Computational Cosmology

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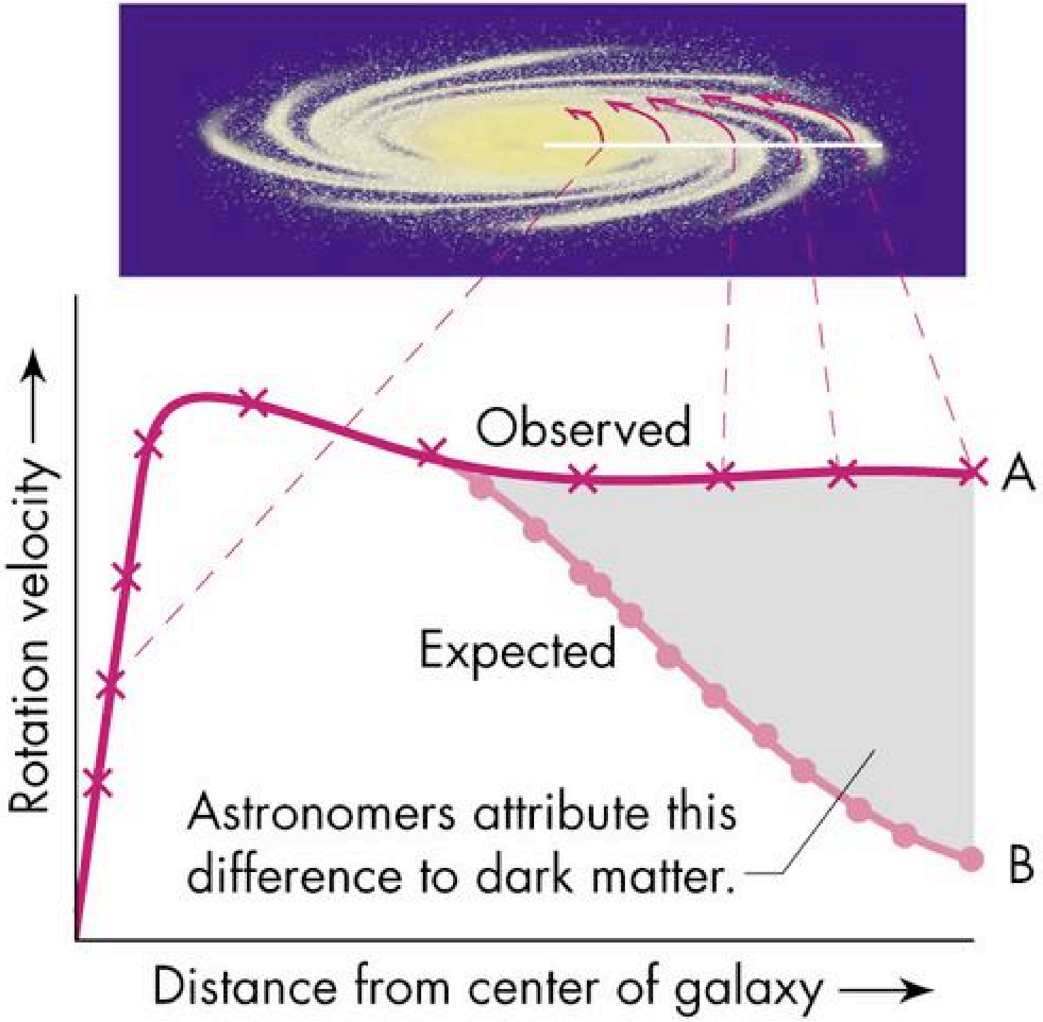
- MOND = **MO**dified **N**ewtonian **D**ynamics
    - why MOND?
    - what is MOND?
    - MOND in cosmology
    - MOND in ***AMIGA***

- MONDian Cosmological Simulations
    - large-scale clustering patterns
    - integral and internal properties galactic haloes

- evidence for dark matter – rotation curves

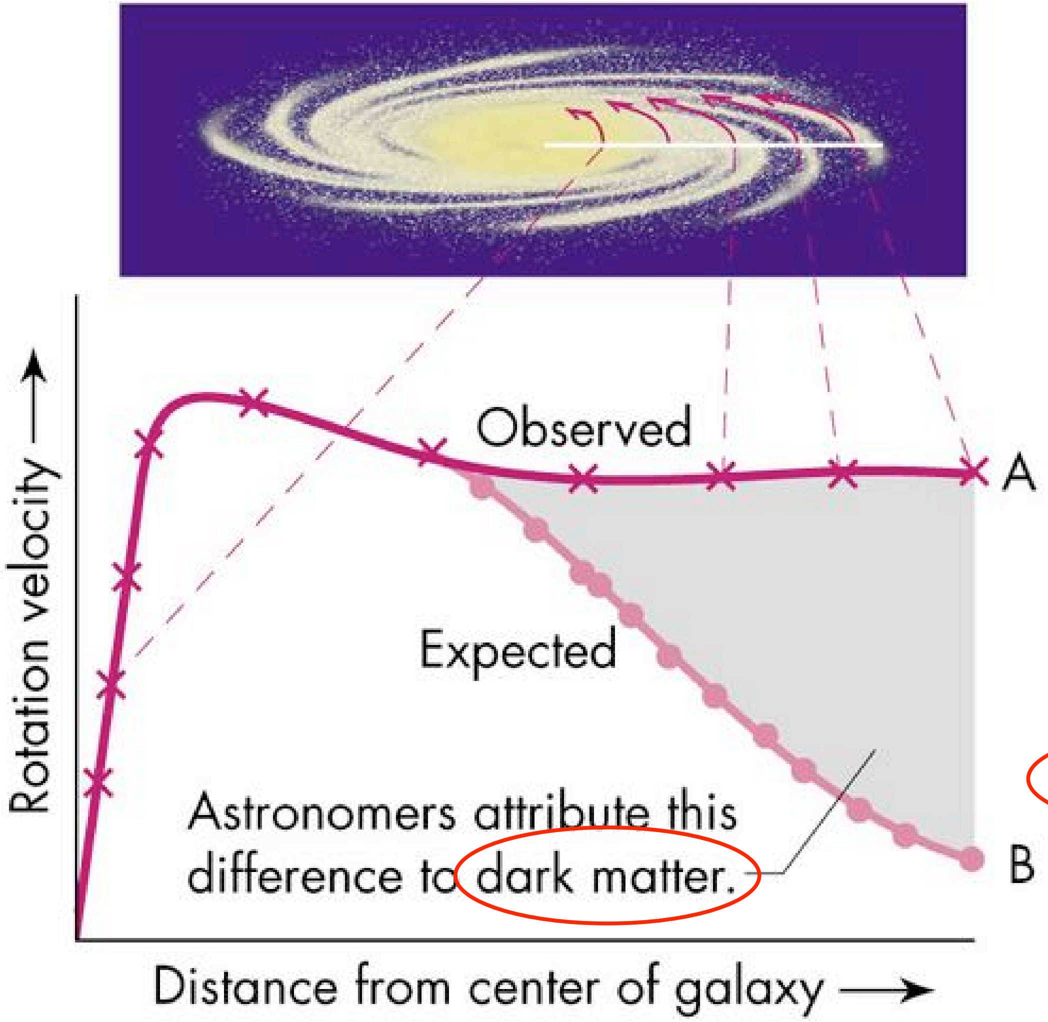


$$v_{circ}^2(r) = \frac{GM(<r)}{r}$$

$$\Big\downarrow \; M = const.$$

$$v_{circ}(r) \propto \frac{1}{\sqrt{r}}$$

- evidence for dark matter – rotation curves



Astronomers attribute this difference to dark matter.

