rules and regulations

- **project:**
  - you can pick one of the projects from the lists provided below or
  - you can propose your own project, but clearly think about:
    - what is the physical problem?
    - what are the relevant equations?
    - what are your objectives and milestones, i.e. what do you plan to do?
    - ***discuss the project and the objectives with your teacher!***

- **project report:**
  - you have to write a report that summarizes your work and the results
    - present your results and conclusions
    - elaborate on the objectives and your milestones

- **presentation:**
  - you have to orally present the results of your project in class:
    - 15min. presentation of your work to the rest of the class

- **distribution of points:**

• quality of project realisation	4 points
• quality of MATLAB script(s)	3 points
• project report	2 point
• presentation of results	1 points

- ***Note***: you need to work in teams of 2 students!

quality guidelines

- **project:**
  - the project is not an exam:
    - ***you should study and investigate a physical problem!***
    - *if you picked one of the suggested projects, the listed workplan is just a guideline: simply presenting plots for the listed points will let you pass the project, but not obtain the highest possible mark: it is certainly necessary to* ***add your own ideas****…*
    - *if you chose to select your own project, you need to have clear objectives in mind and synchronize/define all milestones with your teacher: they need to be well-suited and feasible!*

quality guidelines

- **project report:**

  ▪ the report needs to feature an **introduction**, a description of the **methods**, a presentation of the **results**, a **summary and discussion** as well as a list of **references.** Please also provide an **appendix** where you describe your MATLAB code and how the figures in the main text were generated with it.

  - *the **introduction** needs to explain (to a fellow student!) the idea/history of the project, the relevant equations, and the theory behind the physical system in general. Further, give some motivation: "Why is this project interesting?", "What are its applications?" Give a general introduction into the field, and also mention of the objectives of the actual work ("what are the aims?").*
  - *in the **methods** parts you need to explain what methods you have used, e.g. "What numerical integration scheme?", "What data has been used?", "How do you analyse the data?", etc. Note, you should not describe you MATLAB code here!*
  - *in the **results** part you need to show (key) plots from your investigation and discuss them; you do not need to show all plots generated during the investigation, but you need to explain clearly all results you found. The results part should be well structured primarily following your objectives and milestones. Note that every figure needs to have a proper caption and needs to be explained (and discussed) in the text.*
  - *the **summary** part should contain a brief summary of the main findings and your conclusions about the physical system and **discuss/interpret** the results.*
  - *introduction, results, and summary parts should **not exceed 15 pages**.*
  - *do not forget a **reference** list!*
  - *in an **appendix** please specify which of your scripts serve what purpose and have been used for which figure presented in the report, respectively.*
  - *in general, the report should contain sufficient information so that a fellow student could repeat it (without your scripts and, in fact, in any programming language).*

  - ***Notes**:* MATLAB scripts are **not allowed** to appear in the report or the presentation; but – as mentioned before – you should explain the purpose of each script in an appendix to the report.

quality guidelines

- **presentation:**

  - the presentation is structured similarly to the report: introduction, methods, results & summary/discussion (no appendix!)
  - the presentation should focus on explaining the project to your fellow students and showing ***key results*** from your study of it; the presentation does not necessarily need to contain everything you have put into the report: focus on the most interesting results!
  - the presentation is **15min.** long; substantially running overtime (or using far less time) will make you loose points!

schedule for academic year 2022/23

- Feb 3: choice of project (or submission of proposal) to alexander.knebe@uam.es
- Apr 21: submission of complete project:
  - project report
  - the script(s) used to generate the plots and/or results
- Apr 24 & 25: oral presentation of projects (15 minutes each)

project suggestions

you may pick a project from the provided list,

but you are encouraged to come up with a project on your own!

