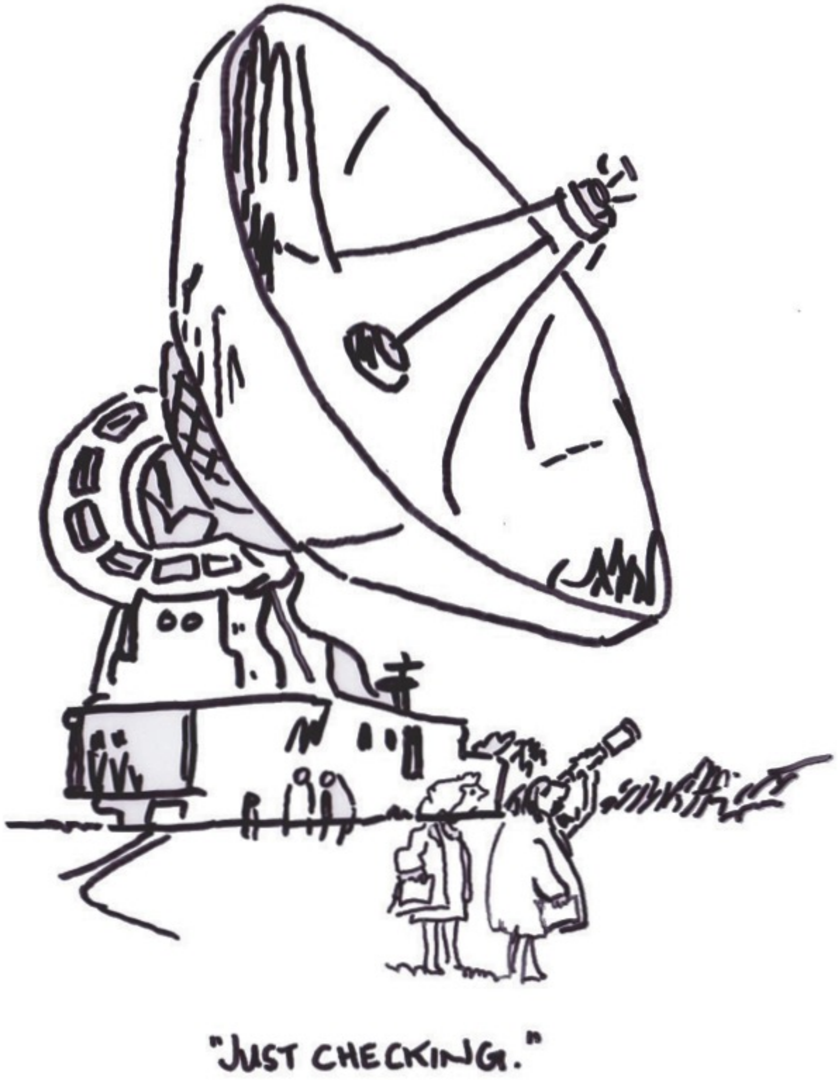
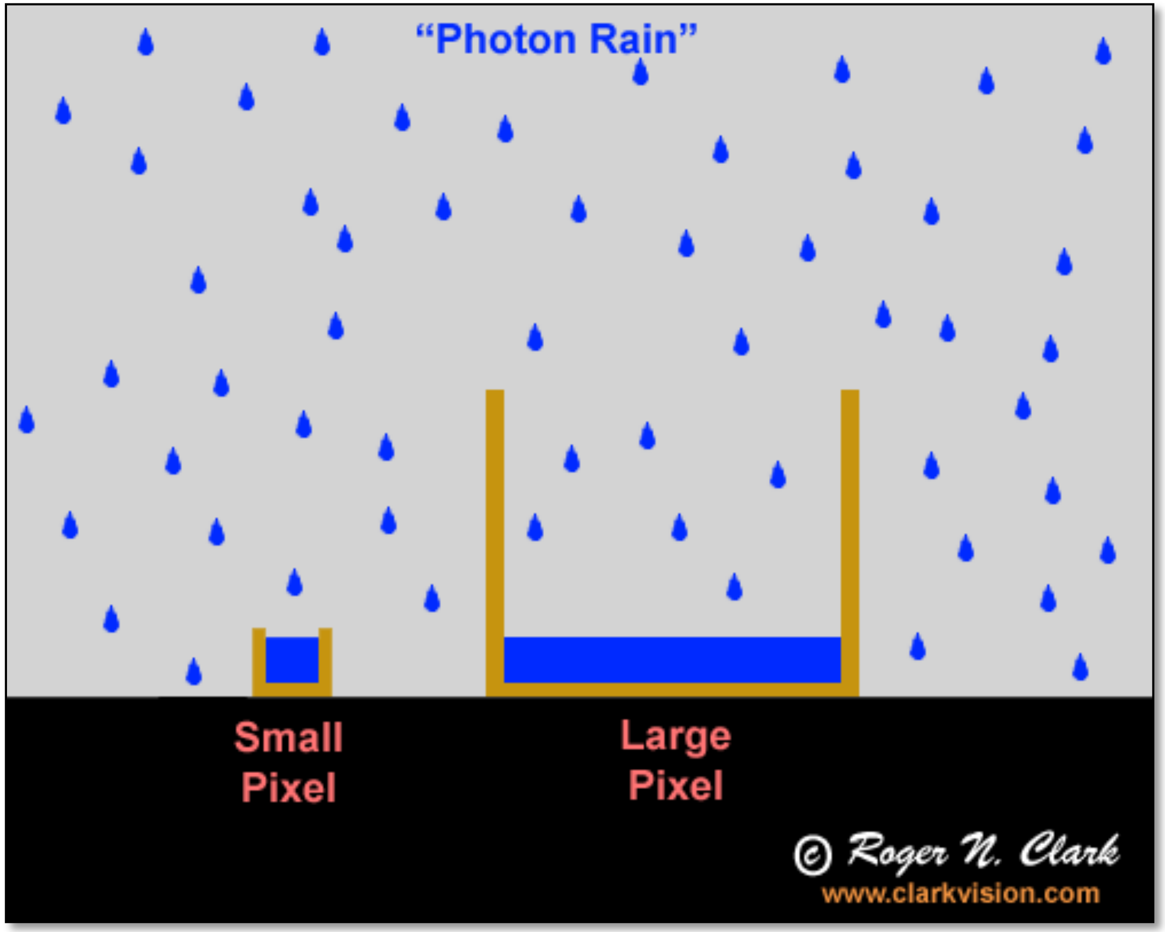
Alexander Knebe (*Universidad Autonoma de Madrid*)



- astronomy is...



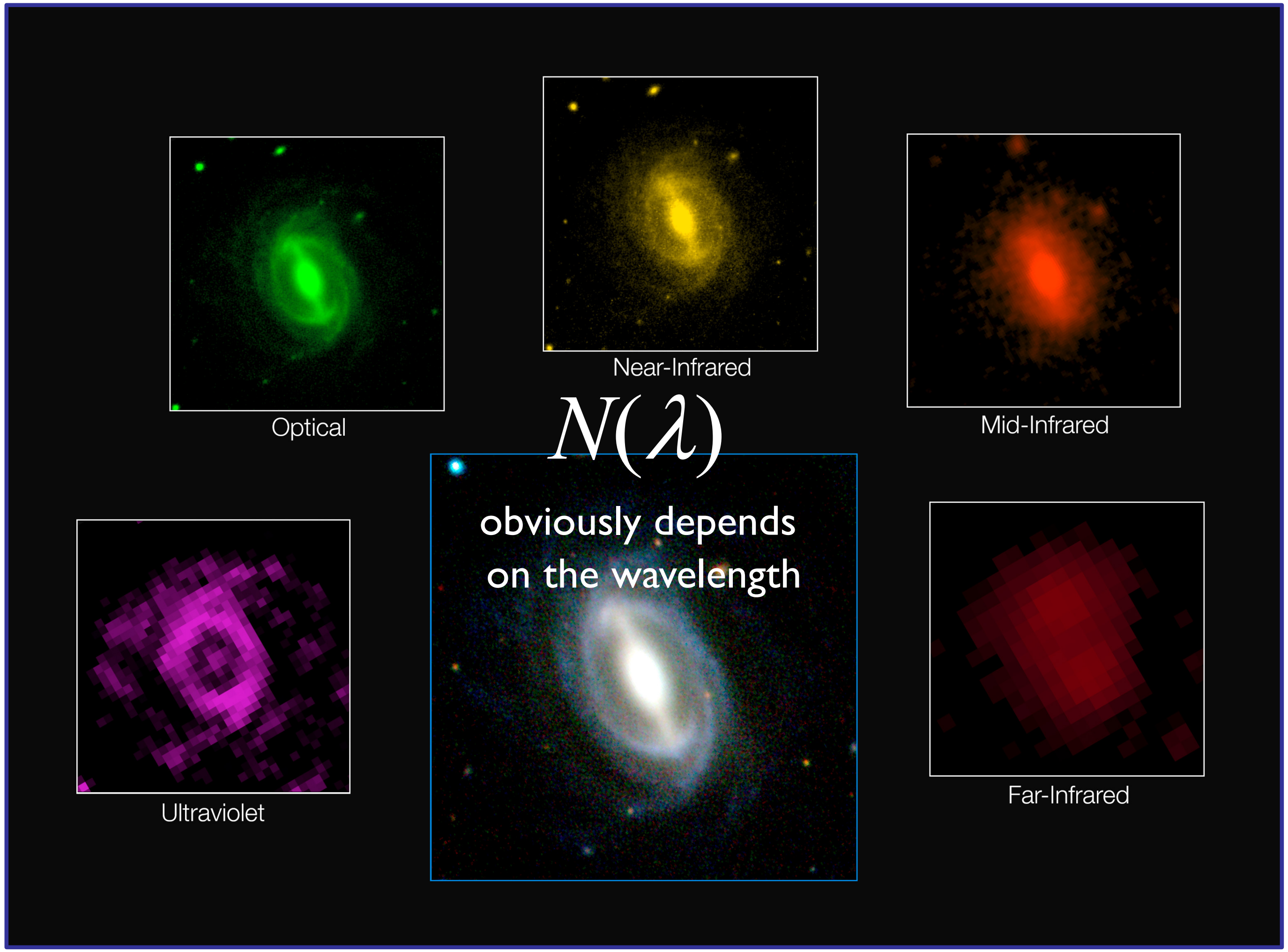
...collecting and counting photons

- astronomy is...

$$N(\lambda)$$

...collecting and counting photons

- astronomy is...



...collecting and counting photons

- astronomy is...

supernova 1994D

$$N(\lambda)$$

obviously depends
on the wavelength,
the observed object

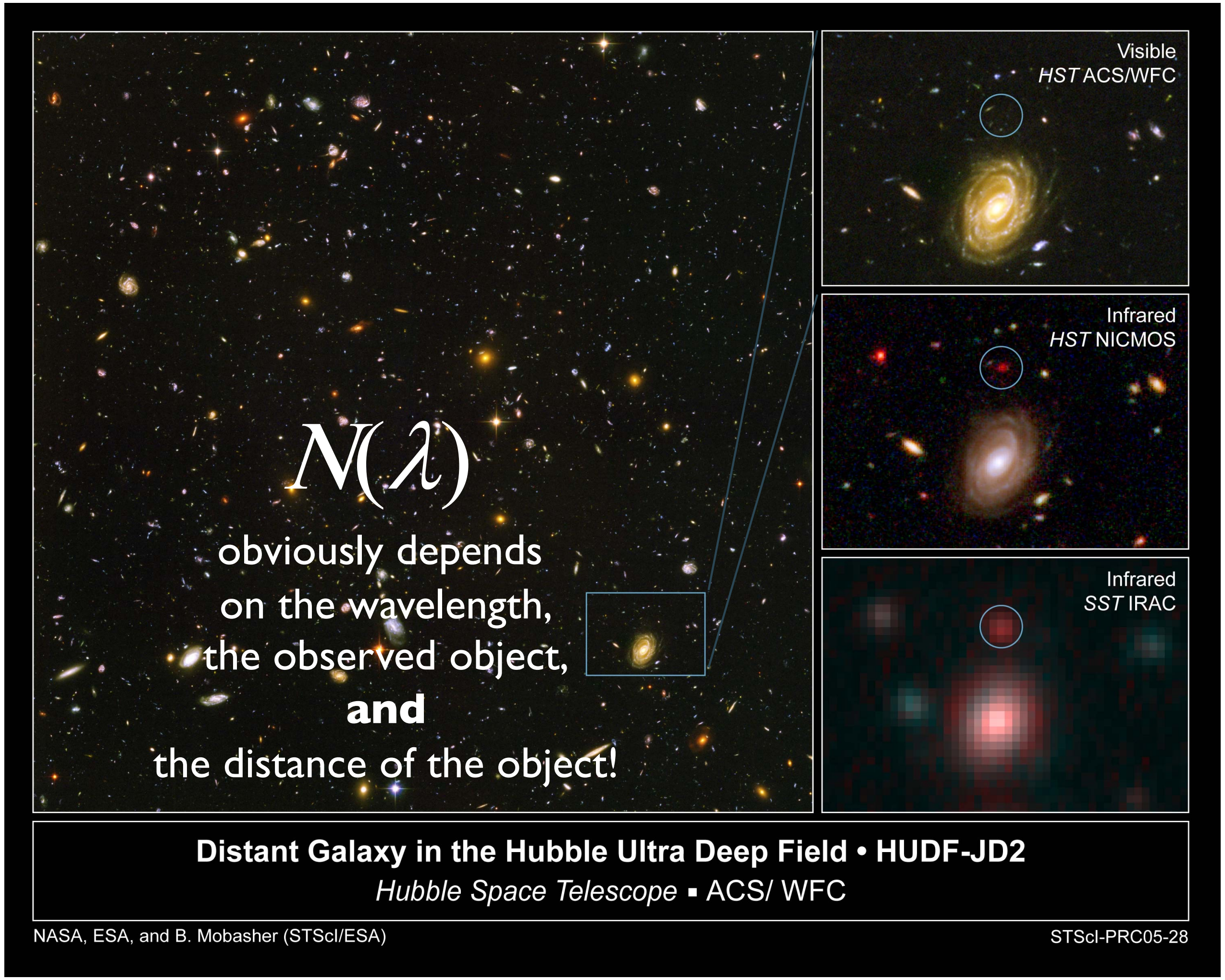
NGC 1232

NGC 1132

...collecting and counting photons

- astronomy is...

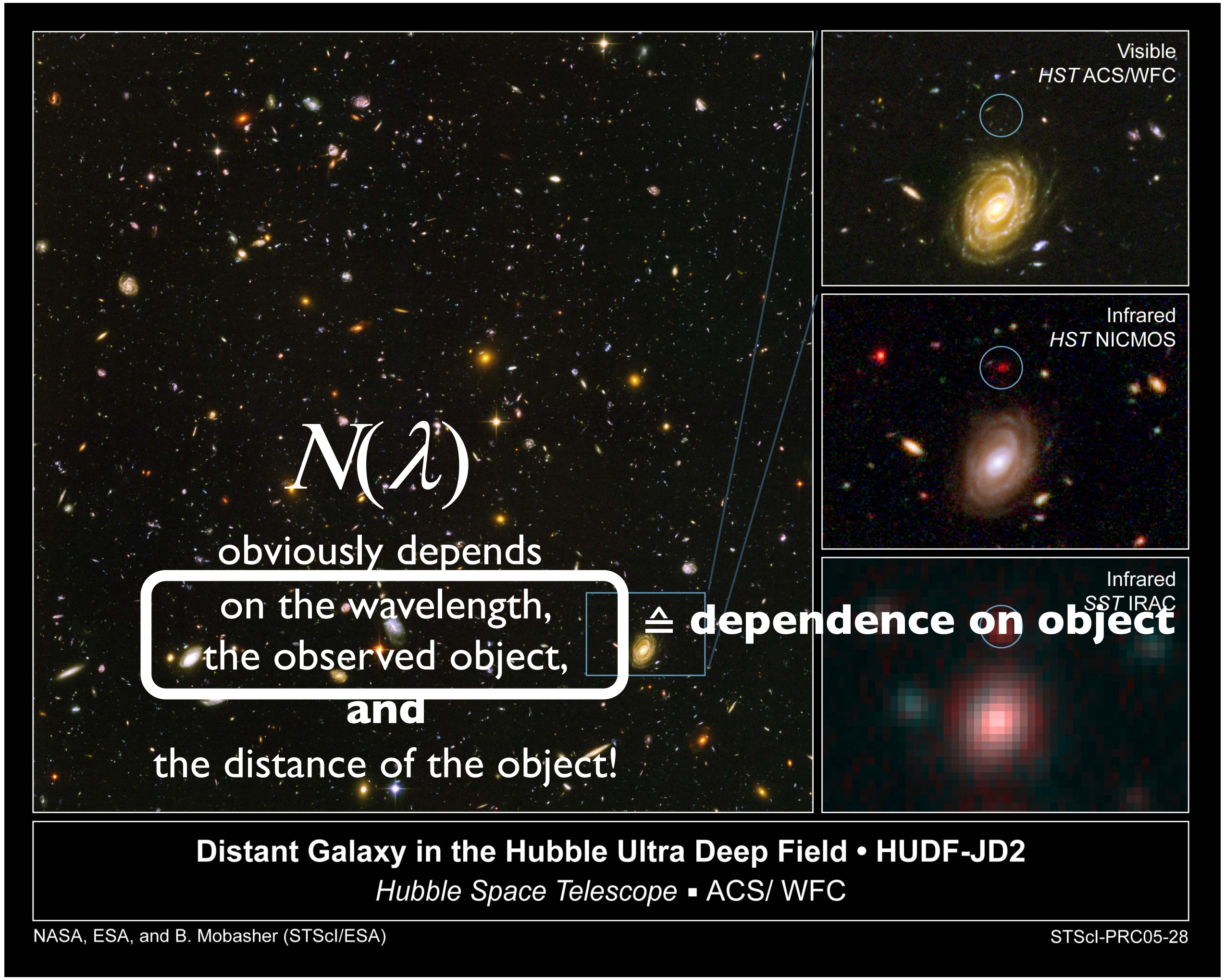
supernova 1994D



...collecting and counting photons

- astronomy is...

supernova 1994D



$N(\lambda)$

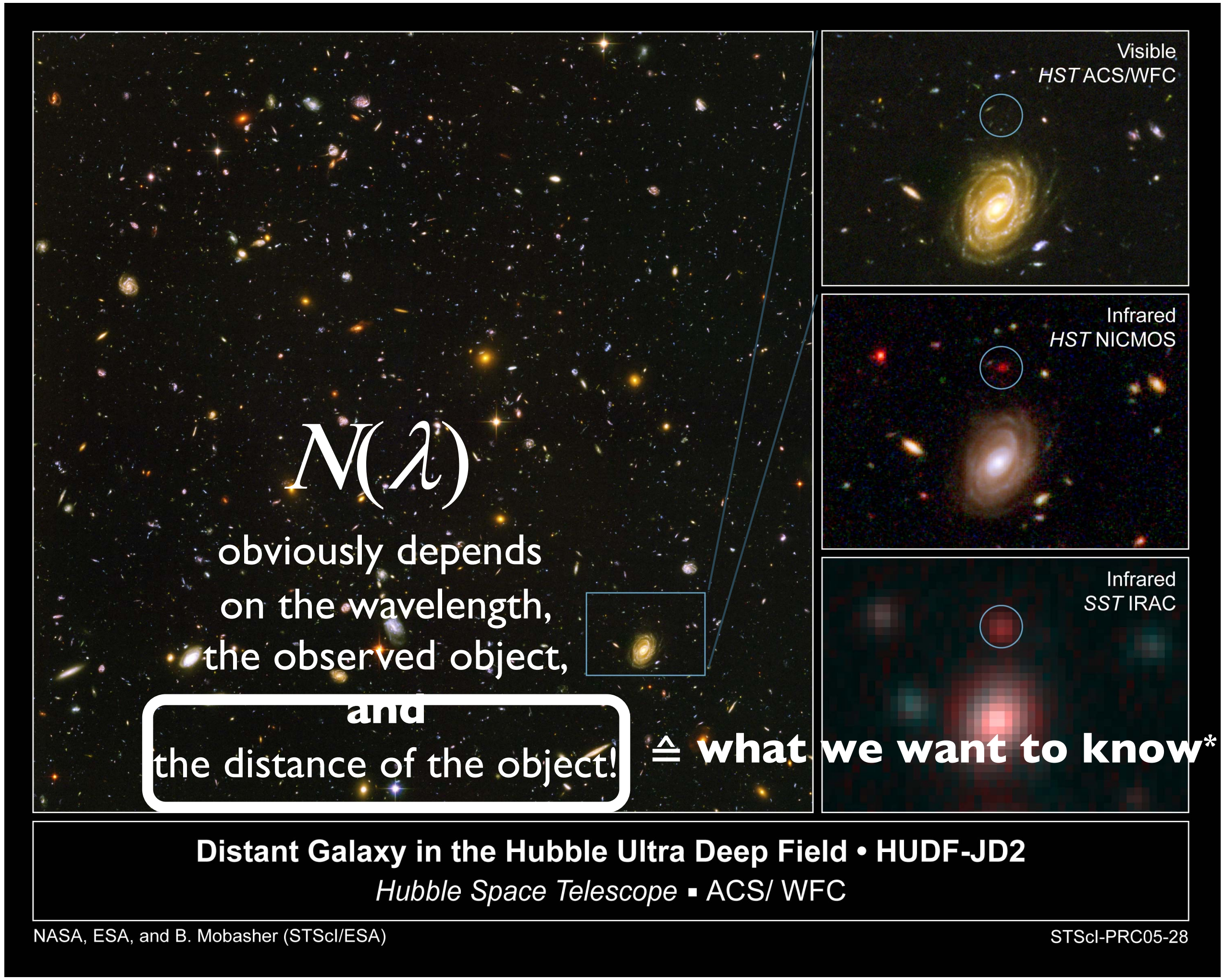
obviously depends
on the wavelength,
the observed object,
and
the distance of the object!

≙ **dependence on object**

...collecting and counting photons

- astronomy is...

supernova 1994D

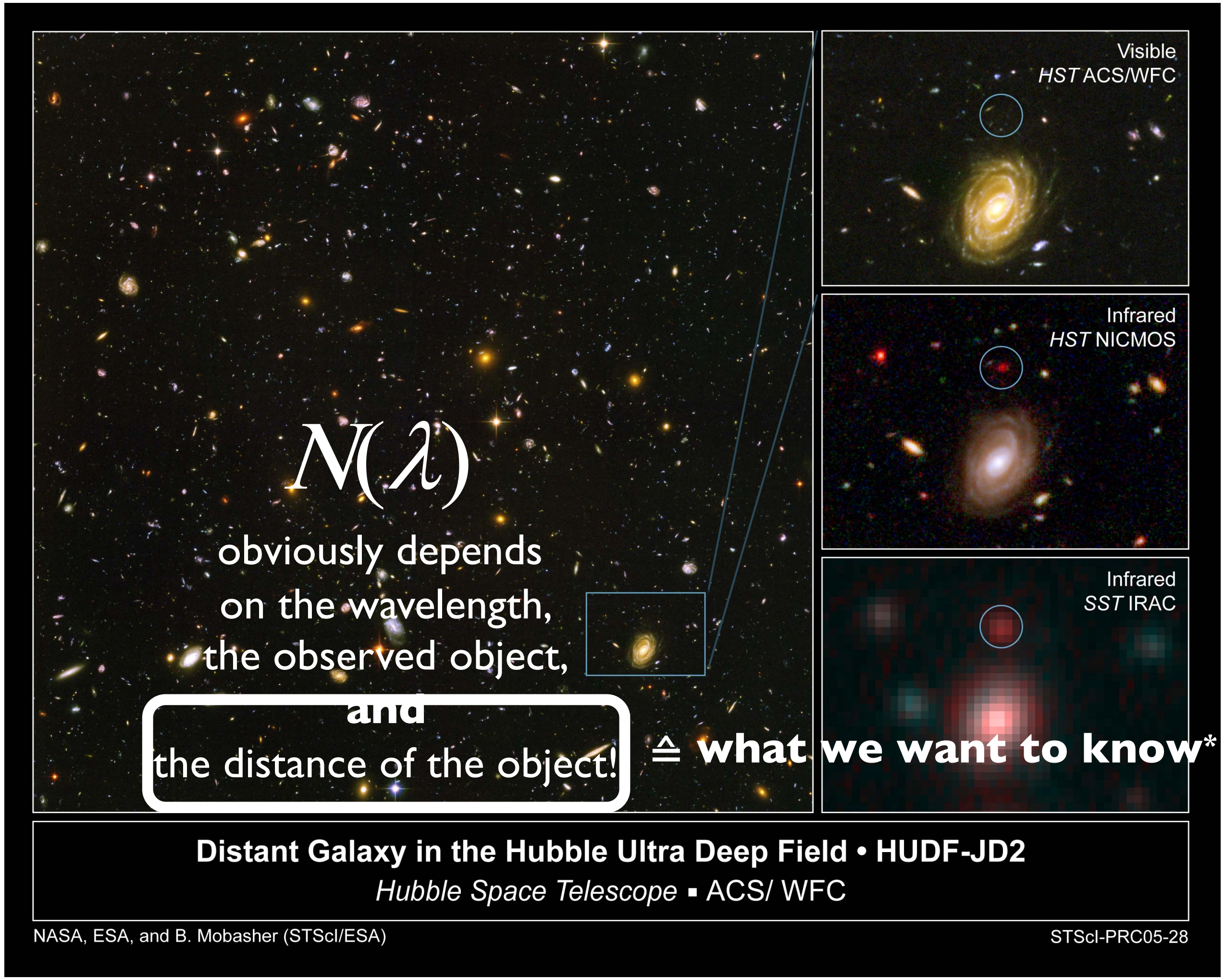


...collecting and counting photons

*redshift z only tells us how much space has expanded since photon emission

- astronomy is...

supernova 1994D



$N(\lambda)$

obviously depends on the wavelength, the observed object, **and** the distance of the object! ≙ **what we want to know***

...collecting and counting photons

*redshift z only tells us how much space has expanded since photon emission: **the redshift is not the distance *per se*!**

- cosmology uses...

supernova 1994D

$$N(\lambda)$$

depends on
the object and
the distance to the object

- cosmology uses...

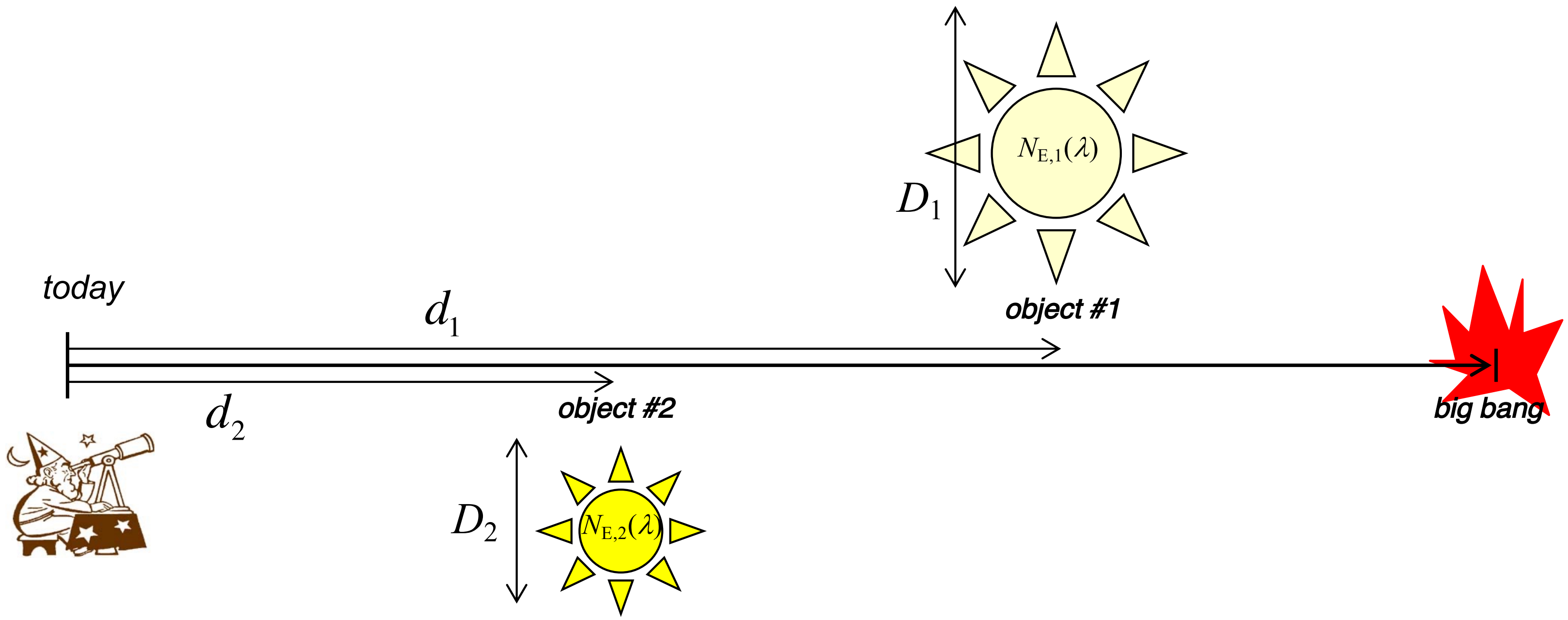
supernova 1994D

$$N(\lambda)$$

depends on
~~the object and~~
the distance to the object

...standard "candles" and "rulers" to eliminate the dependence on the object?

- cosmology uses...