$$v_{circ}^2(r) = \frac{GM(<r)}{r}$$

$$\Big\downarrow \; v_{circ} = const.$$

$$M(<r) \propto r$$

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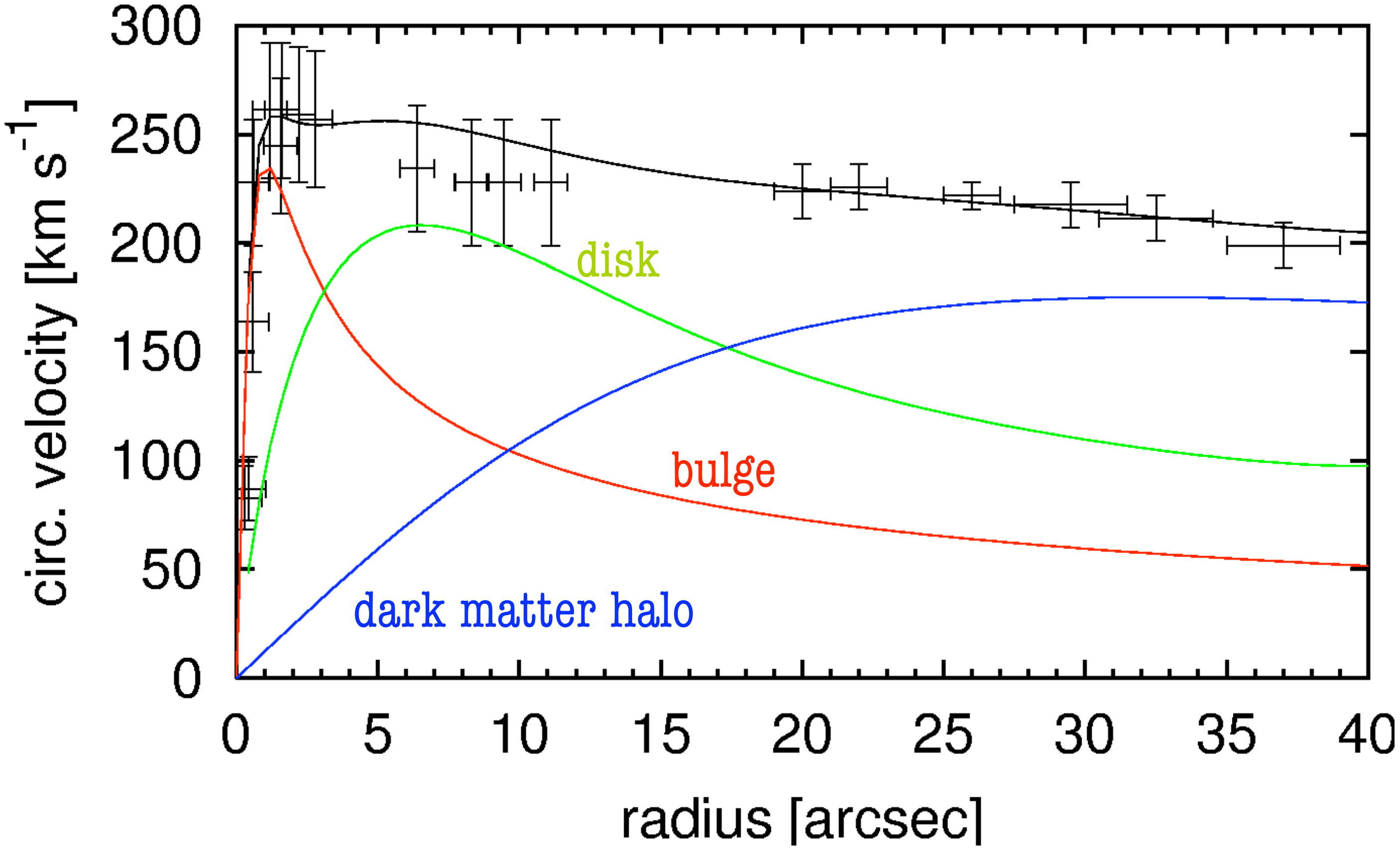

