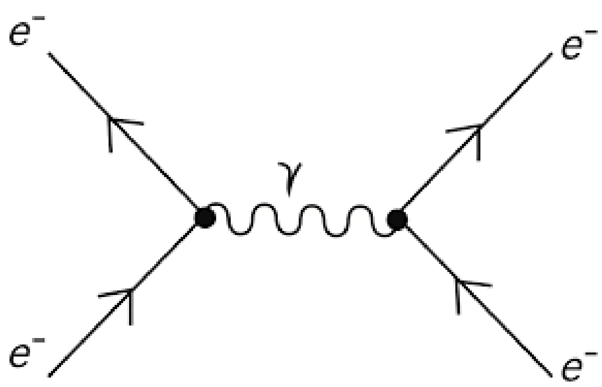
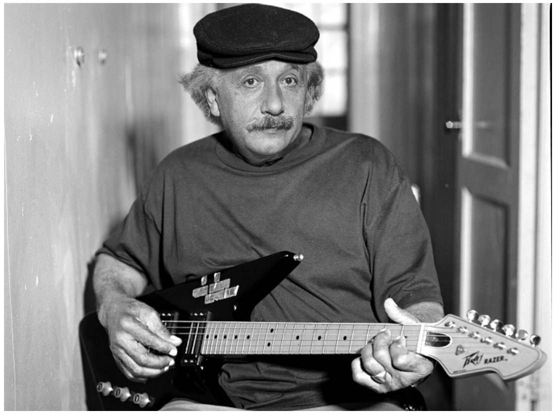
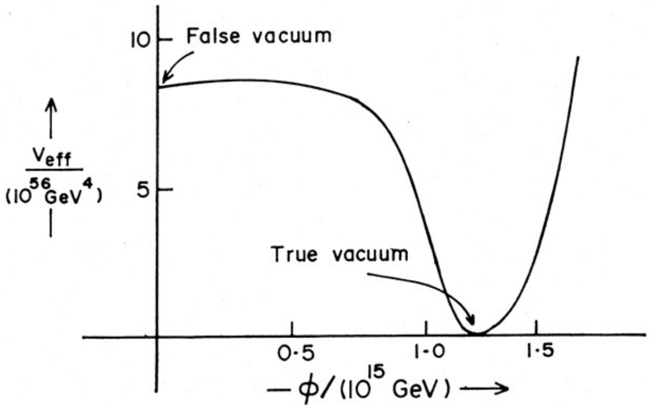
Gauge Invariant perturbations and Baryogenesis







Savvas Nesseris

IFT/UAM-CSIC, Madrid, Spain

Main points of the lecture

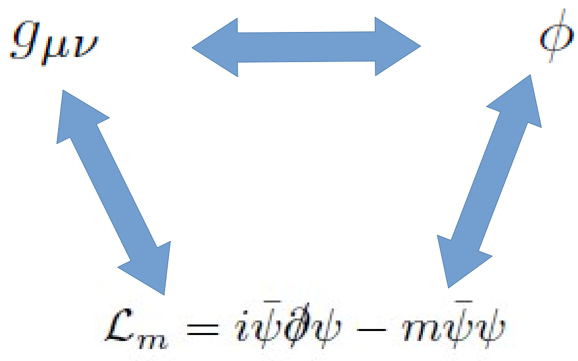
- Theory of Gauge Invariant perturbations
- Transplanckian physics and the power spectrum
- More discussion on Inflation and final notes
- Primordial Black Holes and Inflation
- Baryogenesis
- Summary

Main points of the lecture

- Theory of Gauge Invariant perturbations
- Transplanckian physics and the power spectrum
- More discussion on Inflation and final notes
- Primordial Black Holes and Inflation
- Baryogenesis
- Summary

Theory of Gauge Invariant Perturbations

1) Inflation practically is quantum mechanics on Curved Space Time!



Inflaton perturbations affect metric, which is coupled to matter...

2) Background (known material)

$$
\begin{aligned}
ds^2 &= a^2(\eta)[- d\eta^2 + \gamma_{ij}\, dx^i dx^j] \,, \\
\phi &= \phi(\eta) \,, \\
\eta &= \int dt/a(t)
\end{aligned}
$$

$$
\mathcal{H}^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\phi'^2 + a^2 V(\phi)\right) \,,
$$

$$
\mathcal{H}' - \mathcal{H}^2 = -\frac{\kappa^2}{2}\phi'^2 \,,
$$

$$
\phi'' + 2\mathcal{H}\phi' + a^2 V'(\phi) = 0 \,,
$$

$$
\mathcal{H} = aH \qquad \phi' = a\dot{\phi}
$$

Theory of Gauge Invariant Perturbations

3) Break perturbations into Scalar-Vector-Tensor (SVT) and use SVT decomposition

$$w_i = w_i^{||} + w_i^{\perp}$$

Longitudinal Transverse

$$\vec{\nabla} \times \vec{w}^{||} = 0 \Rightarrow w_i^{||} = \nabla_i A$$
$$\vec{\nabla} \cdot \vec{w}^{\perp} = 0$$

$$S_{ij} = S_{ij}^{||} + S_{ij}^{\perp} + S_{ij}^{T}$$

$$S_{ij}^{\perp} = \nabla_i S_j^{\perp} + \nabla_j S_i^{\perp}$$
$$\partial^i S_{ij}^{T} = 0$$
$$S_{ij}^{||} = \left(\nabla_i \nabla_j - \frac{1}{3} g_{ij} \nabla^2 \right) B$$

4) Perturb FRW metric and inflaton

$$ds^2 = a^2(\eta) \left[-(1+2A)\, d\eta^2 + 2B_{|i}\, dx^i d\eta + \left\{ (1+2\mathcal{R})\gamma_{ij} + 2E_{|ij} + 2h_{ij} \right\} dx^i dx^j \right] ,$$

$$\phi = \phi(\eta) + \delta\phi(\eta, x^i) .$$

$(A, B, \mathcal{R}, E)$

Gauge dependent functions

$$h_{ij} = \frac{h}{3}\delta_{ij} + h_{ij}^{||} + h_{ij}^{\perp} + h_{ij}^{T}$$

Theory of Gauge Invariant Perturbations

5) $g_{\mu\nu}$ degrees of freedom (dof): 4x4 = 16→ (symmetry)→ 10 → 2+4+4

Propagating dof

Gauge freedom

Coordinate freedom

6) Gauge transformation

$$\tilde{\eta} = \eta + \xi^0(\eta, x^i)\,,$$
$$\tilde{x}^i = x^i + \gamma^{ij}\xi_{|j}(\eta, x^i)\,,$$

$$\tilde{A} = A - \xi^{0\prime} - \mathcal{H}\xi^0\,,$$
$$\tilde{\mathcal{R}} = \mathcal{R} - \mathcal{H}\xi^0\,, \qquad \tilde{h}_{ij} = h_{ij}$$
$$\tilde{B} = B + \xi^0 - \xi'$$
$$\tilde{E} = E - \xi$$

7) Bardeen potentials

$$\Phi = A + (B - E')' + \mathcal{H}(B - E')\,,$$
$$\Psi = \mathcal{R} + \mathcal{H}(B - E')\,,$$

Gauge invariant!!!!!!

Prove!

Theory of Gauge Invariant Perturbations

8) End goal of perturbation analysis is to calculate two point function for scalar potentials $\Phi \sim R_k$, where R_k is the curvature perturbation (see later).

$$\langle 0|\mathcal{R}_k^* \mathcal{R}_{k'}|0\rangle = \frac{|u_k|^2}{z^2}\,\delta^3(\mathbf{k}-\mathbf{k}') \equiv \frac{\mathcal{P}_\mathcal{R}(k)}{4\pi k^3}\,(2\pi)^3\,\delta^3(\mathbf{k}-\mathbf{k}')$$

$$\mathcal{P}_\mathcal{R}(k) = \frac{k^3}{2\pi^2}\,\frac{|u_k|^2}{z^2} = \frac{\kappa^2}{2\epsilon}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{3-2\nu} \equiv A_S^2\left(\frac{k}{aH}\right)^{n_s-1}$$

Inflaton perturbation
We need the solution!

$$u \equiv a\delta\phi + z\Phi\,,$$
$$z \equiv a\frac{\phi'}{\mathcal{H}}\,.$$

Primordial power spectrum

And similarly for the tensor perturbations:

$$\sum_\lambda \langle 0|h_{k,\lambda}^* h_{k',\lambda}|0\rangle = \frac{8\kappa^2}{a^2}\,|v_k|^2\delta^3(\mathbf{k}-\mathbf{k}') \equiv \frac{\mathcal{P}_g(k)}{4\pi k^3}\,(2\pi)^3\,\delta^3(\mathbf{k}-\mathbf{k}')$$

$$\mathcal{P}_g(k) = 8\kappa^2\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{3-2\mu} \equiv A_T^2\left(\frac{k}{aH}\right)^{n_T}$$

Theory of Gauge Invariant Perturbations

9) Keep scalar perturbations and find relations between potentials with Einstein eqs

$$\Phi = \Psi\,,$$
$$\frac{k^2 - 3K}{a^2}\Psi = \frac{\kappa^2}{2}\delta\rho$$

No anisotropic stress!