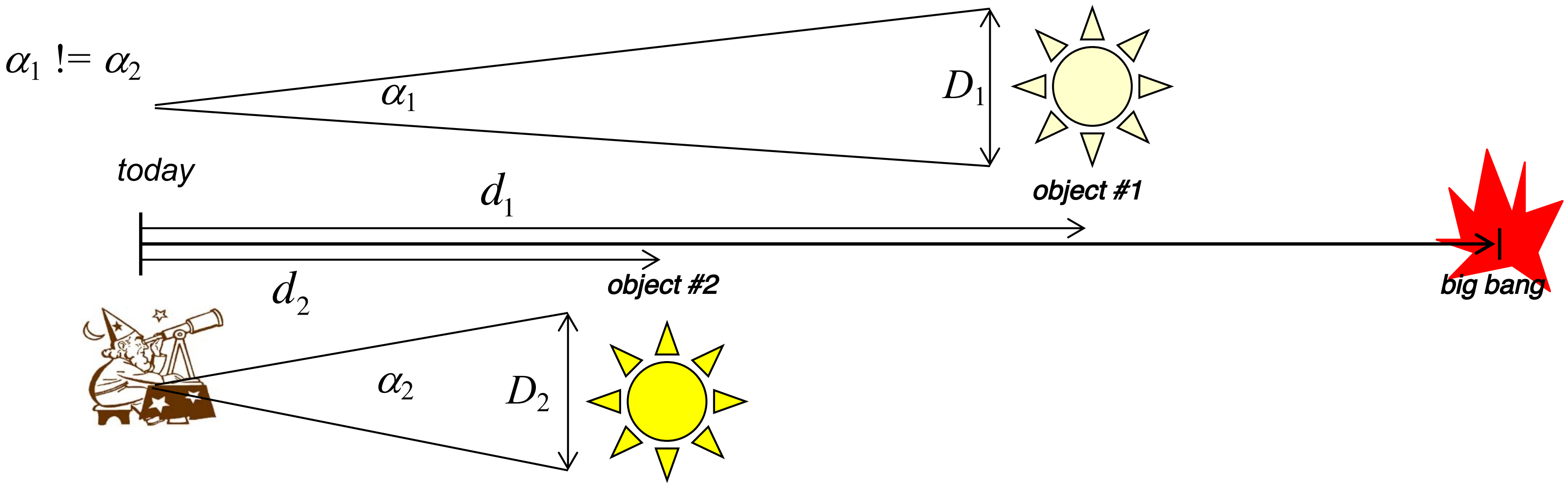


...standard "candles" and "rulers" to eliminate the dependence on the object

- cosmology uses...

standard ruler: objects might have different luminosity, but the same size

$D_1 = D_2$



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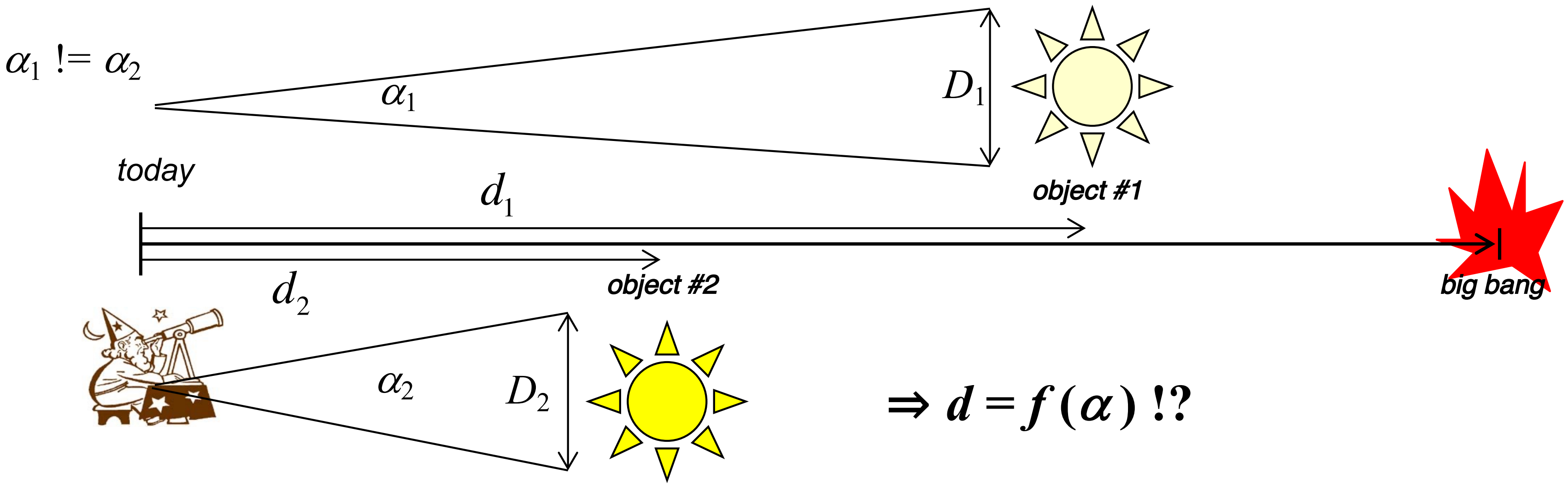
- cosmology uses...

standard ruler: objects might have different luminosity, but the same size

$$D_1 = D_2$$



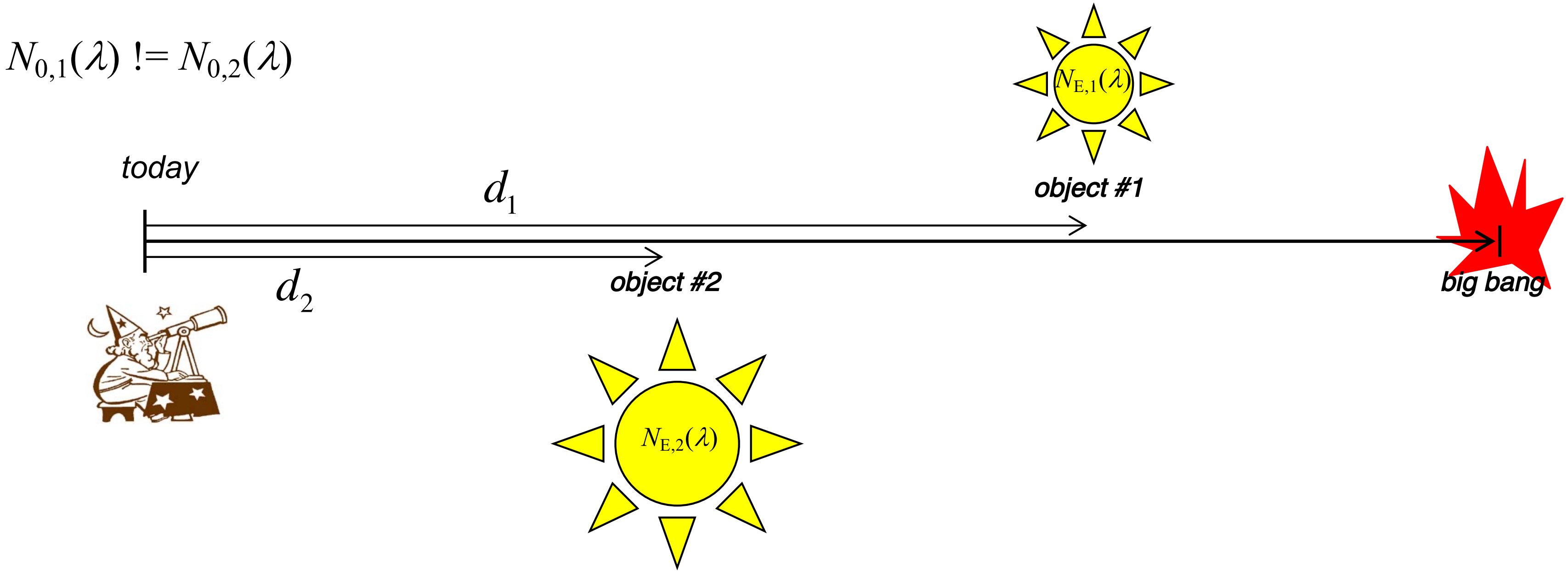
$$\Rightarrow d = f(\alpha) \; !?$$

...standard "rulers" to eliminate the dependence on the object

- cosmology uses...

standard candle: objects might have different sizes, but the same luminosity

$$N_{E,1}(\lambda) = N_{E,2}(\lambda)$$



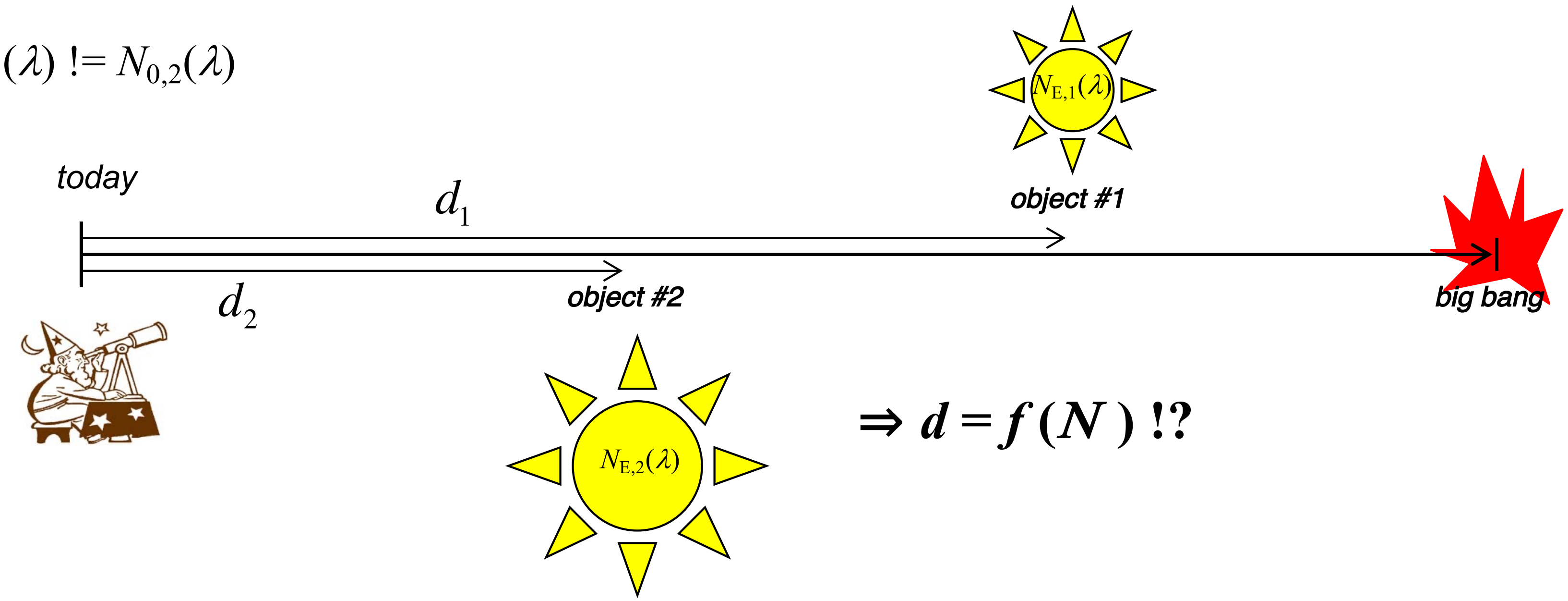
...standard "candles" to eliminate the dependence on the object

- cosmology uses...

standard candle: objects might have different sizes, but the same luminosity

$$N_{E,1}(\lambda) = N_{E,2}(\lambda)$$

$$N_{0,1}(\lambda) \;!= N_{0,2}(\lambda)$$

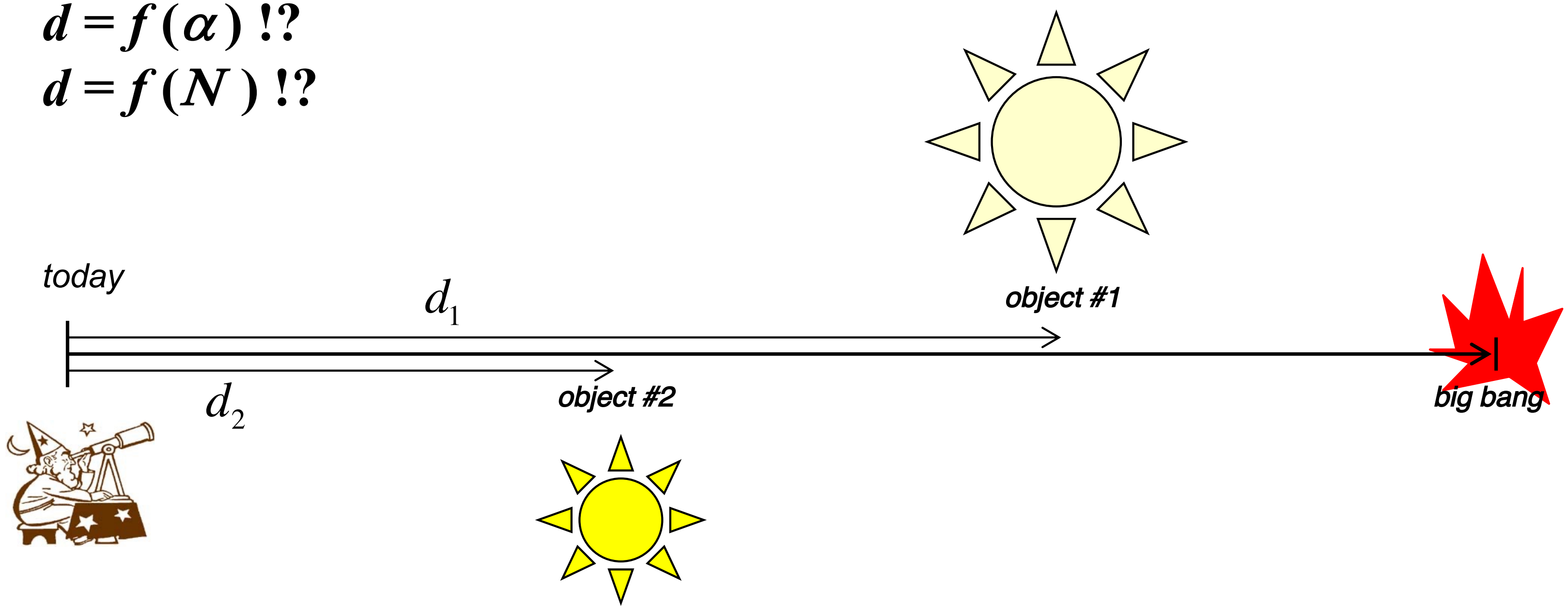


$$\Rightarrow d = f(N)\ !?$$

...standard "candles" to eliminate the dependence on the object

- cosmology uses...

$$d = f(\alpha)\ !?$$
$$d = f(N)\ !?$$

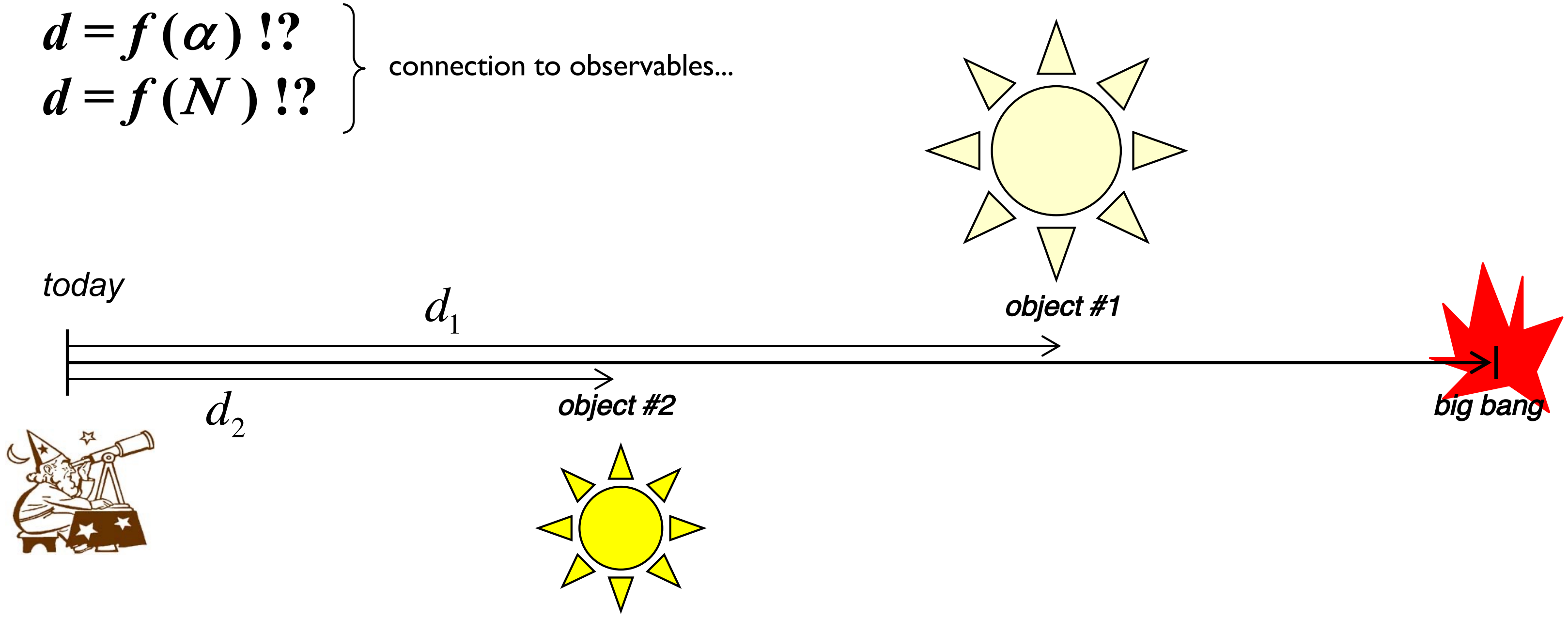


...standard "candles" and "rulers" to eliminate the dependence on the object

- cosmology uses...

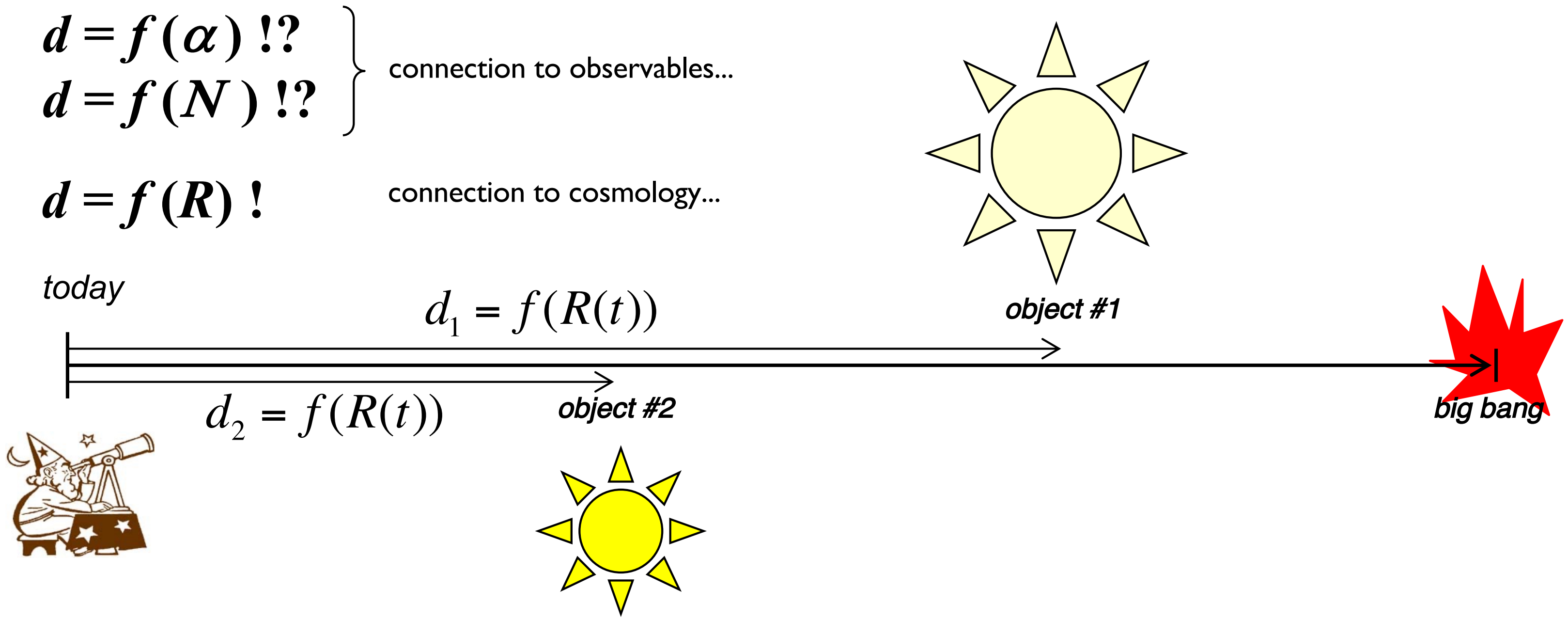
$$d = f(\alpha)\ !?$$
$$d = f(N)\ !?$$

connection to observables...



...standard "candles" and "rulers" to eliminate the dependence on the object

- cosmology uses...

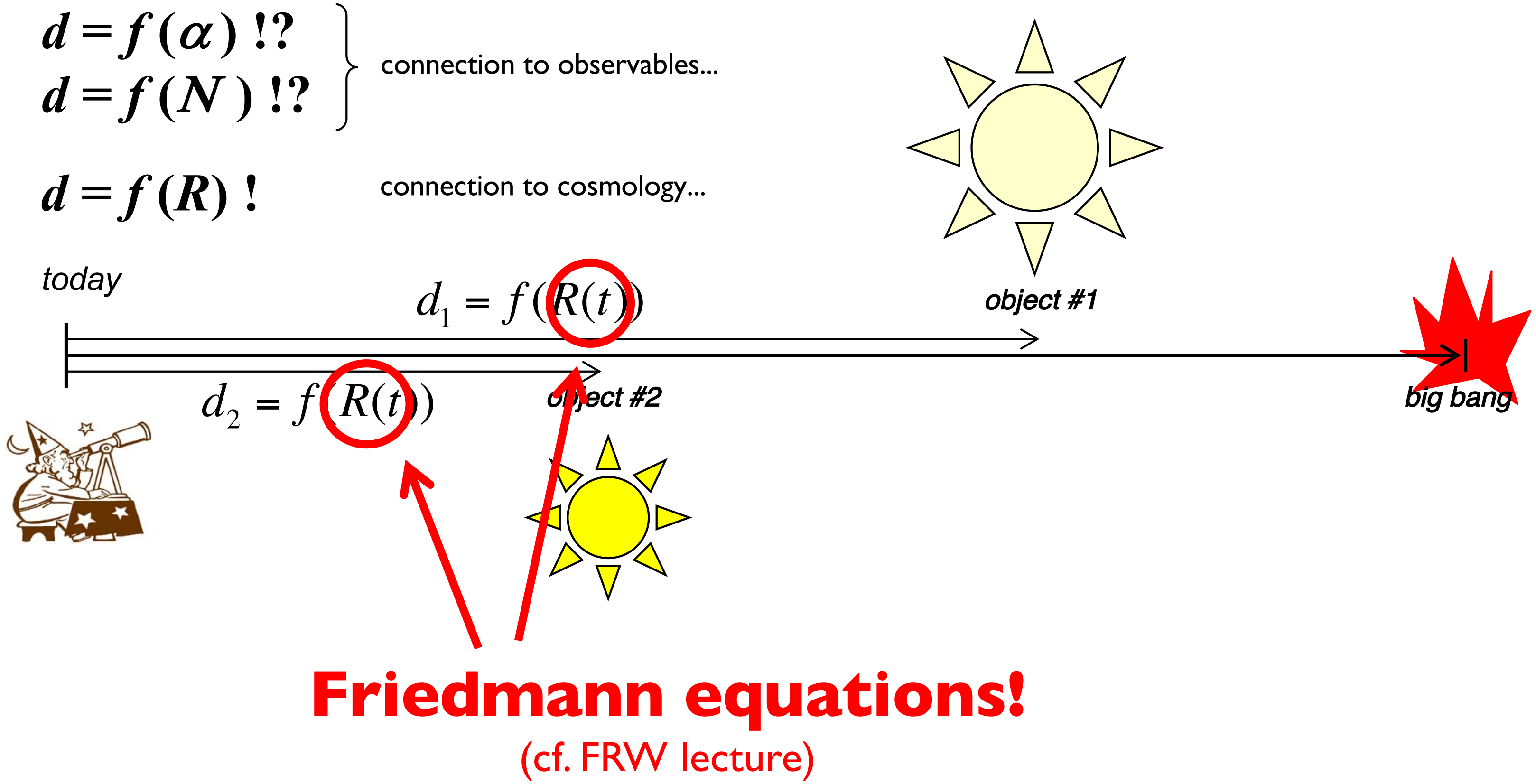


...standard "candles" and "rulers" to eliminate the dependence on the object and to infer **the cosmological parameters**!

- cosmology uses...

$d = f(\alpha)$!?
$d = f(N)$!?
connection to observables...

$d = f(R)$!
connection to cosmology...



...standard "candles" and "rulers" to eliminate the dependence on the object and to infer **the cosmological parameters**!

- cosmology uses...

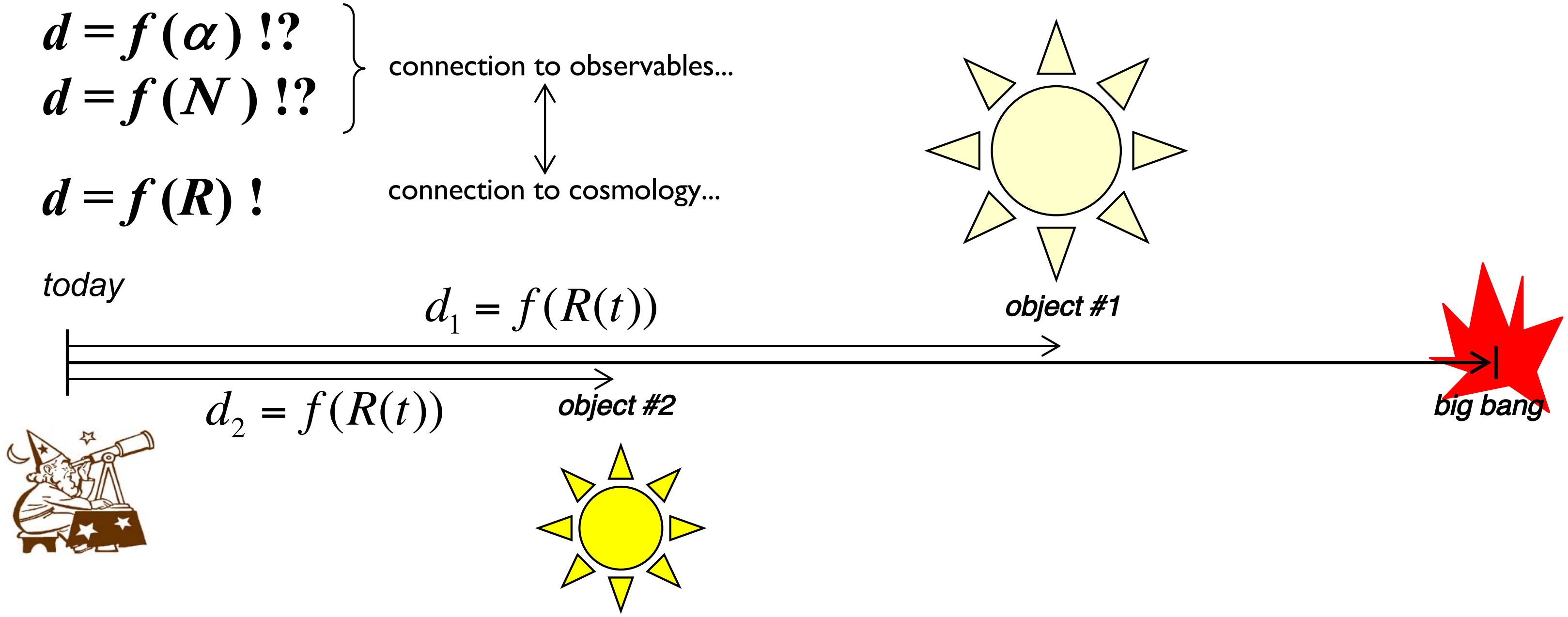
$$d = f(\alpha)\ !?$$
$$d = f(N)\ !?$$

connection to observables...

↕

connection to cosmology...

$$d = f(R)\ !$$



...but we need to have a gauge for the relation between "photon counts" and distance:
Cosmic Distance Ladder

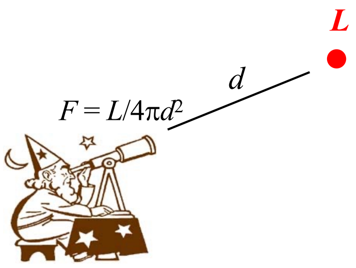
...standard "candles" and "rulers" to eliminate the dependence on the object and to infer **the cosmological parameters**!

- cosmic distance ladder
- cosmological distances
- cosmological horizons & volumes
- supernova cosmology

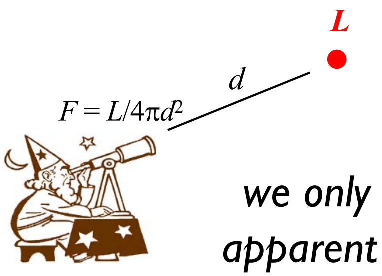
- **cosmic distance ladder**
- cosmological distances
- cosmological horizons & volumes
- supernova cosmology

- cosmological distance ladder...

- cosmological distance **ladder?**



- cosmological distance **ladder?**

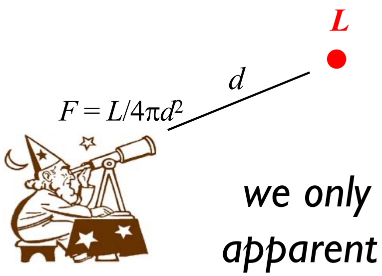


we only ever observe apparent magnitudes F and never absolute magnitudes L!

Cosmic Distance Ladder

idea

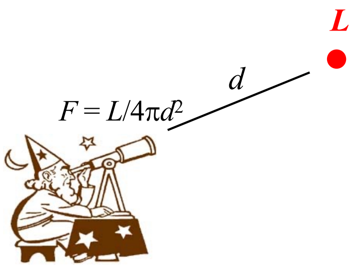
- cosmological distance **ladder?**



we only ever observe
apparent magnitudes F
and never
absolute magnitudes L!

→ standard candles to the rescue...