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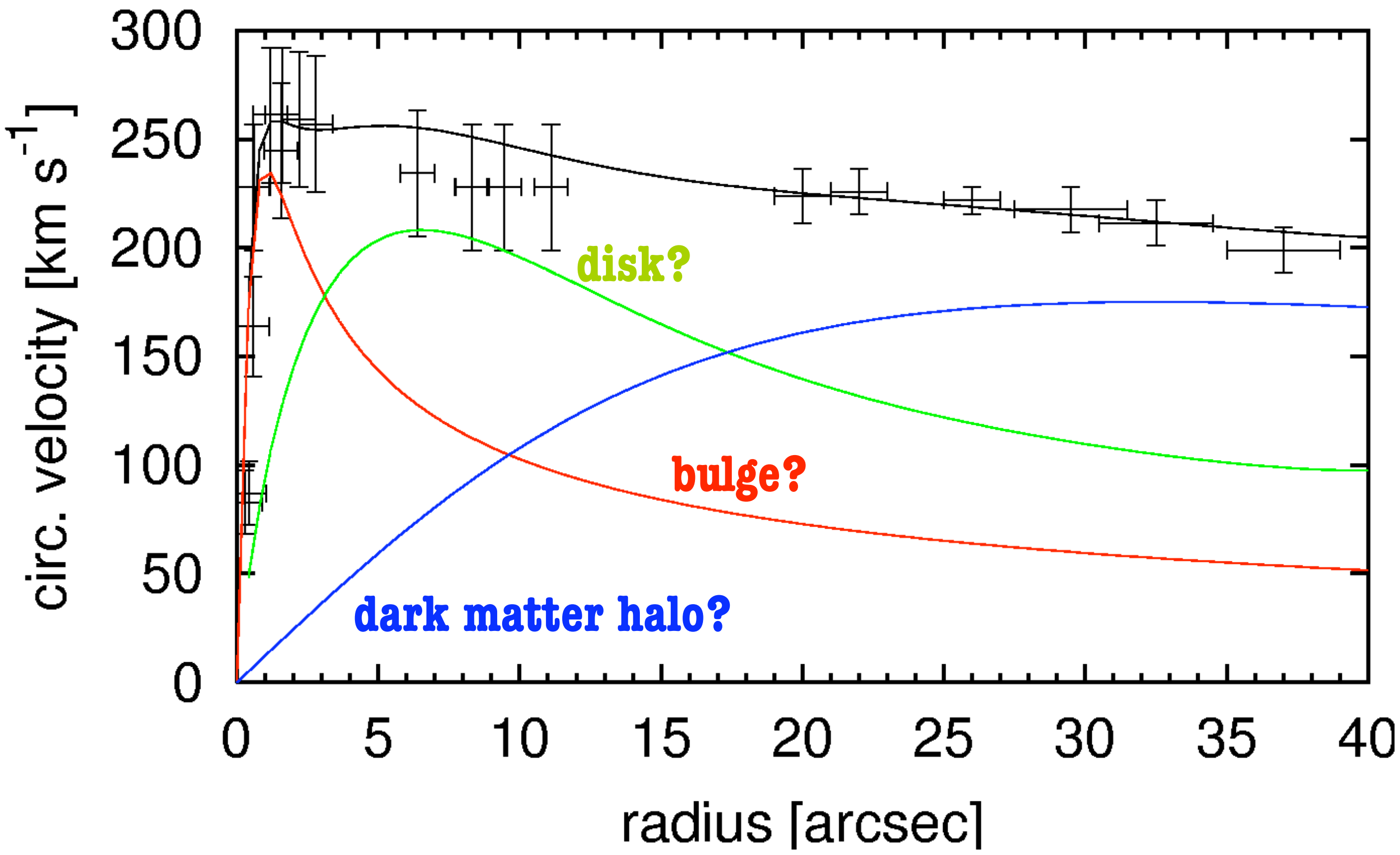
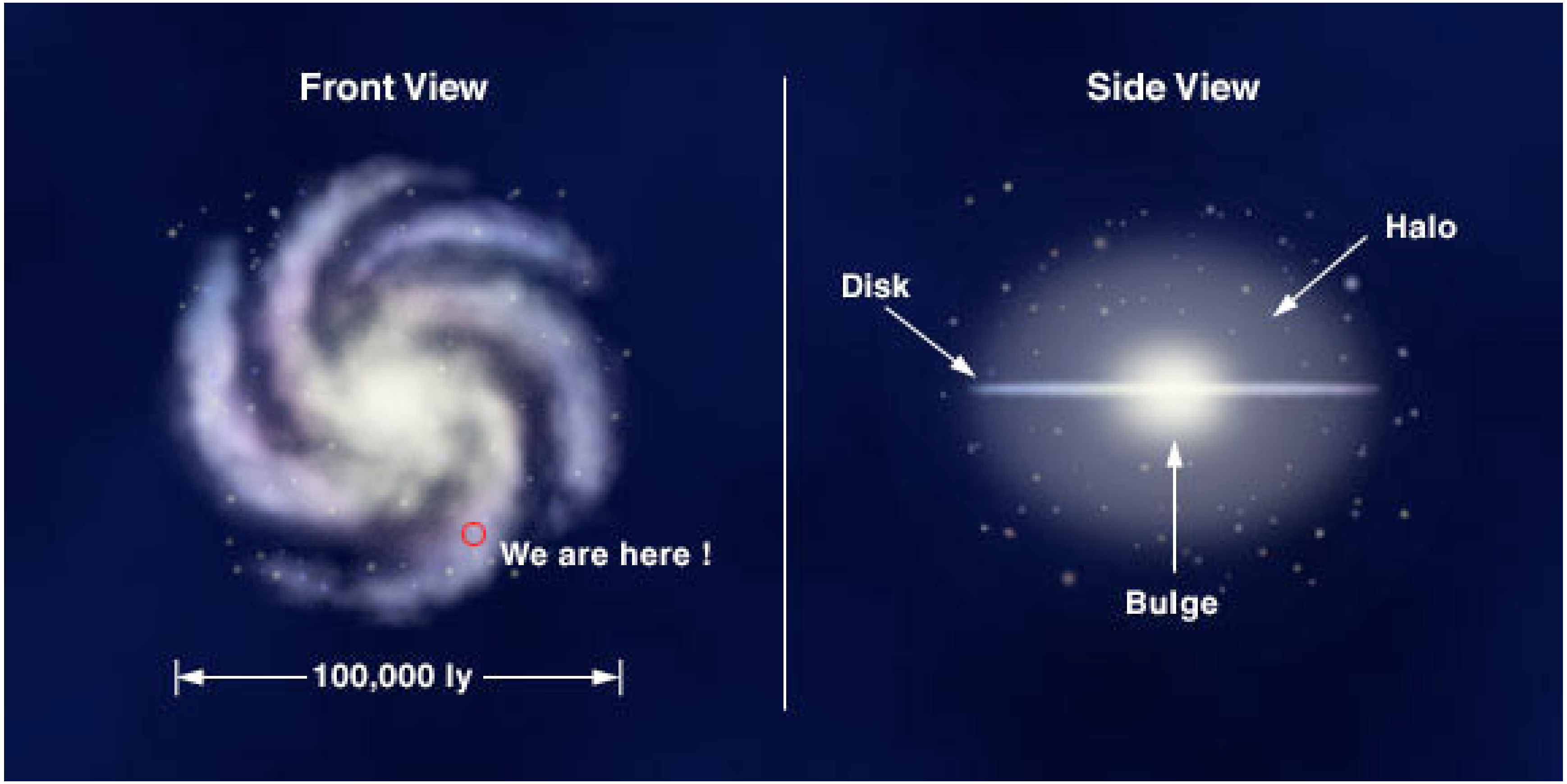
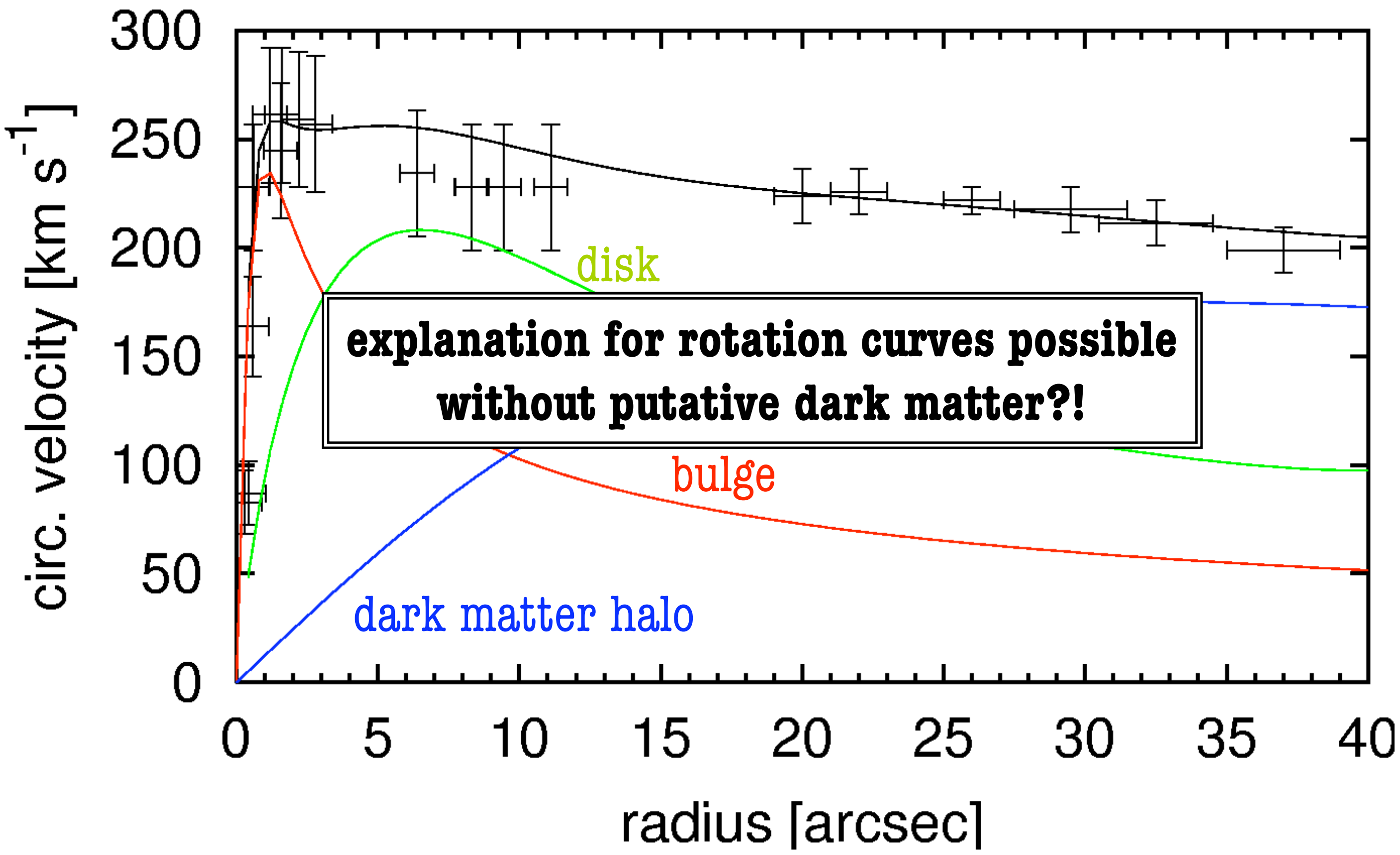


- evidence for dark matter – rotation curves

schematic model of a spiral galaxy…



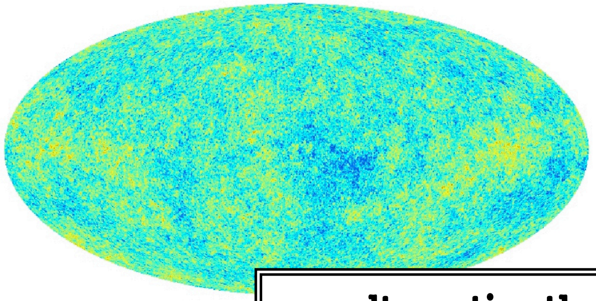
- evidence for dark matter – rotation curves



explanation for rotation curves possible without putative dark matter?!