Poisson equation in GR

10) Rest of Einstein equations

$$\Phi'' + 3\mathcal{H}\Phi' + (\mathcal{H}' + 2\mathcal{H}^2)\Phi = \frac{\kappa^2}{2}[\phi'\delta\phi' - a^2V'(\phi)\delta\phi]\,,$$
$$-\nabla^2\Phi + 3\mathcal{H}\Phi' + (\mathcal{H}' + 2\mathcal{H}^2)\Phi = -\frac{\kappa^2}{2}[\phi'\delta\phi' + a^2V'(\phi)\delta\phi]$$
$$\Phi' + \mathcal{H}\Phi = \frac{\kappa^2}{2}\phi'\delta\phi\,,$$
$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi = 4\phi'\Phi' - 2a^2V'(\phi)\Phi - a^2V''(\phi)\delta\phi$$

Scalar field pert. equation

Theory of Gauge Invariant Perturbations

11) Equations are a bit complicated, let's try to simplify them a bit. Define:

$$u \equiv a\delta\phi + z\Phi\,,$$
$$z \equiv a\frac{\phi'}{\mathcal{H}}\,.$$

Not the redshift!!!!!!!

$$\Longrightarrow$$

$$u'' - \nabla^2 u - \frac{z''}{z}u = 0\,,$$
$$\nabla^2\Phi = \frac{\kappa^2}{2}\frac{\mathcal{H}}{a^2}(zu' - z'u)\,,$$
$$\left(\frac{a^2\Phi}{\mathcal{H}}\right)' = \frac{\kappa^2}{2}zu\,.$$

12) Equations can be solved analytically, eg in Matter Domination (MD), or numerically in general for the classical system:

$$u'' - \nabla^2 u - \frac{z''}{z}u = 0 \Longrightarrow u(z) \Longrightarrow \left(\frac{a^2\Phi}{\mathcal{H}}\right)' = \frac{\kappa^2}{2}zu \Longrightarrow \Phi(z)$$

$$\Longrightarrow \delta\phi(z)$$

Quantum Mechanics in Curved space-time

1) Set up Quantum Mechanical system

$$\delta S = \frac{1}{2}\int d^3x\, d\eta \left[(u')^2 - (\nabla u)^2 + \frac{z''}{z} u^2 \right]$$

Kinetic term

Potential with time dependent mass term

2) Quantization and commutation relations

$$\hat{u}(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[u_k(\eta)\, \hat{a}_{\mathbf{k}}\, e^{i\mathbf{k}\cdot\mathbf{x}} + u_k^*(\eta)\, \hat{a}^\dagger_{\mathbf{k}}\, e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

Annihilation operator

Creation operator

$$[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = \delta^3(\mathbf{k} - \mathbf{k}')\,,$$
$$\hat{a}_{\mathbf{k}}|0\rangle = 0\,.$$

Vacuum annihilation

Quantum Mechanics in Curved space-time

3) Equations of motion for each mode

$$u_k'' + \left(k^2 - \frac{z''}{z}\right)u_k = 0$$

4) Introduce slow roll parameters

$$\epsilon = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2} = \frac{\kappa^2}{2}\frac{z^2}{a^2},$$

$$\delta = 1 - \frac{\phi''}{\mathcal{H}\phi'} = 1 + \epsilon - \frac{z'}{\mathcal{H}z},$$

$$\xi = -\left(2 - \epsilon - 3\delta + \delta^2 - \frac{\phi'''}{\mathcal{H}^2\phi'}\right)$$

See also previous inflation lecture

Quantum Mechanics in Curved space-time

5) Time and potential can be rewritten as

$$\eta = \frac{-1}{\mathcal{H}} + \int \frac{\epsilon da}{a\mathcal{H}},$$

$$\frac{z''}{z} = \mathcal{H}^2\left[(1+\epsilon-\delta)(2-\delta) + \mathcal{H}^{-1}(\epsilon' - \delta')\right]$$

6) Slow roll parameters evolve slowly ;-)

$$\epsilon' = 2\mathcal{H}\left(\epsilon^2 - \epsilon\delta\right) = \mathcal{O}(\epsilon^2),$$

$$\delta' = \mathcal{H}\left(\epsilon\delta - \xi\right) = \mathcal{O}(\epsilon^2).$$

$\Rightarrow$

$$\eta = \frac{-1}{\mathcal{H}}\frac{1}{1-\epsilon}$$

$$\frac{z''}{z} = \frac{1}{\eta^2}\left(\nu^2 - \frac{1}{4}\right)$$

$$\nu = \frac{1+\epsilon-\delta}{1-\epsilon} + \frac{1}{2}$$

Quantum Mechanics in Curved space-time

7) Quasi-de Sitter has characteristic scale → event horizon ~1/H

i) Modes within the horizon have wavelengths λ << 1/H → $k \gg aH$

ii) Modes outside the horizon have wavelengths λ >> 1/H → $k \ll aH$

⇒

$$u_k = \frac{1}{\sqrt{2k}} e^{-ik\eta} \qquad k \gg aH \,,$$
$$u_k = C_1 \, z \qquad k \ll aH \,.$$

8) Proof for k>>a H:

$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0 \qquad \xRightarrow{k \gg aH}$$

$$u_k'' + k^2 \, u_k = 0 \qquad \Rightarrow \qquad u_k = \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

Quantum Mechanics in Curved space-time

9) Proof for k<<a H:

$$u_k'' + \left(k^2 - \frac{z''}{z}\right)u_k = 0 \quad \xrightarrow{k \ll aH} \quad u_k'' - \frac{z''}{z}u_k = 0$$

ODE for mass term:

$$\frac{z''}{z} = \frac{1}{\eta^2}\left(\nu^2 - \frac{1}{4}\right) \quad \Longrightarrow \quad z = c_1\eta^{\frac{1}{2}-\nu} + c_2\eta^{\frac{1}{2}+\nu}$$

Final solution:

$$u_k = \tilde{c}_1\eta^{\frac{1}{2}-\nu} + \tilde{c}_2\eta^{\frac{1}{2}+\nu} \quad \Longrightarrow \quad u_k = \alpha_1 z$$

Quantum Mechanics in Curved space-time

10) General solution

$$u_k(\eta) = \frac{\sqrt{\pi}}{2} e^{i(\nu+\frac{1}{2})\frac{\pi}{2}} (-\eta)^{1/2} H_\nu^{(1)}(-k\eta)$$

Hankel function of the 1st kind
H(x)~J(x)+i Y(x)

11) Find solutions in limit kη→ 0

$$|u_k| = \frac{2^{\nu-\frac{3}{2}}}{\sqrt{2k}} \frac{\Gamma(\nu)}{\Gamma(\frac{3}{2})} (-k\eta)^{\frac{1}{2}-\nu} \equiv \frac{C(\nu)}{\sqrt{2k}} \left(\frac{k}{aH}\right)^{\frac{1}{2}-\nu},$$

$$C(\nu) = 2^{\nu-\frac{3}{2}} \frac{\Gamma(\nu)}{\Gamma(\frac{3}{2})} (1-\epsilon)^{\nu-\frac{1}{2}} \simeq 1 \qquad \text{for} \quad \epsilon, \delta \ll 1$$

Decaying mode, ignore

$$\left(\frac{a^2\Phi}{\mathcal{H}}\right)' = \frac{\kappa^2}{2} z u .$$

⇨

$$\Phi = C_1 \left(1 - \frac{\mathcal{H}}{a^2} \int a^2 d\eta\right) + C_2 \frac{\mathcal{H}}{a^2}$$

$$\delta\phi = \frac{C_1}{a^2} \int a^2 d\eta - \frac{C_2}{a^2} .$$

Quantum Mechanics in Curved space-time

12) Find expression that is constant for superhorizon modes

$$\left(\frac{a^2\Phi}{\mathcal{H}}\right)' = \frac{\kappa^2}{2} z u \,.$$

$$\epsilon = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2} = \frac{\kappa^2}{2}\frac{z^2}{a^2}$$

$$\Rightarrow \quad \zeta \equiv \Phi + \frac{1}{\epsilon\mathcal{H}}\left(\Phi' + \mathcal{H}\Phi\right) = \frac{u}{z}$$

Constant for k<<aH

13) Find solutions for Φ in Radiation/Matter domination

$$\Phi = C_1\left(1 - \frac{\mathcal{H}}{a^2}\int a^2 d\eta\right) \quad \Rightarrow \quad \Phi_k = \left(1 - \frac{\mathcal{H}}{a^2}\int a^2 d\eta\right)\mathcal{R}_k$$

Curvature perturbation

$$\begin{aligned} H^2 &= H_0^2 a^{-3(1+w)} \\ \mathcal{H}^2/a^2 &= H_0^2 a^{-3(1+w)} \\ a &\sim \eta^{\frac{2}{1+3w}} \end{aligned}$$

$$\Rightarrow \quad \Phi_k = \frac{3+3\omega}{5+3\omega}\mathcal{R}_k = \begin{cases} \frac{2}{3}\mathcal{R}_k & \text{radiation era} \\ \frac{3}{5}\mathcal{R}_k & \text{matter era} \end{cases}$$

Quantum Mechanics in Curved space-time

14) Tensor perturbations

$$\delta S = \frac{1}{2}\int d^3x\, d\eta\, \frac{a^2}{2\kappa^2}\Big[(h'_{ij})^2 - (\nabla h_{ij})^2\Big] \Longrightarrow$$

$$\hat{h}_{ij}(\eta,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \sum_{\lambda=1,2} \Big[h_k(\eta)\, e_{ij}(\mathbf{k},\lambda)\, \hat{a}_{\mathbf{k},\lambda}\, e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.\Big]$$

$$e_{ij} = e_{ji}\,, \quad k^i e_{ij} = 0\,, \quad e_{ii} = 0\,,$$
$$e_{ij}(-\mathbf{k},\lambda) = e^*_{ij}(\mathbf{k},\lambda)\,, \quad \sum_\lambda e^*_{ij}(\mathbf{k},\lambda)e^{ij}(\mathbf{k},\lambda) = 4\,,$$

Space time dimensions

15) Find ODE:

$$v_k(\eta) = \frac{a}{\sqrt{2}\kappa} h_k(\eta) \Longrightarrow v''_k + \left(k^2 - \frac{a''}{a}\right) v_k = 0$$

$$\frac{a''}{a} = 2\mathcal{H}^2\left(1 - \frac{\epsilon}{2}\right) = \frac{1}{\eta^2}\left(\mu^2 - \frac{1}{4}\right),$$
$$\mu = \frac{1}{1-\epsilon} + \frac{1}{2}\,.$$

$$\Longrightarrow$$

$$v_k = \frac{1}{\sqrt{2k}} e^{-ik\eta} \qquad k \gg aH\,,$$
$$v_k = C\, a \qquad k \ll aH\,.$$

