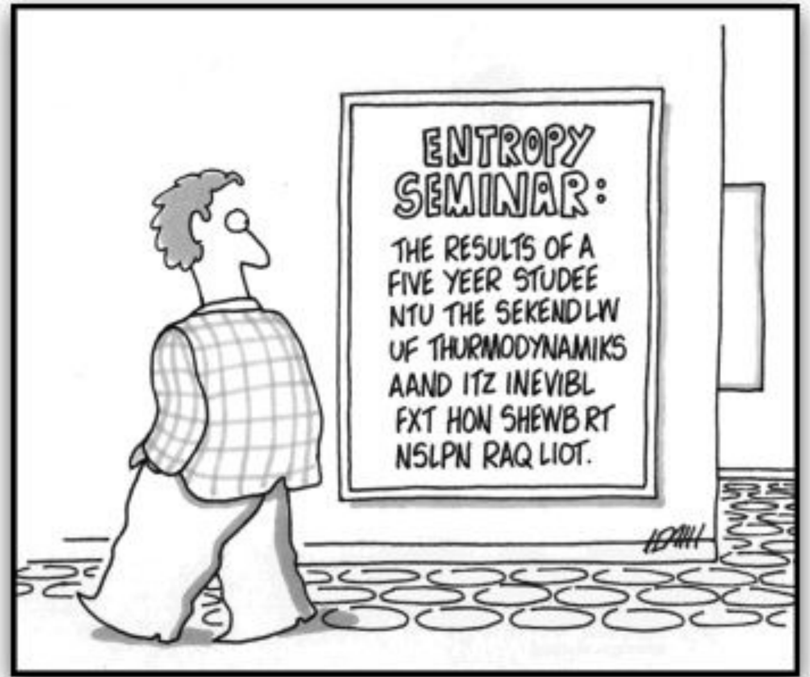
Alexander Knebe (*Universidad Autonoma de Madrid*)



- the hot big bang model
- thermal equilibrium
- entropy of the Universe
- decoupling
- matter radiation equality

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**any heat flow would define a preferred direction (✗ isotropy ✗)*

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$$\Rightarrow \quad \rho R^{3(1+\omega)} = const.$$

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Stefan-Boltzmann law

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Stefan-Boltzmann law

<u>matter</u>

rest-mass energy + thermal energy

$$d(\rho c^2 R^3) = d((nm_p c^2 + 3nk_B T/2)R^3)$$

$$pd(R^3) = nk_B T d(R^3)$$

ideal gas equation

$$d(nm_p R^3 c^2 + 3nk_B TR^3/2) = -3nk_B TR^2 dR$$

$$\boxed{N = nR^3 = const.}$$

$$d(Nm_p c^2 + \frac{3}{2}Nk_B T) = -3Nk_B TR^{-1}dR$$

$$\frac{3}{2}Nk_B dT = -3Nk_B TR^{-1}dR$$

$$\frac{dT}{T} = -2\frac{dR}{R} \qquad \Rightarrow T \propto R^{-2} \quad \propto z^{-2}$$

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the Universe becomes denser *and hotter* when going backwards in time: *Hot Big Bang Model*

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(exercise)

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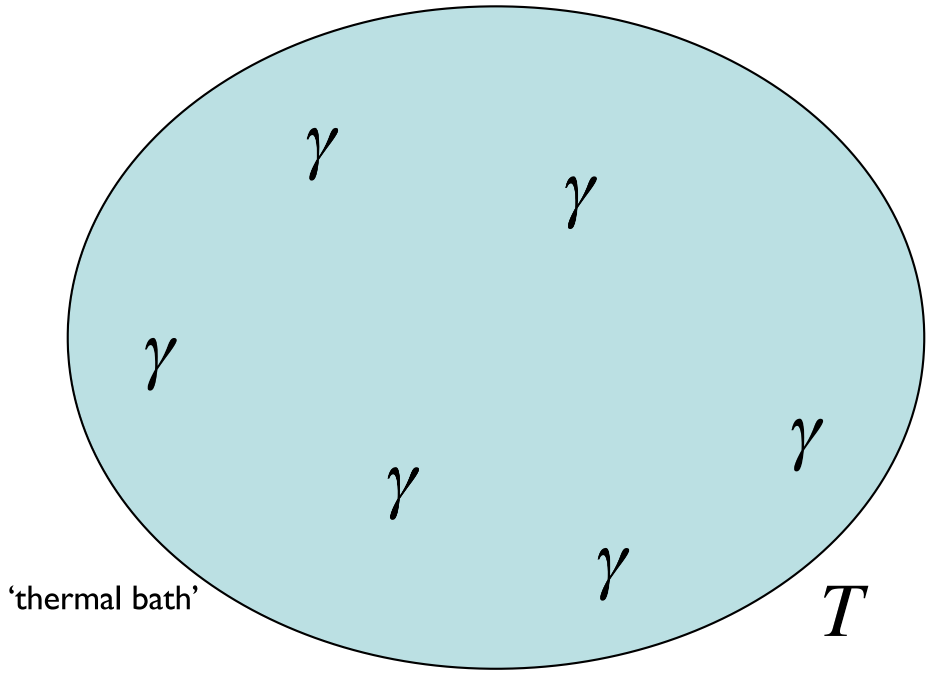
- cosmic plasma in equilibrium



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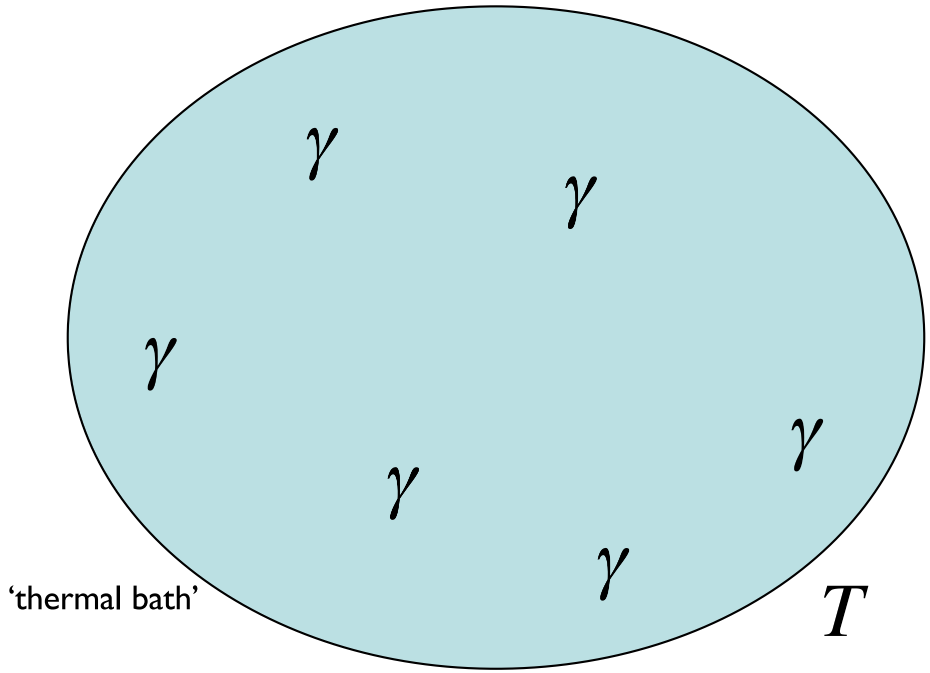

let's add neutrinos, electrons, positions, and protons...

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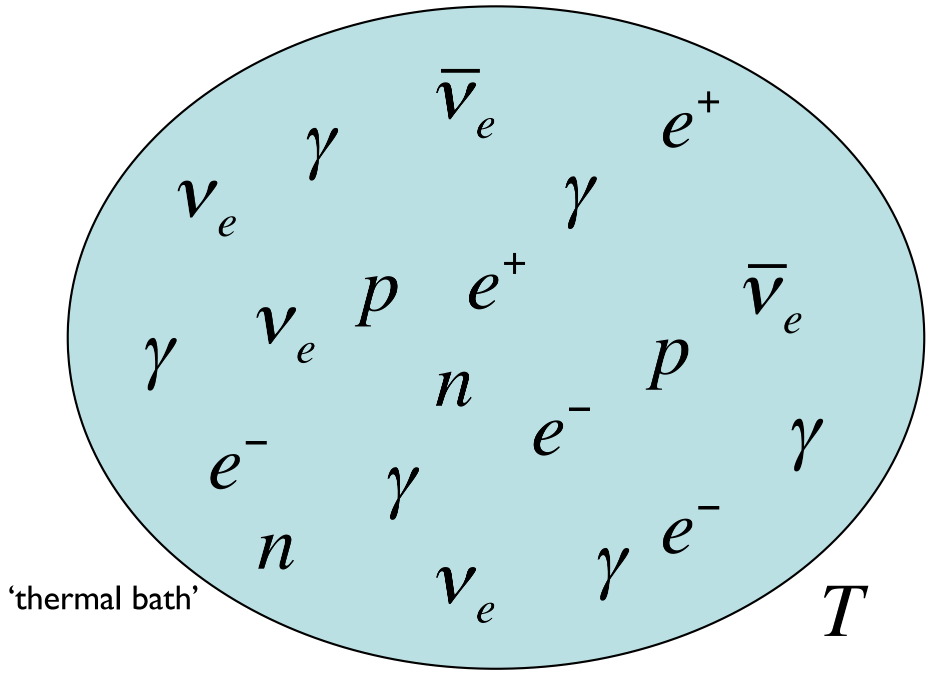

equilibrium maintained by:

- weak interaction

$$\nu_e + n \Leftrightarrow p + e^-$$
$$e^+ + n \Leftrightarrow p + \bar{\nu}_e$$
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- Thomson scattering:

$$e^- + \gamma \Leftrightarrow e^- + \gamma$$

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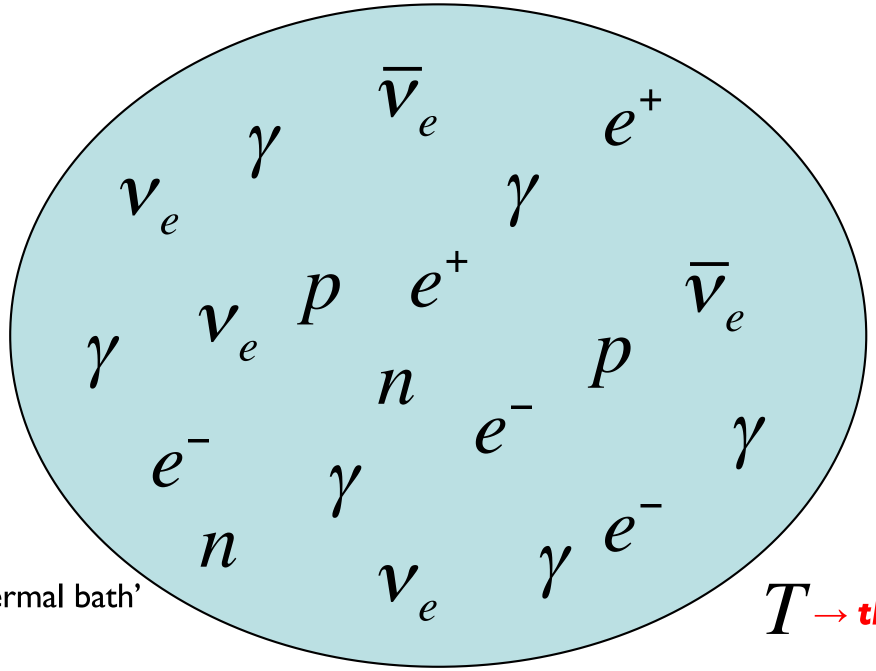

'thermal bath'

T → ***the dominant species determines the equilibrium temperature!***

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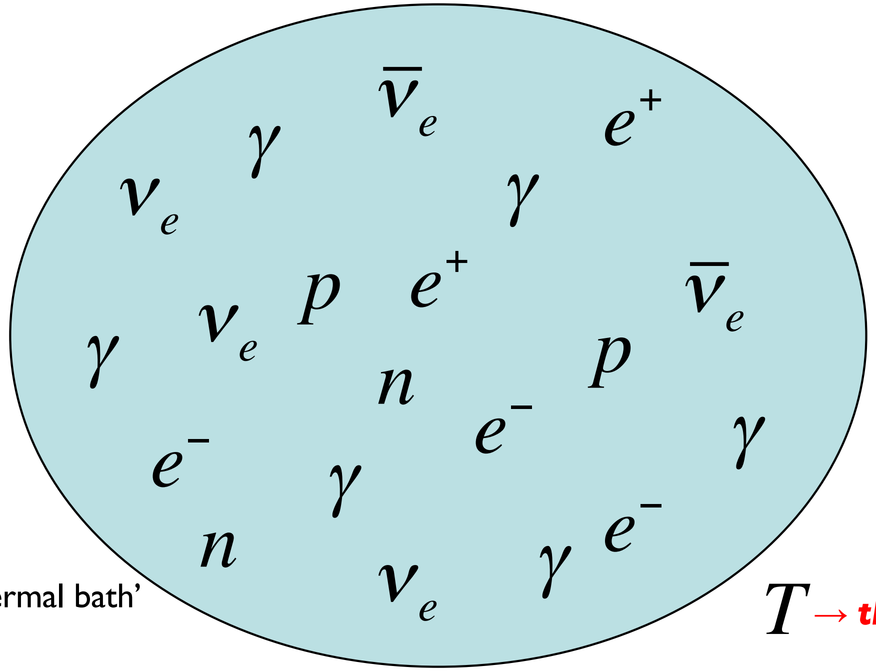

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the dominant species is *photons*