- cosmological distance **ladder?**
    - <u>example</u>:
        - we have a class of stars with identical luminosities
        - we determine the distance to one such star locally (e.g. via parallax)



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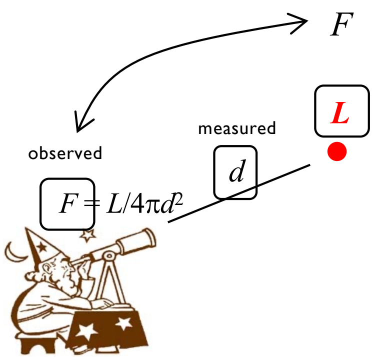

F ≙ apparent magnitude (photons received)

L ≙ absolute magnitude (photons emitted)

- cosmological distance **ladder?**
  - <u>example</u>:
    - we have a class of stars with identical luminosities
    - we determine the distance to one such star locally (e.g. via parallax)
    - observing such star(s) in another type of distant object (globular cluster, galaxy, etc.) we can calculate the distance to that object via $d^2 = L/4\pi F$

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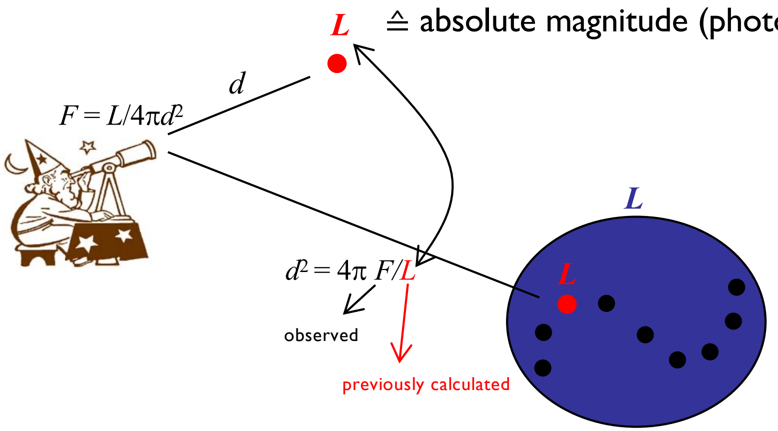

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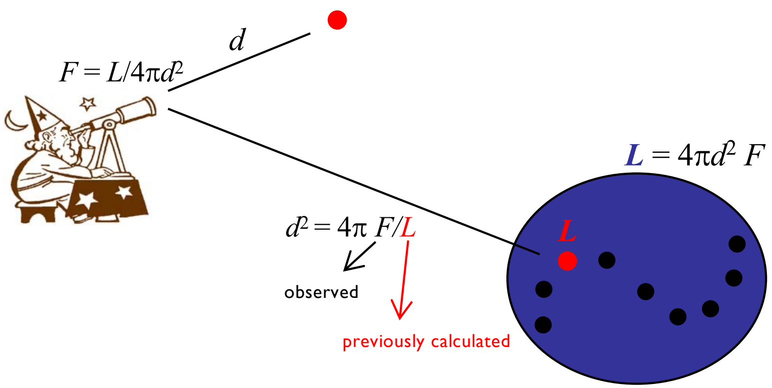

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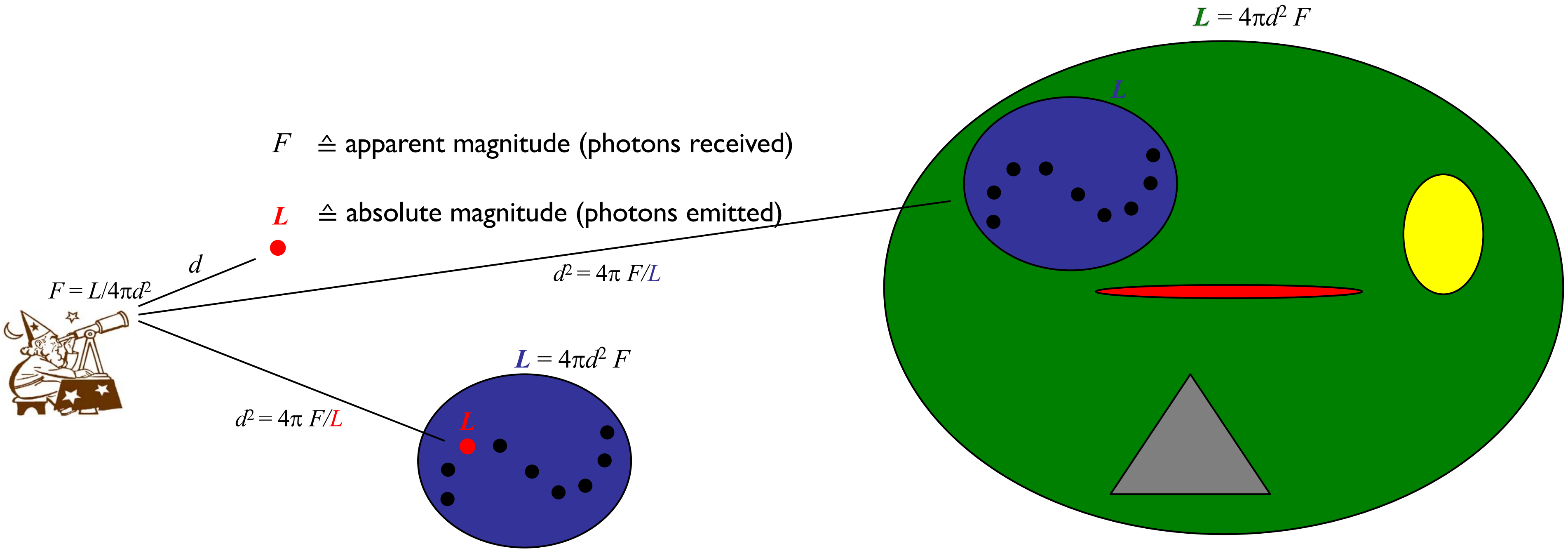
    - that object itself (if "standard" in some sense) can then be used as the next rung…



- cosmological distance **ladder?**
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    - we have a class of stars with identical luminosities
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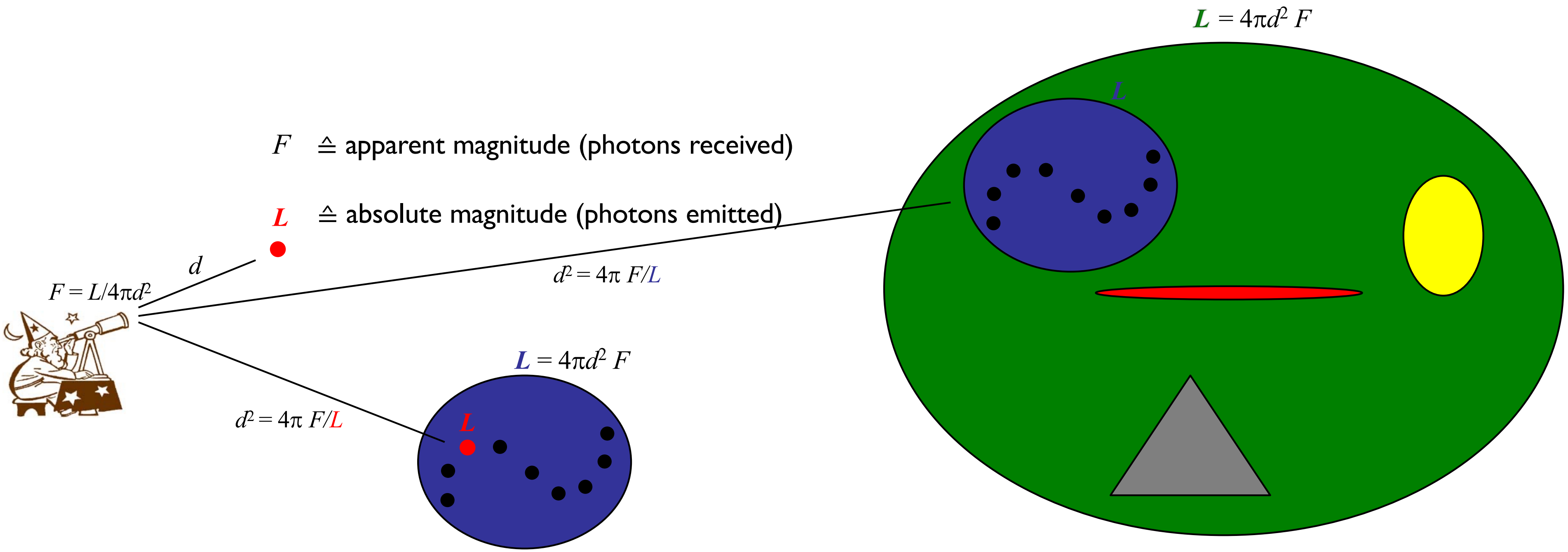
    - that object itself (if "standard" in some sense) can then be used as the next rung…

we still require a gauge!

$F \triangleq$ apparent magnitude (photons received)

$L \triangleq$ absolute magnitude (photons emitted)



- direct parallax:

one of the few possibility to

directly get the distance

without knowing anything about the object

- direct parallax:

$$\sin p = \frac{R_e}{D}$$

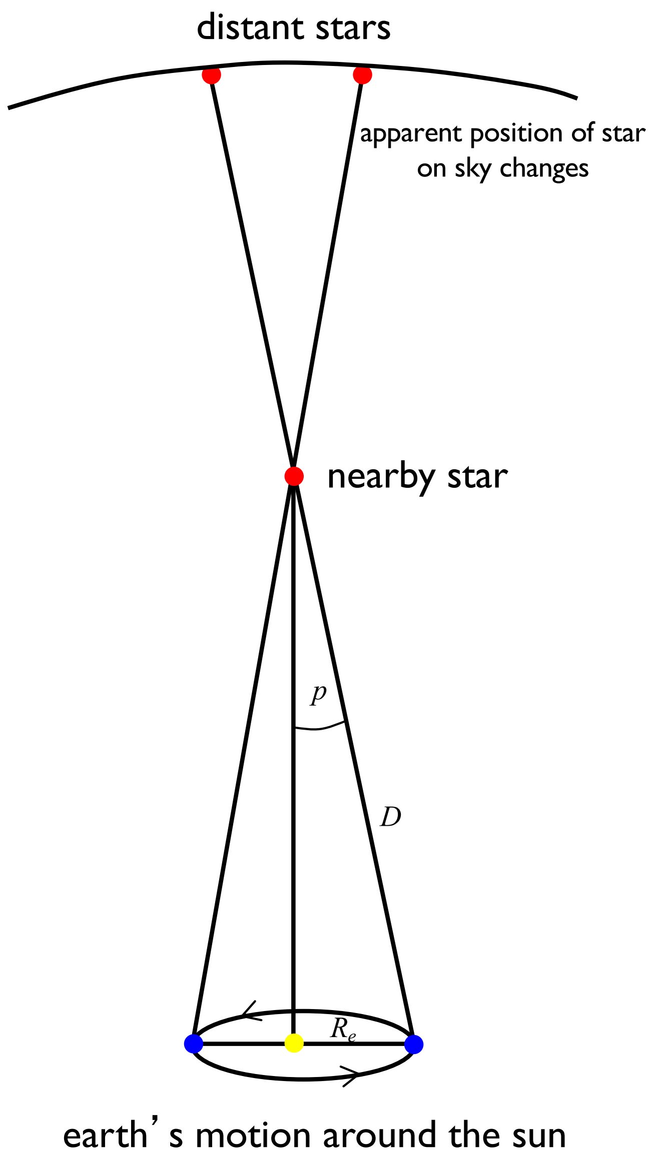
$\sin p \approx p$ [radians] (for small p)

$$p'' = \frac{R_e}{D} \times \frac{360}{2\pi} \times \frac{1}{3600}\ \text{[arcsec]}$$

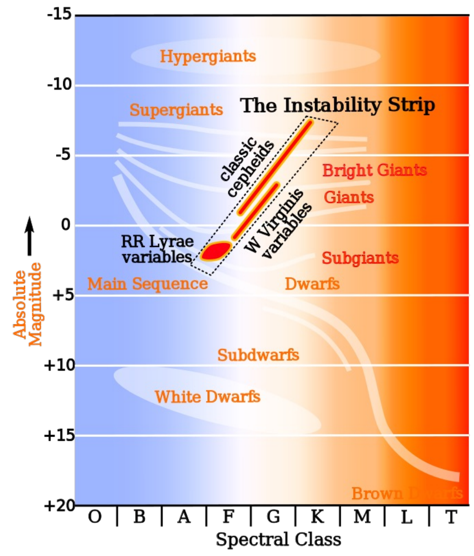
- parsec (definition!):

$$D = \frac{1''}{p''} \quad \text{[pc]}$$

$$1pc = 3.0857 \cdot 10^{16} m$$



- RR Lyrae stars: *pulsating horizontal branch stars*

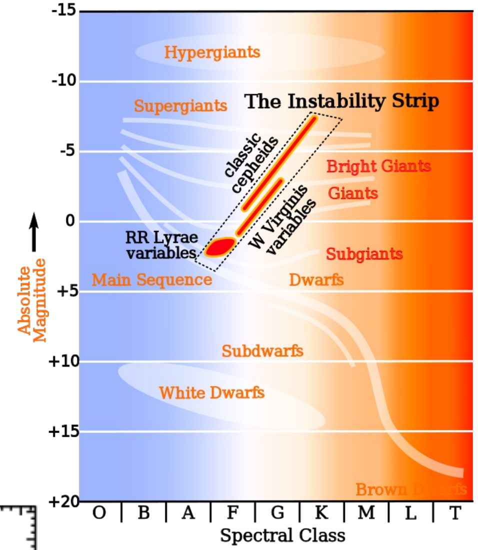


  - similar (mean) absolute luminosity:

standard candle: $\langle L \rangle \approx const.$ (=energy/time)

  - unfortunately not very bright though...

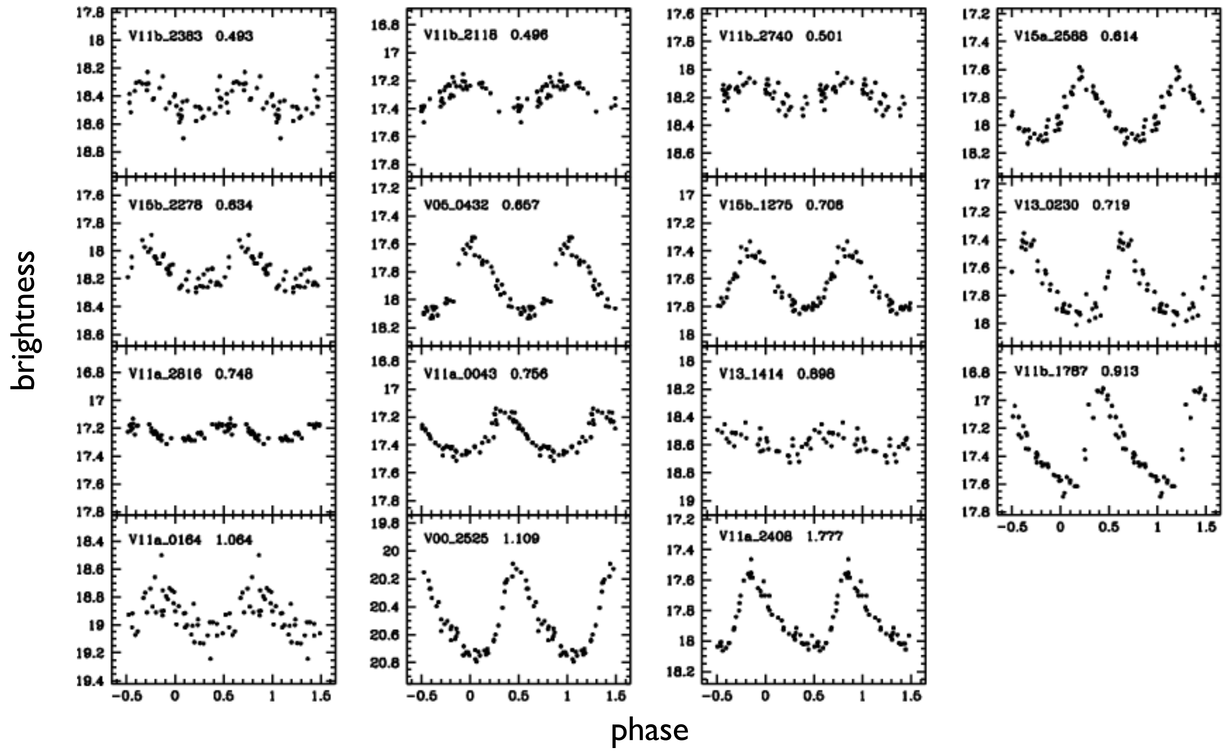
- Cepheid stars: *pulsating stars off the main sequence*



  - much brighter than RR Lyrae stars

  - relation between pulsation period and absolute luminosity:

$$\log L \propto \log P$$



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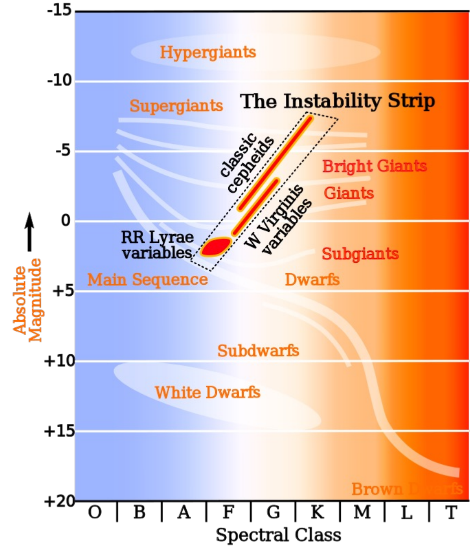

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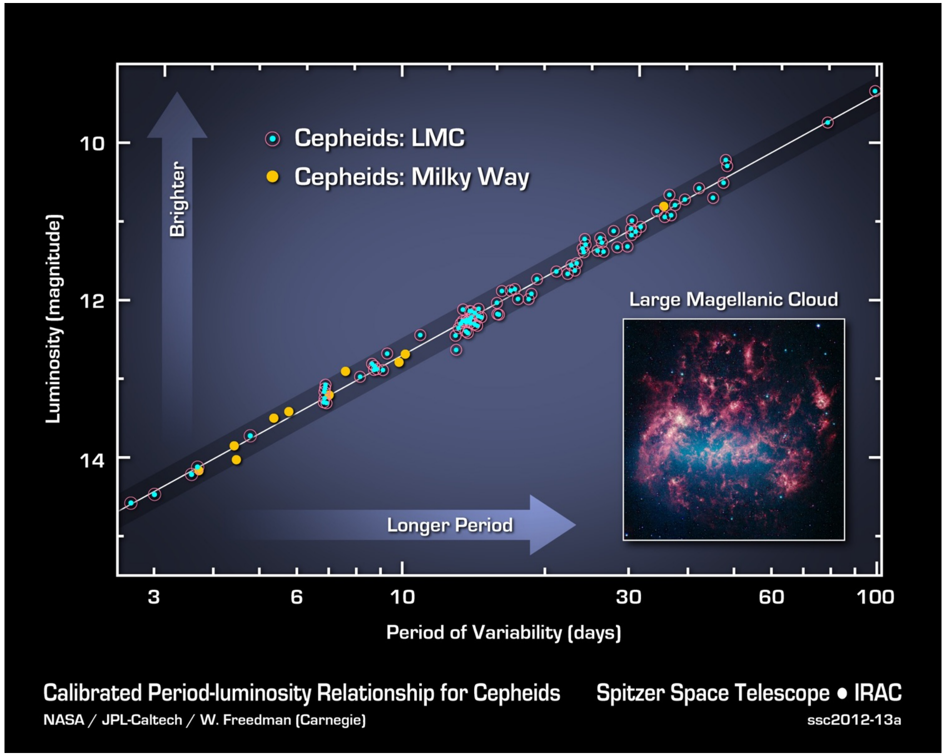
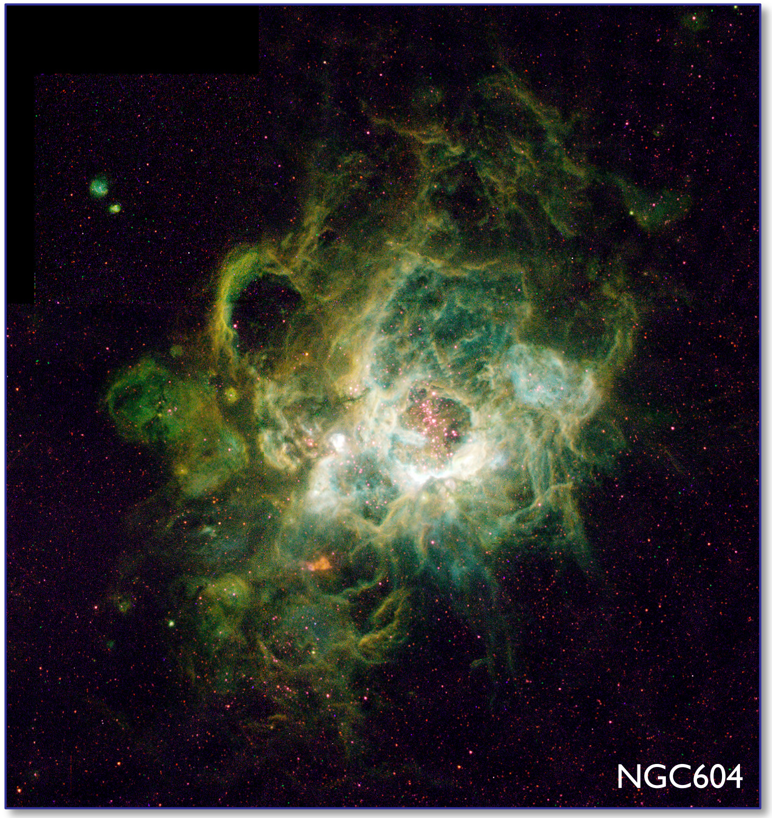


- HII regions
  - large clouds of ionized hydrogen surrounding very hot stars < 30 Mpc

standard ruler: $\langle D \rangle \approx const.$



- HII regions
  - large clouds of ionized hydrogen surrounding very hot stars < 30 Mpc

standard ruler: $\langle D \rangle \approx const.$

- planetary nebulae < 30 Mpc
  - reprocessed light from central star

standard candle: $\langle L \rangle \approx const.$

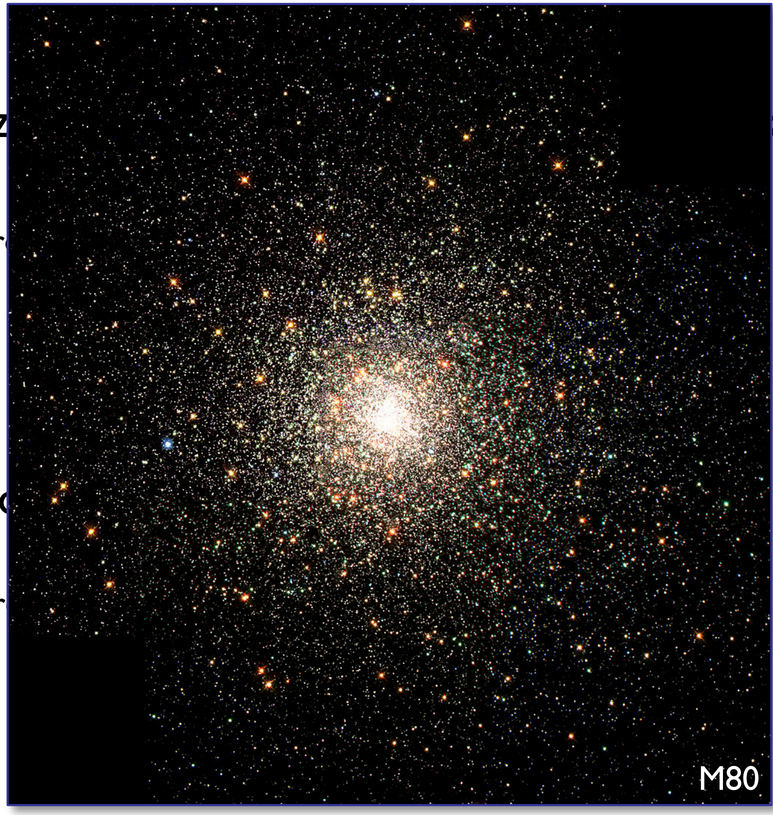

Spirograph Nebula

- HII regions
  - large clouds of ioniz… …ars < 30 Mpc

    standar…

- planetary nebulae < 30 Mpc
  - reprocessed light fr…

    standar…



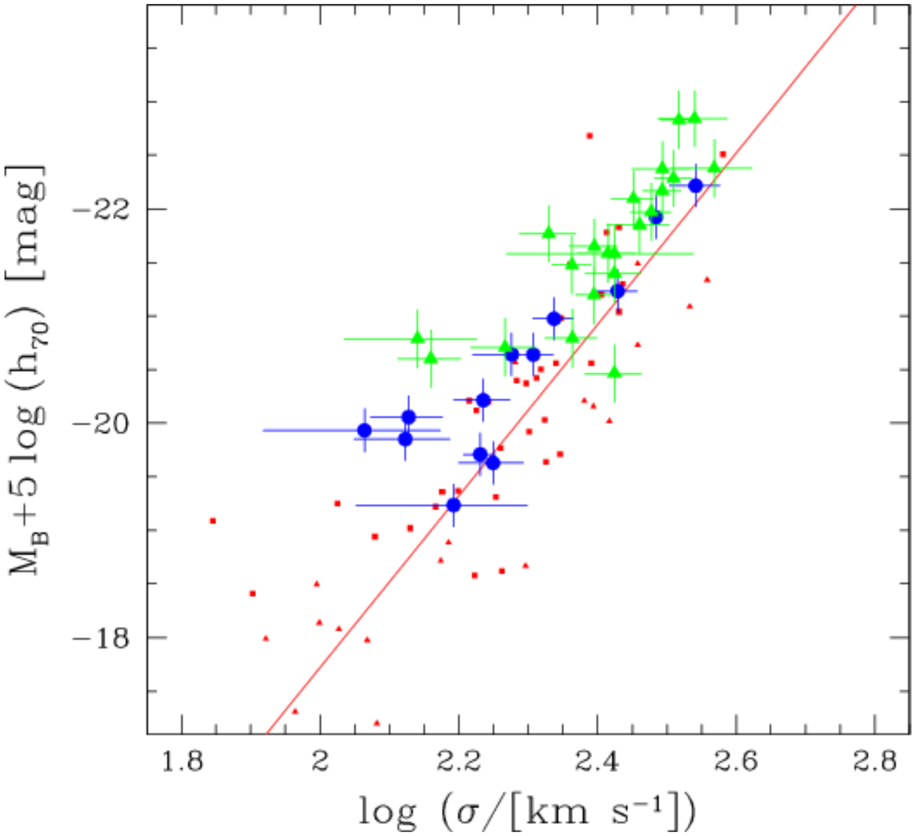
- globular clusters < 50-100 Mpc
  - clusters of around 10^5 to 10^7 stars

    standard candle: $\langle L \rangle \approx const.$

- elliptical galaxies – Faber-Jackson relation

  - empirically determined

$$L \propto \sigma_{los}^{\alpha} \quad \text{with } \alpha \approx 3-4$$



- elliptical galaxies – Faber-Jackson relation

  - empirically determined

$$L \propto \sigma_{los}^{\alpha} \quad \text{with } \alpha \approx 3-4$$

  - explanation:

$$U \propto \frac{M^2}{R}, \quad T \propto M\sigma_{los}^2 \quad \xrightarrow{2T+U=0} \quad \sigma_{los}^2 \propto \frac{M}{R} \quad \longrightarrow \quad \sigma_{los}^2 \propto \frac{L}{R} \quad \longrightarrow \quad \sigma_{los}^2 \propto \frac{L}{\sqrt{L/4\pi\Sigma}}$$

virial theorem

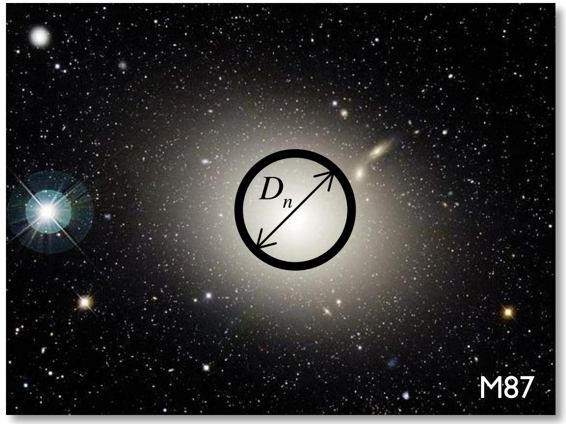
eliminate M in favour of L assuming $M/L = const.$

eliminate R in favour of Σ assuming $\Sigma = L/4\pi R^2 = const.$

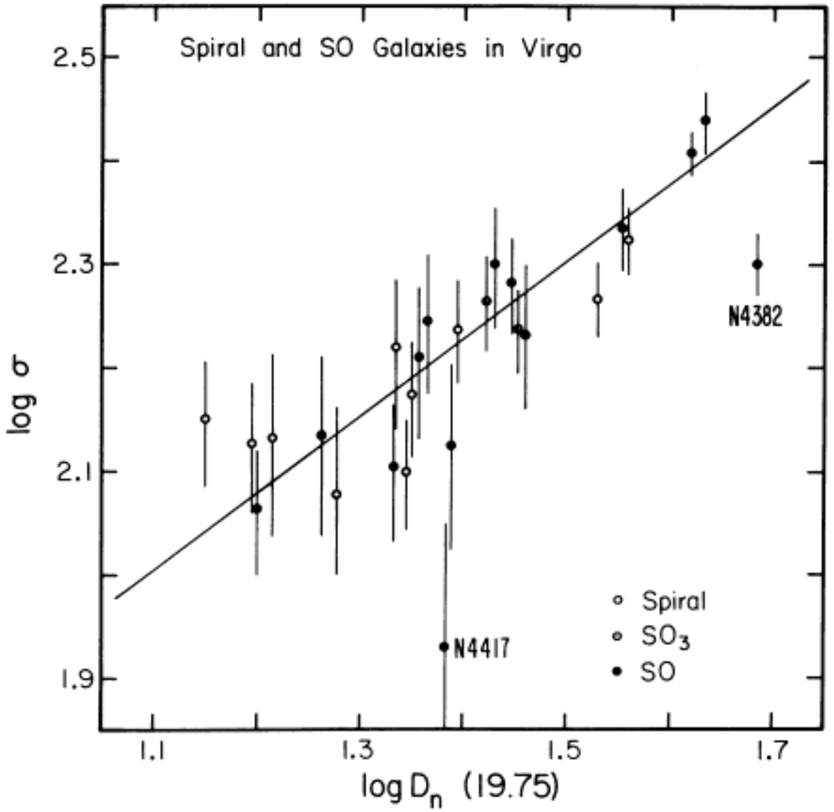
$$\Longrightarrow \quad \sigma_{los}^4 \propto L$$

- elliptical galaxies

  - empirically determined

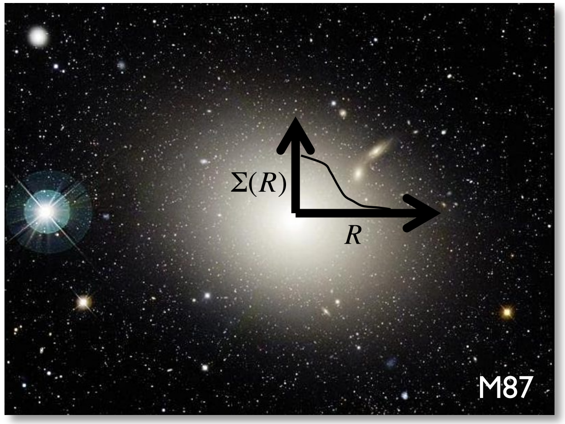


$$D_n \propto \sigma_{los}^{\alpha} \quad \text{with } \alpha \approx 1.2$$



D_n = diameter within which
the mean surface brightness exceeds some threshold

- elliptical galaxies – fundamental plane



  - surface brightness profile

$$\Sigma(R) = \Sigma_0 e^{-\left(R/R_{eff}\right)^4}$$

$\rightarrow \Sigma_0$

$\rightarrow R_{eff}$

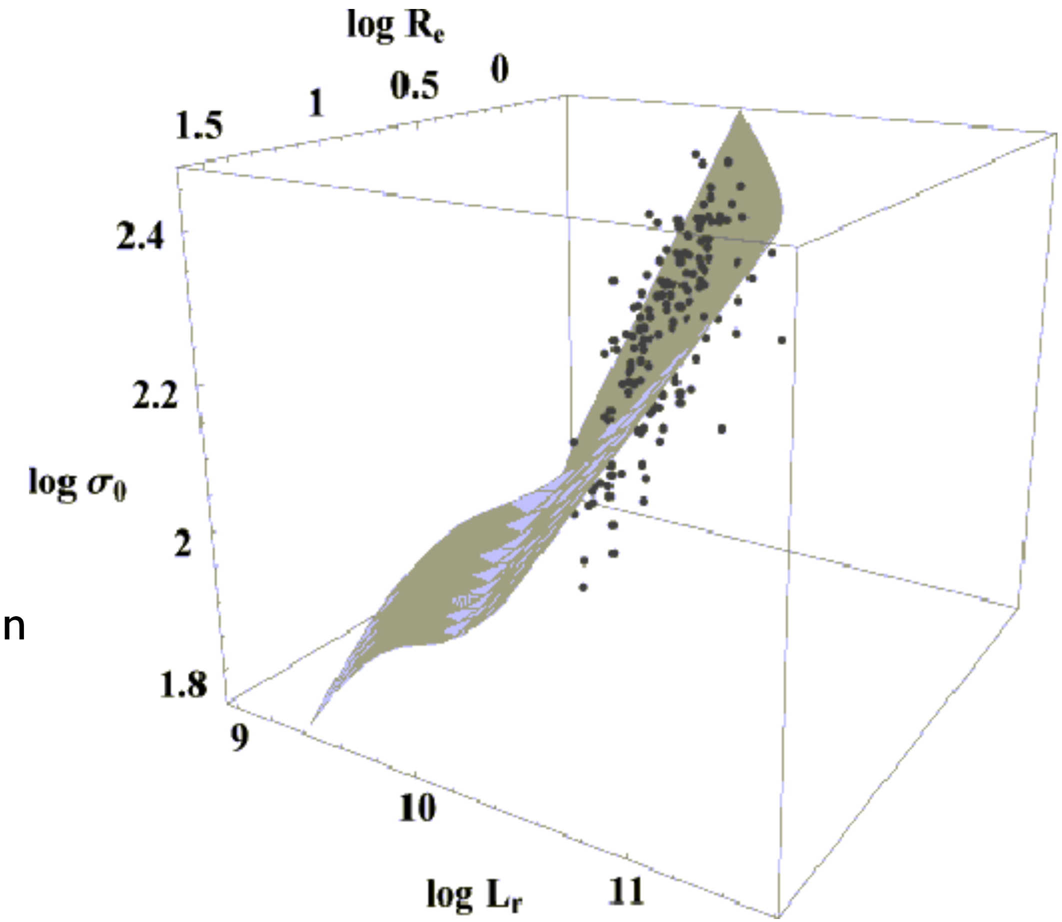
  - line-of-sight velocity dispersion

$\rightarrow \sigma_{los}$

  - fundamental plane:

$$\log_{10} R_{eff} = A \log_{10} \sigma_{los} + B \log_{10} \Sigma_0 + C$$

- elliptical galaxies – fundamental plane



- surface brightness profile

  $\rightarrow \Sigma_0$

  $\rightarrow R_{eff}$

- line-of-sight velocity dispersion

  $\rightarrow \sigma_{los}$

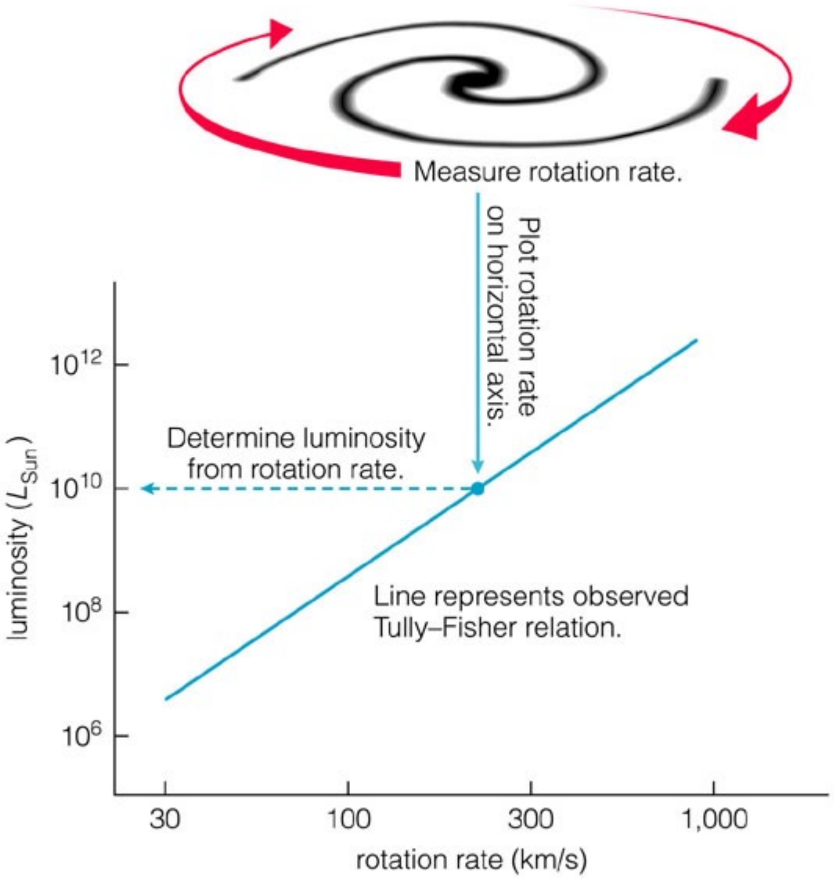
- fundamental plane:

$$\log_{10} R_{eff} = A \log_{10} \sigma_{los} + B \log_{10} \Sigma_0 + C$$

- spiral galaxies – Tully-Fisher relation

  - empirically determined

$$L \propto v_{rot}^{\beta} \quad \text{with } \beta \approx 4$$



- spiral galaxies – Tully-Fisher relation

  - empirically determined

$$L \propto v_{rot}^{\beta} \quad \text{with } \beta \approx 4$$

  - explanation:

$\rightarrow$ same logic as with Faber-Jackson relation...

