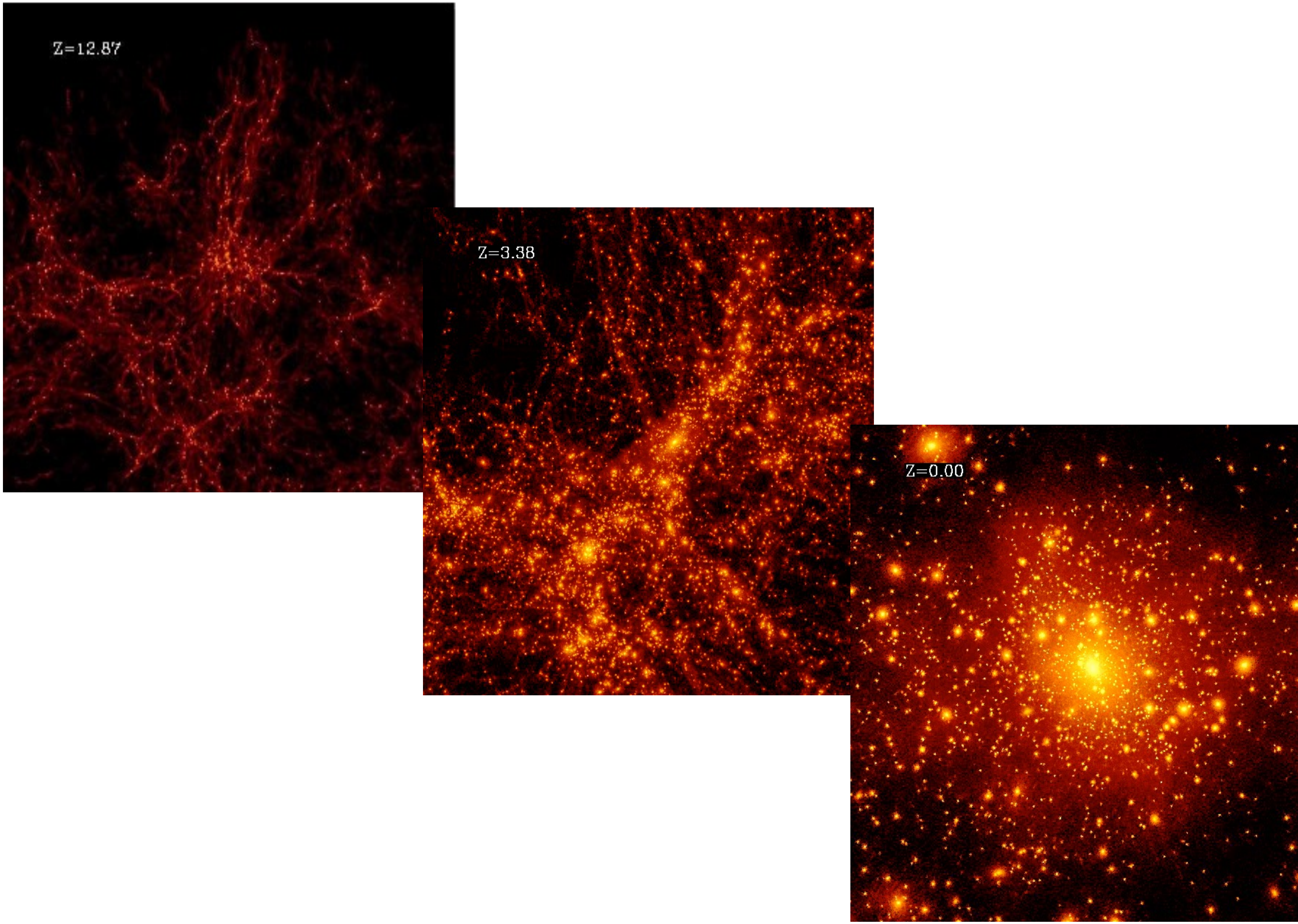
3. halo/galaxy catalogue

- analysing the outputs: comparison and feedback!?



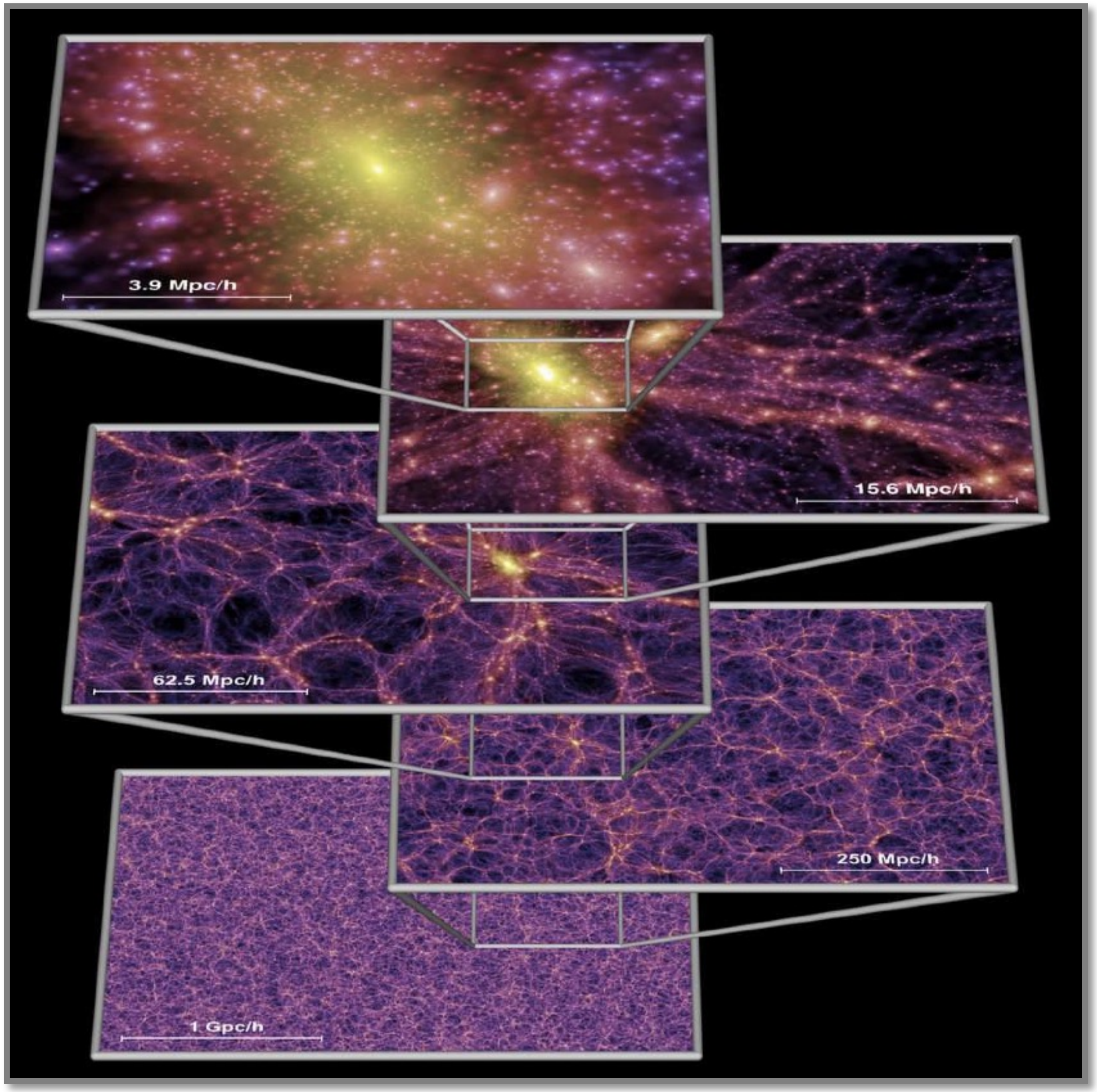
- simulation of cosmic structure formation





Z=12.87
Z=3.38
Z=0.00

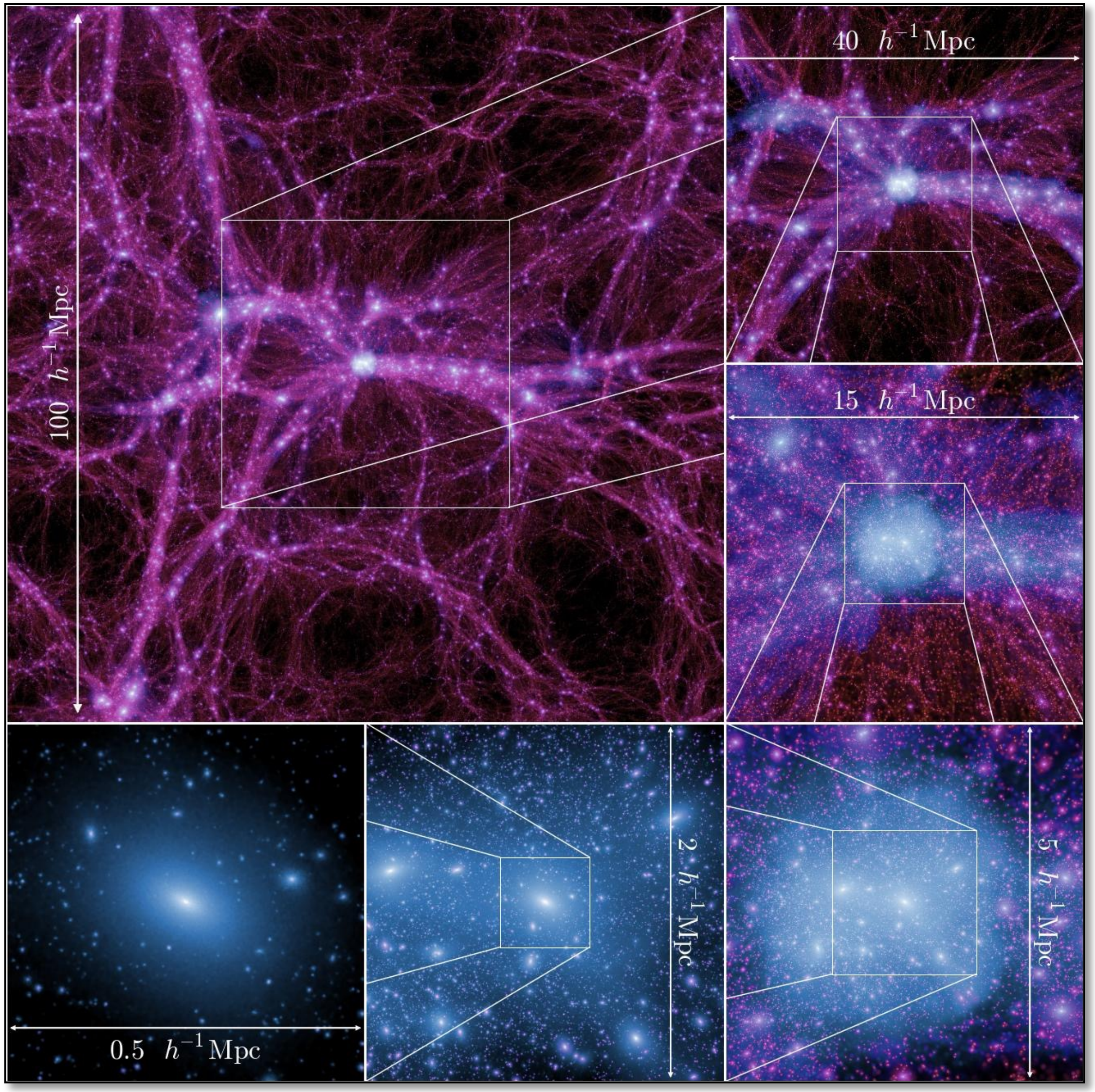
- Millennium simulation (2005)



$N = 2160^3$

$B = 500$ Mpc/h

- Millennium-II simulation (2010)