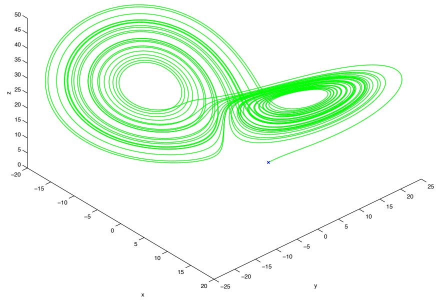
come and see me in C-8-316 if you have questions and/or want to discuss your project!

example projects

- **The Butterfly Effect**

• the convection rolls of air in the Earth's atmosphere can be described by the so-called Lorenz equation (first published by Edward Lorenz in 1963)

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = rx - y - xz$$
$$\frac{dz}{dt} = xy - bz$$



…where σ, r, and b are constants. The standard values are $\sigma = 10$ and $b = 8/3$ and r is treated as a free parameter. These equations are fundamental for the study of *deterministic chaos*; they can be used to demonstrate the so-called butterfly effect where tiny variations in the initial conditions lead to vastly different behaviour/solutions.

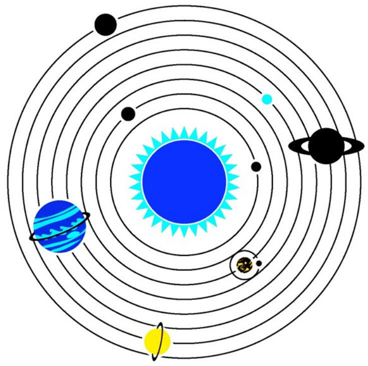
• workplan:

- we aim at writing a program that demonstrates the butterfly effect
- we will study the solutions to the Lorenz equations for various initial conditions and different values for r, b, and σ
- we will graphically represent the solutions $x(t)$, $y(t)$, $z(t)$ individually
- we will graphically represent the trajectories ($x(t)$, $y(t)$, $z(t)$) in a 3D figure
- we will adapt the 4th-order Runge-Scheme to solve the Lorenz equations
- we will use the initial conditions

  x_0=5, y_0=5, z_0=5 and x_0=5.001, y_0=5.001, z_0=5 for r = 28

  to demonstrate the butterfly effect by plotting $x(t)$ for both solutions.
- we will show that the solutions start to differ from $t \approx 13$ onwards.
- we investigate how changes in r, b, and σ affect the solution
- we will visualize various interesting trajectories ($x(t)$, $y(t)$, $z(t)$) in a 3D figure
- we will study interesting cases in phase-space, too
- we check the influence of the number of integration steps for a given solution
- ...

example projects



▪ **The Solar System**

• the movement of the planets in our solar system is described by

$$\frac{d\vec{r}_i}{dt} = \vec{v}_i$$

$$m_i \frac{d\vec{v}_i}{dt} = G \sum_{j \neq i} \frac{m_i m_j}{\left|\vec{r}_j - \vec{r}_i\right|^3} \left(\vec{r}_j - \vec{r}_i\right)$$

…where i represents a given body of the solar system and the summation is over all other bodies $i \neq j$. Note that due to the conservation of angular momentum the motion of all bodies in the solar system is in a plane and hence we can treat it as a 2-dimensional problem:

$$\vec{r}(t) = \left(x(t), y(t)\right)$$

$$\vec{v}(t) = \left(v_x(t), v_y(t)\right)$$

and hence the equations of motion to be numerically integrated read as follows:

$$\dot{x}_i = v_{i,x}$$

$$\dot{y}_i = v_{i,y}$$

$$\dot{v}_{i,x} = G \sum_{j \neq i} \frac{m_j}{\left(\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2\right)^{3/2}} \left(x_j - x_i\right)$$

$$\dot{v}_{i,y} = G \sum_{j \neq i} \frac{m_j}{\left(\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2\right)^{3/2}} \left(y_j - y_i\right)$$

• workplan:

- we aim at writing a program that calculates the orbits of the Sun and the inner planets Mercury, Venus, Earth and Mars.
- we will treat each body identically, i.e. the sun's position is not fixed in the centre but its orbit is also integrated
- we will study the influence of (massive) comets
- we will use the 2^{nd}-order Runge-Scheme to solve the equations-of-motion for each of the five bodies under the common gravity of all bodies
- we will plot the orbits for each body for one Earth year.
- we will confirm energy conservation
- we will confirm Kepler's 2^{nd} law, i.e. angular momentum conservation
- we will confirm Kepler's 3^{rd} law, i.e. T^2/R^3=const. where T is the time a planet requires for one full orbit and R the radius of that orbit.
- we will investigate what happens when changing the gravitational constant
- we will introduce (massive) comets entering the system arbitrarily
- …

example projects

▪ The Lagrange Points of the Sun-Earth System

• there are five equilibrium points found in a system of two orbiting masses M_1 and M_2:

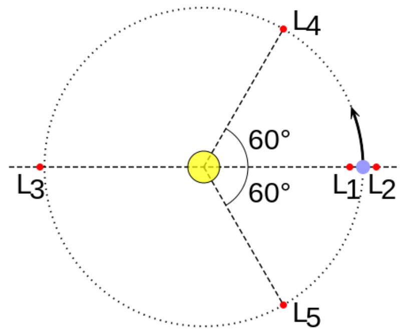
$$L1 = \left(R\left[1 - \left(\frac{\alpha}{3}\right)^{1/3}\right], \quad 0 \right)$$

$$L2 = \left(R\left[1 + \left(\frac{\alpha}{3}\right)^{1/3}\right], \quad 0 \right)$$

$$L3 = \left(-R\left[1 + \frac{5}{12}\alpha\right], \quad 0 \right)$$

$$L4 = \left(\frac{R}{2}\left[\frac{M_1 - M_2}{M_1 + M_2}\right], \quad +\frac{\sqrt{3}}{2}R \right)$$

$$L5 = \left(\frac{R}{2}\left[\frac{M_1 - M_2}{M_1 + M_2}\right], \quad -\frac{\sqrt{3}}{2}R \right)$$



…where R is the distance between M_1 and M_2 and α = 3 x 10^{-6} for the Sun-Earth system.

A satellite placed at one of these positions will remain at a constant distance to the Earth orbiting around the sun with the same orbital period as the Earth.

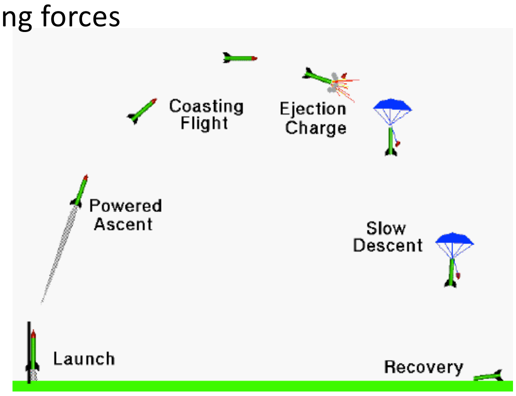
• workplan:

- we aim at writing a program that calculates the orbits of the Sun, the Earth and a satellite $M_{satellite} << M_{earth}$ placed at each of the Lagrange points.
- we will treat each body identically, i.e. the sun's position is not fixed in the centre but its orbit is also integrated
- we will use the 2^{nd}-order Runge-Scheme to solve the equations-of-motion for each of the three bodies under the common gravity of all bodies (see Solar System project!)
- we will plot the orbits for each body
- we will show that L1, L2, and L3 are less stable than L4 or L5 and hence require much more precise numerical integration:
    - a satellite at L4 or L5 can be integrated for 6 months with Δt=30 hours
    - a satellite at L3 can be integrated for 2 months with Δt=30 hours
    - a satellite at L1 or L2 can be integrated for 1 month with Δt=30 hours
- we will show the stability by plotting the distance between the Earth and the satellite
- ...

example projects

▪ Rocket Control System

• a rocket launched on earth will experience the following forces



$$\frac{d\vec{r}}{dt} = \vec{v}$$
$$m\frac{d\vec{v}}{dt} = \vec{v}\frac{dm}{dt} + m\left(\vec{a}_g + \vec{a}_f + \vec{a}_c + \vec{a}_C + \vec{a}_p\right)$$

…where the forces are as follows

$\vec{a}_g = \vec{g}$	gravitational force
$\vec{a}_f = -k\vec{v}$	friction
$\vec{a}_c = \vec{\omega} \times (\vec{\omega} \times \vec{r})$	centripetal force
$\vec{a}_C = 2\vec{\omega} \times \vec{v}$	Coriolis force
$\vec{a}_p = f(?)$	propulsion