Primordial power spectrum

1) Two point correlation function→ power spectrum in Fourier space

$$\langle 0|\mathcal{R}_k^*\mathcal{R}_{k'}|0\rangle = \frac{|u_k|^2}{z^2}\,\delta^3(\mathbf{k}-\mathbf{k}') \equiv \frac{\mathcal{P}_\mathcal{R}(k)}{4\pi k^3}\,(2\pi)^3\,\delta^3(\mathbf{k}-\mathbf{k}')$$

$$\mathcal{P}_\mathcal{R}(k) = \frac{k^3}{2\pi^2}\frac{|u_k|^2}{z^2} = \frac{\kappa^2}{2\epsilon}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{3-2\nu} \equiv A_S^2\left(\frac{k}{aH}\right)^{n_s-1}$$

Primordial power spectrum

2) Notes:

i) ns=1 → equal power on all scales (flat spectrum)

ii) ns is determined from inflationary model

ii) As = amplitude of inflation perturbations, has to be determined from CMB

$$n_s - 1 \equiv \frac{d\ln \mathcal{P}_\mathcal{R}(k)}{d\ln k} = 3-2\nu = 2\left(\frac{\delta-2\epsilon}{1-\epsilon}\right) \simeq 2\eta_V - 6\epsilon_V$$

Primordial power spectrum

3) Primordial power spectrum might have a "running", ie higher order corrections

$$\frac{dn_s}{d\ln k} = -\frac{dn_s}{d\ln\eta} = -\eta\mathcal{H}\left(2\xi + 8\epsilon^2 - 10\epsilon\delta\right) \simeq 2\xi_V + 24\epsilon_V^2 - 16\eta_V\epsilon_V$$

4) Similarly for tensor perturbations

$$\sum_\lambda \langle 0|h^*_{k,\lambda}h_{k',\lambda}|0\rangle = \frac{8\kappa^2}{a^2}|v_k|^2\delta^3(\mathbf{k}-\mathbf{k}') \equiv \frac{\mathcal{P}_g(k)}{4\pi k^3}(2\pi)^3\,\delta^3(\mathbf{k}-\mathbf{k}')$$

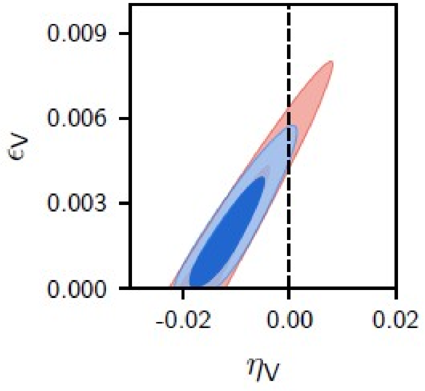
$$\mathcal{P}_g(k) = 8\kappa^2\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{3-2\mu} \equiv A_T^2\left(\frac{k}{aH}\right)^{n_T}$$

$$n_T \equiv \frac{d\ln\mathcal{P}_g(k)}{d\ln k} = 3 - 2\mu = \frac{-2\epsilon}{1-\epsilon} \simeq -2\epsilon_V < 0$$

Primordial power spectrum

5) Primordial power spectrum for tensors might also have a "running"

$$\frac{dn_T}{d\ln k} = -\frac{dn_T}{d\ln\eta} = -\eta\mathcal{H}\left(4\epsilon^2 - 4\epsilon\delta\right) \simeq 8\epsilon_V^2 - 4\eta_V\epsilon_V$$

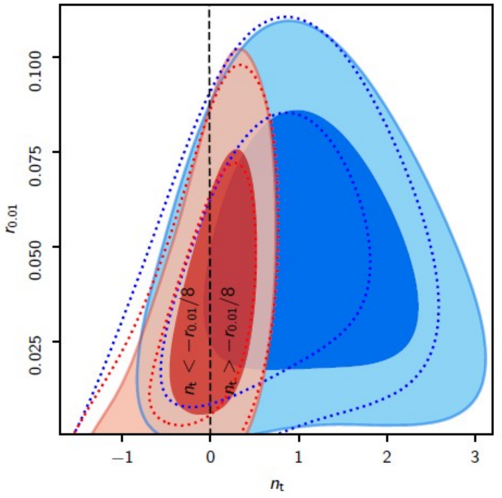


6) In single-field slow-roll models $n_t \sim -r/8$, $r = P_t/P_s$

astro-ph/9303019

$$\ln \mathcal{P}_s(k) = \ln \mathcal{P}_0(k) + \frac{1}{2}\frac{d\ln n_s}{d\ln k}\ln(k/k_*)^2 + \frac{1}{6}\frac{d^2\ln n_s}{d\ln k^2}\ln(k/k_*)^3 + \ldots,$$

$$\ln \mathcal{P}_t(k) = \ln(rA_s) + n_t\ln(k/k_*) + \ldots,$$



Primordial power spectrum

7) Planck 2018 constraints on scalar parameters

Parameter	TT,TE,EE+lowE+lensing
$\Omega_b h^2$	0.02237 ± 0.00015
$\Omega_c h^2$	0.1200 ± 0.0012
$100\theta_{MC}$	1.04092 ± 0.00031
τ	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.044 ± 0.014
n_s	0.9649 ± 0.0042
H_0	67.36 ± 0.54
Ω_m	0.3153 ± 0.0073
σ_8	0.8111 ± 0.0060

8) On running and tensors etc

$$n_s = 0.9587 \pm 0.0056 \ (0.9625 \pm 0.0048)\,,$$
$$dn_s/d\ln k = 0.013 \pm 0.012 \ (0.002 \pm 0.010)\,,$$
$$d^2 n_s/d\ln k^2 = 0.022 \pm 0.012 \ (0.010 \pm 0.013)\,,$$

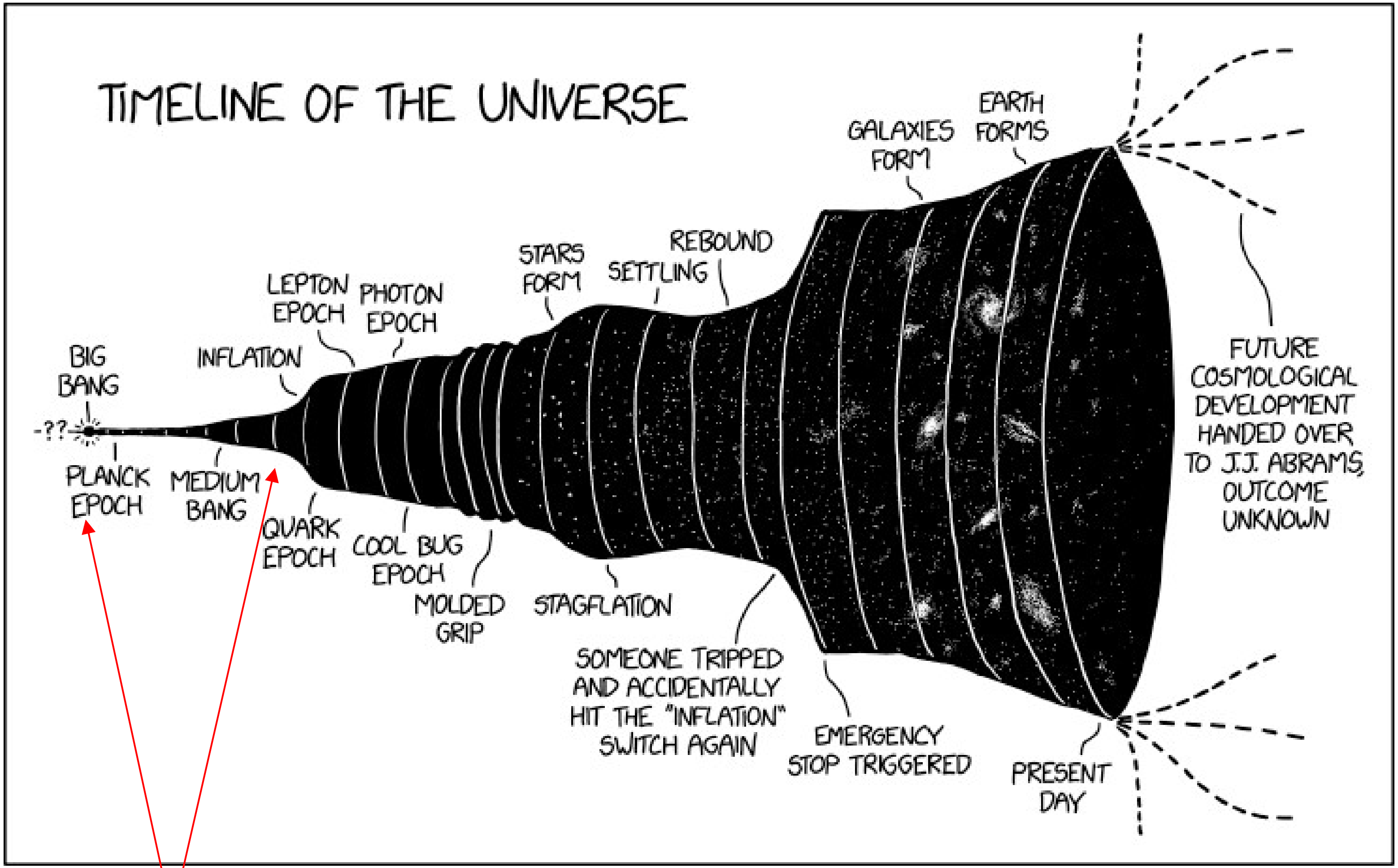
Cosmological model ΛCDM+r	Parameter	*Planck* TT,TE,EE +lowEB+lensing	*Planck* TT,TE,EE +lowE+lensing+BK14	*Planck* TT,TE,EE +lowE+lensing+BK14+BAO
	r	< 0.11	< 0.070	< 0.070
	$r_{0.002}$	< 0.10	< 0.064	< 0.065
	n_s	0.9659 ± 0.0041	0.9653 ± 0.0041	0.9670 ± 0.0037
+$dn_s/d\ln k$	r	< 0.16	< 0.079	< 0.076
	$r_{0.002}$	< 0.16	< 0.077	< 0.072
	n_s	0.9647 ± 0.0044	0.9640 ± 0.0043	0.9658 ± 0.0038
	$dn_s/d\ln k$	-0.0085 ± 0.0073	-0.0071 ± 0.0068	-0.0065 ± 0.0066