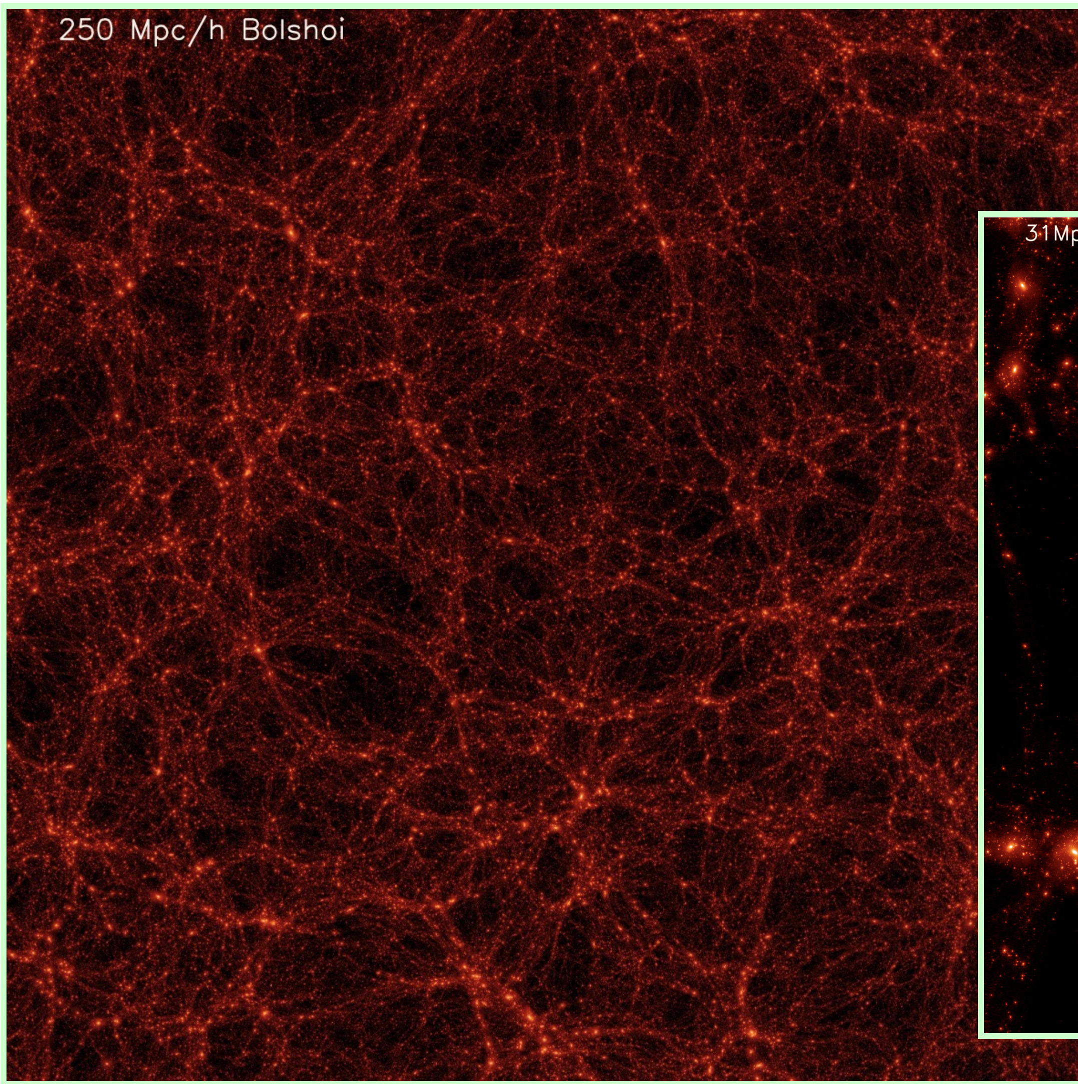


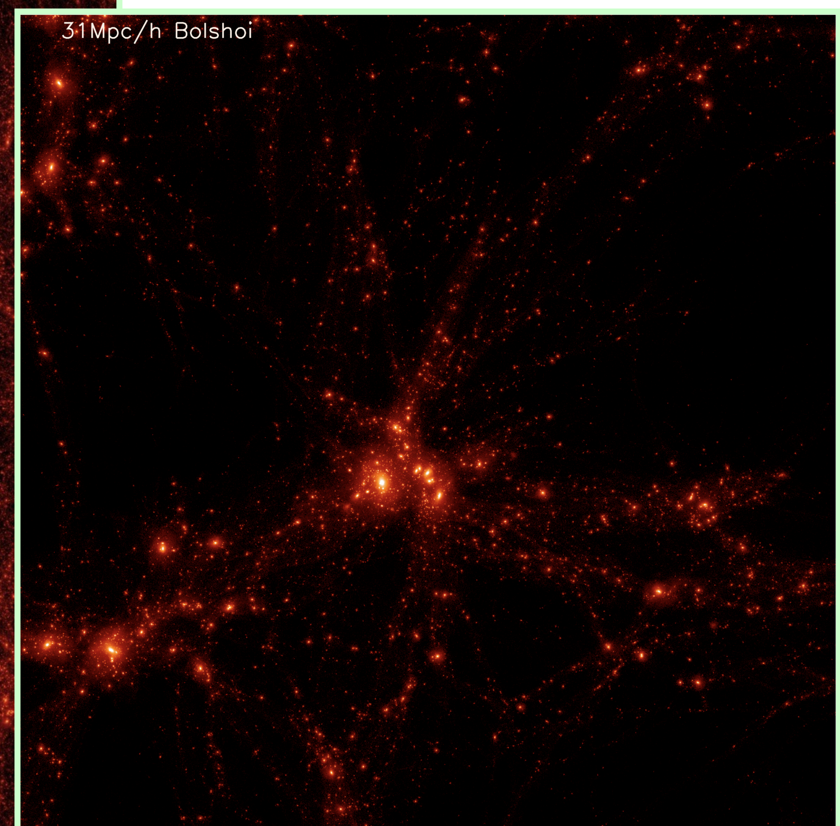
$N = 2160^3$

$B = 100$ Mpc/h

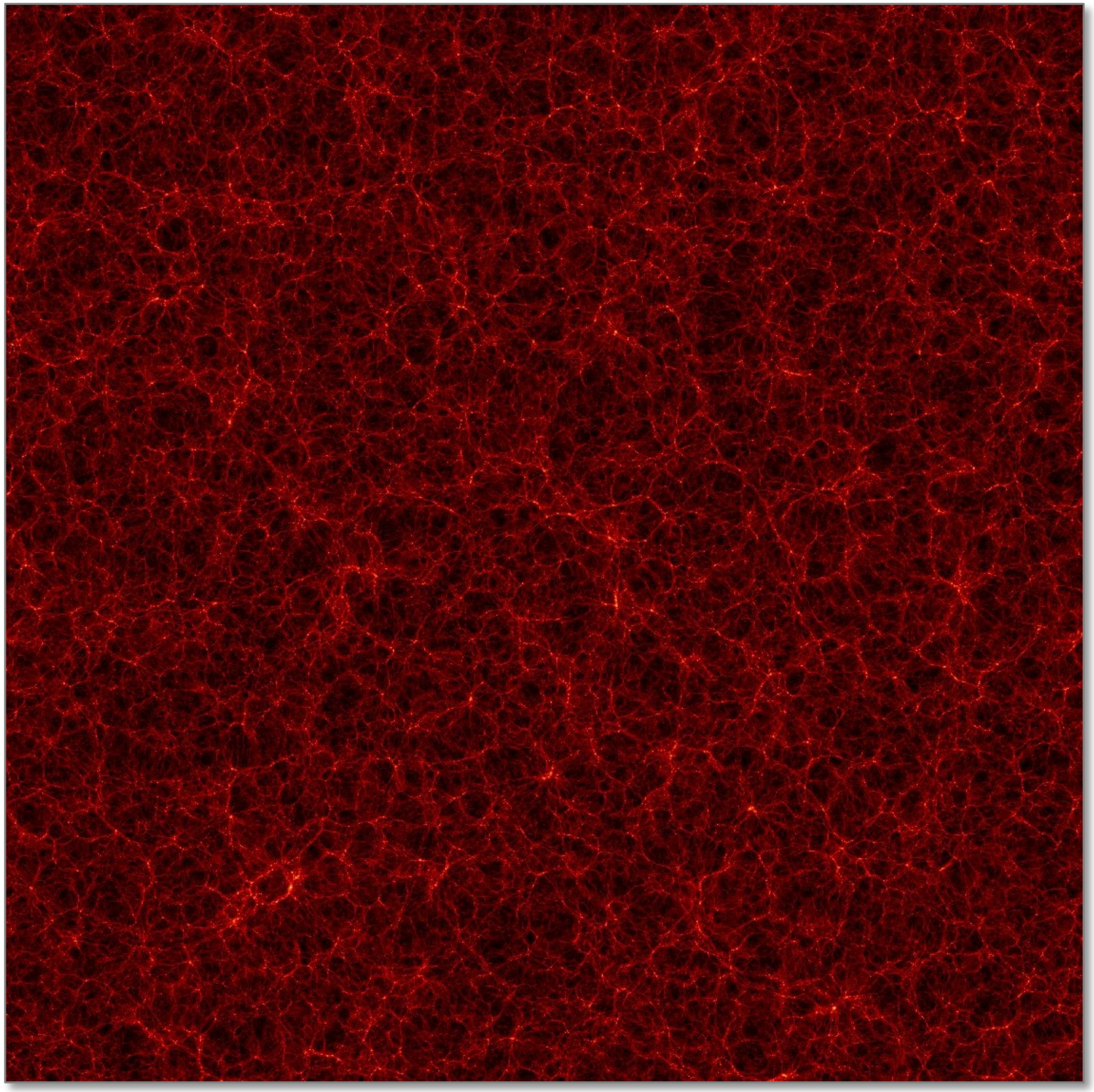
- Bolshoi simulation

$N = 2048^3$
$B = 250 \text{ Mpc}/h$





- MultiDark simulation

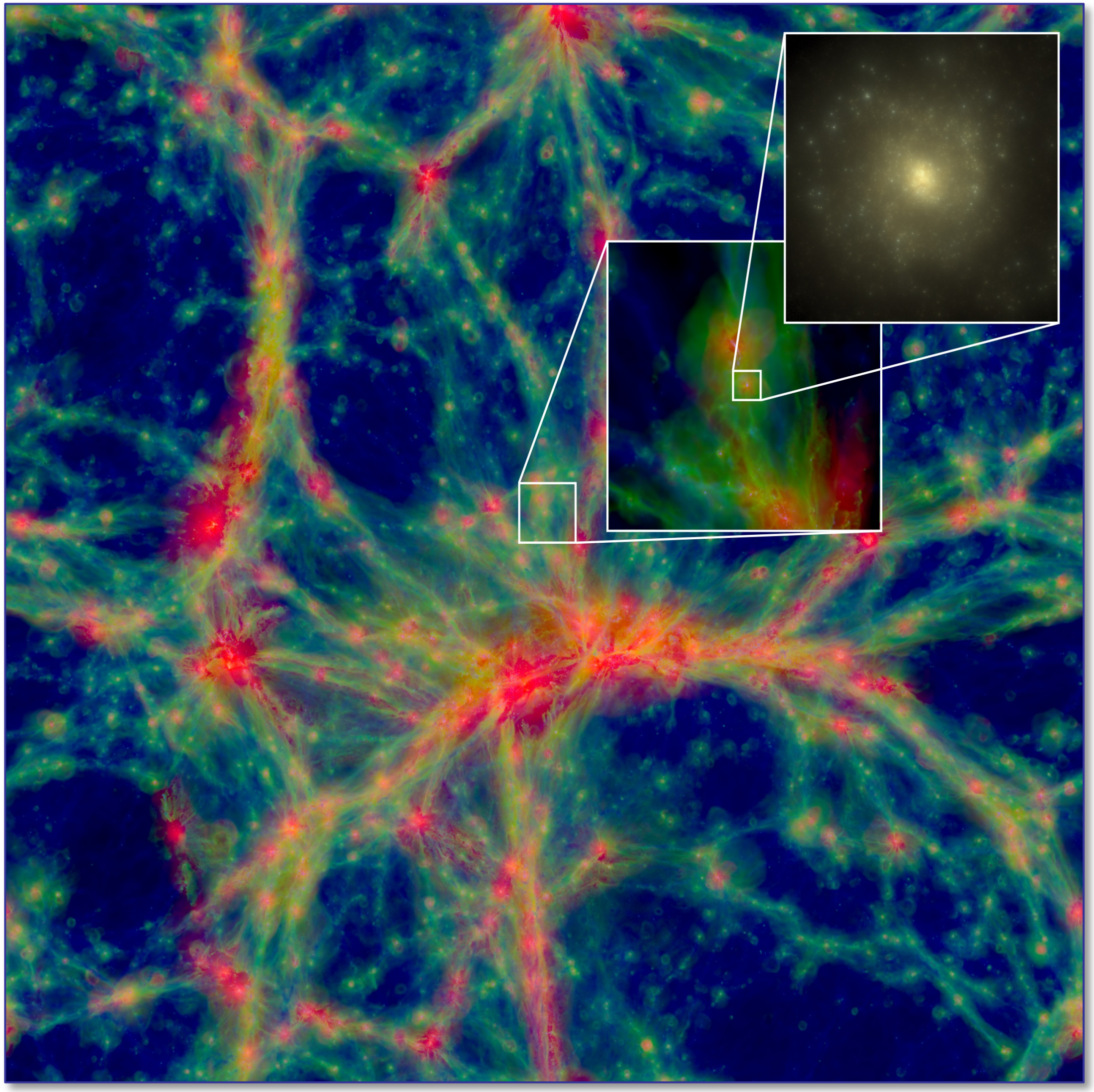


$N = 3840^3$
$B = 1000$ Mpc/h

- MultiDark simulation

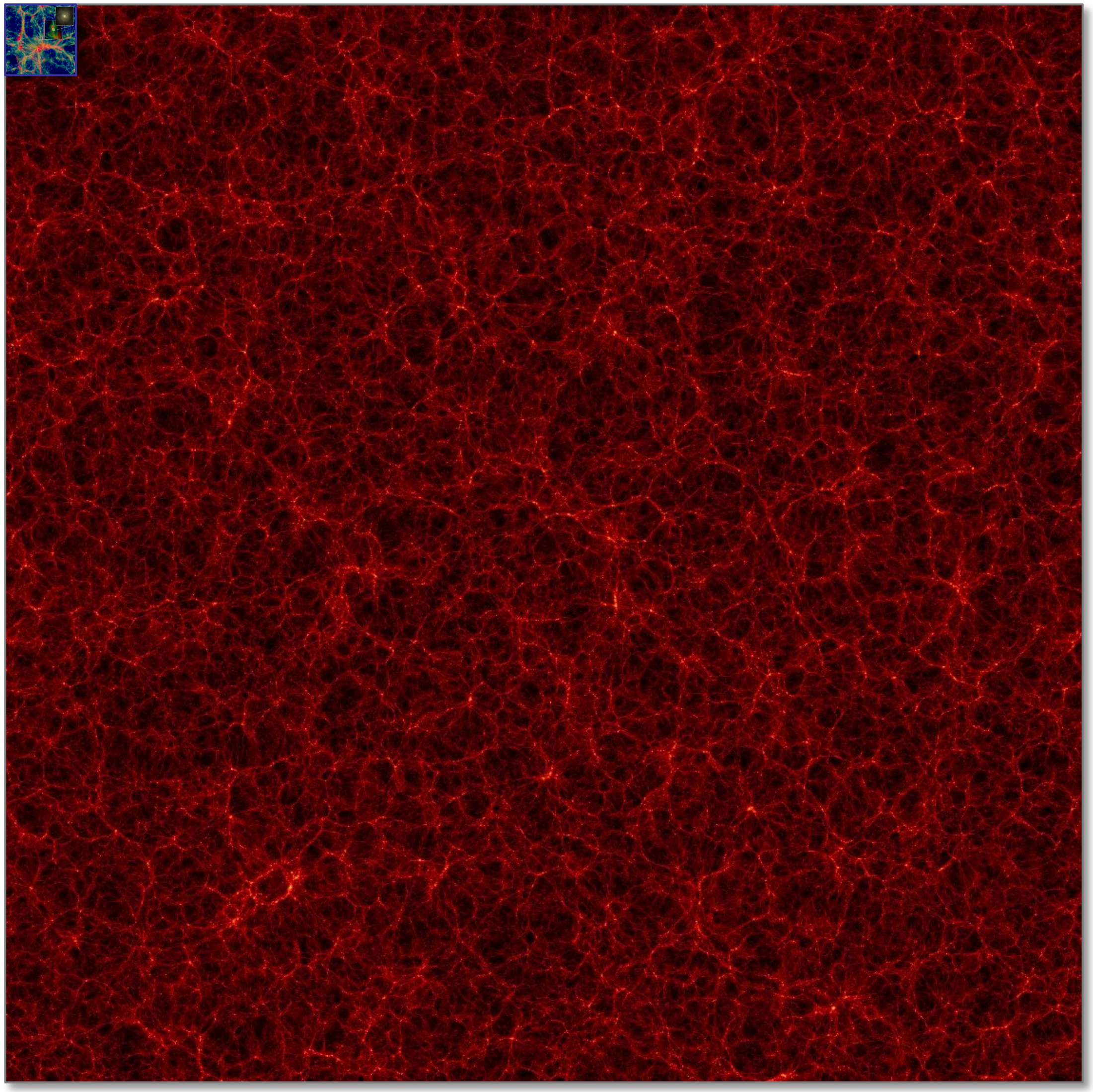
$N = 3840^3$
$B = 1000\ \mathrm{Mpc}/h$

- EAGLE full physics simulation

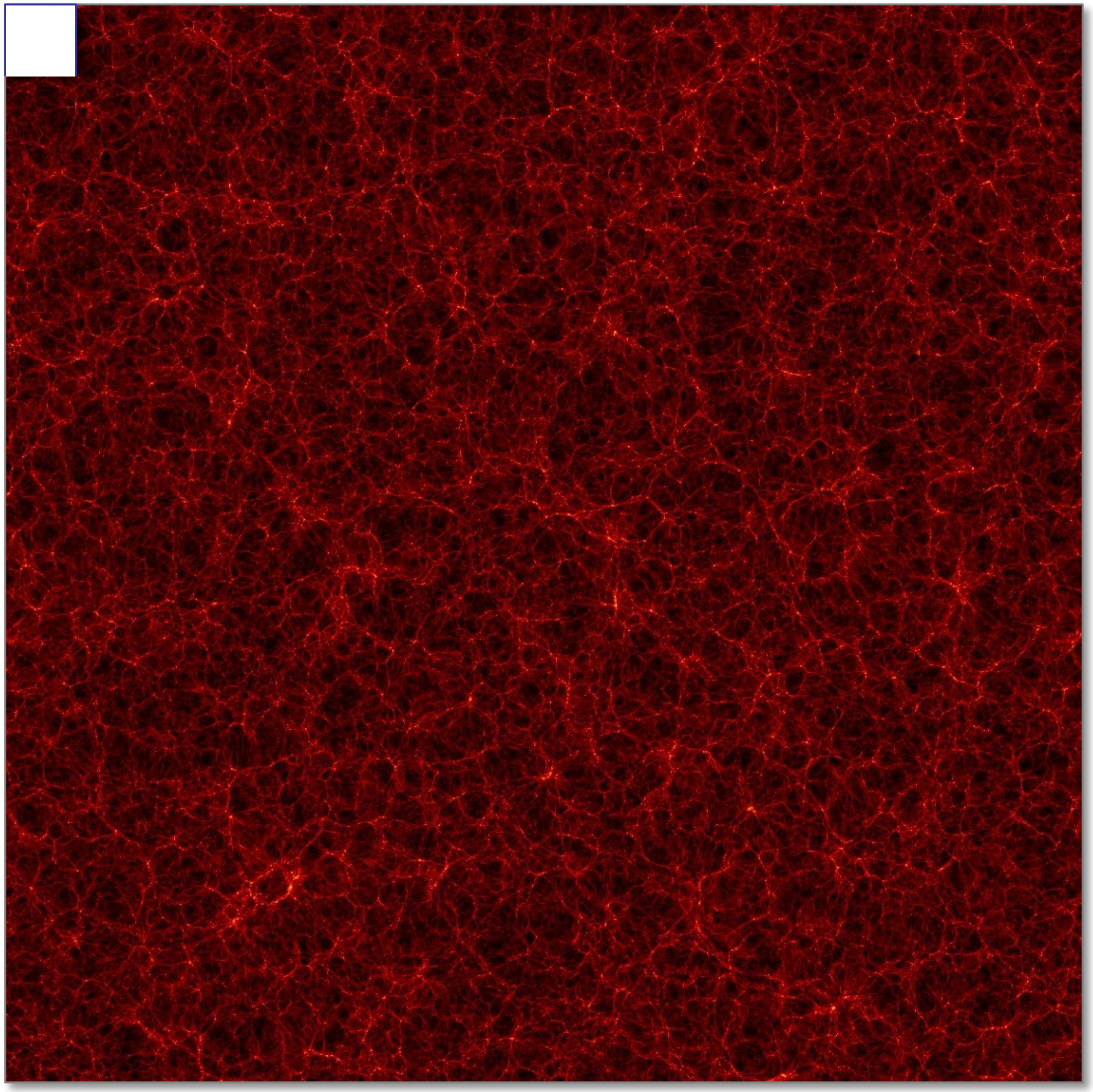


$N = 2000^3$
$B = 75$ Mpc/h

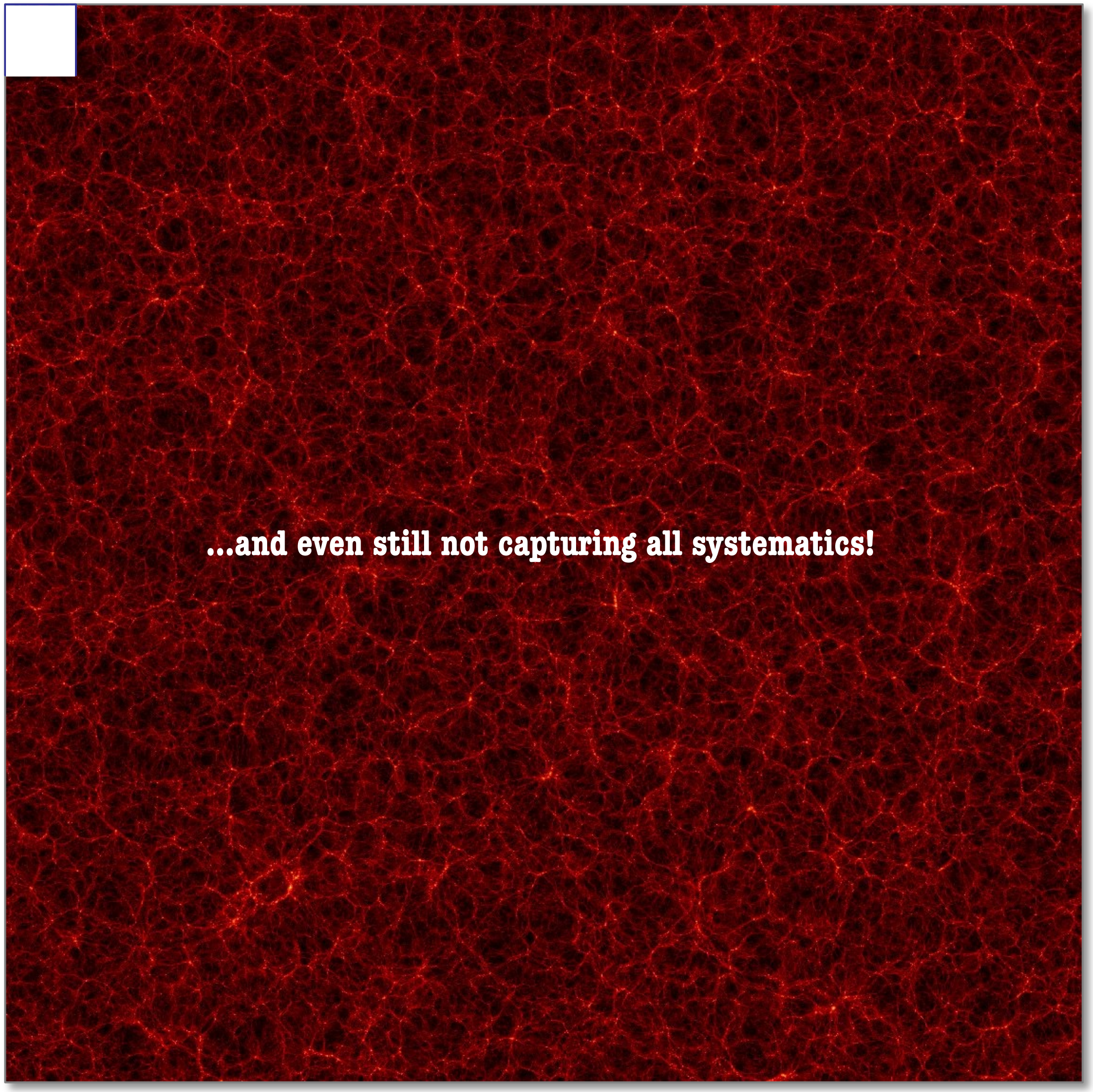
- EAGLE full physics vs. MultiDark dark-matter only simulation