⇒ the whole bath evolves like *radiation* $T \propto R^{-1}$

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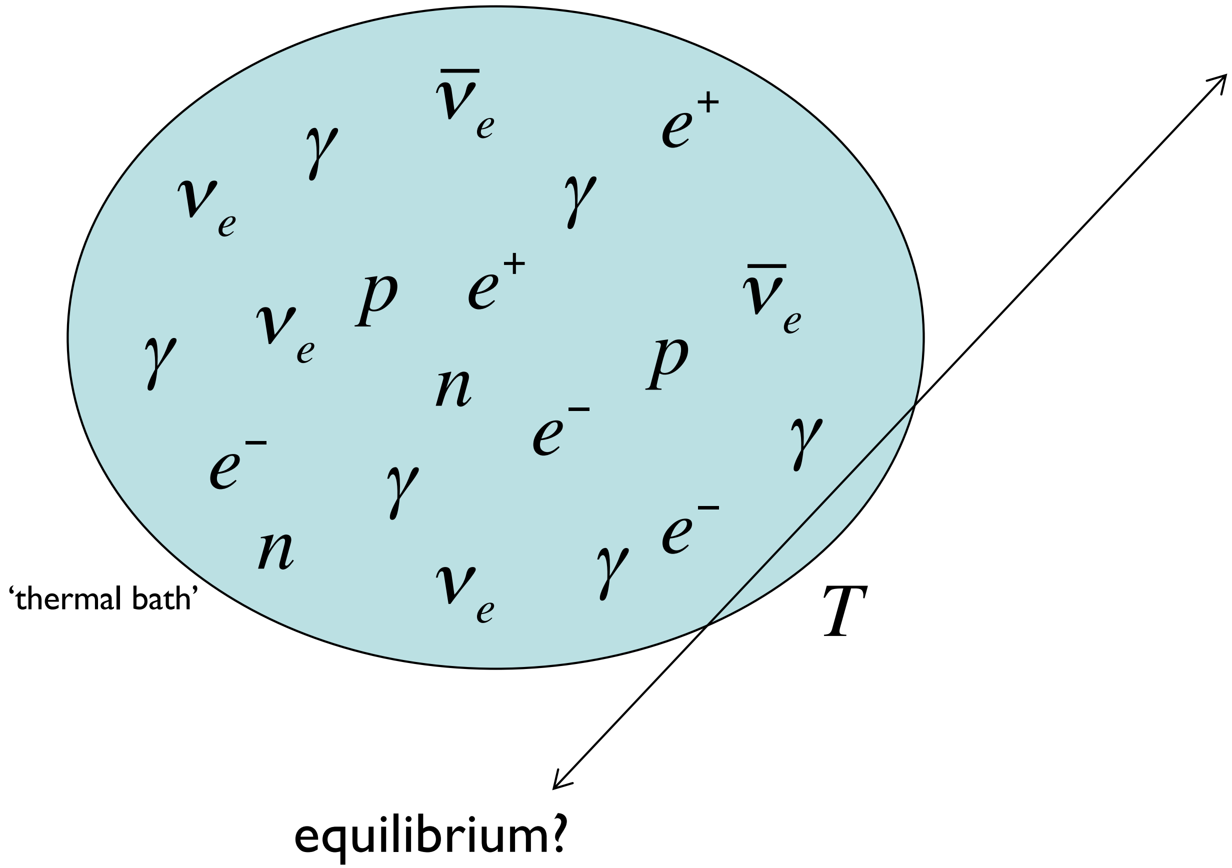

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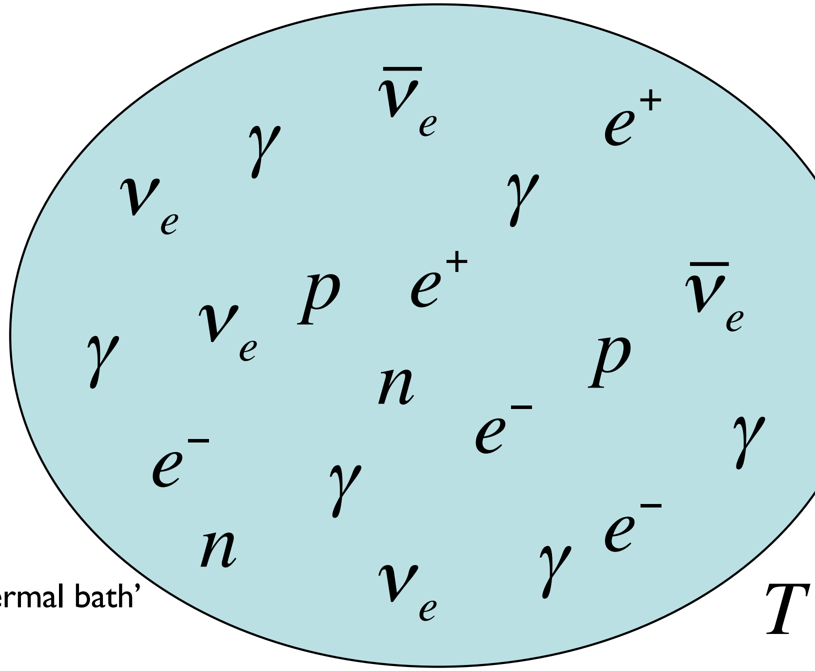

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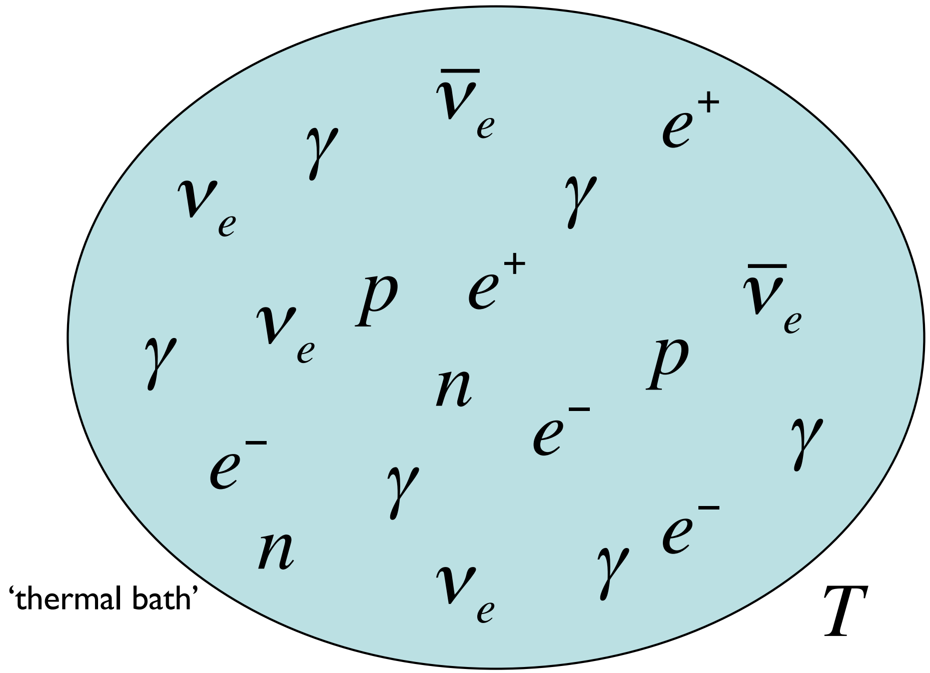
- types of equilibrium:
  - kinetic equilibrium: efficient energy and momentum exchange of particles
  - chemical equilibrium: chemical reactions between particles are in equilibrium
  - thermal equilibrium: kinetic + chemical equilibrium

- cosmic plasma in equilibrium



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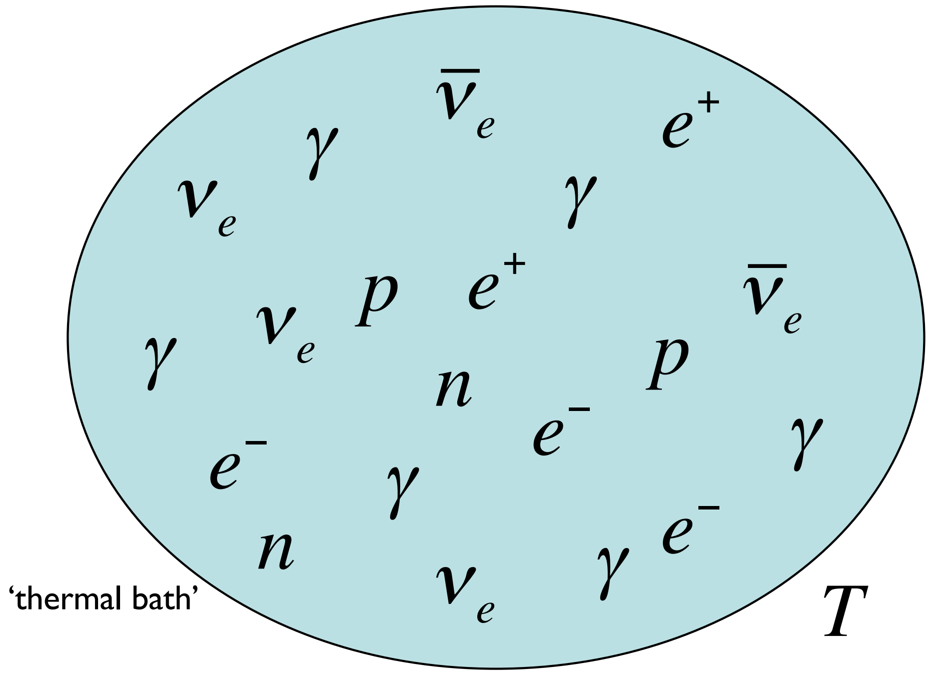
what more could we add?

- cosmic plasma in equilibrium



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- any kind of interaction…

$$\Gamma_c \propto n\,\sigma\,v$$

n : number density
σ : interaction cross-section
v : relative velocity

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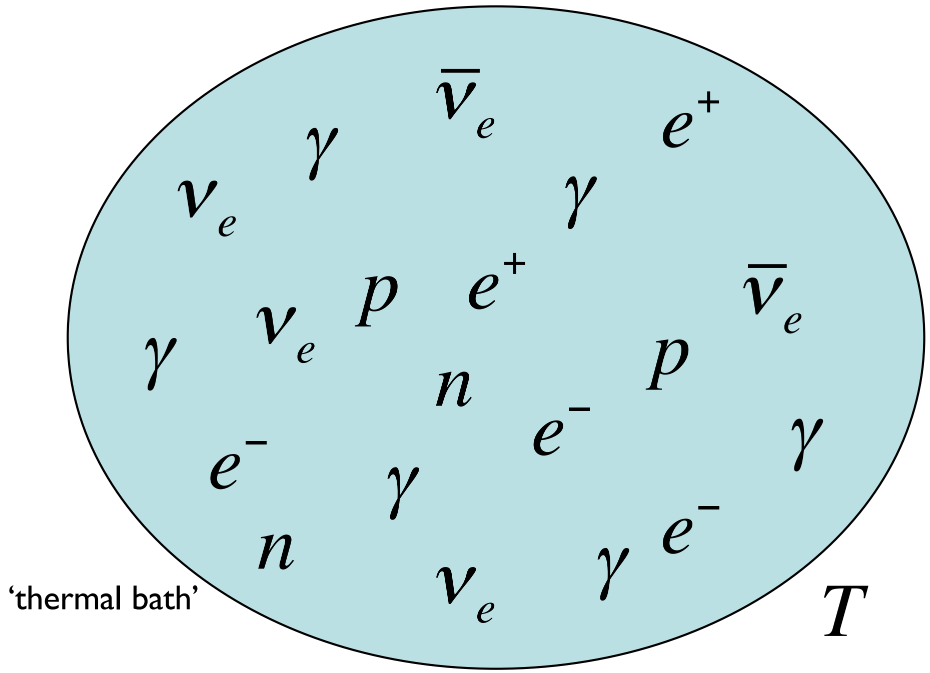

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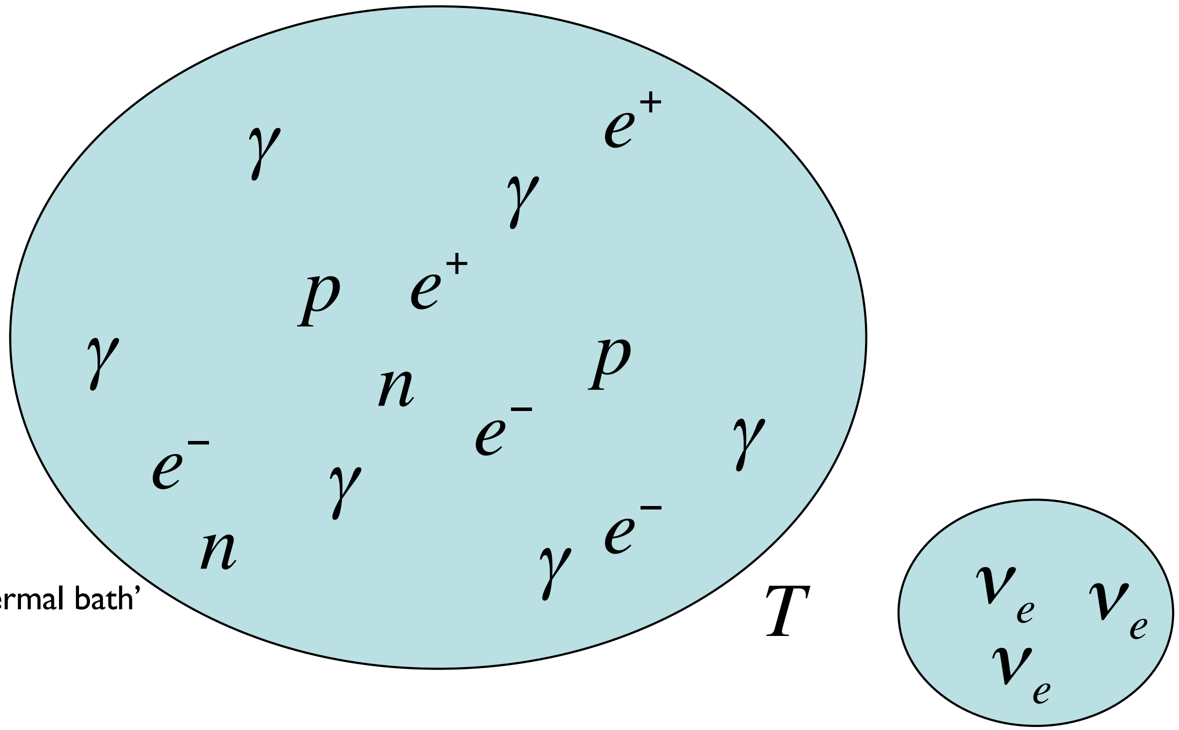