• workplan:

- we aim at writing a program that calculates the trajectory of a given missile
- we will graphically represent several interesting trajectories
- we will carefully study the influence of the friction, centripetal, and Coriolis force
- we integrate the equations-of-motions using the 2nd order Runge-Kutta scheme
- we will compare flight paths with/without friction, centripetal and Coriolis force
- given a certain destination we will calculate the launch conditions
- we include a propulsion term that is coupled to the mass loss dm/dt
- we will add an additional friction term representing the parachute
- ...

example projects

▪ **The Cyclotron**

• in a cyclotron charged particles are accelerated to extremely high velocities which are then used to bombard a certain target (e.g. cancer cells in the case of protons)

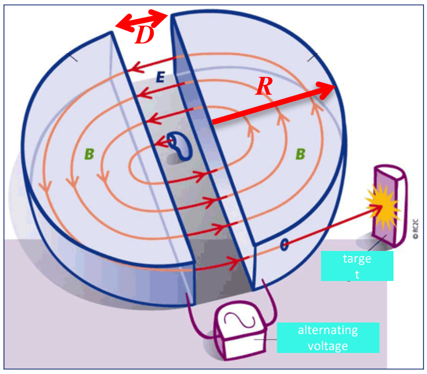
• the motion of the particle is described by the alternating influence of an electric field E

$$m\frac{d^2\vec{r}}{dt^2} = q\vec{E}$$

used to accelerate it and a magnetic field B

$$m\frac{d^2\vec{r}}{dt^2} = q\left(\vec{v} \times \vec{B}\right)$$

used to keep the particle on a (circular) orbit.



• workplan:

• we are going to “construct” a virtual cyclotron in MATLAB

• we will visualize and study the orbits of particles with different charges and masses

• write a script for the movement in the electric field

• write a script for the movement in the magnetic field

• combine both movements using a fixed distance D between the “half-pipes”

• we will visualize the movement of the electron

• we will study the necessary characteristics of a cyclotron capable of accelerating a proton to 1MeV (e.g. magnetic field strength in relation to radius R of “half-pipes”)

• we will extend the previous study to differently charged particles, e.g. alpha-particles, deuterium ions, etc.)

• ...

example projects

▪ The dark side of the Universe

• the observation that supernovae explosions in far distant galaxies are dimmer than expected led to the conclusion that the universe is not only filled with dark matter but also dark energy!

• the equation giving the relation between supernova brightness (m) and distance (z) is

$$m(z) = M + 5\log_{10}\left(\frac{c(1+z)}{\sqrt{|k|}} \; \gamma\left(\sqrt{|k|} \int_0^z \left[(1+z')^2 (1+\Omega_m z') - z'(2+z')\Omega_\Lambda \right]^{-1/2} dz' \right) \right)$$

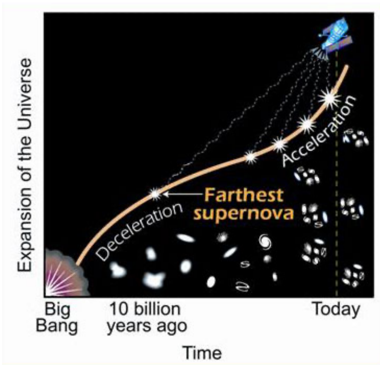
…where the variables/parameters have the following meaning

$$\begin{aligned}
\Omega_m &= \text{measure for matter content of the Universe} \qquad \Omega_m \in [0,2] \\
\Omega_\Lambda &= \text{measure for dark energy content of the Universe} \quad \Omega_\Lambda \in [0,1] \\
c &= \text{speed of light} \\
m &= \text{observed brightness of supernova} \\
M &= \text{actual brightness of supernova} \\
z &= \text{distance to supernova measured by its redshift } z \\
k &= \text{measure for the curvature of the Universe} \begin{cases} 1-\Omega_m-\Omega_\Lambda & \text{for} \quad \Omega_m+\Omega_\Lambda > 1 \\ 1 & \text{for} \quad \Omega_m+\Omega_\Lambda = 1 \\ 1-\Omega_m-\Omega_\Lambda & \text{for} \quad \Omega_m+\Omega_\Lambda < 1 \end{cases}
\end{aligned}$$

$$\gamma(x) = \begin{cases} \sin(x) & \text{for} \quad \Omega_m+\Omega_\Lambda > 1 \\ x & \text{for} \quad \Omega_m+\Omega_\Lambda = 1 \\ \sinh(x) & \text{for} \quad \Omega_m+\Omega_\Lambda < 1 \end{cases}$$