- EAGLE full physics vs. MultiDark dark-matter only simulation

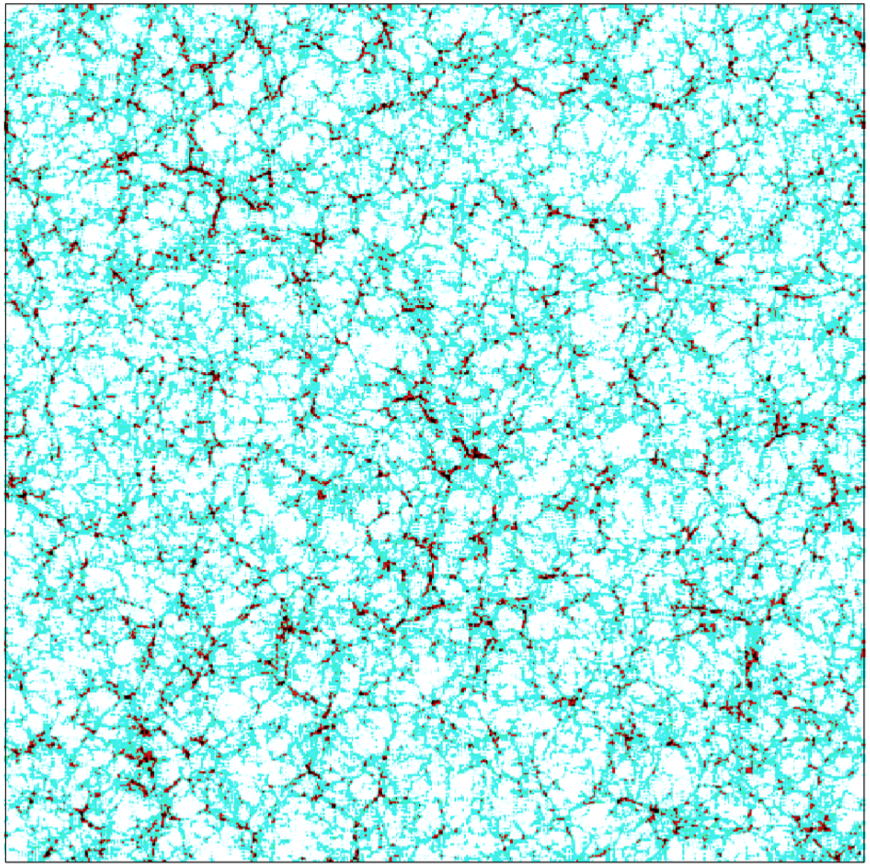


- EAGLE full physics vs. MultiDark dark-matter only simulation



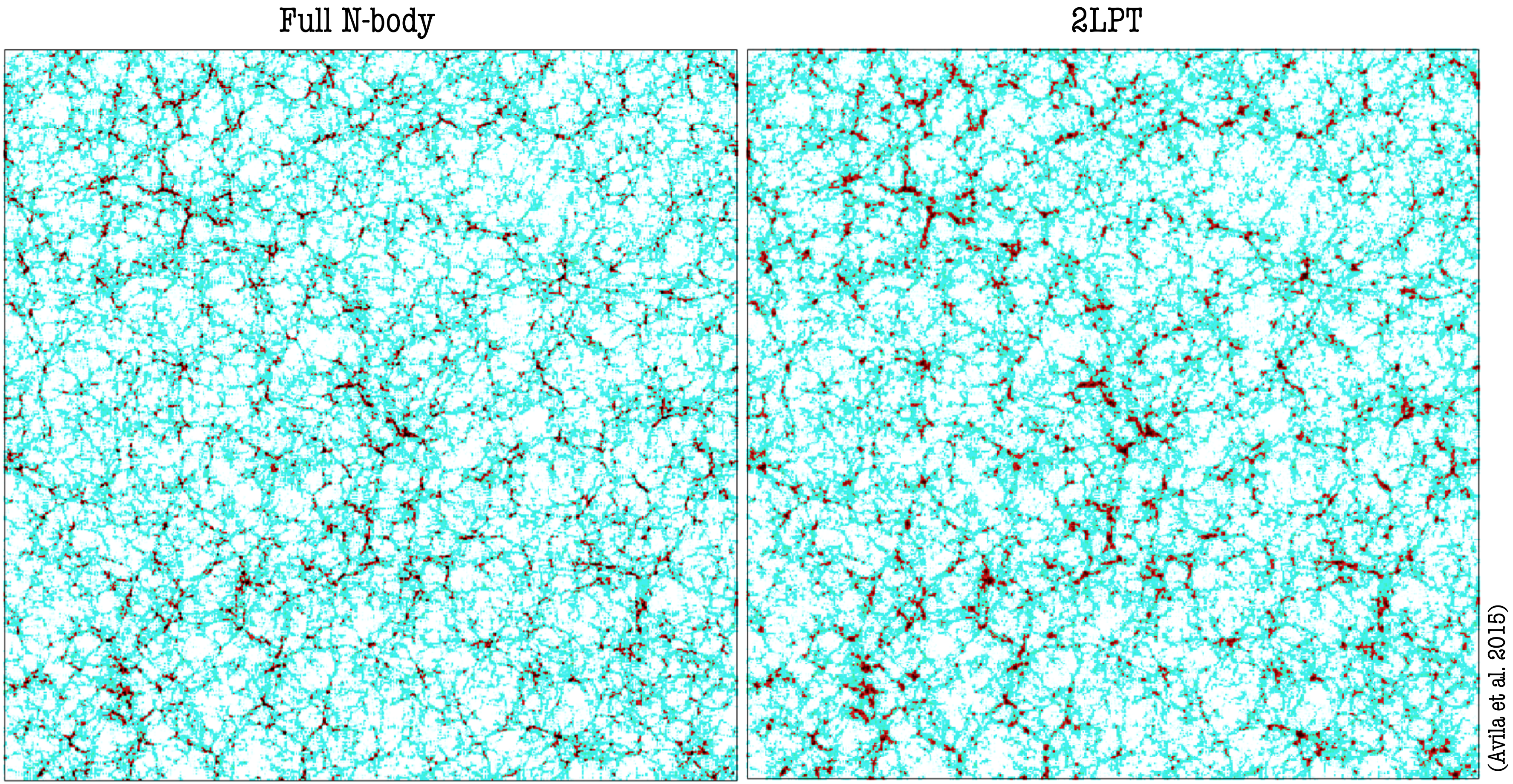
- approximate methods vs. full simulations

Full N-body



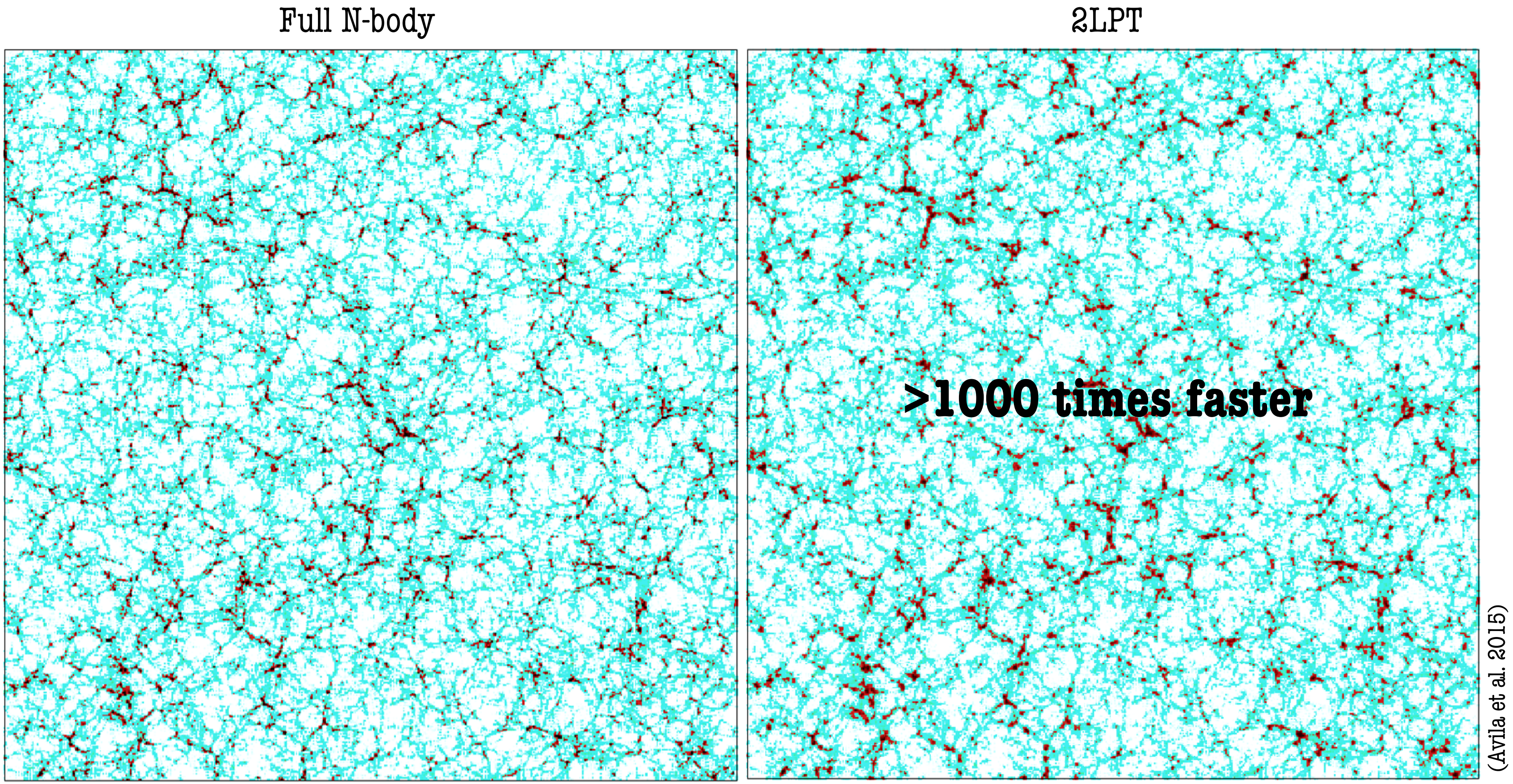
$z = 0.5$

- approximate methods vs. full simulations



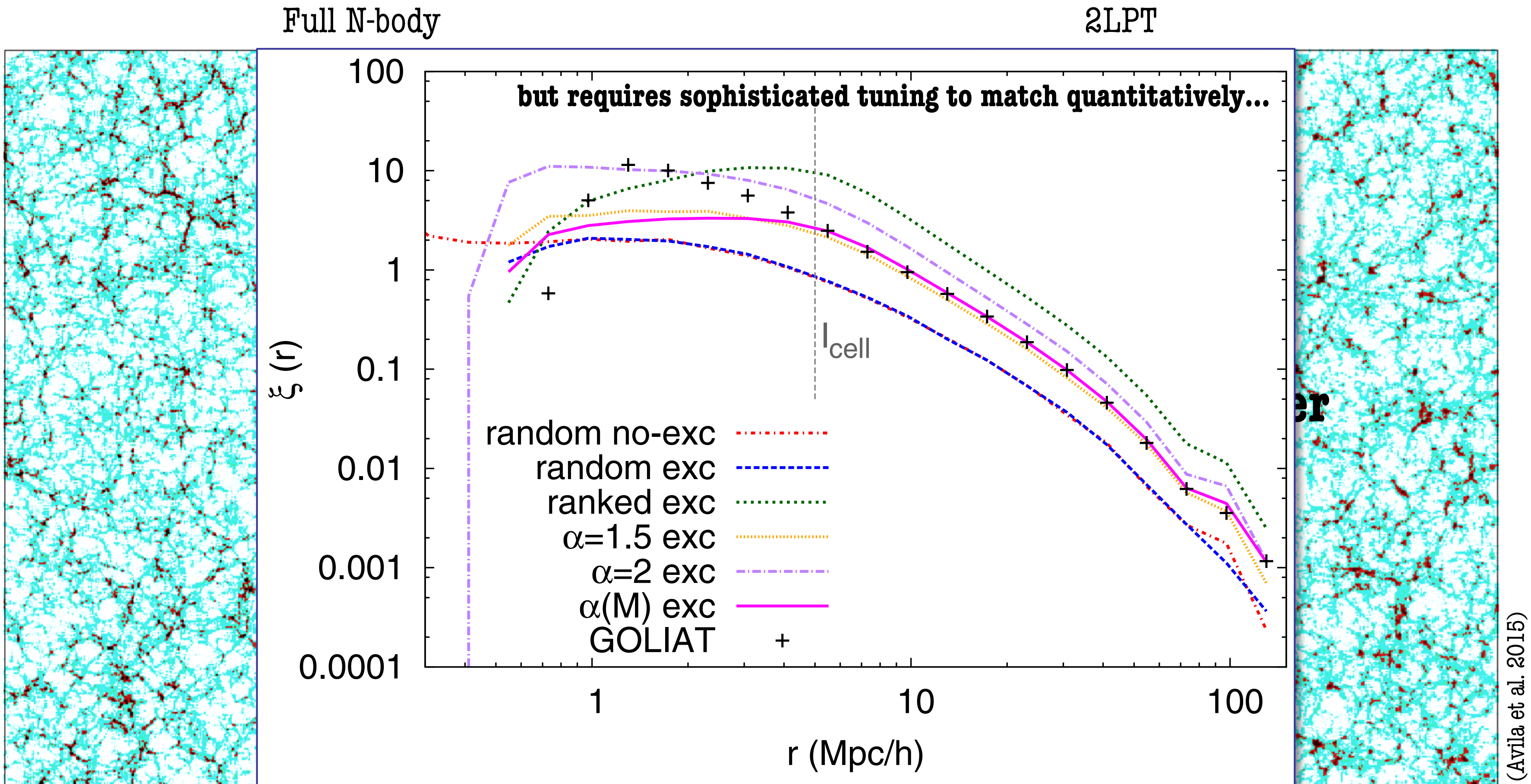
$z = 0.5$

- approximate methods vs. full simulations



$z = 0.5$

- approximate methods vs. full simulations



$z = 0.5$

- introduction
- Boltzmann solver
- initial conditions generators
- simulation codes
  - **the N-body principle**
  - the equations-of-motion
  - the forces

dark matter particles are collisionless!

(i.e. the evolution is driven by the mean field rather than 2-body interactions)

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- mean free path of dark matter particles

$$\lambda = \frac{1}{n\sigma}$$

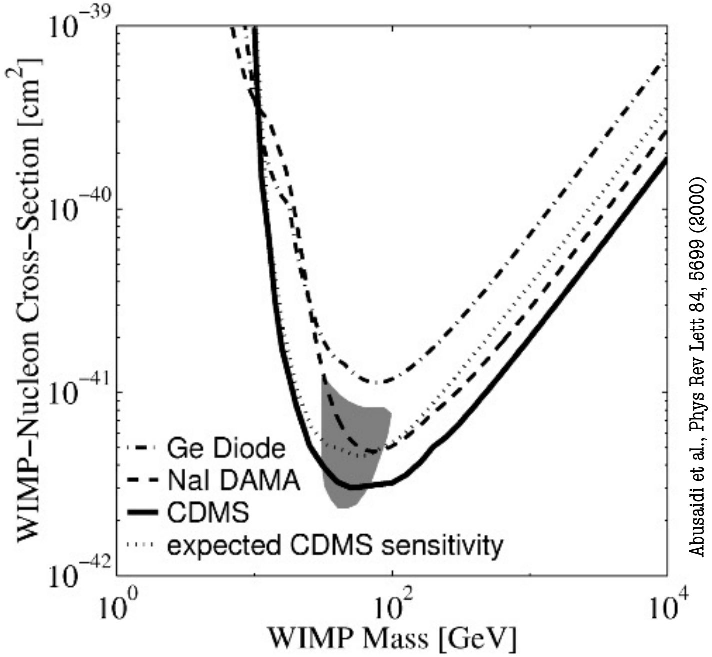
dark matter particles are collisionless!

(i.e. the evolution is driven by the mean field rather than 2-body interactions)

- mean free path of dark matter particles

$$\left.\begin{aligned}\sigma &\approx 10^{-42} cm^2 \\ m_{DM} &\approx 10^2 GeV \approx 10^{-22} g\end{aligned}\right\} \longleftarrow$$

$$\rho_{crit} \approx 10^{-30} \frac{g}{cm^3}$$



$$\rho_{crit} = \frac{N\, m_{DM}}{V} = n\, m_{DM}$$

$$n \approx 10^{-8} \frac{1}{cm^3}$$

$$\lambda = \frac{1}{n\sigma}$$

dark matter particles are collisionless!

(i.e. the evolution is driven by the mean field rather than 2-body interactions)

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$$m_{DM} \approx 10^2 GeV \approx 10^{-22} g$$

$$\rho_{crit} \approx 10^{-30} \frac{g}{cm^3}$$

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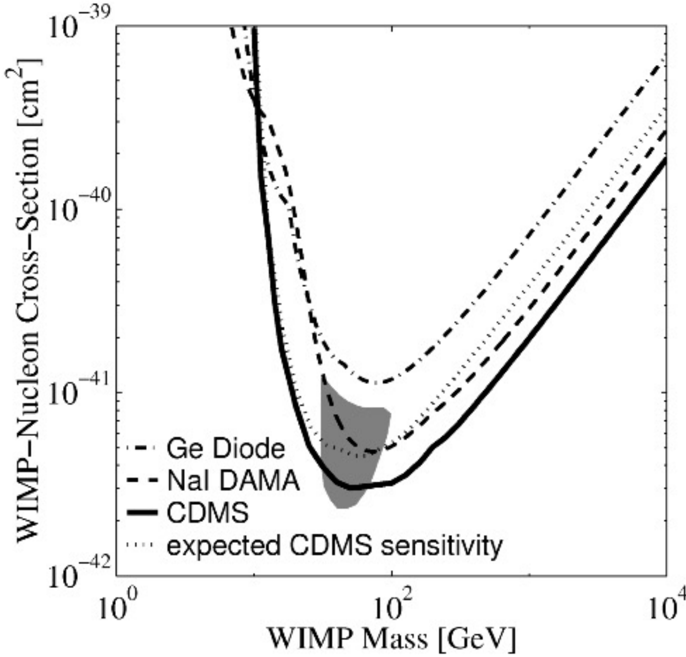


Abusaidi et al., Phys Rev Lett 84, 5699 (2000)

$$\lambda = \frac{1}{n\sigma} \approx \frac{1}{10^{-8} 10^{-42}} cm = 10^{50} cm \approx 10^{30} Mpc$$

how to describe a collisionless system?

- phase-space distribution function

$$f(\vec{r},\vec{v},t)\ d^3r\ d^3v$$

probability* of finding a dark matter particle in the interval:

$$[\vec{r}-\frac{d\vec{r}}{2},\ \vec{r}+\frac{d\vec{r}}{2}]$$
$$[\vec{v}-\frac{d\vec{v}}{2},\ \vec{v}+\frac{d\vec{v}}{2}]$$

e.g., particle with velocity v_1 and coordinate r_1: $f(\vec{r},\vec{v})=\delta(\vec{r}-\vec{r}_1)\delta(\vec{v}-\vec{v}_1)$

* $\int f(\vec{r},\vec{v},t)\ d^3r\ d^3v=1$

continuity, self-gravity and no collisions =>

- collisionless Boltzmann equation (CBE)

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3}\left(v_i \frac{\partial f}{\partial r_i} - \frac{\partial \Phi}{\partial r_i}\frac{\partial f}{\partial v_i}\right) = 0$$

- coupled with Poisson's equation

$$\Delta\Phi(\vec{r}) = 4\pi G\rho(\vec{r})$$

- collisionless Boltzmann equation (CBE)

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3}\left(v_i \frac{\partial f}{\partial r_i} - \frac{\partial \Phi}{\partial r_i}\frac{\partial f}{\partial v_i}\right) = 0$$ ⇒ **difficult to solve numerically!**

- coupled with Poisson's equation

$$\Delta\Phi(\vec{r}) = 4\pi G\rho(\vec{r})$$ ⇒ **we'll deal with it later…**

- collisionless Boltzmann equation (CBE)

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3}\left(v_i \frac{\partial f}{\partial r_i} - \frac{\partial \Phi}{\partial r_i}\frac{\partial f}{\partial v_i}\right) = 0$$

$$\downarrow$$

- "method of characteristics":

$$\frac{\partial f}{\partial t} + \left\{ f, H \right\} = 0 \qquad\qquad H = \frac{1}{2}v^2 + \Phi(\vec{r})$$

$$\downarrow$$

$$\frac{df(\vec{r},\vec{v})}{dt} = 0$$

- "method of characteristics":

$$\frac{\partial f}{\partial t} + \left\{ f, H \right\} = 0 \qquad H = \frac{1}{2} v^2 + \Phi(\vec{r})$$

$$\downarrow$$

$$\frac{df(\vec{r},\vec{v})}{dt} = 0$$

$$\downarrow$$

f is constant along the possible trajectories $[\vec{r}(t),\vec{v}(t)]$

$f(\vec{r},\vec{v},t) = f(\vec{r}_0,\vec{v}_0,0) \quad \forall \vec{r},\vec{v}$ satisfying $\{\vec{r},H\} = \frac{\partial H}{\partial \vec{v}}$, $\{\vec{v},H\} = -\frac{\partial H}{\partial \vec{r}}$

- "method of characteristics":

$$\frac{\partial f}{\partial t} + \left\{ f, H \right\} = 0 \qquad H = \frac{1}{2} v^2 + \Phi(\vec{r})$$

$\downarrow$ solution to CBE

$$f(\vec{r},\vec{v},t) = f(\vec{r}_0,\vec{v}_0,0) \quad \forall [\vec{r},\vec{v}] \text{ satisfying } \begin{matrix} \{\vec{r},H\} = \frac{\partial H}{\partial \vec{v}} \\ \{\vec{v},H\} = -\frac{\partial H}{\partial \vec{r}} \end{matrix}$$

$\downarrow$

the problems "reduces" to finding $[r(t), v(t)]$ for a given initial value problem $f(r_0,v_0)$

- initial value problem

the initial values

$$f(\vec{r}(t_0),\vec{v}(t_0))$$

Hamiltonian of the system

$$H = \frac{1}{2}v^2 + \Phi(\vec{r})$$

the equations of motion

$$\{\vec{r},H\} = \frac{\partial H}{\partial \vec{v}}$$

$$\{\vec{v},H\} = -\frac{\partial H}{\partial \vec{r}}$$

- N-body approach

1. sample $f(r_i(t_0),v_i(t_0))$ with $i=1, \ldots, N$ points $[r_i(t_0),v_i(t_0)]$
2. those $[r_i(t),v_i(t)]$ obeying the equations-of-motion sample $f(r_i(t),v_i(t))$