Main points of the lecture

- Theory of Gauge Invariant perturbations
- Transplanckian physics and the power spectrum
- More discussion on Inflation and final notes
- Primordial Black Holes and Inflation
- Baryogenesis
- Summary

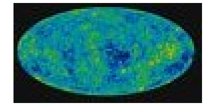
Timeline of the cosmos and transplanckian physics

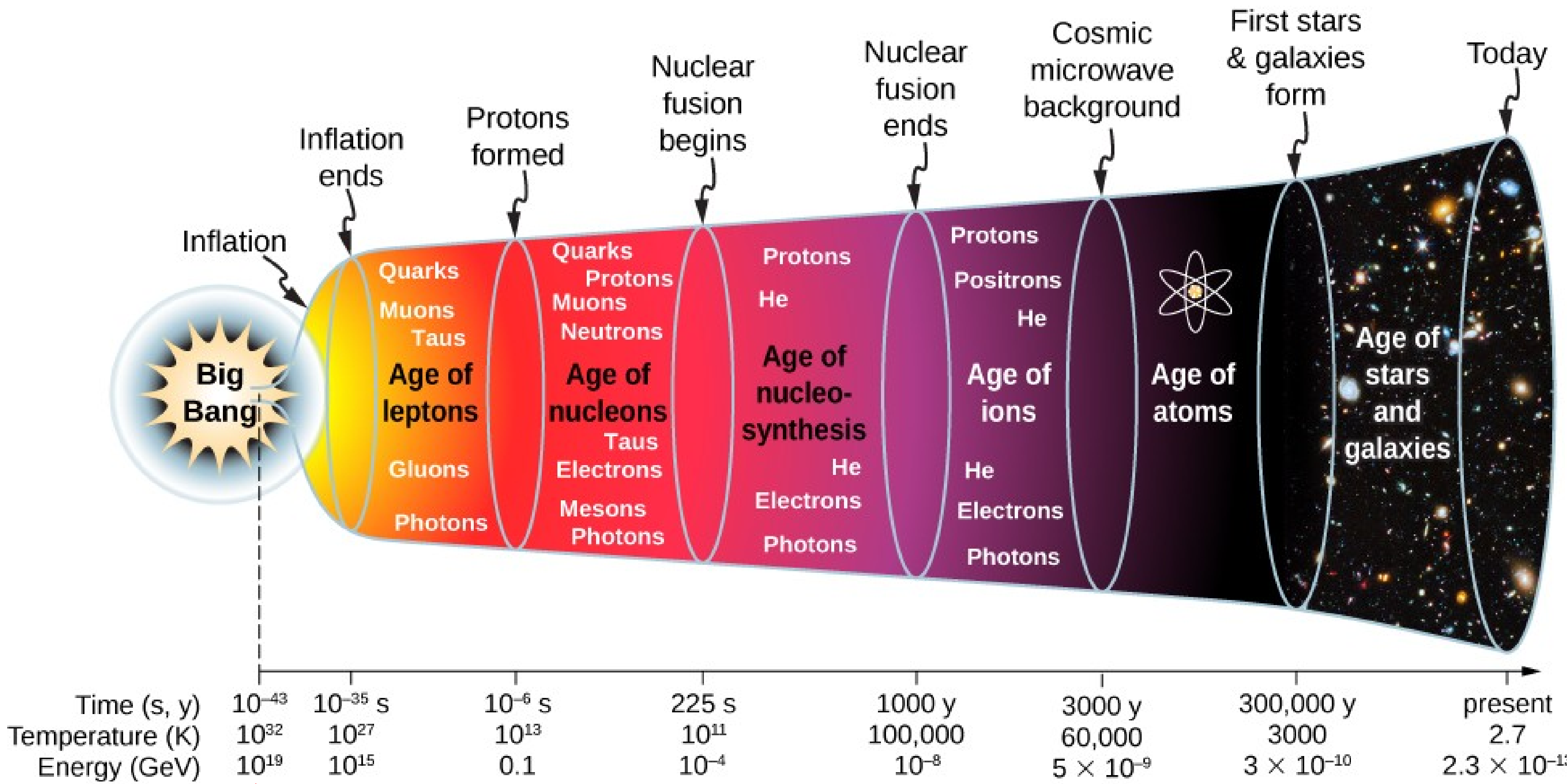
https://xkcd.com/2240/



Brief window for inflation, easy to get above Planck scales (10^19 GeV)!

Timeline of the cosmos and transplanckian physics





Brief window for inflation, easy to get above Planck scales (10^19 GeV)!

Transplanckian physics and the power spectrum

0705.4666

1) High energy (transplanckian) physics affects power spectrum!

   i) We have no solid quantum gravity theory, so check the operators that matter.

   ii) Relevant (dim<d) operators restore symmetry at high energies, irrelevant (dim>d) ones do not!

   iii) Consider various operators (quad in φ) and add inflaton Lagrangian

$\mathscr{D} \equiv (h^{\mu\nu}\nabla_\mu\nabla_\nu - K n^\mu \nabla_\mu)^{1/2}$
$= \frac{1}{a}(-\vec{\nabla}\cdot\vec{\nabla})^{1/2}$

Momentum projection operator (Lorentz invariance violating)

$$\tfrac{1}{3}K\varphi^2, \quad \varphi\mathscr{D}\varphi$$

Dimension 3

$$\tfrac{1}{9}K^2\varphi^2, \ \tfrac{1}{3}K\varphi\mathscr{D}\varphi, \ -h^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi$$

Dimension 4

$K_{\mu\nu}\,dx^\mu dx^\nu = -a^2 H\, d\vec{x}\cdot d\vec{x}$

2) Add the terms and find effect in primordial power spectrum

$$\mathscr{L}_{\rm NR} = \frac{d_1}{M}H^3\varphi^2 + \frac{d_2}{aM}H^2\varphi(-\vec{\nabla}\cdot\vec{\nabla})^{1/2}\varphi + \frac{d_3}{a^2M}H\vec{\nabla}\varphi\cdot\vec{\nabla}\varphi + \frac{d_4}{a^3M}\varphi(-\vec{\nabla}\cdot\vec{\nabla})^{3/2}\varphi$$

$$\langle 0(\eta)|\varphi(\eta,\vec{x})\varphi(\eta,\vec{y})|0(\eta)\rangle = \int\frac{d^3\vec{k}}{(2\pi)^3}e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\left[\frac{2\pi^2}{k^3}P_k(\eta)\right]$$

$|0(\eta)\rangle = Te^{-i\int_{\eta_0}^{\eta}d\eta' H_I(\eta')}|0\rangle$

$H_I(\eta) = -\int d^3\vec{x}\sqrt{-g}\,\mathscr{L}_{\rm NR}$

Transplanckian physics and the power spectrum

3) Effect in primordial power spectrum (term by term)

0705.4666

$K^3\varphi^2 \to H^3\varphi^2$ ⇒

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left[1 + \frac{4}{3}d_1\frac{H}{M}\left[\ln|2k\eta| - 2 + \gamma\right] + \cdots\right]$$
$$= \frac{H^2}{4\pi^2}\frac{4^\nu\,\Gamma^2(\nu)}{2\pi}|k\eta|^{3-2\nu} + \cdots,$$

$K^2\varphi\mathscr{D}\varphi \to H^2\varphi(-\vec{\nabla}\cdot\vec{\nabla})^{1/2}\varphi$ ⇒

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left[1 + d_2\frac{H}{M}\left[\pi + \frac{\cos(2k\eta_0)}{k\eta_0}\right] + \cdots\right]$$

$Kh^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi \to H\vec{\nabla}\varphi\cdot\vec{\nabla}\varphi$ ⇒

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left[1 + d_3\frac{H}{M}\left[3 + \cos\left(2\frac{k}{k_*}\frac{M}{H}\right)\right] + \cdots\right]$$

$\varphi\mathscr{D}^3\varphi \to \varphi(-\vec{\nabla}\cdot\vec{\nabla})^{3/2}\varphi$ ⇒

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left[1 - d_4\frac{k}{k_*}\cos\left(2\frac{k}{k_*}\frac{M}{H}\right) + \cdots\right]$$

Main points of the lecture

- Theory of Gauge Invariant perturbations
- Transplanckian physics and the power spectrum
- More discussion on Inflation and final notes
- Primordial Black Holes and Inflation
- Baryogenesis
- Summary

More discussion on Inflation

1) Scientific American article accusing Inflation of being "non-empirical science":

Ijjas, Steinhardt, Loeb
Scientific American
January 2017
also
arXiv: 1402.6980


COSMOLOGY

POP goes the universe

THE LATEST ASTROPHYSICAL MEASUREMENTS, COMBINED WITH THEORETICAL PROBLEMS, CAST DOUBT ON THE LONG-CHERISHED INFLATIONARY THEORY OF THE EARLY COSMOS AND SUGGEST WE NEED NEW IDEAS