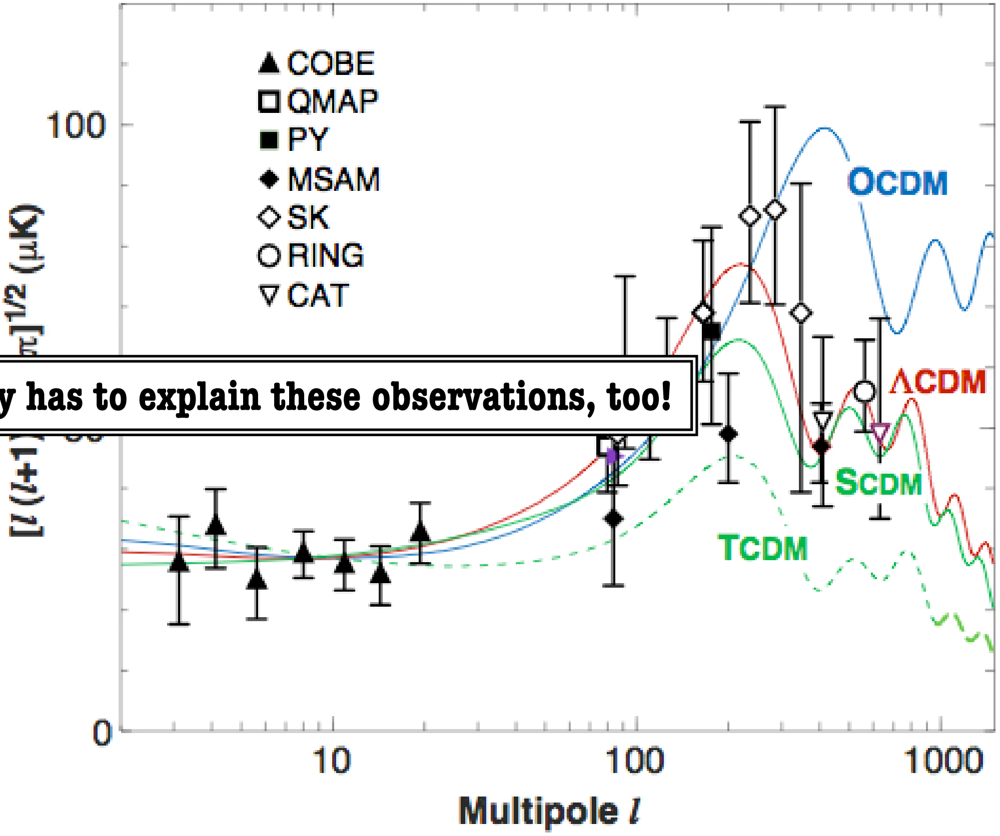
- evidence for dark matter – CMB, ...

CMB temperature map



any alternative theory has to explain these observations, too!

decomposition of $\Delta T/T$
into spherical harmonics...



- MOND = **MO**dified **N**ewtonian **D**ynamics
  - Newton's Second Law of Motion

$$\vec{F} = m\vec{a}_N$$

  - Modified Newtonian Dynamics

$$\vec{F} = m\vec{a}_M \mu\left(a_M / a_0\right)$$

will this give a flat rotation curve?

- MOND = **MO**dified **N**ewtonian **D**ynamics

  - MONDian accelerations

$$\vec{a}_N = \vec{a}_M \mu\left(a_M / a_0\right)$$

  - Milgrom's interpolation function

$$\mu(x) = \frac{x}{\sqrt{1-x^2}} \quad \Rightarrow \quad \begin{cases} \mu(x) \to x & ; x \to 0 \\ \mu(x) \to 1 & ; x \to \infty \end{cases}$$

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- limiting behaviour

x –> 0 : weak acceleration limit $a_M = \sqrt{a_N a_0}$

x –> ∞ : strong acceleration limit $a_M = a_N$

- MOND = **MO**dified **N**ewtonian **D**ynamics
  - MONDian accelerations – strong limit

$$a_M = a_N$$

all is well ;-)

- MOND = **MO**dified **N**ewtonian **D**ynamics
  - MONDian accelerations – weak limit

$$a_M = \sqrt{a_N a_0}$$

$$\left.\begin{aligned} a_M &= \frac{v^2}{r} \\ a_N &= \frac{GM(<r)}{r^2} \end{aligned}\right\} \Rightarrow \quad v^4 = GMa_0 \quad \Rightarrow$$ **flat rotation curve…**

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  - MONDian accelerations – weak limit

$$a_M = \sqrt{a_N a_0}$$

$$\left.\begin{aligned} a_M &= \frac{v^2}{r} \\ a_N &= \frac{GM(<r)}{r^2} \end{aligned}\right\} \Rightarrow \quad v^4 = GMa_0 = Ga_0 \frac{M}{L} L$$

$$\Rightarrow \quad v^4 \propto L$$

Tully-Fisher law!

- MOND = **MO**dified **N**ewtonian **D**ynamics
  - re-interpretation as modified gravity

$$\nabla \cdot \mu(a_M / a_0)\vec{a}_M = -4\pi G\rho(\vec{r})$$

  - relation between $\boldsymbol{a}_{\mathrm{M}}$ and $\boldsymbol{a}_{\mathrm{N}}$

$$\vec{a}_N = \mu(a_M / a_0)\vec{a}_M + \nabla \times \vec{h}$$

MOND

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vanishes for spherical, planar or cylindrical symmetry

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$$\mu(x) = \frac{x}{\sqrt{1-x^2}}$$

$$a_M = a_N \left[\frac{1}{2} + \frac{1}{2}\sqrt{1+\left(\frac{2a_0}{a_N}\right)^2} \right]^{1/2}$$

- assumptions
  - the term $\nabla \times \boldsymbol{h}$ can be neglected
  - MOND only affects *peculiar* accelerations

$$\boldsymbol{r} = R\boldsymbol{x} \qquad \nabla \cdot \boldsymbol{a}_{\mathrm{pec}} = -4\pi G\,(\rho(\boldsymbol{x}) - <\rho>)$$

$$\boldsymbol{r}'' = R\boldsymbol{x}'' + 2R'\boldsymbol{x}' + R''\boldsymbol{x} \qquad R\boldsymbol{x}'' + 2R'\boldsymbol{x}' = \boldsymbol{a}_{\mathrm{pec}}/R^2$$

(R: cosmic expansion factor)