- consistency check…

$$\{\vec{r},H\} = \frac{\partial H}{\partial \vec{v}} \qquad \{\vec{v},H\} = -\frac{\partial H}{\partial \vec{r}} \xrightarrow{H=\frac{1}{2}v^2+\Phi(\vec{r})} \frac{d\vec{r}}{dt} = \vec{v} \qquad \frac{d\vec{v}}{dt} = -\nabla\Phi \xrightarrow{\vec{F}=-\nabla\Phi} \frac{d^2\vec{r}}{dt^2} = \vec{F}$$

- collisionless system of N-bodies
  - equations-of-motion

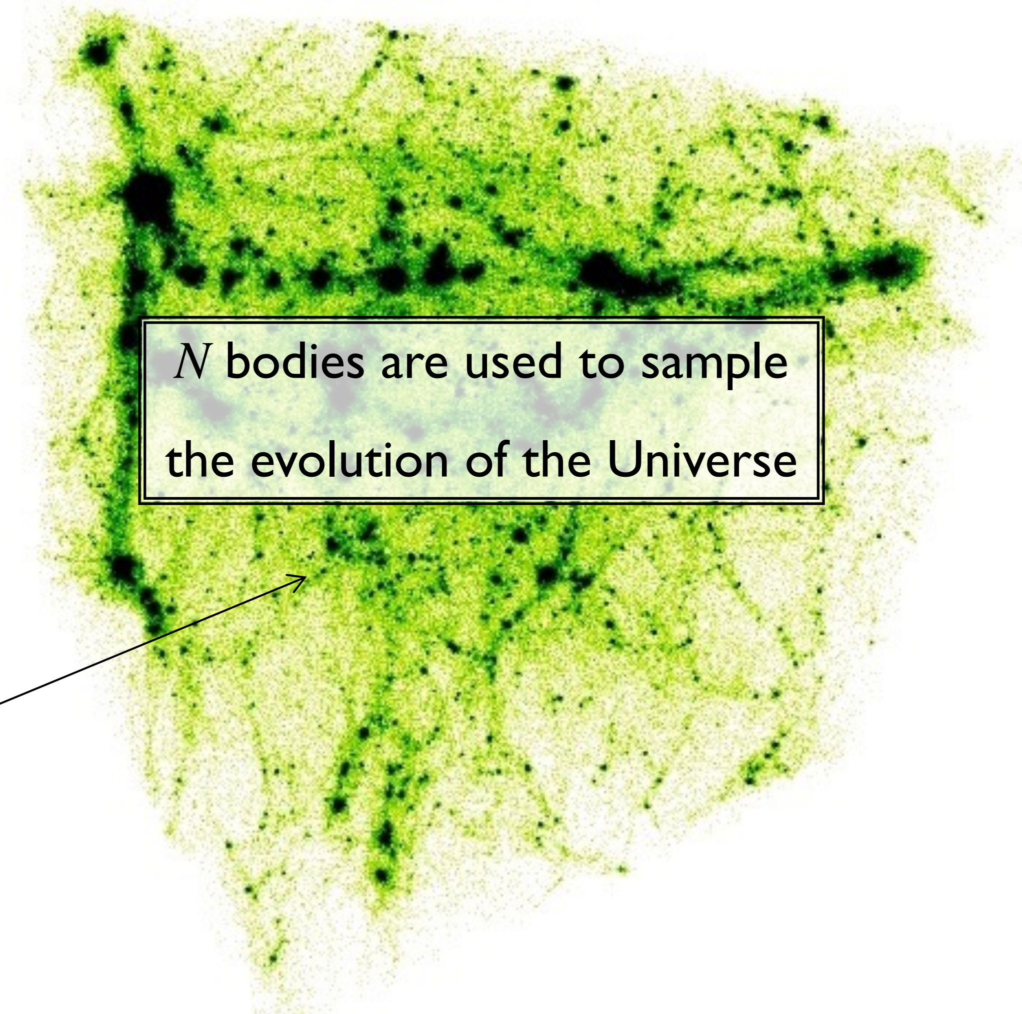
$$\frac{d\vec{r}}{dt} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = -\nabla\Phi = \vec{F}(\vec{r},t)$$

- collisionless system of N-bodies
  - equations-of-motion

$$\frac{d\vec{r}}{dt} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = -\nabla\Phi = \vec{F}(\vec{r},t)$$



one "simulation particle"
represents
billions of dark matter particles:

$m_{\rm simu} \sim 10^7 M_\odot$ vs $m_{\rm DM} << 10^{-60} M_\odot$

- collisionless system of N-bodies
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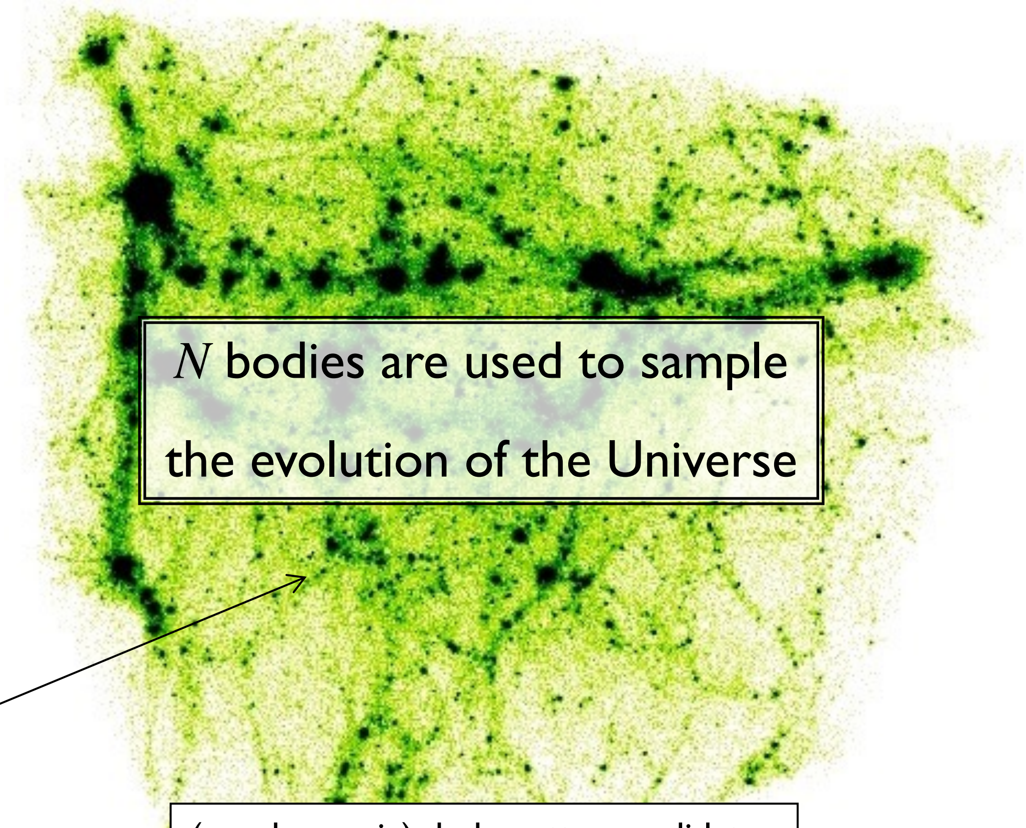

one "simulation particle"
represents
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$m_{\rm simu} \sim 10^7 M_\odot$ vs $m_{\rm DM} << 10^{-60} M_\odot$

(non-baryonic) dark matter candidates

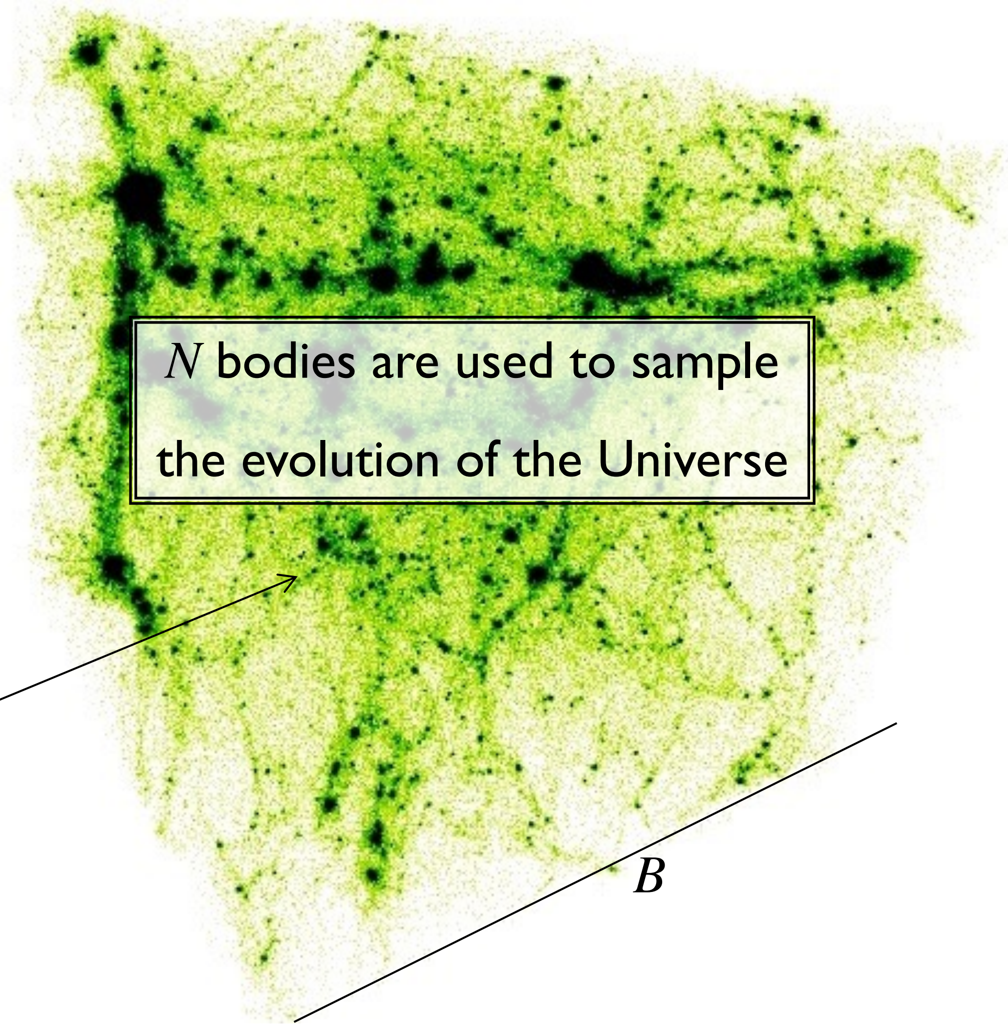
axion:	10^{-5} eV
neutrino:	10eV
WIMP:	$1\text{-}10^3$ GeV
monopoles:	10^{16} GeV
Planck relics:	10^{19} GeV
	???

$0.5 MeV \approx 9 \cdot 10^{-28} g$

- collisionless system of N-bodies
  - equations-of-motion

$$\frac{d\vec{r}}{dt} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = -\nabla\Phi = \vec{F}(\vec{r},t)$$



one "simulation particle"
represents
billions of dark matter particles:

$m_{\rm simu} \sim 10^7 M_\odot$ vs $m_{\rm DM} << 10^{-60} M_\odot$

$$m_{simu} = \frac{\bar{\rho}V}{N} = \Omega_{m,0}\frac{3H_0^2}{8\pi G}\frac{B^3}{N}$$

- collisionless system of N-bodies
  - equations-of-motion

$$\frac{d\vec{r}}{dt} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = \boxed{-\nabla\Phi = \vec{F}(\vec{r},t)} \;\; ?$$

- collisionless system of N-bodies

  - equations-of-motion

$$\frac{d\vec{r}}{dt} = \vec{v}$$
$$\frac{d\vec{v}}{dt} = -\nabla\Phi = \vec{F}(\vec{r},t)$$

  - the forces (details follow later…)

    - ○ "particle" approach: $\vec{F}(\vec{r}_i) = -\sum_{i \neq j} \frac{Gm_j}{(r_i - r_j)^3}(\vec{r}_i - \vec{r}_j)$

    - ○ "grid" approach:

$$\Delta\Phi(\vec{g}_{i,j,k}) = 4\pi G\left(\rho(\vec{g}_{i,j,k}) - \overline{\rho}\right)$$
$$\vec{F}(\vec{g}_{i,j,k}) = -\nabla\Phi(\vec{g}_{i,j,k})$$

- ***collisionless*** system of N-bodies

  - equations-of-motion

$$\frac{d\vec{r}}{dt} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = -\nabla\Phi = \vec{F}(\vec{r},t)$$

  - the forces (details follow later…)

    - "particle" approach: $$\vec{F}(\vec{r}_i) = -\sum_{i \neq j} \frac{Gm_j}{(r_i - r_j)^3}(\vec{r}_i - \vec{r}_j)$$ **manually soften force**

    - "grid" approach: $$\Delta\Phi(\vec{g}_{i,j,k}) = 4\pi G\left(\rho(\vec{g}_{i,j,k}) - \bar{\rho}\right)$$ $$\vec{F}(\vec{g}_{i,j,k}) = -\nabla\Phi(\vec{g}_{i,j,k})$$ **automatically softened on grid-scale**

- collisionless system of N-bodies

  - **equations-of-motion**

$$\frac{d\vec{r}}{dt} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = -\nabla\Phi = \vec{F}(\vec{r},t)$$

  - the forces (details follow later…)

    - "particle" approach: $\vec{F}(\vec{r}_i) = -\sum_{i \neq j} \frac{Gm_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$

    - "grid" approach:

$$\Delta\Phi(\vec{g}_{i,j,k}) = 4\pi G\left(\rho(\vec{g}_{i,j,k}) - \overline{\rho}\right)$$

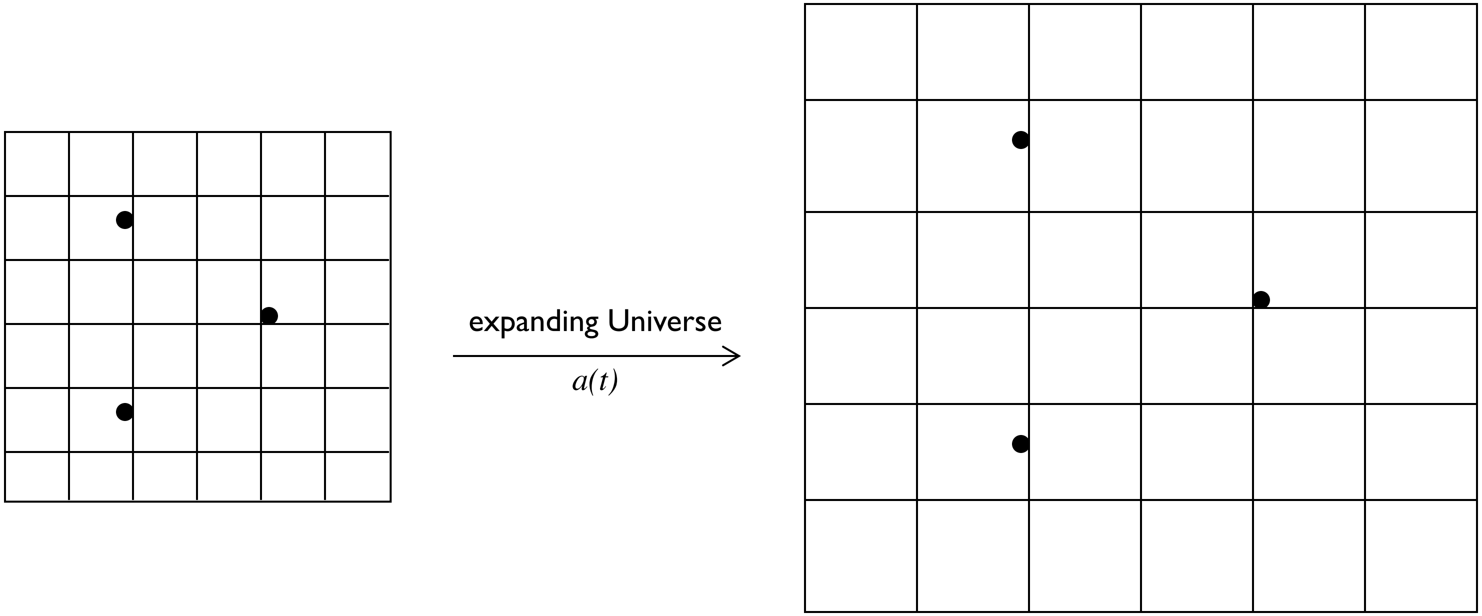
$$\vec{F}(\vec{g}_{i,j,k}) = -\nabla\Phi(\vec{g}_{i,j,k})$$

- introduction
- Boltzmann solver
- initial conditions generators
- simulation codes
  - the N-body principle
  - **the equations-of-motion**
  - the forces

- introduction of comoving coordinates

all length scales scale like $a(t)$

$$\vec{r}(t) = a(t)\vec{x}(t)$$



- introduction of comoving coordinates

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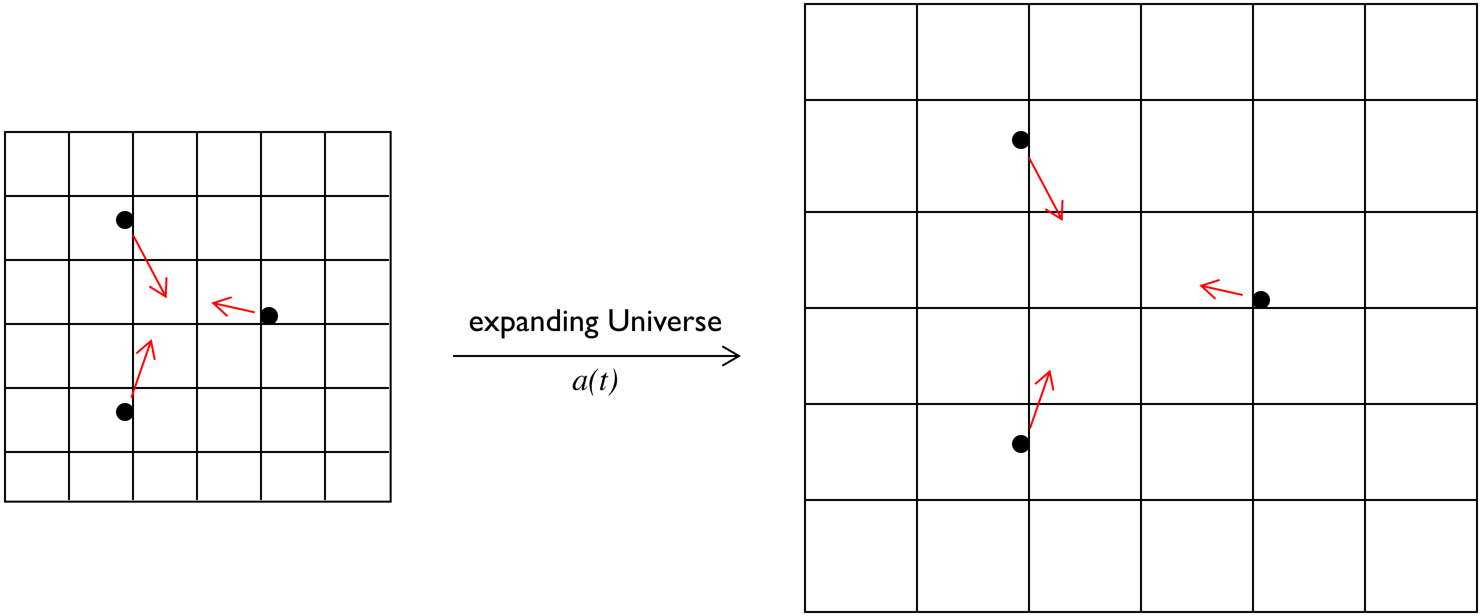
$$\vec{r}(t) = a(t)\vec{x}(t)$$