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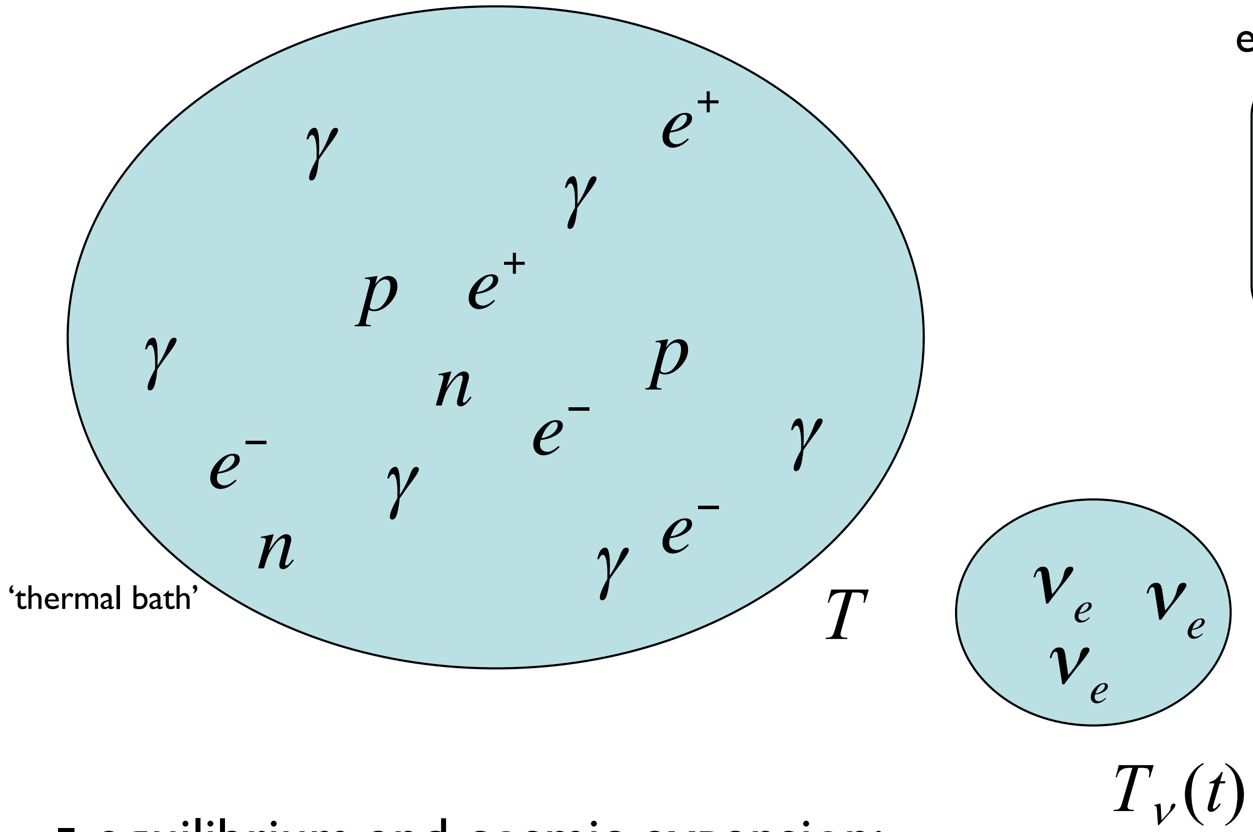

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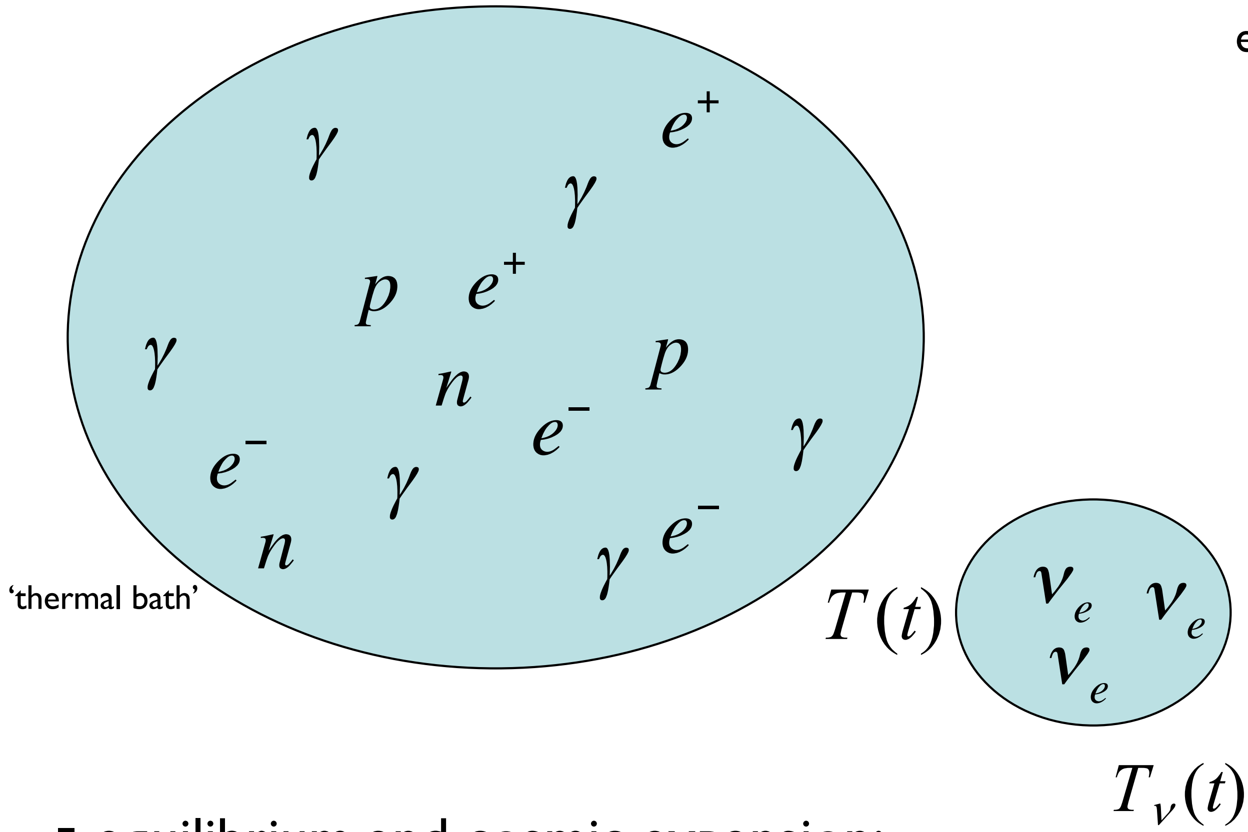

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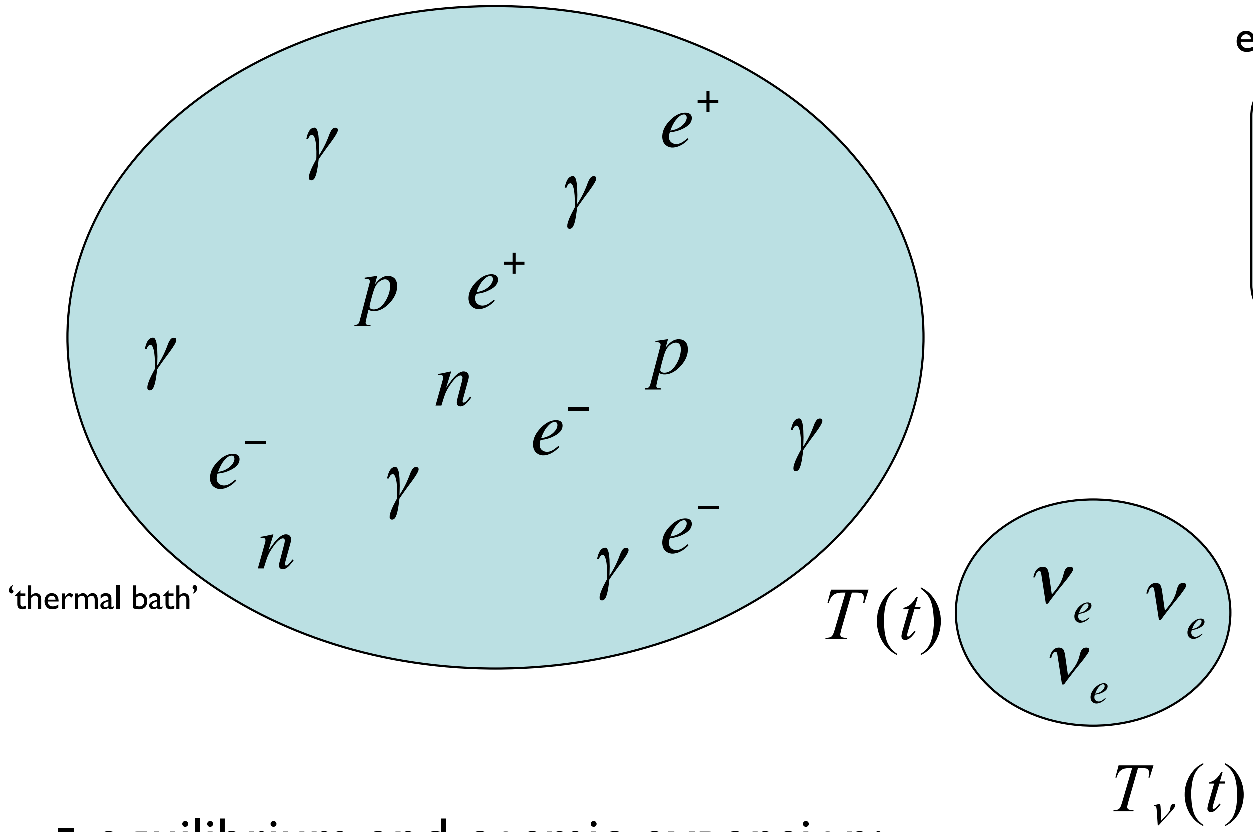

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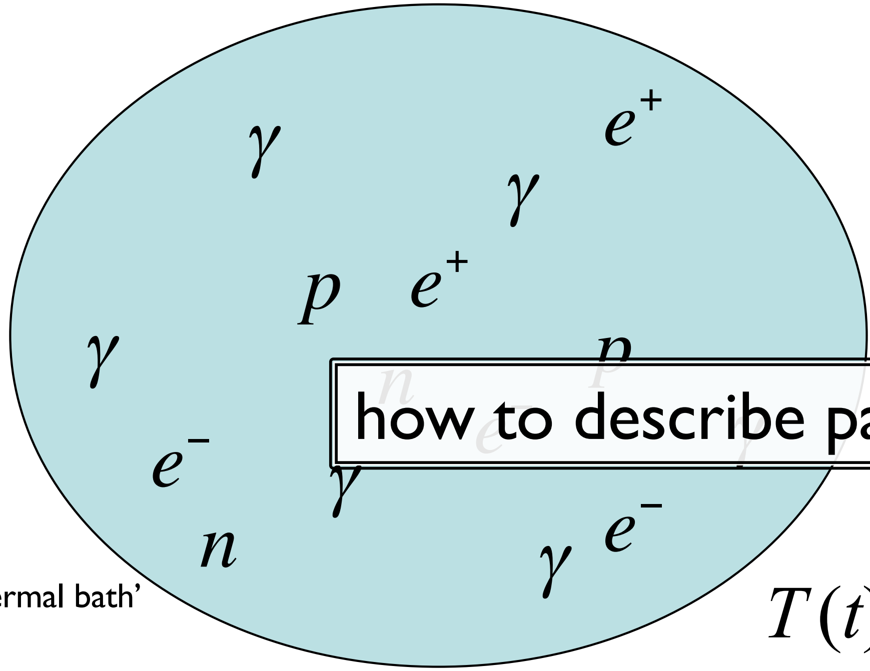
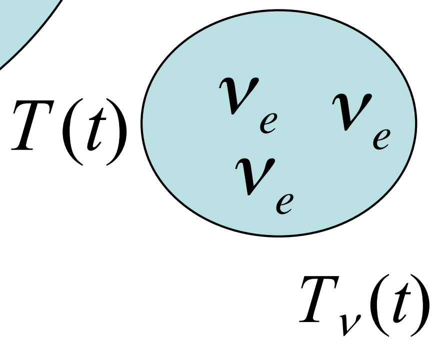


‘thermal bath’



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how to describe particles in equilibrium?

- any kind of interaction…

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- particles…

$f(p)$: phase space distribution function*

*homogeneity drops dependence on $\vec{x}$, isotropy gives dependence on only $p = |\vec{p}|$

- particles…

allows us to calculate all that is of relevance!

↑

$f(p)$: phase space distribution function

- particles…

number density $$n = \frac{g}{(2\pi\hbar)^3}\int f(p)4\pi p^2\, dp$$

energy density $$\rho c^2 = \frac{g}{(2\pi\hbar)^3}\int E(p)\, f(p)4\pi p^2\, dp$$

pressure $$P = \frac{g}{(2\pi\hbar)^3}\int \frac{p^2c^2}{3E}\, f(p)4\pi p^2\, dp$$ $$E^2 = |\vec{p}c|^2 + m^2c^4$$

$f(p)$: phase space distribution function

g : statistical weight

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...but how to get it?

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integro-differential equation for $f(p)$ (Boltzmann equation, more later in Computational Cosmology lecture):

$$\frac{dn}{dt} + 3Hn = \int C\left[f(\vec{p})\right]d^3p$$

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relativistic:

- …in kinetic equilibrium $$f(p) = \frac{1}{e^{(E-\mu)/k_BT} \pm 1}$$

• "+" sign: Fermi-Dirac distribution (FERMIONS)
• "–" sign: Bose-Einstein distribution (BOSONS)

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relativistic:
$$f(p) = \frac{1}{e^{(E-\mu)/k_BT} \pm 1}$$

non-relativistic ($T<E-\mu$):
$$f(p) \approx e^{-\left(mc^2+p^2/2mc^2-\mu\right)/k_BT}$$

- "+" sign: Fermi-Dirac distribution (FERMIONS)
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$$\left(E = \sqrt{|\vec{p}c|^2 + m^2c^4} = mc^2\sqrt{p^2/2mc^2+1} \approx mc^2 + p^2/2mc^2\right)$$

- particles…

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non-relativistic ($T<E-\mu$):
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Note: each particle species m_i, μ_i, T_i has its own distribution function…

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$$P = \frac{g}{(2\pi\hbar)^3}\int \frac{p^2c^2}{3E}\,f(p)4\pi p^2\,dp$$

$$E^2 = |\vec{p}c|^2 + m^2c^4$$

- …in kinetic equilibrium

relativistic:
$$f(p) = \frac{1}{e^{(E-\mu)/k_BT} \pm 1}$$

non-relativistic ($T<E-\mu$):
$$f(p) \approx e^{-\left(mc^2+p^2/2mc^2-\mu\right)/k_BT}$$

Note: each particle species m_i, μ_i, T_i has its own distribution function…

- particles…

number density $$n = \frac{g}{(2\pi\hbar)^3}\int f(p)4\pi p^2\,dp$$

energy density $$\rho c^2 = \frac{g}{(2\pi\hbar)^3}\int E(p)\,f(p)4\pi p^2\,dp$$

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relativistic:

- …in kinetic equilibrium
$$f(p) = \frac{1}{e^{(E-\mu)/k_BT} \pm 1}$$

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0^*$)…

number density
$$n = \frac{g}{2\pi^2\hbar^3}\int \frac{p^2}{e^{c\sqrt{p^2+m^2c^2}/k_BT} \pm 1}dp$$

energy density
$$\rho c^2 = \frac{g}{2\pi^2\hbar^3}\int c\sqrt{p^2+m^2c^2}\,\frac{p^2}{e^{c\sqrt{p^2+m^2c^2}/k_BT} \pm 1}dp$$

pressure
$$P = \frac{g}{6\pi^2\hbar^3}\int \frac{p^2c^2}{c\sqrt{p^2+m^2c^2}}\,\frac{p^2}{e^{c\sqrt{p^2+m^2c^2}/k_BT} \pm 1}dp$$

*In the early universe $\mu << T$ ($\mu_\gamma = 0$ anyways).

