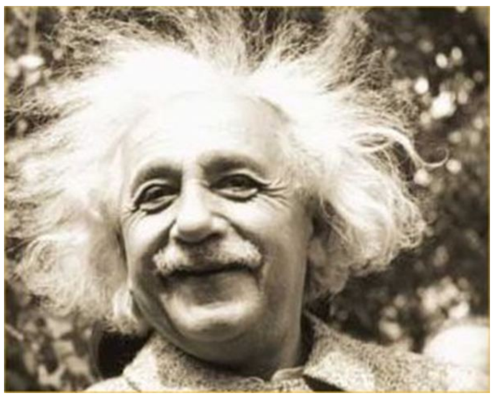
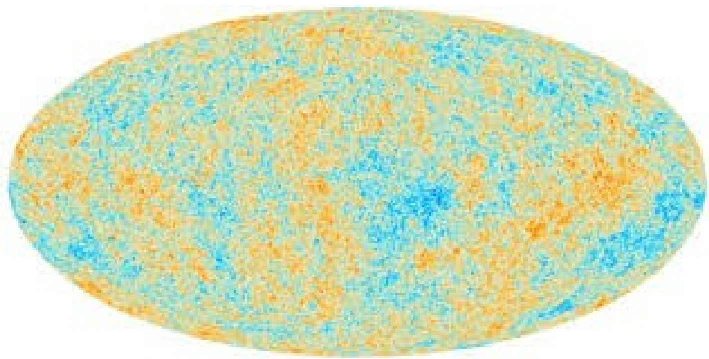
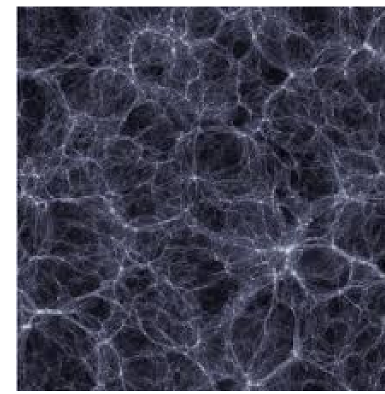
Cosmic Microwave Background, Part II: Theory







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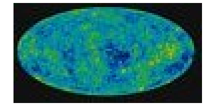
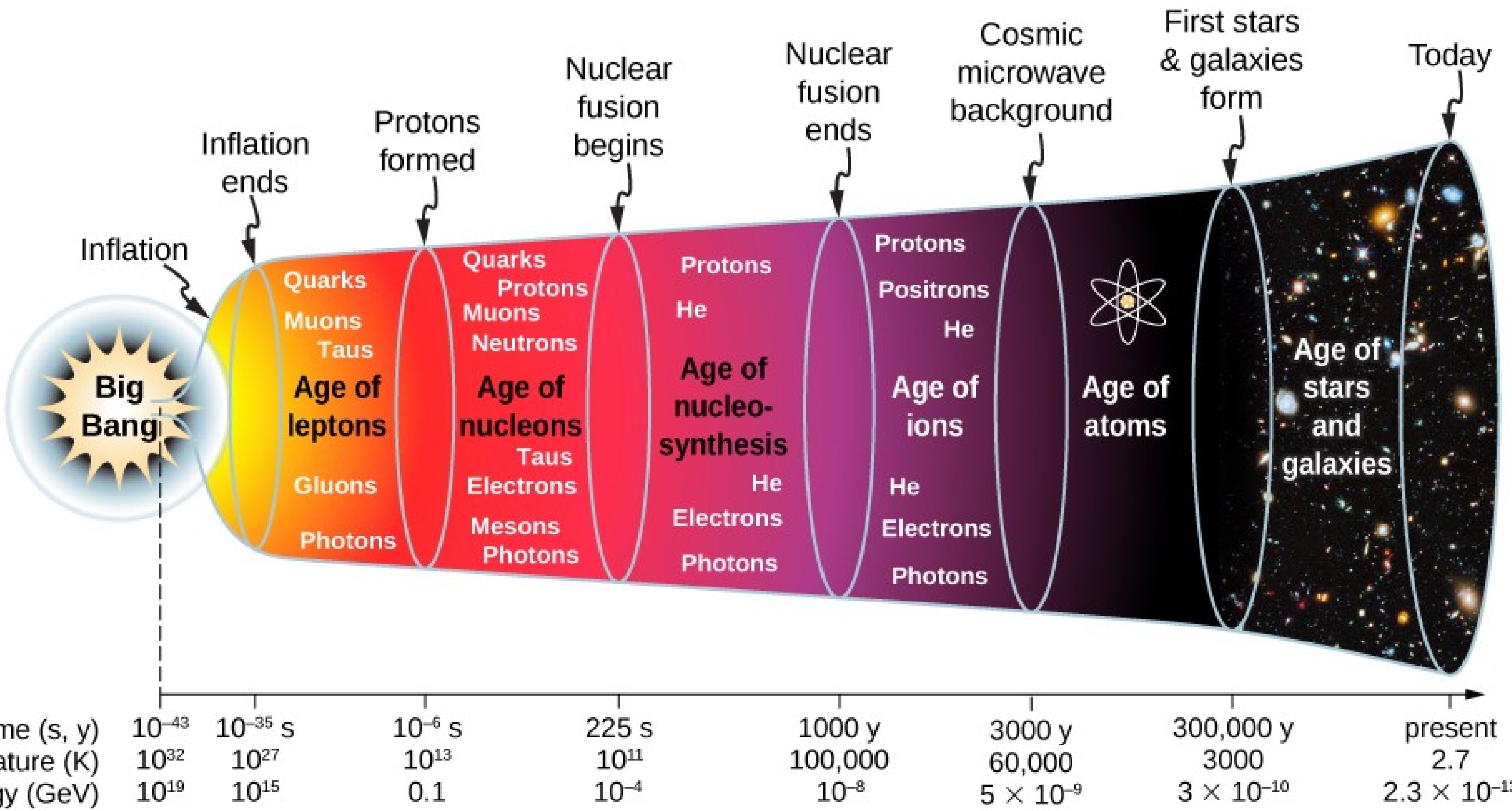
Main points of the lecture

- Overview (theory and basic framework)
- CMB features (effects, parameter sensitivity)
- BAO oscillations (rigorous proof, math codes, P(k) behaviour)
- The Cls (SW and ISW), Planck papers, CLASS
- Summary

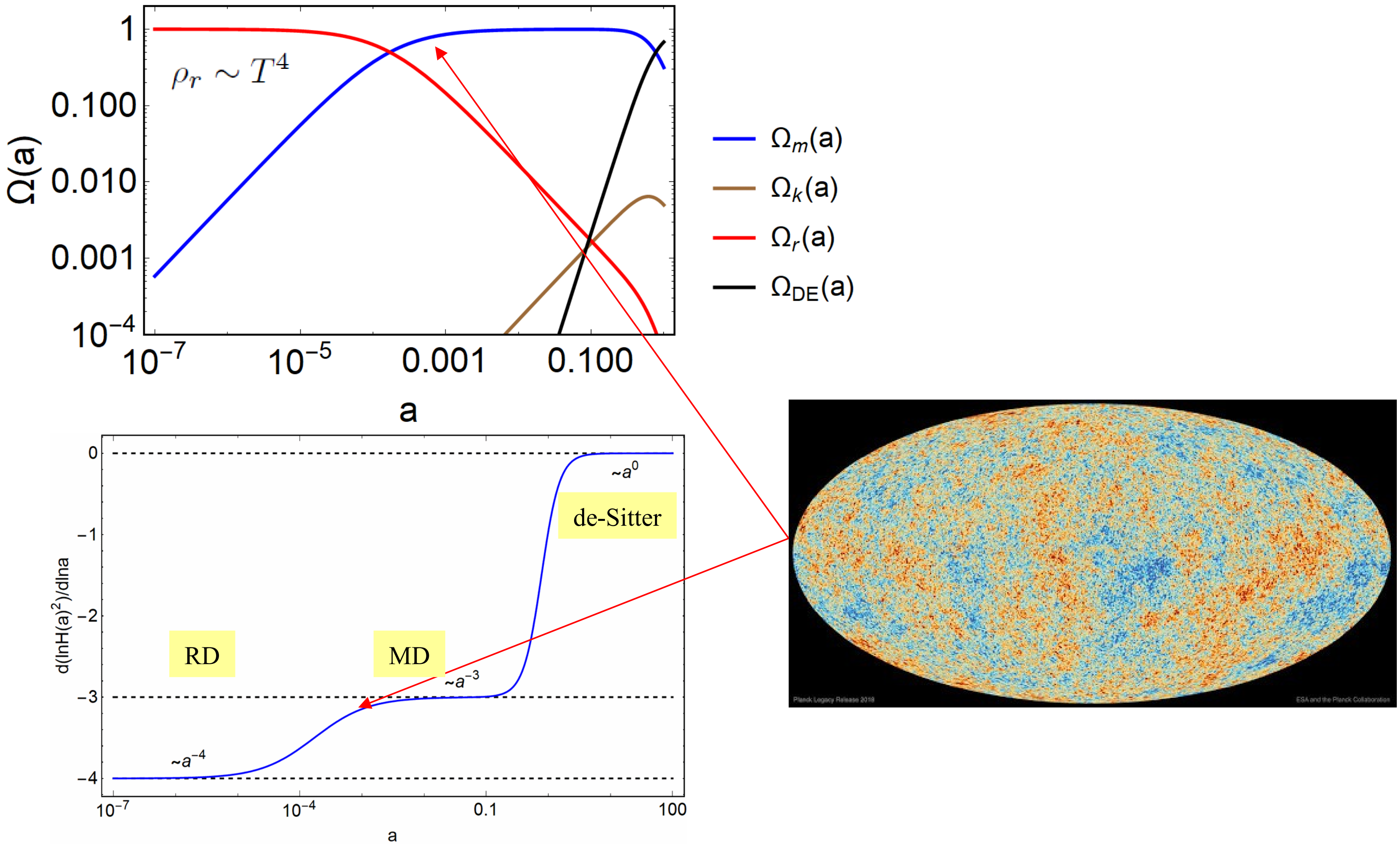
The hot Big Bang theory redux





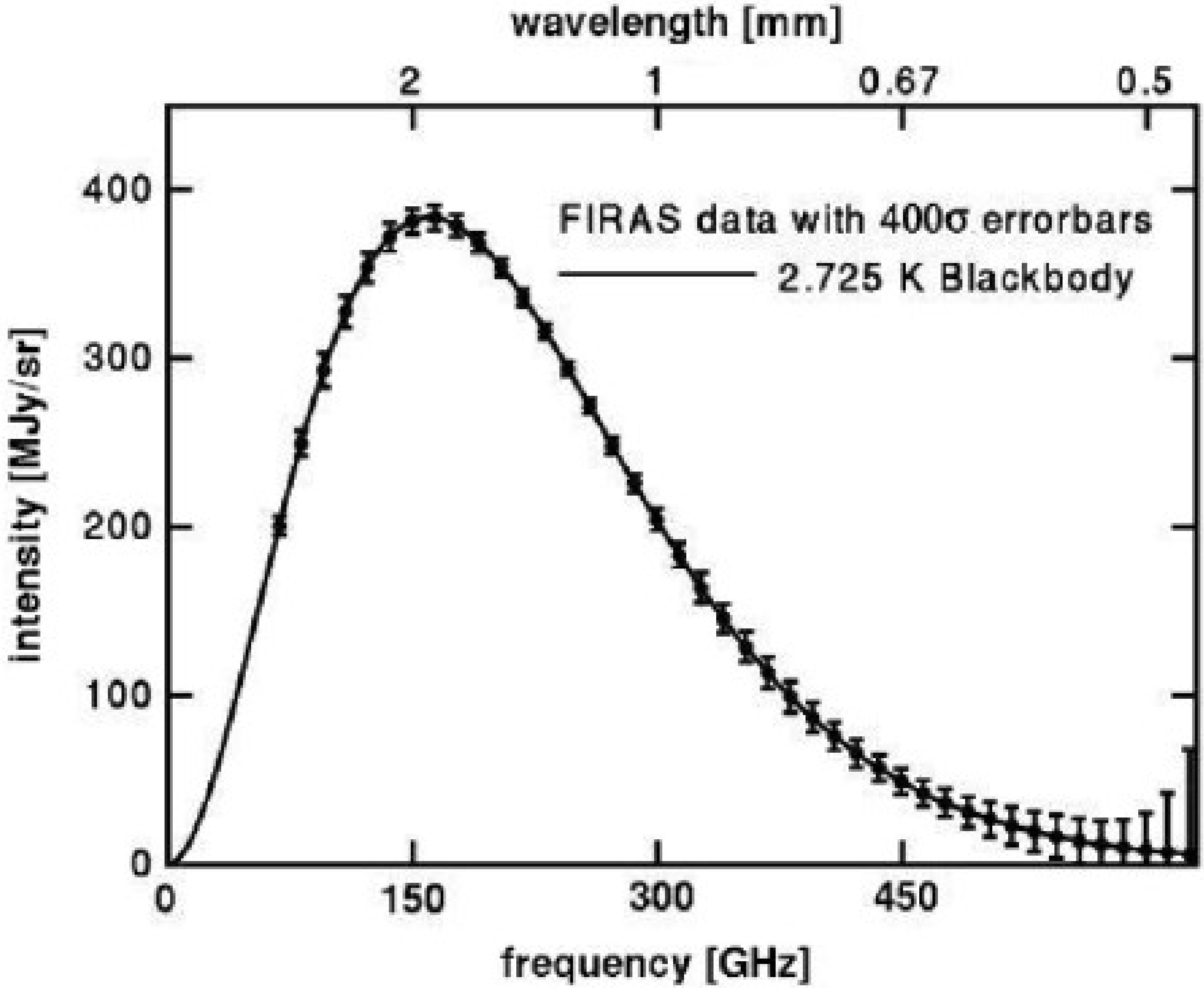
The hot Big Bang theory redux

1) The Cosmic Microwave Background (CMB) is a relic of the hot Big Bang!