→ we are only interested in the peculiar motion…

- introduction of comoving coordinates

$$\frac{d\vec{r}}{dt} = \vec{v}$$
$$\frac{d\vec{v}}{dt} = -\nabla\Phi$$

$$\vec{r}(t) = a(t)\vec{x}(t)$$
$$\vec{p}(t) = a(t)[\vec{v}(t) - H(t)\vec{r}(t)]$$

$$\frac{d\vec{x}}{dt} = \frac{\vec{p}}{a^2}$$
$$\frac{d\vec{p}}{dt} = -\nabla_x\psi$$

- introduction of comoving coordinates

$$\frac{d\vec{r}}{dt} = \vec{v}$$
$$\frac{d\vec{v}}{dt} = -\nabla\Phi$$

$$\vec{r}(t) = a(t)\vec{x}(t)$$
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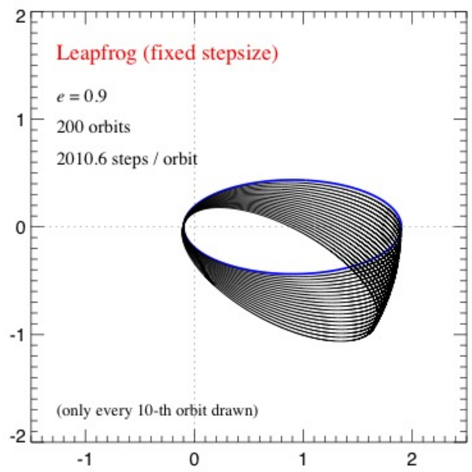

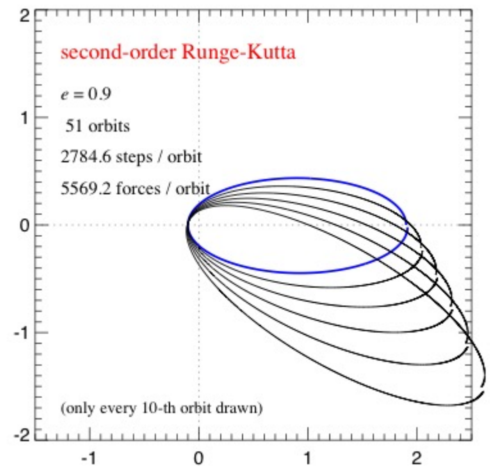

integrated via symplectic 2nd order accurate leap-frog scheme

$$\vec{x}^{n+1/2} = \vec{x}^{n} + \frac{\Delta t}{2}\frac{\vec{p}^{n}}{a^2 m}$$
$$\vec{p}^{n+1} = \vec{p}^{n} - \Delta t\,\vec{\nabla}_x\Psi^{n+1/2}$$
$$\vec{x}^{n+1} = \vec{x}^{n+1/2} + \frac{\Delta t}{2}\frac{\vec{p}^{n+1}}{a^2 m}$$

- introduction of comoving coordinates

$$\vec{r}(t) = a(t)\vec{x}(t)$$
$$\vec{p}(t) = a(t)[\vec{v}(t) - H(t)\vec{r}(t)]$$

$$\frac{d\vec{r}}{dt} = \vec{v} \qquad \longrightarrow \qquad \frac{d\vec{x}}{dt} = \frac{\vec{p}}{a^2}$$

$$\frac{d\vec{v}}{dt} = -\nabla\Phi \qquad \qquad \frac{d\vec{p}}{dt} = -\nabla_x\psi$$

$$\Delta\Phi = 4\pi G\rho \qquad \qquad \Delta_x\psi = \frac{4\pi G}{a}(\rho_x - \overline{\rho}_x)$$

(ψ : peculiar potential!)

- introduction of comoving coordinates

$$\frac{d\vec{r}}{dt} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = -\nabla\Phi$$

$$\Delta\Phi = 4\pi G\rho$$

$$\vec{r}(t) = a(t)\vec{x}(t)$$

$$\vec{p}(t) = a(t)[\vec{v}(t) - H(t)\vec{r}(t)]$$

$$\frac{d\vec{x}}{dt} = \frac{\vec{p}}{a^2}$$

$$\frac{d\vec{p}}{dt} = -\nabla_x\psi$$

$$\Delta_x\psi = \frac{4\pi G}{a}(\rho_x - \overline{\rho}_x)$$

(ψ : peculiar potential!)

how to obtain the forces/potential?

- introduction
- Boltzmann solver
- initial conditions generators
- simulation codes
  - the N-body principle
  - the equations-of-motion
  - **the forces**

- the forces – in comoving coordinates

$$\Delta_x \psi = \frac{4\pi G}{a}(\rho_x - \overline{\rho}_x)$$

particle approach

$$\vec{F}(\vec{x}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{(x_i - x_j)^3}(\vec{x}_i - \vec{x}_j)$$

grid approach

$$\Delta\psi(\vec{g}_{k,l,m}) = 4\pi G \rho(\vec{g}_{k,l,m})$$

$$\vec{F}(\vec{g}_{k,l,m}) = -\nabla\psi(\vec{g}_{k,l,m})$$

- the forces – in comoving coordinates

$$\Delta_x \psi = \frac{4\pi G}{a}(\rho_x - \overline{\rho}_x)$$

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$$\vec{F}(\vec{g}_{k,l,m}) = -\nabla\psi(\vec{g}_{k,l,m})$$

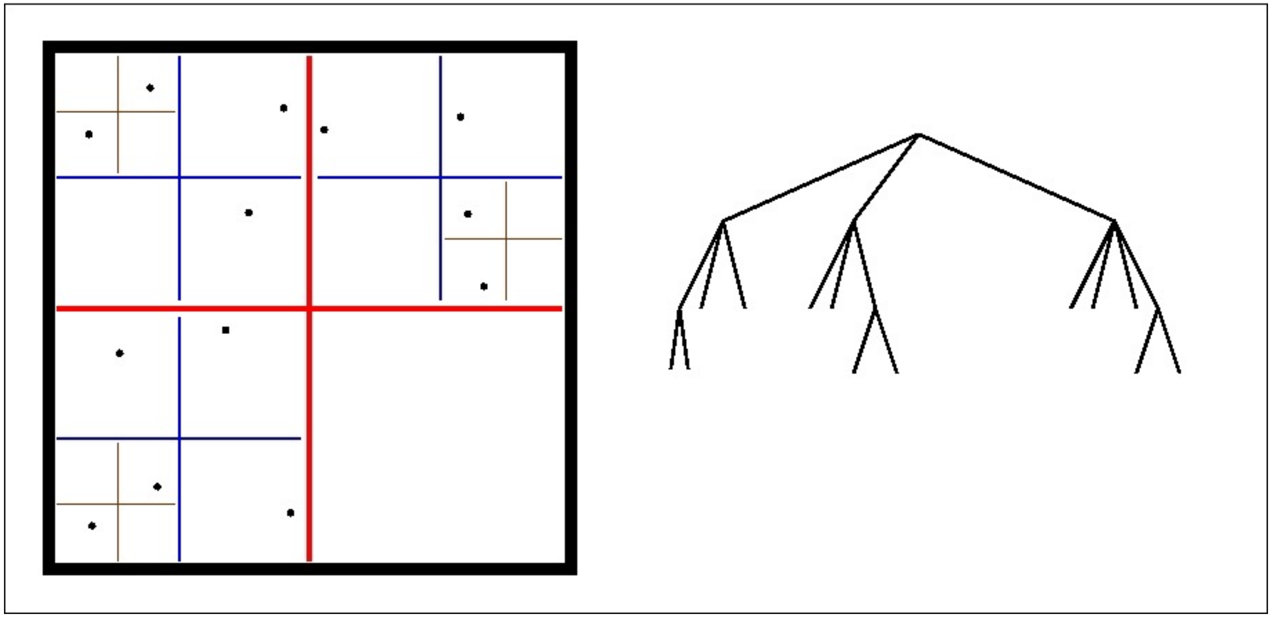
- particle approach

$$\vec{F}_i(\vec{x}_i) = -\sum_{i \neq j} \frac{Gm_j}{(x_i - x_j)^3}(\vec{x}_i - \vec{x}_j) \quad \forall i \in N$$

- particle approach

$$\vec{F}_i(\vec{x}_i) = -\sum_{i \neq j} \frac{G m_j}{(x_i - x_j)^3} (\vec{x}_i - \vec{x}_j) \quad \forall i \in N$$

- overcoming the N^2 bottleneck by using a "tree"



- particle approach

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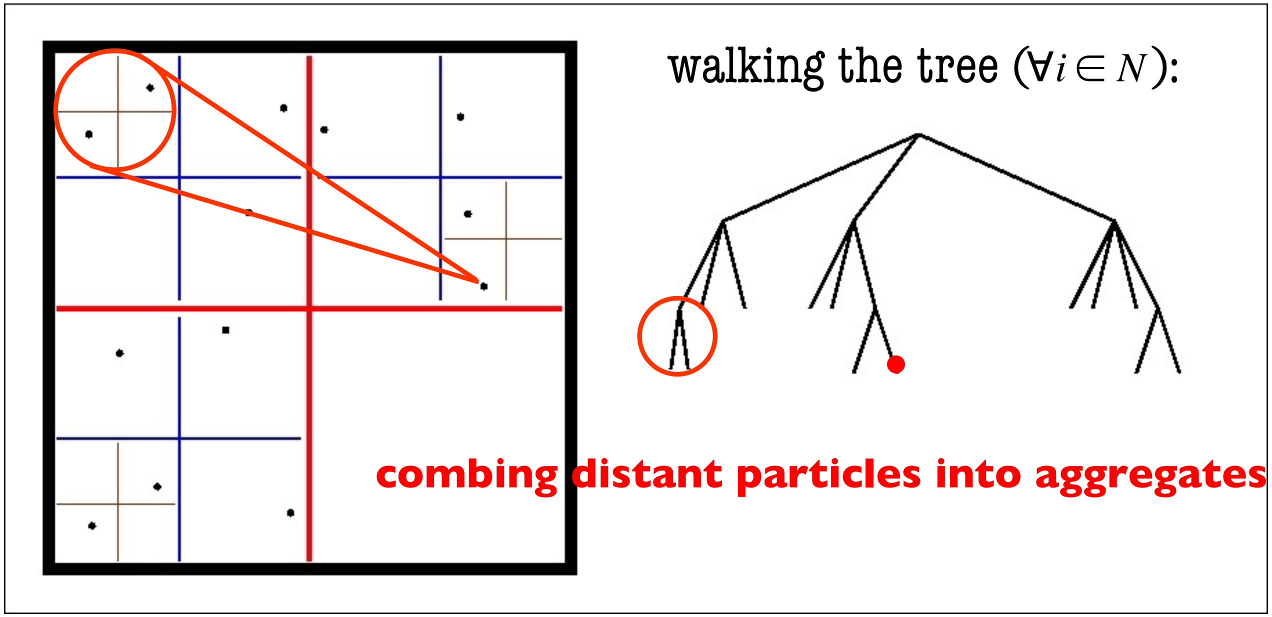


- particle approach

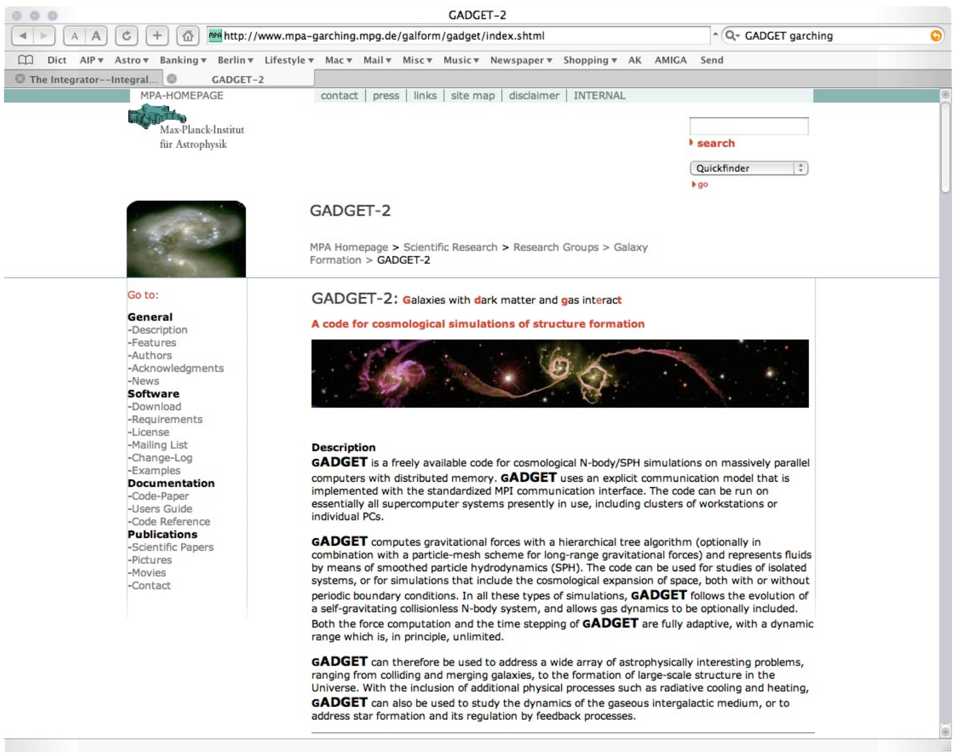
$$\vec{F}_i(\vec{x}_i) = -\sum_{i \neq j} \frac{Gm_j}{(x_i - x_j)^3}(\vec{x}_i - \vec{x}_j) \quad \forall i \in N$$

- other subtleties:
  - need to avoid singularity for $x_i = x_j$ => force softening
  - period boundary conditions => Ewald summation

- particle approach

$$\vec{F}_i(\vec{x}_i) = -\sum_{i \neq j} \frac{Gm_j}{(x_i - x_j)^3}(\vec{x}_i - \vec{x}_j) \quad \forall i \in N$$

- open-source code:
  - GADGET2



- the forces – in comoving coordinates

$$\Delta_x \psi = \frac{4\pi G}{a}(\rho_x - \overline{\rho}_x)$$

<u>particle approach</u>

$$\vec{F}(\vec{x}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{(x_i - x_j)^3}(\vec{x}_i - \vec{x}_j)$$

<u>grid approach</u>

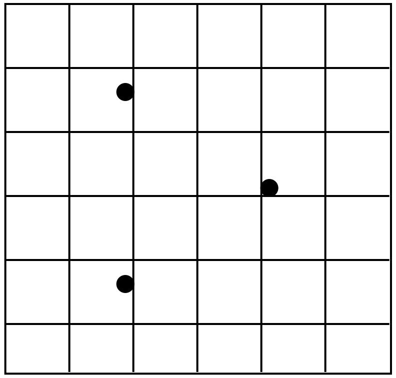
$$\Delta \psi(\vec{g}_{k,l,m}) = 4\pi G \rho(\vec{g}_{k,l,m})$$

$$\vec{F}(\vec{g}_{k,l,m}) = -\nabla \psi(\vec{g}_{k,l,m})$$

- grid approach

$$\Delta\psi(\vec{g}_{k,l,m}) = \frac{4\pi G}{a}\left(\rho(\vec{g}_{k,l,m}) - \bar{\rho}\right)$$

$$\vec{F}(\vec{g}_{k,l,m}) = -\nabla\psi(\vec{g}_{k,l,m})$$

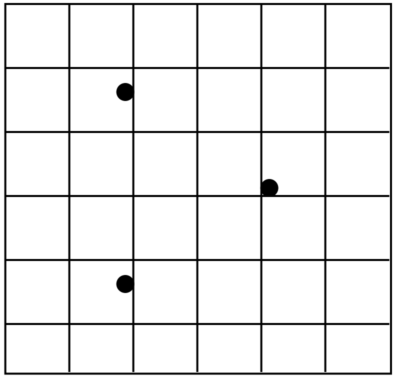


1. calculate mass density on grid $\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$
2. solve Poisson's equation on grid $\Phi(\vec{g}_{k,l,m})$
3. differentiate potential to get forces $\vec{F}(\vec{g}_{k,l,m})$
4. interpolate forces back to particles $\vec{F}(\vec{g}_{k,l,m}) \rightarrow \vec{F}(\vec{x}_i)$

- grid approach

$$\Delta\psi(\vec{g}_{k,l,m}) = \frac{4\pi G}{a}\left(\rho(\vec{g}_{k,l,m}) - \bar{\rho}\right)$$

$$\vec{F}(\vec{g}_{k,l,m}) = -\nabla\psi(\vec{g}_{k,l,m})$$



1. calculate mass density on grid $\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$
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sounds like a waste of time and computer resources,
but **exceptionally fast** in practice

- grid approach

$$\Delta\psi(\vec{g}_{k,l,m}) = \frac{4\pi G}{a}\left(\rho(\vec{g}_{k,l,m}) - \bar{\rho}\right)$$

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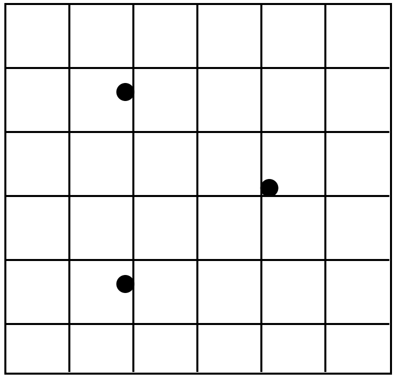


1. calculate mass density on grid — $\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$
2. solve Poisson's equation on grid — $\Phi(\vec{g}_{k,l,m})$ **?**
3. differentiate potential to get forces — $\vec{F}(\vec{g}_{k,l,m})$
4. interpolate forces back to particles — $\vec{F}(\vec{g}_{k,l,m}) \rightarrow \vec{F}(\vec{x}_i)$

sounds like a waste of time and computer resources,
but **exceptionally fast** in practice

- grid approach

$$\Delta\psi(\vec{g}_{k,l,m}) = \frac{4\pi G}{a}\left(\rho(\vec{g}_{k,l,m}) - \overline{\rho}\right)$$

- numerically solve Poisson's equation via Fourier Transforms

$$\Delta\psi = S \qquad \rightarrow \text{equation we wish to solve}$$

- grid approach

$$\Delta\psi(\vec{g}_{k,l,m}) = \frac{4\pi G}{a}\left(\rho(\vec{g}_{k,l,m}) - \overline{\rho}\right)$$

- numerically solve Poisson's equation via Fourier Transforms

$\Delta\psi = S$ → equation we wish to solve

$\Delta\mathcal{G} = \delta$ → equation way easier to solve…

(δ = Dirac's delta-function)

- grid approach

$$\Delta\psi(\vec{g}_{k,l,m}) = \frac{4\pi G}{a}\left(\rho(\vec{g}_{k,l,m}) - \bar{\rho}\right)$$

- numerically solve Poisson's equation via Fourier Transforms

$\Delta\psi = S$ → equation we wish to solve

$\Delta\mathcal{G} = \delta$ → equation way easier to solve…

(δ = Dirac's delta-function)

  - Green's function of Poisson's equation:

$\hat{\mathcal{G}}(\vec{k}) = -\frac{1}{k^2}$ → Fourier Space

$\mathcal{G}(\vec{x}) = \frac{1}{4\pi x}$ → Real Space

- grid approach

$$\Delta\psi(\vec{g}_{k,l,m}) = \frac{4\pi G}{a}\left(\rho(\vec{g}_{k,l,m}) - \overline{\rho}\right)$$

- numerically solve Poisson's equation via Fourier Transforms

$$\Delta\psi = S$$

$$\psi(\vec{x}) = \iiint \mathcal{G}(\vec{x} - \vec{x}')\, S(\vec{x}')d^3x' \quad \rightarrow \text{Real Space}$$

$$\hat{\psi}(\vec{k}) = \hat{\mathcal{G}}(\vec{k})\hat{S}(\vec{k}) \quad \rightarrow \text{Fourier Space}$$

- grid approach

$$\Delta\psi(\vec{g}_{k,l,m}) = \frac{4\pi G}{a}\left(\rho(\vec{g}_{k,l,m}) - \overline{\rho}\right)$$

