Supercomoving Gastrophysics

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\vec{u}\right) = 0$$

$$\frac{\partial\left(\rho\vec{u}\right)}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\vec{\vec{1}}\right) = \rho\ \left(-\nabla\Phi\right)$$

$$\frac{\partial\left(\rho E\right)}{\partial t} + \nabla\cdot\left(\left[\rho E + p\right]\vec{u}\right) = \rho\vec{u}\cdot\left(-\nabla\Phi\right) + \left(\Gamma - L\right)$$

$$\frac{\partial S}{\partial t} + \nabla\cdot\left(S\vec{u}\right) = 0$$

$$\Delta\Phi = 4\pi G\left(\rho_{tot} + \frac{3p_{tot}}{c^2}\right) - \Lambda$$

$$\frac{d\vec{r}_{DM}}{dt} = \vec{u}_{DM}$$

$$\frac{d\vec{u}_{DM}}{dt} = -\nabla\Phi$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot \left(\rho \vec{u} \otimes \vec{u} + p \vec{1}\right) = \rho \; (-\nabla \Phi)$$

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot \left([\rho E + p] \vec{u}\right) = \rho \vec{u} \cdot (-\nabla \Phi) + (\Gamma - L)$$

$$\frac{\partial S}{\partial t} + \nabla \cdot (S \vec{u}) = 0$$

gas physics
via
Euler equations

$$\Delta \Phi = 4 \pi G \left(\rho_{tot} + \frac{3 p_{tot}}{c^2} \right) - \Lambda$$

dark matter
via
Monte Carlo integration

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gas physics via Euler equations

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coupling between gas + dark matter

dark matter via Monte Carlo integration

$$\frac{d\vec{r}_{DM}}{dt} = \vec{u}_{DM}$$

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$$\frac{d\vec{r}_{DM}}{dt} = \vec{u}_{DM}$$

$$\frac{d\vec{u}_{DM}}{dt} = -\nabla\Phi$$

ρ = gas density
$\vec{u}$ = gas velocity
p = gas pressure
E = total gas energy
S = gas entropy
ε = internal gas energy
Γ = cooling
L = heating
Φ = total gravitational potential
ρ_{tot} = total matter density (DM + gas)
p_{tot} = total pressure (DM + gas)
Λ = cosmological constant
$\vec{r}_{DM}$ = dark matter particle position
$\vec{u}_{DM}$ = dark matter particle velocity

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\vec{u}\right) = 0$$

$$\frac{\partial\left(\rho\vec{u}\right)}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\vec{1}\right) = \rho\ \left(-\nabla\Phi\right)$$

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$$\frac{d\vec{r}_{DM}}{dt} = \vec{u}_{DM}$$

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additional/closure equations:

$$E = \varepsilon + \frac{1}{2}u^2$$

$$p = \left(\gamma - 1\right)\rho\varepsilon$$

$$S = \frac{p}{\rho^{\gamma-1}}$$

$$\varepsilon = \frac{1}{\left(\gamma-1\right)}\frac{1}{\mu}\frac{k_B}{m_p}T$$

AIP

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \otimes \vec{u} + p \vec{1}) = \rho \ (-\nabla \Phi)$$

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot ([\rho E + p] \vec{u}) = \rho \vec{u} \cdot (-\nabla \Phi) + (\Gamma - L)$$

$$\frac{\partial S}{\partial t} + \nabla \cdot (S \vec{u}) = 0$$

$$\Delta \Phi = 4\pi G \left(\rho_{tot} + \frac{3 p_{tot}}{c^2} \right) - \Lambda$$

$$\frac{d\vec{r}_{DM}}{dt} = \vec{u}_{DM}$$

$$\frac{d\vec{u}_{DM}}{dt} = -\nabla \Phi$$

Note:
the total energy E does not contain gravitational energy,
but it is being taken care of in the energy conservation equation!

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$$E = \varepsilon + \frac{1}{2} u^2$$

$$p = (\gamma - 1) \rho \varepsilon$$

$$S = \frac{p}{\rho^{\gamma - 1}}$$

$$\varepsilon = \frac{1}{(\gamma - 1)} \frac{1}{\mu} \frac{k_B}{m_p} T$$

supercomoving variable		physical variable
dT	=	$\frac{dt}{a^2}$
$\vec{x}$	=	$\frac{\vec{r}}{a}$
$\vec{v}$	=	$a\vec{u} - \dot{a}\vec{r}$
ρ_x	=	$a^3\rho$
ϕ_x	=	$a^2\left(\Phi + \frac{1}{2}a\ddot{a}x^2\right)$
p_x	=	$a^5 p$
ε_x	=	$a^2\varepsilon$
T_x	=	$a^2 T$
S_x	=	$a^{3\gamma-8}S$
$\mathcal{H}$	=	$a\dot{a}$

$a(t)$ can be **any** function of time!

useful relations:

$$\vec{v} = a^2\dot{\vec{x}}$$

$$\left.\frac{\partial f}{\partial t}\right|_r = \left.\frac{\partial f}{\partial t}\right|_x - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x f$$

$$\nabla_r = \frac{1}{a}\nabla_x$$

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we now have to re-write all differential equations…

useful relations:

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$a(t)$ can be **any** function of time!

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{u} \right) = 0$$

$$\boxed{\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0}$$

A B

A

$$\left.\frac{\partial\rho}{\partial t}\right|_r = \left.\frac{\partial\rho}{\partial t}\right|_x - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x\rho$$
$$= \frac{\partial\rho}{\partial T}\frac{\partial T}{\partial t} - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x\rho$$
$$= \frac{\partial\rho}{\partial T}\frac{1}{a^2} - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x\rho$$
$$= \frac{1}{a^2}\left[\frac{1}{a^3}\frac{\partial\rho_x}{\partial T} - 3\rho_x\frac{1}{a^4}\frac{da}{dT}\right] - \frac{\dot{a}}{a^4}\vec{x}\cdot\nabla_x\rho_x$$
$$= \frac{1}{a^2}\left[\frac{1}{a^3}\frac{\partial\rho_x}{\partial T} - 3\rho_x\frac{1}{a^3}\mathcal{H}\right] - \frac{\dot{a}}{a^4}\vec{x}\cdot\nabla_x\rho_x$$
$$= \frac{1}{a^5}\left(\frac{\partial\rho_x}{\partial T} - 3\rho_x\mathcal{H} - \mathcal{H}\vec{x}\cdot\nabla_x\rho_x\right)$$

B

$$\nabla\cdot(\rho\vec{u}) = \frac{1}{a}\nabla_x\cdot\left(\frac{1}{a^3}\rho_x\left[\frac{1}{a}\left(\vec{v} + \mathcal{H}\vec{x}\right)\right]\right)$$
$$= \frac{1}{a^5}\nabla_x\cdot\left(\rho_x\left[\vec{v} + \mathcal{H}\vec{x}\right]\right)$$
$$= \frac{1}{a^5}\nabla_x\cdot(\rho_x\vec{v}) + \frac{\mathcal{H}}{a^5}\nabla_x\cdot(\rho_x\vec{x})$$
$$= \frac{1}{a^5}\nabla_x\cdot(\rho_x\vec{v}) + \frac{\mathcal{H}}{a^5}\left[\vec{x}\cdot\nabla_x\rho_x + \rho_x\nabla_x\cdot\vec{x}\right]$$
$$= \frac{1}{a^5}\nabla_x\cdot(\rho_x\vec{v}) + \frac{\mathcal{H}}{a^5}\left[\vec{x}\cdot\nabla_x\rho_x + \rho_x 3\right]$$
$$= \frac{1}{a^5}\left(\nabla_x\cdot(\rho_x\vec{v}) + \mathcal{H}\vec{x}\cdot\nabla_x\rho_x + \mathcal{H}\rho_x 3\right)$$