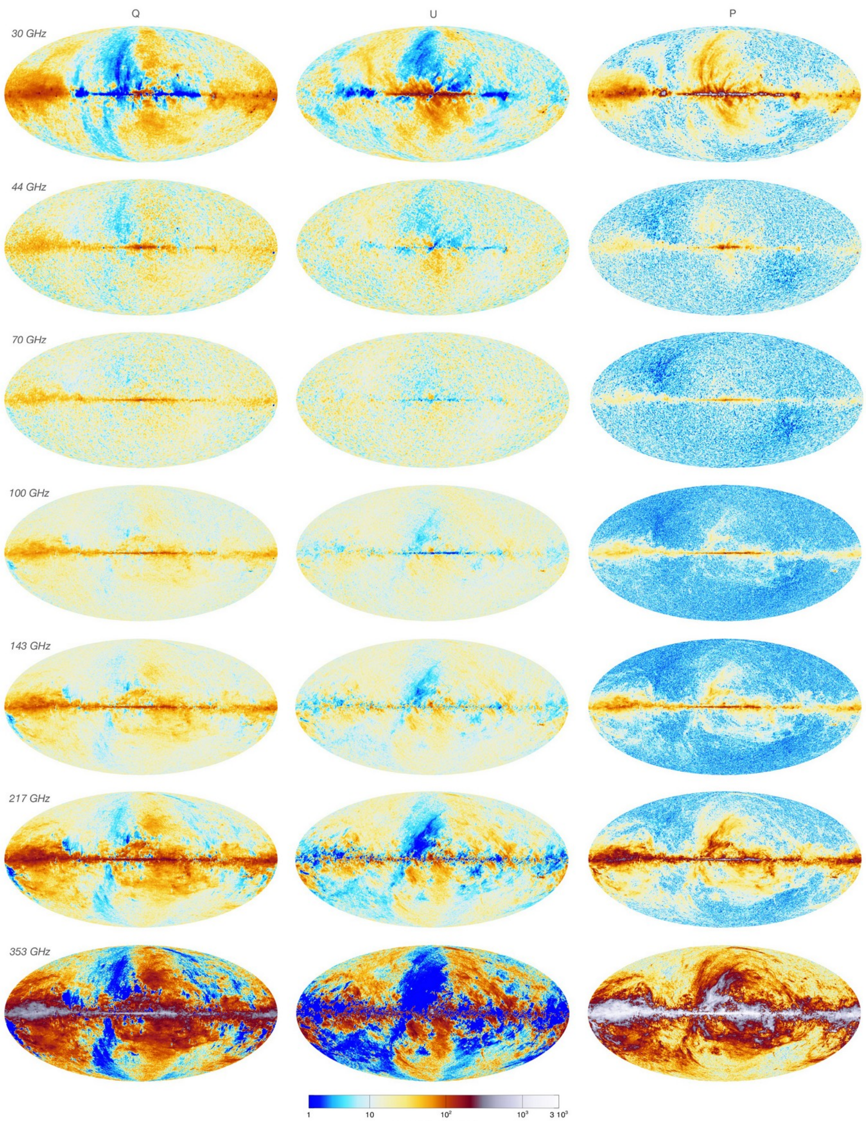
CMB anisotropies and spectrum

2) The CMB is an almost perfect black-body! Note: these are 400σ errors...

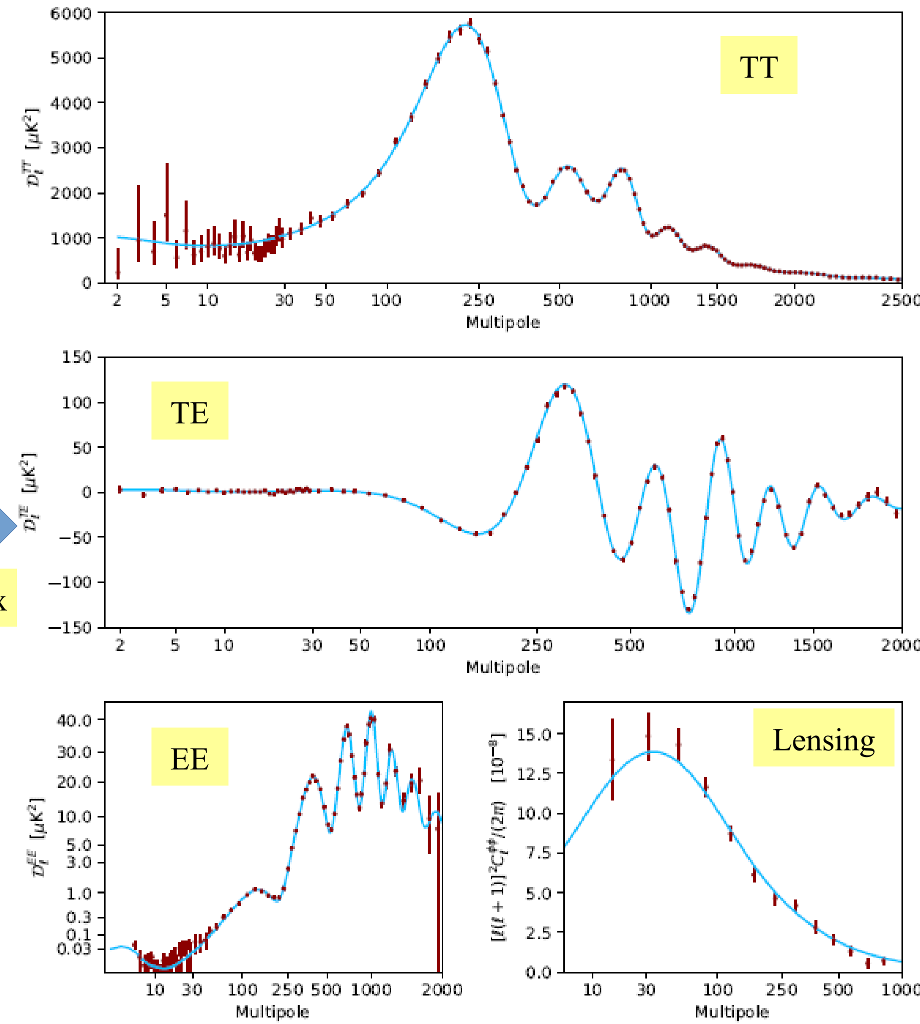


CMB anisotropies and spectrum

3) Planck 2018 CMB maps and power spectrum (1807.06205):



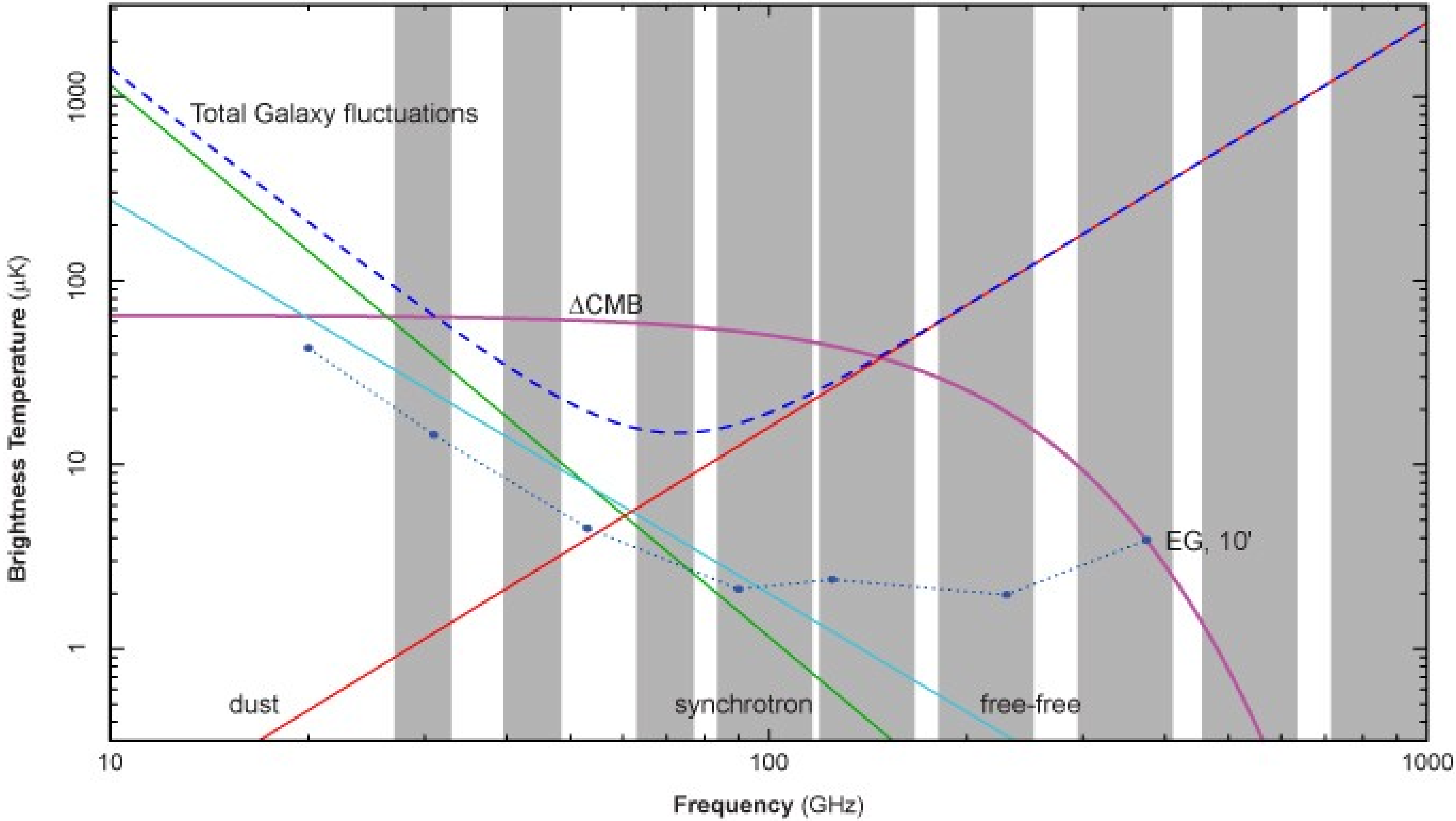
Healpix



CMB anisotropies and spectrum

4) Why all these frequencies (different channels)? The channels can be used to eliminate (subtract out) the Galactic noise!

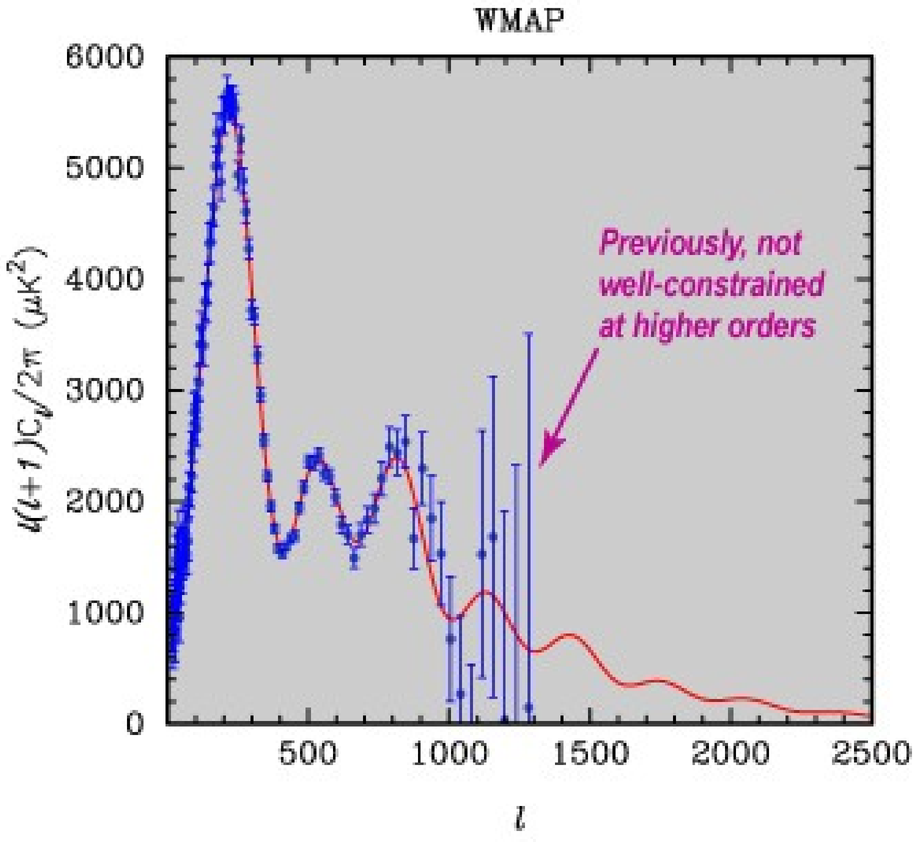
http://planck.caltech.edu

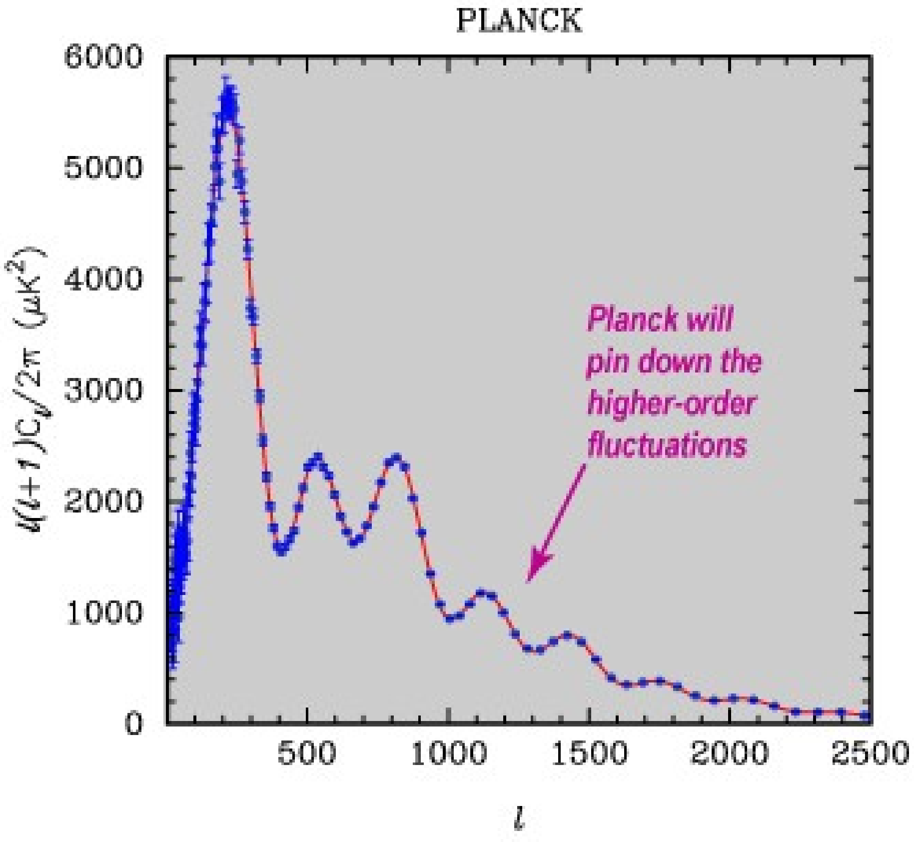


CMB anisotropies and spectrum

5) The extra channels also increase resolution, which means we can go to higher multipoles!