Further, for relativistic particles which are continuously created and annihilated there is no net change in particle number and hence their chemical potential can be neglected in general.

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $\boldsymbol{m \ll T}$)…

number density $$n = \frac{g}{2\pi^2\hbar^3}\int \frac{p^2}{e^{c\sqrt{p^2+m^2c^2}/k_BT} \pm 1}dp$$

energy density $$\rho c^2 = \frac{g}{2\pi^2\hbar^3}\int c\sqrt{p^2+m^2c^2}\, \frac{p^2}{e^{c\sqrt{p^2+m^2c^2}/k_BT} \pm 1}dp$$

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Further, for relativistic particles which are continuously created and annihilated there is no net change in particle number and hence their chemical potential can be neglected in general.

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $m << T$)…

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$$n = \frac{g}{2\pi^2\hbar^3}\int \frac{p^2}{e^{c\sqrt{p^2}/k_B T} \pm 1} dp$$

energy density
$$\rho c^2 = \frac{g}{2\pi^2\hbar^3}\int c\sqrt{p^2}\, \frac{p^2}{e^{c\sqrt{p^2}/k_B T} \pm 1} dp$$

pressure
$$P = \frac{g}{6\pi^2\hbar^3}\int c\frac{p^2}{\sqrt{p^2}}\, \frac{p^2}{e^{c\sqrt{p^2}/k_B T} \pm 1} dp$$

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $m << T$)…

number density
$$n = \frac{g}{2\pi^2\hbar^3}\int \frac{p^2}{e^{cp/k_BT} \pm 1}dp$$

energy density
$$\rho c^2 = \frac{g}{2\pi^2\hbar^3}\int cp\,\frac{p^2}{e^{cp/k_BT} \pm 1}dp$$

pressure
$$P = \frac{g}{6\pi^2\hbar^3}\int cp\,\frac{p^2}{e^{cp/k_BT} \pm 1}dp$$

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pressure $$P = \frac{g}{6\pi^2\hbar^3}\int cp\,\frac{p^2}{e^{cp/k_BT} \pm 1}dp$$

combine to eliminate integral and get P=…

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $m \ll T$)…

number density $$n = \frac{g}{2\pi^2\hbar^3}\int \frac{p^2}{e^{cp/k_BT} \pm 1}\,dp$$

energy density $$\rho c^2 = \frac{gc}{2\pi^2\hbar^3}\int \frac{p^3}{e^{cp/k_BT} \pm 1}\,dp$$

pressure $$P = \frac{1}{3}\rho c^2$$

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pressure
$$P = \frac{1}{3}\rho c^2$$

$$\xi = cp / k_BT \quad \Rightarrow \quad p = k_BT\xi / c\,, \quad dp = k_BTd\xi / c$$

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $m << T$)…

number density
$$n = \frac{g}{2\pi^2\hbar^3}\int\left(\frac{k_B T}{c}\right)^2 \frac{\xi^2}{e^\xi \pm 1}\frac{k_B T}{c}d\xi$$

energy density
$$\rho c^2 = \frac{gc}{2\pi^2\hbar^3}\int\left(\frac{k_B T}{c}\right)^3 \frac{\xi^3}{e^\xi \pm 1}\frac{k_B T}{c}d\xi$$

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- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $m << T$)…

number density $$n = \frac{g}{2\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 T^3 \int \frac{\xi^2}{e^\xi \pm 1} d\xi$$

energy density $$\rho c^2 = \frac{g}{2\pi^2}\frac{k_B^4}{\hbar^3 c^3} T^4 \int \frac{\xi^3}{e^\xi \pm 1} d\xi$$

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$$\int \frac{\xi^n}{e^\xi - 1} d\xi = \Gamma(n+1)\zeta(n+1)$$

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$$\int \frac{\xi^2}{e^\xi + 1} d\xi = \frac{3}{4}\int \frac{\xi^n}{e^\xi - 1} d\xi$$

$$\int \frac{\xi^3}{e^\xi + 1} d\xi = \frac{7}{8}\int \frac{\xi^n}{e^\xi - 1} d\xi$$

$$\frac{1}{e^\xi + 1} = \frac{1}{e^\xi - 1} - \frac{2}{e^{2\xi} - 1}$$

because of yet another substitution $u = 2\xi$…

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $m << T$)…

number density
$$n = \frac{g}{2\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 T^3 \int \frac{\xi^2}{e^\xi \pm 1} d\xi$$

energy density
$$\rho c^2 = \frac{g}{2\pi^2}\frac{k_B^4}{\hbar^3 c^3} T^4 \int \frac{\xi^3}{e^\xi \pm 1} d\xi$$

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$$\int \frac{\xi^3}{e^\xi + 1} d\xi = \frac{7}{8}\int \frac{\xi^n}{e^\xi - 1} d\xi$$

$n = 2$
$\Gamma(n) = (n-1)!$ $\Rightarrow$

$= 2\zeta(3)$

$= \frac{3}{4} 2\zeta(3)$

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $m << T$)…

number density
$$n = \frac{g}{2\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 T^3 \left[\frac{3}{4}\right] 2\zeta(3)$$

energy density
$$\rho c^2 = \frac{g}{2\pi^2}\frac{k_B^4}{\hbar^3 c^3} T^4 \int \frac{\xi^3}{e^\xi \pm 1} d\xi$$

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$$n = 2$$
$$\Gamma(n) = (n-1)! \quad \Rightarrow$$

$$= 2\zeta(3)$$

$$= \frac{3}{4} 2\zeta(3)$$

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $m<<T$)…

number density
$$n = \left[\frac{3}{4}\right]\frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$$

energy density
$$\rho c^2 = \frac{g}{2\pi^2}\frac{k_B^4}{\hbar^3 c^3}T^4\int\frac{\xi^3}{e^\xi \pm 1}d\xi$$

pressure
$$P = \frac{1}{3}\rho c^2$$

$$\int\frac{\xi^n}{e^\xi - 1}d\xi = \Gamma(n+1)\zeta(n+1)$$

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$$\int\frac{\xi^3}{e^\xi + 1}d\xi = \frac{7}{8}\int\frac{\xi^n}{e^\xi - 1}d\xi$$

$$n = 3$$
$$\Gamma(n) = (n-1)! \quad \Rightarrow$$

$$= 6\zeta(4) = 6\frac{\pi^4}{90}$$

$$= \frac{7}{8}6\zeta(4) = \frac{7}{8}6\frac{\pi^4}{90}$$

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $m << T$)…

number density $$n = \left[\frac{3}{4}\right]\frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$$

energy density $$\rho c^2 = \left[\frac{7}{8}\right]\frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3} gT^4$$

pressure $$P = \frac{1}{3}\rho c^2$$

- ***relativistic*** particles in kinetic equilibrium ($\mu = 0$, $m << T$)…

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$$n = \left[\frac{3}{4}\right]\frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$$

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$$P = \frac{1}{3}\rho c^2$$

$$n = \frac{g}{(2\pi\hbar)^3}\int f(p) 4\pi p^2 \, dp$$

$$\rho c^2 = \frac{g}{(2\pi\hbar)^3}\int E(p)\, f(p) 4\pi p^2 \, dp$$

$$P = \frac{g}{(2\pi\hbar)^3}\int \frac{p^2 c^2}{3E}\, f(p) 4\pi p^2 \, dp$$

$$f(p) = \frac{1}{e^{(E-\mu)/k_B T} \pm 1}$$

- particles…

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$$E^2 = |\vec{p}c|^2 + m^2c^4$$

- …in kinetic equilibrium

non-relativistic ($T<E-\mu$):

$$f(p) \approx e^{-\left(mc^2+p^2/2mc^2-\mu\right)/k_BT}$$

- ***non-relativistic*** particles in kinetic equilibrium ($m>>T$)…

number density

$$n = g\left(\frac{mk_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2} e^{-\left(mc^2-\mu\right)/k_B T}$$

energy density

$$\rho c^2 = nmc^2 + \frac{3}{2} nk_B T$$

pressure

$$P = nk_B T$$

- particles in kinetic equilibrium

	bosons	fermions	
number density	$n = \frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = \frac{3}{4}\frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = g\left(\frac{mk_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2} e^{-\left(mc^2-\mu\right)/k_B T}$
energy density	$\rho c^2 = \frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3} gT^4$	$\rho c^2 = \frac{7}{8}\frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3} gT^4$	$\rho c^2 = nmc^2 + \frac{3}{2} nk_B T$
pressure	$P = \frac{1}{3}\rho c^2$	$P = \frac{1}{3}\rho c^2$	$P = nk_B T$
	(*non-degenerate relativistic gas*) $k_B T >> mc^2, \mu = 0$		**(*non-relativistic gas*)** $k_B T << mc^2$

- particles in kinetic equilibrium

	bosons	fermions	
number density	$n = \frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = \frac{3}{4}\frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = g\left(\frac{mk_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2} e^{-\left(mc^2-\mu\right)/k_B T}$
energy density	$\rho c^2 = \frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3} gT^4$	$\rho c^2 = \frac{7}{8}\frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3} gT^4$	$\rho c^2 = nmc^2 + \frac{3}{2} nk_B T$
pressure	$P = \frac{1}{3}\rho c^2$	$P = \frac{1}{3}\rho c^2$	$P = nk_B T$

does anything look familiar here?

(*non-degenerate relativistic gas*)
$k_B T >> mc^2,\ \mu = 0$

(*non-relativistic gas*)
$k_B T << mc^2$

- particles in kinetic equilibrium

	bosons	fermions	
number density	$n = \frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = \frac{3}{4}\frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = g\left(\frac{mk_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2} e^{-(mc^2-\mu)/k_B T}$
energy density	$\rho c^2 = \frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3} gT^4$	$\rho c^2 = \frac{7}{8}\frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3} gT^4$	$\rho c^2 = nmc^2 + \frac{3}{2} nk_B T$
pressure	$P = \frac{1}{3}\rho c^2$	$P = \frac{1}{3}\rho c^2$	$P = nk_B T$
	(non-degenerate relativistic gas) $k_B T >> mc^2$, $\mu = 0$		**(non-relativistic gas)** $k_B T << mc^2$

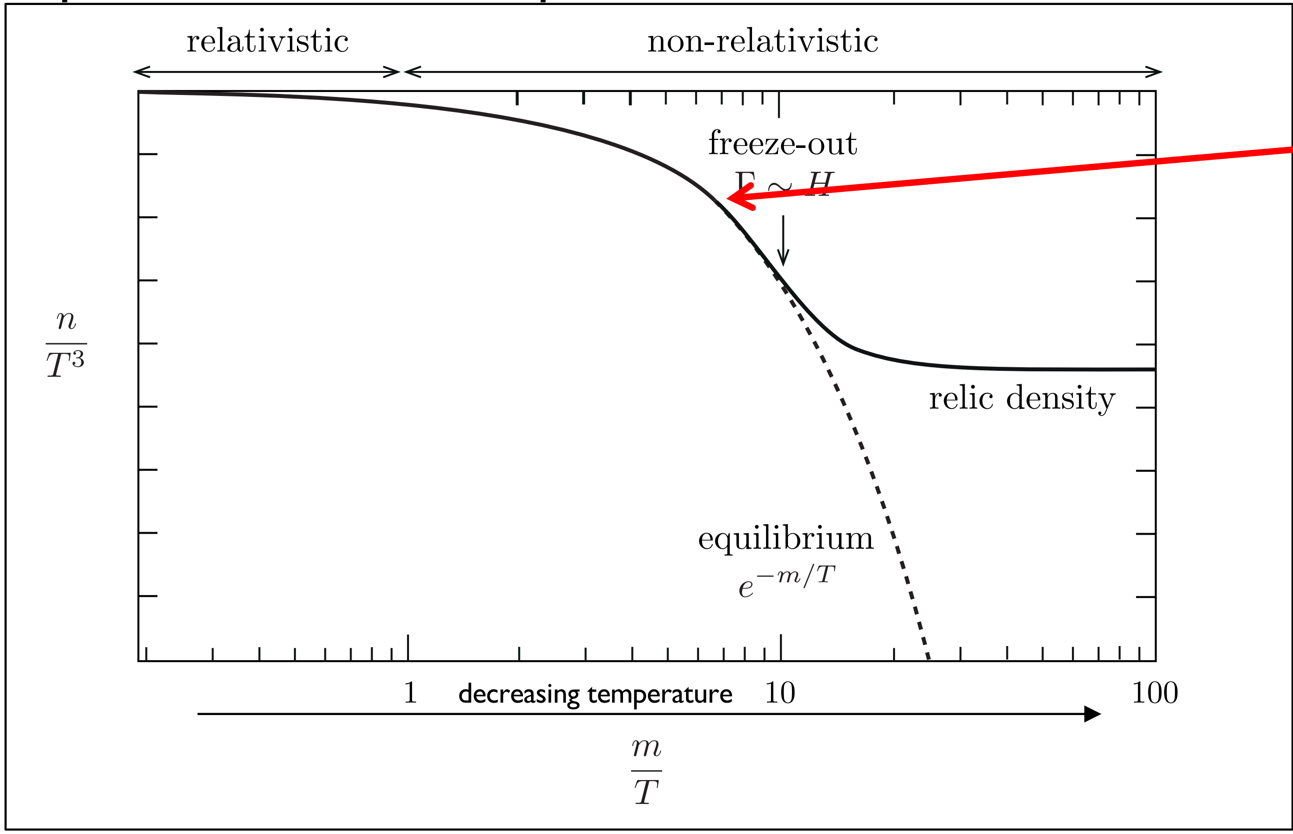
radiation energy $\propto T^4$ (= Stefan-Boltzmann law)