A+B=0

$$0 = \frac{1}{a^5}\left(\frac{\partial\rho_x}{\partial T} - 3\rho_x\mathcal{H} - \mathcal{H}\vec{x}\cdot\nabla_x\rho_x\right) + \frac{1}{a^5}\left(\nabla_x\cdot(\rho_x\vec{v}) + \mathcal{H}\vec{x}\cdot\nabla_x\rho_x + 3\mathcal{H}\rho_x\right)$$
$$= \frac{\partial\rho_x}{\partial T} - 3\rho_x\mathcal{H} - \mathcal{H}\vec{x}\cdot\nabla_x\rho_x + \nabla_x\cdot(\rho_x\vec{v}) + \mathcal{H}\vec{x}\cdot\nabla_x\rho_x + 3\mathcal{H}\rho_x$$
$$= \frac{\partial\rho_x}{\partial T} + \nabla_x\cdot(\rho_x\vec{v})$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\vec{u}\right) = 0 \longrightarrow \frac{\partial\rho_x}{\partial T} + \nabla_x\cdot\left(\rho_x\vec{v}\right) = 0$$

$$\begin{aligned}
\left.\frac{\partial\rho}{\partial t}\right|_r &= \left.\frac{\partial\rho}{\partial t}\right|_x - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x\rho \\
&= \frac{\partial\rho}{\partial T}\frac{\partial T}{\partial t} - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x\rho \\
&= \frac{\partial\rho}{\partial T}\frac{1}{a^2} - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x\rho \\
&= \frac{1}{a^2}\left[\frac{1}{a^3}\frac{\partial\rho_x}{\partial T} - 3\rho_x\frac{1}{a^4}\frac{da}{dT}\right] - \frac{\dot{a}}{a^4}\vec{x}\cdot\nabla_x\rho_x \\
&= \frac{1}{a^2}\left[\frac{1}{a^3}\frac{\partial\rho_x}{\partial T} - 3\rho_x\frac{1}{a^3}\mathcal{H}\right] - \frac{\dot{a}}{a^4}\vec{x}\cdot\nabla_x\rho_x \\
&= \frac{1}{a^5}\left(\frac{\partial\rho_x}{\partial T} - 3\rho_x\mathcal{H} - \mathcal{H}\vec{x}\cdot\nabla_x\rho_x\right)
\end{aligned}$$

$$\begin{aligned}
\nabla\cdot\left(\rho\vec{u}\right) &= \frac{1}{a}\nabla_x\cdot\left(\frac{1}{a^3}\rho_x\left[\frac{1}{a}\left(\vec{v} + \mathcal{H}\vec{x}\right)\right]\right) \\
&= \frac{1}{a^5}\nabla_x\cdot\left(\rho_x\left[\vec{v} + \mathcal{H}\vec{x}\right]\right) \\
&= \frac{1}{a^5}\nabla_x\cdot\left(\rho_x\vec{v}\right) + \frac{\mathcal{H}}{a^5}\nabla_x\cdot\left(\rho_x\vec{x}\right) \\
&= \frac{1}{a^5}\nabla_x\cdot\left(\rho_x\vec{v}\right) + \frac{\mathcal{H}}{a^5}\left[\vec{x}\cdot\nabla_x\rho_x + \rho_x\nabla_x\cdot\vec{x}\right] \\
&= \frac{1}{a^5}\nabla_x\cdot\left(\rho_x\vec{v}\right) + \frac{\mathcal{H}}{a^5}\left[\vec{x}\cdot\nabla_x\rho_x + \rho_x 3\right] \\
&= \frac{1}{a^5}\left(\nabla_x\cdot\left(\rho_x\vec{v}\right) + \mathcal{H}\vec{x}\cdot\nabla_x\rho_x + \mathcal{H}\rho_x 3\right)
\end{aligned}$$

$$\begin{aligned}
0 &= \frac{1}{a^5}\left(\frac{\partial\rho_x}{\partial T} - 3\rho_x\mathcal{H} - \mathcal{H}\vec{x}\cdot\nabla_x\rho_x\right) + \frac{1}{a^5}\left(\nabla_x\cdot\left(\rho_x\vec{v}\right) + \mathcal{H}\vec{x}\cdot\nabla_x\rho_x + 3\mathcal{H}\rho_x\right) \\
&= \frac{\partial\rho_x}{\partial T} - 3\rho_x\mathcal{H} - \mathcal{H}\vec{x}\cdot\nabla_x\rho_x + \nabla_x\cdot\left(\rho_x\vec{v}\right) + \mathcal{H}\vec{x}\cdot\nabla_x\rho_x + 3\mathcal{H}\rho_x \\
&= \frac{\partial\rho_x}{\partial T} + \nabla_x\cdot\left(\rho_x\vec{v}\right)
\end{aligned}$$

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\overleftrightarrow{1}\right) = \rho\ \left(-\nabla\Phi\right)$$

A B C

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\overleftrightarrow{1}\right) = \rho\ \left(-\nabla\Phi\right)$$

A $$\left.\frac{\partial(\rho\vec{u})}{\partial t}\right|_r = \rho\left.\frac{\partial\vec{u}}{\partial t}\right|_r + \vec{u}\left.\frac{\partial\rho}{\partial t}\right|_r$$

A.1 A.2

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\overleftrightarrow{1}\right) = \rho\ (-\nabla\Phi)$$

A.1

$$\begin{aligned}
\rho\left.\frac{\partial\vec{u}}{\partial t}\right|_r &= \rho\left(\left.\frac{\partial\vec{u}}{\partial t}\right|_x - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x\vec{u}\right) \\
&= \frac{1}{a^3}\rho_x\left(\left.\frac{\partial\vec{u}}{\partial t}\right|_x - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x\vec{u}\right) \\
&= \frac{1}{a^3}\rho_x\left(\left.\frac{\partial\vec{u}}{\partial t}\right|_x - K\right) \\
&= \frac{1}{a^3}\rho_x\left(\frac{\partial}{\partial t}\left[\frac{1}{a}\left\{\vec{v} + \mathcal{H}\vec{x}\right\}\right] - K\right) \\
&= \frac{1}{a^3}\rho_x\left(\left[-\frac{(\vec{v} + \mathcal{H}\vec{x})}{a^2}\dot{a} + \frac{1}{a}\left\{\frac{\partial\vec{v}}{\partial t} + \frac{\partial(\mathcal{H}\vec{x})}{\partial t}\right\}\right] - K\right) \\
&= \frac{1}{a^3}\rho_x\left(\left[-\frac{(\vec{v} + \mathcal{H}\vec{x})}{a^2}\dot{a} + \frac{1}{a}\left\{\frac{1}{a^2}\frac{\partial\vec{v}}{\partial T} + \frac{\partial(\mathcal{H}\vec{x})}{\partial t}\right\}\right] - K\right) \\
&= \frac{1}{a^3}\rho_x\left(\left[-\frac{(\vec{v} + \mathcal{H}\vec{x})}{a^2}\dot{a} + \frac{1}{a}\left\{\frac{1}{a^2}\frac{\partial\vec{v}}{\partial T} + \vec{x}\frac{\partial\mathcal{H}}{\partial t} + \mathcal{H}\frac{d\vec{x}}{dt}\right\}\right] - K\right) \\
&= \frac{1}{a^3}\rho_x\left(\left[-\frac{(\vec{v} + \mathcal{H}\vec{x})}{a^2}\dot{a} + \frac{1}{a}\left\{\frac{1}{a^2}\frac{\partial\vec{v}}{\partial T} + \vec{x}\frac{\partial\mathcal{H}}{\partial t}\right\}\right] - K\right) \\
&= \frac{1}{a^3}\rho_x\left(\left[-\frac{(\vec{v} + \mathcal{H}\vec{x})}{a^2}\dot{a} + \frac{1}{a}\left\{\frac{1}{a^2}\frac{\partial\vec{v}}{\partial T} + \vec{x}\left[\dot{a}^2 + a\ddot{a}\right]\right\}\right] - K\right) \\
&= \frac{1}{a^3}\rho_x\left(\left[-\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a}\left\{\frac{1}{a^2}\frac{\partial\vec{v}}{\partial T} + \vec{x}\left[\dot{a}^2 + a\ddot{a}\right]\right\}\right] - K\right) \\
&= \frac{1}{a^3}\rho_x\left(\left[-\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \frac{\vec{x}\left[\dot{a}^2 + a\ddot{a}\right]}{a}\right] - K\right) \\
&= \frac{1}{a^3}\rho_x\left(\left[-\frac{\mathcal{H}}{a^3}\vec{v} + \frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \ddot{a}\vec{x}\right] - K\right)
\end{aligned}$$