http://planck.caltech.edu





The CMB power spectrum

1) TT fluctuations, expand on Legendre polynomials and spherical harmonics

$$\Delta \equiv \Delta T/T$$

Temperature anisotropy = the CMB maps!

$$\Delta(\vec{x},\hat{n},\tau) = \int d^3k\, e^{i\vec{k}\cdot\vec{x}}\, \Delta(\vec{k},\hat{n},\tau) \equiv \int d^3k\, e^{i\vec{k}\cdot\vec{x}} \sum_{l=0}^{\infty} (-i)^l\,(2l+1)\,\Delta_l(\vec{k},\tau)\, P_l(\hat{k}\cdot\hat{n})$$

$$\Delta(\hat{n}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_{lm}(\hat{n})\ , \quad a_{lm} = (-i)^l\, 4\pi \int d^3k\, Y^*_{lm}(\hat{k})\,\Delta_l(\vec{k},\tau)$$

Legendre polynomials

Spherical harmonics

2) Properties of the Legendre polynomials

$$\int_{-1}^{1} dx P_\ell(x) P_{\ell'}(x) = \delta_{\ell\ell'} \frac{2}{2\ell+1}$$

$$(\ell+1)P_{\ell+1}(x) = (2\ell+1)xP_\ell(x) - \ell P_{\ell-1}(x)$$

$$-1 \leq\ x\ \leq 1$$

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \frac{3x^2-1}{2}$$

3) Properties of the Spherical harmonics

$$\int d\Omega Y^*_{\ell m}(\Omega) Y_{\ell' m'}(\Omega) = \delta_{\ell\ell'}\delta_{mm'}$$

$$P_\ell(\hat{x}\cdot\hat{x}') = \frac{4\pi}{2\ell+1}\sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{x}) Y^*_{\ell m}(\hat{x}')$$

$$Y_{0,0}(\theta,\phi) = \frac{1}{\sqrt{4\pi}}$$
$$Y_{1,\pm1}(\theta,\phi) = \mp i\sqrt{\frac{3}{8\pi}}\sin(\theta)e^{\pm i\phi}$$

The CMB power spectrum

4) Define two point correlation for the temperature anisotropy

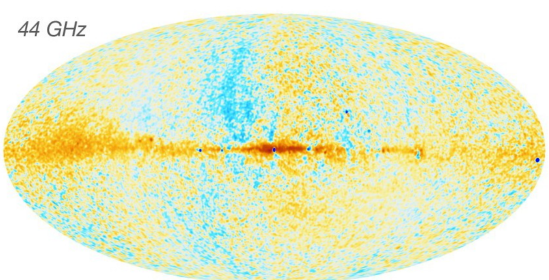
$$C(\theta) \equiv \langle \Delta(\hat{n}_1)\, \Delta(\hat{n}_2) \rangle = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1)\, C_l\, P_l(\hat{n}_1 \cdot \hat{n}_2) \;\Rightarrow\; \langle a_{lm} a^*_{l'm'} \rangle = C_l\, \delta_{ll'}\, \delta_{mm'}$$

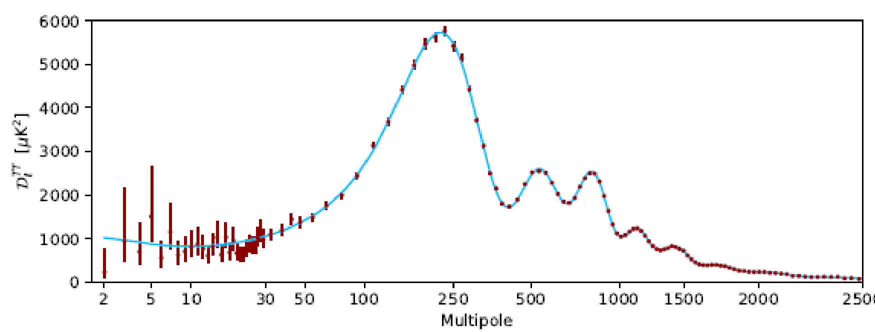
5) Consider initial perturbation

$$\Delta_l(\vec{k}, \tau) = \psi_i(\vec{k}\,)\, \Delta_l(k, \tau) \;\Rightarrow\; \langle \psi_i(\vec{k}_1)\, \psi_i(\vec{k}_2) \rangle = P_\psi(k)\, \delta_D(\vec{k}_1 + \vec{k}_2)$$

$$\Rightarrow\; C_l = 4\pi \int d^3k\, P_\psi(k)\, \Delta_l^2(k, \tau)$$

6) The Cls compress information! From 5*10^7 px (nside=2048, npix =12*nside^2) to ~2500 multipoles

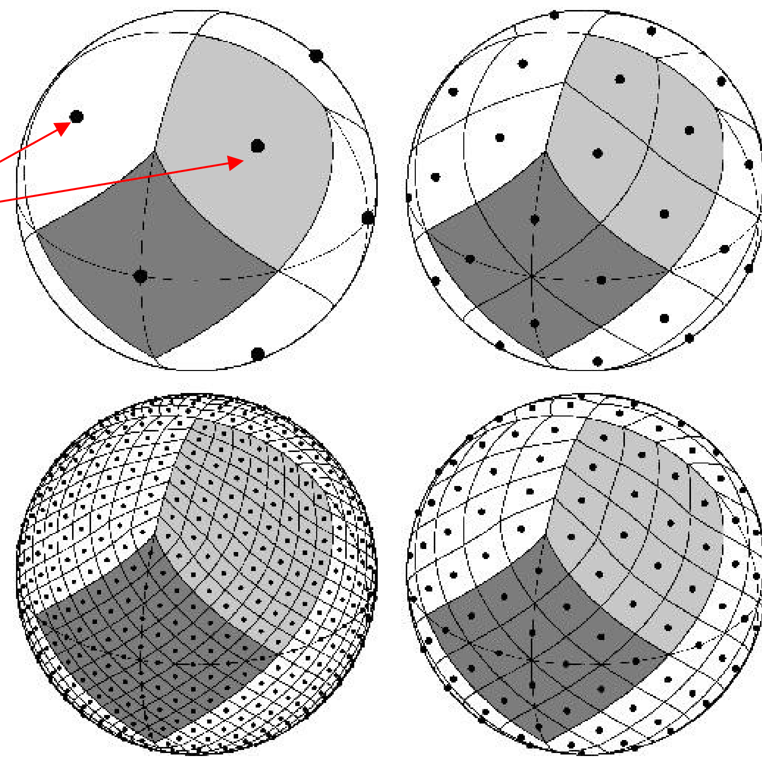




The CMB power spectrum

7) HEALPix (Hierarchical Equal Area isoLatitude Pixelisation). A 2-sphere is tessellated into curvilinear quadrilaterals with the lowest resolution 12 pixels and the resolution is increased by partitioning every pixel into 4 new.

$N_{\text{side}} = 1,\ 2,\ 4,\ 8$



Pixels are distributed on lines of constant latitude.

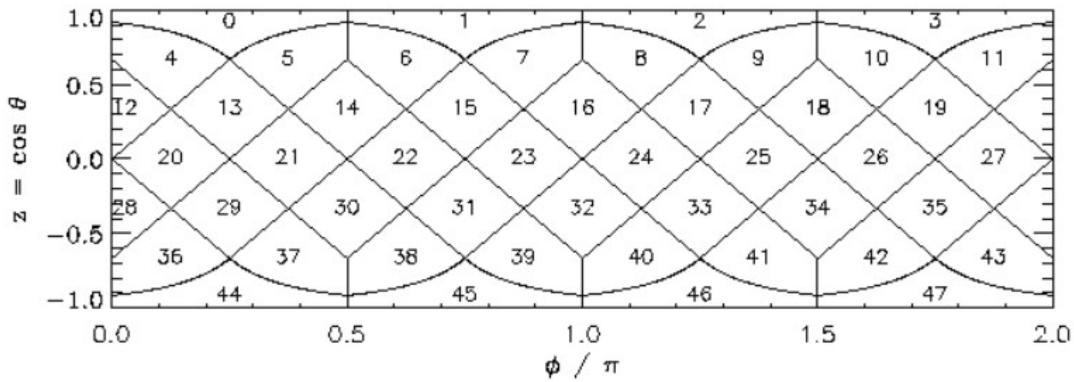
The sphere is partitioned, respectively, into 12, 48, 192, and 768 pixels.

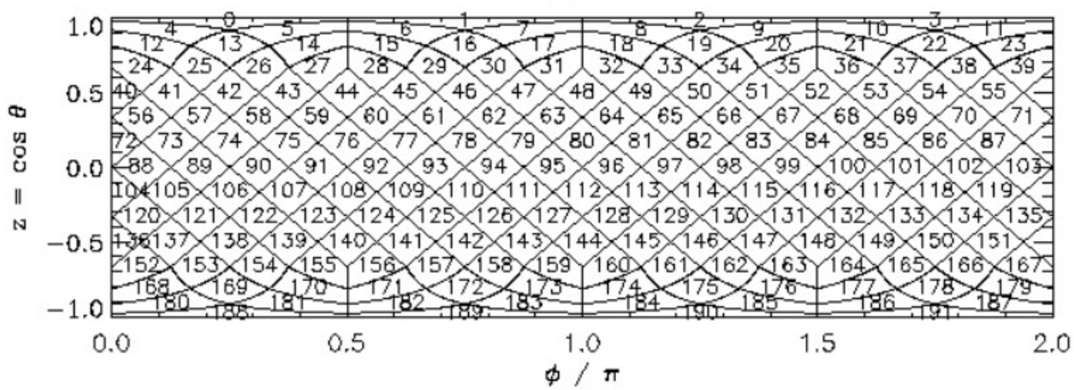
Areas of all pixels at a given resolution are identical!

$$N_{\text{pix}} = 12 \times N_{\text{side}}^2 = 12,\ 48,\ 192,\ 768.$$

The CMB power spectrum

8) HEALPix converts CMB maps to Cls!

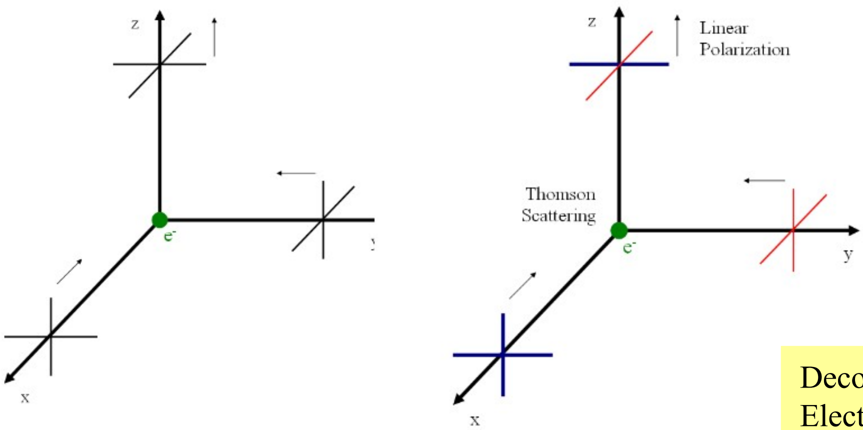




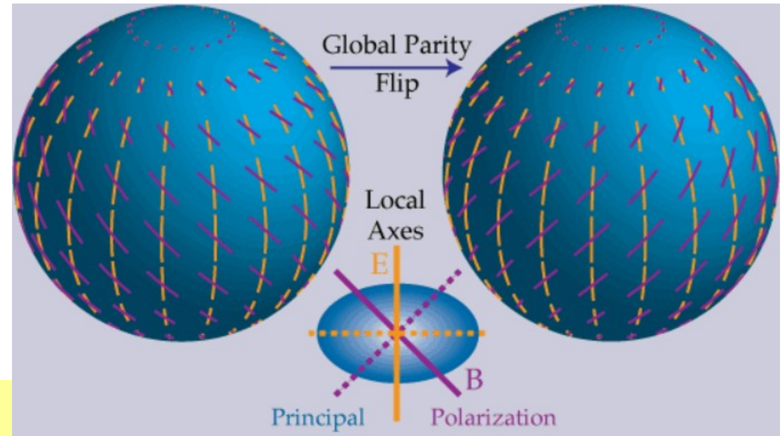
$$\hat{a}_{\ell m} = \frac{4\pi}{N_{\rm pix}} \sum_{p=0}^{N_{\rm pix}-1} Y^*_{\ell m}(\gamma_p) f(\gamma_p), \qquad \hat{C}_\ell = \frac{1}{2\ell+1} \sum_m |\hat{a}_{\ell m}|^2.$$

The CMB power spectrum

9) Polarization from Thomson scattering (E and B modes)