• workplan:

- we plot the function $m(z)$ for various possible combinations of Ω_m and Ω_Λ
- we will use the observational data for m and z as found on the Supernova Cosmology Project website (http://supernova.lbl.gov/union) to obtain the best values for Ω_m and Ω_Λ



- write a script-function for the integral
- write a script-function for $\gamma(x)$
- write a script-function for $m(z)$
- read and plot the observational data
- write a script obtaining the best Ω_m and Ω_Λ values using as error estimate

$$\chi^2 = \sum_{i=1}^{N} (m_i - m(z_i))^2$$

where m_i is the observational value and $m(\Omega_m, \Omega_\Lambda, z_i)$ the analytical value at the observed redshift z_i

- …

example projects

- **Planck photons**

• the distribution of photons emitted by a so-called black body is given by the Planck curve

$$I(\nu,T)=\frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

• the Planck curve for different temperatures follows the following laws

• Wien's displacement law: $\lambda_{peak}=\frac{b}{T}$

• Stefan-Boltzman law: $\sigma T^4=\pi\int_0^\infty I(\nu,T)d\nu$

• workplan:

• numerically verify Wien's displacement law

• numerically verify the Stefan-Boltzman law

• generate a pool of N photons whose wavelengths are distributed according to the Planck curve

• mix two such pools of photons and verify the resulting distribution: is it again a Planck curve?

• write a function that determines the constant in Wien's law

• write a function that determines the constant in the Stefan-Boltzmann law

• write a function that returns the wavelengths of N photons according to an input T

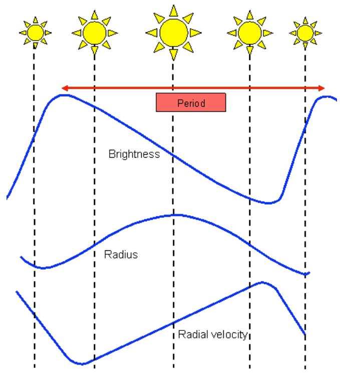
• write a function that finds the best-fit Planck curve to a given pool of photons

• ...

example projects

▪ Cepheid stars

- Cepheids are special stars who change their radius (and luminosity) periodically
- these pulsations are described by the differential equation



$$\frac{d^2\left(\frac{R}{R_0}\right)}{dt^2} = \frac{GM}{R_0^3}\left[\left(\frac{P}{P_0}\right)\left(\frac{R}{R_0}\right) - \left(\frac{R_0}{R}\right)^2\right]$$

…where the variables and parameter have the following meaning

$$\begin{aligned} R &= \text{radius of Cepheid star} \\ P &= \text{surface pressure of Cepheid star} \\ M &= \text{mass of Cepheid star} \\ R_0, P_0 &= \text{equilibrium solution} \\ G &= \text{gravitational constant} \end{aligned}$$

- there further exist a relation between the pulsation period and absolute luminosity:

$$M_{\text{absolute}} = -2.81\log(T) - (1.43 \pm 0.1)$$

…where M_{absolute} is the absolute magnitude (related to the luminosity) and T the period in days.