- supernovae type Ia (SN Ia) *standard candle*

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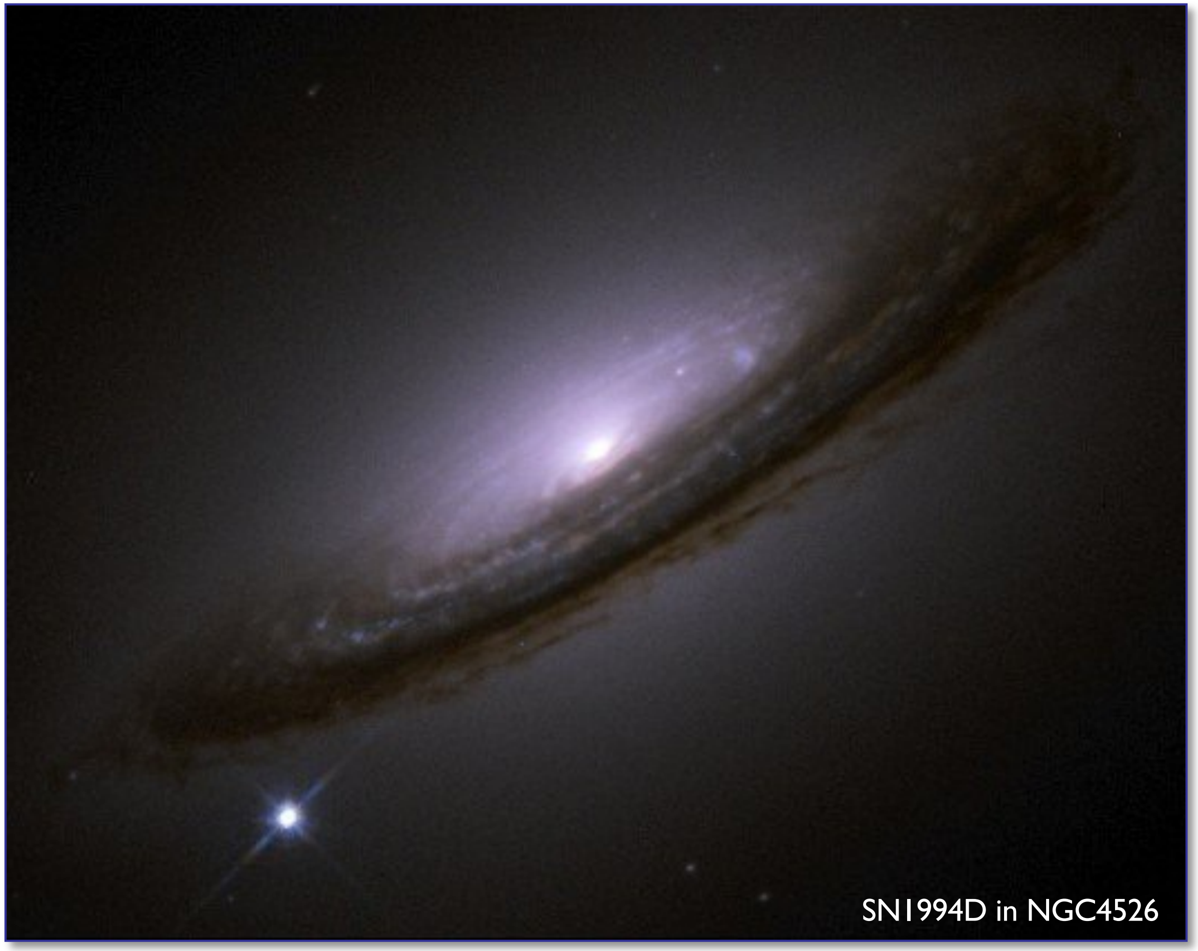
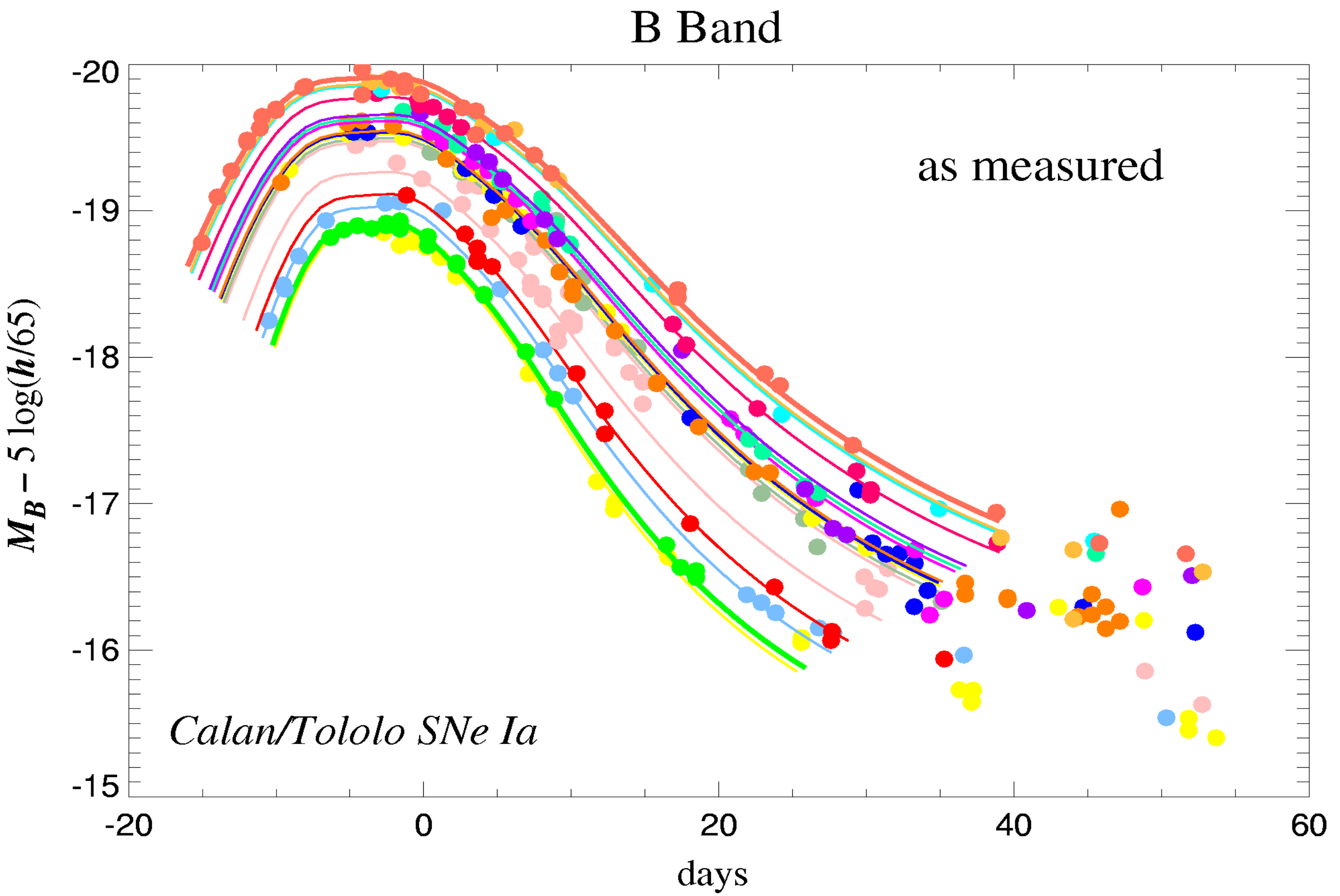


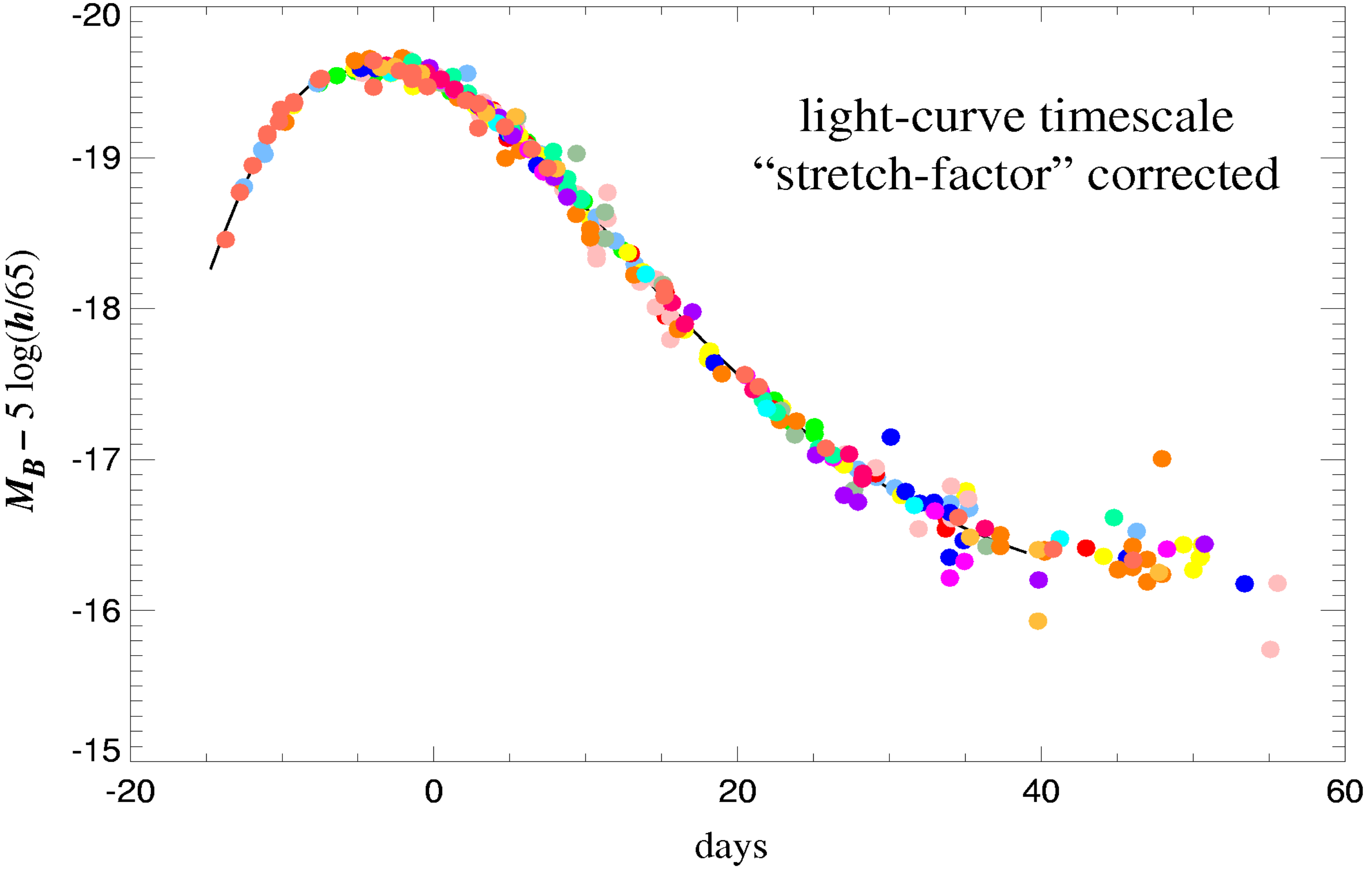
- supernovae type Ia (SN Ia) *standard candle*

  - characteristic light curve



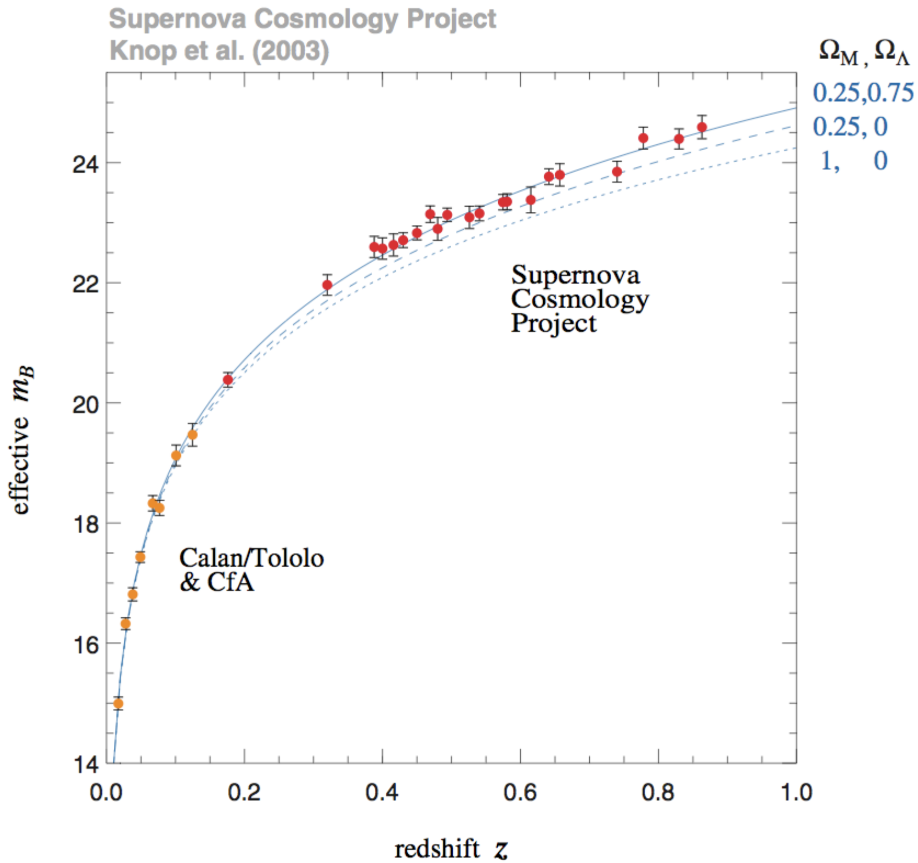
- supernovae type Ia (SN Ia) *standard candle*

  - characteristic light curve (corrected for redshift…)

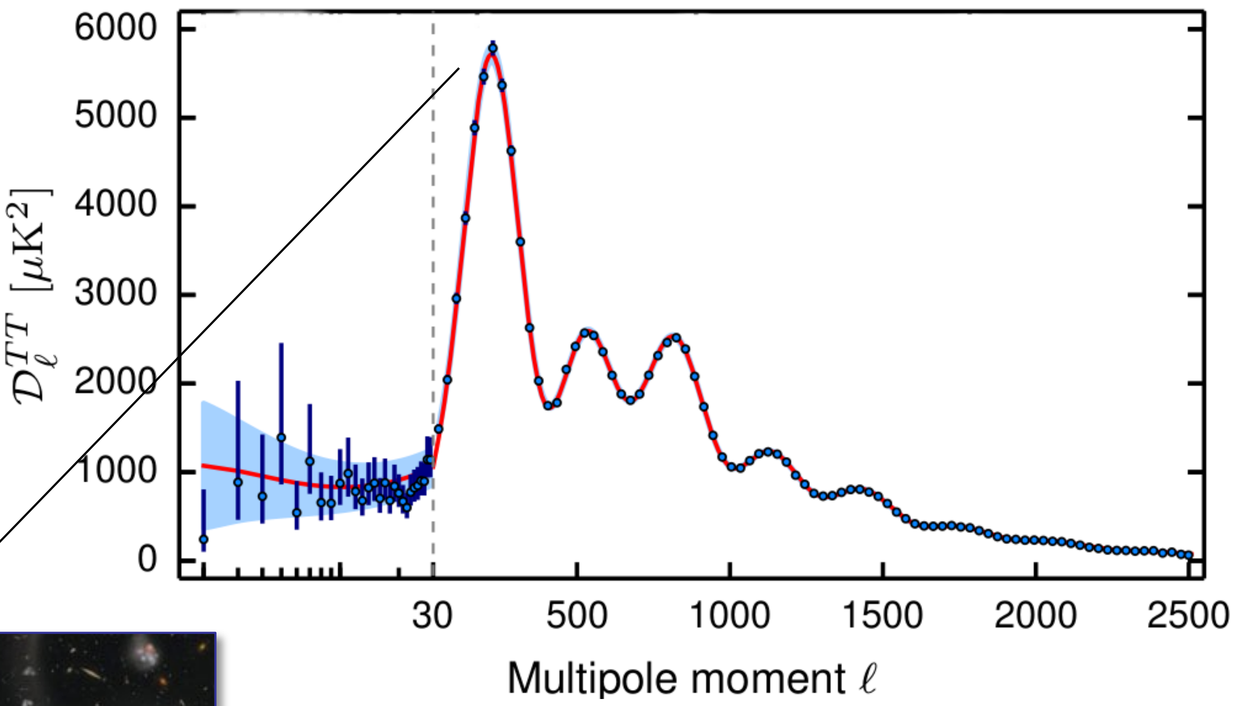


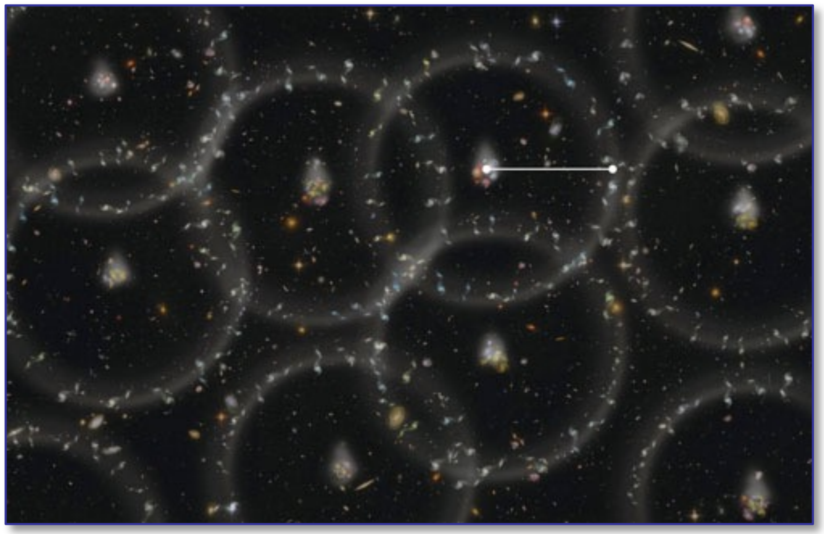
- supernovae type Ia (SN Ia) *standard candle*

  - characteristic light curve
  - observable out to great distances



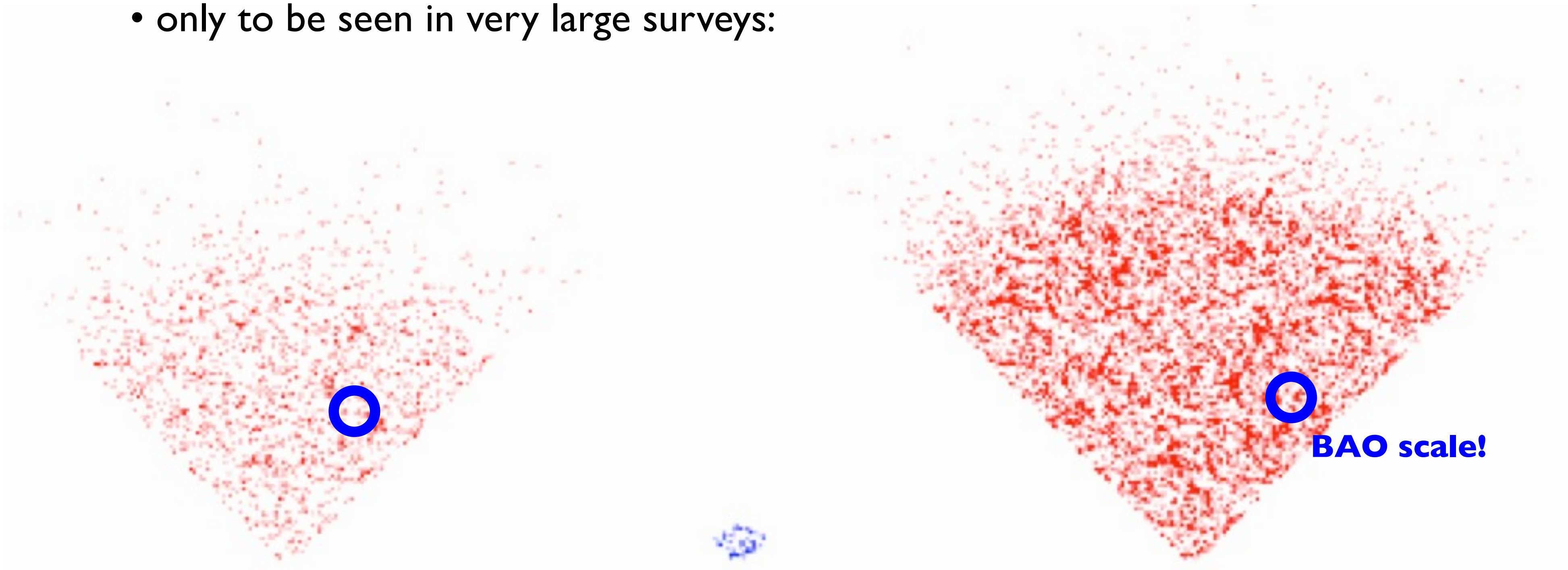
- baryonic acoustic oscillations *standard ruler*





- baryonic acoustic oscillations *standard ruler*

  - regular, periodic fluctuations in baryonic matter
  - originating from acoustic oscillations in pre-recombination plasma
  - only to be seen in very large surveys:



SDSS CfA2 BOSS (Baryon Oscillations Spectroscopic Survey)

- the distance ladder

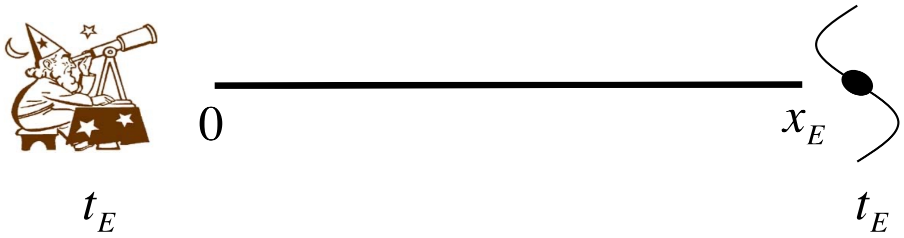
- cosmic distance ladder
- **cosmological distances**
- cosmological horizons & volumes
- supernova cosmology

- cosmic distance ladder
- **cosmological distances:**
  - **proper/comoving distance**
  - luminosity distance
  - angular diameter distance
  - travel-time distance
  - summary
- cosmological horizons & volumes
- supernova cosmology

- cosmological distances:

we are after a relation $d = f(R) = f(z)$

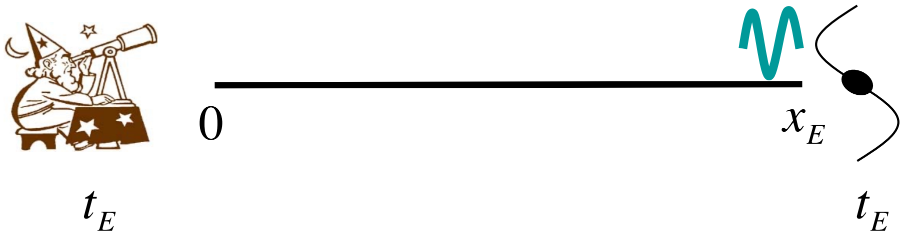
- cosmological distances:



x_E is the comoving coordinate, it is not *per se* the distance to the object!

- cosmological distances:

x_E: comoving coordinate



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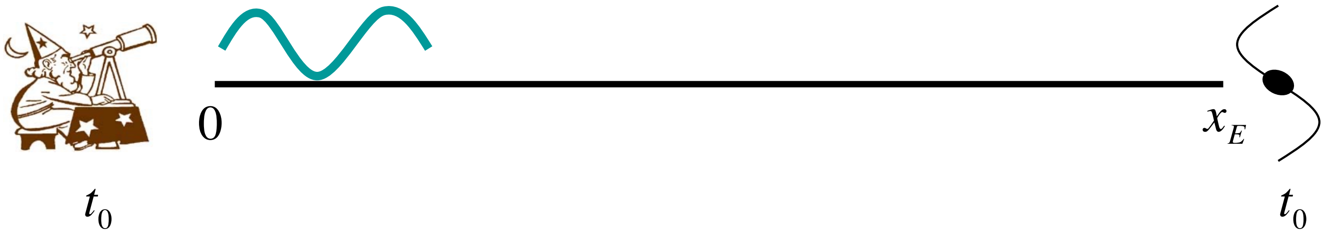

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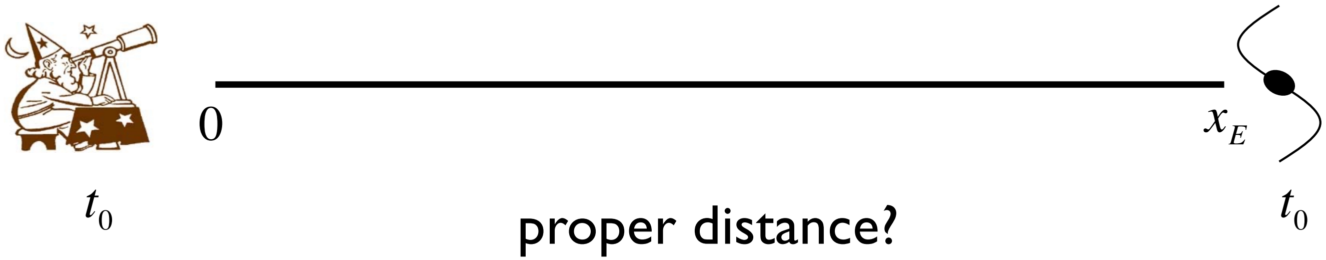
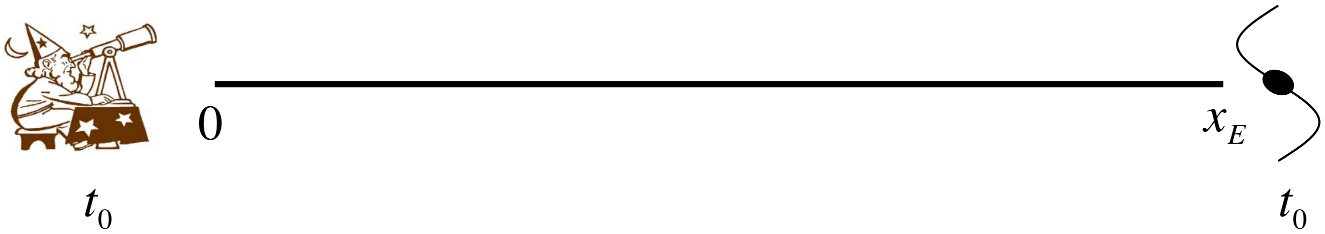


- proper distance:



• FRW metric:

$$ds^2 = (cdt)^2 - R^2(t)\left[\frac{dx^2}{1-kx^2} + x^2\left(d\vartheta^2 + \sin^2(\vartheta)d\varphi^2\right)\right]$$

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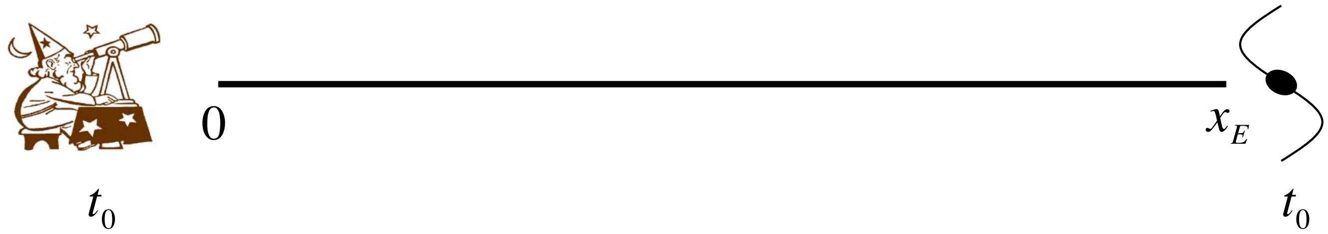

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proper distance separates two events happening at constant cosmic time.