AIP

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\overleftrightarrow{1}\right) = \rho\ (-\nabla\Phi)$$

A.2

$\vec{u}\left.\frac{\partial\rho}{\partial t}\right|_r$ leave it as it is...will cancel automatically ;-)

AIP

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\vec{\vec{1}}\right) = \rho\ (-\nabla\Phi)$$

B

leave it as it is...will cancel automatically ;-)

$$\begin{aligned}
\nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\vec{\vec{1}}\right) &= \nabla p + \nabla\cdot\rho\vec{u}\otimes\vec{u} \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \rho\vec{u}\cdot\nabla\vec{u} \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a}\vec{u}\cdot\nabla_x\vec{u}\right) \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a}\vec{u}\cdot\nabla_x\vec{u}\right) \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a^2}(\vec{v}+\mathcal{H}\vec{x})\cdot\nabla_x\vec{u}\right) \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a^2}(\vec{v}\cdot\nabla_x\vec{u} + \mathcal{H}\vec{x}\cdot\nabla_x\vec{u})\right) \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a^2}\vec{v}\cdot\nabla_x\vec{u} + \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x\vec{u}\right) \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a^2}\vec{v}\cdot\nabla_x\vec{u} + K\right) \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a^2}\vec{v}\cdot\nabla_x\left(\frac{1}{a}\left[\vec{v}+\mathcal{H}\vec{x}\right]\right) + K\right) \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \frac{1}{a^3}\vec{v}\cdot\nabla_x\mathcal{H}\vec{x} + K\right) \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \frac{1}{a^3}\mathcal{H}\vec{v}\cdot\nabla_x\vec{x} + K\right) \\
&= \nabla p + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \frac{1}{a^3}\mathcal{H}\vec{v} + K\right) \\
&= \frac{1}{a^6}\nabla_x p_x + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \frac{1}{a^3}\mathcal{H}\vec{v} + K\right)
\end{aligned}$$

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\overleftrightarrow{1}\right) = \rho\,\left(-\nabla\Phi\right)$$

A.1+A.2+B

$$\frac{1}{a^3}\rho_x\left(\left[-\frac{\mathcal{H}}{a^3}\vec{v} + \frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \ddot{a}\vec{x}\right] - K\right) + \vec{u}\frac{\partial\rho}{\partial t} + \frac{1}{a^6}\nabla_x p_x + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \frac{1}{a^3}\mathcal{H}\vec{v} + K\right)$$

$$= \vec{u}\frac{\partial\rho}{\partial t} + \vec{u}\nabla\cdot(\rho\vec{u}) + \frac{1}{a^3}\rho_x\left(\left[-\frac{\mathcal{H}}{a^3}\vec{v} + \frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \ddot{a}\vec{x}\right] - K\right) + \frac{1}{a^6}\nabla_x p_x + \frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \frac{1}{a^3}\mathcal{H}\vec{v} + K\right)$$

$$= \vec{u}\left(\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{u})\right) + \frac{1}{a^3}\rho_x\left(\left[-\frac{\mathcal{H}}{a^3}\vec{v} + \frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \ddot{a}\vec{x}\right] - K\right) + \frac{1}{a^6}\nabla_x p_x + \frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \frac{1}{a^3}\mathcal{H}\vec{v} + K\right)$$

$$= \frac{1}{a^3}\rho_x\left(\left[-\frac{\mathcal{H}}{a^3}\vec{v} + \frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \ddot{a}\vec{x}\right] - K\right) + \frac{1}{a^6}\nabla_x p_x + \frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \frac{1}{a^3}\mathcal{H}\vec{v} + K\right)$$

$$= \frac{1}{a^3}\rho_x\left(\left[-\frac{\mathcal{H}}{a^3}\vec{v} + \frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \ddot{a}\vec{x}\right]\right) + \frac{1}{a^6}\nabla_x p_x + \frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \frac{1}{a^3}\mathcal{H}\vec{v}\right)$$

$$= \frac{1}{a^3}\rho_x\left(-\frac{\mathcal{H}}{a^3}\vec{v} + \frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \ddot{a}\vec{x} + \frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \frac{1}{a^3}\mathcal{H}\vec{v}\right) + \frac{1}{a^6}\nabla_x p_x$$

$$= \frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \ddot{a}\vec{x}\right) + \frac{1}{a^6}\nabla_x p_x$$

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\overleftrightarrow{1}\right) = \rho\ (-\nabla\Phi)$$

C

$$\begin{aligned}
-\rho\nabla\Phi &= -\frac{1}{a^3}\rho_x\frac{1}{a}\nabla_x\left(\frac{\phi_x}{a^2} - \frac{1}{2}a\ddot{a}x^2\right) \\
&= -\frac{1}{a^6}\rho_x\nabla_x\phi_x + \frac{1}{a^4}\rho_x\nabla_x\left(\frac{1}{2}a\ddot{a}x^2\right) \\
&= -\frac{1}{a^6}\rho_x\nabla_x\phi_x + \frac{\ddot{a}}{2a^3}\rho_x\nabla_x\left(x^2\right) \\
&= -\frac{1}{a^6}\rho_x\nabla_x\phi_x + \frac{\ddot{a}}{a^3}\rho_x\vec{x} \\
&= \frac{1}{a^3}\rho_x\left(-\frac{1}{a^3}\nabla_x\phi_x + \ddot{a}\vec{x}\right)
\end{aligned}$$

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\overleftrightarrow{1}\right) = \rho\;(-\nabla\Phi)$$