- particles in kinetic equilibrium

	bosons	fermions	
number density	$n = \frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = \frac{3}{4}\frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = g\left(\frac{mk_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2} e^{-(mc^2-\mu)/k_BT}$
energy density	$\rho c^2 = \frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3} gT^4$	$\rho c^2 = \frac{7}{8}\frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3} gT^4$	$\rho c^2 = nmc^2 + \frac{3}{2} nk_B T$
pressure	$P = \frac{1}{3}\rho c^2$	$P = \frac{1}{3}\rho c^2$	$P = nk_B T$
	(non-degenerate relativistic gas) $k_BT >> mc^2,\ \mu = 0$		**(non-relativistic gas)** $k_BT << mc^2$

number density of non-relativistic particles is exponentially suppressed

- particles in kinetic equilibrium



$$n = g\left(\frac{mk_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2} e^{-\left(mc^2-\mu\right)/k_BT}$$

$$\rho c^2 = nmc^2 + \frac{3}{2} nk_BT$$

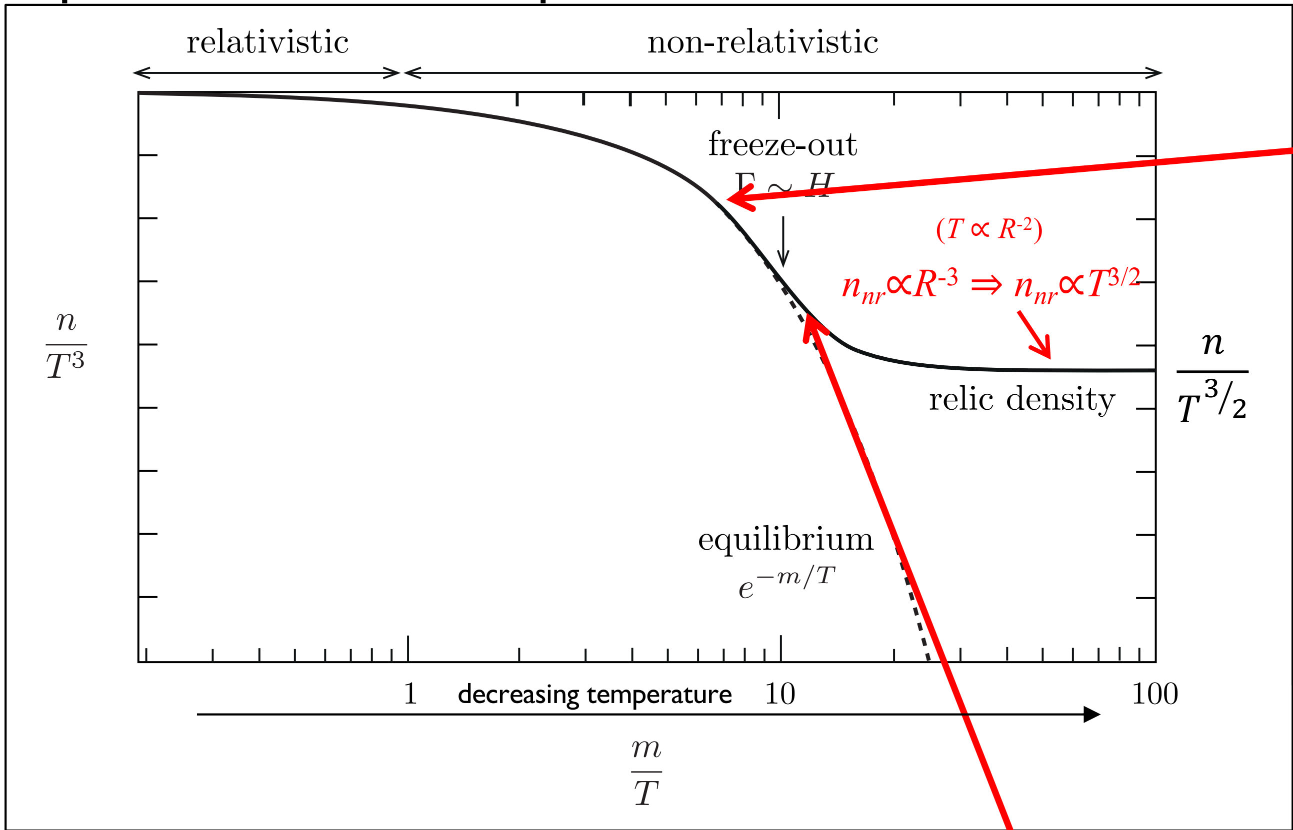
$$P = nk_BT$$

(non-relativistic gas)

$\boldsymbol{k_BT \ll mc^2}$

number density of non-relativistic particles is exponentially suppressed

- particles in kinetic equilibrium



$$n = g\left(\frac{mk_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2} e^{-\left(mc^2-\mu\right)/k_BT}$$

$$\rho c^2 = nmc^2 + \frac{3}{2}nk_BT$$

$$P = nk_BT$$

(non-relativistic gas)

$k_BT << mc^2$

number density of non-relativistic particles is exponentially suppressed, until they decouple from the thermal bath… (more later)

- particles in kinetic equilibrium

	bosons	fermions	
number density	$n = \frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = \frac{3}{4}\frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = g\left(\frac{mk_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2} e^{-\left(mc^2-\mu\right)/k_B T}$
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(*non-degenerate relativistic gas*)
$k_B T >> mc^2, \mu = 0$

(*non-relativistic gas*)
$k_B T << mc^2$

all particles in thermal bath share the same temperature*,
but have their own distribution (*g, m, [7/8]*)

*the temperature dictated by the dominant species

- particles in kinetic equilibrium

	bosons	fermions	
number density	$n = \frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = \frac{3}{4}\frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar c}\right)^3 gT^3$	$n = g\left(\frac{mk_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2} e^{-\left(mc^2-\mu\right)/k_BT}$
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pressure	$P = \frac{1}{3}\rho c^2$	$P = \frac{1}{3}\rho c^2$	$P = nk_BT$
	(non-degenerate relativistic gas) $k_BT >> mc^2, \mu = 0$		**(non-relativistic gas)** $k_BT << mc^2$

all particles in thermal bath share the same temperature,
but have their own distribution (g, m, [7/8])

…but for ***relativistic species*** they can be combined via an effective g_* !

- total energy density:

$$\rho = \rho_{rel}^{th} + \rho_{nr}^{th} + \rho_{rel}^{dec} + \rho_{nr}^{dec}$$

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$$\rho = \rho_{rel}^{th} + \rho_{nr}^{th} + \rho_{rel}^{dec} + \rho_{nr}^{dec}$$

1. relativistic species

$$\rho_{rel} c^2 = \sum_i \rho_{rel,i} c^2 = \frac{\pi^2}{30} \frac{k_B^4}{\hbar^3 c^3} g_*(T)\, T^4$$

$$g_*^{th}(T) = \sum_B g_i^B + \frac{7}{8} \sum_F g_i^F$$

- total energy density:

$$\rho = \underline{\rho_{rel}^{th}} + \rho_{nr}^{th} + \rho_{rel}^{dec} + \rho_{nr}^{dec}$$

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$$\rho_{rel} c^2 = \sum_i \rho_{rel,i} c^2 = \frac{\pi^2}{30} \frac{k_B^4}{\hbar^3 c^3} g_*(T)\, T^4 \qquad g_*^{th}(T) = \sum_B g_i^B + \frac{7}{8} \sum_F g_i^F$$

<u>$k_B T$ >> 175 GeV</u> $\rightarrow$ all particles of the standard model are relativistic

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the early Universe is a thermal bath in equilibrium
in which photons dominate and dictate the temperature (evolution)

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the early Universe is a thermal bath in equilibrium
in which photons dominate and dictate the temperature (evolution)

$k_B T$ >> 175 GeV $\rightarrow$ all particles of the standard model are relativistic

g_B = gluons + photons + $W^{\pm}$ + Z^0 + Higgs = 8x2 + 2 + 3x3 + 1 = 28
g_F = quarks + leptons + neutrinos = 12x6 + 6x2 + 3x2 = 90

=> g_* = 28 + 7/8 x 90 = 106.75

- total energy density:

$$\rho = \underline{\rho_{rel}^{th}} + \rho_{nr}^{th} + \rho_{rel}^{dec} + \rho_{nr}^{dec}$$

1. relativistic species

$$\rho_{rel} c^2 = \sum_i \rho_{rel,i} c^2 = \frac{\pi^2}{30} \frac{k_B^4}{\hbar^3 c^3} g_*(T)\, T^4$$

$$g_*^{th}(T) = \sum_B g_i^B + \frac{7}{8} \sum_F g_i^F$$

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as T drops, various of those relativistic species become non-relativistic (and/or annihilate)
=> they are removed from g_*
***careful**: neutrinos, for instance, continue to exist and remain relativistic after decoupling…*

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the equilibrium temperature of the decoupled species T_i can be different to the equilibrium temperature T of the photon bath!
(as is the case for decoupled neutrinos!)

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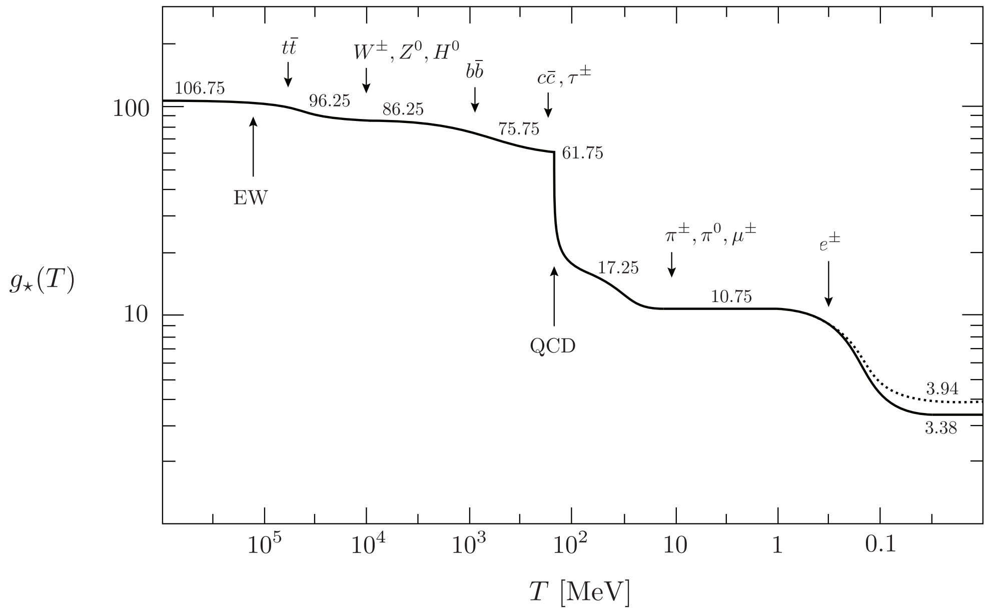

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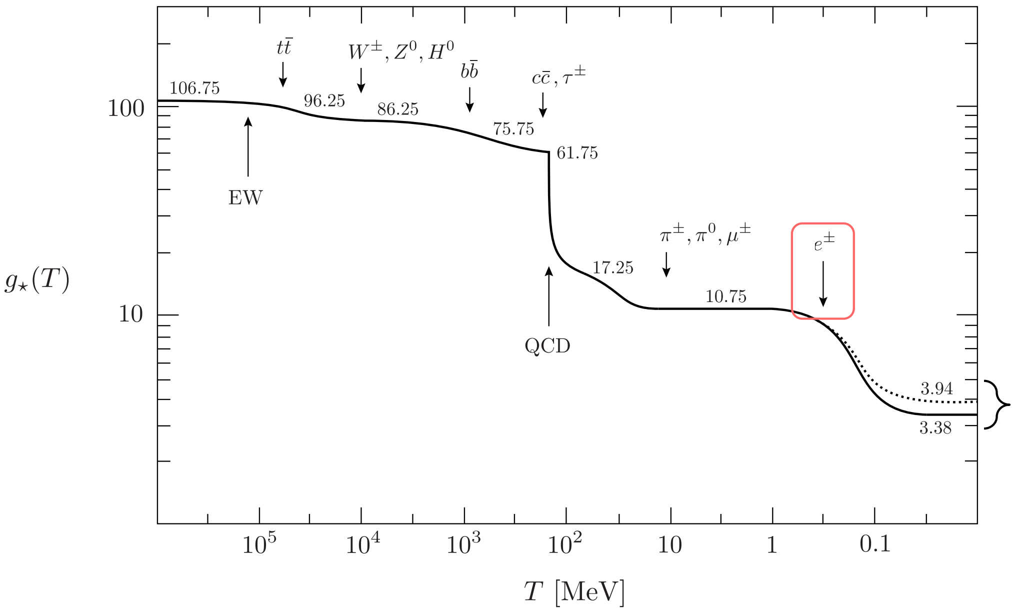

that g_*^{th} and g_*^{dec} are different means that some decoupled species has $T_i \neq T$

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Partícula	Espín	Grados de libertad (g)	Naturaleza
Higgs	0	1	Escalar masivo
fotón	1	2	Vector sin masa
gravitón	2	2	Tensor sin masa
gluón	1	2	Vector sin masa
W y Z	1	3	Vector masivo
leptones y quarks	1/2	4	Fermión de Dirac
neutrinos	1/2	4 (2)	Fermión de Dirac (de Majorana)