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combine with 2nd Friedmann equation

$$r'' = \frac{a_{pec}}{R^2} - \frac{4\pi G}{3}\frac{<\rho>}{R^2}x$$

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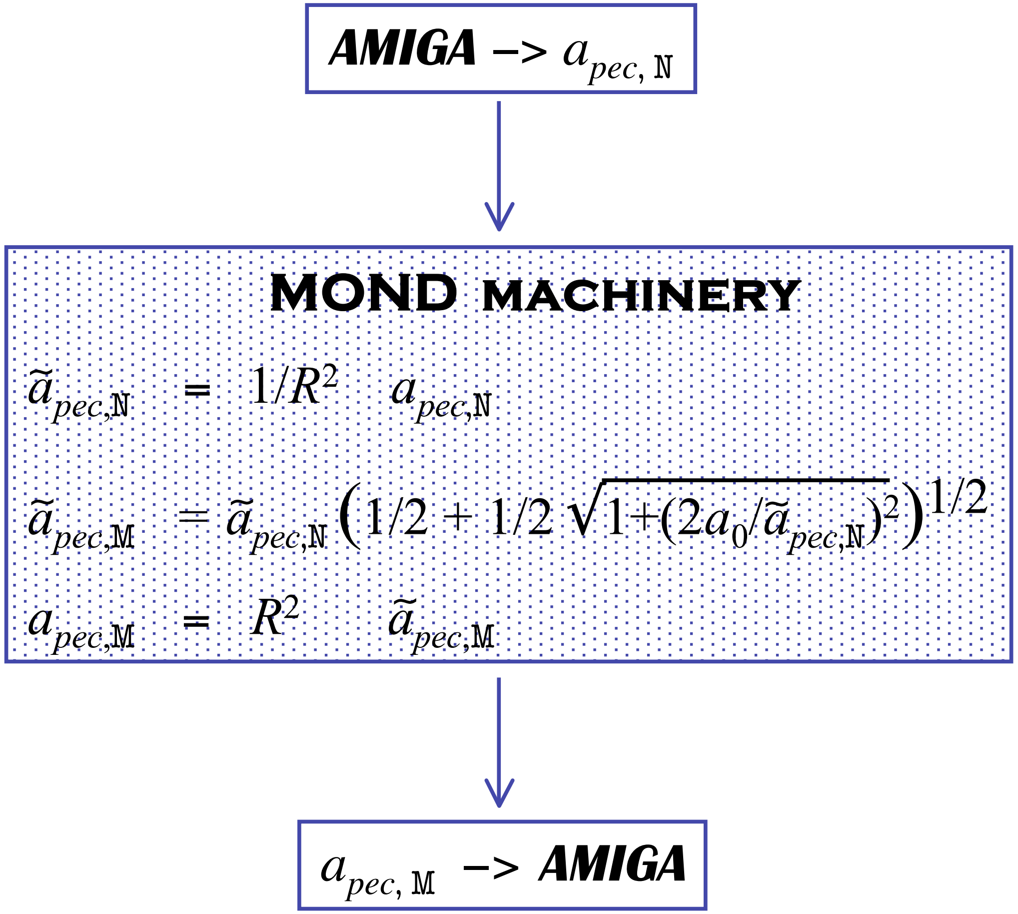
MOND in *AMIGA*

- implementation in N-body code ***AMIGA***[1] (***AMIGA*** = successor of **MLAPM**)



AMIGA –> $a_{pec,\mathtt{N}}$

MOND machinery

$$\tilde{a}_{pec,\mathtt{N}} = 1/R^2 \quad a_{pec,\mathtt{N}}$$

$$\tilde{a}_{pec,\mathtt{M}} = \tilde{a}_{pec,\mathtt{N}} \left(1/2 + 1/2\sqrt{1+(2a_0/\tilde{a}_{pec,\mathtt{N}})^2}\right)^{1/2}$$

$$a_{pec,\mathtt{M}} = R^2 \quad \tilde{a}_{pec,\mathtt{M}}$$

$a_{pec,\mathtt{M}}$ –> ***AMIGA***

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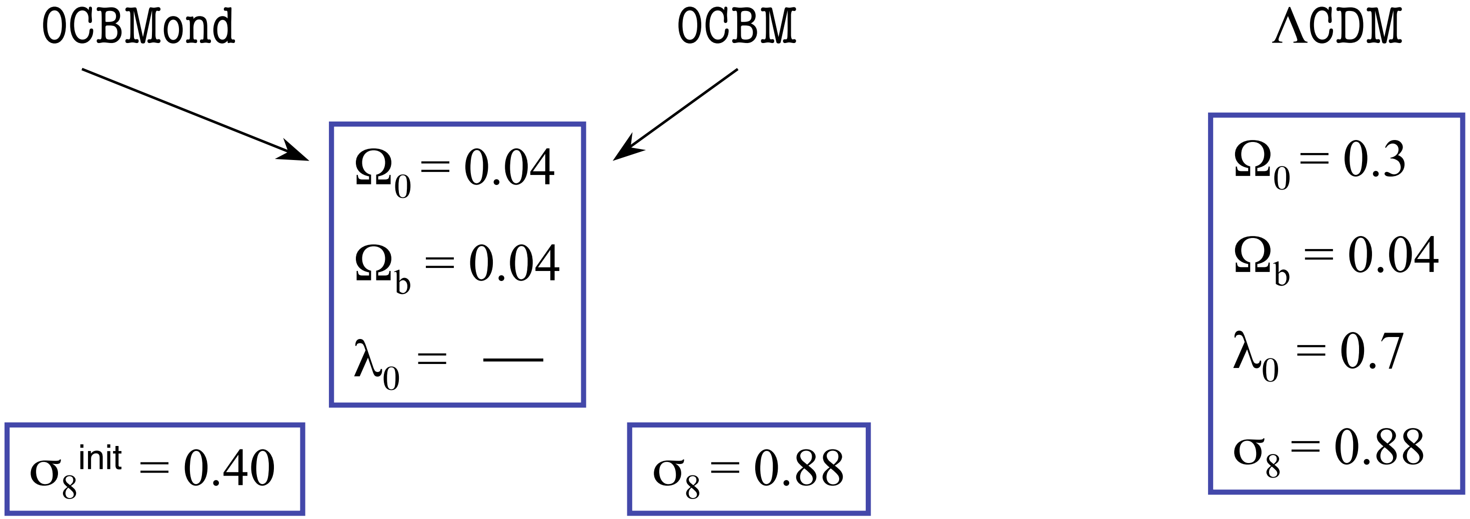
[1] http://www.aip.de/People/aknebe/AMIGA

- the competing cosmological models
- the large-scale structure
- galactic haloes:
  - abundance in raw numbers
  - rotational properties
  - internal structures
  - density profiles
- summary
  - any consensus?