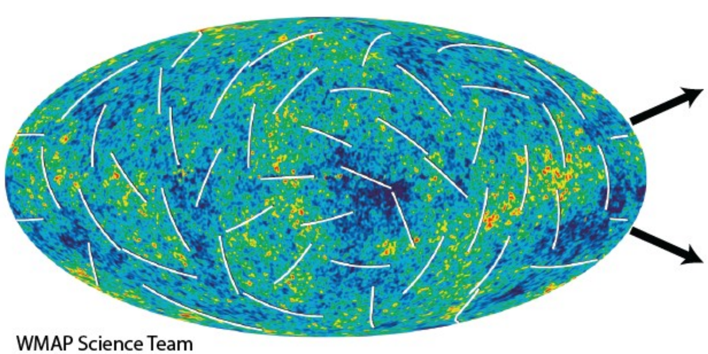


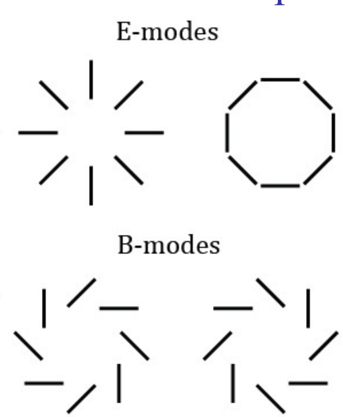
Decompose in Electric & Magnetic parts



10) E mode is caused by thermal over/under-densities

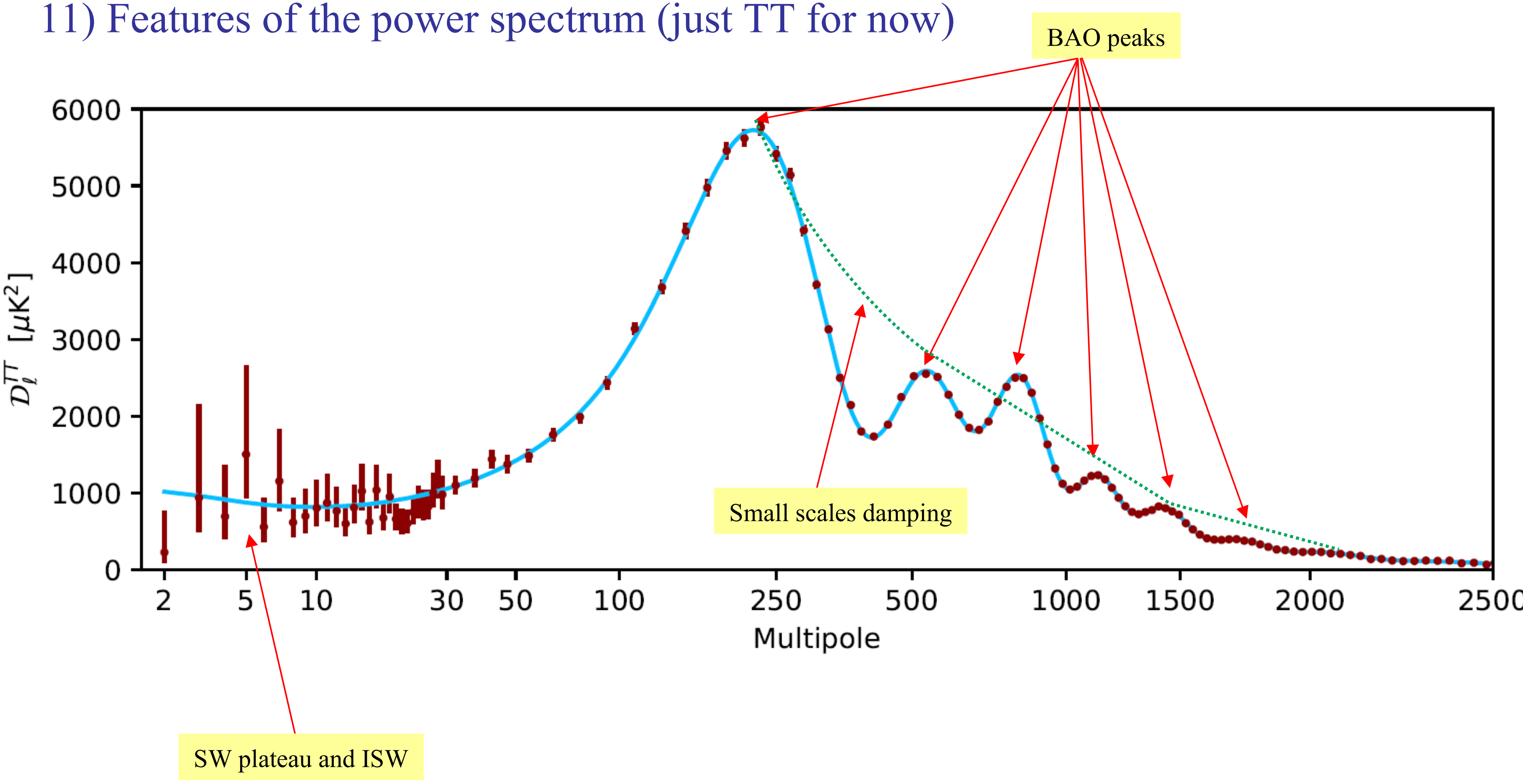
B mode is caused by GWs and dust (due to magnetic fields & imperfect alignment)!





CMB anisotropies and spectrum

11) Features of the power spectrum (just TT for now)



Features of the TT CMB spectrum

1) Baryon Acoustic Oscillations

Peaks at specific multipoles due to competition between baryons and photons

2) Diffusion damping

Damping at small scales (large l) due to increase in mean free path of photons

3) Primary anisotropies

i) Sachs-Wolfe effect (flat Cls for $l<30$)
ii) Adiabatic/isocurvature perturbations
iii) Doppler shift

4) Secondary anisotropies

i) Integrated Sachs-Wolfe effect (enhances anisotropies at $l<10$)
ii) Reionization at $z\sim10$

5) Cosmological parameters sensitivities

Features of CMB spectrum depend on parameters like Ω_m, Ω_b, Ω_{DE}, Ω_k, n_s etc

Baryon Acoustic Oscillations

1) Perturbation equations for Baryon-Photon plasma

$$\begin{aligned}
\dot{\delta}_\gamma &= -\frac{4}{3}\theta_\gamma + 4\dot{\phi}\,, \\
\dot{\theta}_\gamma &= k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) + k^2\psi + a n_e \sigma_T(\theta_b - \theta_\gamma)\,, \\
\dot{\delta}_b &= -\theta_b + 3\dot{\phi}\,, \\
\dot{\theta}_b &= -\frac{\dot{a}}{a}\theta_b + c_s^2 k^2 \delta_b + \frac{4\bar{\rho}_\gamma}{3\bar{\rho}_b} a n_e \sigma_T(\theta_\gamma - \theta_b) + k^2\psi
\end{aligned}$$

See Advanced Cosmo class (CMB Module) next semester!

$$\delta \equiv \delta\rho/\bar{\rho}$$

$$\theta = i k^j v_j$$

BAO_calculations.nb

2) Define S as below and eliminate all except δγ:

$$\delta_\gamma - 4\phi \equiv 4S$$

$$\ddot{S} + \frac{\dot{R}}{1+R}\dot{S} + k^2 c_s^2 S = \left(-\frac{k^2}{3}\psi - \frac{k^2}{3}\phi/(1+R)\right)$$

Damping term

Oscillatory term

"Driving force"

Baryon Acoustic Oscillations

3) Zero order approximate solution (ignore damping and force):

$$\ddot{S} + k^2 c_s^2 S \simeq 0 \quad \Rightarrow \quad S = A\cos(kr_s + \theta_0)$$

Sound horizon!

$$r_s = \int_0^\eta d\eta' c_s(\eta') \simeq c_s(\eta)\eta$$

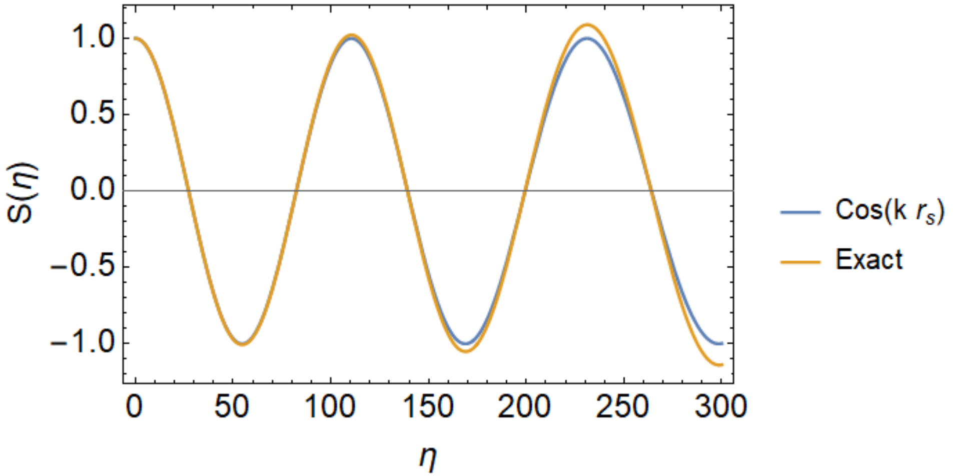
$$c_s^2 = \frac{1}{3(1+R)}$$

Sound speed of baryon-photon plasma

$$R = \frac{3}{4}\frac{\Omega_b}{\Omega_\gamma} = \frac{a^{-3}}{a^{-4}} R_0 = R_0 a$$

4) Comparison and location of the peaks

$$k_p = \frac{n\pi}{r_s}$$



BAO.nb

Baryon Acoustic Oscillations

5) More accurate comparison (D is distance to recombination)

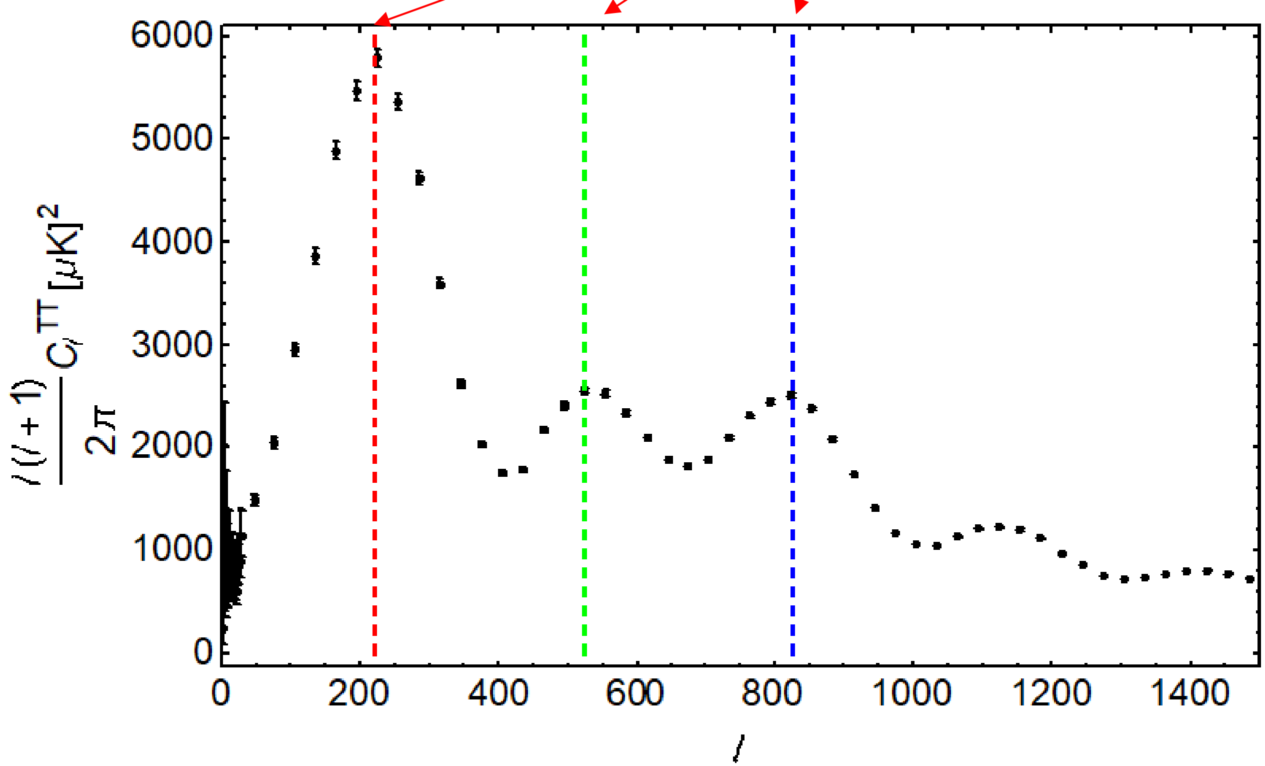
astro-ph/0006436

$$\ell_A \equiv \pi D/s_*$$

$$\ell_A \approx 172 d \left(\frac{z_*}{10^3}\right)^{1/2} \times \left(\frac{1}{\sqrt{R_*}} \ln \frac{\sqrt{1+R_*}+\sqrt{R_*+r_*R_*}}{1+\sqrt{r_*R_*}}\right)^{-1}$$

$$\ell_m = \ell_A(m-\phi)$$

$$\phi \approx 0.267 \left(\frac{r_*}{0.3}\right)^{0.1}$$



CAMB_plots.py

CMB_theory.nb

plot_cls.nb

Hot spots vs cold spots

1) Consider perturbed FRW metric with Newtonian potentials φ,ψ

astro-ph/9506072

$$ds^2 = a^2(\tau)\left\{-(1+2\psi)d\tau^2 + (1-2\phi)dx^i dx_i\right\}$$

2) Photon four momentum, given the FRW metric:

$$P^\mu = \left(a^{-1}p(1-\psi), a^{-1}p^i(1+\phi)\right) \Rightarrow P^0 = a^{-1}p(1-\psi) \sim \frac{1}{\lambda}$$

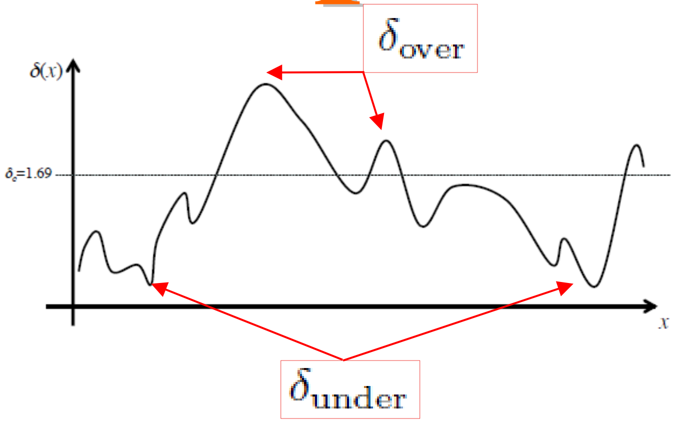
3) Einstein equations (0,0) and (i,j) parts give Poisson equations:

$$\begin{aligned} k^2\phi &= -4\pi G_N a^2 \rho_m \delta_m \\ \phi &= \psi \end{aligned} \Rightarrow \psi = -4\pi G_N \frac{a^2}{k^2}\rho_m \delta_m$$

Hot spots vs cold spots

4) Definition of over-density

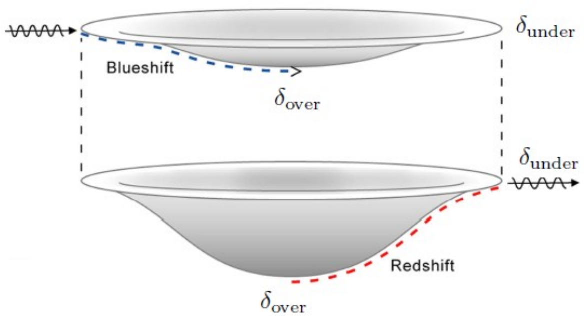
$$\delta_{\rm over} \gg \delta_{\rm under}$$



5) Given the above this translates to redshift for photon trying to escape

$$\delta_{\rm over} > \delta_{\rm under} \Rightarrow \psi_{\rm over} < \psi_{\rm under} \Rightarrow$$

$$P^0_{\rm over} > P^0_{\rm under} \Rightarrow \lambda_{\rm over} < \lambda_{\rm under}$$

