Exercise 4: Cosmological Simulations

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Problem 7: Momentum Conservation in PM codes

Think of a PM code where mass assignment and interpolation of the forces back to the particle positions follow the same scheme utilizing a particular assignment function $W(|\vec{x}|)$. That means, the "overdensity" $\delta_{i,j,k}$ at the centre of cell i, j, k and the force $\vec{F}_p$ at a particle's position $\vec{x}_p$ can be written as follows:

$$\begin{aligned}\delta_{i,j,k} &= \textstyle\sum_{p=1}^{N_p} \frac{m_p}{M} W(\vec{q}_{i,j,k} - \vec{x}_p) - 1 \\ \vec{F}_p &= \textstyle\sum_{i,j,k=1}^{N_\mathrm{grid}} F_{i,j,k} W(\vec{q}_{i,j,k} - \vec{x}_p)\end{aligned}$$

where $\vec{q}_{i,j,k}$ is the position of the cell centre, $\vec{x}_p$ the particle's position m_p the particle's mass and M the total mass of all particles.

You further deal with periodic boundary conditions and derivatives can be approximated by finite differences. Please show that this scheme satisfies momentum conservation.

Tip: Derive a formula for the total force of all particles in relation to the assignment function $W(|\vec{x}|)$. Use the discretized Nabla operator to obtain the potential $\Phi_{i,j,k} = \sum_{i,j,k}^{N_\mathrm{grid}} G(\vec{q}_{i,j,k} - \vec{q}_{lmn})\delta_{l,m,n}$ where G is the Green's function of Poisson's equation. What happens to the sign when changing (inverting) the coordinates (e.g. $\vec{q}_1 \leftrightarrow \vec{q}_{N_p}$, $\vec{q}_2 \leftrightarrow \vec{q}_{N_p-1}$, etc.)

(5 points)

Problem 8: Number of Particles vs. Grid Cells

How many grid cells do you require for a pure PM code to reliably resolve all initially present waves? Derive a formula that involves the number of particles.

(3 points)

Problem 9: Simulations using `AMIGA`

`AMIGA` is an open source N-body code designed for cosmological simulations. It is based upon the idea of a PM code, but enhances its resolution in high density regions by dividing those cells into equal sib-cells on an individual basis; this scheme is also referred to as "adaptive mesh refinement" as the mesh adapts to the problem. It then solves Poisson's equation on those refined meshes to obtain the forces used to update the particle's positions and velocities. This exercise aims at getting you acquainted with `AMIGA`[1] and to run cosmological simulations, respectively.

1. **initial conditions**

   Generate initial conditions for a ΛCDM and a ΛWDM cosmological model using the already familiar code `PMstartM`. Use 32^3 particles and adjust the box size in a way to capture all relevant features of the input power spectrum $P(k)$. Store the IC's in ASCII format using `PM2asciiM`.

   Use the conversion tool `ascii2amiga` to convert the ASCII files to a format readable by `AMIGA`.

   (4 points)

2. **snapshots**

   When started, `AMIGA` asks for the redshifts at which you like to write an output file to disk. Use the tool `outputs` to generate a list of 100 redshifts that are equally spaced in time t inbetween your starting redshift and $z = 0$ (Note: do not forget to check your initial redshift and that it complies with the criterion "`DRho/rho`$\approx 0.1 - 0.2$").

   Run the code and carefully monitor the log-file. It should take a couple of hours to finish.

   (4 points)

3. **results**

   To view your simulation convert the files written to disk back to ASCII format using `amiga2ascii`. Try to use `gnuplot` to generate a movie out of those files.

   (4 points)

[1] `AMIGA` can be downloaded from the following web page `http://www.aip.de/People/aknebe/AMIGA`