- the competitors



OCBMond

OCBM

ΛCDM

$\Omega_0 = 0.04$

$\Omega_b = 0.04$

$\lambda_0 = $ —

$\sigma_8^{\text{init}} = 0.40$

$\sigma_8 = 0.88$

$\Omega_0 = 0.3$

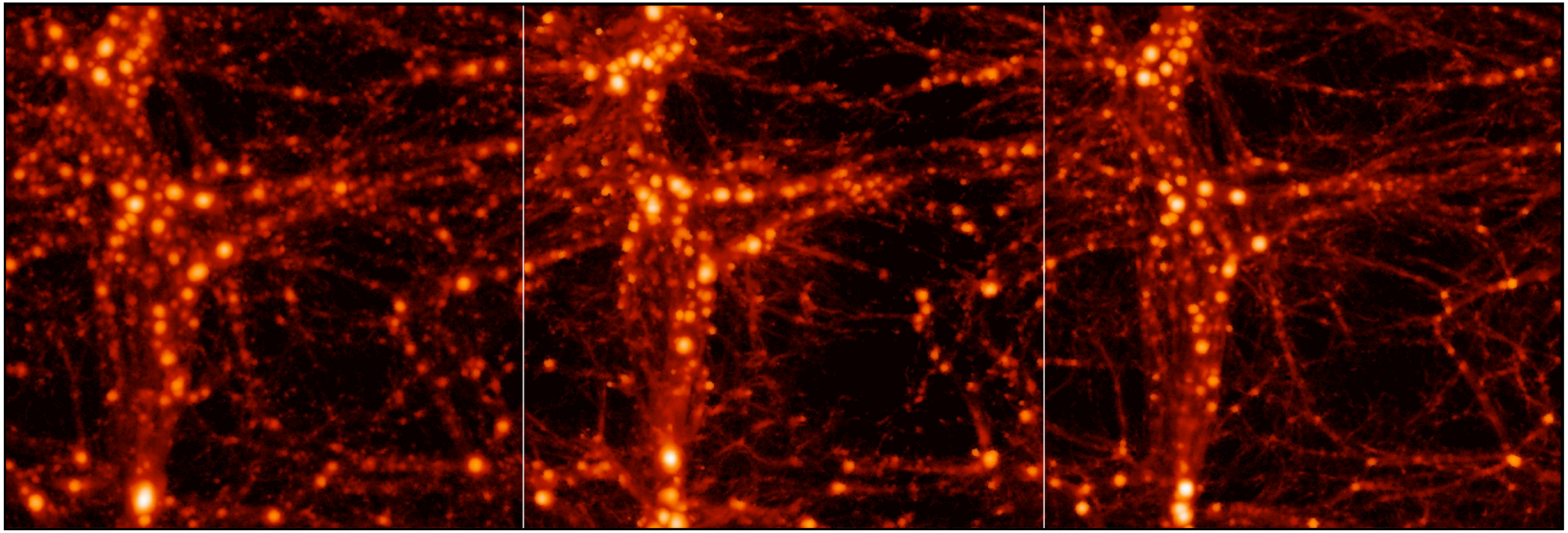
$\Omega_b = 0.04$

$\lambda_0 = 0.7$

$\sigma_8 = 0.88$

- other parameters:
  - 128^3 particles
  - 32 h^{-1}Mpc box
  - force resolution 11 h^{-1}kpc

- large scale clustering pattern



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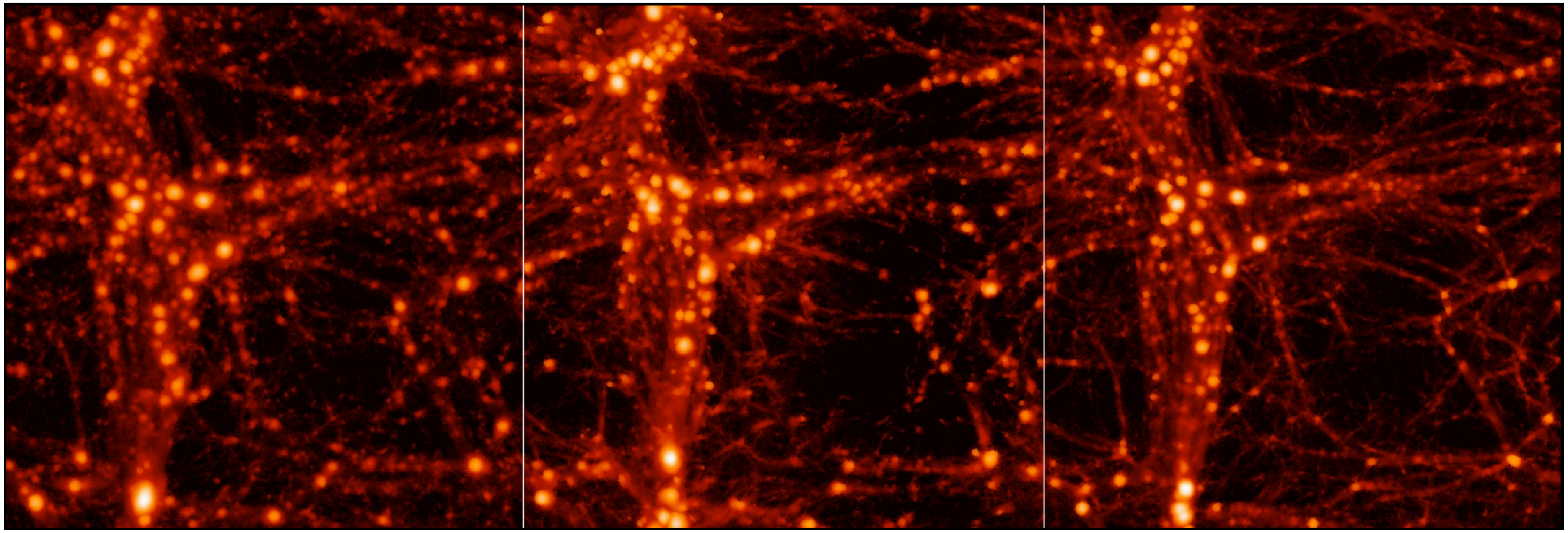

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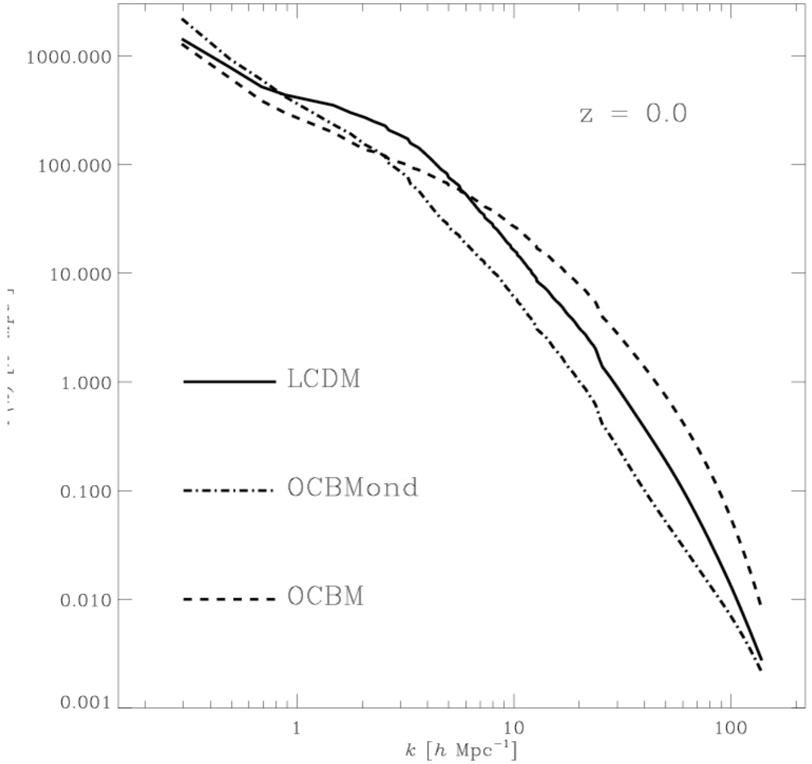
- large scale clustering pattern

dark matter power spectrum

- MOND features:
  - ahead on large scales
  - behind on small scales
  - no “non-linear break”