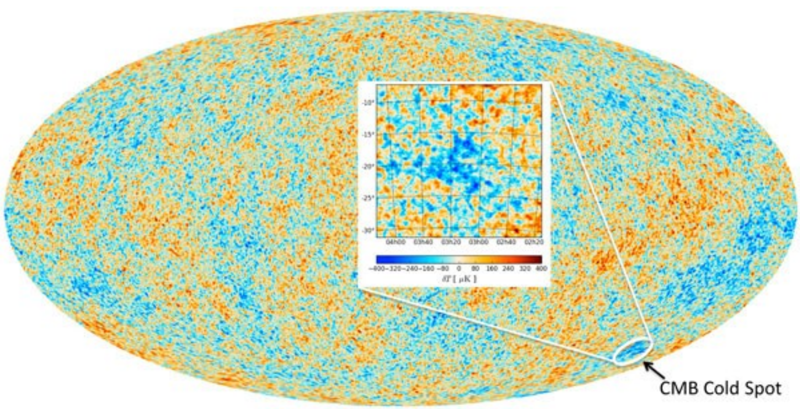


6) This leads to temperature decrease (coldspot) between overdensity and underdensity!

$$\frac{\Delta T}{T} \sim \frac{1}{3}\delta\psi \Rightarrow \frac{\Delta T}{T} < 0$$

$\Delta T = T_{\rm over} - T_{\rm under}$

$\delta\psi = \psi_{\rm over} - \psi_{\rm under} < 0$



Derivation of Sachs-Wolfe effect

1) SW effect→ photon escapes static potential. To zero order the SW effect contribution is a Spherical Bessel (derive or see Dodelson 8.6)

SW_cls.nb

$$\Delta(\hat{n},\tau_0) \approx \tfrac{1}{3}\psi(\vec{x} = -\vec{n}\chi, \tau_{\rm rec}) \quad \Rightarrow \quad \Delta_l(k,\tau) = \tfrac{1}{3} j_l(k\chi)$$

2) Assume power-law power spectrum

$$P_\psi(k) = A\chi^3 (k\chi)^{n-4} \propto k^{n-4} \quad \Rightarrow \quad C_l \approx \frac{2^n \pi^3}{9} A \frac{\Gamma(3-n)\,\Gamma\left(\frac{2l+n-1}{2}\right)}{\Gamma^2\left(\frac{4-n}{2}\right)\,\Gamma\left(\frac{2l+5-n}{2}\right)}$$

n=1

$$C_l \approx (8\pi^2/9) A/[l(l+1)]$$

3) Be careful with notation

$$C(\theta) = \left\langle \frac{\delta T^*}{T}(\mathbf{n}) \frac{\delta T}{T}(\mathbf{n}') \right\rangle_{\mathbf{n}\cdot\mathbf{n}' = \cos\theta} = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1)\, C_l\, P_l(\cos\theta)$$

JGB notes

$$\frac{\delta T}{T}(\theta,\phi) \; = \; \frac{1}{3}\Phi(\eta_{\rm LS})\,Q \; = \; \frac{1}{5}\mathcal{R}\,Q(\eta_0,\theta,\phi) \; \equiv \; \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm}\, Y_{lm}(\theta,\phi)\,,$$

$$\Phi = \tfrac{3}{5}\mathcal{R}$$

$$C_l^{(S)} = \frac{4\pi}{25} \int_0^\infty \frac{dk}{k} \mathcal{P}_{\mathcal{R}}(k)\, j_l^2(k\eta_0) \quad \Rightarrow$$

$$C_l^{(S)} = \frac{2\pi}{25} A_S^2 \frac{\Gamma[\frac{3}{2}]\,\Gamma[1 - \frac{n-1}{2}]\,\Gamma[l + \frac{n-1}{2}]}{\Gamma[\frac{3}{2} - \frac{n-1}{2}]\,\Gamma[l + 2 - \frac{n-1}{2}]}\,,$$

$$\frac{l(l+1)\,C_l^{(S)}}{2\pi} = \frac{A_S^2}{25} = \text{constant}\,, \qquad \text{for } n = 1$$

Derivation of Sachs-Wolfe effect

4) Similarly for tensors. They obey the following ODE:

JGB notes

$$h_k'' + 3\mathcal{H}\, h_k' + (k^2 + 2K)\, h_k = 0$$

5) The contribution in the spectrum is

SW_cls.nb

$$\frac{\delta T}{T}(\theta,\phi) = \int_{\eta_{\rm LS}}^{\eta_0} dr\, h'(\eta_0 - r)\, Q_{rr}(r,\theta,\phi)$$

$$Q_{kl}^{rr}(r) = \left[\frac{(l-1)l(l+1)(l+2)}{\pi k^2}\right]^{1/2} \frac{j_l(kr)}{r^2}$$

6) A similar calculation gives:

$$C_l^{(T)} = \frac{9\pi}{4}(l-1)l(l+1)(l+2) \int_0^\infty \frac{dk}{k}\mathcal{P}_g(k)\, I_{kl}^2\,,$$

$$I_{kl} = \int_0^{x_0} dx\, \frac{j_2(x_0 - x) j_l(x)}{(x_0 - x)x^2}\,,$$

→

$$l(l+1)\, C_l^{(T)} = \frac{\pi}{36}(1 + \frac{48\pi^2}{385})\, A_T^2\, B_l\,,$$

$$B_l = (1.1184, 0.8789, \ldots, 1.00) \quad \text{for } l = 2, 3, \ldots, 30.$$

The Integrated Sachs-Wolfe effect

1) ISW effect → photon escapes time-varying potential due to accelerated expansion caused by DE → late time effect at large scales (l<20)!

Dodelson 8.5.1

$$\frac{\Delta T}{T} \simeq \int_0^{\eta_0} \left(\dot{\phi} + \dot{\psi}\right) d\eta \quad \Longrightarrow \quad \Delta_\ell \simeq \int_0^{\eta_0} e^{-\tau} \left(\dot{\phi} + \dot{\psi}\right) j_\ell \left[k(\eta_0 - \eta)\right] d\eta$$

$$\tau = \int_{\eta_{rec}}^{\eta_0} d\eta n_e \sigma_\tau a(\eta)$$

Optical depth, see later

2) The Cls depend strongly on DE!

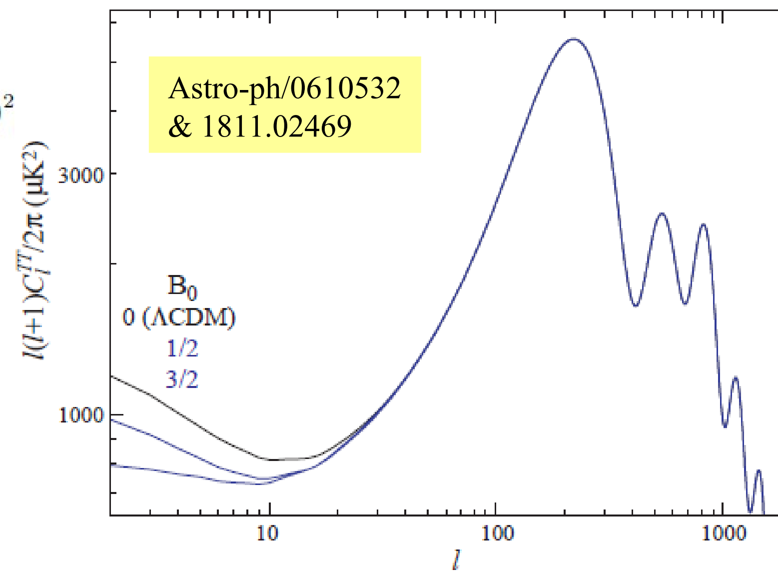
$$C_\ell^{\rm ISW} = 4\pi \int \frac{dk}{k} I_\ell^{\rm ISW}(k)^2 \frac{9}{25} \frac{k^3 P_\zeta}{2\pi^2}$$

$$\frac{k^3 P_\zeta}{2\pi^2} = A_s \left(\frac{k}{k_0}\right)^{n_s - 1} T(k)^2$$

Transfer function, see Eq (7.71) in Dodelson

$$I_\ell^{\rm ISW}(k) = 2 \int dz \frac{dG}{dz} j_\ell(k\, r(z))$$

$$G(a,k) = \frac{\Phi(a,k) + \Psi(a,k)}{\Phi(a_{ini},k) + \Psi(a_{ini},k)}$$



Other effects

1) Diffusion damping= Damping at small scales (large l) due to increase in mean free path of photons

$$\ddot{S} + \frac{\dot{R}}{1+R}\dot{S} + k^2 c_s^2 S = \left(-\frac{k^2}{3}\psi - \frac{k^2}{3}\phi/(1+R)\right)$$

Damping term

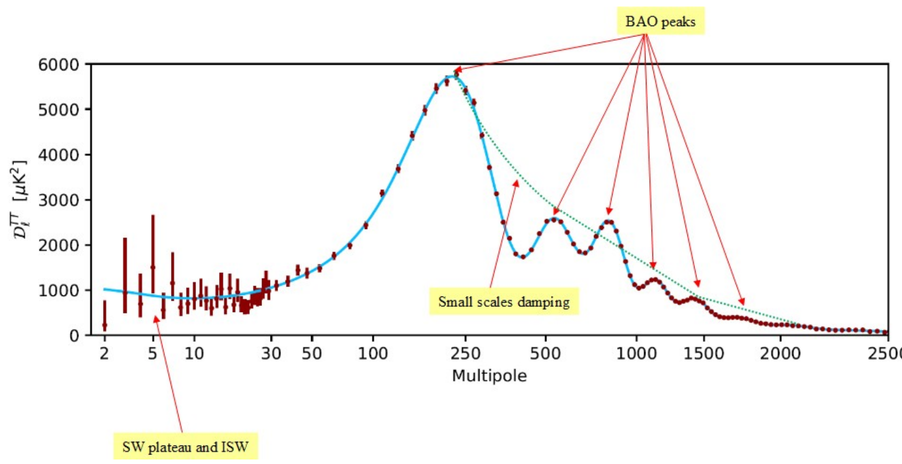
Oscillatory term

"Driving force"

2) Damping term gives rise to exponential suppression in Cls (Dodelson 8.4/pg 230)

$$\delta_\gamma \simeq \cos\left(k r_s(\tau)\right) e^{-\frac{k^2}{k_D^2}}$$

$$k_D^{-2} \equiv \int_0^\tau \frac{d\eta'}{6(1+R) n_e \sigma_T a(\eta')} \left[\frac{R^2}{1+R} + \frac{8}{9}\right]$$



Other effects

3) Adiabatic/isocurvature perturbations. Consider volume with equal distribution of matter and radiation. Two ways to perturb:

i) Change volume adiabatically (conserve entropy)→ number density the same

$$\delta_\gamma = \frac{\delta\rho_\gamma}{\rho_\gamma} = \frac{\delta n_\gamma}{n_\gamma} \xrightarrow{n_\gamma \sim T^3} \frac{\delta T}{T} = \delta_\gamma/3 \longrightarrow \delta_\gamma = 3\frac{\delta T}{T}$$

ii) Perturb entropy, keep energy density the same $\rho_m\delta_m=\rho_\gamma\,\delta_\gamma$:

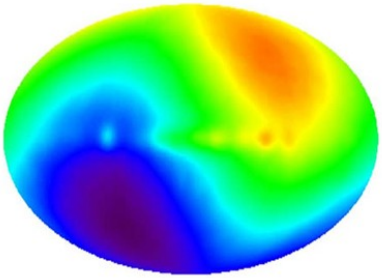
$$\delta_\gamma = 3\frac{\delta T}{T} + \text{const.}$$

4) Doppler shift (dipole):

i) Plasma had non-zero velocity at recombination

ii) Milky Way moves at 600km/h wrt CMB

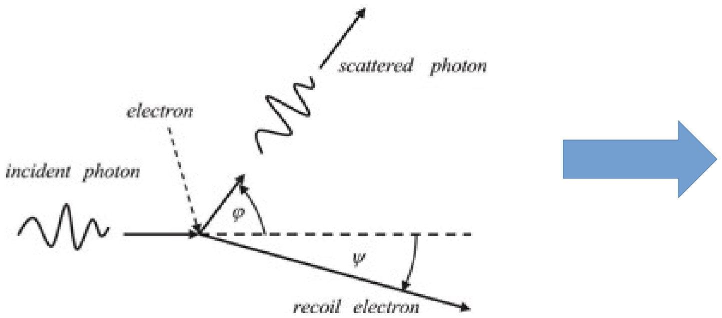
$$\frac{\delta T}{T}(\mathbf{r}) = -\frac{\mathbf{r}\cdot\mathbf{v}}{c} \longrightarrow$$



Other effects

5) Reionization at z~10:

From quazar spectra we know Universe reionized at z~[6,20] → more scattering with electrons (Thomson scattering). This affects modes within the horizon at the time of re-ionization or l>>1 (small scales) by reducing the Cls:



$$\Delta_\ell \rightarrow \Delta_\ell e^{-\tau}$$

$$\tau = \int_{\eta_{rec}}^{\eta_0} d\eta n_e \sigma_\tau a(\eta)$$

6) Cosmic Variance:

For each l we have 2l+1 a_{lm} coefficients, of which we can only predict the distribution not actual values, ie they are random variables

l=100 → 201 a_{lm} (good for statistics!)

$$a_{lm} = (-i)^l \, 4\pi \int d^3k \, Y^*_{lm}(\hat{k}) \, \Delta_l(\vec{k}, \tau)$$

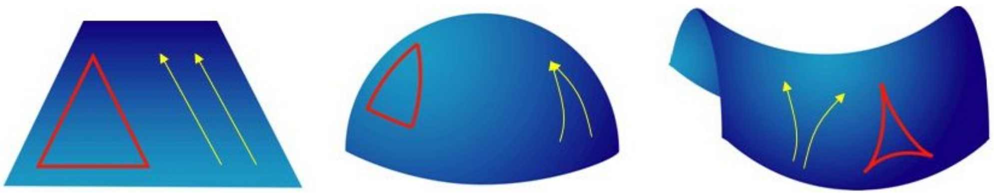
l=2 → 5 a_{lm} (not good for statistics!)

$$\langle a_{lm} a^*_{l'm'} \rangle = C_l \, \delta_{ll'} \, \delta_{mm'}$$

Cosmological parameters

1) Curvature changes distances:

$$d_A = \frac{1}{1+z}\frac{c}{H_0\sqrt{\Omega_K^{(0)}}}\sinh\left(\sqrt{\Omega_K^{(0)}}\int_0^z \frac{d\tilde{z}}{E(\tilde{z})}\right)$$