- workplan:
  - we solve the equation under the assumption that the oscillation are adiabatic
  - we use the period-luminosity relation to calculate the distance to various Cepheids
  - study the extreme case where the Cepheid is a “black hole”
  - write a script to solve the differential equation
  - plot $R(t)$ for various Cepheids stars
  - write a script to obtain the period from the solution $R(t)$
  - plot the observed luminosity as a function of distance for various Cepheids using the period-luminosity relation and the so-called “distance modulus” relating observed luminosity and distance d in parsecs: $M_{\text{absolute}} = 5 + M_{\text{observed}} - 5\log(d)$
  - study the extreme case where the radius approximates the Schwarzschild radius of a black hole: $R_S = 2GM/c^2$
  - …

example projects

▪ The Coupled Pendulum

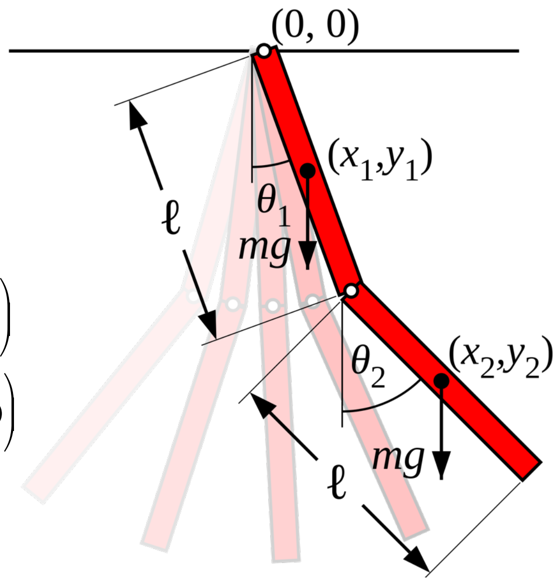
• two pendulums attached to each other are described by the following set of equations:

$$\frac{d\theta_1}{dt} = \frac{6}{ml^2}\frac{2p_1 - 3\cos(\theta_1-\theta_2)p_2}{16-9\cos^2(\theta_1-\theta_2)}$$

$$\frac{d\theta_2}{dt} = \frac{6}{ml^2}\frac{8p_2 - 3\cos(\theta_1-\theta_2)p_1}{16-9\cos^2(\theta_1-\theta_2)}$$

$$\frac{dp_1}{dt} = -\frac{1}{2}ml^2\left(\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2) + 3\frac{g}{l}\sin(\theta_1)\right)$$

$$\frac{dp_2}{dt} = -\frac{1}{2}ml^2\left(-\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2) + \frac{g}{l}\sin(\theta_2)\right)$$



• workplan:

• we aim at numerically solving the coupled system of equations

• we will visualize and study the movement of the two pendulums

• we modify the 2nd order Runge-Kutta scheme to solve the equations

• we will plot $\theta_1(t)$ and $\theta_2(t)$ for various initial conditions

• we plot $x(t)$ and $y(t)$ for the centre-of-mass of each pendulum

• we plot $x_l(t)$ and $y_l(t)$ for the end-points of each pendulum

• we will verify energy conservation

• we will additionally determine and visualize a situation where the attached second pendulum "tips over"

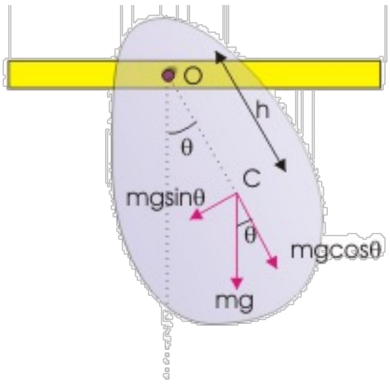
• ...

example projects

- **The Physical Pendulum**

• if a solid body is pivoted about any other point than its centre-of-mass and displaced by a small angle it will start to oscillate corresponding to the following equation