By Anna Ijjas, Paul J. Steinhardt and Abraham Loeb

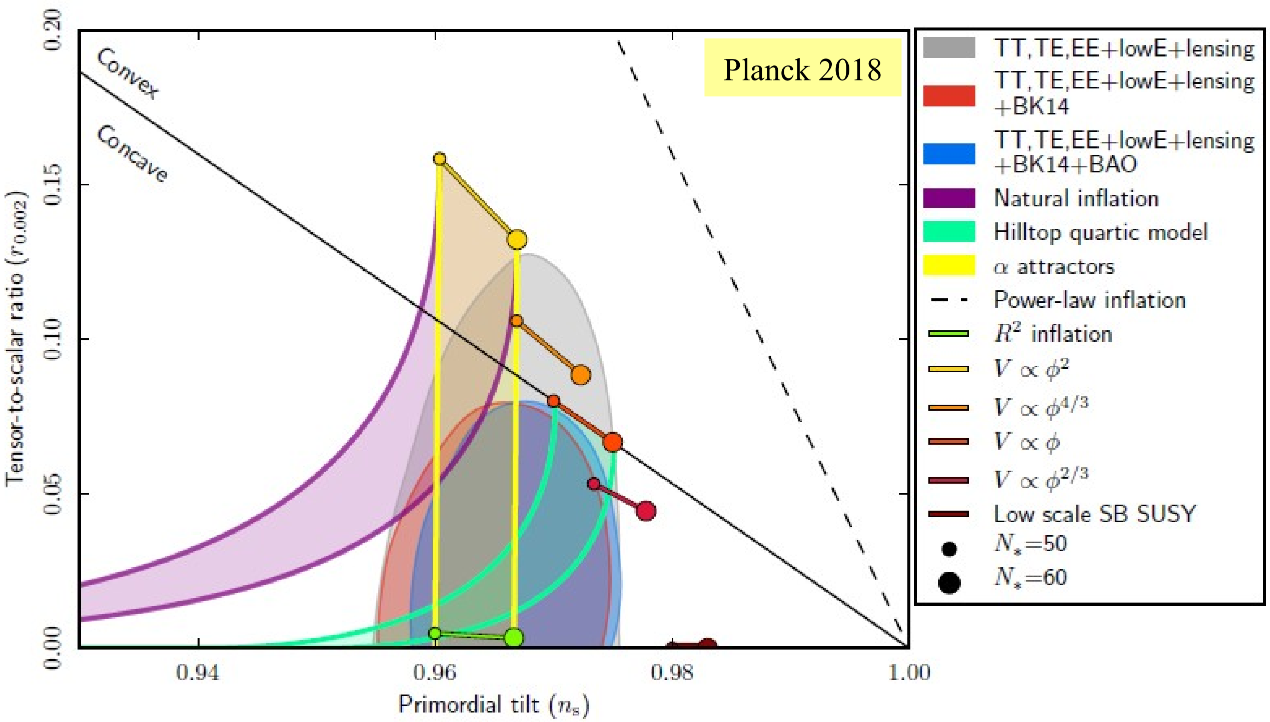
?????

More discussion on Inflation

2) Response on Scientific American article:

agreed, however, with the interpretation. If anything, the Planck data disfavored the simplest inflation models and exacerbated long-standing foundational problems with the theory, providing new reasons to consider competing ideas about the origin and evolution of the universe.

No! Many inflation models still viable, eg Starobinsky Inflation



More discussion on Inflation

3) Various ad hominem attacks and/or trivially flawed descriptions

FOLLOWING THE ORACLE

TO DEMONSTRATE inflation's problems, we will start by following the edict of its proponents: assume inflation to be true without

No comment…

referred to as inflationary energy,

Inflaton/scalar field???

idea that any outcome is possible. Does inflation tell us why the big bang happened or how the initial patch of space was created that eventually evolved into the universe observed today? The answer, again, is no.

Inflation solves classical problems, sets up ICs and random Gaussian fluct.

energy density one assumes. Thus, the arrangement Planck saw cannot be taken as confirmation of inflation.

Planck measured ns=0.967!

Planck 2018

More discussion on Inflation

4) Flawed conclusions…

NONEMPIRICAL SCIENCE?

GIVEN THE ISSUES with inflation and the possibilities of bouncing cosmologies, one would expect a lively debate among scientists today focused on how to distinguish between these theories through observations. Still, there is a hitch: inflationary cosmology, as we currently understand it, cannot be evaluated using the scientific method. As we have discussed, the expected outcome of inflation can easily change if we vary the initial conditions, change the shape of the inflationary energy density curve, or simply note that it leads to eternal inflation and a multimess. Individually and collectively, these features make inflation so flexible that no experiment can ever disprove it.

I can do MCMCs and determine model parameters or even rule out some models!

5) See response from rest of community:

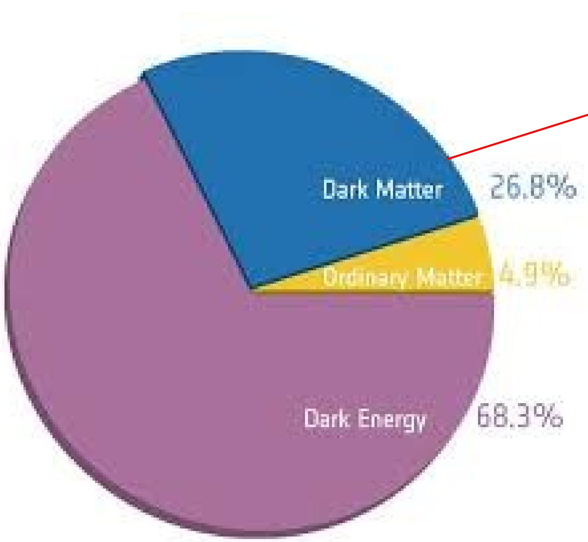
Scientific American blog:
A cosmic Constroversy by
Guth, Linde, Carroll, Efstathiou, Hawking, Maldacena et al

Also videos on inflation and Mathematica code!

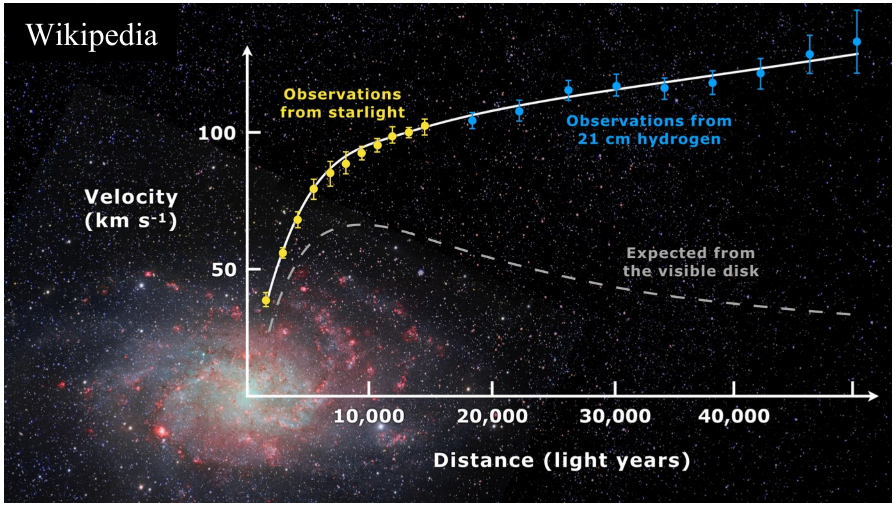
Main points of the lecture

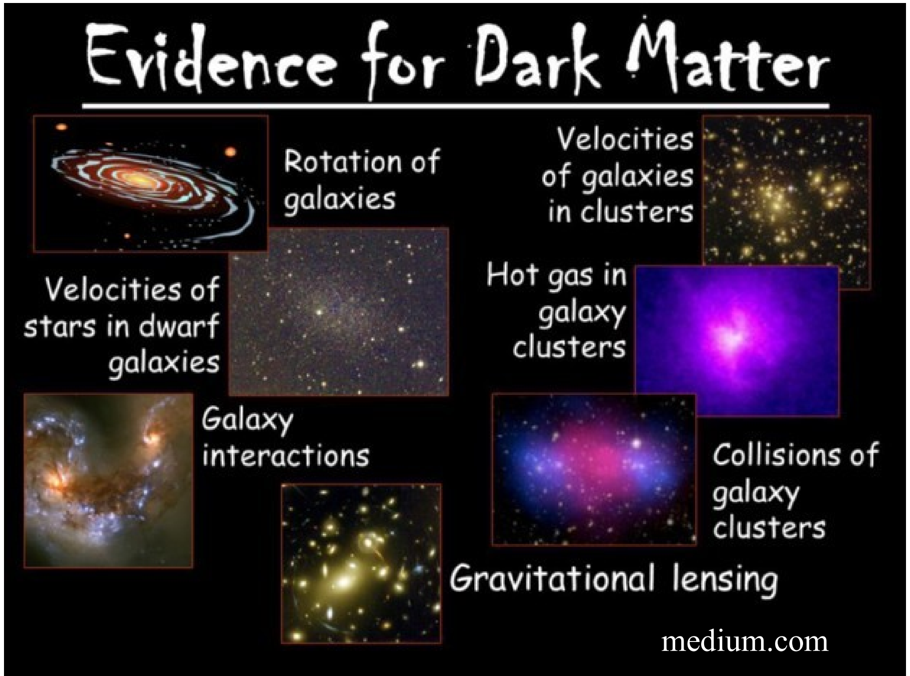
- Theory of Gauge Invariant perturbations
- Transplanckian physics and the power spectrum
- More discussion on Inflation and final notes
- Primordial Black Holes and Inflation
- Baryogenesis
- Summary