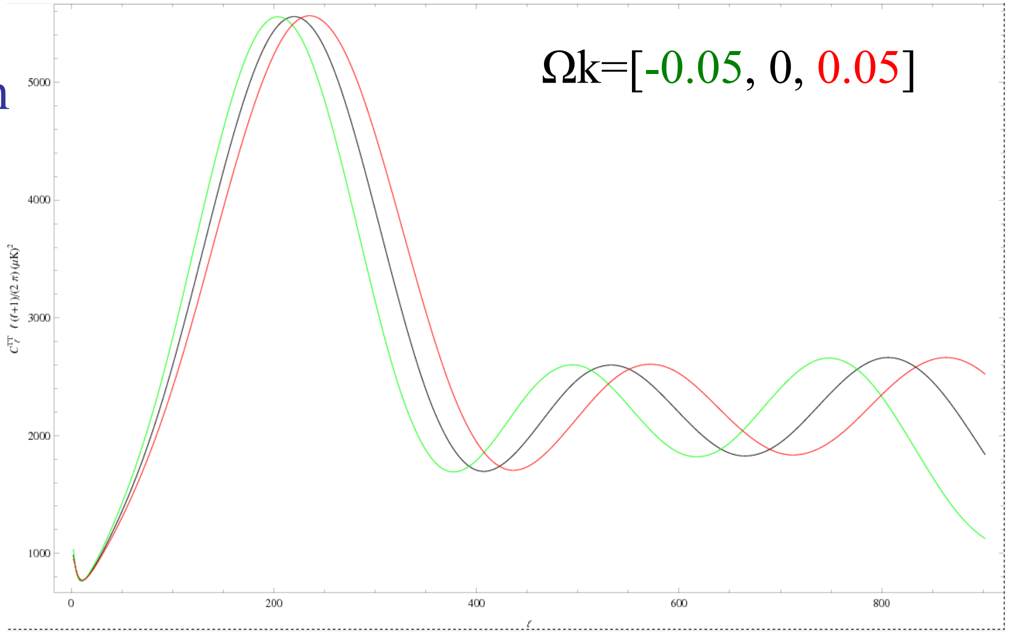
Main effect is on the location of the 1st peak ~distance to recombination

$$k_p = \frac{n\pi}{r_s}$$

$$\ell_A \equiv \pi D/s_*$$

$$\ell_m = \ell_A(m-\phi)$$

$$\phi \approx 0.267\left(\frac{r_*}{0.3}\right)^{0.1}$$



Cosmological parameters

2) Spectral index n_s affects normalization

$$C_l = 4\pi \int d^3k \, P_\psi(k) \, \Delta_l^2(k,\tau)$$

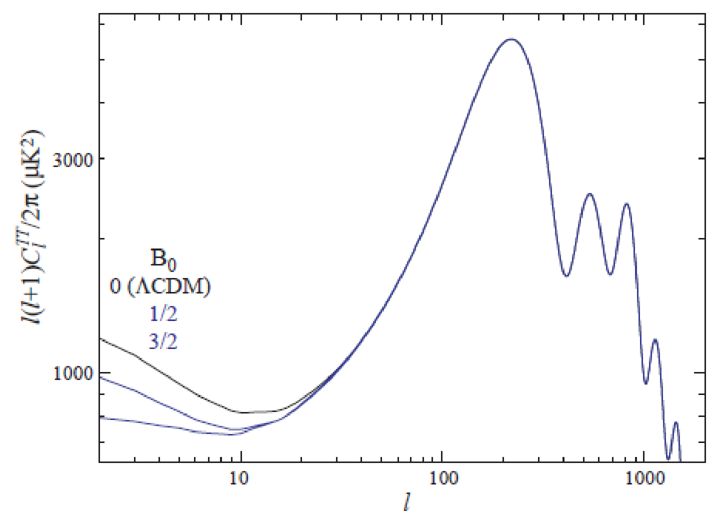
$$P_\psi \sim k^{n_s - 1}$$

$$\frac{C_\ell(n_s)}{C_\ell(n_s = 1)} \simeq \left(\frac{\ell}{\ell_0}\right)^{n_s - 1}$$

3) Dark energy→ late time effect (z<1) at large scales (l<10) → ISW effect

$$C_\ell^{\rm ISW} = 4\pi \int \frac{dk}{k} I_\ell^{\rm ISW}(k)^2 \frac{9}{25} \frac{k^3 P_\zeta}{2\pi^2}$$

$$I_\ell^{\rm ISW}(k) = 2 \int dz \frac{dG}{dz} j_\ell(k\, r(z))$$



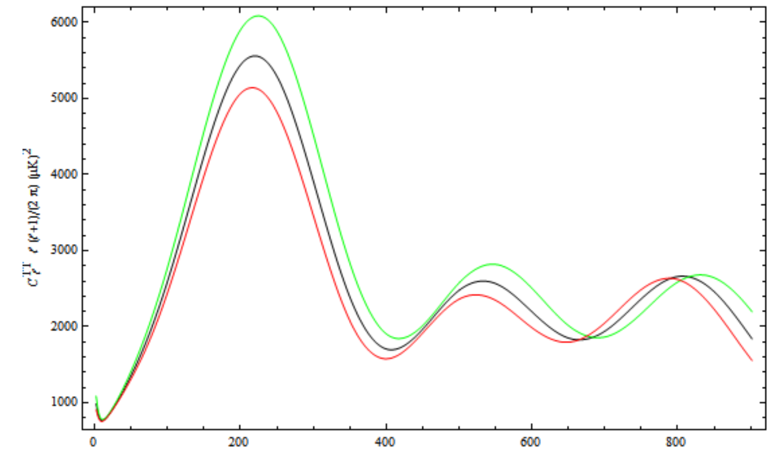
Cosmological parameters

4) Ω_m affects DM potentials
(deeper potentials → less BAO)

$$\psi = -4\pi G_N \frac{a^2}{k^2}\rho_m \delta_m$$

Ωm=[0.2038, 0.2538, 0.3038] and H0=70

Ωmh^2=[0.099862, 0.124362, 0.148862]



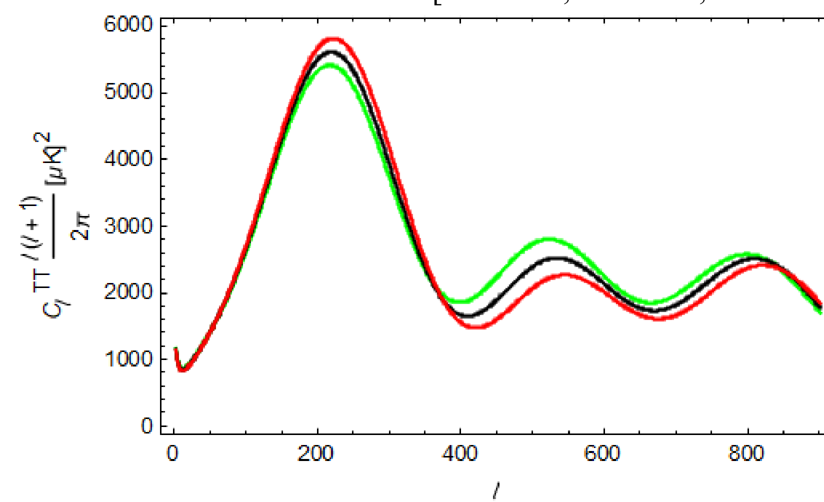
5) Ω_b affects height of peaks

$$\ddot{S} + \frac{\dot{R}}{1+R}\dot{S} + k^2 c_s^2 S = \left(-\frac{k^2}{3}\psi - \frac{k^2}{3}\phi/(1+R)\right)$$

$$R = \frac{3}{4}\frac{\Omega_b}{\Omega_\gamma} = \frac{a^{-3}}{a^{-4}}R_0 = R_0 a$$

Ωb=[0.0362, 0.0462, 0.0562] and H0=70

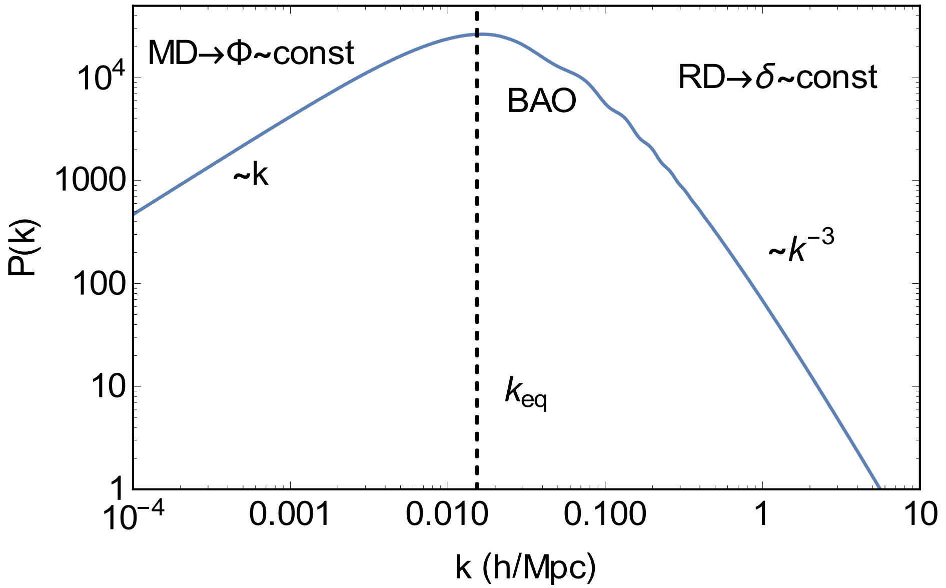
Ωbh^2=[0.017738, 0.022638, 0.027538]



Behaviour of the power spectrum P(k)

1) Matter power spectrum is an important quantity, affecting the CMB

$$P(k) \equiv \langle |\delta_k|^2 \rangle$$



2) The potential can be written as

$$\Phi(k,a) = \Phi_p(k) \times T(k) \times \delta(a)$$

Initial value from inflation

$$\langle \Phi_p^2 \rangle \sim k^{-3} k^{n_s - 1}$$

Transfer function

$$T(k) = \frac{\Phi(k, a_{\rm late})}{\Phi_{\rm large}(k, a_{\rm late})}$$

$$k_{eq} = 0.073 \Omega_{m0} h \; (h/\mathrm{Mpc})$$

3) With these, express P(k) as

$$\begin{aligned} P(k) &= \langle \delta_k^2 \rangle \\ &= k^4 \langle \Phi_p^2 \rangle T(k)^2 \delta(a)^2 \\ &\sim k^4 k^{-3} k^{n_s - 1} T(k)^2 \\ &\sim k^{n_s} T(k)^2 \end{aligned}$$

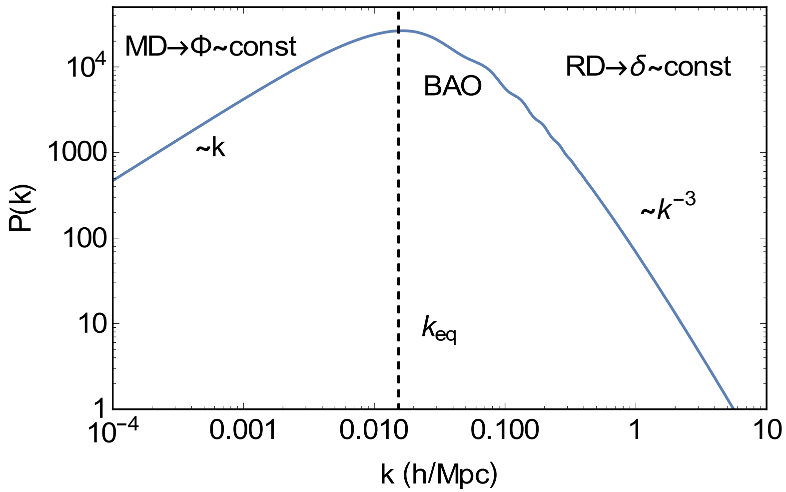
$$k >> k_{eq} \rightarrow \delta \sim const \rightarrow \Phi \sim 1/k^2 \rightarrow T \sim 1/k^2 \rightarrow P(k) \sim k^{-3}$$

$$k << k_{eq} \rightarrow \Phi \sim const \rightarrow \delta \sim k^2 \rightarrow T \sim 1 \rightarrow P(k) \sim k$$

Behaviour of the power spectrum P(k)

4) Behavior of transfer function

$$T(k) = \begin{cases} 1/k^2, & k >> k_{eq} \\ 1, & k << k_{eq} \end{cases}$$



5) Behavior of P(k)

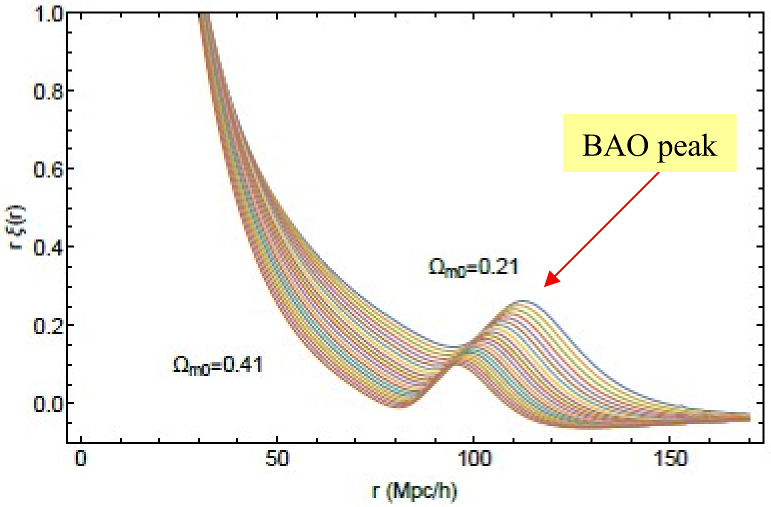
$$P(k) = \begin{cases} 1/k^3, & k >> k_{eq} \\ k, & k << k_{eq} \end{cases}$$

$$k_{eq} = 0.073\Omega_{m0}h \ (h/\mathrm{Mpc})$$

6) Fourier transform of P(k) is ξ(r),
the correlation function (~prob of galaxies at r)

$$\xi(r) = \frac{1}{2\pi^2}\int_0^\infty P(k) j_0(kr) k^2 dk$$

$$\xi(r) = r^{-n-3}, \ n = (1, -3)$$



Discussion of Planck papers

1) Main Planck papers (for us!): 1807.06205, 1807.0629, 1807.06211

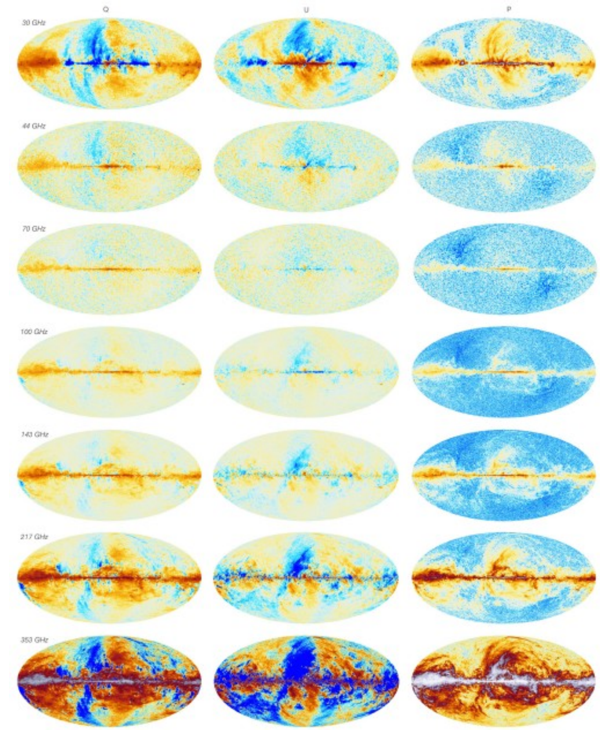
***Planck* 2018 results. I.**
Overview, and the cosmological legacy of *Planck*