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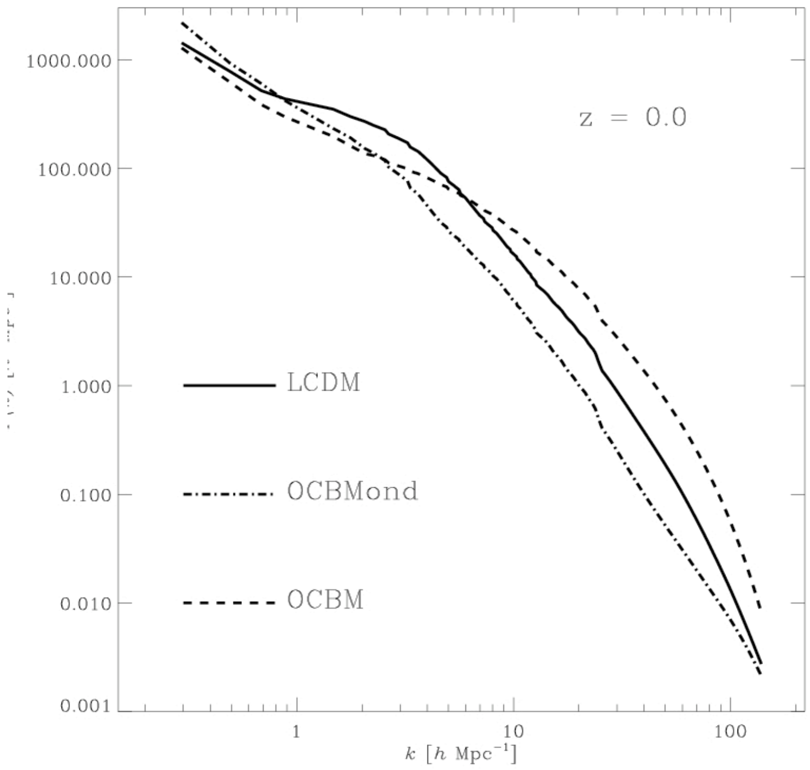


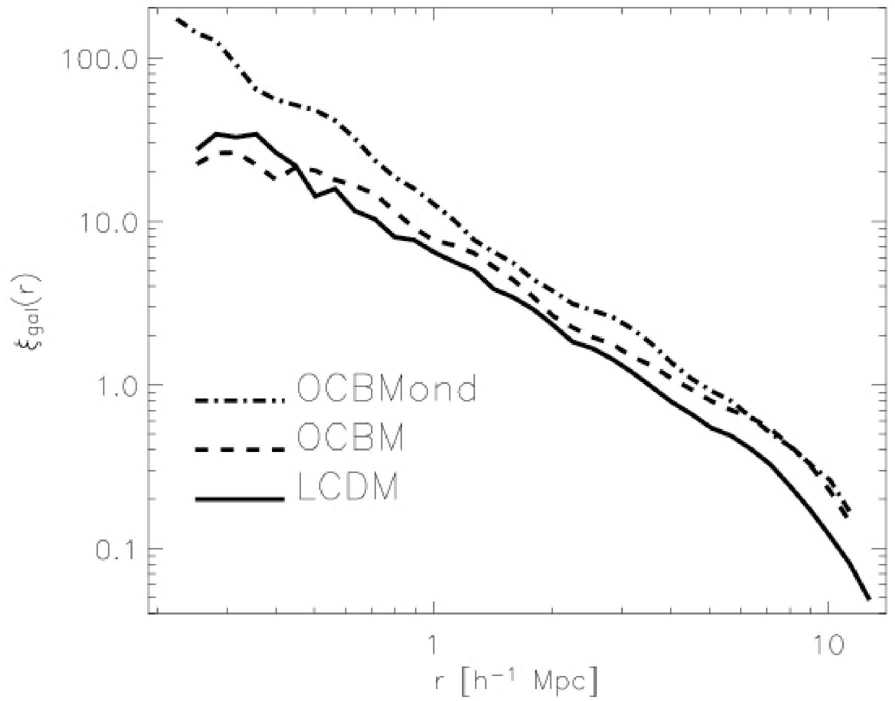
- MOND features:
  - ahead on large scales
  - behind on small scales
  - no "non-linear break"

indicating late evolution

- large scale clustering pattern

2-point correlation of virialized objects

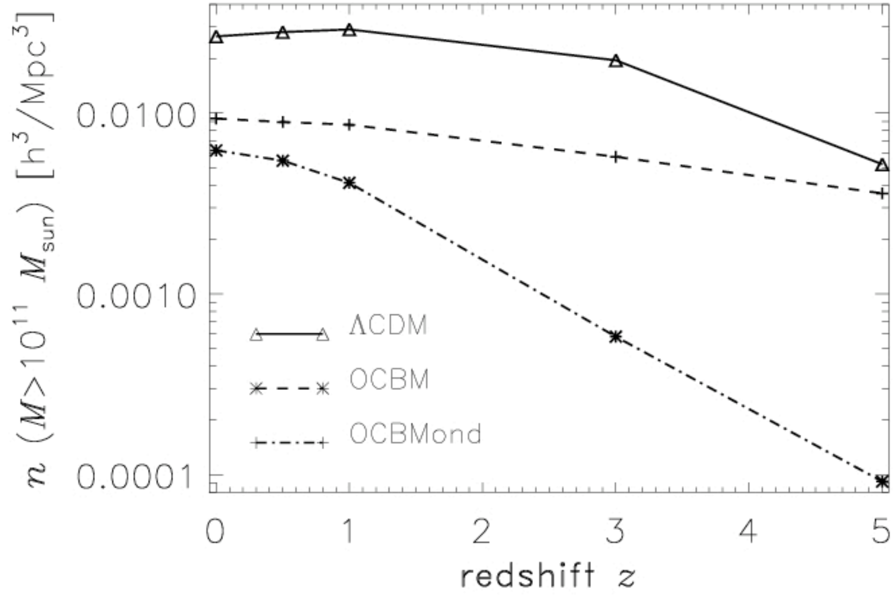
- MOND features:
  - – stronger correlation on small scales



- galactic haloes

abundance of objects with $M > 10^{11} M_\odot$

- MOND features:
  - very late evolution
  - hardly any objects beyond $z \geq 3$



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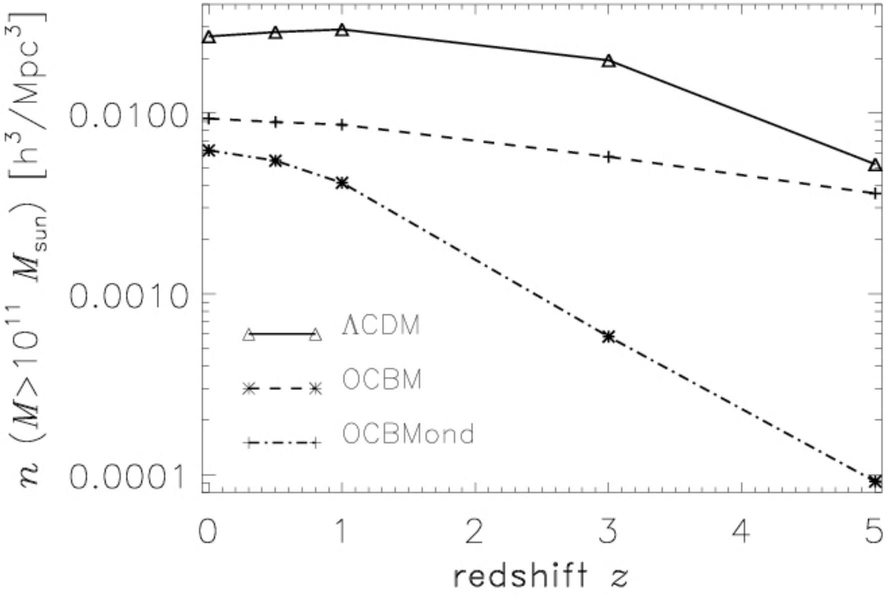
abundance of objects with $M > 10^{11} M_\odot$