$$I\frac{d^2\theta}{dt^2} = -mgh\sin(\theta)$$



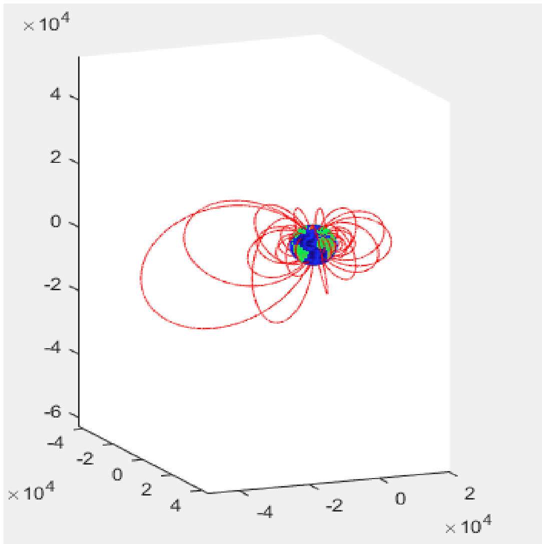
…where I is the moment of inertia tensor, m the total mass of the pendulum, h the distance of the centre-of-mass to the suspension point and g the gravitational acceleration.

• workplan:

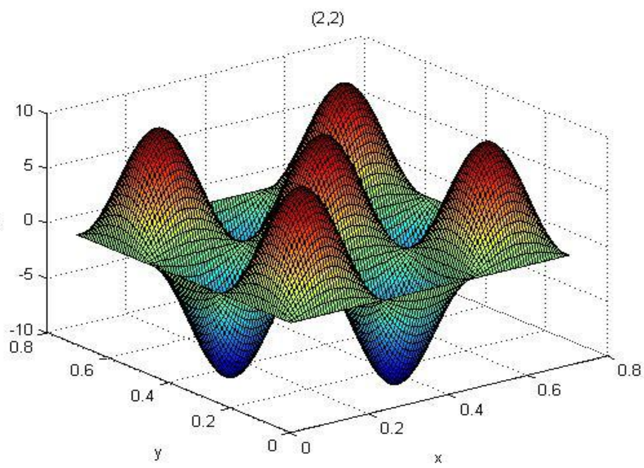
- we aim at writing a program that calculates the movement of the physical pendulum
- we will also include a damping term that is proportional to the velocity
- we will generalize the program to also allow for an external (oscillatory) force
- we will study resonances for the external forces
- we will plot $\theta(t)$ for small angles $\theta \approx \sin(\theta)$
- in the absence of an external force (other than gravity) we determine the best integration interval when using the Euler method
- we will verify energy conservation
- we will obtain the solution without the small angle assumption
- we graphically illustrate the function $\theta(t)$ for the damped and undamped case
- we will determine the critical damping term
- we will study the system in phase-space
- ...

more possible projects…

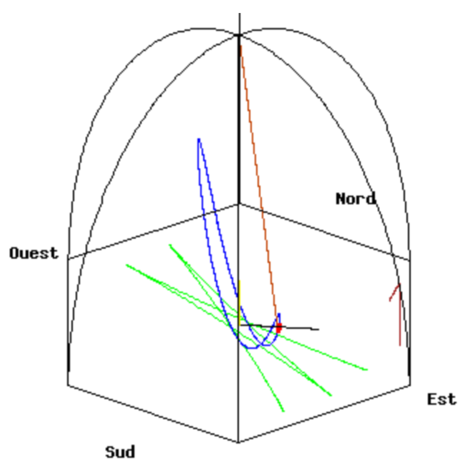
- **Earth’s Magnetic Field**
    - we will visualize Earth’s magnetic field (e.g. IGRF)
    - we will calculate the trajectories of charged particles in Earth’s magnetic field
    - ...



- **Chladni Figures**
    - we will find solutions to the (differential) equations describing Chladni figures (i.e. various modes of vibration on a rigid surface)
    - we will represent those figures for various configurations
    - ...

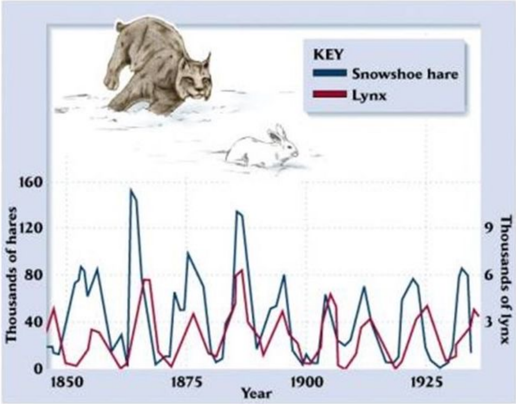


- **Foucault Pendulum**
    - we will study the movement of a pendulum in the rotating coordinate system of the Earth (i.e. the Foucault Pendulum)
    - we will place the pendulum at various locations (e.g. North Pole, Equator, Madrid, ...)
    - ...