(impossible to measure as it is defined only at one particular moment in time)

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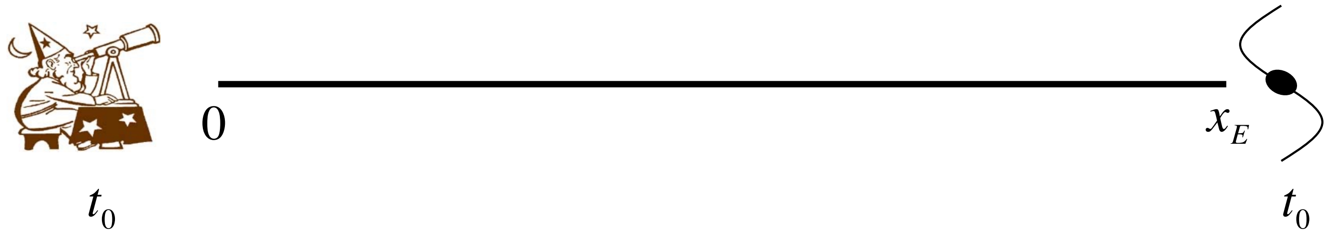

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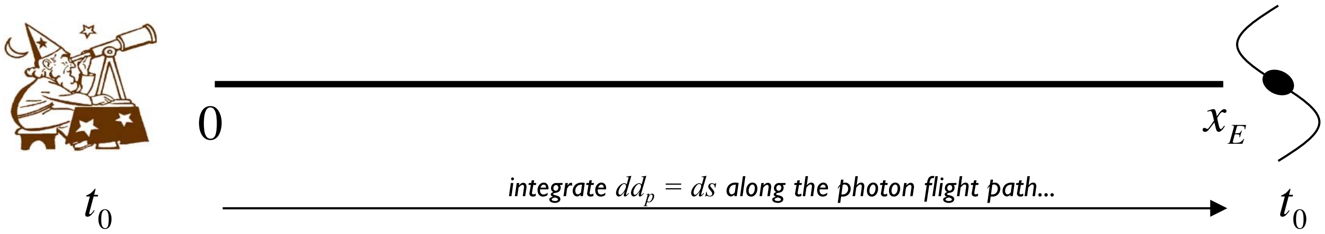

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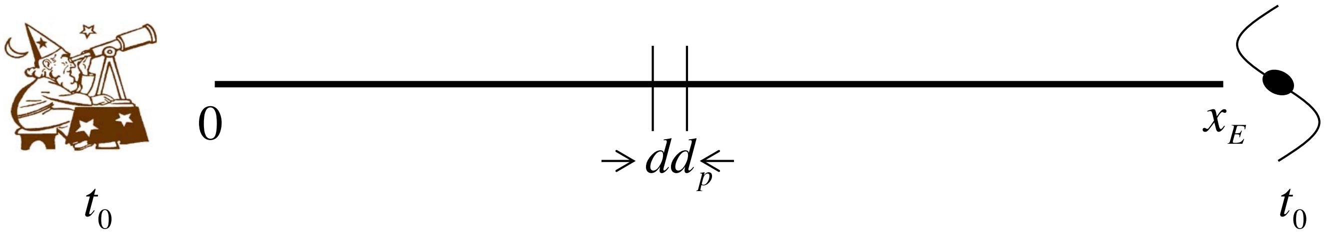

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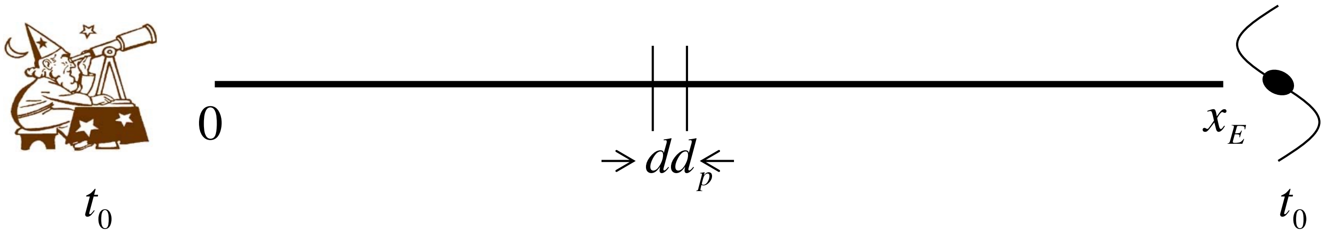

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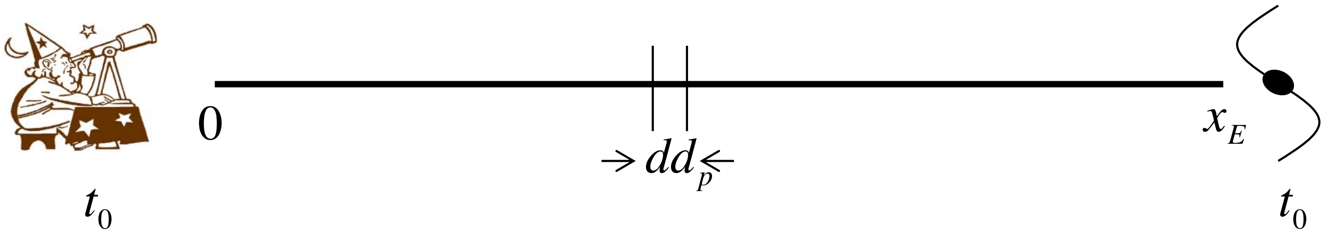

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$$\Rightarrow \qquad d_p = R(t)\int_0^{x_E}\frac{dx}{\sqrt{1-kx^2}} = R(t)f(x_E)$$

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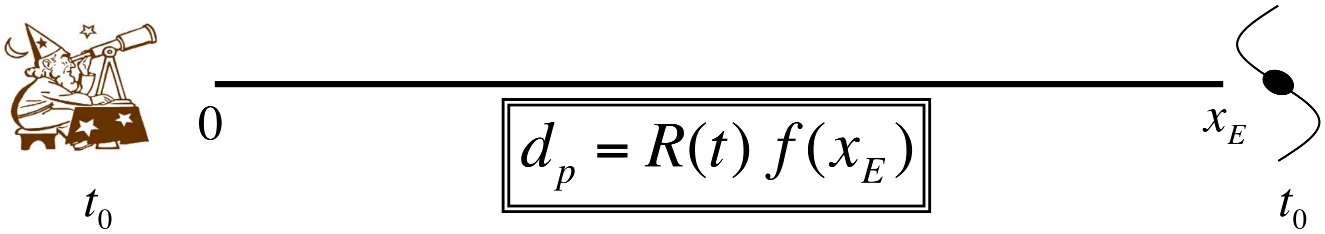

$$d_p = R(t)\, f(x_E)$$

$$\text{with } f(x_E) = \begin{cases} x_E & k=0 \\ \frac{1}{\sqrt{|k|}} \arcsin(\sqrt{|k|}\, x_E) & k=1 \\ \frac{1}{\sqrt{|k|}} \operatorname{arcsinh}(\sqrt{|k|}\, x_E) & k=-1 \end{cases}$$

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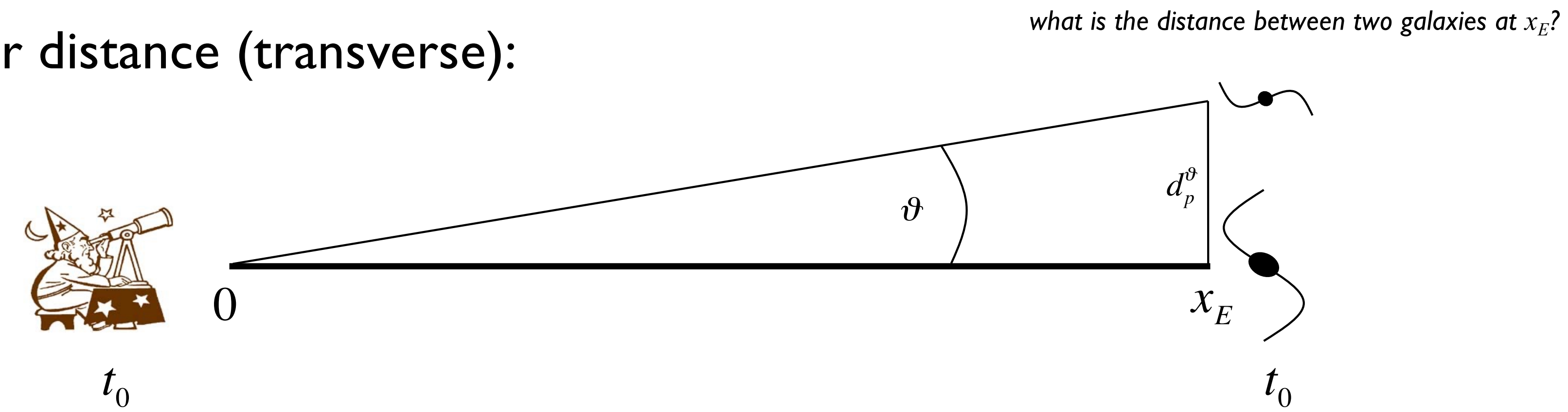
- proper distance (transverse):



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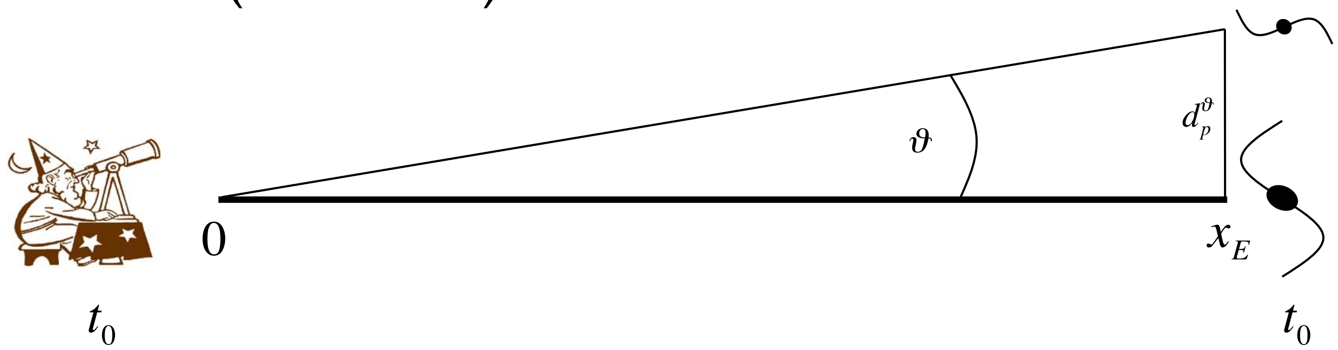


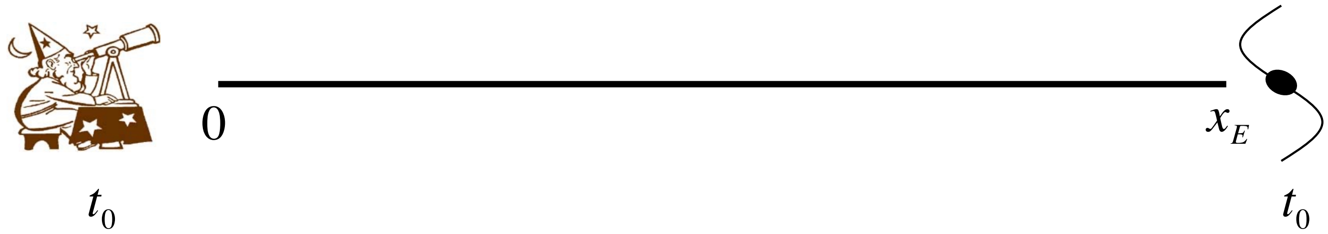
• FRW metric ($dt = 0$): $$ds^2 = R^2(t)\left[\frac{dx^2}{1-kx^2} + x^2\left(d\vartheta^2 + \sin^2(\vartheta)d\varphi^2\right)\right]$$

$$\overset{dx=0;\, d\varphi=0}{=>} \quad dd_p^\vartheta = R(t)x_E d\vartheta$$

$$=> \quad \boxed{d_p^\vartheta = R(t)x_E\int_0^{\vartheta_E} d\vartheta} \quad \text{with*} \quad x_E = \begin{cases} d_p/R & ;k=0 \\ \frac{1}{\sqrt{|k|}}\sin\left(\sqrt{|k|}\,d_p/R\right) & ;k=1 \\ \frac{1}{\sqrt{|k|}}\sinh\left(\sqrt{|k|}\,d_p/R\right) & ;k=-1 \end{cases}$$

*simple inversion of $f(x_E)$ from previous slide…

- comoving distance:

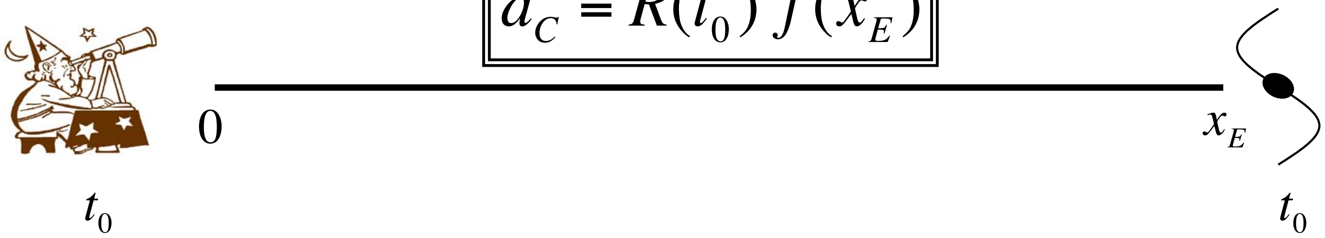


proper distance at some pre-defined reference time

(common practice is to use today's time as reference)

- comoving distance:

$$d_C = R(t_0)\, f(x_E)$$



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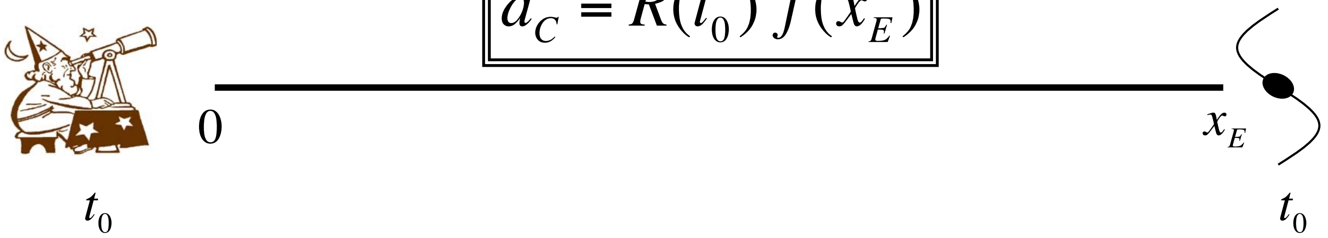

if setting $R(t_0)=1$, then $f(x_E)$ is in fact the comoving distance...

proper distance at some pre-defined reference time

(common practice is to use today's time as reference)

- comoving/proper distance:

t_0

$$\boxed{d_C = R(t_0)\, f(x_E)}$$

0 —————————— x_E

$$\boxed{d_p = R(t)\, f(x_E)}$$

t_0

$$\begin{aligned} d_p &= R(t) f(x_E) \\ d_c &= R_0 f(x_E) \end{aligned} \quad \Rightarrow \quad f(x_E) = \frac{d_p}{R(t)} = \frac{d_c}{R_0} \quad \Rightarrow \quad \boxed{d_p = \frac{R(t)}{R_0} d_c}$$

proper distance at some pre-defined reference time

(common practice is to use today's time as reference)

- comoving/proper distance:

$$d_c = R_0 f(x_E)$$

0 — x_E: comoving coordinate

$$d_p = \frac{R(t)}{R_0} d_C$$

$$f(x_E) = \begin{cases} x_E & \text{k=0} \\ \frac{1}{\sqrt{|k|}} \arcsin(\sqrt{|k|}\, x_E) & \text{k=1} \\ \frac{1}{\sqrt{|k|}} \operatorname{arcsinh}(\sqrt{|k|}\, x_E) & \text{k=-1} \end{cases}$$

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…but how to calculate $f(x_E)$ for object at given redshift z_E?

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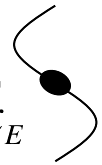


$$d_p = \frac{R(t)}{R_0} d_C$$

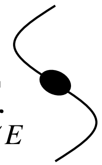
• null geodesic for photons*: $ds^2 = 0 = (cdt)^2 - R^2(t)\left[\frac{dx^2}{1-kx^2}\right]$

*remember: we are counting photons...

- comoving/proper distance:

$$d_c = R_0 f(x_E)$$

0 —————————————— x_E



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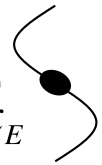

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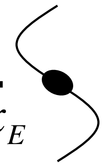

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$$f(x_E) = \int_0^{x_E} \frac{dx}{\sqrt{1-kx^2}} = \int_{t_E}^{t_0} \frac{cdt}{R(t)}$$

side note for later...

$$= \text{const.} \quad \Rightarrow \quad 0 = \frac{df(x_E)}{dt_E} = \frac{cdt}{R(t)}\bigg|_{t_E}^{t_0} = \frac{cdt_0}{R_0} - \frac{cdt_E}{R(t_E)}$$

$$\Rightarrow \frac{dt_0}{R_0} = \frac{dt_E}{R(t_E)}$$

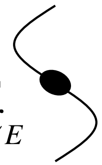
time intervals are changed in proportion to the expansion

(this agrees with an energy change, to be used below...)

- comoving/proper distance:

$$d_c = R_0 f(x_E)$$

0 ———————— x_E



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$$f(x_E) = \int_0^{x_E} \frac{dx}{\sqrt{1-kx^2}} = \int_{t_E}^{t_0} \frac{cdt}{R(t)}$$ replace with Friedmann equation…

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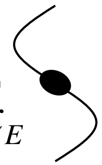

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$H^2 = H_0^2 E^2(z)$

$$= c\int_{R_E}^{R_0} \frac{dR}{\dot{R}R} = c\int_{R_E}^{R_0} \frac{dR}{R^2 H_0 E(z)} \qquad ; E^2(z) = \sum_i \Omega_{i,0}\left(1+z\right)^{3(1+w_i)}$$

$\frac{R}{R_0} = \frac{1}{1+z}$

$$= \frac{c}{H_0}\int_{z_E}^{0} \frac{(1+z)^2}{R_0 E(z)}\left(-\frac{1}{(1+z)^2}\right)dz = \frac{c}{H_0}\int_0^{z_E} \frac{R_0}{R^2 E(z)}\frac{R^2}{R_0^2}dz = \frac{c}{H_0 R_0}\int_0^{z_E} \frac{1}{E(z)}dz$$

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$$d_c = R_0 f(x_E)$$

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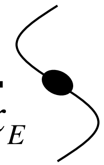
we eventually replaced x_E with z_E

- comoving/proper distance:

$$d_c = R_0 f(x_E)$$

0 ——— x_E



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with $E^2(z) = \sum_i \Omega_{i,0} (1+z)^{3(1+w_i)}$

$$w_i = \begin{cases} 0 & \text{dust} \\ 1/3 & \text{radiation} \\ -1/3 & \text{curvature} \\ -1 & \Lambda \end{cases}$$

- comoving/proper distance: *we were after the relation $d = f(z)$ ...and found it!*

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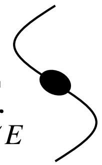

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...and it sensitively depends on the cosmological parameters...

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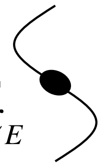

$$d_p = \frac{R(t)}{R_0} d_C$$

how to connect it to observabales (other than z)?

• null geodesic

$$f(x_E) = \frac{c}{H_0 R_0} \int_0^{z_E} \frac{1}{E(z)} dz$$

with $E^2(z) = \sum_i \Omega_{i,0} (1+z)^{3(1+w_i)}$ $\qquad w_i = \begin{cases} 0 & \text{dust} \\ 1/3 & \text{radiation} \\ -1/3 & \text{curvature} \\ -1 & \Lambda \end{cases}$

...and it sensitively depends on the cosmological parameters.

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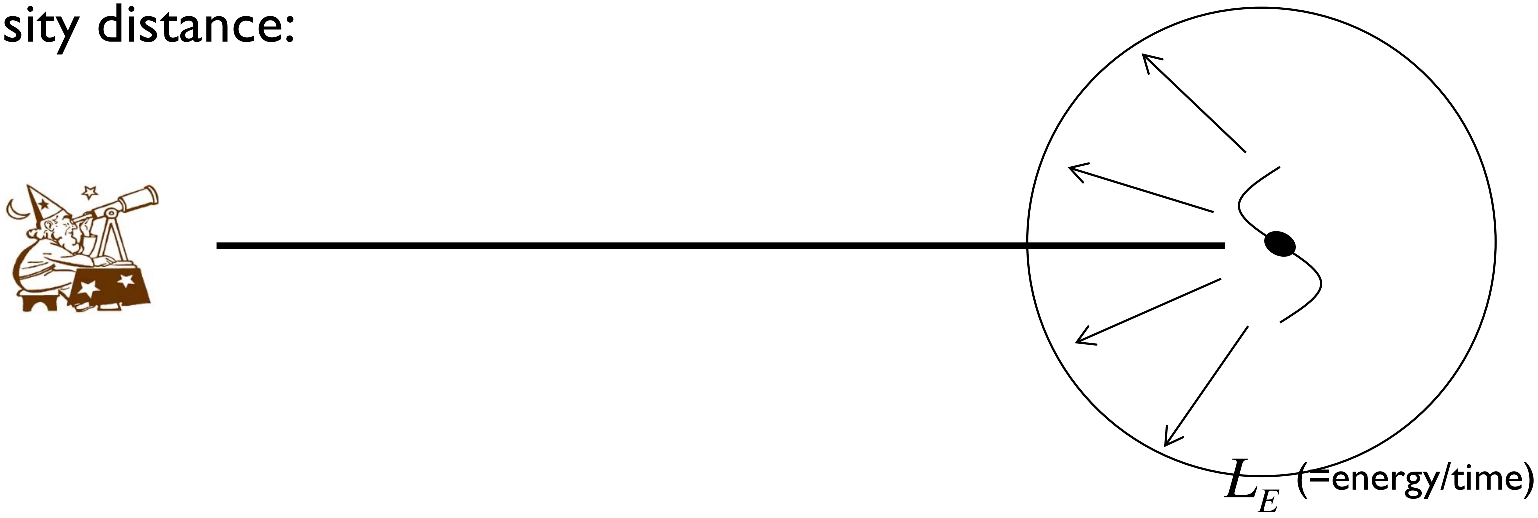

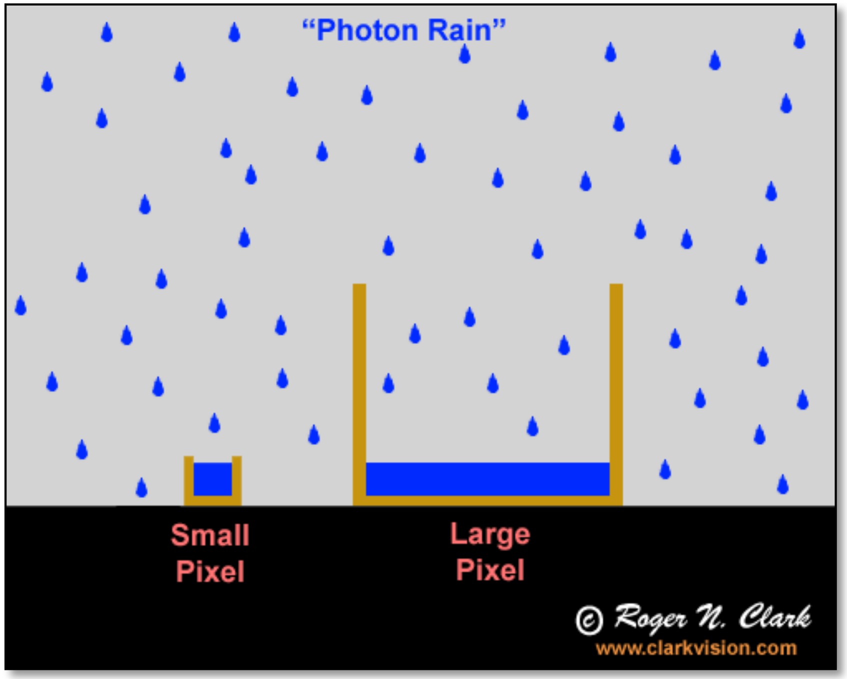

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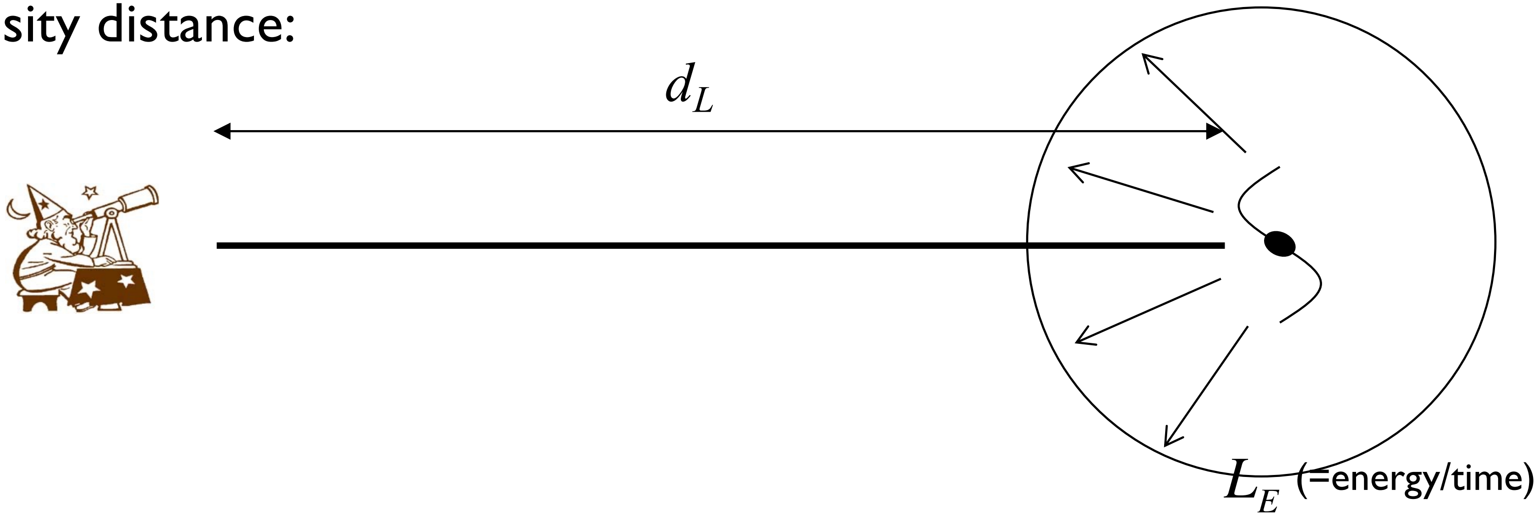
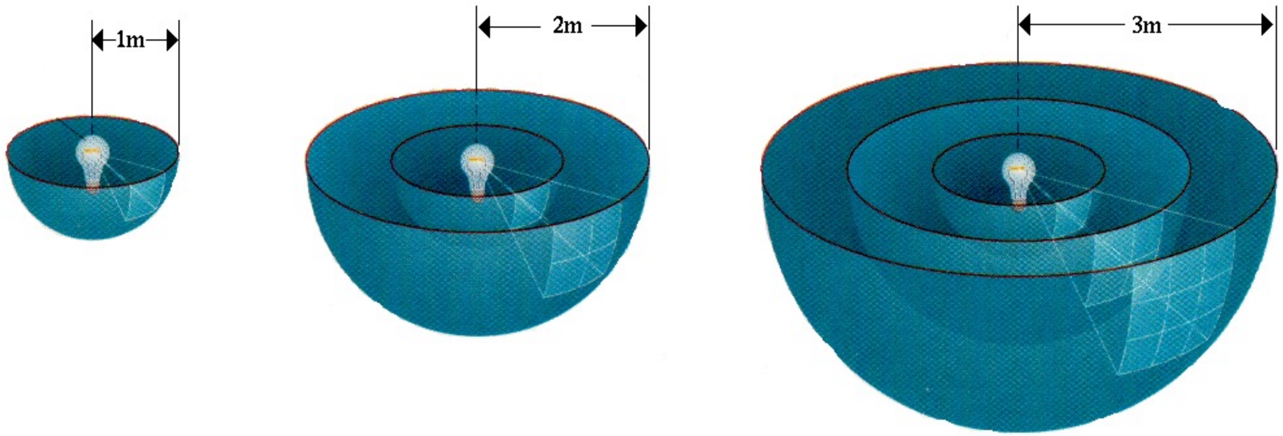
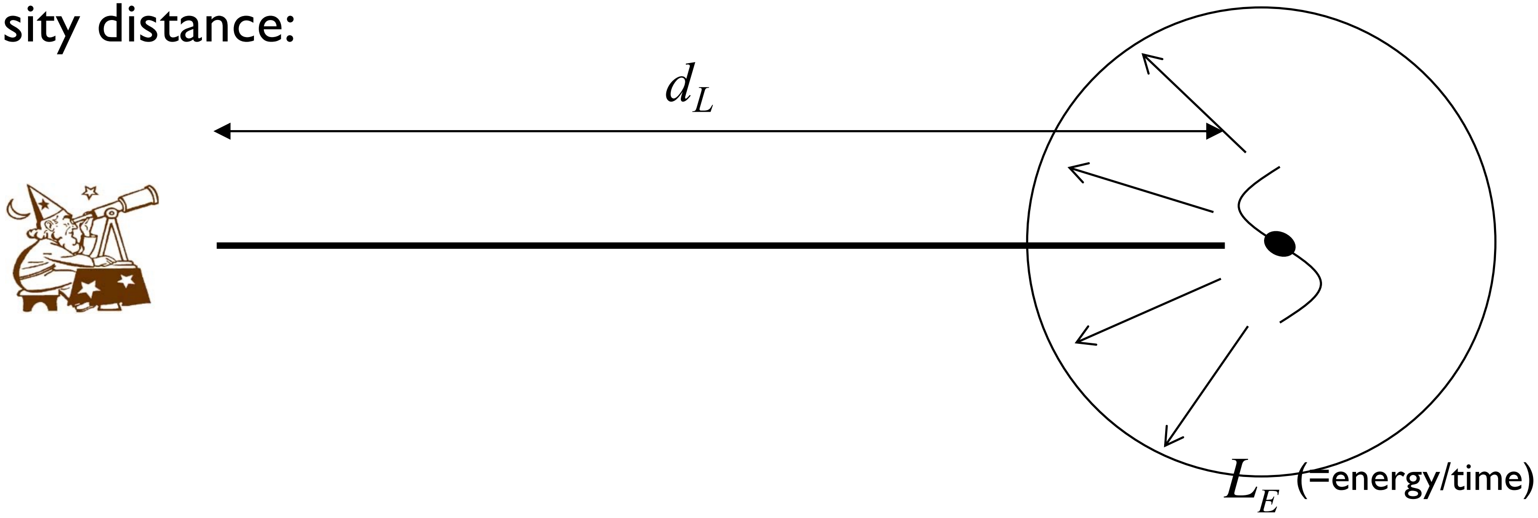