A+B=C

$$\frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \ddot{a}\vec{x}\right) + \frac{1}{a^6}\nabla_x p_x = \frac{1}{a^3}\rho_x\left(-\frac{1}{a^3}\nabla_x\phi_x + \ddot{a}\vec{x}\right)$$

$$\rho_x\left(\frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \ddot{a}\vec{x}\right) + \frac{1}{a^3}\nabla_x p_x = \rho_x\left(-\frac{1}{a^3}\nabla_x\phi_x + \ddot{a}\vec{x}\right)$$

$$\rho_x\left(\frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v}\right) + \frac{1}{a^3}\nabla_x p_x = \rho_x\left(-\frac{1}{a^3}\nabla_x\phi_x\right)$$

$$\rho_x\left(\frac{\partial\vec{v}}{\partial T} + \vec{v}\cdot\nabla_x\vec{v}\right) + \nabla_x p_x = \rho_x(-\nabla_x\phi_x)$$

$$\rho_x\frac{\partial\vec{v}}{\partial T} + \rho_x\vec{v}\cdot\nabla_x\vec{v} + \nabla_x p_x = \rho_x(-\nabla_x\phi_x)$$

$$\frac{\partial(\rho_x\vec{v})}{\partial T} + \nabla_x(\rho_x\vec{v}\otimes\vec{v}) + \nabla_x p_x = \rho_x(-\nabla_x\phi_x)$$

$$\frac{\partial(\rho_x\vec{v})}{\partial T} + \nabla_x\left(\rho_x\vec{v}\otimes\vec{v} + p_x\vec{1}\right) = \rho_x(-\nabla_x\phi_x)$$

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla\cdot\left(\rho\vec{u}\otimes\vec{u} + p\vec{1}\right) = \rho\,(-\nabla\Phi) \longrightarrow \frac{\partial(\rho_x\vec{v})}{\partial T} + \nabla_x\cdot\left(\rho_x\vec{v}\otimes\vec{v} + p_x\vec{1}\right) = \rho_x(-\nabla_x\phi_x)$$

A+B=C

$$\frac{1}{a^3}\rho_x\left(\frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \ddot{a}\vec{x}\right) + \frac{1}{a^6}\nabla_x p_x = \frac{1}{a^3}\rho_x\left(-\frac{1}{a^3}\nabla_x\phi_x + \ddot{a}\vec{x}\right)$$

$$\rho_x\left(\frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v} + \ddot{a}\vec{x}\right) + \frac{1}{a^3}\nabla_x p_x = \rho_x\left(-\frac{1}{a^3}\nabla_x\phi_x + \ddot{a}\vec{x}\right)$$

$$\rho_x\left(\frac{1}{a^3}\frac{\partial\vec{v}}{\partial T} + \frac{1}{a^3}\vec{v}\cdot\nabla_x\vec{v}\right) + \frac{1}{a^3}\nabla_x p_x = \rho_x\left(-\frac{1}{a^3}\nabla_x\phi_x\right)$$

$$\rho_x\left(\frac{\partial\vec{v}}{\partial T} + \vec{v}\cdot\nabla_x\vec{v}\right) + \nabla_x p_x = \rho_x\left(-\nabla_x\phi_x\right)$$

$$\rho_x\frac{\partial\vec{v}}{\partial T} + \rho_x\vec{v}\cdot\nabla_x\vec{v} + \nabla_x p_x = \rho_x\left(-\nabla_x\phi_x\right)$$

$$\frac{\partial(\rho_x\vec{v})}{\partial T} + \nabla_x\left(\rho_x\vec{v}\otimes\vec{v}\right) + \nabla_x p_x = \rho_x\left(-\nabla_x\phi_x\right)$$

$$\frac{\partial(\rho_x\vec{v})}{\partial T} + \nabla_x\left(\rho_x\vec{v}\otimes\vec{v} + p_x\vec{1}\right) = \rho_x\left(-\nabla_x\phi_x\right)$$

$$\frac{\partial(\rho E)}{\partial t} + \nabla\cdot\left(\left[\rho E + p\right]\vec{u}\right) = \rho\vec{u}\cdot\left(-\nabla\Phi\right) + \left(\Gamma - L\right)$$

$$\frac{\partial(\rho E)}{\partial t} + \nabla \cdot \left(\left[\rho E + p \right] \vec{u} \right) = \rho \vec{u} \cdot \left(-\nabla \Phi \right) + \left(\Gamma - L \right)$$

rather than inserting supercomoving coordinates
we are going to derive the energy conservation...

(for simplicity we drop all subscripts)

$$\frac{\partial(\rho E)}{\partial t} + \nabla\cdot\left([\rho E + p]\vec{u}\right) = \rho\vec{u}\cdot(-\nabla\Phi) + (\Gamma - L)$$

$$\frac{\partial v}{\partial T} + (\vec{v}\cdot\nabla)\vec{v} = -\nabla\phi - \frac{1}{\rho}\nabla p$$

$$\rho\vec{v}\cdot\frac{\partial v}{\partial T} + \rho\vec{v}\cdot(\vec{v}\cdot\nabla)\vec{v} = -\rho\vec{v}\cdot\nabla\phi - \frac{1}{\rho}\rho\vec{v}\cdot\nabla p$$

$$\rho\vec{v}\cdot\frac{\partial v}{\partial T} + \rho\vec{v}\cdot(\vec{v}\cdot\nabla)\vec{v} = -\rho\vec{v}\cdot\nabla\phi - \vec{v}\cdot\nabla p$$

$$\nabla\cdot(\rho\vec{v}\phi) = \rho\vec{v}\cdot\nabla\phi + \phi\nabla\cdot(\rho\vec{v})$$

$$\nabla\cdot(p\vec{v}) = \vec{v}\cdot\nabla p + p\nabla\cdot\vec{v}$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = \rho\vec{v}\cdot\frac{\partial\vec{v}}{\partial T} + \frac{1}{2}v^2\frac{\partial\rho}{\partial T}$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} - \frac{1}{2}v^2\frac{\partial\rho}{\partial T} + \frac{1}{2}\nabla\cdot(\rho v^2\vec{v}) - \frac{1}{2}v^2\nabla\cdot(\rho\vec{v}) = -\nabla\cdot(\rho\vec{v}\phi) + \phi\nabla\cdot(\rho\vec{v}) - \nabla\cdot(p\vec{v}) + p\nabla\cdot\vec{v}$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} - \frac{1}{2}v^2\left[\frac{\partial\rho}{\partial T} + \nabla\cdot(\rho\vec{v})\right] + \frac{1}{2}\nabla\cdot(\rho v^2\vec{v}) = -\nabla\cdot(\rho\vec{v}\phi) + \phi\nabla\cdot(\rho\vec{v}) - \nabla\cdot(p\vec{v}) + p\nabla\cdot\vec{v}$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} + \frac{1}{2}\nabla\cdot(\rho v^2\vec{v}) = -\nabla\cdot(\rho\vec{v}\phi) + \phi\nabla\cdot(\rho\vec{v}) - \nabla\cdot(p\vec{v}) + p\nabla\cdot\vec{v}$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v}\right) + \phi\nabla\cdot(\rho\vec{v}) + p\nabla\cdot\vec{v}$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} + p\nabla\cdot\vec{v}$$