- numerically solve Poisson's equation via Fourier Transforms

$$\psi = \rho \otimes \mathcal{G}$$

FFT → convolution becomes multiplication

$$\hat{\psi} = \hat{\rho} \quad \hat{\mathcal{G}}$$

FFT^{-1}

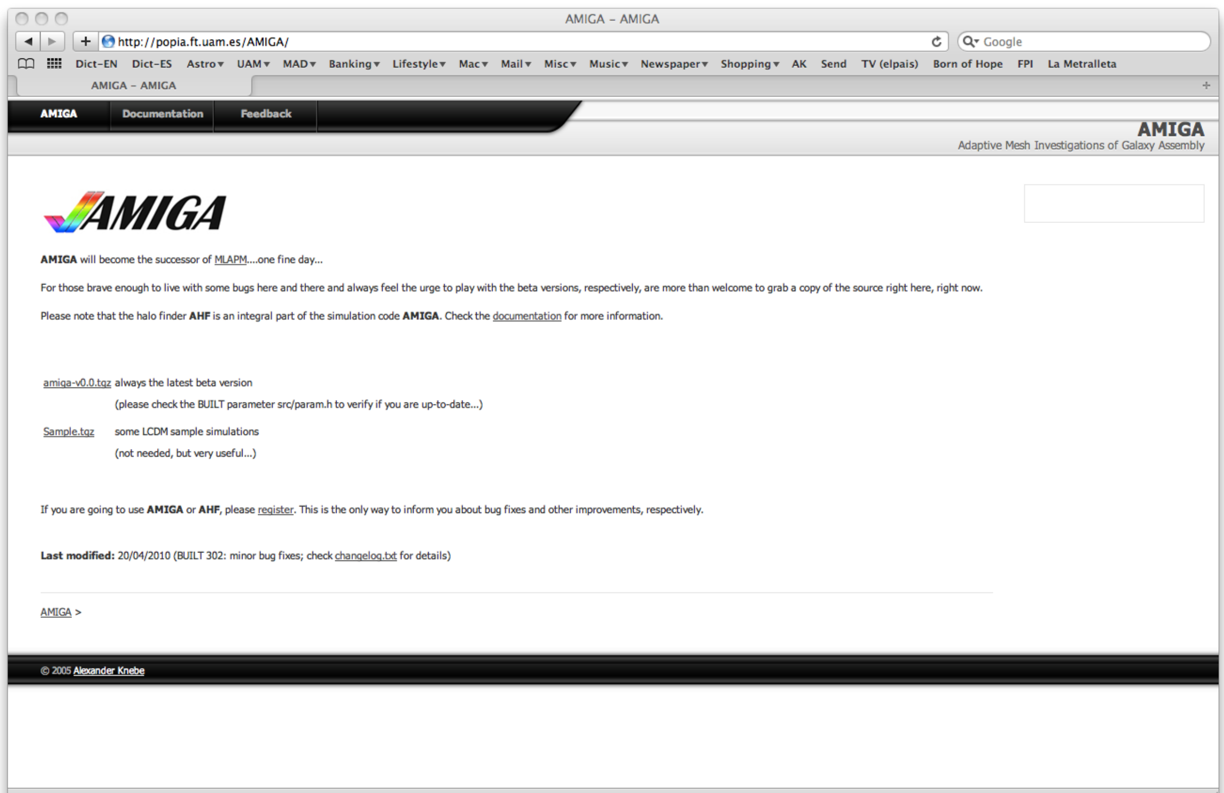
$$\psi$$

FFT demands a *regular* grid though!

- grid approach

$$\Delta\psi(\vec{g}_{k,l,m}) = \frac{4\pi G}{a}\left(\rho(\vec{g}_{k,l,m}) - \bar{\rho}\right)$$

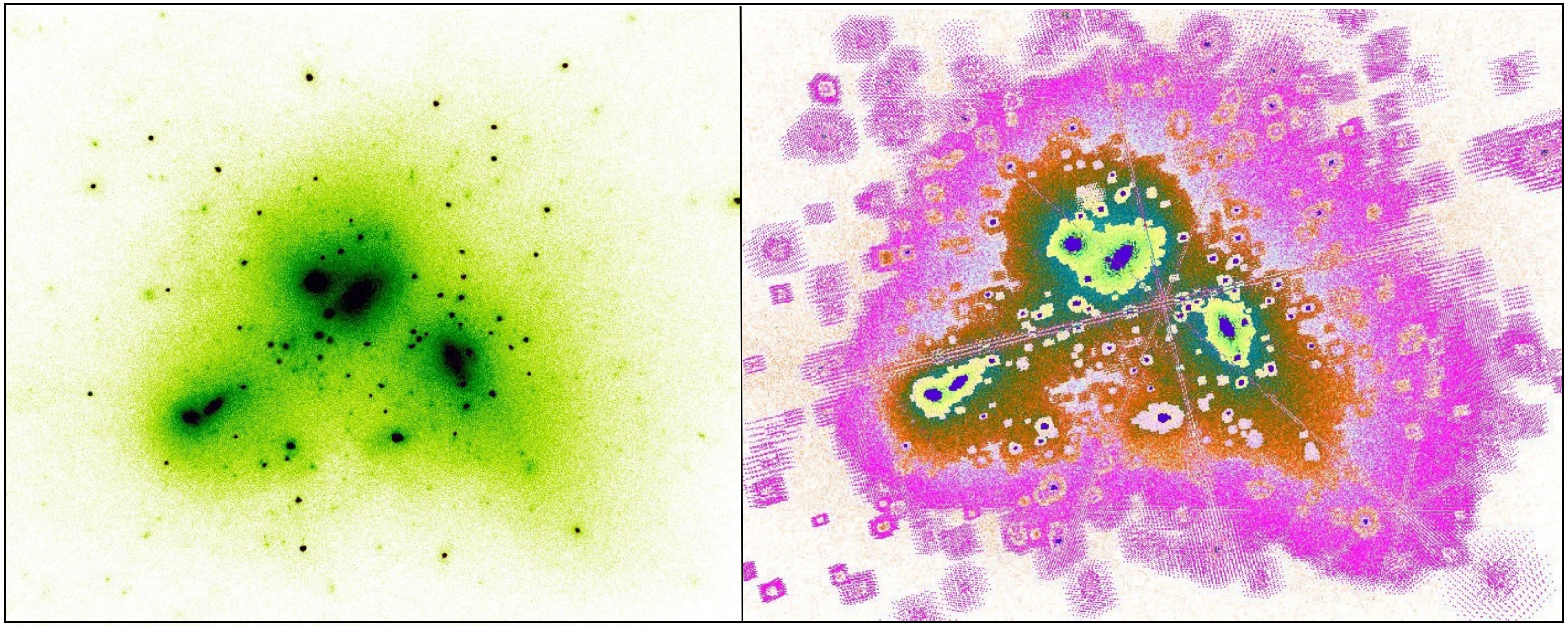
- open-source code:
  - ***AMIGA***



- hybrid approach
  - treePM
    - long-range force = PM method
    - short-range force = tree method
  - P^3M
    - long-range force = PM method
    - short-range force = PP method (direct summation)
  - AMR
    - PM method, but recursively refining cells

- hybrid approach

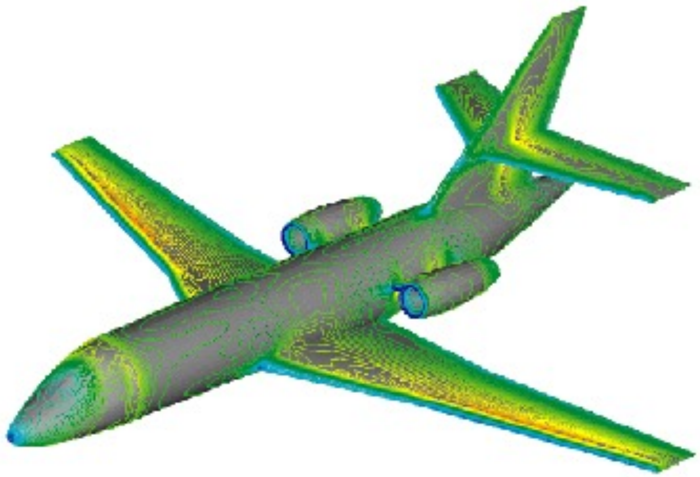
density field of simulated galaxy cluster

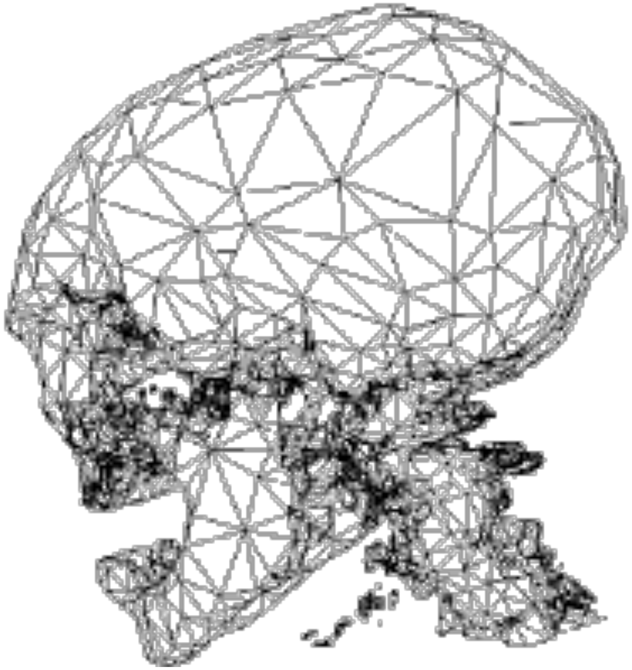


adaptive grid hierarchy

- AMR
  - PM method, but recursively refining cells

- hybrid approach





not limited to astrophysics

- AMR
  - PM method, but recursively refining cells

- full set of equations

  - **collisionless matter** (e.g. dark matter)

$$\frac{d\vec{x}_{DM}}{dt} = \vec{v}_{DM}$$

$$\frac{d\vec{v}_{DM}}{dt} = -\nabla\phi$$

$$\Delta\phi = 4\pi G\rho_{tot}$$

- full set of equations

  - **collisionless matter** (e.g. dark matter)

$$\frac{d\vec{x}_{DM}}{dt} = \vec{v}_{DM}$$

$$\frac{d\vec{v}_{DM}}{dt} = -\nabla\phi$$

$$\Delta\phi = 4\pi G\rho_{tot}$$

  - **collisional matter** (e.g. gas)

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\vec{v}\right) = 0$$

$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times\left(\vec{v}\times\vec{B}\right)$$

$$\frac{\partial\left(\rho\vec{v}\right)}{\partial t} + \nabla\cdot\left(\rho\vec{v}\otimes\vec{v} + \left(p + \frac{1}{2\mu}B^2\right)\overleftrightarrow{1} - \frac{1}{\mu}\vec{B}\otimes\vec{B}\right) = \rho\ \left(-\nabla\phi\right)$$

$$\frac{\partial\left(\rho E\right)}{\partial t} + \nabla\cdot\left(\left[\rho E + p + \frac{1}{2\mu}B^2\right]\vec{v} - \frac{1}{\mu}\left[\vec{v}\cdot\vec{B}\right]\vec{B}\right) = \rho\vec{v}\cdot\left(-\nabla\phi\right) + \left(\Gamma - L\right)$$

$$p = \left(\gamma - 1\right)\rho\varepsilon$$

$$\rho\varepsilon = \rho E - \frac{1}{2}\rho v^2$$

- full set of equations

  - **collisionless matter** (e.g. dark matter)

$$\frac{d\vec{x}_{DM}}{dt} = \vec{v}_{DM}$$

$$\frac{d\vec{v}_{DM}}{dt} = -\nabla\phi$$

dominated by long-range interactions!

$$\Delta\phi = 4\pi G\rho_{tot}$$

  - **collisional matter** (e.g. gas)

$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times\left(\vec{v}\times\vec{B}\right)$$

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$$\frac{\partial\left(\rho E\right)}{\partial t} + \nabla\cdot\left(\left[\rho E + p + \frac{1}{2\mu}B^2\right]\vec{v} - \frac{1}{\mu}\left[\vec{v}\cdot\vec{B}\right]\vec{B}\right) = \rho\vec{v}\cdot\left(-\nabla\phi\right) + \left(\Gamma - L\right)$$

$$p = \left(\gamma - 1\right)\rho\varepsilon$$

$$\rho\varepsilon = \rho E - \frac{1}{2}\rho v^2$$

- full set of equations

  - **collisionless matter** (e.g. dark matter)

$$\frac{d\vec{x}_{DM}}{dt} = \vec{v}_{DM}$$

$$\frac{d\vec{v}_{DM}}{dt} = -\nabla\phi$$

$$\Delta\phi = 4\pi G\rho_{tot}$$

  - **collisional matter** (e.g. gas)

dominated by short-range/local interactions!

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times\left(\vec{v}\times\vec{B}\right)$$

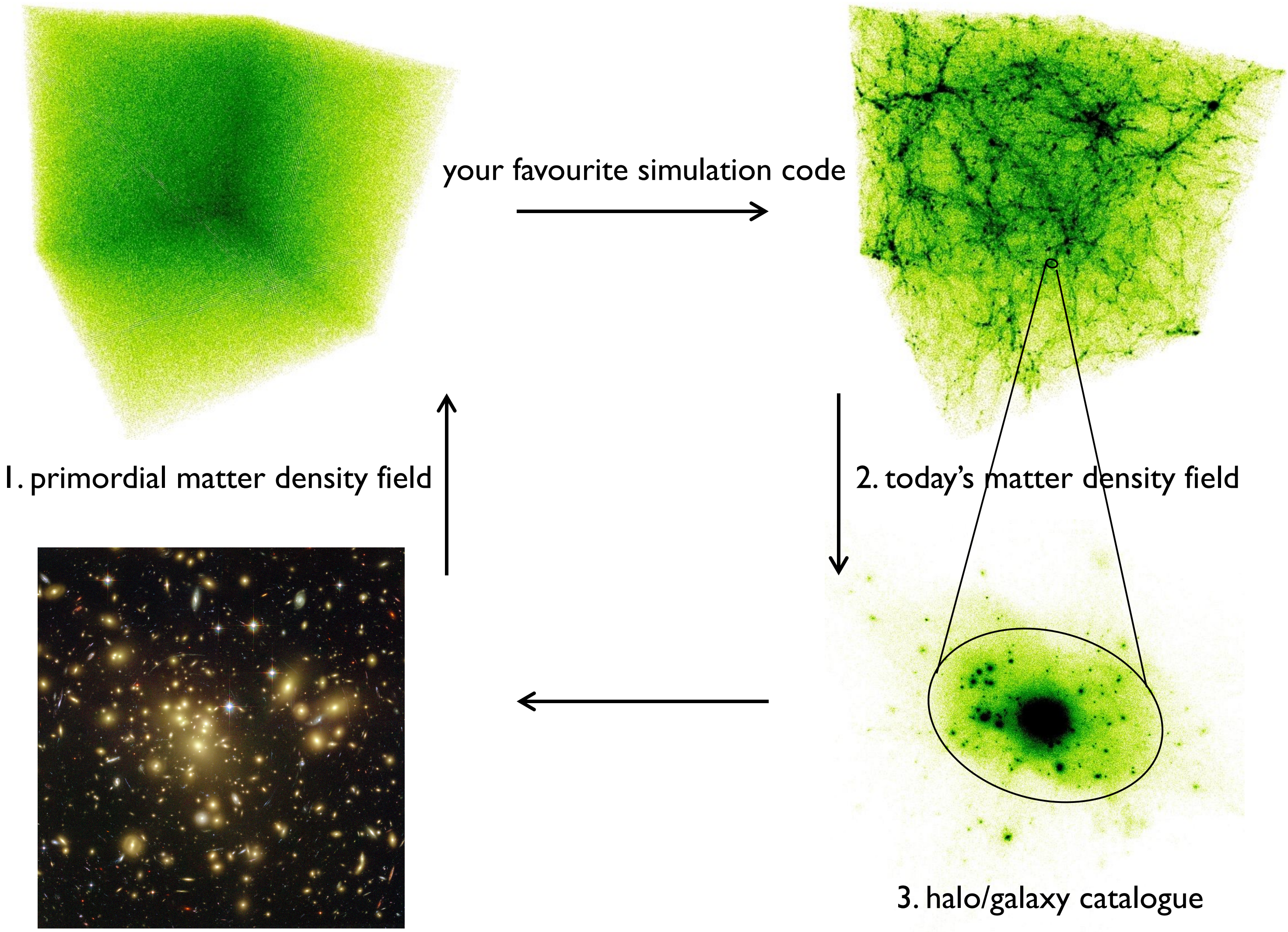
$$\frac{\partial(\rho\vec{v})}{\partial t} + \nabla\cdot\left(\rho\vec{v}\otimes\vec{v} + \left(p + \frac{1}{2\mu}B^2\right)\overleftrightarrow{1} - \frac{1}{\mu}\vec{B}\otimes\vec{B}\right) = \rho\ (-\nabla\phi)$$

$$\frac{\partial(\rho E)}{\partial t} + \nabla\cdot\left(\left[\rho E + p + \frac{1}{2\mu}B^2\right]\vec{v} - \frac{1}{\mu}\left[\vec{v}\cdot\vec{B}\right]\vec{B}\right) = \rho\vec{v}\cdot(-\nabla\phi) + (\Gamma - L)$$