Evidence for Dark Matter

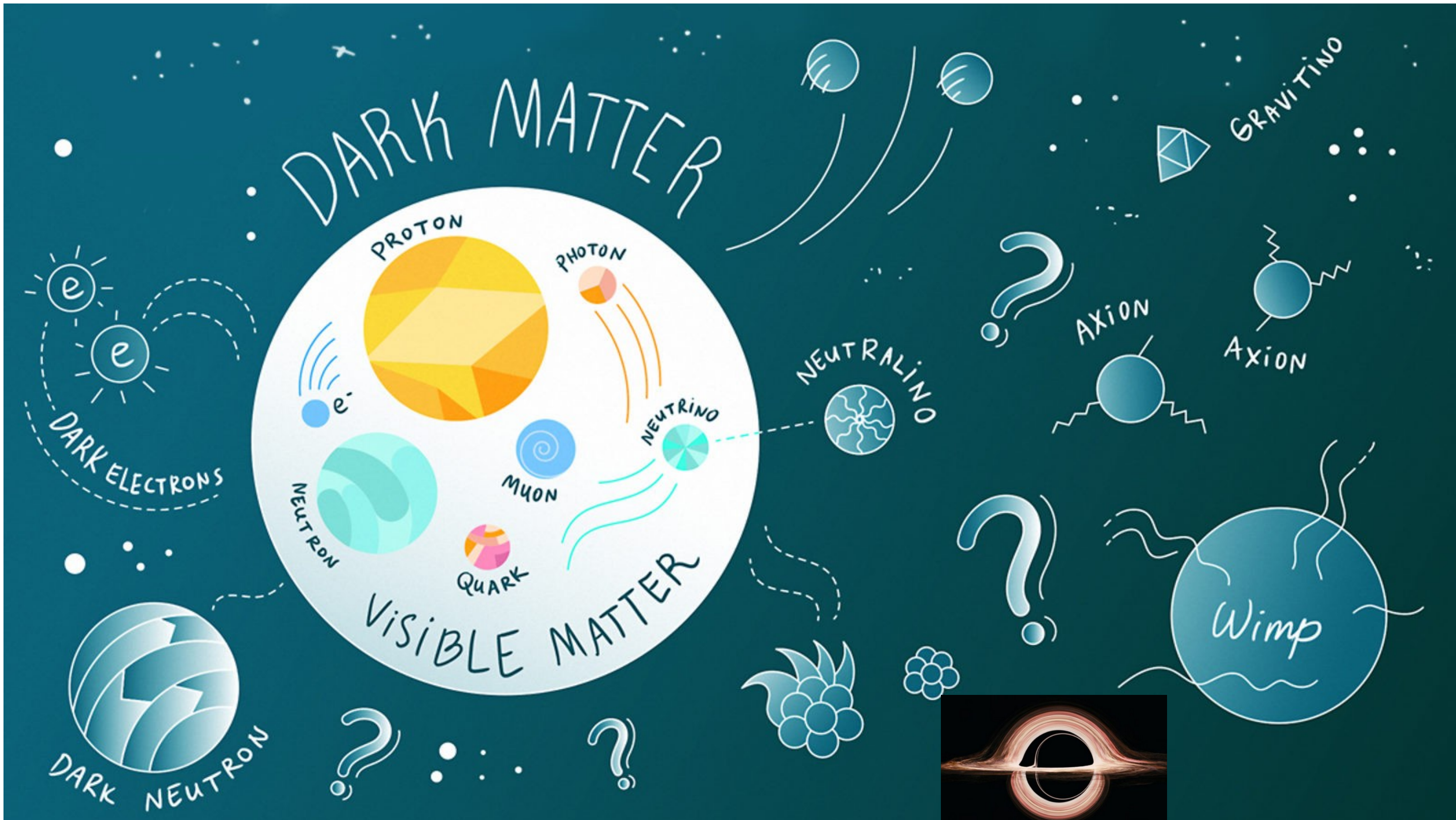


Planck 2018





Dark Matter candidates



Primordial Black Holes!

Primordial Black Holes redux

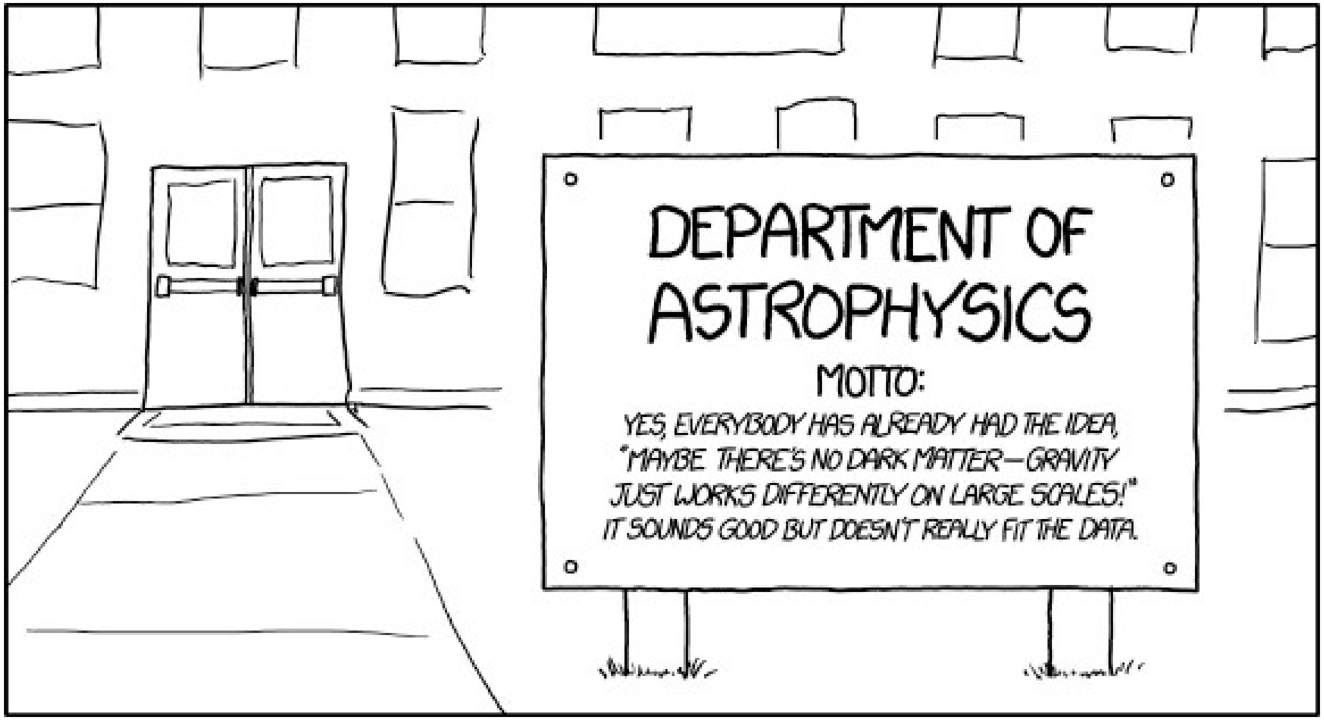
1) This is an old idea (Garcia-Bellido, Linde and Wands 1996) that became hot recently after GWs discovery (see also GW lecture for PBH hyperbolic encounters).

2) PBHs are formed after inflation when broad peaks in the primordial curvature power spectrum P(k) collapse gravitationally during the radiation era and form clusters of BHs that merge and increase in mass after recombination until today.

3) Masses range from 0.01-10^5 Msun and could jump-start structure formation

Primordial Black Holes redux

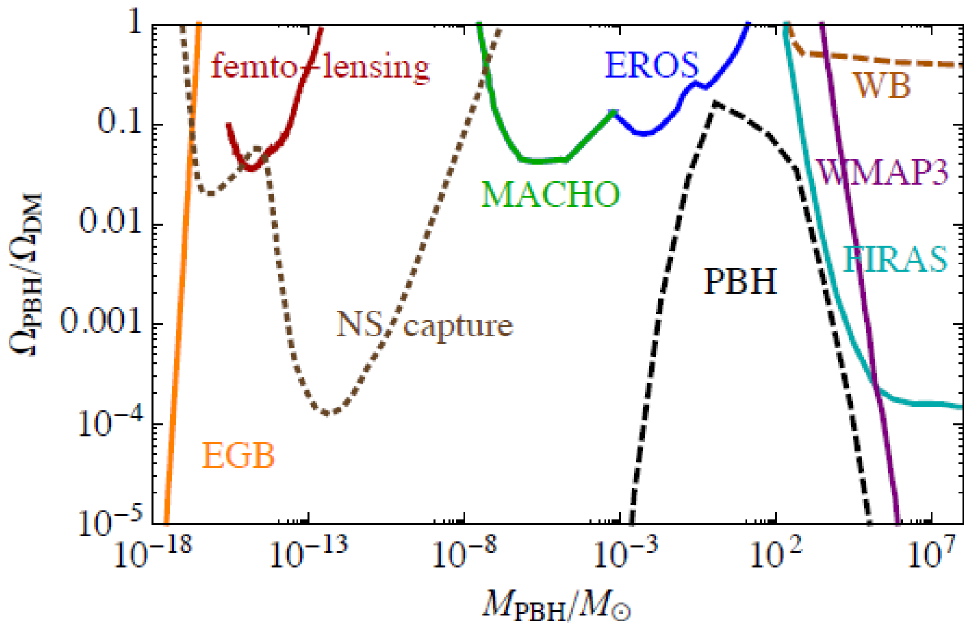
4) They are a plausible DM candidate, besides particle DM (axions, SUSY stuff etc) or modifications of gravity (MOND, TeVeS, MoGs, DM-DE interactions).