***Planck* 2018 results. VI. Cosmological parameters**

***Planck* 2018 results. X. Constraints on inflation**

2) Main characteristics and frequencies

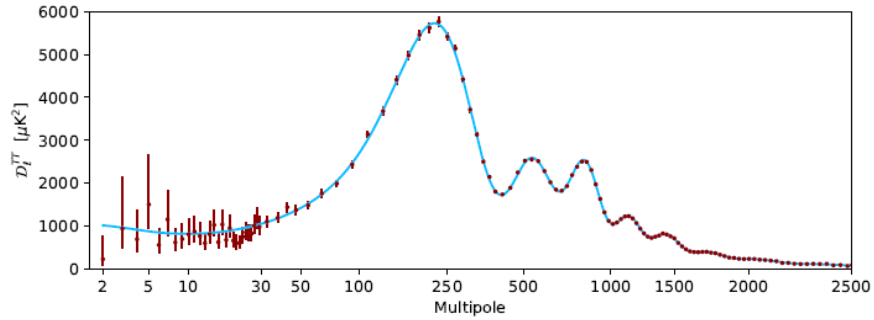
Property	Frequency [GHz] 30	44	70	100	143	217	353	545	857
Frequency [GHz][a]	28.4	44.1	70.4	100	143	217	353	545	857
Effective beam FWHM [arcmin][b]	32.29	27.94	13.08	9.66	7.22	4.90	4.92	4.67	4.22
Temperature Sensitivity [μK_{CMB} deg][c]	2.5	2.7	3.5	1.29	0.55	0.78	2.56		
[kJy sr^{-1} deg][c]								0.78	0.72
Polarization Sensitivity [μK_{CMB} deg][c]	3.5	4.0	5.0	1.96	1.17	1.75	7.31		
Dipole-based calibration uncertainty [%][d]	0.17	0.12	0.20	0.008	0.021	0.028	0.024	~1	
Planet submm inter-calibration accuracy [%][e]	...	...	...	...	...	...	...	...	~3
Temperature transfer function uncertainty [%][f]	0.25	0.11	Ref.	Ref.	0.12	0.36	0.78	4.3	
Polarization calibration uncertainty [%][g]	< 0.01 %	< 0.01 %	< 0.01 %	1.0	1.0	1.0	...	...	...
Zodiacal emission monopole level [μK_{CMB}][h]	0	0	0	0.43	0.94	3.8	34.0	...	...
[MJy sr^{-1}][h]	...	...	...	...	...	...	...	0.04	0.12
LFI zero level uncertainty [μK_{CMB}][i]	±0.7	±0.7	±0.6	...	...	...	...	...	...
HFI Galactic emission zero level uncertainty [MJy sr^{-1}][j]	...	...	...	±0.0008	±0.0010	±0.0024	±0.0067	±0.0165	±0.0147
HFI CIB monopole assumption [MJy sr^{-1}][k]				0.0030	0.0079	0.033	0.13	0.35	0.64
HFI CIB zero level uncertainty [MJy sr^{-1}][l]	...	...	...	±0.0031	±0.0057	±0.016	±0.038	±0.066	±0.077



Discussion of Planck papers

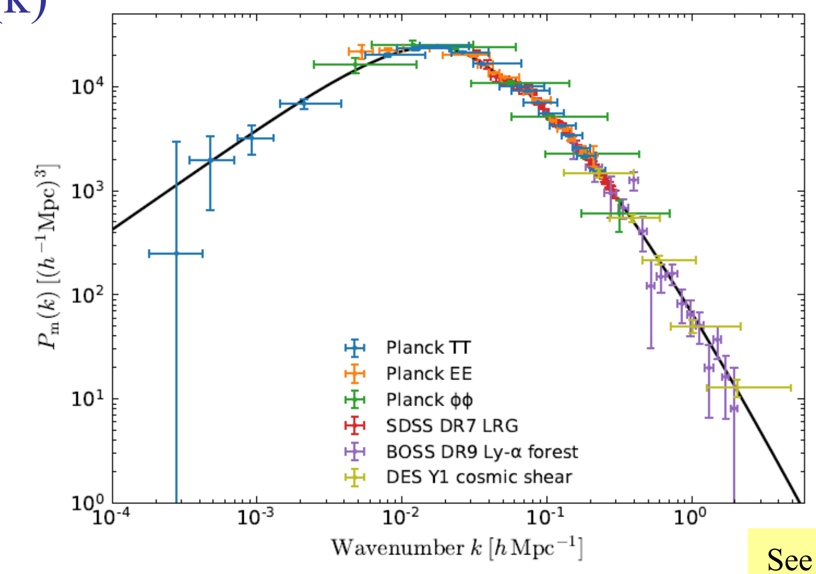
3) Position of peaks

Extremum	Multipole	Amplitude $[\mu K^2]$
***TT* power spectrum**		
Peak 1	220.6 ± 0.6	5733 ± 39
Trough 1	416.3 ± 1.1	1713 ± 20
Peak 2	538.1 ± 1.3	2586 ± 23
Trough 2	675.5 ± 1.2	1799 ± 14
Peak 3	809.8 ± 1.0	2518 ± 17
Trough 3	1001.1 ± 1.8	1049 ± 9
Peak 4	1147.8 ± 2.3	1227 ± 9
Trough 4	1290.0 ± 1.8	747 ± 5
Peak 5	1446.8 ± 1.6	799 ± 5
Trough 5	1623.8 ± 2.1	399 ± 3
Peak 6	1779 ± 3	378 ± 3
Trough 6	1919 ± 4	249 ± 3
Peak 7	2075 ± 8	227 ± 6
Trough 7	2241 ± 24	120 ± 6



4) Six-parameter ΛCDM model and P(k)

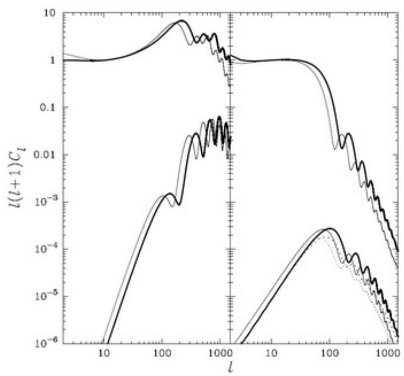
Parameter	*Planck* alone	*Planck* + BAO
$\Omega_b h^2$	0.022383	0.022447
$\Omega_c h^2$	0.12011	0.11923
$100\theta_{MC}$	1.040909	1.041010
τ	0.0543	0.0568
$\ln(10^{10} A_s)$	3.0448	3.0480
n_s	0.96605	0.96824
H_0 [km s^{-1}Mpc^{-1}] . . .	67.32	67.70
Ω_Λ	0.6842	0.6894
Ω_m	0.3158	0.3106
$\Omega_m h^2$	0.1431	0.1424
$\Omega_m h^3$	0.0964	0.0964
σ_8	0.8120	0.8110
$\sigma_8(\Omega_m/0.3)^{0.5}$	0.8331	0.8253
z_{re}	7.68	7.90
Age [Gyr]	13.7971	13.7839



See Planck papers for more details!

Boltzmann codes

Calculating all the previous stuff is tedious!
There are a few codes though (CAMB, CLASS etc)



Code for Anisotropies in the Microwave Background

by Antony Lewis and Anthony Challinor

Pros: Code in f90, fast, recently updated, forum support.
Cons: Code in f90, not very modular…

Code in C++, recently updated, very modular
Documentation a bit confusing sometimes

For now we focus in CLASS

Quick tour of CLASS

Compile/run CLASS (Reqs: make, gcc)

The various files (.c, .ini etc)

The equations are in both synchronous & conformal Newtonian gauge (see Advanced Cosmo next semester).

Quick tour of CLASS

1) Get CLASS from :
https://lesgourg.github.io/class_public/class.html
https://github.com/lesgourg/class_public

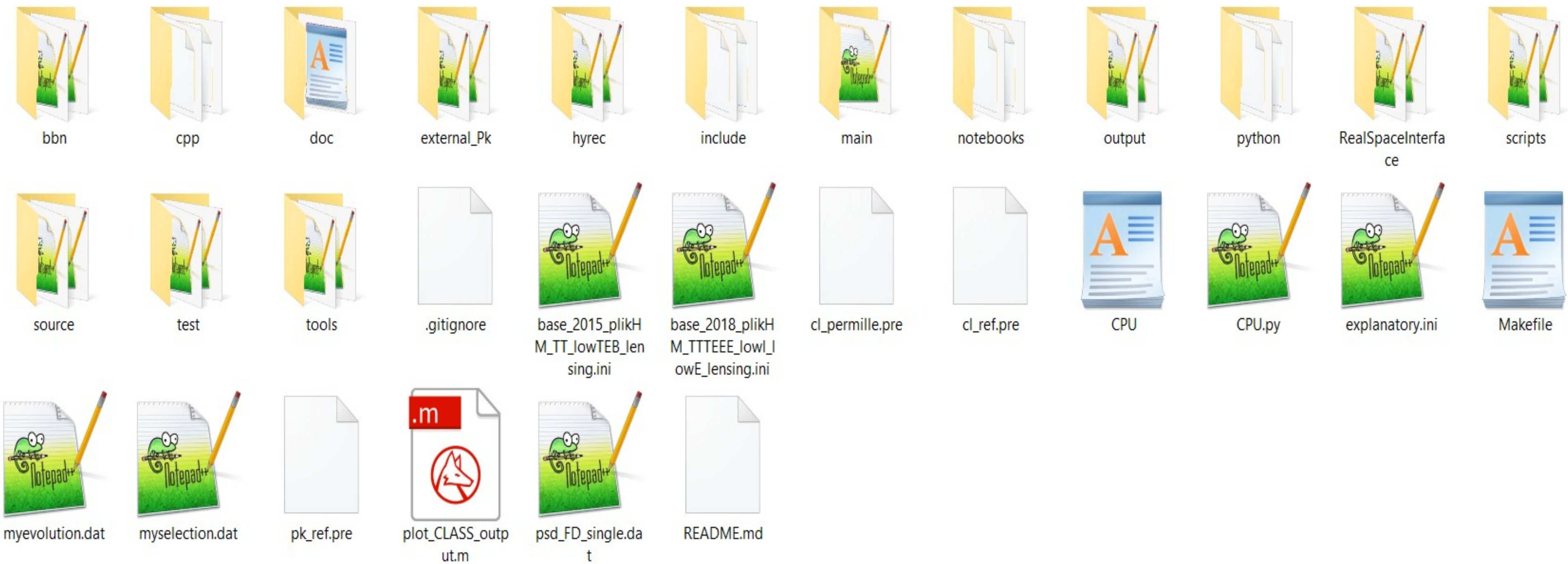
2) Unzip with a tool (WinZip, 7 Zip etc) or on Macs, Linux:
tar xfv CLASS.tar.gz

3) Navigate to the CLASS directory and have a look at the files

i) cd CLASS

ii) on Windows just navigate to the folder!

Quick tour of CLASS



\source: Folder with c files that numerically solve CMB equations

explanatory.ini: File with cosmological parameters

\doc: Folder with manual you **SHOULD read!!! DISCUSS**

***.pre: Files with higher-precision settings**

\bbn, \main, \tools, \hyrec, \python: Folders with Utilities

\output: output files are saved, python

Makefile: Main compiler options

Quick tour of CLASS

background.c input.c lensing.c nonlinear.c output.c perturbations.c primordial.c spectra.c thermodynamics.c transfer.c

background.c: Solves background aka Friedmann equations.

input.c: Reads the parameters from the ini files.

lensing.c: Applies CMB lensing to spectra.

nonlinear.c: Applies non-linear corrections to P(k) at k>0.1 h/Mpc.

output.c: Writes the final spectra to txt files.

perturbations.c: Solves perturbation equations!!!

primordial.c: Contains the primordial power spectrum P(k) from inflation.

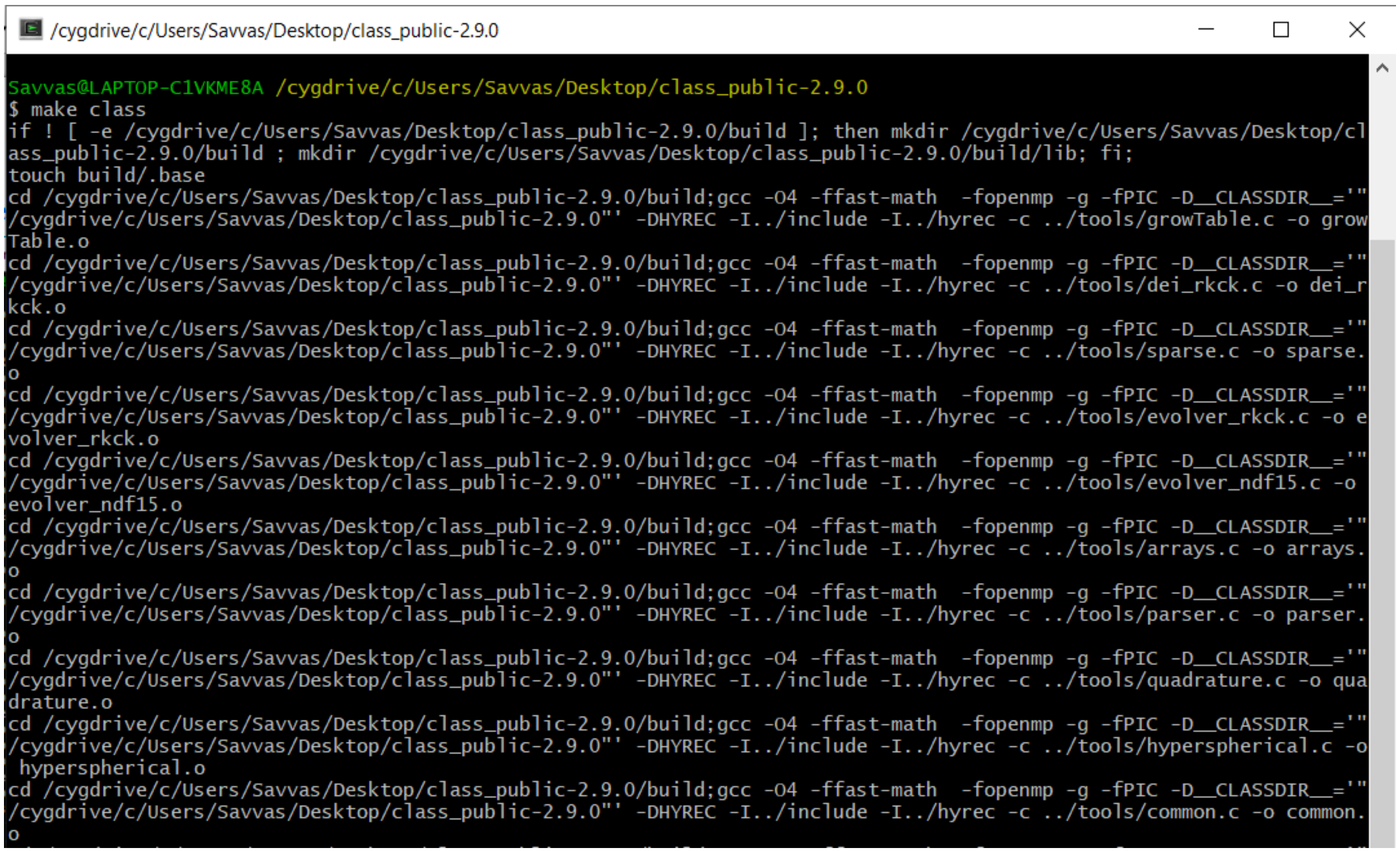
spectra.c : Calculates the spectra Cls.

thermodynamics.c: Does the recombination stuff etc.

transfer.c: Calculates transfer functions T(k).