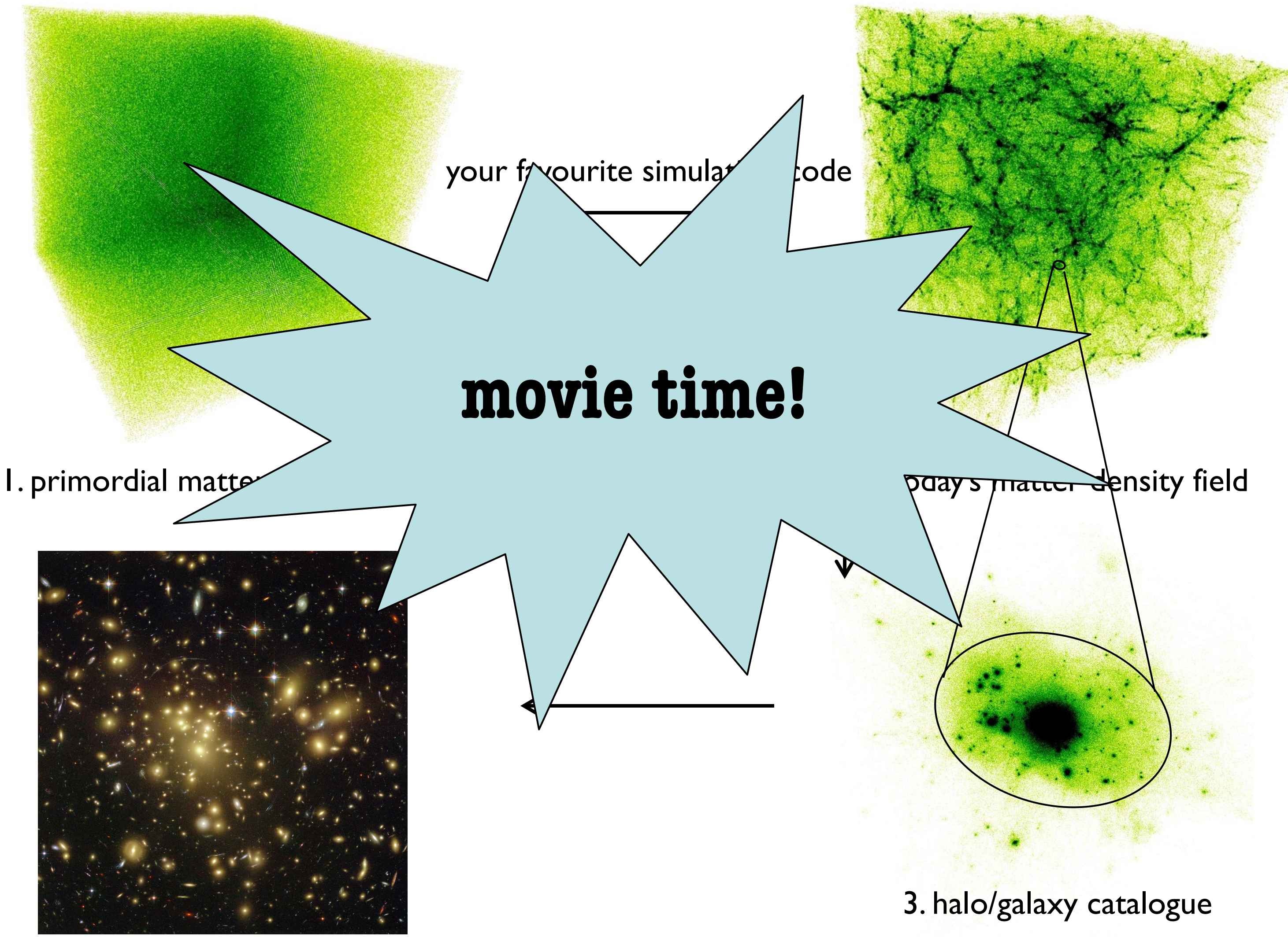
$$p = (\gamma - 1)\rho\varepsilon$$

$$\rho\varepsilon = \rho E - \frac{1}{2}\rho v^2$$

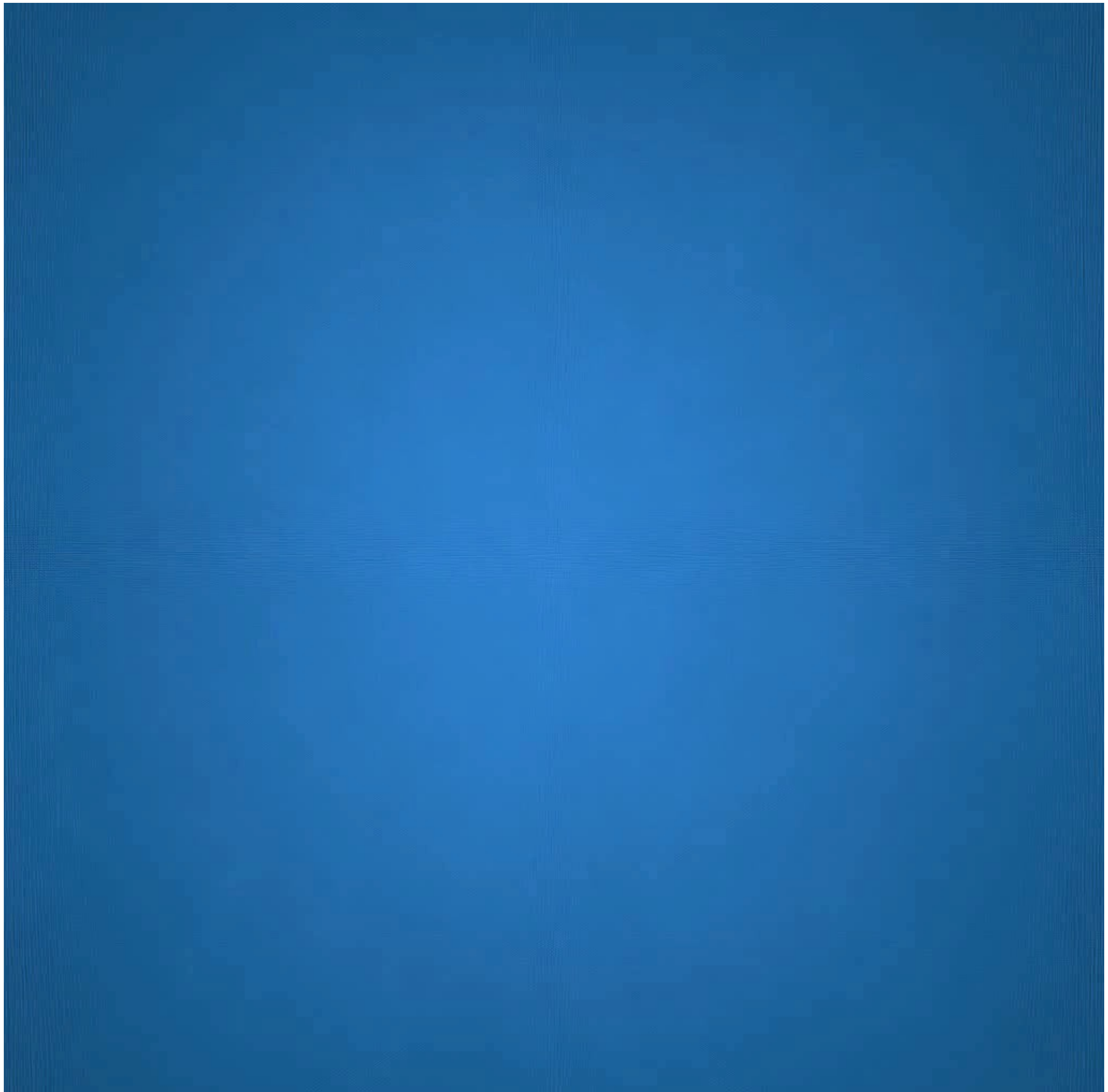
- simulation of cosmic structure formation



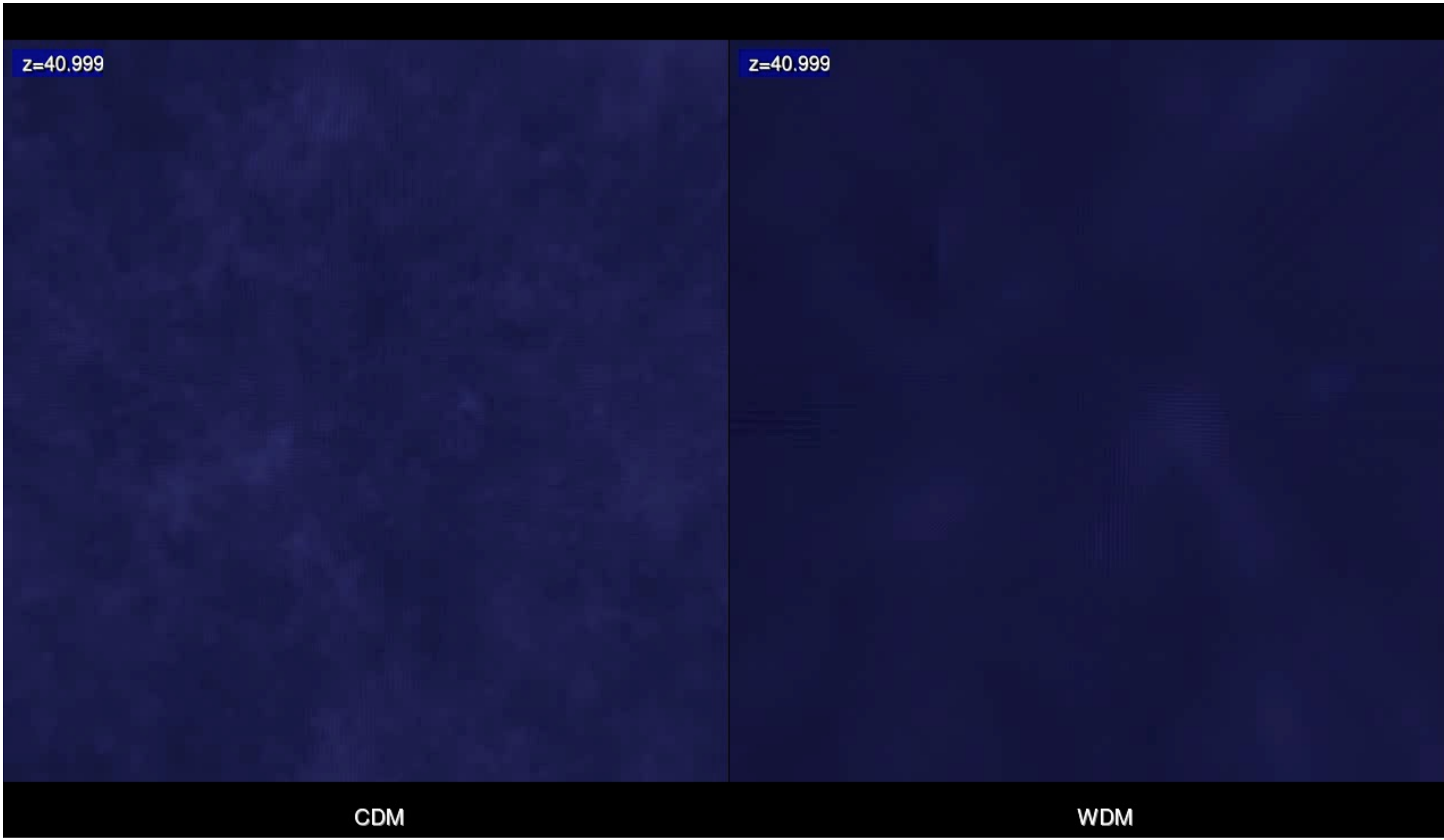
- simulation of cosmic structure formation



- formation of large-scale structure (Local Universe!)

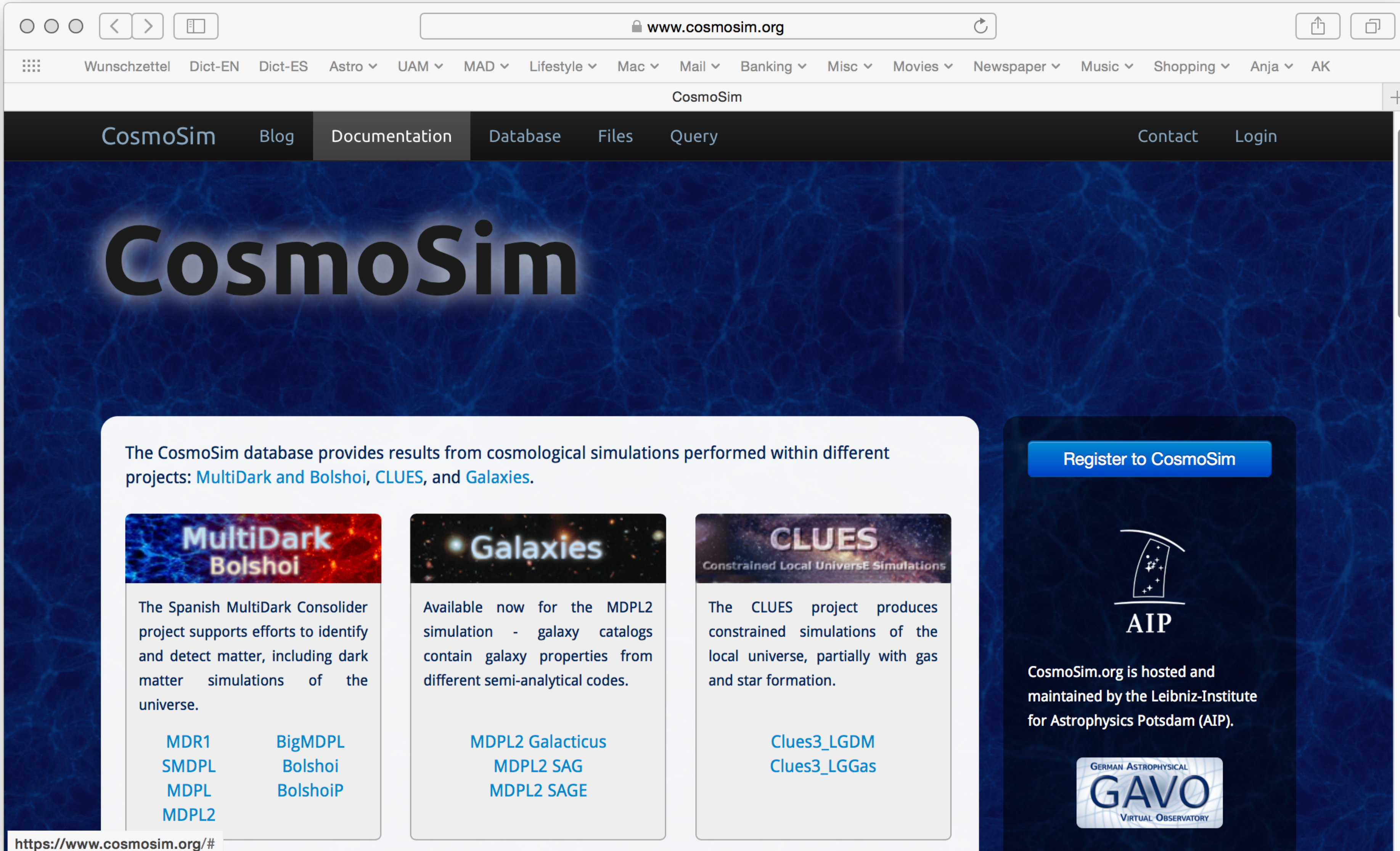


- formation of Local Group – incl. gas (CDM vs. WDM)



- formation of MW-type object – incl. gas (CDM)

(courtesy C. Brook, NIHAO collaboration)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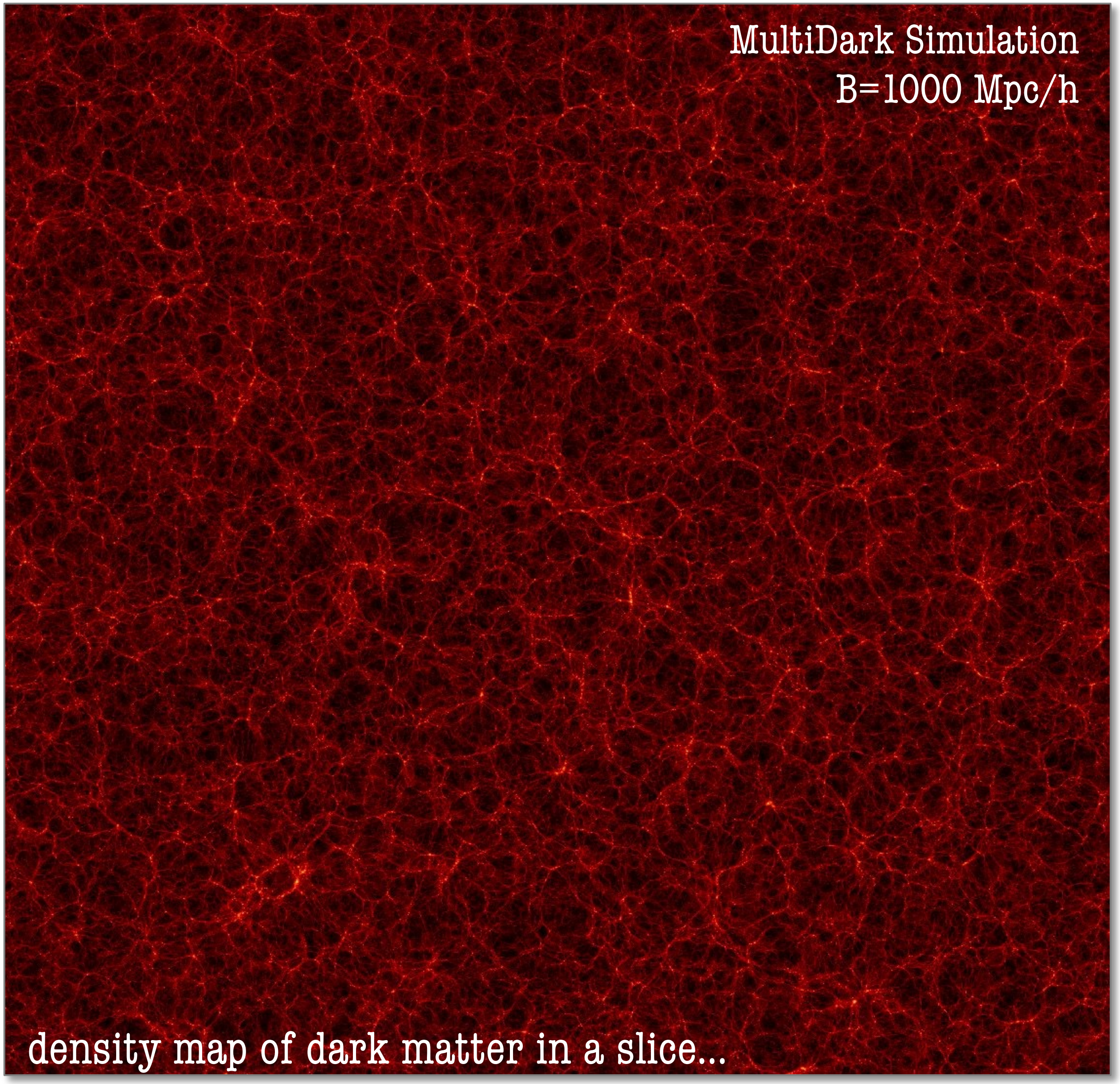
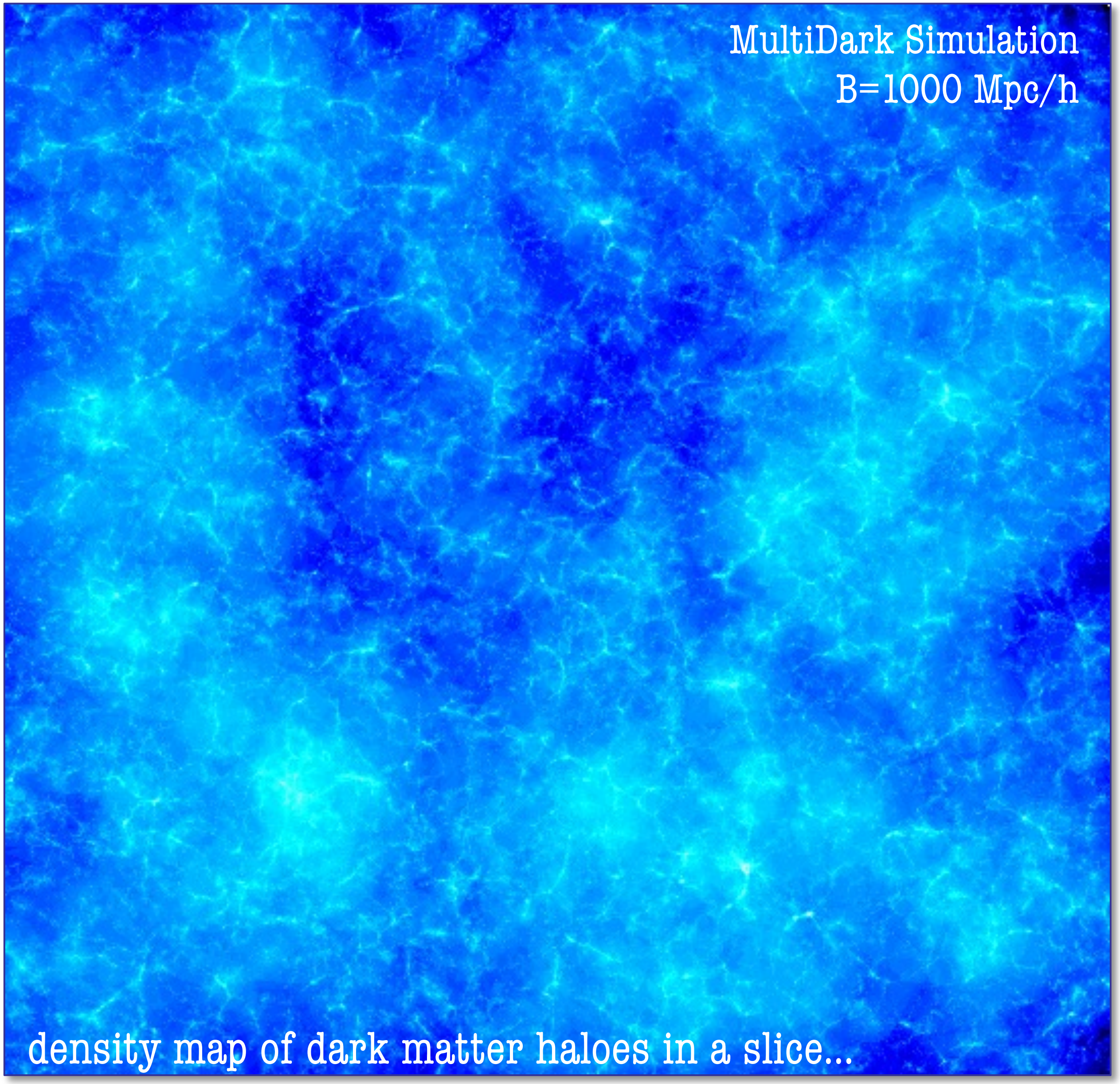
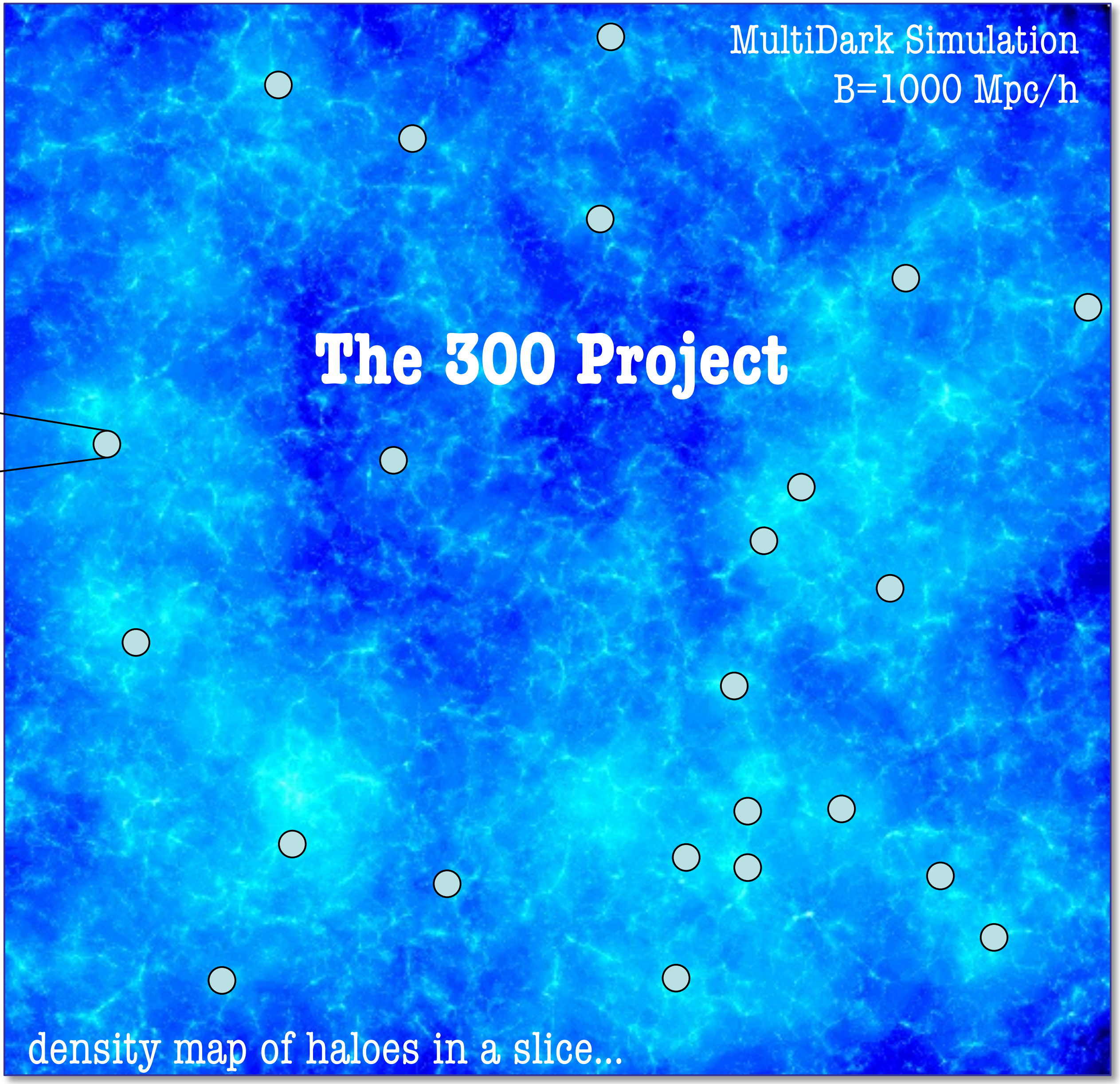
CosmoSim
The CosmoSim database provides results from cosmological simulations performed within different projects: MultiDark and Bolshoi, CLUES, and Galaxies.
MultiDark Bolshoi
The Spanish MultiDark Consolider project supports efforts to identify and detect matter, including dark matter simulations of the universe.
MDR1 SMDPL MDPL MDPL2
BigMDPL Bolshoi BolshoiP
Galaxies
Available now for the MDPL2 simulation - galaxy catalogs contain galaxy properties from different semi-analytical codes.
MDPL2 Galacticus
MDPL2 SAG
MDPL2 SAGE
CLUES
Constrained Local UniversE Simulations
The CLUES project produces constrained simulations of the local universe, partially with gas and star formation.
Clues3_LGDM
Clues3_LGGas
Register to CosmoSim
AIP
CosmoSim.org is hosted and maintained by the Leibniz-Institute for Astrophysics Potsdam (AIP).
German Astrophysical GAVO Virtual Observatory
https://www.cosmosim.org/#