https://xkcd.com/1758/

Primordial Black Holes redux

5) PBHs could make up almost all of DM with a non-monochromatic distribution



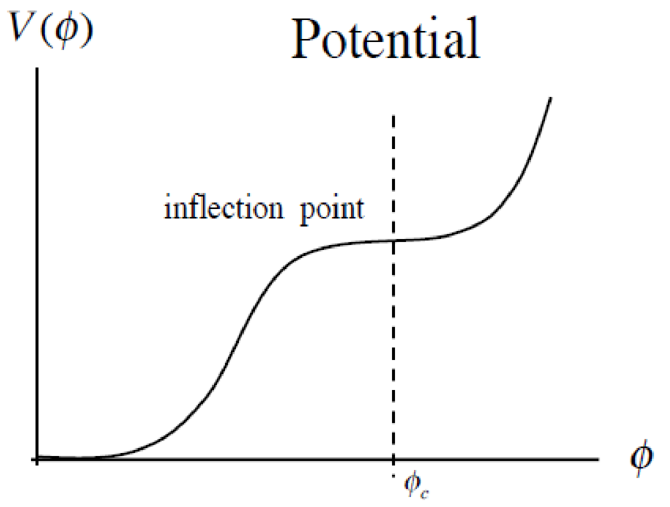
1702.08275 and 1501.07565

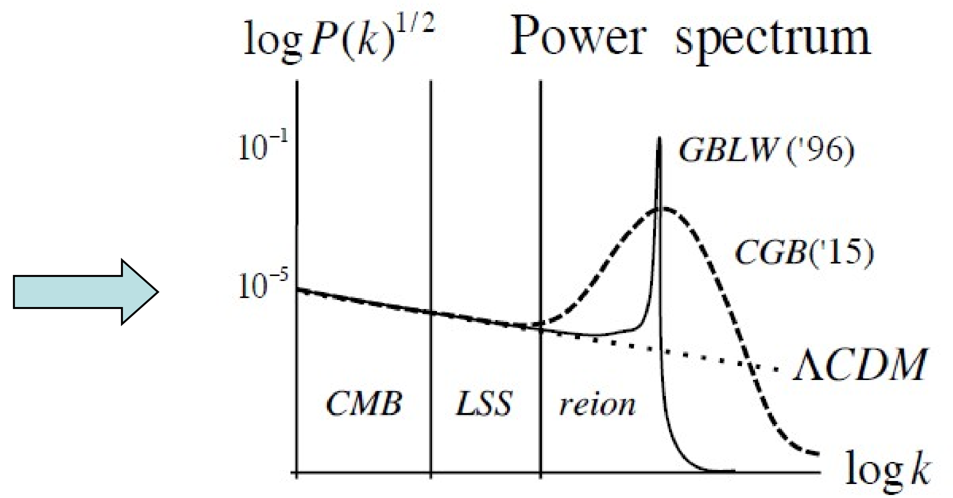
6) Other constraints come from extragalactic photon background (orange), femto-lensing (red), micro-lensing by MACHO (green) and EROS (blue), from wide binaries (light brown), and CMB distortions by FIRAS (cyan) and WMAP3 (purple).

Primordial Black Holes redux

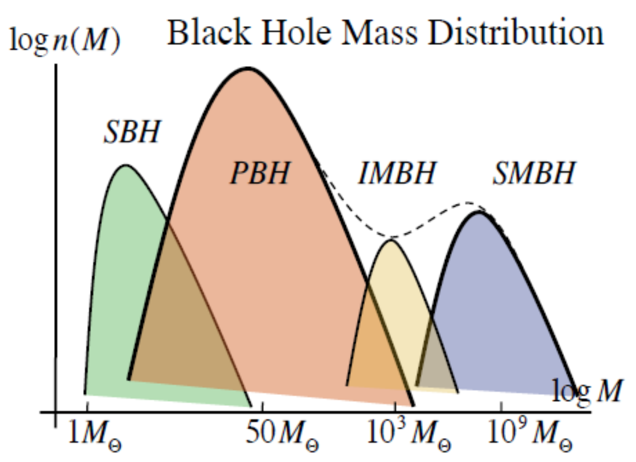
7) Peaks in the spectrum can be formed by inflection points in the potential ($P \sim 1/\varepsilon$, $\varepsilon \rightarrow 0 \rightarrow P>>!$)

1702.08275





8) PBHs can have range of masses (are not "mono-chromatic")



Primordial Black Holes redux

9) They have many potential signatures and side-effects:

1702.08275

i) PBHs have no spin!

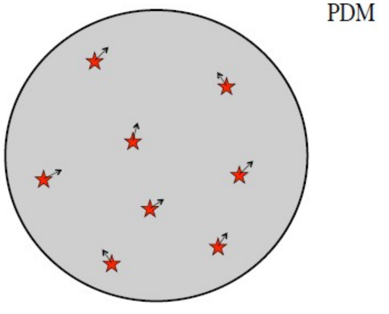
ii) Emission of GWs in binaries and hyperbolic encounters (see GW lecture).

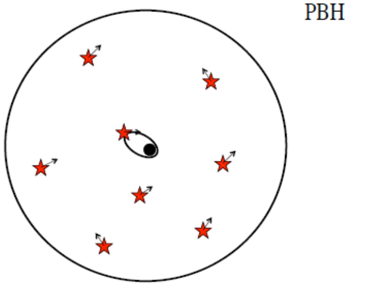
iii) Microlensing of SnIa → possible explanation for superluminal SnIa (or super-Chandrasekhar).

iv) Missing-baryons problem (see Open Problems lecture): PBHs might have eaten up the baryons!

v) Stochastic background of GWs: uniform distro of GW sources creates a background → could be visible by LISA!

vi) Anomalous motion of stars: compare PDM vs PBH-DM (could be seen by GAIA).





10) Very hot topic, lots of activity happening also here at the IFT!

Main points of the lecture

- Theory of Gauge Invariant perturbations
- Transplanckian physics and the power spectrum
- More discussion on Inflation and final notes
- Primordial Black Holes and Inflation
- Baryogenesis
- Summary

Baryogenesis

1) Origin of word baryon:

baryon

/ˈbarɪɒn/

noun PHYSICS

noun: **baryon**; plural noun: **baryons**

a subatomic particle, such as a nucleon or hyperon, that has a mass equal to or greater than that of a proton.

Origin

GREEK
barus
heavy

ENGLISH
-on

→ baryon
1950s

1950s: from Greek *barus* 'heavy' + -on.

Translate baryon to Greek

1. βαρυονίου

Use over time for: baryon

Baryogenesis

1) Origin of word baryon:

baryon
/ˈbarɪɒn/

noun PHYSICS
noun: **baryon**; plural noun: **baryons**

a subatomic particle, such as a nucleon or hyperon, that has a mass equal to or greater than that of a proton.

Origin

GREEK
barus
heavy

ENGLISH
-on

→ baryon
1950s

1950s: from Greek *barus* 'heavy' + -on.

Translate baryon to Greek

1. βαρυονίου

Use over time for: baryon

2) Origin of word genesis:

genesis
/ˈdʒɛnɪsɪs/

noun
noun: **genesis**; plural noun: **genesises**

the origin or mode of formation of something.
"this tale had its genesis in fireside stories"
synonyms: origin, source, root, beginning, commencement, start, outset

Origin

GREEK

ENGLISH
Genesis

→ genesis
early 17th century

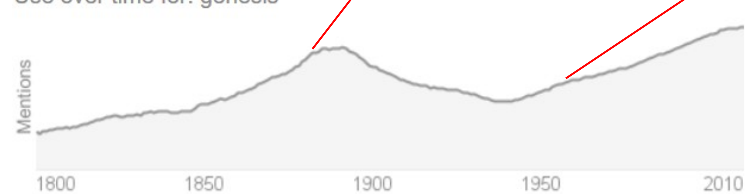
early 17th century: from Greek (see Genesis).

Translate genesis to Greek

noun
1. γένεση
2. γένεσις

Evolution theory started being accepted

Use over time for: genesis

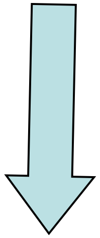


Show less

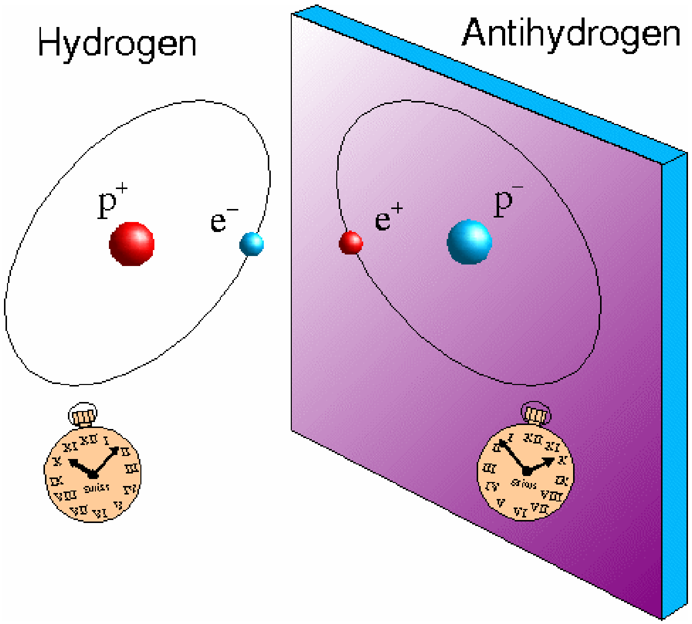
Baryogenesis

1) Motivations to study Baryogenesis:
   i) Understand origin+properties of baryons, d'oh!
   ii) Solve matter-antimatter asymmetry (see below).
   iii) Test extensions of Standard Model, eg GUT models etc.