I. relativistic species

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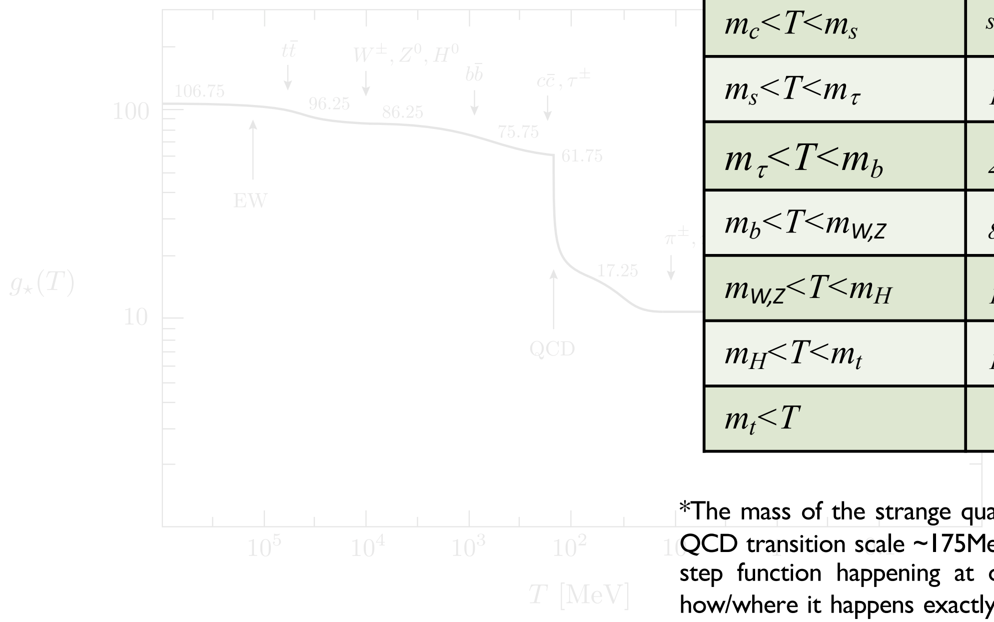

temperature	T	particles	g_*	$4g_*$
$T<T_{dec}$		γ's + 3 ν's	3.36	13.45=4*((2+(7/8) *2 * 3 * (4/11)$^{(4/3)}$))
$T_{dec}<T<m_e$	*0.5 MeV*	γ's + 3 ν's	7.25	29=4*(2+(7/8)*2 * 3)
$m_e<T<m_\mu$	*95 MeV*	+ e^-, e^+	10.75	43=29 + 4*((7/8)*2 * 2)
$m_\mu<T<m_\pi$	*139 MeV*	+ μ^-, μ^+	44.25	57=43 + 4*((7/8)*2 * 2)
$m_\pi<T<T_{QCD}$	*150 MeV*	+ π^+, π^-, π^0	17.25	69=57 + 4*(3) ***remark***: now the 3 pions annihilate again…
$T_{QCD}<T<m_c$	*1.3 GeV*	+ u,u, d,d, + g's – π^+, π^-, π^0	61.75	205= 69 + 4*(8*2 + (7/8)*(2*3*2*2) – 3*1) ***remark***: the 3 pions (w/ g*=1) are formed!
$m_c<T<m_s$	*see below**	s, $\underline{s}$		247=205 + 4*((7/8)*1*3*2*2)
$m_s<T<m_\tau$	*1.8 GeV*	c, $\underline{c}$	72.25	280=247 + 4*((7/8)*2*3 * 2)
$m_\tau<T<m_b$	*4.2 GeV*	τ^-, τ^+	75.75	303=289 + 4*((7/8)*2 * 2)
$m_b<T<m_{W,Z}$	*85 GeV*	b, $\underline{b}$	86.25	345=303 + 4*((7/8)*2*3 * 2)
$m_{W,Z}<T<m_H$	*125 GeV*	$W^\pm$, Z^0	95.25	381=345 + 4*(3*3)
$m_H<T<m_t$	*173 GeV*	H	96.25	385=345 + 4*(1)
$m_t<T$		t, $\underline{t}$	106.75	427=385 + 4*((7/8)*2*3 * 2)

*The mass of the strange quark is 95Mev at the 1GeV scale and in general is of course running with energy. So, at the QCD transition scale ~175MeV it is quite higher, eg around 125MeV or so. The transition scale is a bit fuzzy, ie it's not a step function happening at one value only, so without a very difficult numerical simulation we cannot say exactly how/where it happens exactly. The system is strongly coupled, so counting degrees of freedom in the range of Tc to the bottom quark mass does not make much sense anyway. Also, any simulation is very difficult to do to begin with.

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change in photon temperature due to electron decoupling…

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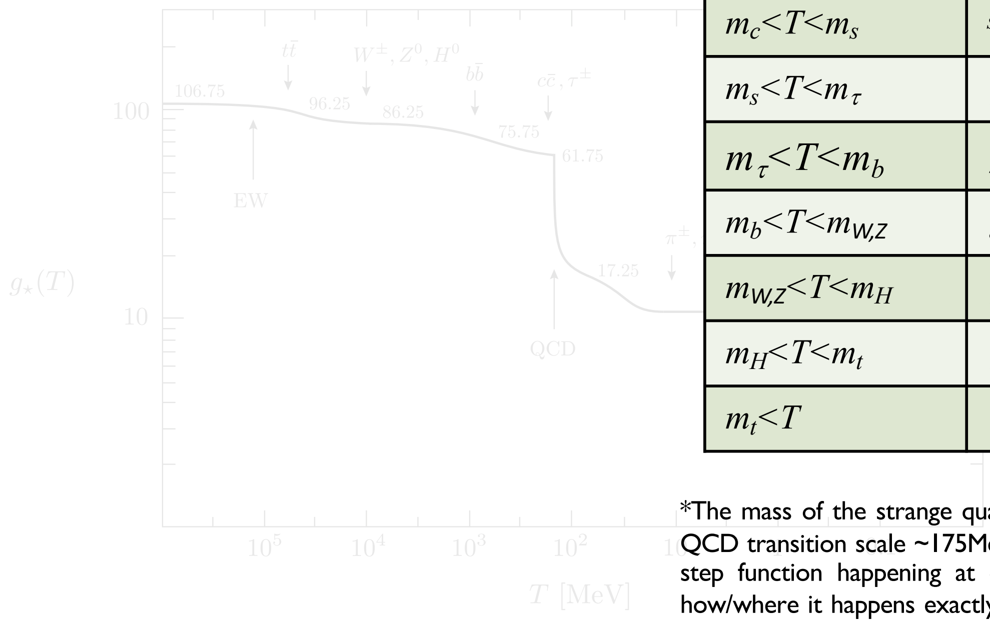

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2. non-relativistic species

$$\rho_{nr}c^2 \propto \sum_i m_i c^2\, n_i + \frac{3}{2} n_i k_B T$$

$$n_i^{th} = g_i \left(\frac{m_i k_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2} e^{-\left(m_i c^2 - \mu_i\right)/k_B T}$$

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*remember: $\rho_{rel}^{dec} \propto R^{-4}$, $T_{rel}^{dec} \propto R^{-1}$

$\rho_m^{dec} \propto R^{-3}$, $T_m^{dec} \propto R^{-2}$

- the hot big bang model
- thermal equilibrium
- **entropy of the Universe**
- decoupling
- matter radiation equality

- entropy is being conserved* $TdS = dU + pdV = 0$

but what is it value?

*see FRW lecture

- entropy is being conserved $TdS = dU + pdV = 0$

$$dV = d(R^3)$$
$$dU = d(V\rho c^2) = d(R^3\rho c^2)$$

- entropy is being conserved $TdS = dU + pdV = 0$

$$\left.\begin{aligned} dV &= d(R^3) \\ dU &= d(R^3\rho c^2) \end{aligned}\right\} \quad \begin{aligned} dS &= \frac{1}{T}\left[dU + pdV\right] \\ &= \frac{1}{T}\left[d\left(R^3\rho c^2\right) + pd\left(R^3\right)\right] \\ &= \frac{1}{T}\left[d\left(R^3\left(\rho c^2 + p\right)\right) - R^3 dp\right] \end{aligned}$$

$$pd(R^3) = d(pR^3) - R^3 dp$$

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replace in favour of dT

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* $w = 1/3$

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$$= \frac{4R^3}{3T}\frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3} g_{*S} T^4$$

$$= \frac{2\pi^2}{45}\frac{k_B^4}{\hbar^3 c^3} g_{*S} \left(RT\right)^3$$

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- temperature evolution:

$$T \propto g_{*S}^{-1/3} R^{-1}$$

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- when particles decouple and become non-relativistic, g_{*S} drops and its entropy is transferred to heat bath.
- when particles decouple but remain relativistic, g_{*S} also drops, but they keep their entropy in the form of g_{*S}^{dec}

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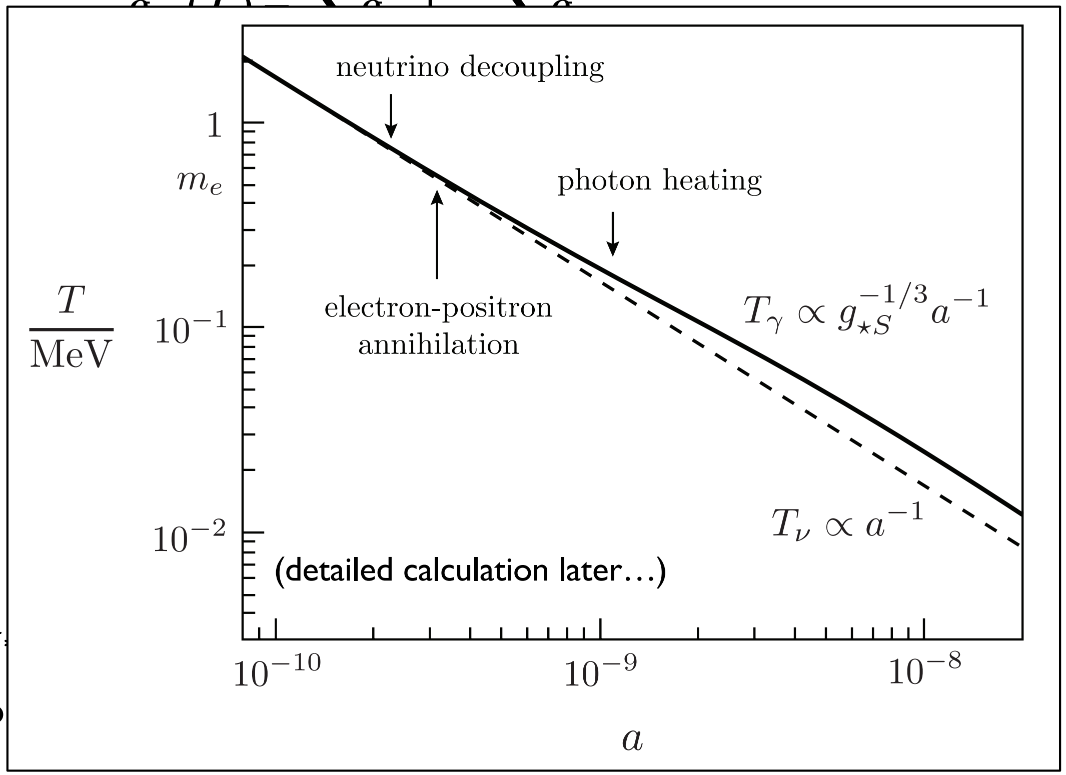
- relativistic species:

$$S(T) = \frac{2\pi^2}{45} \frac{k_B^4}{\hbar^3 c^3} g_{*S} \left(RT\right)^3$$

$$g^{th}(T) = \sum g^B + \frac{7}{\ldots} \sum g^F$$



- temperature evolution:

$$T \propto g_{*S}^{-1/3} R^{-1}$$

- when particles decouple and become non-relativistic, g
- when particles decouple but remain relativistic, g_{*S} also

- entropy is being conserved $TdS = dU + pdV = 0$

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FRW lecture: $\underline{\Omega_r = 1:}$

$$H = \sqrt{\frac{8\pi G}{3}\rho_r} = \sqrt{\frac{8\pi G}{3} \frac{\pi^2}{30} \frac{k_B^4}{\hbar^3 c^3} g_*(T)\, T^4} \propto \sqrt{g_*(T)}\, T^2 \propto \frac{1}{t}$$

$R(t) \propto t^{1/2} \Rightarrow H \propto 1/t$

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• particle numbers:

$$n_i = \left[\frac{3}{4}\right] \frac{\zeta(3)}{\pi^2} \left(\frac{k_B}{\hbar c}\right)^3 g_i T^3 \quad \Rightarrow \quad \frac{n_i}{S} = \left[\frac{3}{4}\right] \frac{45\zeta(3)}{2\pi^4} \frac{g_i}{g_{*S}} \frac{1}{R^3}$$

$\left[\frac{3}{4}\right]$ for fermions

- the hot big bang model
- thermal equilibrium
- entropy of the Universe
- **decoupling**
- matter radiation equality

interaction rate of particles vs. expansion rate of Universe

interaction rate of particles << expansion rate of Universe

=> particles drop out of thermal equilibrium

$$\Gamma_c \quad \ll \quad H$$

interaction rate of particles << expansion rate of Universe

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$$\overset{?}{T^{\alpha} \propto} \; \Gamma_c \quad \ll \quad H \; \overset{?}{\propto} T^{\beta}$$

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- interaction rate of particles: $\Gamma_c \propto n \, \sigma \, v$

n : number density
σ : interaction cross-section
v : relative velocity

- interaction rate of particles: $\Gamma_c \propto n\,\sigma\,v$

cross-section of interaction keeping species in equilibrium

n : number density
σ : interaction cross-section
v : relative velocity

- interaction rate of particles: $\Gamma_c \propto n\,\sigma\,v$
  - interaction mediated by massless gauge bosons: $\Gamma_c \propto T$ (gluon, photon)
  - interaction mediated by massive gauge bosons ($T<M_X$): $\Gamma_c \propto T^5$ (W, Z)

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  - interaction mediated by massless gauge bosons: $\frac{\Gamma_c}{H} \propto T^{1/4}$
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...and simultaneously solving the Boltzmann equation $\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle\left(n^2 - n_{eq}^2\right)$