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$$\frac{\partial(\rho E)}{\partial t} + \nabla\cdot\left([\rho E + p]\vec{u}\right) = \rho\vec{u}\cdot(-\nabla\Phi) + (\Gamma - L)$$

internal energy conservation:

$$p\nabla\cdot\vec{v} = \left(-\rho\frac{\partial\varepsilon}{\partial T} - \rho\vec{v}\cdot\nabla\varepsilon - \rho\varepsilon\mathcal{H}[3\gamma-5]\right)$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} + p\nabla\cdot\vec{v}$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} + \left(-\rho\frac{\partial\varepsilon}{\partial T} - \rho\vec{v}\cdot\nabla\varepsilon - \rho\varepsilon\mathcal{H}[3\gamma-5]\right)$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \rho\frac{\partial\varepsilon}{\partial T} - \rho\vec{v}\cdot\nabla\varepsilon - \rho\varepsilon\mathcal{H}[3\gamma-5]$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \left(\frac{\partial(\rho\varepsilon)}{\partial T} - \varepsilon\frac{\partial\rho}{\partial T}\right) - \rho\vec{v}\cdot\nabla\varepsilon - \rho\varepsilon\mathcal{H}[3\gamma-5]$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \left(\frac{\partial(\rho\varepsilon)}{\partial T} - \varepsilon\frac{\partial\rho}{\partial T}\right) - \left(\nabla\cdot(\rho\varepsilon\vec{v}) - \varepsilon\nabla\cdot(\rho\vec{v})\right) - \rho\varepsilon\mathcal{H}[3\gamma-5]$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v} + \rho\varepsilon\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \left(\frac{\partial(\rho\varepsilon)}{\partial T} - \varepsilon\frac{\partial\rho}{\partial T}\right) + \varepsilon\nabla\cdot(\rho\vec{v}) - \rho\varepsilon\mathcal{H}[3\gamma-5]$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v} + \rho\varepsilon\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \frac{\partial(\rho\varepsilon)}{\partial T} + \varepsilon\frac{\partial\rho}{\partial T} + \varepsilon\nabla\cdot(\rho\vec{v}) - \rho\varepsilon\mathcal{H}[3\gamma-5]$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v} + \rho\varepsilon\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \frac{\partial(\rho\varepsilon)}{\partial T} + \varepsilon\left[\frac{\partial\rho}{\partial T} + \nabla\cdot(\rho\vec{v})\right] - \rho\varepsilon\mathcal{H}[3\gamma-5]$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v} + \rho\varepsilon\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \frac{\partial(\rho\varepsilon)}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma-5]$$

continued on next page...

$$\frac{\partial(\rho E)}{\partial t} + \nabla\cdot\left([\rho E + p]\vec{u}\right) = \rho\vec{u}\cdot(-\nabla\Phi) + (\Gamma - L)$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v} + \rho\varepsilon\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \frac{\partial(\rho\varepsilon)}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho\varepsilon)}{\partial T} + \frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2\vec{v} + \rho\varepsilon\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial\left(\rho\varepsilon + \frac{1}{2}\rho v^2\right)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + p\vec{v} + \left[\rho\varepsilon + \frac{1}{2}\rho v^2\right]\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial T} = -\nabla\cdot\left(\rho\vec{v}\phi + [\rho E + p]\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial T} + \nabla\cdot\left([\rho E + p]\vec{v}\right) = -\nabla\cdot(\rho\vec{v}\phi) - \phi\frac{\partial\rho}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial T} + \nabla\cdot\left([\rho E + p]\vec{v}\right) = -\nabla\cdot(\rho\vec{v}\phi) + \phi\nabla\cdot(\rho\vec{v}) - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial T} + \nabla\cdot\left([\rho E + p]\vec{v}\right) = -\phi\nabla\cdot(\rho\vec{v}) - \rho\vec{v}\cdot\nabla\phi + \phi\nabla\cdot(\rho\vec{v}) - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial T} + \nabla\cdot\left([\rho E + p]\vec{v}\right) = -\rho\vec{v}\cdot\nabla\phi - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial t} + \nabla \cdot \left([\rho E + p]\vec{u}\right) = \rho\vec{u} \cdot (-\nabla\Phi) + (\Gamma - L)$$

$$\longrightarrow \frac{\partial(\rho_x E_x)}{\partial T} + \nabla_x \cdot \left([E_x + p_x]\vec{v}\right) = \rho_x \vec{v} \cdot (-\nabla_x \phi_x) - H\rho_x \varepsilon_x [3\gamma - 5] + (\Gamma_x - L_x)$$

$$\frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla \cdot \left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2 \vec{v} + \rho\varepsilon\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \frac{\partial(\rho\varepsilon)}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho\varepsilon)}{\partial T} + \frac{1}{2}\frac{\partial(\rho v^2)}{\partial T} = -\nabla \cdot \left(\rho\vec{v}\phi + p\vec{v} + \frac{1}{2}\rho v^2 \vec{v} + \rho\varepsilon\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial\left(\rho\varepsilon + \frac{1}{2}\rho v^2\right)}{\partial T} = -\nabla \cdot \left(\rho\vec{v}\phi + p\vec{v} + \left[\rho\varepsilon + \frac{1}{2}\rho v^2\right]\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial T} = -\nabla \cdot \left(\rho\vec{v}\phi + [\rho E + p]\vec{v}\right) - \phi\frac{\partial\rho}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial T} + \nabla \cdot \left([\rho E + p]\vec{v}\right) = -\nabla \cdot (\rho\vec{v}\phi) - \phi\frac{\partial\rho}{\partial T} - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial T} + \nabla \cdot \left([\rho E + p]\vec{v}\right) = -\nabla \cdot (\rho\vec{v}\phi) + \phi\nabla \cdot (\rho\vec{v}) - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial T} + \nabla \cdot \left([\rho E + p]\vec{v}\right) = -\phi\nabla \cdot (\rho\vec{v}) - \rho\vec{v} \cdot \nabla\phi + \phi\nabla \cdot (\rho\vec{v}) - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial(\rho E)}{\partial T} + \nabla \cdot \left([\rho E + p]\vec{v}\right) = -\rho\vec{v} \cdot \nabla\phi - \rho\varepsilon\mathcal{H}[3\gamma - 5]$$

$$\frac{\partial S}{\partial t} + \nabla \cdot (S\vec{u}) = 0$$

A B

A

$$\left.\frac{\partial S}{\partial t}\right|_r = \left.\frac{\partial S}{\partial t}\right|_x - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x S$$
$$= \frac{\partial S}{\partial T}\frac{\partial T}{\partial t} - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x S$$
$$= \frac{\partial S}{\partial T}\frac{1}{a^2} - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x S$$
$$= \frac{1}{a^2}\frac{\partial}{\partial T}\left(a^{3\gamma-8}S_x\right) - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x S$$
$$= \frac{1}{a^2}\left[(3\gamma-8)a^{3\gamma-9}\dot{a}S_x + a^{3\gamma-8}\frac{\partial S_x}{\partial T}\right] - \frac{\dot{a}}{a}a^{3\gamma-8}\vec{x}\cdot\nabla_x S_x$$
$$= a^{3\gamma-10}\left[(3\gamma-8)\mathcal{H}S_x + \frac{\partial S_x}{\partial T} - \mathcal{H}\vec{x}\cdot\nabla_x S_x\right]$$

B

$$\nabla\cdot(S\vec{u}) = \frac{1}{a}\nabla_x\left[a^{3\gamma-8}S_x\frac{1}{a}\left(\vec{v}+\mathcal{H}\vec{x}\right)\right]$$
$$= a^{3\gamma-10}\nabla_x\left[S_x\left(\vec{v}+\mathcal{H}\vec{x}\right)\right]$$
$$= a^{3\gamma-10}\left[\nabla_x\cdot(S_x\vec{v}) + \mathcal{H}\nabla_x\cdot(S_x\vec{x})\right]$$
$$= a^{3\gamma-10}\left[\nabla_x\cdot(S_x\vec{v}) + \mathcal{H}S_x\nabla_x\cdot\vec{x} + \mathcal{H}\vec{x}\cdot\nabla_x S_x\right]$$
$$= a^{3\gamma-10}\left[\nabla_x\cdot(S_x\vec{v}) + 3\mathcal{H}S_x + \mathcal{H}\vec{x}\cdot\nabla_x S_x\right]$$