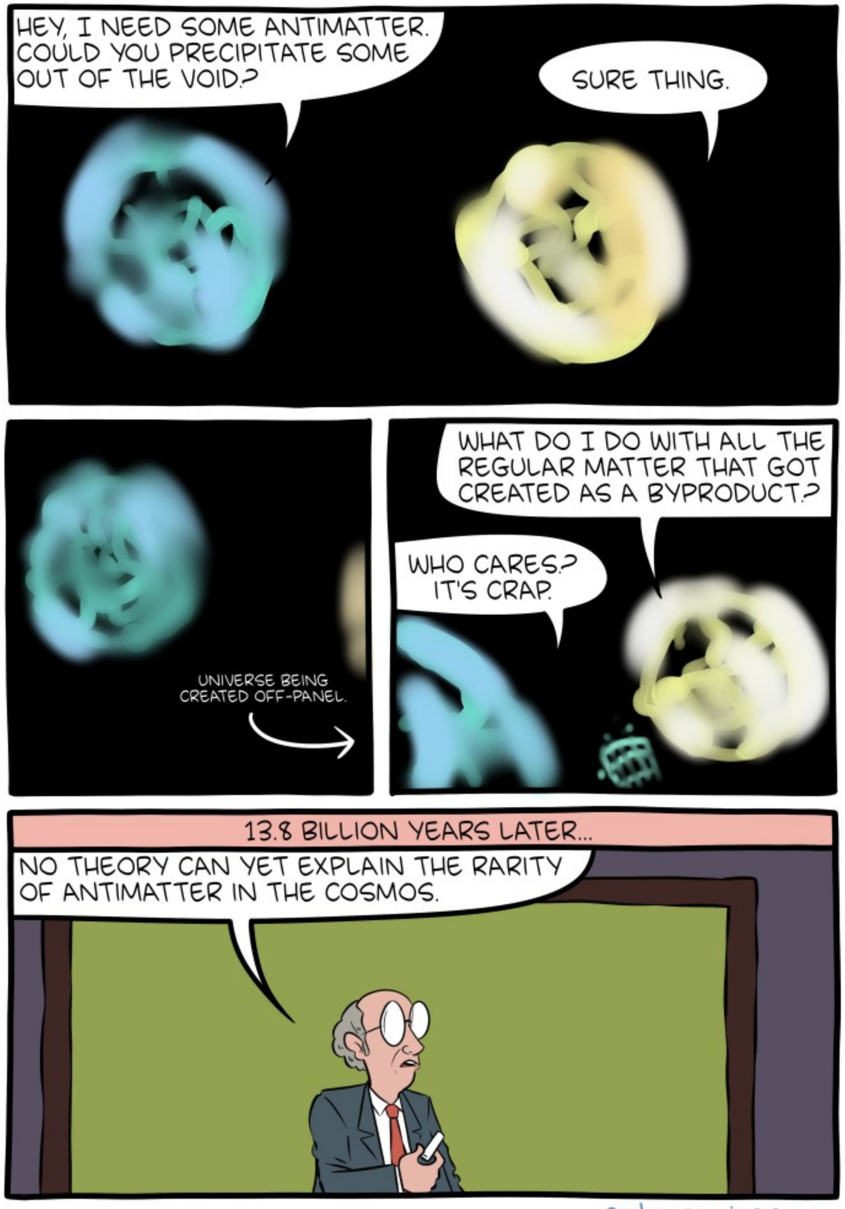
2) Antimatter is same as matter under CPT, but we don't see any in the Universe!!!



Matter-antimatter asymmetry!



Baryogenesis



Baryogenesis

3) Baryon asymmetry and photon to baryon ratio:

$$\eta \equiv \frac{n_B - n_{\overline{B}}}{n_\gamma}$$

Constant as $n_B \sim a^{-3}$ and $n_\gamma \sim a^{-3}$

$$n_\gamma = \frac{1}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3 \int_0^\infty \frac{x^2}{e^x - 1}\,\mathrm{d}\,x = \frac{2\zeta(3)}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3 \approx 20.3\left(\frac{T}{1\mathrm{K}}\right)^3 \mathrm{cm}^{-3}$$

At least at late times... But at early times, and high temperatures, many heavy particles were in thermal equilibrium, which later annihilated to produce more photons but not baryons.

4) Better to use entropy!

$$s \stackrel{\text{def}}{=} \frac{\text{entropy}}{\text{volume}} = \frac{p+\rho}{T} = \frac{2\pi^2}{45} g_* T^3 \quad \Rightarrow \quad \eta \equiv \frac{n_B - n_{\overline{B}}}{s} \approx \text{const.}$$

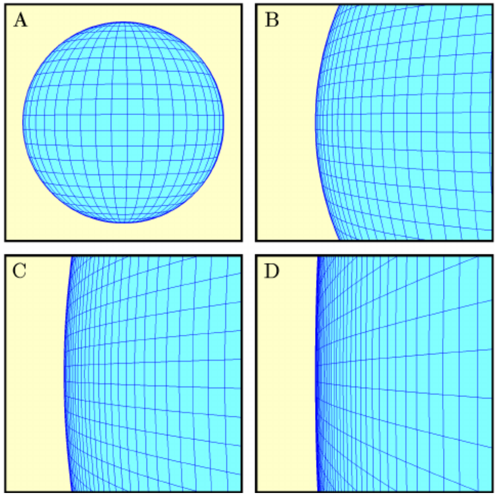
5) Commonly quotes parameter is η_{10}

$$\eta_{10} = 10^{10}\eta = 273.7\,\Omega_B h^2$$

BBN (2014) $\quad \eta_{10} = 6.2 \pm 0.5$

Planck (2015) $\quad \eta_{10} = 6.103 \pm 0.038$

Baryogenesis



6) Baryon asymmetry (BA) is generated after inflation, as inflation washes out all initial asymmetries (also reheating helps)!

Universe grows by ~e^60~10^27→ 1m→ 12 Gly!

7) Way out: Shakharov conditions to have BA

i) Baryon number violating interactions

Obviously require more baryons than antibaryons

ii) C and CP violating interactions

C violation: excess of $b>\bar{b}$ must not balanced by $\bar{b}>b$

CP violation: $b_L>\bar{b}_R$ different from $\bar{b}_L>b_R$

iii) Out of equilibrium interactions

Interactions must not happen in both directions equally $\Gamma(X\rightarrow A+B)=\Gamma(A+B\rightarrow X)$

Baryon number violation

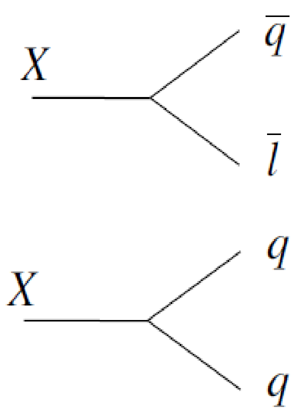
1) Standard model is invariant under B. Proton lifetime > 10^34 yrs! Some B-number violating operators are

$$\frac{qqql}{\Lambda^2}, \quad \frac{d^c u^c u^c e^c}{\Lambda^2}, \qquad \Delta B = \Delta L = 1; \qquad \Delta(B-L) = 0$$

$$p^+ \rightarrow e^+ + \pi^0 \rightarrow e^+ + 2\gamma$$

Example of operators mediating proton decay and typical process

2) GUTs, eg SU(5), create B viol. as quarks and leptons are in the same multiplet!



$$\tau_p \sim \alpha_{GUT}^{-2} M_X^4 m_p^{-5} \; > \; 1.29 \times 10^{34} \text{ yrs}$$

$$\Rightarrow \quad M_X > 10^{16} \; GeV$$

C, CP and CPT symmetries

1) Scalars

$$C: \quad \phi \rightarrow \phi^*$$

$$P: \quad \phi(t,\vec{x}) \rightarrow \pm\phi(t,-\vec{x})$$

$$CP: \quad \phi(t,\vec{x}) \rightarrow \pm\phi^*(t,-\vec{x})$$

2) Fermions

$$C: \quad \psi_L \rightarrow i\sigma_2\psi_R^*, \quad \psi_R \rightarrow -i\sigma_2\psi_L^*, \quad \psi \rightarrow i\gamma_2\psi^*$$

$$P: \quad \psi_L \rightarrow \psi_R(t,-\vec{x}), \quad \psi_R \rightarrow \psi_L(t,-\vec{x}), \quad \psi \rightarrow \gamma^0\psi(t,-\vec{x})$$

$$CP: \quad \psi_L \rightarrow i\sigma_2\psi_R^*(t,-\vec{x}), \quad \psi_R \rightarrow -i\sigma_2\psi_L^*(t,-\vec{x}), \quad \psi \rightarrow i\gamma^2\gamma^0\psi^*(t,-\vec{x})$$

3) Vectors

$$C: \quad A^\mu \rightarrow -A^\mu$$

$$P: \quad A^\mu(t,\vec{x}) \rightarrow (A^0,-\vec{A})(t,-\vec{x})$$

$$CP: \quad A^\mu(t,\vec{x}) \rightarrow (-A^0,\vec{A})(t,-\vec{x})$$

C, CP violationing interactions

1) B violation is not enough! Consider X→ A+B and cc. If C is not broken

$$\Gamma(\overline{X} \rightarrow \overline{Y} + \overline{B}) = \Gamma(X \rightarrow Y + B)$$

2) If C is broken, then the rate increases as

$$\frac{dB}{dt} \propto \Gamma(X \rightarrow Y + B) - \Gamma(\overline{X} \rightarrow \overline{Y} + \overline{B})$$

3) When C holds, then

dB/dt~0

C, CP violationing interactions

4) Consider the decay X→ q_L+q_L and X→q_R and q_R

Under CP $q_L \rightarrow \bar{q}_R$

Under C $q_L \rightarrow \bar{q}_L$

5) C violation implies → $\Gamma(X \rightarrow q_L + q_L) \neq \Gamma(\bar{X} \rightarrow \bar{q}_L + \bar{q}_L)$

6) CP conservation implies $\Gamma(X \rightarrow q_L + q_L) = \Gamma(\bar{X} \rightarrow \bar{q}_R + \bar{q}_R)$

$\Gamma(X \rightarrow q_R + q_R) = \Gamma(\bar{X} \rightarrow \bar{q}_L + \bar{q}_L)$

7) Which for equal X and Xb means no asymmetry because

$$\Gamma(X \rightarrow q_L + q_L) + \Gamma(X \rightarrow q_R + q_R) = \Gamma(\bar{X} \rightarrow \bar{q}_R + \bar{q}_R) + \Gamma(\bar{X} \rightarrow \bar{q}_L + \bar{q}_L)$$

Out of equilibrium interactions and SM

1) All interactions must be out of equilibrium (OoE), since for X→ Y+B:

$$\Gamma(X \rightarrow Y + B) = \Gamma(Y + B \rightarrow X)$$

This means rates in both directions are the same → no asymmetry!

2) In SM we have violations of B, C and CP but no heavy particle to decay OoE:

i) B violation via chiral anomalies (t'Hooft 76)

ii) CP violation possible with complex phase in CKM matrix for quark mixing
(but too small as it's suppressed by quark masses)

iii) C violation as anti-neutrino always right handed!

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$
$$\pi^- \rightarrow \mu^- + \overline{\nu_\mu}$$

Always right-handed!

Summary

1) Inflation is a part of the Standard Cosmological model that can give unique & verifiable predictions.

2) Inflation can probe high energy (also transplanckian!) physics.

3) Planck 2018 has provided stringent constraints, but still room for improvement!

4) Baryogenesis→ matter-antimatter asymmetry: probe of new BSM physics

5) We need 3 Sakharov conditions (off-equilibrium, Baryon and C+CP violation).