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
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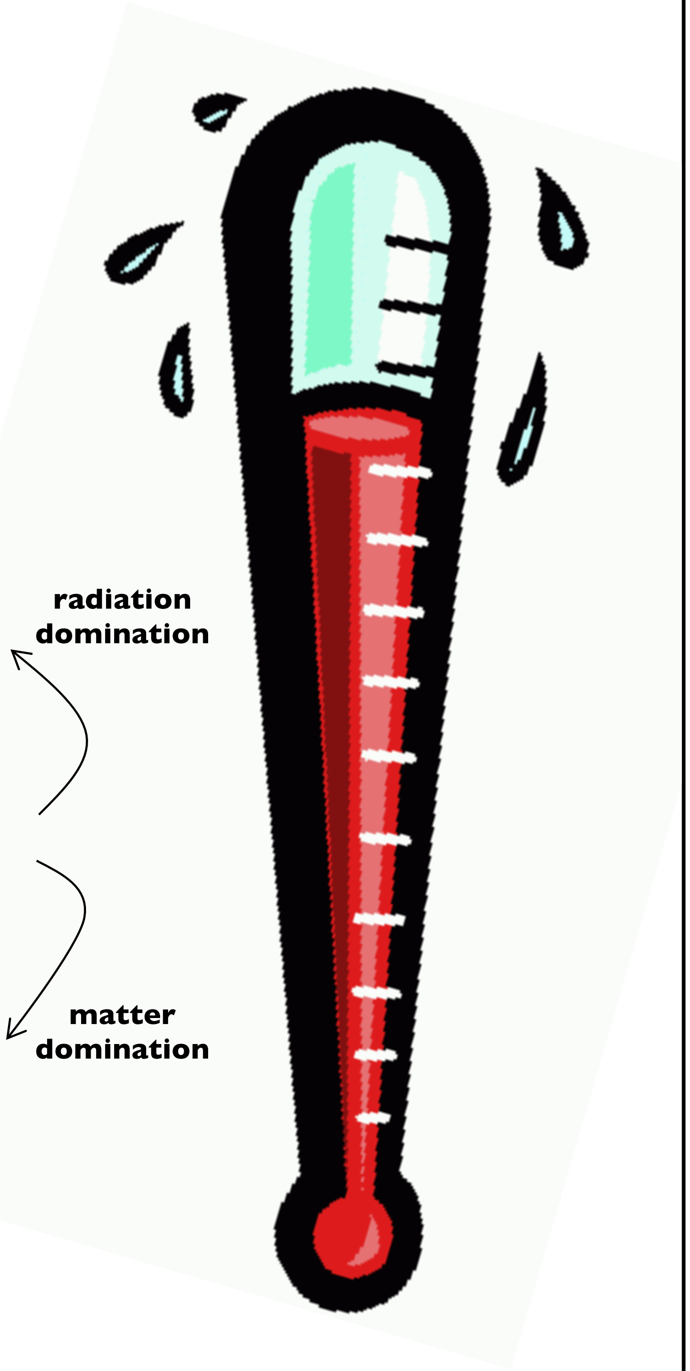

Thermal History of the Universe

decoupling

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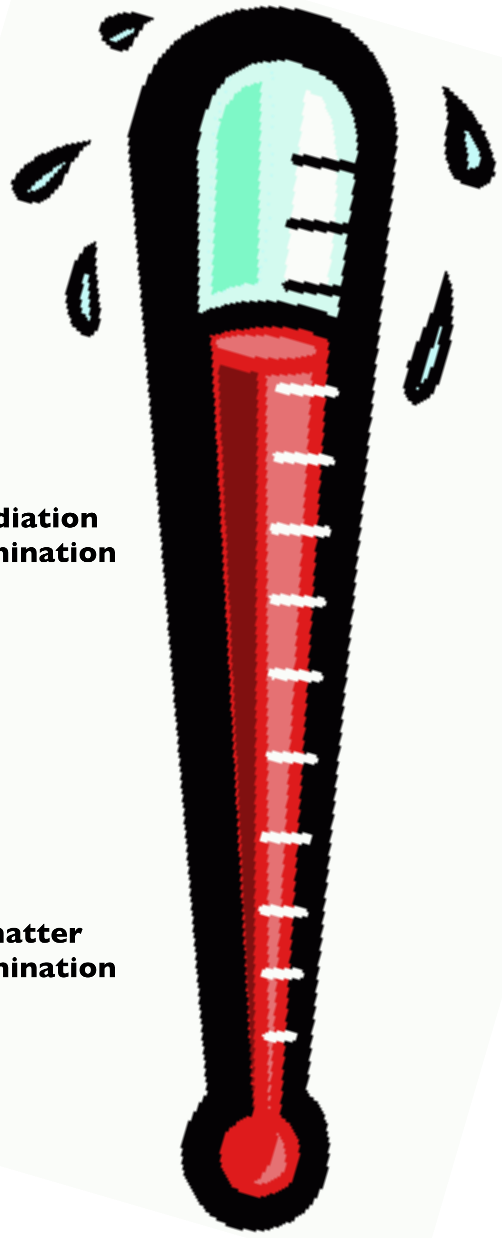
radiation domination

matter domination



- neutrino decoupling
  - coupled to thermal bath via $n + \nu \leftrightarrow p + e^-$ (weak interaction)

$$p + \bar{\nu} \leftrightarrow n + e^+$$

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radiation domination: $H^2 = H_0^2 \Omega_{r,0} \left(\frac{R}{R_0}\right)^{-4}$ because of $T_\nu^{dec} > 0.511 MeV$ $> T^{eq} \approx 0.75 eV$

electrons are obviously still around… matter-radiation equality will be calculated below…

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$T \in [0.8, 0.511]\, MeV$: $T \propto g_{*S}^{-1/3} R^{-1}$ $g_{*S} = 2 + \frac{7}{8} 4$

electrons & positrons (→ $\frac{7}{8}4$)

photons (→ 2)

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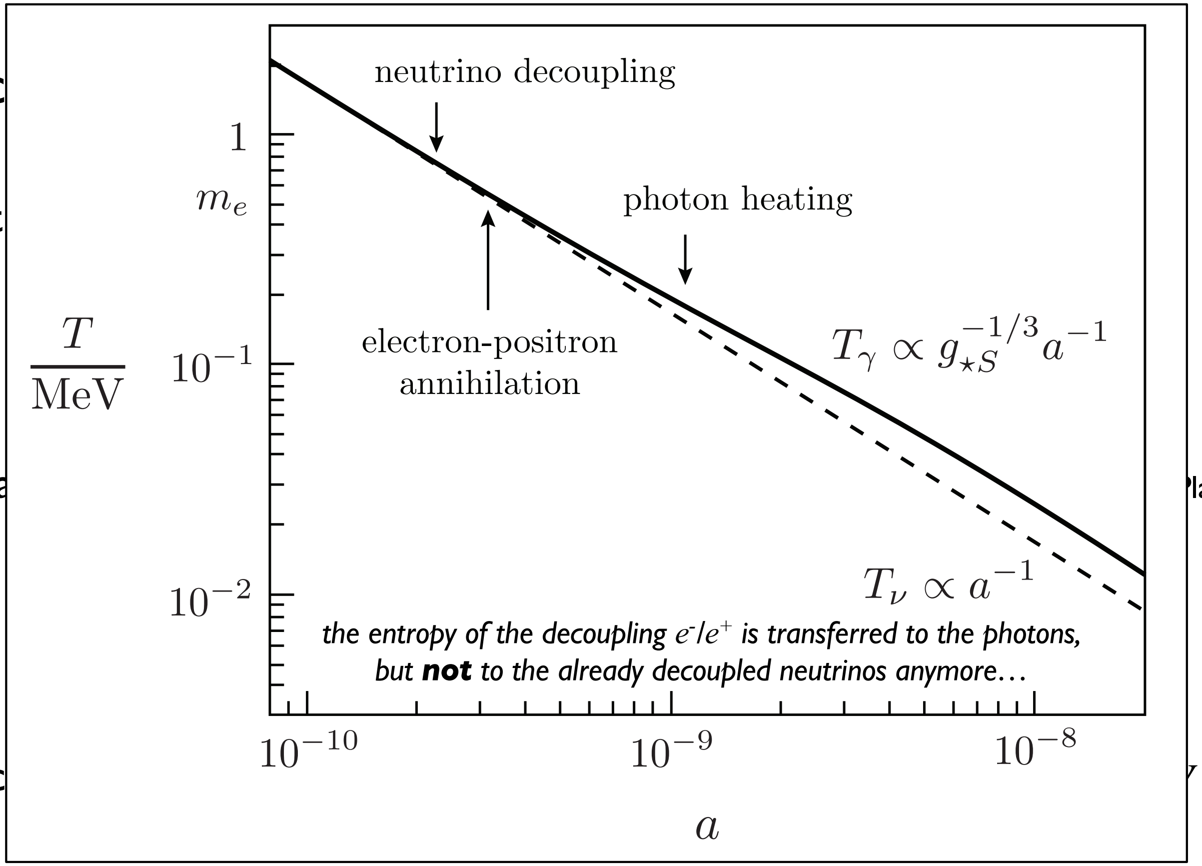
(part of exercise)

- entropy conservation: $T_0 / T_\nu = (11/4)^{1/3} \underset{T_0 = 2.725\, K}{\Rightarrow} T_\nu = 1.945\, K$

- neutrino dec
  - coupled to t
  - interaction ra

    lanck mass
  - decoupling c

    (electron rest mass)



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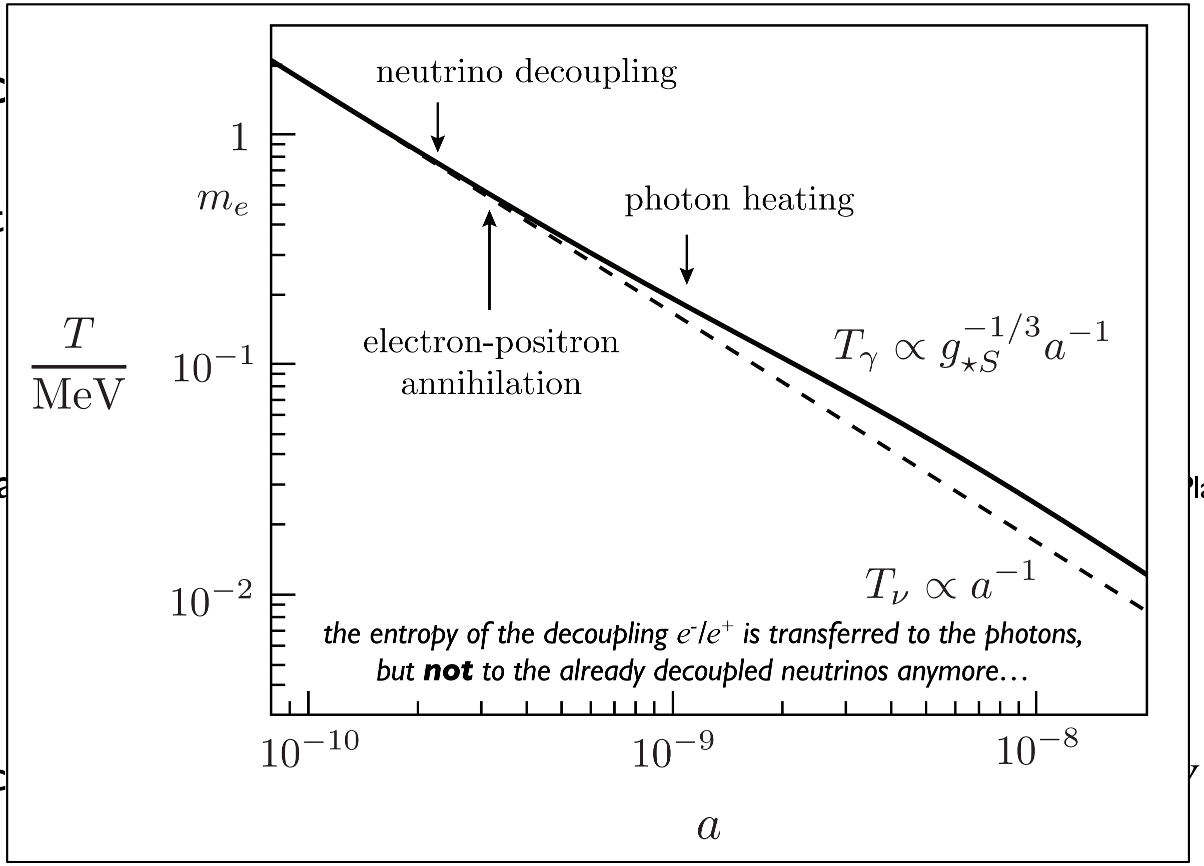

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$T < 0.511\, MeV$: $T \propto g_{*S}^{-1/3} R^{-1}$ $\quad g_{*S} = 2$ (**electrons**-positrons annihilated)

some electrons remain (cf. Baryogenesis)

(part of exercise)

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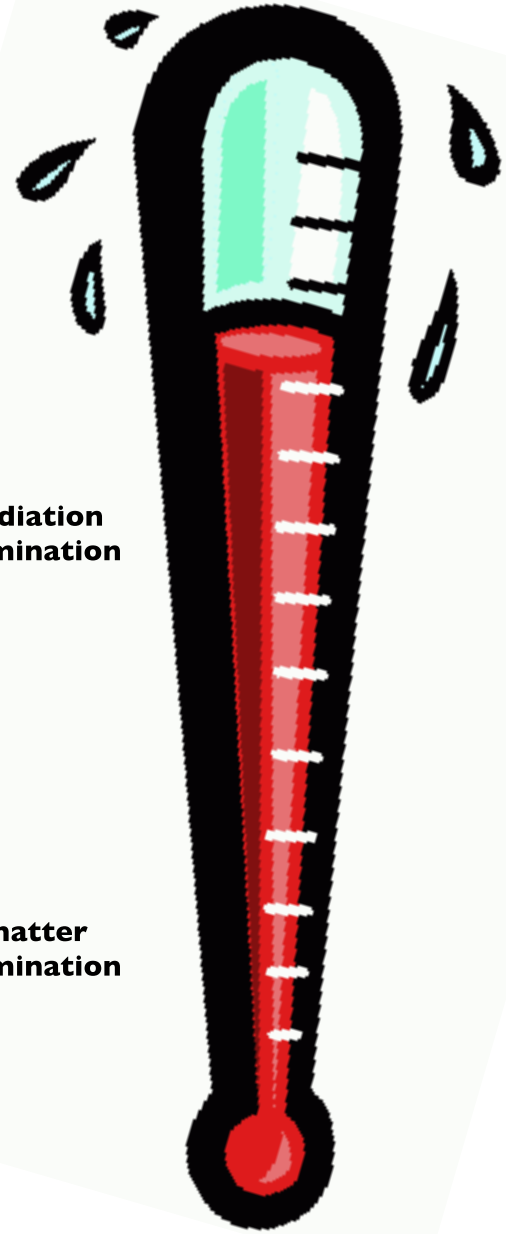