A+B=0

$$0 = a^{3\gamma-10}\left[(3\gamma-8)\mathcal{H}S_x + \frac{\partial S_x}{\partial T} - \mathcal{H}\vec{x}\cdot\nabla_x S_x\right] + a^{3\gamma-10}\left[\nabla_x\cdot(S_x\vec{v}) + 3\mathcal{H}S_x + \mathcal{H}\vec{x}\cdot\nabla_x S_x\right]$$
$$= (3\gamma-8)\mathcal{H}S_x + \frac{\partial S_x}{\partial T} - \mathcal{H}\vec{x}\cdot\nabla_x S_x + \nabla_x\cdot(S_x\vec{v}) + 3\mathcal{H}S_x + \mathcal{H}\vec{x}\cdot\nabla_x S_x$$
$$= (3\gamma-8)\mathcal{H}S_x + \frac{\partial S_x}{\partial T} + \nabla_x\cdot(S_x\vec{v}) + 3\mathcal{H}S_x$$
$$= \frac{\partial S_x}{\partial T} + \nabla_x\cdot(S_x\vec{v}) + (3\gamma-5)\mathcal{H}S_x$$

$$\boxed{\frac{\partial S}{\partial t} + \nabla\cdot\left(S\vec{u}\right) = 0} \longrightarrow \boxed{\frac{\partial S_x}{\partial T} + \nabla_x\cdot\left(S_x\vec{v}\right) = -\mathcal{H}S_x\left[3\gamma-5\right]}$$

$$\begin{aligned}
\left.\frac{\partial S}{\partial t}\right|_r &= \left.\frac{\partial S}{\partial t}\right|_x - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x S \\
&= \frac{\partial S}{\partial T}\frac{\partial T}{\partial t} - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x S \\
&= \frac{\partial S}{\partial T}\frac{1}{a^2} - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x S \\
&= \frac{1}{a^2}\frac{\partial}{\partial T}\left(a^{3\gamma-8}S_x\right) - \frac{\dot{a}}{a}\vec{x}\cdot\nabla_x S \\
&= \frac{1}{a^2}\left[(3\gamma-8)a^{3\gamma-9}\dot{a}S_x + a^{3\gamma-8}\frac{\partial S_x}{\partial T}\right] - \frac{\dot{a}}{a}a^{3\gamma-8}\vec{x}\cdot\nabla_x S_x \\
&= a^{3\gamma-10}\left[(3\gamma-8)\mathcal{H}S_x + \frac{\partial S_x}{\partial T} - \mathcal{H}\vec{x}\cdot\nabla_x S_x\right]
\end{aligned}$$

$$\begin{aligned}
\nabla\cdot\left(S\vec{u}\right) &= \frac{1}{a}\nabla_x\left[a^{3\gamma-8}S_x\frac{1}{a}\left(\vec{v}+\mathcal{H}\vec{x}\right)\right] \\
&= a^{3\gamma-10}\nabla_x\left[S_x\left(\vec{v}+\mathcal{H}\vec{x}\right)\right] \\
&= a^{3\gamma-10}\left[\nabla_x\cdot\left(S_x\vec{v}\right) + \mathcal{H}\nabla_x\cdot\left(S_x\vec{x}\right)\right] \\
&= a^{3\gamma-10}\left[\nabla_x\cdot\left(S_x\vec{v}\right) + \mathcal{H}S_x\nabla_x\cdot\vec{x} + \mathcal{H}\vec{x}\cdot\nabla_x S_x\right] \\
&= a^{3\gamma-10}\left[\nabla_x\cdot\left(S_x\vec{v}\right) + 3\mathcal{H}S_x + \mathcal{H}\vec{x}\cdot\nabla_x S_x\right]
\end{aligned}$$

$$\begin{aligned}
0 &= a^{3\gamma-10}\left[(3\gamma-8)\mathcal{H}S_x + \frac{\partial S_x}{\partial T} - \mathcal{H}\vec{x}\cdot\nabla_x S_x\right] + a^{3\gamma-10}\left[\nabla_x\cdot\left(S_x\vec{v}\right) + 3\mathcal{H}S_x + \mathcal{H}\vec{x}\cdot\nabla_x S_x\right] \\
&= (3\gamma-8)\mathcal{H}S_x + \frac{\partial S_x}{\partial T} - \mathcal{H}\vec{x}\cdot\nabla_x S_x + \nabla_x\cdot\left(S_x\vec{v}\right) + 3\mathcal{H}S_x + \mathcal{H}\vec{x}\cdot\nabla_x S_x \\
&= (3\gamma-8)\mathcal{H}S_x + \frac{\partial S_x}{\partial T} + \nabla_x\cdot\left(S_x\vec{v}\right) + 3\mathcal{H}S_x \\
&= \frac{\partial S_x}{\partial T} + \nabla_x\cdot\left(S_x\vec{v}\right) + (3\gamma-5)\mathcal{H}S_x
\end{aligned}$$

$$\Delta\Phi = 4\pi G\left(\rho_{tot} + \frac{3p_{tot}}{c^2}\right) - \Lambda$$

$$\Delta\Phi = 4\pi G\left(\rho_{tot} + \frac{3p_{tot}}{c^2}\right) - \Lambda$$

neglect total pressure...

$$\Delta\Phi = 4\pi G\left(\rho_{tot} - \rho_{\Lambda}\right)$$

A B

$$\Delta\Phi = 4\pi G(\rho_{tot} - \rho_\Lambda)$$

A

$$\begin{aligned}\Delta\Phi &= \frac{1}{a^2}\Delta_x\left(\frac{\phi_x}{a^2} - \frac{1}{2}a\ddot{a}x^2\right)\\ &= \frac{1}{a^4}\Delta_x\phi_x - \frac{\ddot{a}}{2a}\Delta_x x^2\\ &= \frac{1}{a^4}\Delta_x\phi_x - \frac{\ddot{a}}{2a}6\\ &= \frac{1}{a^4}\Delta_x\phi_x - 3\frac{\ddot{a}}{a}\\ &= \frac{1}{a^4}\Delta_x\phi_x - 3\frac{1}{a}\left(\frac{4\pi G}{3}a(\bar{\rho}_{tot} - \rho_\Lambda)\right)\\ &= \frac{1}{a^4}\Delta_x\phi_x - 4\pi G(\bar{\rho}_{tot} - \rho_\Lambda)\\ &= \frac{1}{a^4}\Delta_x\phi_x - \frac{4\pi G}{a^3}(\bar{\rho}_{x,tot} - \rho_{x,\Lambda})\end{aligned}$$

2nd Friedmann equation*

B

$$4\pi G(\rho_{tot} - \rho_\Lambda) = \frac{4\pi G}{a^3}(\rho_{x,tot} - \rho_{x,\Lambda})$$

A=B

$$\frac{1}{a^4}\Delta_x\phi_x - \frac{4\pi G}{a^3}(\bar{\rho}_{x,tot} - \rho_{x,\Lambda}) = \frac{4\pi G}{a^3}(\rho_{x,tot} - \rho_{x,\Lambda})$$

$$\Delta_x\phi_x = 4\pi G a(\rho_{x,tot} - \bar{\rho}_{x,tot})$$

*this is the only point where cosmology enters!

$$\Delta\Phi = 4\pi G(\rho_{tot} - \rho_\Lambda) \longrightarrow \Delta_x \phi_x = 4\pi G a(\rho_{x,tot} - \bar{\rho}_{x,tot})$$