$$F_{obs} \stackrel{!}{=} \frac{L_E}{4\pi d_L^2}$$

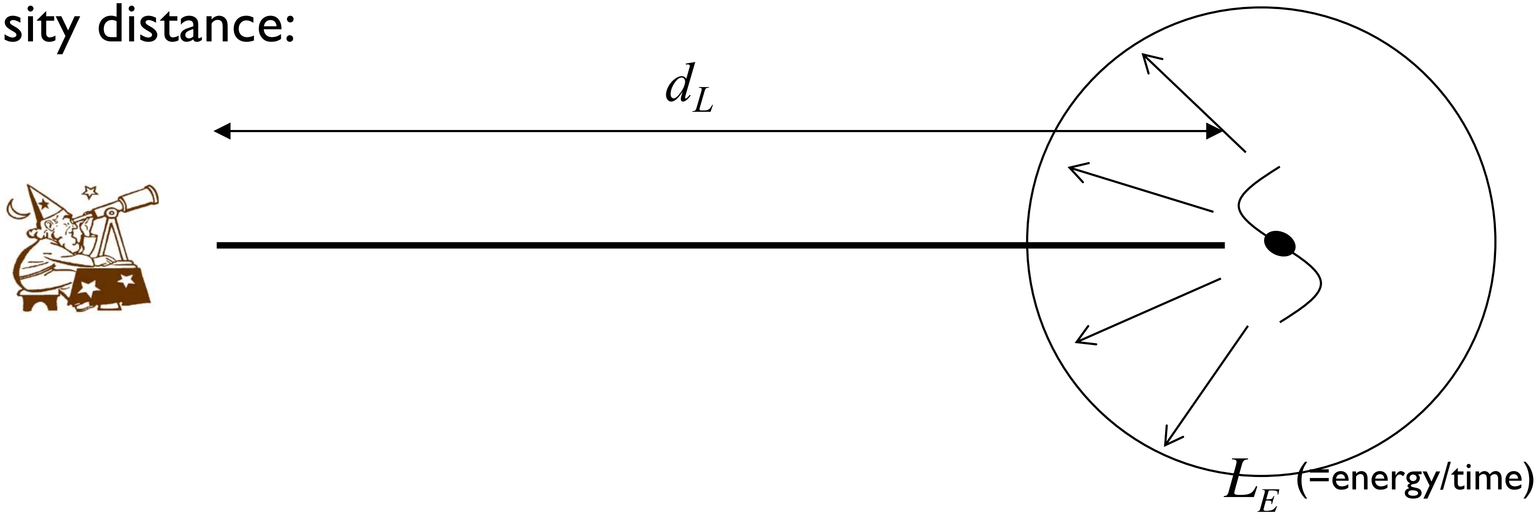


- luminosity distance:



$$\sqrt{\frac{4\pi F_{obs}}{L_E}} = d_L$$

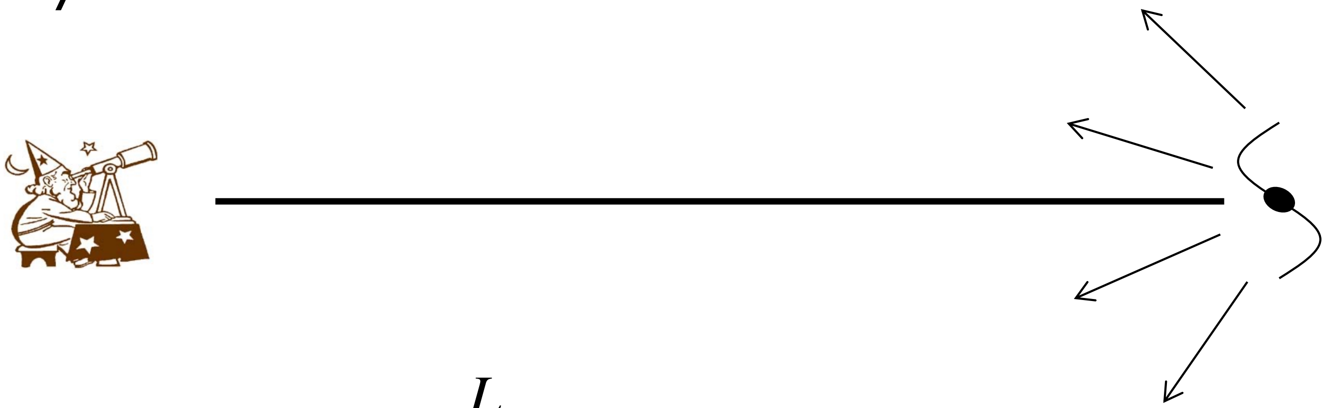
- luminosity distance:



$$\sqrt{\frac{4\pi F_{obs}}{L_E}} = d_L = h(x_E)?$$

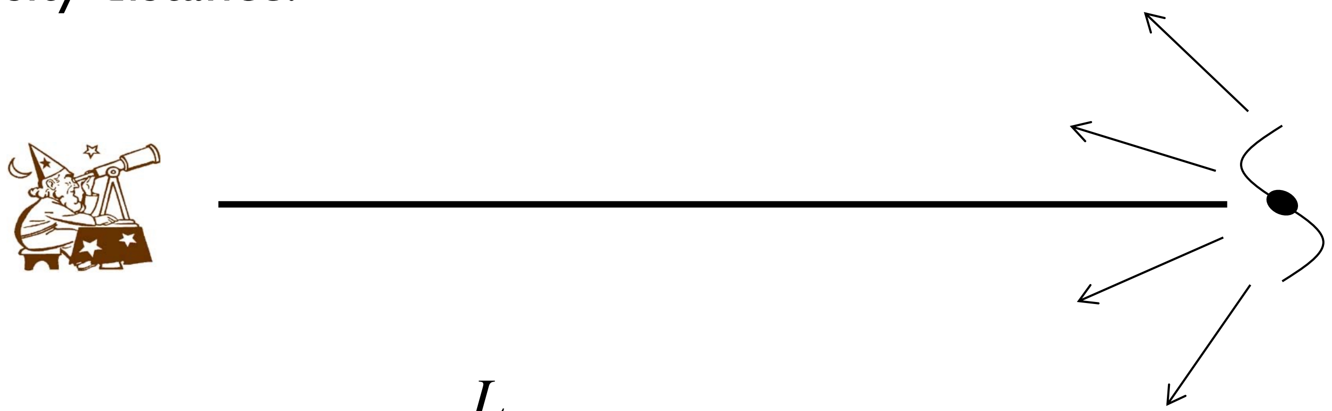
*we are not using $f(x_E)$ here as it might be confused with the comoving distance...

- luminosity distance:



1. photons: $L_0 = \frac{L_E}{(1+z)^2}$ L_E (=energy/time)

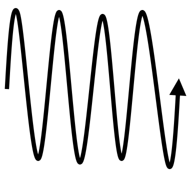
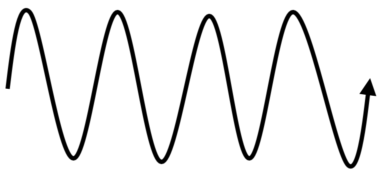
- luminosity distance:



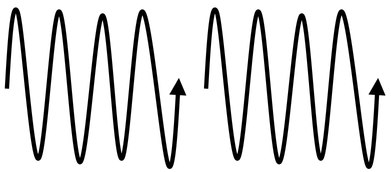
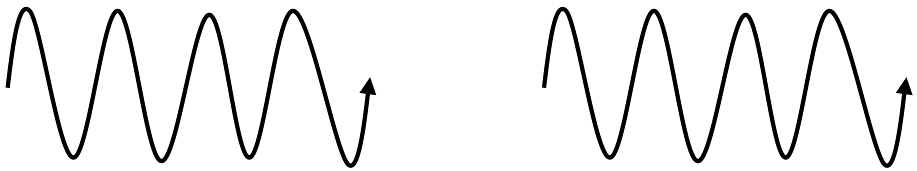
1. photons: $L_0 = \frac{L_E}{(1+z)^2}$ L_E (=energy/time)

---

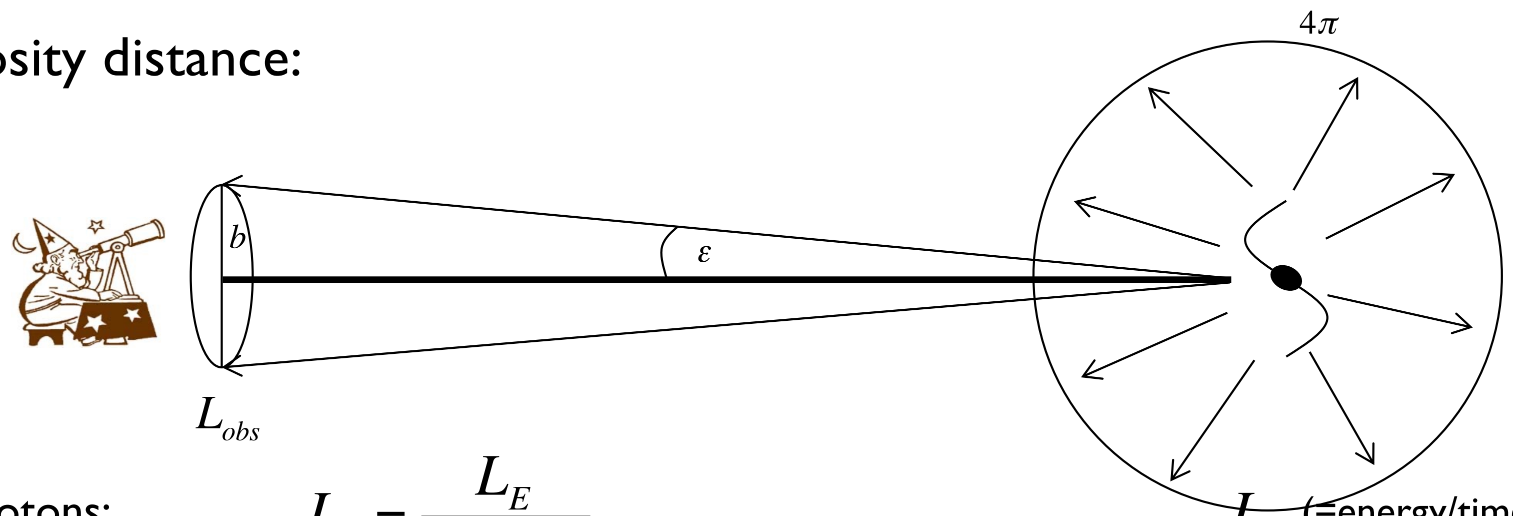
1. change of wavelength

$(1+z)^{-1}:$  $\frac{\lambda_0}{R_0} = \frac{\lambda_E}{R_E}$ --------> 

2. change of distance between photons

$(1+z)^{-1}:$  $\frac{dt_0}{R_0} = \frac{dt_E}{R_E}$ --------> 

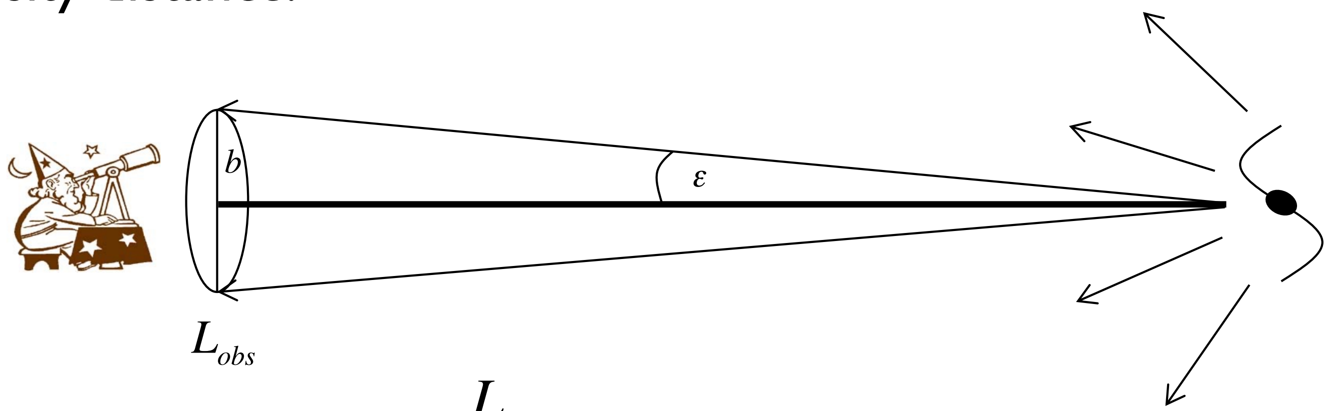
- luminosity distance:



1. photons: $$L_0 = \frac{L_E}{(1+z)^2}$$ L_E (=energy/time)

2. geometry: $$L_{obs} = L_0 \times f$$ with $$f = \frac{\pi\varepsilon^2}{4\pi}$$ (ratio of solid angles)

- luminosity distance:



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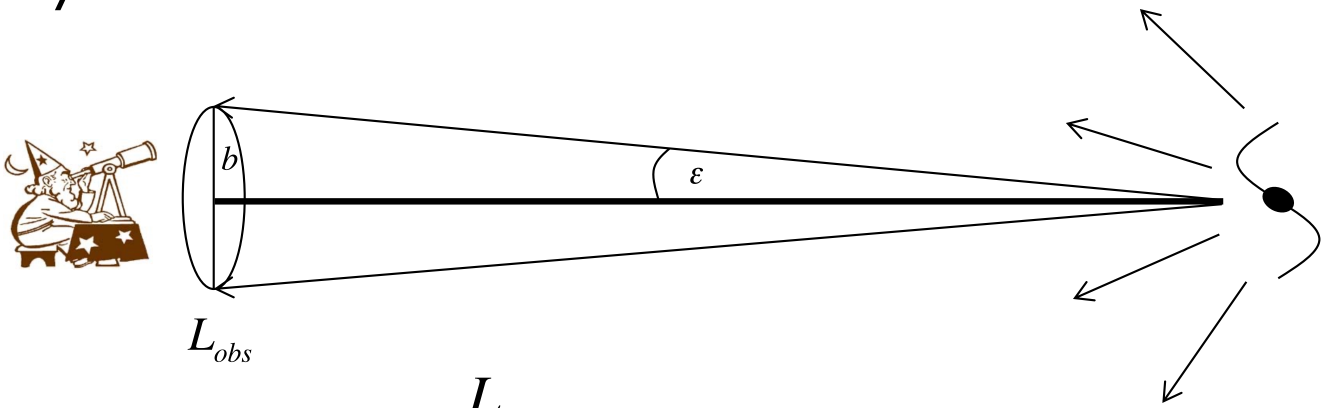
2. geometry: $$L_{obs} = L_0 \times f$$ with $$f = \frac{\pi\varepsilon^2}{4\pi} =$$

$$b = R(t_0)x_E \int_0^{\varepsilon} d\vartheta = R(t_0)x_E\varepsilon$$

($R(t_0)$ because of "telescope size today",
cf. "proper transverse distance" in formula for b)

- luminosity distance:



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L_E (=energy/time)

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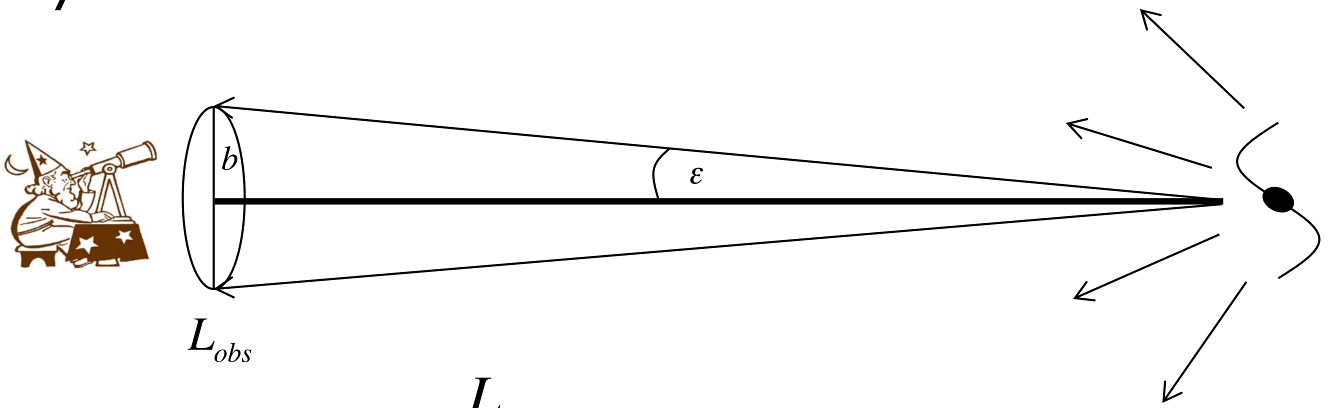

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3. measurement: (energy/time/area) $$F_{obs} = \frac{L_{obs}}{\pi b^2}$$

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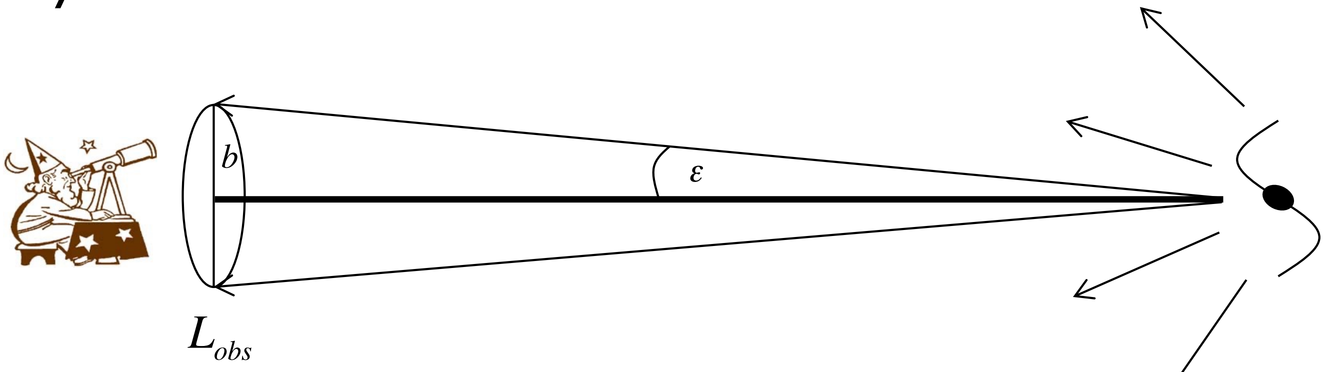

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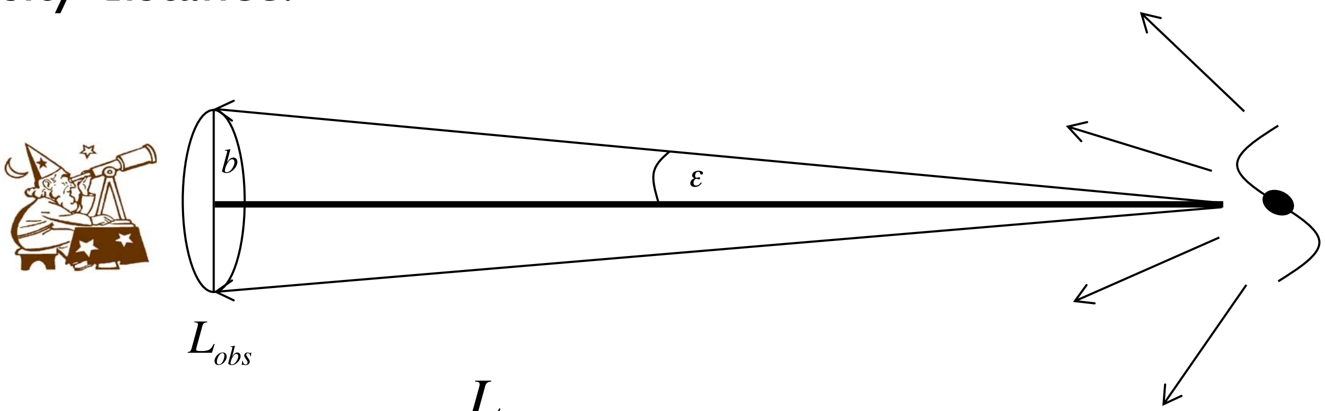

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$$\sqrt{\frac{4\pi F_{obs}}{L_E}} \overset{!}{=} d_L \qquad =>$$

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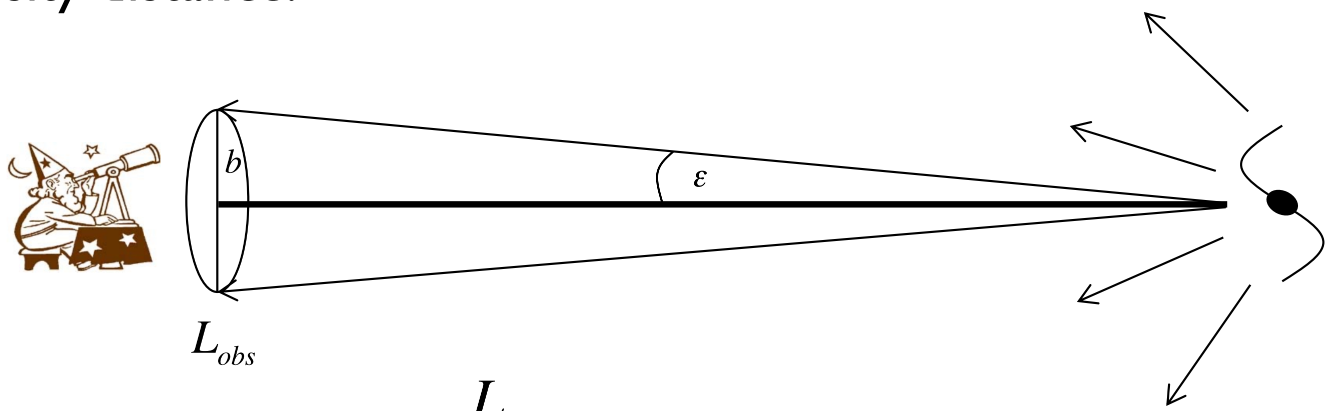

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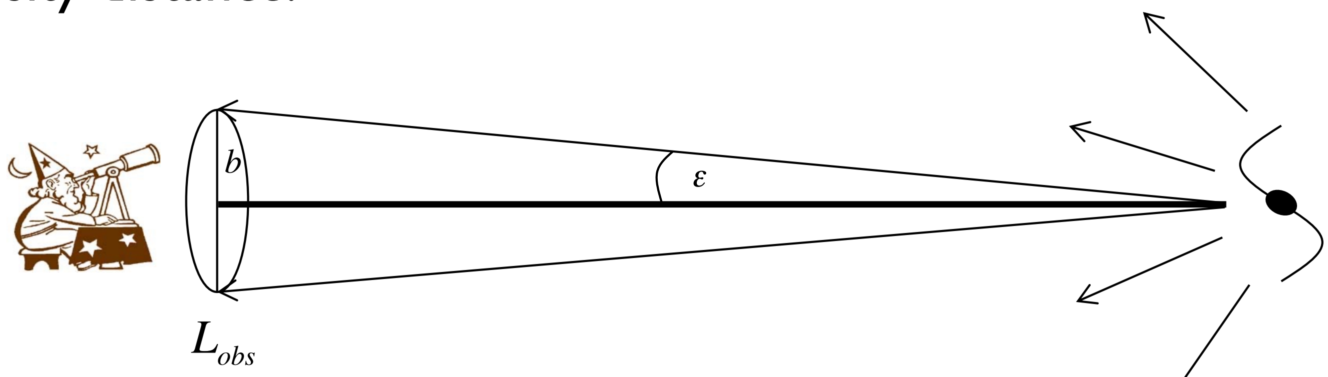
$$\sqrt{\frac{4\pi F_{obs}}{L_E}} \overset{!}{=} d_L \quad \Rightarrow \quad \boxed{d_L = \sqrt{\frac{L_E/4\pi}{F_{obs}}} = \frac{R^2(t_0)}{R(t_E)}x_E}$$

- luminosity distance:



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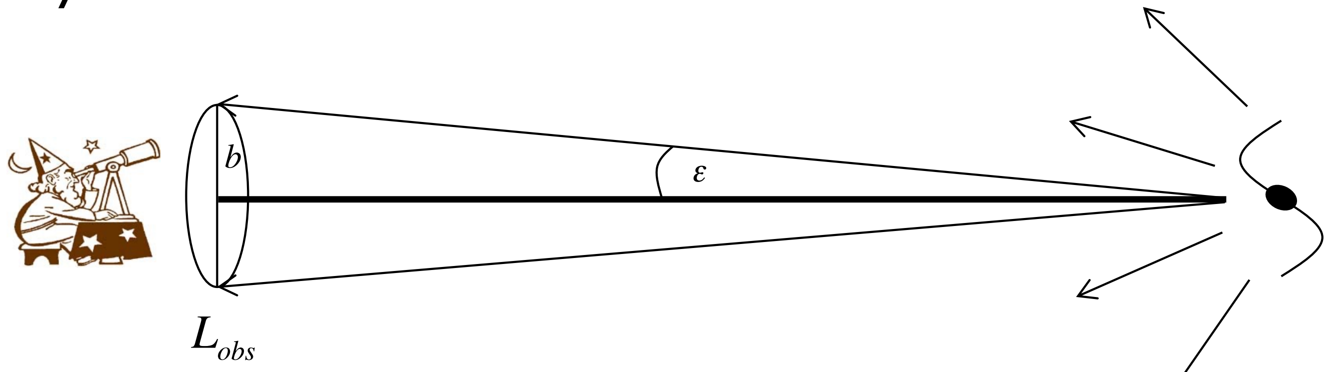

1. photons: $L_0 = \dfrac{L_E}{(1+z)^2}$ $\qquad L_E$ (=energy/time)

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(energy/time/area)

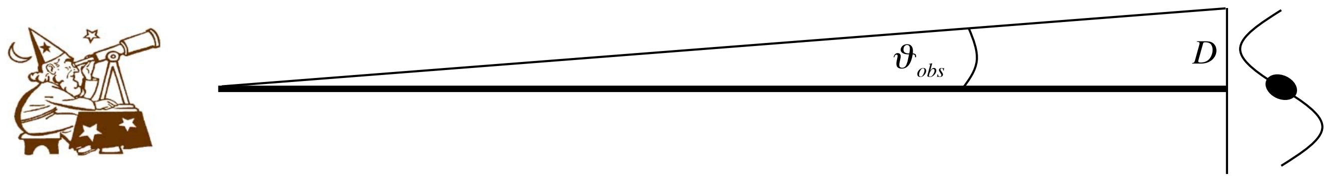
we require standard candles!

$$d_L = \sqrt{\frac{L_E/4\pi}{F_{obs}}} = \frac{R^2(t_0)}{R(t_E)} x_E$$

$$d_L = h(x_E)!$$

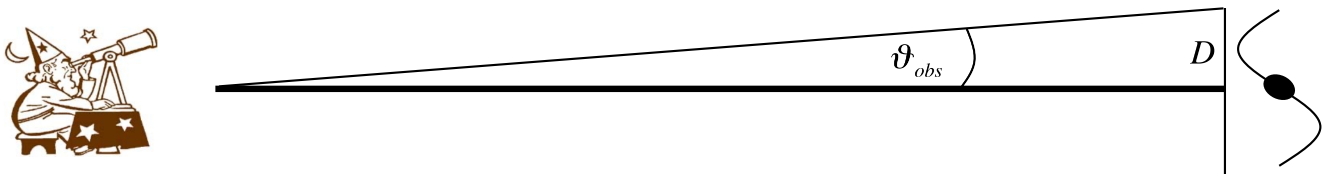
- cosmic distance ladder
- **cosmological distances:**
  - proper/comoving distance
  - luminosity distance
  - **angular diameter distance**
  - travel-time distance
  - summary
- cosmological horizons & volumes
- supernova cosmology

- angular diameter distance:



$$\vartheta_{obs} \stackrel{!}{=} \frac{D}{d_A}$$

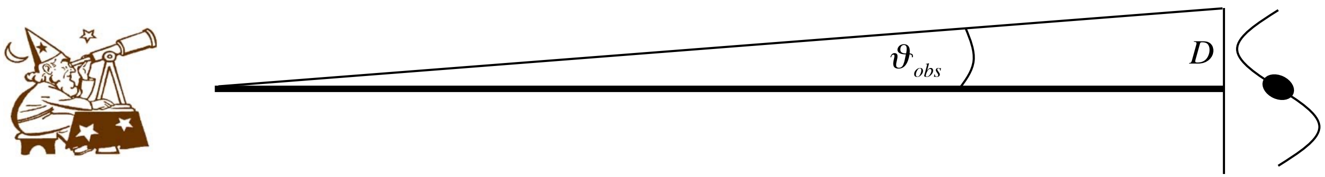
- angular diameter distance:



$$\vartheta_{obs} \stackrel{!}{=} \frac{D}{d_A}$$

$$d_A = h(x_E)?$$

- angular diameter distance:



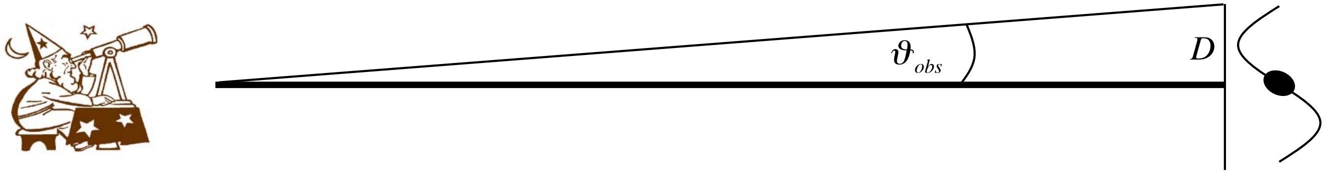
$$D = R(t_E)x_E \int_0^{\vartheta_E} d\vartheta = R(t_E)x_E\vartheta_E$$

($R(t_E)$ because of "galaxy size at time of emission")

$$\vartheta_{obs} \equiv \vartheta_E$$

$$\vartheta_{obs} \overset{!}{=} \frac{D}{d_A} \qquad => \qquad \boxed{d_A = \frac{D}{\vartheta_{obs}} = R(t_E)x_E}$$

- angular diameter distance:



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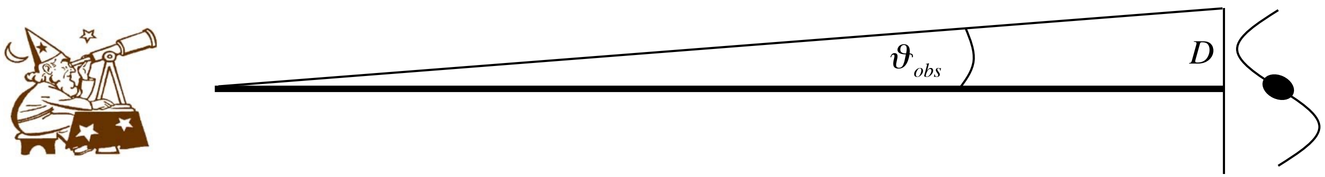

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we require standard rulers!

$$\boxed{d_A = \frac{D}{\vartheta_{obs}} = R(t_E)x_E}$$

$$d_A = h(x_E)!$$

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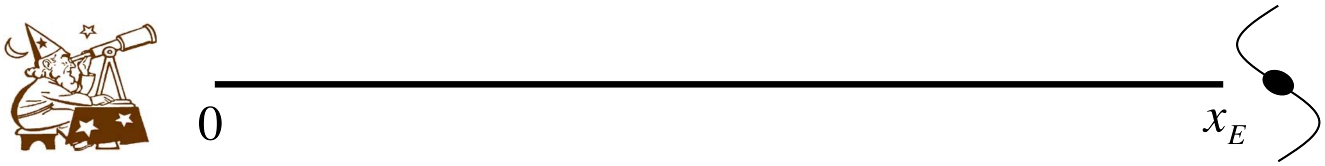
- travel-time distance:



$$d_T = \int_{t_E}^{t_0} c\,dt = \ldots = \frac{c}{H_0}\int_0^{z_E} \frac{1}{(1+z)E(z)}\,dz$$

- cosmic distance ladder
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- inter-relation:
  - comoving distance: $d_c = \frac{c}{H_0}\int_0^{z_E} \frac{1}{E(z)}dz$
  - proper distance: $d_p = \frac{R(t)}{R_0} d_C$
  - luminosity distance: $d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = \frac{R_0}{R(t)} R_0 x_E$
  - angular diameter distance: $d_A = \frac{D}{\vartheta_{obs}} = \frac{R(t)}{R_0} R_0 x_E$

$$E^2(z) = \sum_i \Omega_{i,0} \left(1+z\right)^{3(1+w_i)}$$

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$$d_A = \left(\frac{R(t)}{R_0}\right)^2 d_L$$

$$E^2(z) = \sum_i \Omega_{i,0}\left(1+z\right)^{3(1+w_i)}$$

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?