MultiDark Simulation
B=1000 Mpc/h
density map of dark matter in a slice...

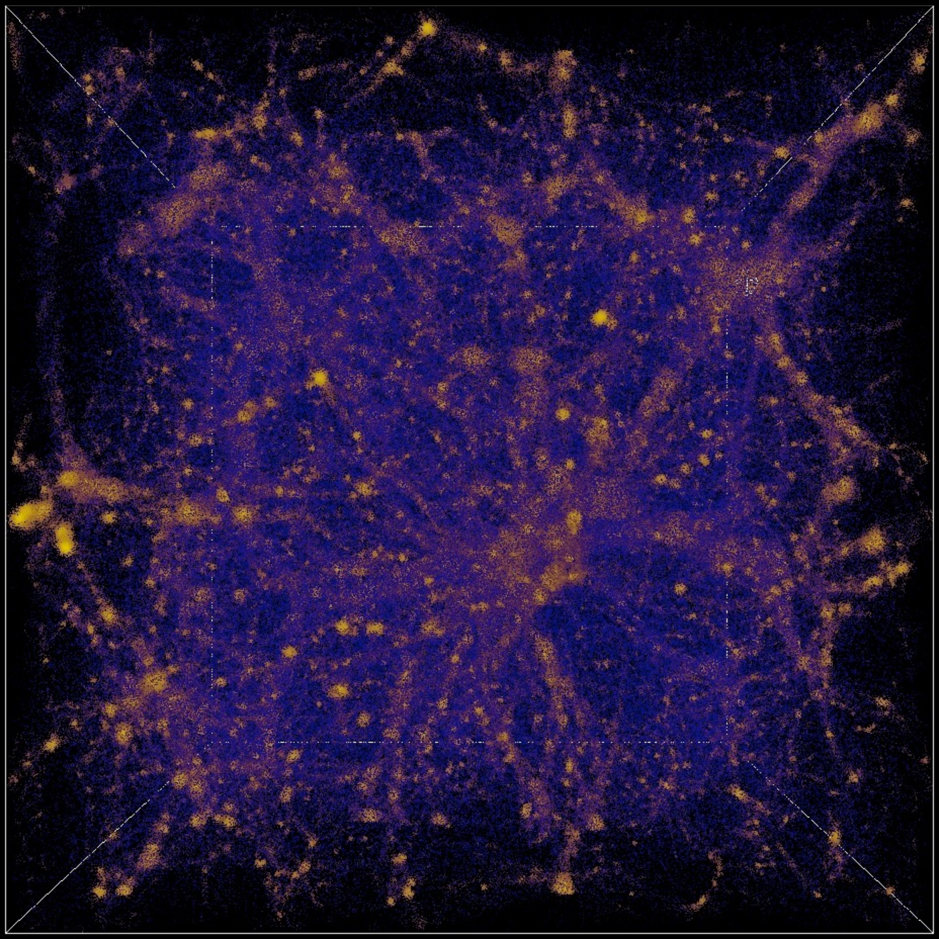

MultiDark Simulation
B=1000 Mpc/h
density map of dark matter haloes in a slice...



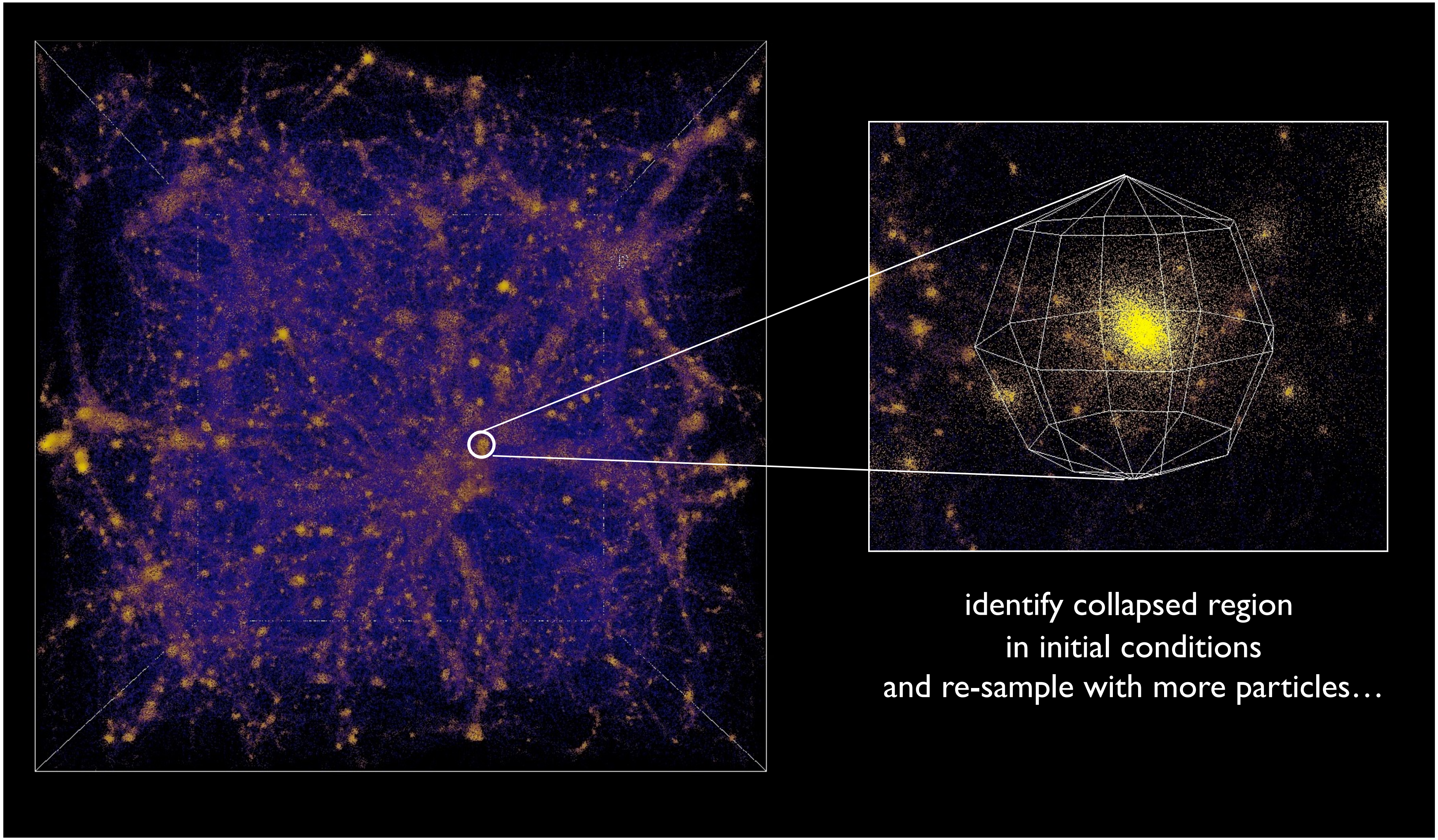
to be re-simulated
w/ full physics...

- run a low resolution simulation
- identify an interesting object
- trace back particles of that object to Lagrangian positions in IC's
- re-sample waves in that area with more particles
- re-run the whole simulation

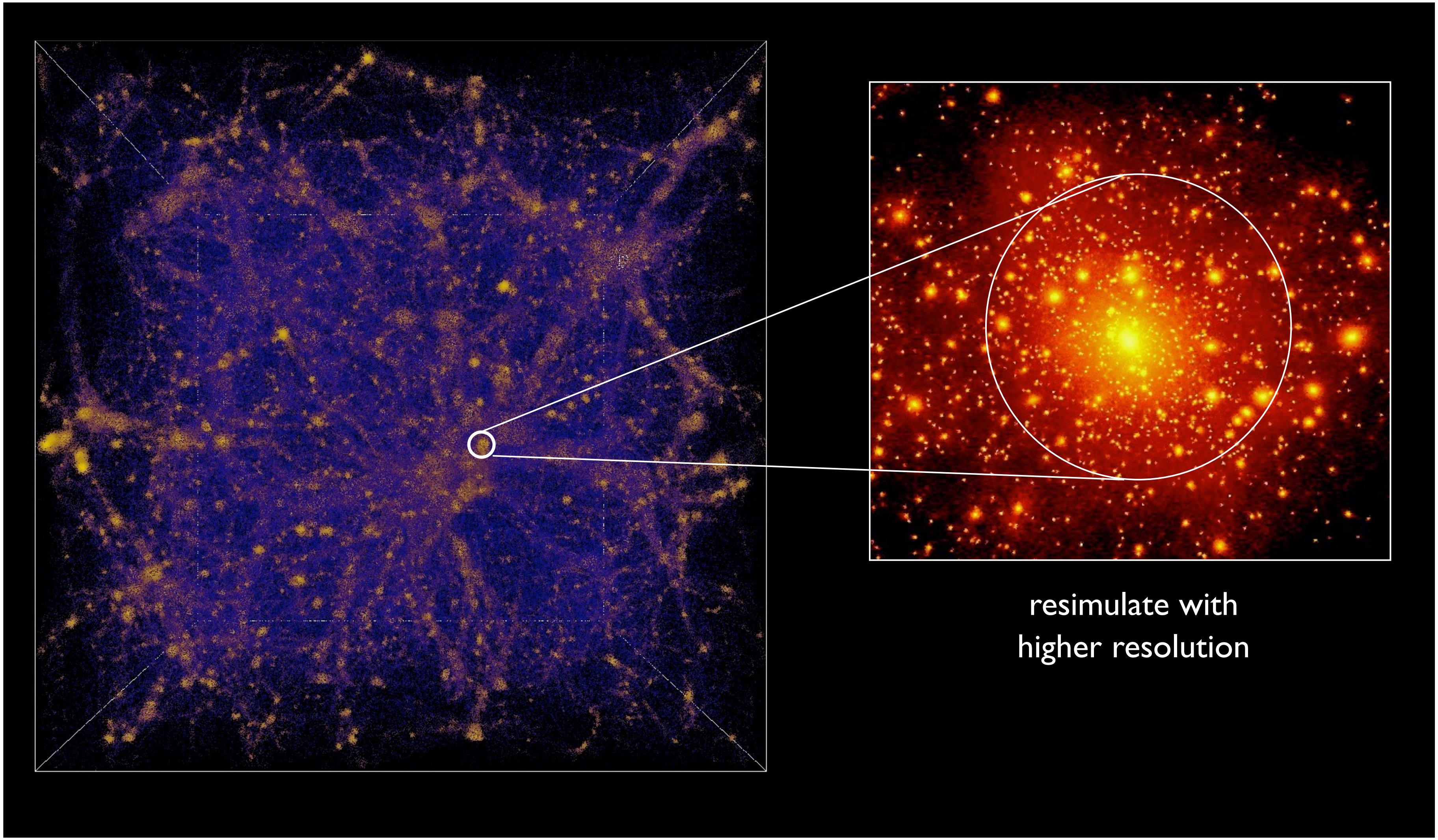
- low resolution simulation



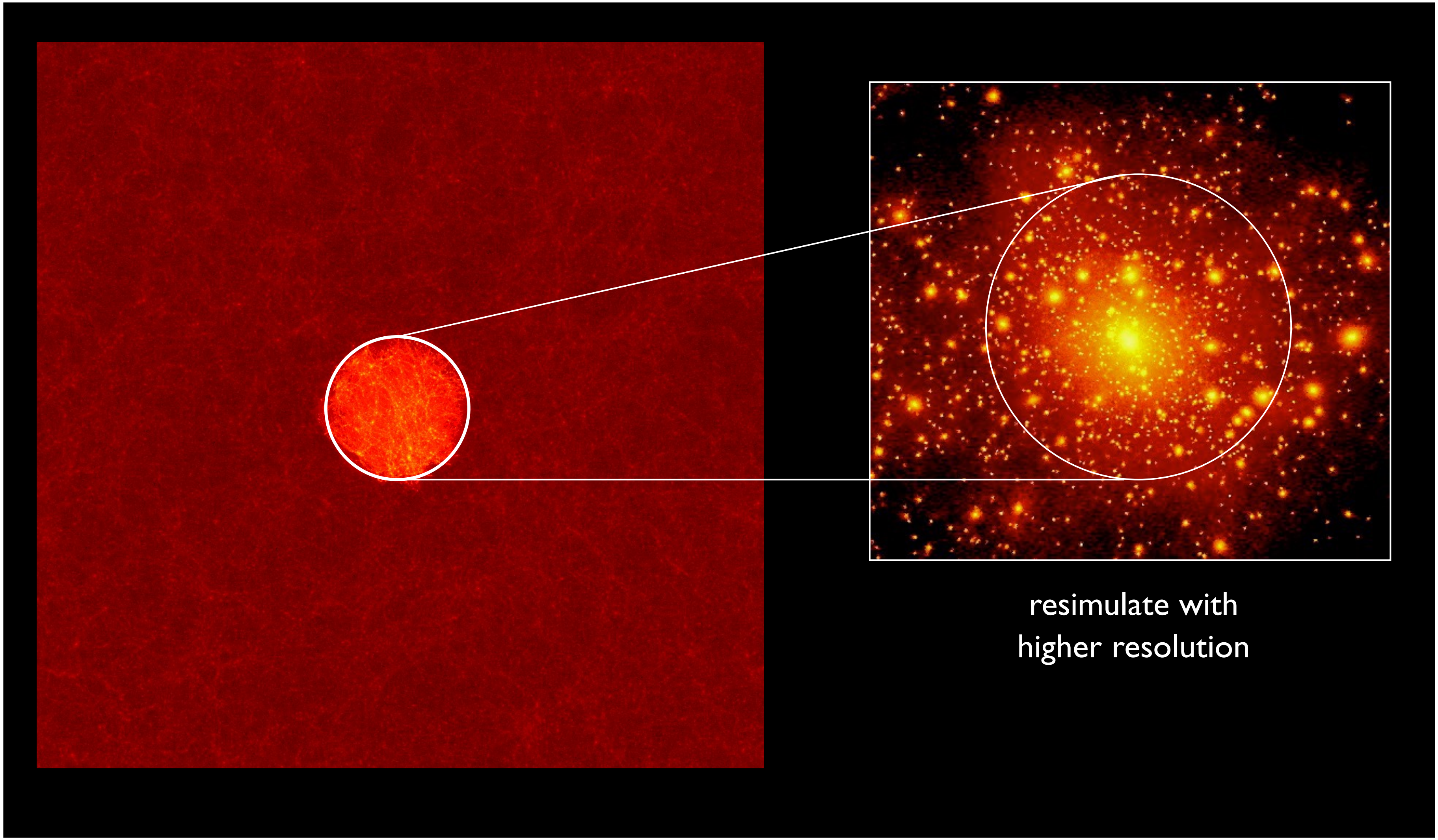
- low resolution simulation



- low resolution simulation



- zoom initial conditions



- zoom simulation