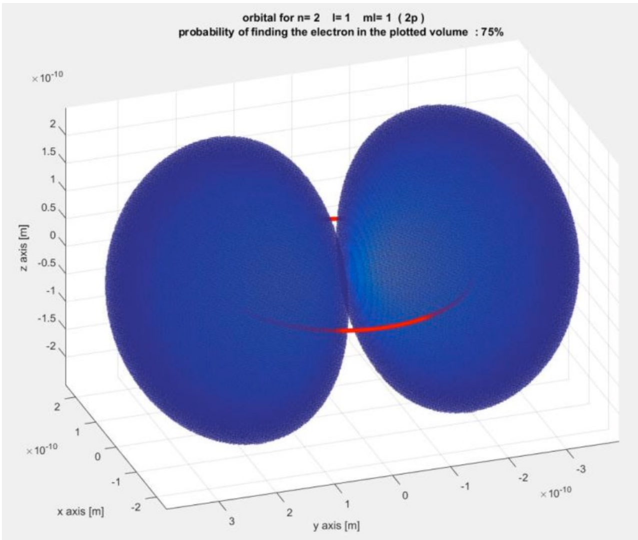


more possible projects…

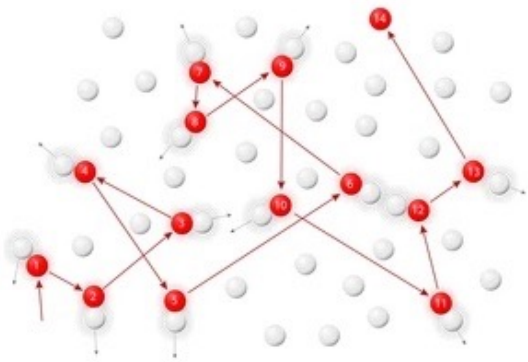
- **Predator-Prey**
  - we will study the dynamics and evolution of a diverse set of biological systems, integrated by different setups of predator and prey species through the so-called Lotka-Voltera equations.
  - ...



- **Schroedinger Equation**
  - we will study the time-independent Schrödinger equation for the hydrogen atom, understanding its theoretical basis
  - we will search numerically for wave functions that satisfy the boundary conditions, and we will compare their associated energies with those obtained experimentally in the series of Lyman, Balmer, Paschen, Brackett, ...
  - ...



- **Brownian Motion**
  - we will study movement of a particle inside a fluid by solving the equations of Langevin.
  - ...

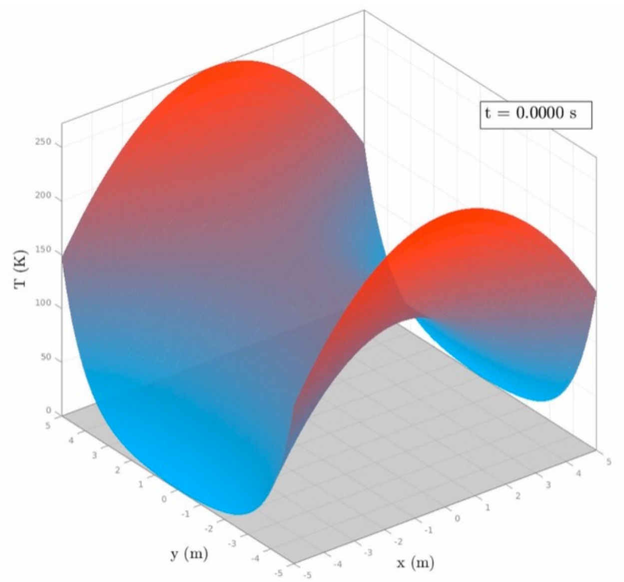


more possible projects…

- **Heat Diffusion**
  - we will temperature changes on a given surface by solving the equations of heat diffusion

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

  - ...



- **Your Faviourite Project**
  - we will study ???.
  - ...