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x_E via inversion of $f(x_E) = \frac{c}{H_0 R_0}\int_0^{z_E}\frac{1}{E(z)}dz = \begin{cases} x_E & \text{k=0} \\ \frac{1}{\sqrt{|k|}}\arcsin(\sqrt{|k|}\,x_E) & \text{k=1} \\ \frac{1}{\sqrt{|k|}}\text{arcsinh}(\sqrt{|k|}\,x_E) & \text{k=-1} \end{cases}$

$$E^2(z) = \sum_i \Omega_{i,0}\left(1+z\right)^{3(1+w_i)}$$

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  - comoving distance: $d_c = \frac{c}{H_0}\int_0^{z_E} \frac{1}{E(z)} dz$
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  - luminosity distance: $d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = \frac{R_0}{R(t)} R_0 x_E$
  - angular diameter distance: $d_A = \frac{D}{\vartheta_{obs}} = \frac{R(t)}{R_0} R_0 x_E$

$$x_E = \begin{cases} \frac{1}{R_0} \quad d_c & ;k=0 \\ \frac{1}{R_0}\frac{c}{H_0\sqrt{|\Omega_{k,0}|}}\sin\left(\frac{\sqrt{|\Omega_{k,0}|}H_0}{c}d_c\right) & ;k=1 \\ \frac{1}{R_0}\frac{c}{H_0\sqrt{|\Omega_{k,0}|}}\sinh\left(\frac{\sqrt{|\Omega_{k,0}|}H_0}{c}d_c\right) & ;k=-1 \end{cases}$$

($\Omega_{k,0} = -\frac{c^2 k}{R_0^2 H_0^2}$, cf. FRW lecture)

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$$E^2(z) = \sum_i \Omega_{i,0}\left(1+z\right)^{3(1+w_i)}$$

- examples for x_E:

  - $k = 0,\ \Omega_r << \Omega_m,\ \Omega_\Lambda = 1 - \Omega_m$ (ΛCDM model)

$$x_E = \frac{c}{H_0 R_0} \int_0^{z_E} \frac{dz}{[\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}]^{1/2}}$$

  - $\Omega_\Lambda = 0,\ \Omega_r = 0,\ \Omega_m = 2q_0$

$$x_E = \frac{z_E q_0 + (q_0 - 1)(-1 + \sqrt{2 q_0 z_E + 1})}{H_0 R_0 q_0^2 (1 + z_E)}$$

  - $\Omega_\Lambda = 1,\ \Omega_m = 0,\ k = 0$

$$x_E = \frac{c z_E}{H_0 R_0}$$

- examples for x_E:

  • $k = 0,\ \Omega_r << \Omega_m,\ \Omega_\Lambda = 1 - \Omega_m$ (ΛCDM model)

$$x_E = \frac{c}{H_0 R_0} \int_0^{z_E} \frac{dz}{[\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}]^{1/2}}$$

$$d_C(z) = \frac{c}{H_0} \int_0^{z} \frac{dz'}{E(z')}$$

$$d_L(z) = d_C(1+z)$$

$$d_A(z) = \frac{d_C}{(1+z)}$$

simple relation of d_L and d_A to d_C

- inter-relation:
  - comoving distance: $$d_c = \frac{c}{H_0}\int_0^{z_E}\frac{1}{E(z)}dz$$
  - proper distance: $$d_p = \frac{R(t)}{R_0}d_C$$

can be measured observationally!
(for standard ruler/candle)

  - luminosity distance: $$d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = \frac{R_0}{R(t)}R_0 x_E$$
  - angular diameter distance: $$d_A = \frac{D}{\vartheta_{obs}} = \frac{R(t)}{R_0}R_0 x_E$$

$$x_E = \begin{cases} \frac{1}{R_0} d_c & ;k=0 \\ \frac{1}{R_0}\frac{c}{H_0\sqrt{|\Omega_{k,0}|}}\sin\left(\frac{\sqrt{|\Omega_{k,0}|}H_0}{c}d_c\right) & ;k=1 \\ \frac{1}{R_0}\frac{c}{H_0\sqrt{|\Omega_{k,0}|}}\sinh\left(\frac{\sqrt{|\Omega_{k,0}|}H_0}{c}d_c\right) & ;k=-1 \end{cases}$$

($\Omega_{k,0} = -\frac{c^2 k}{R_0^2 H_0^2}$, cf. FRW lecture)

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  - angular diameter distance: $d_A = \frac{D}{\vartheta_{obs}} = \frac{R(t)}{R_0} R_0 x_E$

provides the link to "quantify cosmology"!

$$x_E = \begin{cases} \frac{1}{R_0} d_c & ;k=0 \\ \frac{1}{R_0}\frac{c}{H_0\sqrt{|\Omega_{k,0}|}}\sin\left(\frac{\sqrt{|\Omega_{k,0}|}H_0}{c}d_c\right) & ;k=1 \\ \frac{1}{R_0}\frac{c}{H_0\sqrt{|\Omega_{k,0}|}}\sinh\left(\frac{\sqrt{|\Omega_{k,0}|}H_0}{c}d_c\right) & ;k=-1 \end{cases}$$

($\Omega_{k,0} = -\frac{c^2 k}{R_0^2 H_0^2}$, cf. FRW lecture)

$$E^2(z) = \sum_i \Omega_{i,0}\left(1+z\right)^{3(1+w_i)}$$

- inter-relation:
  - comoving distance: $d_c = \frac{c}{H_0}\int_0^{z_E} \frac{1}{E(z)}dz$
  - proper distance: $d_p = \frac{R(t)}{R_0} d_C$
  - luminosity dist… x_E
  - angular diameter distance: $d_A = \frac{D}{\vartheta_{obs}} = \frac{R(t)}{R_0} R_0 x_E$

can we find a simple/approximate relation between redshift z and distance?

$$x_E = \begin{cases} \frac{1}{R_0} \quad d_c & ;k=0 \\ \frac{1}{R_0}\frac{c}{H_0\sqrt{|\Omega_{k,0}|}}\sin\left(\frac{\sqrt{|\Omega_{k,0}|}H_0}{c}d_c\right) & ;k=1 \\ \frac{1}{R_0}\frac{c}{H_0\sqrt{|\Omega_{k,0}|}}\sinh\left(\frac{\sqrt{|\Omega_{k,0}|}H_0}{c}d_c\right) & ;k=-1 \end{cases}$$

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- distance and redshift: Hubble's Law - revisited

$$z = \frac{R(t_0)}{R(t_E)} - 1$$

- distance and redshift: Hubble's Law - revisited

$$z = \frac{R(t_0)}{R(t_E)} - 1$$

• Taylor expanding z:

$$z = \frac{R(t_0)}{R(t_E)} - 1 = \left(\frac{R(t_0)}{R(t_E)} - 1 \right)_0 + \frac{d}{dt_E}\left(\frac{R(t_0)}{R(t_E)} - 1 \right)_0 \left(t_E - t_0\right) + \ldots$$

$$\approx -\left(\frac{R(t_0)}{R^2(t_E)} \dot{R}(t_E) \right)_0 \left(t_E - t_0\right) = \frac{\dot{R}(t_0)}{R(t_0)}\left(t_0 - t_E\right) = H_0\left(t_0 - t_E\right)$$

- distance and redshift: Hubble's Law - revisited

$$z = \frac{R(t_0)}{R(t_E)} - 1$$

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- distance and redshift: Hubble's Law - revisited

$$z = \frac{R(t_0)}{R(t_E)} - 1$$

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• Taylor expanding d_c: $$f(x_E) = \int_{t_E}^{t_0} \frac{cdt}{R(t)} \approx c\,\frac{t_0 - t_E}{R(t_0)}$$

- distance and redshift: Hubble's Law - revisited

$$z = \frac{R(t_0)}{R(t_E)} - 1$$

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- Taylor expanding d_c: $f(x_E) = \int_{t_E}^{t_0} \frac{cdt}{R(t)} \approx c\,\frac{t_0 - t_E}{R(t_0)}$

- proper distance: $d_p = R(t_0) f(x_E) \approx R(t_0) c \frac{t_0 - t_E}{R(t_0)} = c(t_0 - t_E)$

- distance and redshift: Hubble's Law - revisited

$$z = \frac{R(t_0)}{R(t_E)} - 1$$

• Taylor expanding z:

$$z = \frac{R(t_0)}{R(t_E)} - 1 = \left(\frac{R(t_0)}{R(t_E)} - 1\right)_0 + \frac{d}{dt_E}\left(\frac{R(t_0)}{R(t_E)} - 1\right)_0 \left(t_E - t_0\right) + \ldots$$

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- Taylor expanding d_c: $f(x_E) = \int_{t_E}^{t_0} \frac{c\,dt}{R(t)} \approx c\,\frac{t_0 - t_E}{R(t_0)}$

- proper distance: $d_p \approx \frac{cz}{H_0}$ ("Hubble-law distance")

$$\Rightarrow \quad \boxed{cz \approx H_0 d_p}$$

(only valid for nearby sources)

- cosmic distance ladder
- cosmological distances
- **cosmological horizons & volumes**
- supernova cosmology

- cosmic distance ladder
- cosmological distances
- **cosmological horizons & volumes**
  - **horizons**
  - volumes
- supernova cosmology

- horizons (see FRW lecture)

  - particle horizon: max. distance particle can have travelled since decoupling

$$R_p(t) = R(t)\int_{t_{dec}}^{t} \frac{cdt'}{R(t')}$$

  - "particle horizon": max. distance photon can have travelled since big bang (there are events we have not yet seen…)

$$R_p(t) = R(t)\int_{0}^{t} \frac{cdt'}{R(t')}$$

  - event horizon: max. distance particle can travel from now onwards (there may be events we will never see…)

$$R_e(t) = R(t)\int_{t}^{\infty} \frac{cdt'}{R(t')}$$

  - (comoving) Hubble radius: distance at which recessional velocity equals speed of light

$$R_H(t) = \frac{c}{H}; \qquad R_{cH}(t) = \frac{R_0}{R}\frac{c}{H}$$

- horizons (see FRW lecture)

• different bounds define different horizons

• all based upon proper distance

- particle horizon: max. distance particle can have travelled since decoupling

$$R_p(t) = R(t)\int_{t_{dec}}^{t} \frac{cdt'}{R(t')}$$

- "particle horizon": max. distance photon can have travelled since big bang (there are events we have not yet seen…)

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- cosmic distance ladder
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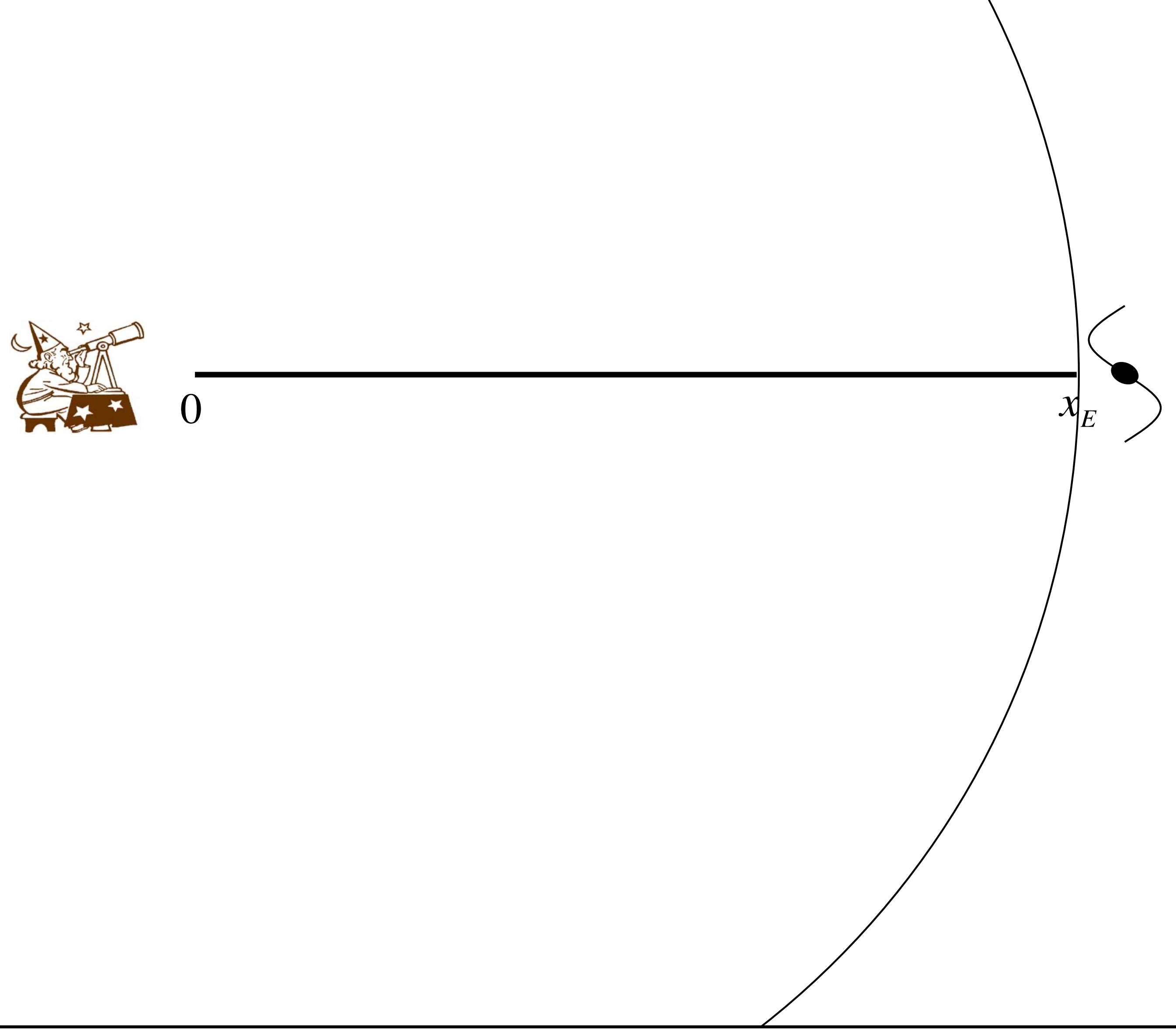
- proper volume at t_0

$$dV_p(t_0) = \sqrt{\det\left(g_{ij}\right)}\,dr\,d\vartheta\,d\varphi$$



- proper volume at t_0

$$dV_p(t_0) = \sqrt{\det\left(g_{ij}\right)}\,dr\,d\vartheta\,d\varphi$$

$$\xrightarrow[d\Omega = d\theta^2 + \sin^2\theta d\phi^2]{t = t_0} = R_0^3 x^2 \frac{dx}{\sqrt{1-kx^2}}\,d\Omega$$

- proper volume at t_0

$$dV_p(t_0) = \sqrt{\det\left(g_{ij}\right)}\, dr\, d\vartheta\, d\varphi$$

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how to relate to one of our distances?

- proper volume at t_0

$$dV_p(t_0) = \sqrt{\det\left(g_{ij}\right)}\,dr\,d\vartheta\,d\varphi$$

$$= R_0^3 x^2 \frac{dx}{\sqrt{1-kx^2}} d\Omega$$

$$\frac{dx}{\sqrt{1-kx^2}} = \frac{cdt}{R(t)} = \frac{dt}{dz}\frac{cdz}{R(t)} \qquad \frac{dt}{dz} = -\frac{R^2}{R_0\dot{R}}$$

$$= R_0^3 x^2 \frac{-cdz}{H_0 R_0 E(z)} d\Omega$$

$$= R_0^2 x^2 \frac{-cdz}{H_0 E(z)} d\Omega$$

$$= R_0^2 x^2 \frac{R_0^2 R_E^2}{R_0^2 R_E^2} \frac{-cdz}{H_0 E(z)} d\Omega$$

$$= \frac{R_0^4 x^2}{R_E^2} \frac{R_E^2}{R_0^2} \frac{-cdz}{H_0 E(z)} d\Omega$$

$d\Omega = d\theta^2 + \sin^2\theta d\phi^2$

- proper volume at t_0

$$dV_p(t_0) = \sqrt{\det\left(g_{ij}\right)}\, dr\, d\vartheta\, d\varphi$$

$$= R_0^3 x^2 \frac{dx}{\sqrt{1-kx^2}} d\Omega$$

$$\frac{dx}{\sqrt{1-kx^2}} = \frac{cdt}{R(t)} = \frac{dt}{dz}\frac{cdz}{R(t)} \qquad \frac{dt}{dz} = -\frac{R^2}{R_0 \dot{R}}$$

$$= R_0^3 x^2 \frac{-cdz}{H_0 R_0 E(z)} d\Omega$$

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$$= R_0^2 x^2 \frac{R_0^2 R_E^2}{R_0^2 R_E^2} \frac{-cdz}{H_0 E(z)} d\Omega$$

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$d\Omega = d\theta^2 + \sin^2\theta\, d\phi^2$

- proper volume at t_0

$$dV_p(t_0) = \sqrt{\det\left(g_{ij}\right)}\, dr\, d\vartheta\, d\varphi$$

$$= R_0^3 x^2 \frac{dx}{\sqrt{1-kx^2}} d\Omega$$

$$\frac{dx}{\sqrt{1-kx^2}} = \frac{cdt}{R(t)} = \frac{dt}{dz}\frac{cdz}{R(t)} \qquad \frac{dt}{dz} = -\frac{R^2}{R_0 \dot{R}}$$

$$= R_0^3 x^2 \frac{-cdz}{H_0 R_0 E(z)} d\Omega$$

$$= R_0^2 x^2 \frac{-cdz}{H_0 E(z)} d\Omega$$

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$$= \frac{R_0^4 x^2}{R_E^2} \frac{R_E^2}{R_0^2} \frac{-cdz}{H_0 E(z)} d\Omega$$

$$d_L = \frac{R_0^2}{R_E} x$$

$$= d_L^2 \frac{1}{(1+z)^2} \frac{-cdz}{H_0 E(z)} d\Omega$$

$$d\Omega = d\theta^2 + \sin^2\theta\, d\phi^2$$

- proper volume at t_0

$$dV_p(t_0) = \sqrt{\det\left(g_{ij}\right)}\,dr\,d\vartheta\,d\varphi$$

$$= R_0^3 x^2 \frac{dx}{\sqrt{1-kx^2}} d\Omega$$

$\frac{dx}{\sqrt{1-kx^2}} = \frac{cdt}{R(t)} = \frac{dt}{dz}\frac{cdz}{R(t)}$ →

$$= R_0^3 x^2 \frac{-cdz}{H_0 R_0 E(z)} d\Omega$$

$$= R_0^2 x^2 \frac{-cdz}{H_0 E(z)} d\Omega$$

$$= R_0^2 x^2 \frac{R_0^2 R_E^2}{R_0^2 R_E^2} \frac{-cdz}{H_0 E(z)} d\Omega$$

$$= \frac{R_0^4 x^2}{R_E^2} \frac{R_E^2}{R_0^2} \frac{-cdz}{H_0 E(z)} d\Omega$$

$d_L = \frac{R_0^2}{R_E} x$ →

$$= d_L^2 \frac{1}{(1+z)^2} \frac{-cdz}{H_0 E(z)} d\Omega$$

integration ↓ (from the d_L form) integration ↓ (from the x form)

=>

$$V_p(t_0) = \frac{4\pi}{H_0} \int_0^{z_E} \frac{d_L^2(z)}{(1+z)^2 E(z)} dz = 4\pi R_0^3 \int_0^{x_E} \frac{x^2}{\sqrt{1-kx^2}} dx$$

$d\Omega = d\theta^2 + \sin^2\theta\, d\phi^2$

- proper volume at t_0

$$V_p(t_0) = \frac{4\pi}{H_0}\int_0^{z_E} \frac{d_L^2(z)}{(1+z)^2 E(z)}\,dz = 4\pi R_0^3 \int_0^{x_E} \frac{x^2}{\sqrt{1-kx^2}}\,dx$$

$$\Rightarrow \quad V_p(t_0) = \begin{cases} \dfrac{4\pi}{3}\left(\dfrac{d_L}{1+z}\right)^3 & k=0 \\[2ex] \dfrac{2\pi}{H_0^3\Omega_{k,0}}\left[H_0\dfrac{d_L}{1+z}\sqrt{1+\left[\dfrac{H_0 d_L}{1+z}\right]^3\Omega_{k,0}} - \dfrac{1}{\sqrt{|\Omega_{k,0}|}}\arcsin\left(H_0 d_L\sqrt{|\Omega_{k,0}|}\right)\right] & k=1 \\[2ex] \dfrac{2\pi}{H_0^3\Omega_{k,0}}\left[H_0\dfrac{d_L}{1+z}\sqrt{1+\left[\dfrac{H_0 d_L}{1+z}\right]^3\Omega_{k,0}} - \dfrac{1}{\sqrt{|\Omega_{k,0}|}}\operatorname{arcsinh}\left(H_0 d_L\sqrt{|\Omega_{k,0}|}\right)\right] & k=-1 \end{cases}$$

- $V_p(t_0)$ is a function of H_0, Ω_m, Ω_Λ, and z
- $V_p(t_0)$ gets corrected by the solid angle Ω at z via $V_p^{\Omega} = V_p \dfrac{\Omega}{4\pi}$

- proper volume at t != t_0

$$dV_p(t) = \sqrt{\det(g_{ij})}\, dr\, d\vartheta\, d\varphi$$

difference to previous calculation…

$$= R^3(t) x^2 \frac{dx}{\sqrt{1-kx^2}} d\Omega$$

$$= \ldots$$

$$= (1+z)^3 dV_p(t_0)$$

- comoving volume

$$dV_p = R^3(t)x^2 \frac{dx}{\sqrt{1-kx^2}} d\Omega$$

$$dV_c = x^2 \frac{dx}{\sqrt{1-kx^2}} d\Omega$$

=>

$$\boxed{V_c(z) = \frac{V_p(z)}{R^3(t(z))}}$$

- cosmic distance ladder
- **cosmological distances**
- **cosmological horizons & volumes**
- supernova cosmology

http://cosmocalc.icrar.org/

- cosmic distance ladder
- cosmological distances
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- cosmological parameters

$$H_0, \quad \Omega_{m,0}, \quad \Omega_{k,0}, \quad \Omega_{\Lambda,0}$$

- cosmological parameters

$$H_0, \quad \Omega_{m,0}, \quad \Omega_{k,0}, \quad \Omega_{\Lambda,0}$$

$$1 = \Omega_{m,0} + \Omega_{k,0} + \Omega_{\Lambda,0}$$

$(\Omega_{r,0} \approx 0)$

- cosmological parameters

$$H_0, \quad \Omega_{m,0}, \quad \Omega_{k,0}, \quad \Omega_{\Lambda,0}$$

$$1 = \Omega_{m,0} + \Omega_{k,0} + \Omega_{\Lambda,0}$$

$$\downarrow$$

only three parameters remain...

$$\boxed{H_0, \quad \Omega_{m,0}, \quad \Omega_{\Lambda,0}}$$

$(\Omega_{r,0} \approx 0)$

- cosmological parameters

ain...

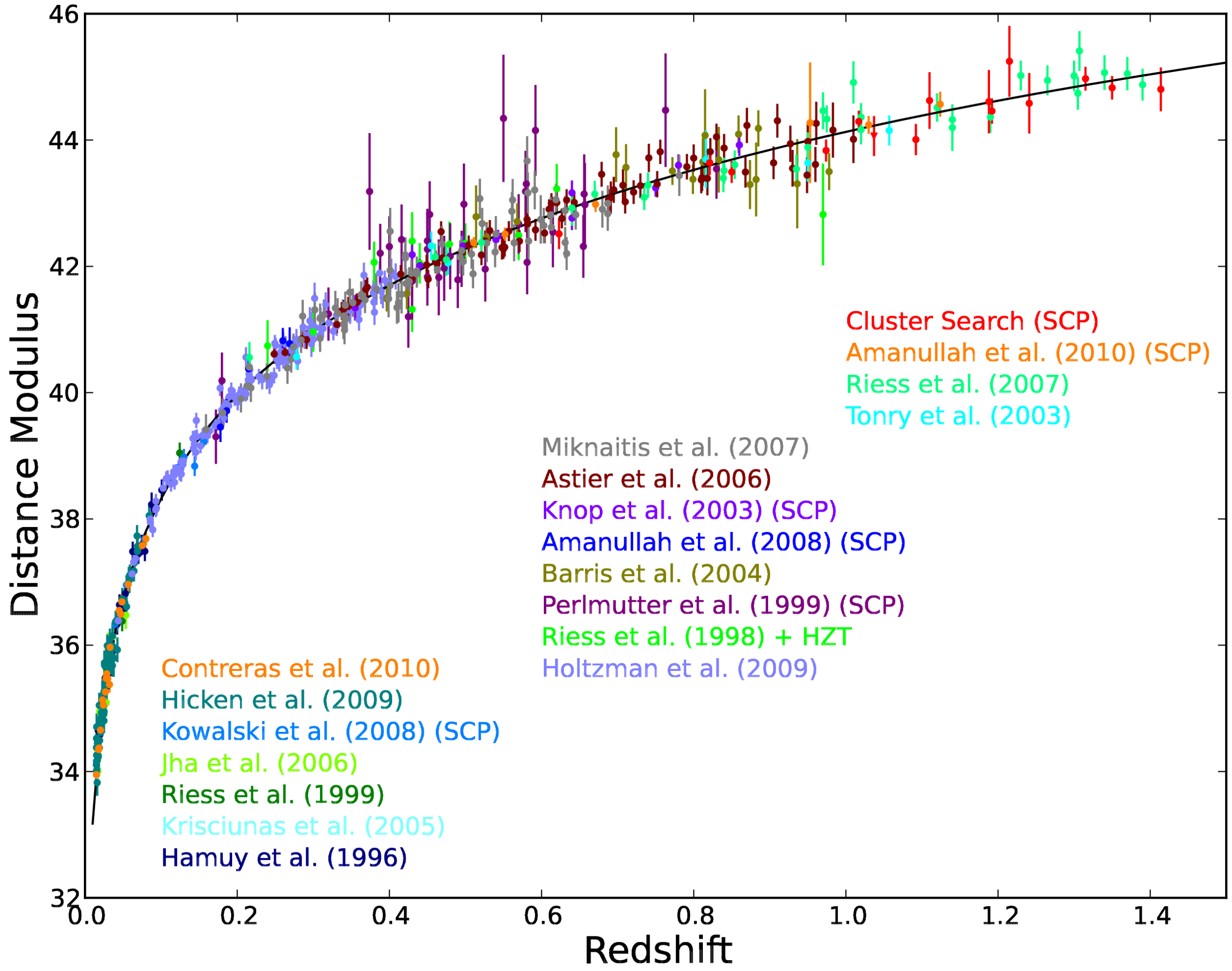
$$\boxed{H_0, \quad \Omega_{m,0}, \quad \Omega_{\Lambda,0}}$$

how to use supernovae Ia to obtain these parameters?

$(\Omega_{r,0} \approx 0)$

- $m(z)$-relation for Union 2.1 SN-Ia data set

$$m - M = 25 - 5\log\left(H_0\right) + 5\log\left(\mathcal{D}(z, \Omega_{m,0}, \Omega_{\Lambda,0})\right)$$

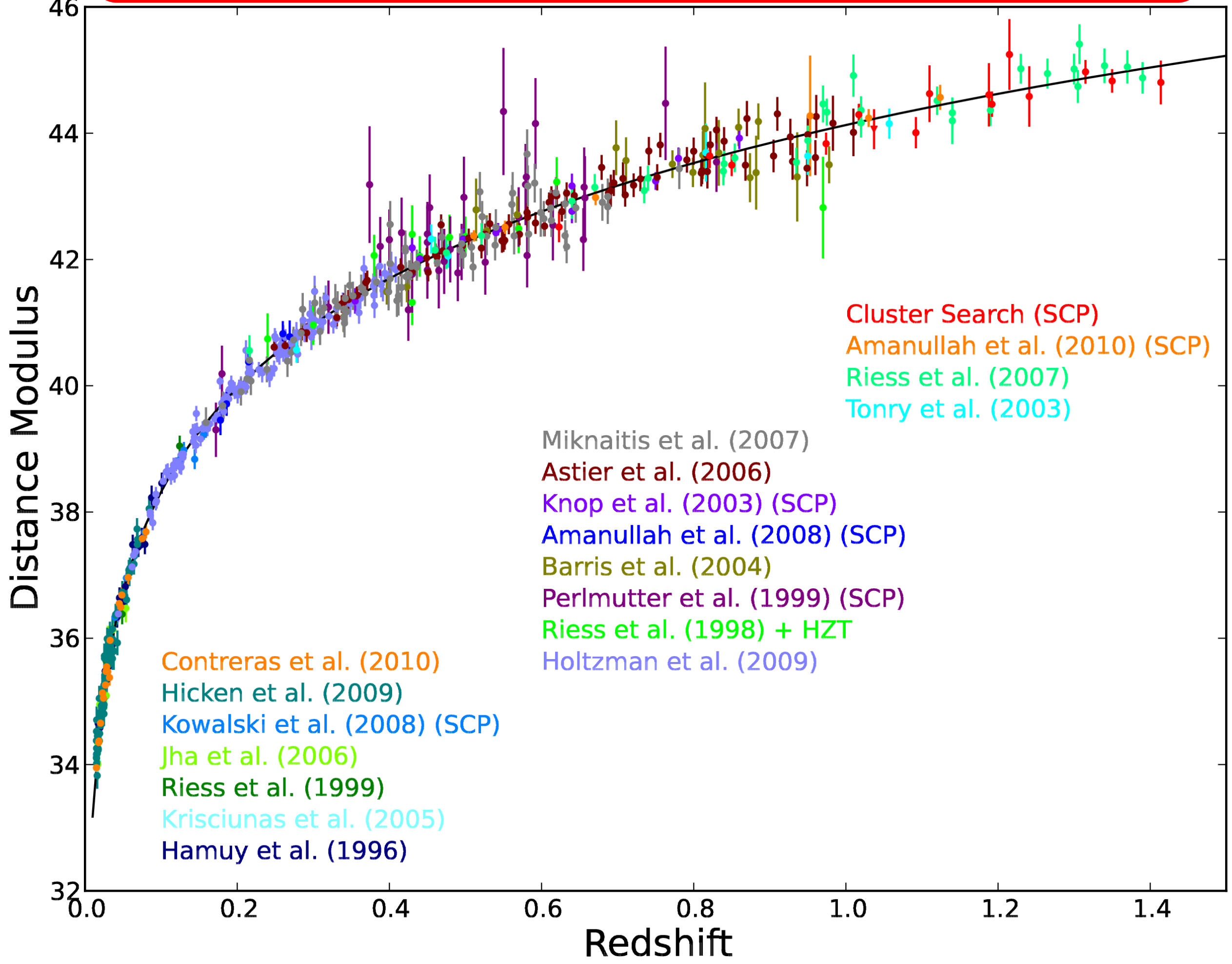


*http://supernova.lbl.gov/union

- $m(z)$-relation for Union 2.1 SN-Ia data set

where does this equation come from?

$$m - M = 25 - 5\log\left(H_0\right) + 5\log\left(\mathcal{D}(z, \Omega_{m,0}, \Omega_{\Lambda,0})\right)$$



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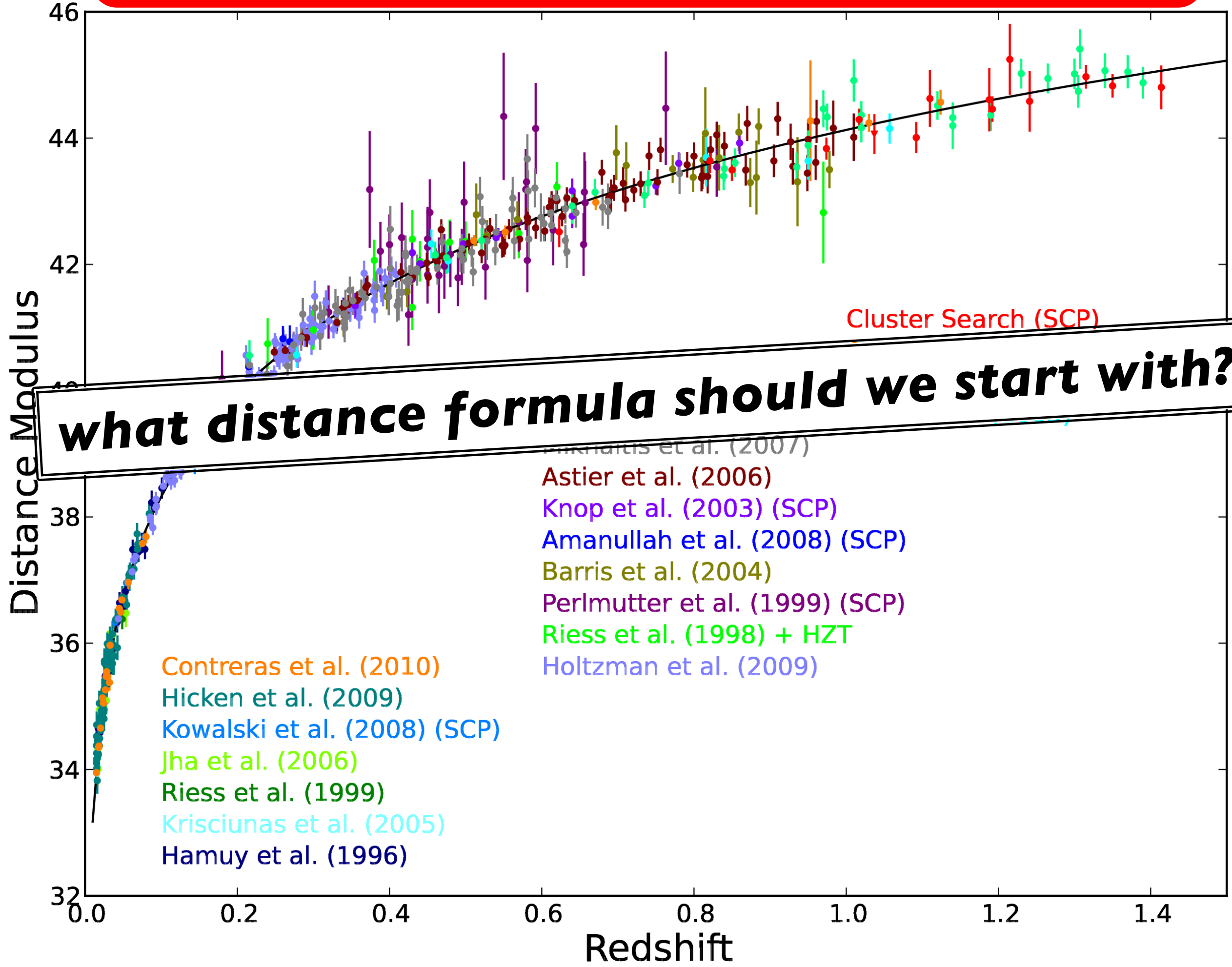

what distance formula should we start with?