A

$$\begin{aligned}
\Delta\Phi &= \frac{1}{a^2}\Delta_x\left(\frac{\phi_x}{a^2} - \frac{1}{2}a\ddot{a}x^2\right) \\
&= \frac{1}{a^4}\Delta_x\phi_x - \frac{\ddot{a}}{2a}\Delta_x x^2 \\
&= \frac{1}{a^4}\Delta_x\phi_x - \frac{\ddot{a}}{2a}6 \\
&= \frac{1}{a^4}\Delta_x\phi_x - 3\frac{\ddot{a}}{a} \\
&= \frac{1}{a^4}\Delta_x\phi_x - 3\frac{1}{a}\left(\frac{4\pi G}{3}a(\bar{\rho}_{tot} - \rho_\Lambda)\right) \\
&= \frac{1}{a^4}\Delta_x\phi_x - 4\pi G(\bar{\rho}_{tot} - \rho_\Lambda) \\
&= \frac{1}{a^4}\Delta_x\phi_x - \frac{4\pi G}{a^3}(\bar{\rho}_{x,tot} - \rho_{x,\Lambda})
\end{aligned}$$

B

$$4\pi G(\rho_{tot} - \rho_\Lambda) = \frac{4\pi G}{a^3}(\rho_{x,tot} - \rho_{x,\Lambda})$$

A=B

$$\frac{1}{a^4}\Delta_x\phi_x - \frac{4\pi G}{a^3}(\bar{\rho}_{x,tot} - \rho_{x,\Lambda}) = \frac{4\pi G}{a^3}(\rho_{x,tot} - \rho_{x,\Lambda})$$

$$\Delta_x\phi_x = 4\pi G a(\rho_{x,tot} - \bar{\rho}_{x,tot})$$

$$\Delta\Phi = 4\pi G\left(\rho_{tot} - \rho_{\Lambda}\right) \longrightarrow \Delta_x \phi_x = 4\pi G a\left(\rho_{x,tot} - \bar{\rho}_{x,tot}\right)$$

$$\frac{d\vec{r}_{DM}}{dt} = \vec{u}_{DM}$$

$$\frac{d\vec{u}_{DM}}{dt} = -\nabla\Phi$$

A

$$\frac{d\vec{u}}{dt} = -\frac{\dot{a}}{a^2}\left(\vec{v} + \mathcal{H}\vec{x}\right) + \frac{1}{a}\left(\frac{d\vec{v}}{dt} + \frac{d\left(\mathcal{H}\vec{x}\right)}{dt}\right)$$

$$= -\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a}\frac{d\vec{v}}{dt} + \frac{1}{a}\frac{d\left(\mathcal{H}\vec{x}\right)}{dt}$$

$$= -\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a^3}\frac{d\vec{v}}{dT} + \frac{1}{a}\frac{d\left(\mathcal{H}\vec{x}\right)}{dt}$$

$$= -\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a^3}\frac{d\vec{v}}{dT} + \frac{1}{a}\left(\vec{x}\frac{d\mathcal{H}}{dt} + \mathcal{H}\frac{d\vec{x}}{dt}\right)$$

$$= -\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a^3}\frac{d\vec{v}}{dT} + \frac{1}{a}\left(\vec{x}\left[a\ddot{a} + \dot{a}^2\right] + \frac{\dot{a}}{a}\frac{d\vec{x}}{dT}\right)$$

$$= -\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a^3}\frac{d\vec{v}}{dT} + \vec{x}\ddot{a} + \frac{\dot{a}^2}{a}\vec{x} + \frac{\dot{a}}{a^2}\frac{d\vec{x}}{dT}$$

$$= \frac{1}{a^3}\frac{d\vec{v}}{dT} + \vec{x}\ddot{a}$$

$$\frac{d\vec{x}}{dT} = \vec{v}$$

B

$$-\nabla\Phi = -\frac{1}{a}\nabla_x\left(\frac{\phi_x}{a^2} - \frac{1}{2}a\ddot{a}x^2\right)$$

$$= -\left(\frac{1}{a^3}\nabla_x\phi_x - \nabla_x\frac{1}{2}\ddot{a}x^2\right)$$

$$= -\left(\frac{1}{a^3}\nabla_x\phi_x - \ddot{a}x\right)$$

A=B

$$\frac{1}{a^3}\frac{d\vec{v}}{dT} + \vec{x}\ddot{a} = -\left(\frac{1}{a^3}\nabla_x\phi_x - \ddot{a}x\right)$$

$$\frac{d\vec{v}}{dT} = -\nabla_x\phi_x$$

$$\frac{d\vec{r}_{DM}}{dt} = \vec{u}_{DM}$$
$$\frac{d\vec{u}_{DM}}{dt} = -\nabla\Phi$$

$\longrightarrow$

$$\frac{d\vec{x}_{DM}}{dT} = \vec{v}_{DM}$$
$$\frac{d\vec{v}_{DM}}{dT} = -\nabla_x \phi_x$$

A

$$\begin{aligned}
\frac{d\vec{u}}{dt} &= -\frac{\dot{a}}{a^2}\left(\vec{v} + \mathcal{H}\vec{x}\right) + \frac{1}{a}\left(\frac{d\vec{v}}{dt} + \frac{d\left(\mathcal{H}\vec{x}\right)}{dt}\right) \\
&= -\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a}\frac{d\vec{v}}{dt} + \frac{1}{a}\frac{d\left(\mathcal{H}\vec{x}\right)}{dt} \\
&= -\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a^3}\frac{d\vec{v}}{dT} + \frac{1}{a}\frac{d\left(\mathcal{H}\vec{x}\right)}{dt} \\
&= -\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a^3}\frac{d\vec{v}}{dT} + \frac{1}{a}\left(\vec{x}\frac{d\mathcal{H}}{dt} + \mathcal{H}\frac{d\vec{x}}{dt}\right) \\
&= -\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a^3}\frac{d\vec{v}}{dT} + \frac{1}{a}\left(\vec{x}\left[a\ddot{a} + \dot{a}^2\right] + \frac{\dot{a}}{a}\frac{d\vec{x}}{dT}\right) \\
&= -\frac{\dot{a}}{a^2}\vec{v} - \frac{\dot{a}^2}{a}\vec{x} + \frac{1}{a^3}\frac{d\vec{v}}{dT} + \vec{x}\ddot{a} + \frac{\dot{a}^2}{a}\vec{x} + \frac{\dot{a}}{a^2}\frac{d\vec{x}}{dT} \\
&= \frac{1}{a^3}\frac{d\vec{v}}{dT} + \vec{x}\ddot{a}
\end{aligned}$$

$$\frac{d\vec{x}}{dT} = \vec{v}$$

B

$$\begin{aligned}
-\nabla\Phi &= -\frac{1}{a}\nabla_x\left(\frac{\phi_x}{a^2} - \frac{1}{2}a\ddot{a}x^2\right) \\
&= -\left(\frac{1}{a^3}\nabla_x\phi_x - \nabla_x\frac{1}{2}\ddot{a}x^2\right) \\
&= -\left(\frac{1}{a^3}\nabla_x\phi_x - \ddot{a}x\right)
\end{aligned}$$