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$H = H_0 \Omega_{m,0} (R_0 / R)^{3/2}$

matter domination: $H^2 = H_0^2 \, \Omega_{m,0} \left(\frac{R}{R_0}\right)^{-3}$ because of… (detailed proof in CMB lecture)

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  - decoupling condition $$\frac{\Gamma_\gamma}{H} = 1 \xrightarrow[T \propto R^{-1}]{} T_\gamma^{dec} \approx 0.27 eV$$

(photons are relativistic)

$$n_e = g_e \left(\frac{m_e k T}{2\pi\hbar^2} \right)^{3/2} e^{-(m_e - \mu_e)c^2/kT}$$

(electrons are non-relativistic)

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling 1.	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
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Matter-radiation equality 3.	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
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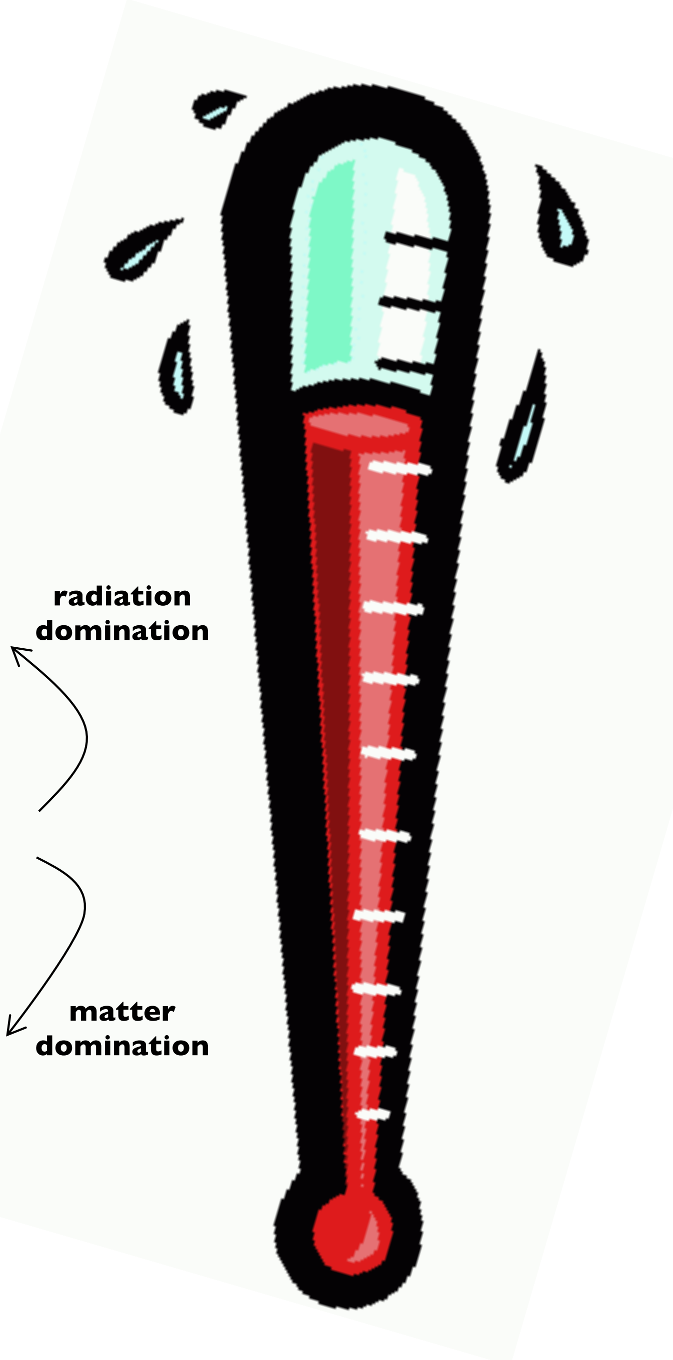
Present	13.8 Gyr	0	0.24 meV



- the hot big bang model
- thermal equilibrium
- entropy of the Universe
- decoupling
- **matter radiation equality**

- barotropic fluids $p = \omega \rho c^2$:
  - radiation $w = 1/3 \Rightarrow \rho_{rel} \propto R^{-4}$
  - matter $w = 0 \Rightarrow \rho_{nr} \propto R^{-3}$
  - vacuum energy $w = -1 \Rightarrow \rho_{\Lambda} = const.$

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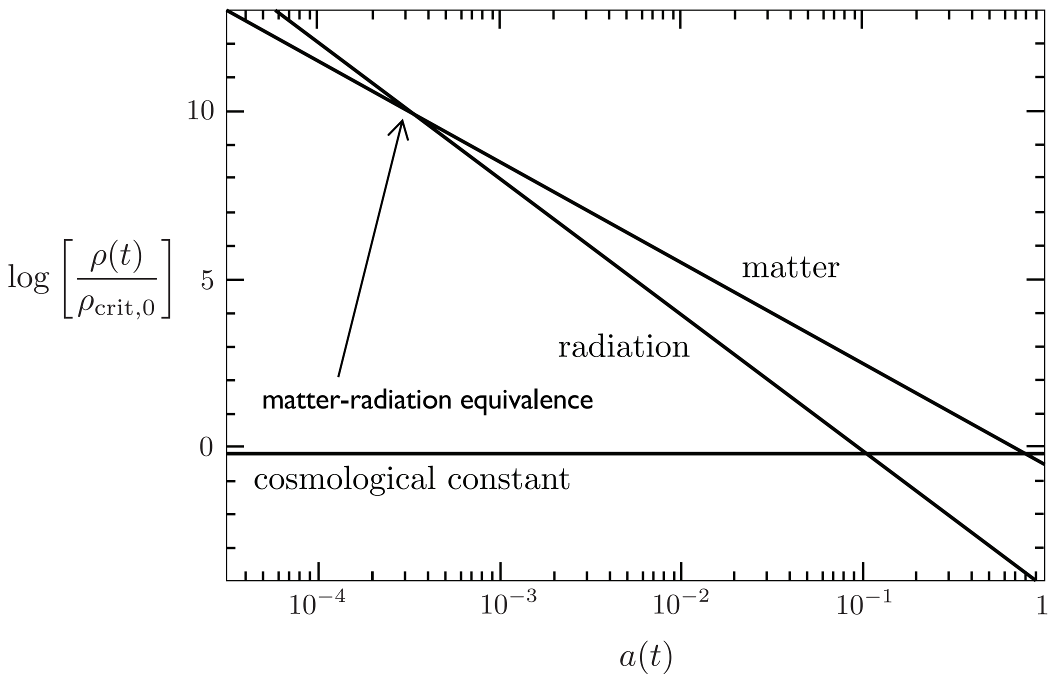

- matter-radiation equality

$$\rho_{rel}(R_{eq}) = \rho_{nr}(R_{eq}) \quad \Rightarrow \quad 1 + z_{eq} = \frac{\rho_{m,0}}{\rho_{r,0}}$$

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<u>proof:</u>

$\rho_{nr} R^3$

$\rho_{rel} R^4$

Note: $\rho_{nr} \equiv \rho_m$; $\rho_{rel} \equiv \rho_r$

- matter-radiation equality

$$\rho_{rel}(R_{eq}) = \rho_{nr}(R_{eq}) \quad \Rightarrow \quad 1 + z_{eq} = \frac{\rho_{m,0}}{\rho_{r,0}}$$

proof:

$$\rho_{nr} R^3 = \rho_{nr,eq} R_{eq}^3 = \rho_{nr,0} R_0^3$$

$$\rho_{rel} R^4 = \rho_{rel,eq} R_{eq}^4 = \rho_{rel,0} R_0^4$$

Note: $\rho_{nr} \equiv \rho_m$; $\rho_{rel} \equiv \rho_r$

- matter-radiation equality

$$\rho_{rel}(R_{eq}) = \rho_{nr}(R_{eq}) \quad \Rightarrow \quad 1+z_{eq} = \frac{\rho_{m,0}}{\rho_{r,0}}$$

<u>proof:</u>

$$\begin{aligned} \rho_{nr,eq}R_{eq}^3 &= \rho_{nr,0}R_0^3 \\ \rho_{rel,eq}R_{eq}^4 &= \rho_{rel,0}R_0^4 \end{aligned} \quad \Longrightarrow \quad \frac{1}{R_{eq}} = \frac{\rho_{nr,0}}{\rho_{rel,0}}\frac{1}{R_0} \quad \Longrightarrow \quad \frac{R_0}{R_{eq}} = 1+z_{eq} = \frac{\rho_{m,0}}{\rho_{r,0}}$$

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- matter-radiation equality

$$\rho_{rel}(R_{eq}) = \rho_{nr}(R_{eq}) \quad \Rightarrow \quad 1 + z_{eq} = \frac{\rho_{m,0}}{\rho_{r,0}}$$

$$\rho_{m,0} = \Omega_{m,0} \frac{3H_0^2}{8\pi G} = 1.9 \times 10^{-29} \Omega_{m,0} h^2 \frac{g}{cm^3}$$

$$\rho_{r,0} = \ ?$$

Note: $\rho_{nr} \equiv \rho_m$; $\rho_{rel} \equiv \rho_r$; H_0=100h km/s/Mpc

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$$\rho_{r,0} = \rho_{CMB,0} + \rho_{\nu,0}$$

Note: $\rho_{nr} \equiv \rho_m$; $\rho_{rel} \equiv \rho_r$; $H_0 = 100h$ km/s/Mpc

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just as for the photons, there is a neutrino background radiation!

Note: $\rho_{nr} \equiv \rho_m$; $\rho_{rel} \equiv \rho_r$; H_0=100h km/s/Mpc

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$$\rho_{CMB,0}c^2 = \frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3}g_{CMB}T_{CMB}^4$$

$$\rho_\nu c^2 = \frac{7}{8}\frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3}g_\nu\left(\left(\frac{4}{11}\right)^{1/3}T_{CMB}\right)^4$$

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$$\frac{T_\nu}{T_{CMB}} = \left(\frac{4}{11}\right)^{1/3}$$ remember neutrino decoupling…

Note: $\rho_{nr} \equiv \rho_m$; $\rho_{rel} \equiv \rho_r$; H_0=100h km/s/Mpc

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$$\rho_{r,0} = \frac{1}{c^2}\frac{\pi^2}{30}\frac{k_B^4}{\hbar^3 c^3}T_{CMB}^4\left(2 + \frac{7}{8}\times 2N_\nu \times \left(\frac{4}{11}\right)^{4/3}\right)$$

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$$1+z_{eq} = \frac{\rho_{m,0}}{\rho_{r,0}} = 24000\,\Omega_{m,0}h^2$$

$$z_{eq} \cong 3440$$ (Planck cosmology)

Note: $\rho_{nr} \equiv \rho_m$; $\rho_{rel} \equiv \rho_r$; $H_0 = 100h$ km/s/Mpc

- matter-radiation equality

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$$\downarrow$$

$$T_{\gamma,eq} \cong \ ?$$

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- matter-radiation equality

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$$z_{eq} \cong 3440$$ (Planck cosmology)

$$T_{\gamma,0} = 2.73K$$
$$T_{\gamma} \propto (1+z)$$

$$\downarrow$$

$$T_{\gamma,eq} \cong 0.8eV$$

Note: $\rho_{nr} \equiv \rho_m$; $\rho_{rel} \equiv \rho_r$; H_0=100h km/s/Mpc

Thermal History of the Universe

matter-radiation equality

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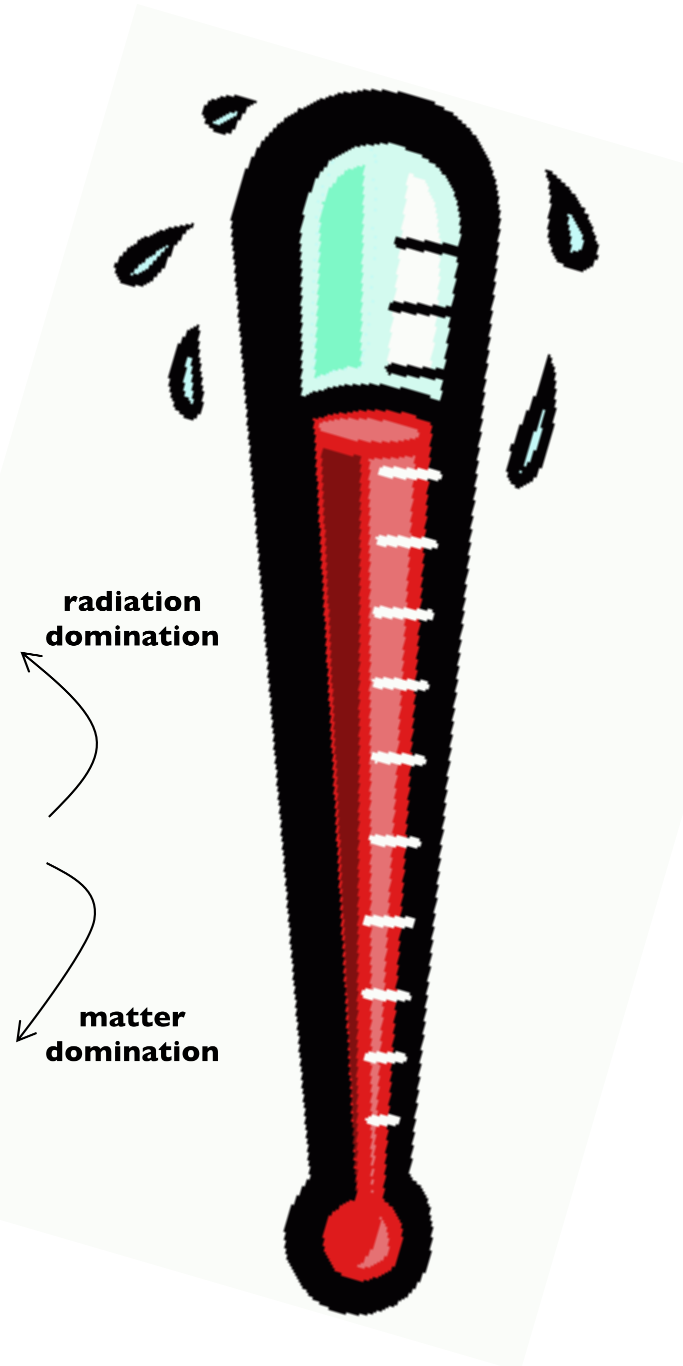

Thermal History of the Universe

hot big bang model

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up next...