*http://supernova.lbl.gov/union

- luminosity distance

$$d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = \frac{R_0}{R(t_E)} R_0 x_E = (1 + z_E) R_0 x_E$$

$(\Omega_{r,0} \approx 0)$

- luminosity distance

$$d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = \frac{R_0}{R(t_E)} R_0 x_E = (1+z_E) R_0 x_E$$

$$x_E = \begin{cases} \dfrac{1}{R_0} \, d_c & ;k=0 \\ \dfrac{c}{R_0 H_0} \dfrac{1}{\sqrt{|1-\Omega_{m,0}-\Omega_{\Lambda,0}|}} \sin\left(\sqrt{|1-\Omega_{m,0}-\Omega_{\Lambda,0}|}\dfrac{H_0}{c} d_c\right) & ;k=1 \\ \dfrac{c}{R_0 H_0} \dfrac{1}{\sqrt{|1-\Omega_{m,0}-\Omega_{\Lambda,0}|}} \sinh\left(\sqrt{|1-\Omega_{m,0}-\Omega_{\Lambda,0}|}\dfrac{H_0}{c} d_c\right) & ;k=-1 \end{cases}$$

$(\Omega_{r,0} \approx 0, \quad \Omega_k = 1 - \Omega_m - \Omega_\Lambda)$

- luminosity distance

$$d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = \frac{R_0}{R(t_E)} R_0 x_E = (1+z_E) R_0 x_E$$

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$$d_c = \frac{c}{H_0}\int_0^{z_E} \frac{1}{E(z)} dz$$

$(\Omega_{r,0} \approx 0, \quad \Omega_k = 1 - \Omega_m - \Omega_\Lambda)$

- luminosity distance

$$d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = \frac{R_0}{R(t_E)} R_0 x_E = (1+z_E) R_0 x_E$$

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$$d_c = \frac{c}{H_0}\int_0^{z_E} \frac{1}{E(z)} dz$$

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0}-\Omega_{\Lambda,0})(1+z)^2 + \Omega_{\Lambda,0}}$$

$(\Omega_{r,0} \approx 0, \quad \Omega_k = 1-\Omega_m-\Omega_\Lambda)$

right-hand side under control, but what about d_L itself (e.g. how do m and M enter)?

- luminosity distance

$$d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = \frac{R_0}{R(t_E)} R_0 x_E = (1+z_E) R_0 x_E$$

$$x_E = \begin{cases} \frac{1}{R_0} d_c & ;k=0 \\ \frac{c}{R_0 H_0} \frac{1}{\sqrt{|1-\Omega_{m,0}-\Omega_{\Lambda,0}|}} \sin\left(\sqrt{|1-\Omega_{m,0}-\Omega_{\Lambda,0}|}\frac{H_0}{c} d_c\right) & ;k=1 \\ \frac{c}{R_0 H_0} \frac{1}{\sqrt{|1-\Omega_{m,0}-\Omega_{\Lambda,0}|}} \sinh\left(\sqrt{|1-\Omega_{m,0}-\Omega_{\Lambda,0}|}\frac{H_0}{c} d_c\right) & ;k=-1 \end{cases}$$

$$d_c = \frac{c}{H_0}\int_0^{z_E} \frac{1}{E(z)} dz$$

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0}-\Omega_{\Lambda,0})(1+z)^2 + \Omega_{\Lambda,0}}$$

$(\Omega_{r,0} \approx 0, \quad \Omega_k = 1 - \Omega_m - \Omega_\Lambda)$

- distance modulus

$$d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = (1 + z_E) R_0 x_E$$

- distance modulus

$$d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = (1+z_E)R_0 x_E$$

☑ formula to relate to cosmology

- distance modulus

☑ theory of SN Ia

$$d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = (1+z_E)R_0 x_E$$

☑ formula to relate to cosmology

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?

☑ formula to relate to cosmology

- distance modulus

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- apparent magnitudes m:* $m_1 - m_2 = -2.5\log_{10}\left(\frac{F_1}{F_2}\right)$ where $F = \frac{L}{4\pi d^2}$

*we require normalisation point: "Vega has apparent magnitude 0"

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- absolute magnitudes M: $m - M = -2.5\log_{10}\left(\frac{L}{4\pi d^2}\frac{4\pi(10\text{pc})^2}{L}\right)$ placing light source L at 10pc

$$= -2.5\log_{10}\left(\frac{(10\text{pc})^2}{d^2}\right) = -5\log\left(\frac{10\text{pc}}{\text{d}}\right)$$

- distance modulus

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=> $d = 10^{1+\frac{m-M}{5}}\,\text{pc} = 10^{-5+\frac{m-M}{5}}\,\text{Mpc}$

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$$\overset{[d_L]=Mpc}{\Rightarrow} \quad m - M = 25 + 5\log(d_L)$$

- distance modulus

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$$\overset{[d_L]=Mpc}{\Rightarrow} \; m - M = 25 + 5\log(d_L) - 5\log(H_0) + 5\log(H_0)$$

(x_E contains $1/H_0$)

- distance modulus

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$$\overset{[d_L]=Mpc}{\Rightarrow} \quad m - M = 25 - 5\log\left(H_0[\mathrm{km/sec/Mpc}]\right) + 5\log\left(H_0 d_L\right)$$

- distance modulus

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cosmology

$$\overset{[d_L]=Mpc}{\Rightarrow} \quad m - M = 25 - 5\log\left(H_0[\text{km/sec/Mpc}]\right) + 5\log\left(H_0 d_L\right)$$

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observation **(relation between F & L and m & M?)**

$$\Rightarrow \quad d = 10^{1+\frac{m-M}{5}}\,\mathrm{pc} = 10^{-5+\frac{m-M}{5}}\,\mathrm{Mpc} \equiv \mathrm{d}_L$$

$[d_L] = Mpc$

$$\Rightarrow \; m - M = 25 - 5\log\left(H_0[\mathrm{km/sec/Mpc}]\right) + 5\log\left(H_0 d_L\right)$$

- distance modulus

$$d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = (1+z_E)R_0 x_E$$

- observation m:

$$F_{obs} = 10^{-2m/5} \times 2.52 \times 10^{-5} \frac{\mathrm{erg}}{\mathrm{cm}^2\,\mathrm{sec}}$$

- standard candle M:

$$L_E = 10^{-2M/5} \times 3.02 \times 10^{35} \frac{\mathrm{erg}}{\mathrm{sec}}$$

$$m - M = 25 - 5\log\left(H_0[\mathrm{km/sec/Mpc}]\right) + 5\log\left(H_0 d_L\right)$$

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=> invert to get m and M

$$m - M = 25 - 5\log\left(H_0[\text{km/sec/Mpc}]\right) + 5\log\left(H_0 d_L\right)$$

- $m(z)$-relation

$$m - M = 25 - 5\log\left(H_0\right) + 5\log\left(\mathcal{D}(z,\Omega_{m,0},\Omega_{\Lambda,0})\right)$$

$$\mathcal{D}(z,\Omega_{m,0},\Omega_{\Lambda,0}) = \frac{c(1+z)}{\sqrt{|k|}}\ \operatorname{sinn}\left(\sqrt{|k|}\int_0^z \left[\left(1+z'\right)^2\left(1+\Omega_{m,0}z'\right) - z'\left(2+z'\right)\Omega_{\Lambda,0}\right]^{-1/2} dz'\right)$$

$\mathcal{D}(z,\Omega_{m,0},\Omega_{\Lambda,0}) = H_0 d_L$ is independent of H_0

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- m, z : observables
- M : standard candle

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- H_0, $\Omega_{m,0}$, $\Omega_{\Lambda,0}$: cosmology

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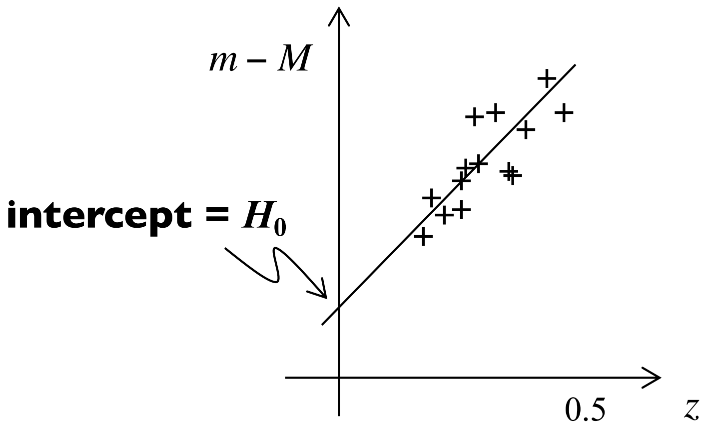
$\rightarrow$ measuring H_0



- $m(z)$-relation

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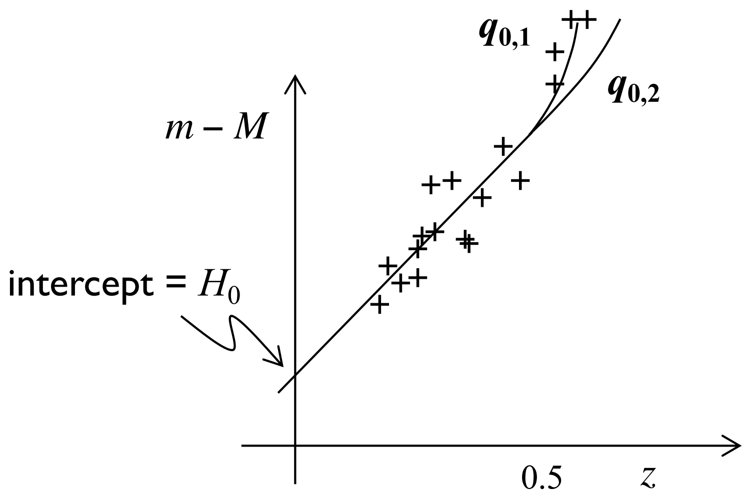
→ measuring q_0



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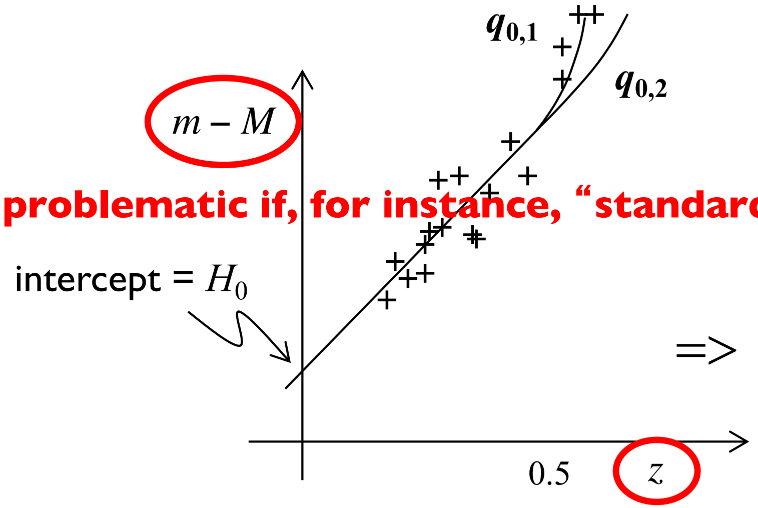
→ measuring q_0



problematic if, for instance, "standard galaxies" are used!

$$\Rightarrow \quad q_{0,eff} = q_0 - \frac{1}{H_0}\frac{\dot{L}}{L}$$

- $m(z)$-relation

$$m - M = 25 - 5\log(H_0) + 5\log\left(\mathcal{D}(z,\Omega_{m,0},\Omega_{\Lambda,0})\right)$$

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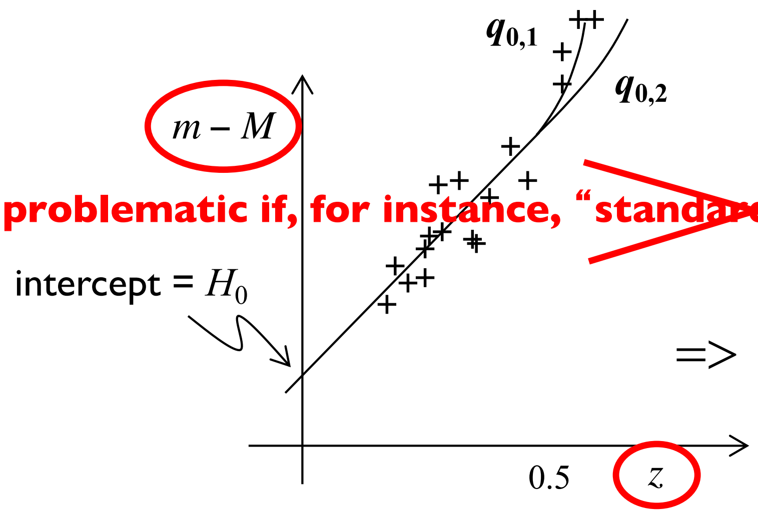
• M : standard candle
• $H_0, \Omega_{m,0}, \Omega_{\Lambda,0}$: cosmology

→ measuring q_0



problematic if, for instance, "standard galaxies" are used!

$$\Rightarrow\quad q_{0,eff} = q_0 - \underbrace{\frac{1}{H_0}\frac{\dot{L}}{L}}_{\approx q_0}$$

- $m(z)$-relation
  - SN Ia are feasible standard candles:
    - – visible out to $z \approx 1$
    - – small dispersion of light curve maximum
    - – light curve independent on redshift

    - – Perlmutter et al. (1997, ApJ, 483, 565*)
    - – Garnavich et al. (1997, AAS presentation†)

$$\left.\begin{array}{l}\text{Perlmutter et al.}\\ \text{Garnavich et al.}\end{array}\right\} q_0 < 0 \quad \Rightarrow \quad \Omega_{\Lambda,0} \neq 0$$

* based upon 7 SN
† based upon 3 SN

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  - SN Ia are feasible standard candles:
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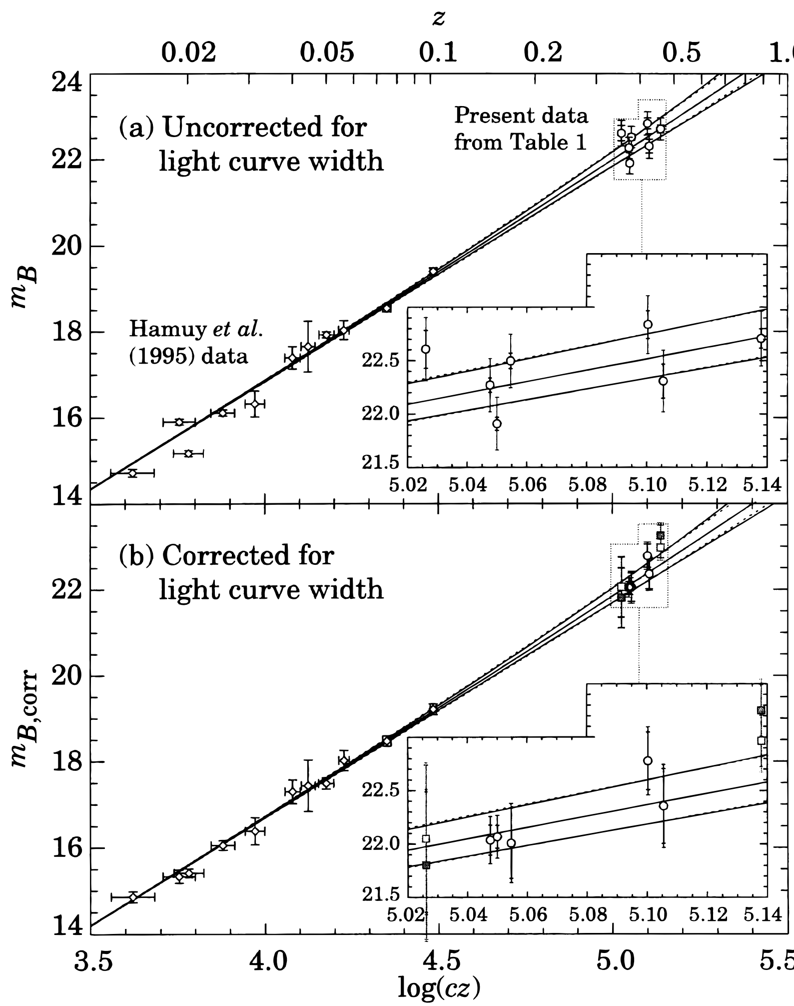
– Riess, Schmidt et al. (1998, AJ, 116, 1009✷)

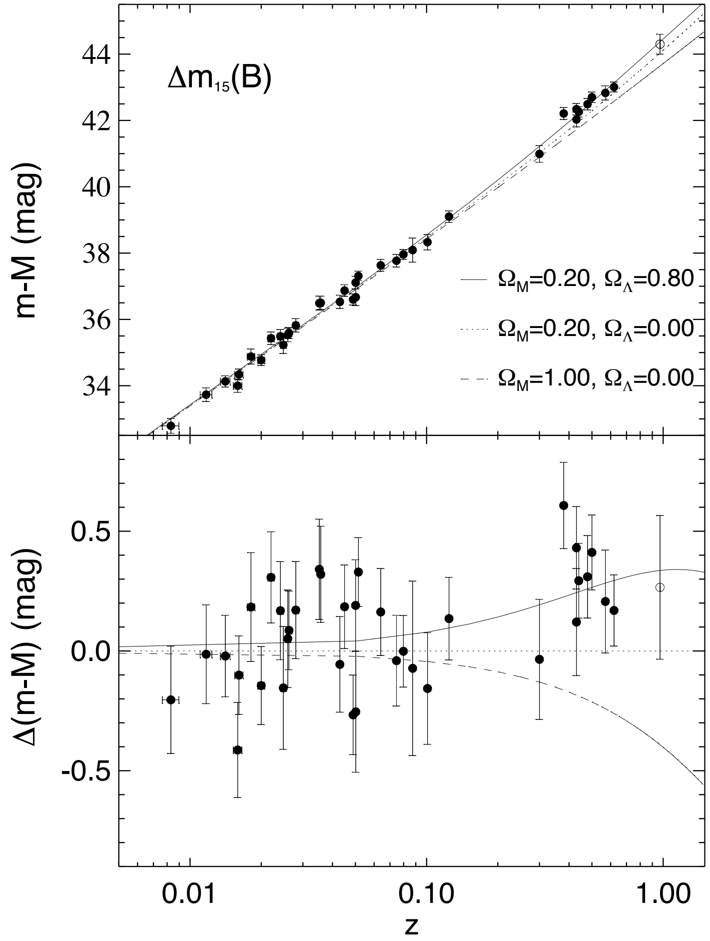
$$\left.\right\} \quad q_0 < 0 \quad \Rightarrow \quad \Omega_{\Lambda,0} \neq 0$$

* based upon 7 high-z SN
✝ based upon 3 high-z SN
✷ based upon 10 high-z SN

- $m(z)$-relation for SN-Ia – the money plots…

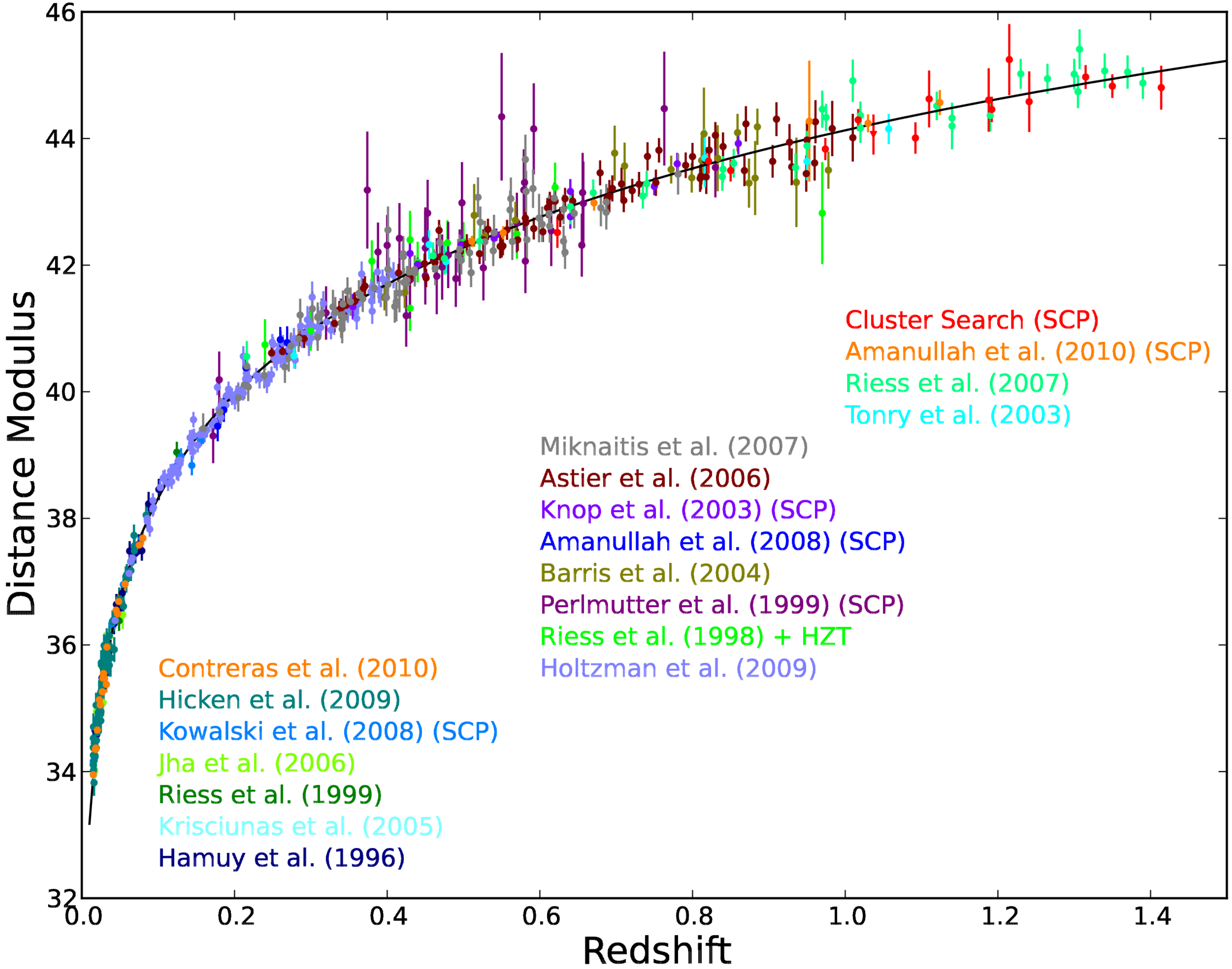


Perlmutter et al. (1997, ApJ, 483, 565)



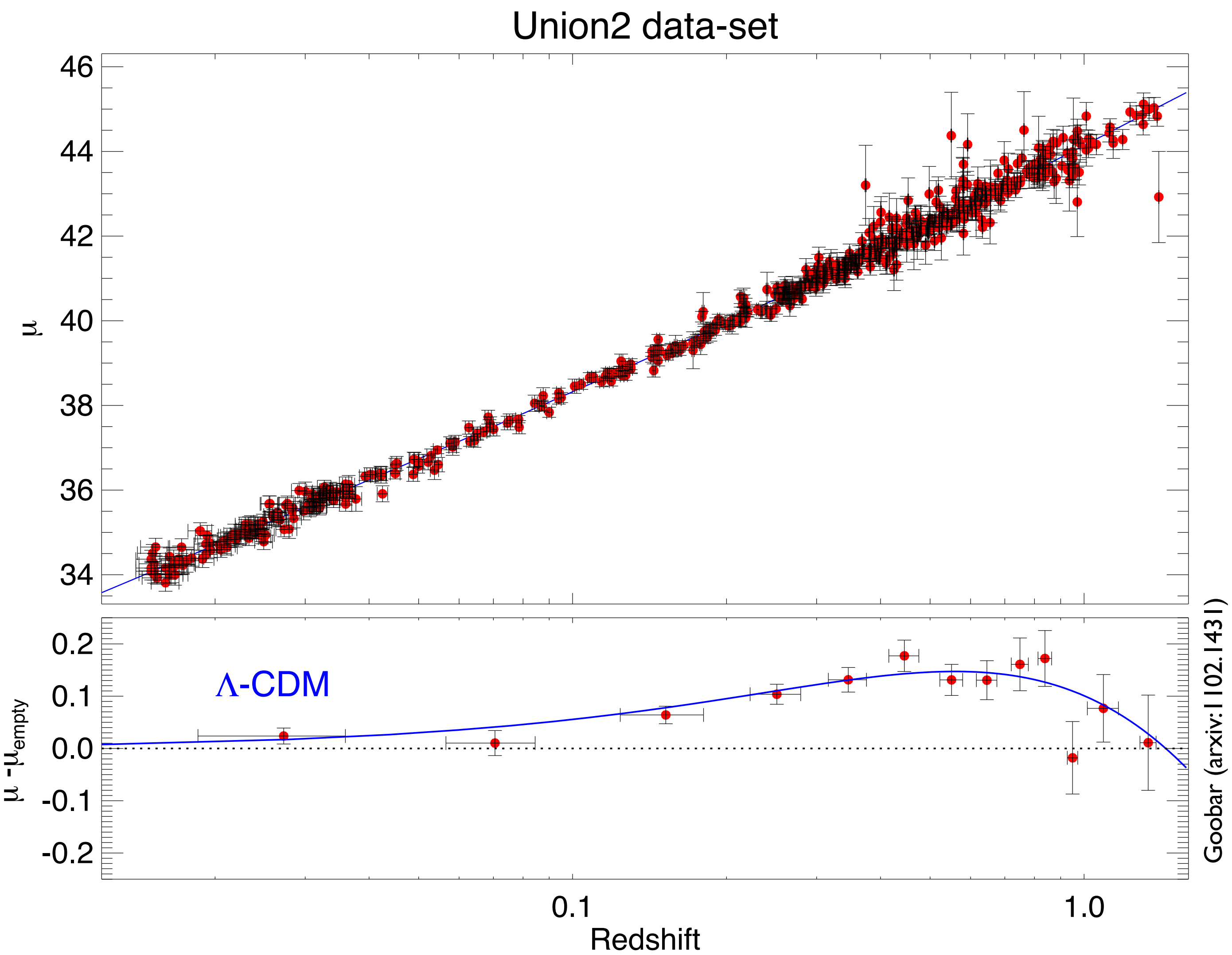
Riess, Schmidt et al. (1998, AJ, 116, 1009)

- $m(z)$-relation for SN-Ia – Union 2* data set



*http://supernova.lbl.gov/union

- $m(z)$-relation for SN-Ia – Union 2 data set vs. ΛCDM



- comoving distance: $d_c = \frac{c}{H_0}\int_0^{z_E}\frac{1}{E(z)}dz$

- proper distance: $d_p = \frac{R(t)}{R_0}d_C$

- luminosity distance: $d_L = \sqrt{\frac{L_E}{4\pi F_{obs}}} = \frac{R_0}{R(t)}R_0 x_E$

- angular diameter distance: $d_A = \frac{D}{\vartheta_{obs}} = \frac{R(t)}{R_0}R_0 x_E$

$$x_E = \begin{cases} \frac{1}{R_0} \quad d_c & ;k=0 \\ \frac{1}{R_0}\frac{c}{H_0\sqrt{|\Omega_{k,0}|}}\sin\left(\frac{\sqrt{|\Omega_{k,0}|}H_0}{c}d_c\right) & ;k=1 \\ \frac{1}{R_0}\frac{c}{H_0\sqrt{|\Omega_{k,0}|}}\sinh\left(\frac{\sqrt{|\Omega_{k,0}|}H_0}{c}d_c\right) & ;k=-1 \end{cases}$$

$$E^2(z) = \sum_i \Omega_{i,0}\left(1+z\right)^{3(1+w_i)}$$

$\Omega_{k,0} = -\frac{c^2 k}{R_0^2 H_0^2}$, cf. FRW lecture