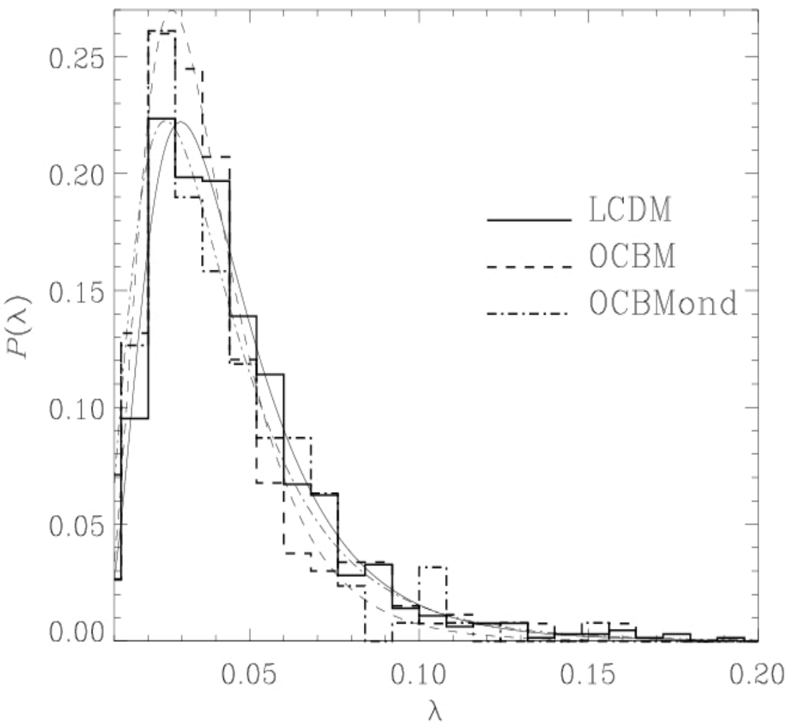


- MOND features:
  - very late evolution
  - hardly any objects beyond $z \geq 3$

→ **serious trouble for MOND?!**

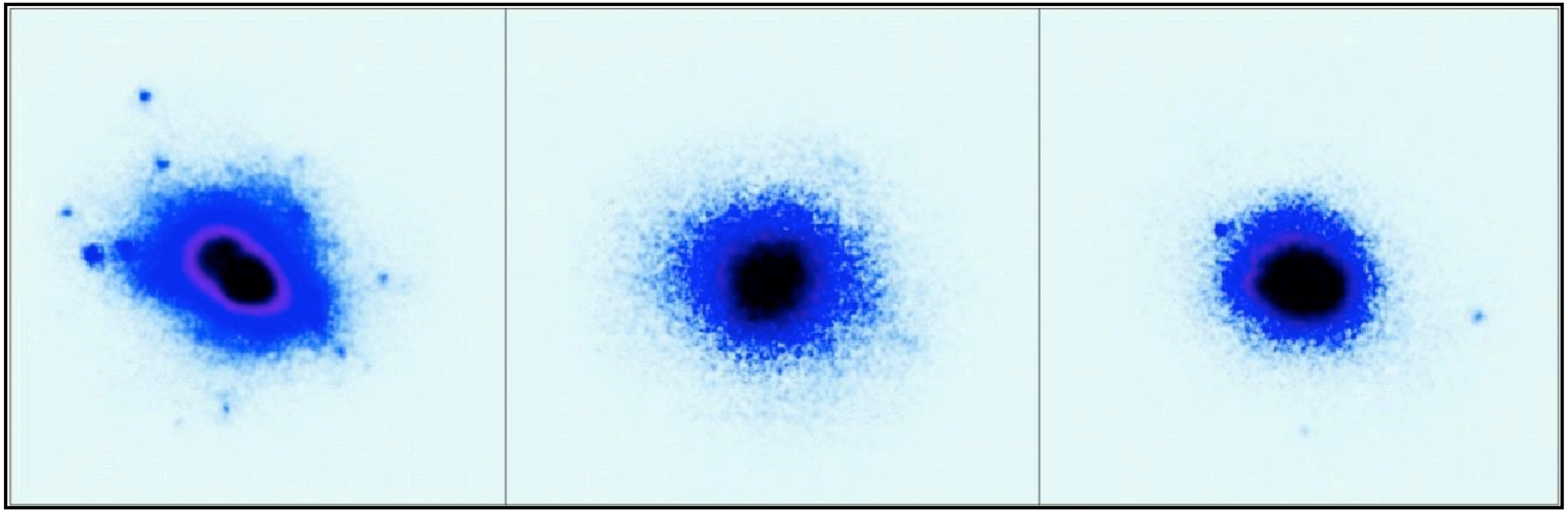
- galactic haloes

spin parameter distribution



  - MOND features:
    - hardly any differences

- galactic halo of mass $M \approx 10^{13} M_{\odot}$



ΛCDM
$z_{\text{stop}} \approx 2$

OCBMond
$z_{\text{stop}} \approx 11$

OCBM
$z_{\text{stop}} \approx 11$

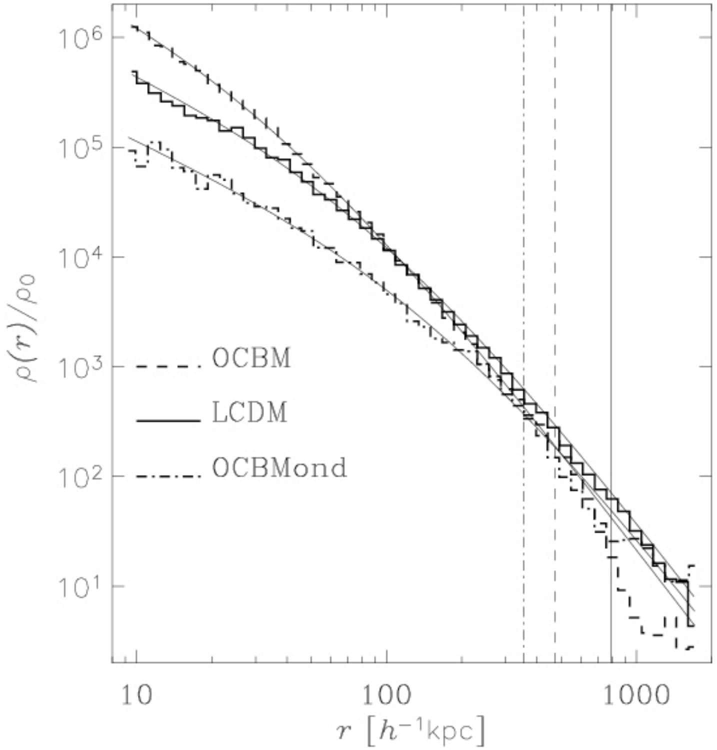
- MOND features:
  - less substructure
  - ...but remember $z_{\text{stop}} \approx 1/\Omega_0$ - 1 ?!

- galactic halo of mass $M \approx 10^{13} M_{\odot}$

density profile of the most massive object

- MOND features:
  - lower central density
  - lower concentration
  - **... but sill NFW shape!**

ΛCDM	$c_{\mathrm{NFW}} = 8.7$
OCBMond	$c_{\mathrm{NFW}} = 2.1$
OCBM	$c_{\mathrm{NFW}} = 14.1$



Knebe & Gibson, 2004, MNRAS 347, 1055

- summary
  - similar LSS clustering patterns yet indicating late evolution
  - stronger correlation of objects on small scales
  - less substructure
  - same spin parameter distribution

  - **hardly any objects beyond $z \geq 5$**

  - **still NFW density profiles**

KNEBE & GIBSON, 2004, MNRAS 347, 1055

- summary
  - similar LSS clustering patterns yet indicating late evolution
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  - **hardly any objects beyond $z \geq 5$**
  - **still NFW density profiles**

howling at the moon?!