Quick tour of CLASS

Run: make class



Compilation

-O4: Optimization O, O2, O3,O4

-fopenmp: parallelization (export OMP_NUM_THREADS=4)

-ffast-math: do fast math optimizations!

Quick tour of CLASS

Run: ./class ./explanatory.ini

/cygdrive/c/Users/Savvas/Desktop/class_public-2.9.0

```
Savvas@LAPTOP-C1VKME8A /cygdrive/c/Users/Savvas/Desktop/class_public-2.9.0
$ ./class.exe ./explanatory_m.ini
Reading input parameters
 -> matched budget equations by adjusting Omega_Lambda = 7.080411e-01
Running CLASS version v2.9.0
Computing background
 -> age = 13.565089 Gyr
 -> conformal age = 14045.490063 Mpc
 -> pba->Neff = 3.046000
 -> radiation/matter equality at z = 3417.449671
    corresponding to conformal time = 112.322062 Mpc
 ---------------------------- Budget equation -----------------------
 ---> Nonrelativistic Species
-> Bayrons                          Omega = 0.0462          , omega = 0.022638
-> Cold Dark Matter                 Omega = 0.245673        , omega = 0.12038
 ---> Relativistic Species
-> Photons                          Omega = 5.0469e-05      , omega = 2.47298e-05
-> Ultra-relativistic relics        Omega = 3.49129e-05     , omega = 1.71073e-05
 ---> Other Content
-> Cosmological Constant            Omega = 0.708041        , omega = 0.34694
 ---> Total budgets
 Radiation                          Omega = 8.53818e-05     , omega = 4.18371e-05
 Non-relativistic                   Omega = 0.291873        , omega = 0.143018
 Other Content                      Omega = 0.708041        , omega = 0.34694
 TOTAL                              Omega = 1               , omega = 0.49
 ---------------------------------------------------------------------
Computing thermodynamics with Y_He=0.2455
 -> recombination at z = 1088.551266 (max of visibility function)
    corresponding to conformal time = 280.411071 Mpc
    with comoving sound horizon = 144.229573 Mpc
    angular diameter distance = 12.633714 Mpc
    and sound horizon angle 100*theta_s = 1.047793
    Thomson optical depth crosses one at z_* = 1081.158356
    giving an angle 100*theta_* = 1.052587
 -> baryon drag stops at z = 1060.493214
    corresponding to conformal time = 285.911598 Mpc
    with comoving sound horizon rs = 146.710586 Mpc
 -> reionization with optical depth = 0.094851
    corresponding to conformal time = 4246.795541 Mpc
Computing sources
Computing primordial spectra (analytic spectrum)
No Fourier spectra nor nonlinear corrections requested. Nonlinear module skipped.
Computing transfers
Computing unlensed harmonic spectra
Computing lensed spectra (fast mode)
Writing output files in output/test_m...
```

explanatory.ini File containing the cosmological parameters etc

Discuss the file!

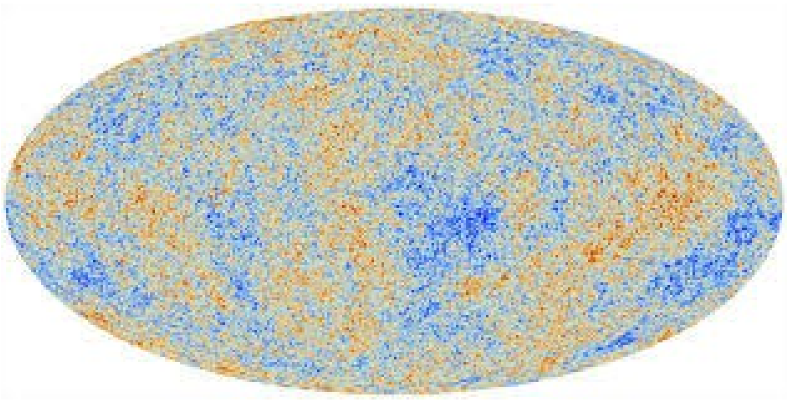
← Various results

explanatory00_cl.dat

```
# dimensionless total [l(l+1)/2pi] C_l's
# for l=2 to 3000, i.e. number of multipoles equal to 2999
#
# -> if you prefer output in CAMB/HealPix/LensPix units/order, set 'format' to 'camb' in input file
# -> if you don't want to see such a header, set 'headers' to 'no' in input file
# -> for CMB lensing (phi), these are C_l^phi-phi for the lensing potential.
#    Remember the conversion factors:
#    C_l^dd (deflection) = l(l+1) C_l^phi-phi
#    C_l^gg (shear/convergence) = 1/4 (l(l+1))^2 C_l^phi-phi
#
# 1:l     2:TT                 3:EE                 4:TE                 5:BB
   2      1.495437883318e-10   7.309026172212e-15   4.815171198139e-13   0.000000000000e+00
   3      1.409365941810e-10   1.238791651610e-14   6.068719691435e-13   0.000000000000e+00
   4      1.324622779164e-10   1.446512635826e-14   6.277976520644e-13   0.000000000000e+00
   5      1.257504771174e-10   1.337853519781e-14   5.879551646450e-13   0.000000000000e+00
   6      1.208053896034e-10   1.032310747877e-14   5.173990840385e-13   0.000000000000e+00
   7      1.173297567911e-10   6.841600365737e-15   4.365335860679e-13   0.000000000000e+00
   8      1.150037082721e-10   4.042374146265e-15   3.576623509953e-13   0.000000000000e+00
   9      1.135829021421e-10   2.336357197455e-15   2.880590507426e-13   0.000000000000e+00
  10      1.128526855214e-10   1.571653533762e-15   2.317799678437e-13   0.000000000000e+00
  11      1.126728139692e-10   1.345905907873e-15   1.905574136941e-13   0.000000000000e+00
  12      1.129346488028e-10   1.288312421471e-15   1.640636552601e-13   0.000000000000e+00
  13      1.135181551011e-10   1.200674797498e-15   1.505300359639e-13   0.000000000000e+00
  14      1.143248480530e-10   1.054790799694e-15   1.474046702054e-13   0.000000000000e+00
  15      1.153611533224e-10   9.105397353921e-16   1.516730195347e-13   0.000000000000e+00
  16      1.165881733448e-10   8.293959335366e-16   1.602914704804e-13   0.000000000000e+00
  17      1.179228800749e-10   8.287731570392e-16   1.707806930138e-13   0.000000000000e+00
  18      1.193693326096e-10   8.811664402440e-16   1.812740695758e-13   0.000000000000e+00
  19      1.209354684649e-10   9.538926660639e-16   1.907919252756e-13   0.000000000000e+00
```

The Cls and the correlation function

Result is txt files with the Cls...



$$T(\vec{x},\hat{p},\eta) = T(\eta)\left[1+\Theta(\vec{x},\hat{p},\eta)\right]$$

$$\Theta(\vec{x},\hat{p},\eta) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} a_{lm}(\vec{x},\eta)Y_{lm}(\hat{p})$$

$$\langle a_{lm}\rangle = 0 \qquad ; \qquad \langle a_{lm}a^*_{l'm'}\rangle = \delta_{ll'}\delta_{mm'}C_l$$

… and the matter power spectrum P(k)

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x})-\langle\rho\rangle}{\langle\rho\rangle} \qquad P(k) \equiv \langle|\delta_k|^2\rangle$$

$$\xi(\vec{r}) \equiv \langle\delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle$$

$$\xi(r) = \frac{1}{(2\pi)^3}\int P(k)\frac{\sin(kr)}{kr}\,4\pi k^2 dk$$

Correlation function:
Denotes probability to find
galaxy at position r

The variables and the equations

For more details see Advanced Cosmo class next semester!

1) Friedman equations in GR (dot is conformal time)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G a^2 \bar{\rho} - \kappa \,,$$
$$\frac{d}{d\tau}\left(\frac{\dot{a}}{a}\right) = -\frac{4\pi}{3} G a^2 (\bar{\rho} + 3\bar{P}) \,,$$

2) More than one ways to perturb the FRW metric!

i) Conformal Newtonian gauge: $ds^2 = a^2(\tau)\left\{-(1+2\psi)d\tau^2 + (1-2\phi)dx^i dx_i\right\}$

ii) Synchronous gauge: $ds^2 = a^2(\tau)\{-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\}$

CLASS implements both synchronous and conformal gauges ¯_(ツ)_/¯

The variables and the equations

For more details see Advanced Cosmo class next semester!

3) The perturbation equations in Conformal Newtonian gauge

$$
\begin{aligned}
k^2\phi + 3\frac{\dot a}{a}\left(\dot\phi + \frac{\dot a}{a}\psi\right) &= 4\pi G a^2 \delta T^0{}_0(\mathrm{Con})\,,\\
k^2\left(\dot\phi + \frac{\dot a}{a}\psi\right) &= 4\pi G a^2 (\bar\rho + \bar P)\theta(\mathrm{Con})\,,\\
\ddot\phi + \frac{\dot a}{a}(\dot\psi + 2\dot\phi) + \left(2\frac{\ddot a}{a} - \frac{\dot a^2}{a^2}\right)\psi + \frac{k^2}{3}(\phi - \psi) &= \frac{4\pi}{3} G a^2 \delta T^i{}_i(\mathrm{Con})\,,\\
k^2(\phi - \psi) &= 12\pi G a^2 (\bar\rho + \bar P)\sigma(\mathrm{Con})\,,
\end{aligned}
$$

4) The perturbation equations in Synchronous gauge

$$
\begin{aligned}
k^2\eta - \frac{1}{2}\frac{\dot a}{a}\dot h &= 4\pi G a^2 \delta T^0{}_0(\mathrm{Syn})\,,\\
k^2\dot\eta &= 4\pi G a^2 (\bar\rho + \bar P)\theta(\mathrm{Syn})\,,\\
\ddot h + 2\frac{\dot a}{a}\dot h - 2k^2\eta &= -8\pi G a^2 \delta T^i{}_i(\mathrm{Syn})\,,\\
\ddot h + 6\ddot\eta + 2\frac{\dot a}{a}\left(\dot h + 6\dot\eta\right) - 2k^2\eta &= -24\pi G a^2 (\bar\rho + \bar P)\sigma(\mathrm{Syn})\,.
\end{aligned}
$$

The potentials

For more details see Advanced Cosmo class next semester!

$$k^2(\phi - \psi) \quad = \quad 12\pi G a^2(\bar{\rho} + \bar{P})\sigma(\mathrm{Con}) \,,$$

```
/* equation for psi */
ppw->pvecmetric[ppw->index_mt_psi] = y[ppw->pv->index_pt_phi] - 4.5 * (a2/k2) * ppw->rho_plus_p_shear;

/* equation for phi' */
ppw->pvecmetric[ppw->index_mt_phi_prime] = -a_prime_over_a * ppw->pvecmetric[ppw->index_mt_psi] + 1.5 * (a2/k2)
* ppw->rho_plus_p_theta;
```

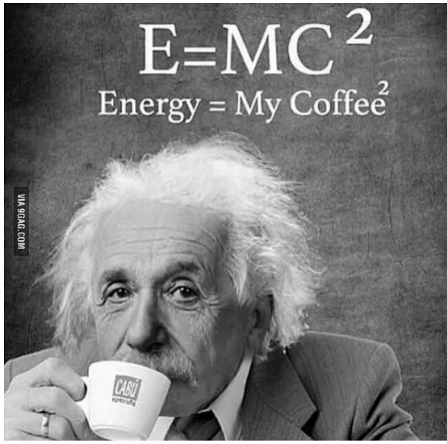
$$k^2\left(\dot{\phi} + \frac{\dot{a}}{a}\psi\right) \quad = \quad 4\pi G a^2(\bar{\rho} + \bar{P})\theta(\mathrm{Con}) \,,$$

Etc for the rest...

Basic code flowchart

1) User inputs main cosmological parameter Ω_m, Ω_b, n_s, H_0 etc

2) Calculate background evolution H(z) and a(t)...



3) Wait for code to solve perturbation equations of Boltzmann hierarchy and mulitpoles $\Delta l(k)$ for grid of values of k, usually in k→ [0.0001,10]h/Mpc

4) Calculate matter power spectrum $P(k) \equiv \langle|\delta_k|^2\rangle$ and $C_l = 4\pi \int d^3k\, P_\psi(k)\, \Delta_l^2(k,\tau)$ Also include other secondary effects as discussed earlier.

5) Output results or feed to MCMC code to estimate best-fit parameters!

Summary

1) CMB revolutionized modern cosmology, helped transition to precision science

2) Main features of CMB: primary and secondary anisotropies

3) BAO is the main feature of CMB and contains most of information.

4) Planck papers are the pinnacle of several years worth of hard work. Contain all info on experiment and results of analysis.

5) Boltzmann codes (CAMB, CLASS) do the heavy lifting of calculating the spectra.

6) Outlook: Tons of things to do with CMB: tensors and BB spectra, more accurate polarization, spectral distortions etc.