A=B

$$\frac{1}{a^3}\frac{d\vec{v}}{dT} + \vec{x}\ddot{a} = -\left(\frac{1}{a^3}\nabla_x\phi_x - \ddot{a}x\right)$$
$$\frac{d\vec{v}}{dT} = -\nabla_x\phi_x$$

$$E = \rho\varepsilon + \frac{1}{2}\rho u^2$$
$$p = (\gamma - 1)\rho\varepsilon$$
$$S = \frac{p}{\rho^{\gamma-1}}$$
$$\varepsilon = \frac{1}{(\gamma-1)}\frac{1}{\mu}\frac{k_B}{m_p}T$$

$\longrightarrow$

$$E_x = \rho_x\varepsilon_x + \frac{1}{2}\rho_x v^2$$
$$p_x = (\gamma - 1)\rho_x\varepsilon_x$$
$$S_x = \frac{p_x}{\rho_x^{\gamma-1}}$$
$$\varepsilon_x = \frac{1}{(\gamma-1)}\frac{1}{\mu}\frac{k_B}{m_p}T_x$$

straight forward to proof...

$$\frac{\partial \rho_x}{\partial T} + \nabla_x \cdot (\rho_x \vec{v}) = 0$$

$$\frac{\partial (\rho_x \vec{v})}{\partial T} + \nabla_x \cdot \left(\rho_x \vec{v} \otimes \vec{v} + p_x \overleftrightarrow{1}\right) = \rho_x \, (-\nabla_x \phi_x)$$

$$\frac{\partial (\rho_x E_x)}{\partial T} + \nabla_x \cdot \left([\rho_x E_x + p_x]\vec{v}\right) = \rho_x \vec{v} \cdot (-\nabla_x \phi_x) - \mathcal{H} \rho_x \varepsilon_x [3\gamma - 5] + (\Gamma_x - L_x)$$

$$\frac{\partial S_x}{\partial T} + \nabla_x \cdot (S_x \vec{v}) = -\mathcal{H} S_x [3\gamma - 5]$$

$$\Delta_x \phi_x = 4\pi G a \left(\rho_{x,tot} - \overline{\rho}_x\right)$$

$$\frac{d\vec{x}_{DM}}{dT} = \vec{v}_{DM}$$

$$\frac{d\vec{v}_{DM}}{dT} = -\nabla_x \phi_x$$

$$\frac{\partial \rho_x}{\partial T} + \nabla_x \cdot (\rho_x \vec{v}) = 0$$

$$\frac{\partial (\rho_x \vec{v})}{\partial T} + \nabla_x \cdot \left(\rho_x \vec{v} \otimes \vec{v} + p_x \overleftrightarrow{1}\right) = \rho_x \left(-\nabla_x \phi_x\right)$$

$$\frac{\partial (\rho_x E_x)}{\partial T} + \nabla_x \cdot \left([\rho_x E_x + p_x]\vec{v}\right) = \rho_x \vec{v} \cdot \left(-\nabla_x \phi_x\right) \boxed{- \mathcal{H} \rho_x \varepsilon_x [3\gamma - 5]} + (\Gamma_x - L_x)$$

$$\frac{\partial S_x}{\partial T} + \nabla_x \cdot (S_x \vec{v}) = \boxed{-\mathcal{H} S_x [3\gamma - 5]}$$

$$\boxed{\Delta_x \phi_x = 4\pi G a \left(\rho_{x,tot} - \bar{\rho}_x\right)}$$

$$\frac{d\vec{x}_{DM}}{dT} = \vec{v}_{DM}$$

$$\frac{d\vec{v}_{DM}}{dT} = -\nabla_x \phi_x$$

the **only** functional difference!
(btw, 3γ-5=0 for γ=5/3)

AIP

$$\frac{\partial \rho_x}{\partial T} + \nabla_x \cdot \left(\rho_x \vec{v}\right) = 0$$

$$\frac{\partial \left(\rho_x \vec{v}\right)}{\partial T} + \nabla_x \cdot \left(\rho_x \vec{v} \otimes \vec{v} + p_x \overleftrightarrow{1}\right) = \rho_x \left(-\nabla_x \phi_x\right)$$

$$\frac{\partial \left(\rho_x E_x\right)}{\partial T} + \nabla_x \cdot \left(\left[\rho_x E_x + p_x\right]\vec{v}\right) = \rho_x \vec{v} \cdot \left(-\nabla_x \phi_x\right) - \mathcal{H}\rho_x \varepsilon_x \left[3\gamma - 5\right] + \left(\Gamma_x - L_x\right)$$

$$\frac{\partial S_x}{\partial T} + \nabla_x \cdot \left(S_x \vec{v}\right) = -\mathcal{H} S_x \left[3\gamma - 5\right]$$

$$\Delta_x \phi_x = 4\pi G a \left(\rho_{x,tot} - \overline{\rho}_x\right)$$

$$\frac{d\vec{x}_{DM}}{dT} = \vec{v}_{DM}$$

$$\frac{d\vec{v}_{DM}}{dT} = -\nabla_x \phi_x$$

additional/closure equations:

$$E_x = \varepsilon_x + \frac{1}{2} v^2$$

$$p_x = \left(\gamma - 1\right)\rho_x \varepsilon_x$$

$$S_x = \frac{p_x}{\rho_x^{\gamma - 1}}$$

$$\varepsilon_x = \frac{1}{\left(\gamma - 1\right)} \frac{1}{\mu} \frac{k_B}{m_p} T_x$$

to allow for the greatest flexibility we are
going to introduce new functions
allowing for an easy coding of:

1. cosmological equations
2. regular equations

...and we drop all subscripts

to allow for the greatest flexibility we are going to introduce new functions allowing for an easy coding of:

1. cosmological equations

2. regular equations

Note: all quantities are still "supercomoving"!

...and we drop all subscripts

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot \left(\rho \vec{v} \otimes \vec{v} + p \overleftrightarrow{1}\right) = \rho \; (-\nabla \phi)$$

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot \left([\rho E + p]\vec{v}\right) = \rho \vec{v} \cdot (-\nabla \phi) - \mathcal{H} \rho \varepsilon [3\gamma - 5] + (\Gamma - L)$$

$$\frac{\partial S}{\partial t} + \nabla \cdot (S \vec{v}) = -\mathcal{H} S [3\gamma - 5]$$

$$\Delta \phi = \mathcal{G} a \mathcal{D}$$

$$\frac{d\vec{x}_{DM}}{dt} = \vec{v}_{DM}$$

$$\frac{d\vec{v}_{DM}}{dt} = (-\nabla \phi)$$

additional/closure equations:

$$E = \varepsilon + \frac{1}{2} v^2$$

$$p = (\gamma - 1) \rho \varepsilon$$

$$S = \frac{p}{\rho^{\gamma - 1}}$$

$$\varepsilon = \frac{1}{(\gamma - 1)} \frac{1}{\mu} \frac{k_B}{m_p} T$$

1. cosmological equations

$$\mathcal{G} = \frac{3}{2}\Omega_0 H_0^2$$ dimension $1/T^2$

$$\mathcal{D} = \frac{\rho_{tot} - \bar{\rho}}{\bar{\rho}}$$ dimensionless

$$a = a(t)$$

$$\mathcal{H} = a\dot{a}$$

1. cosmological equations

$$\rho_{crit}^{phys} = \frac{3H^2}{8\pi G}$$

$$\mathcal{G} = \frac{3}{2}\Omega_0 H_0^2$$

dimension $1/T^2$

$$\Rightarrow G = \frac{3H^2}{8\pi\rho_{crit}^{phys}}$$

$$= \frac{3\Omega H^2}{8\pi\overline{\rho}^{phys}}$$

$$= \Omega H^2 \frac{3}{8\pi\overline{\rho}^{phys}}$$

$$= \frac{\Omega_0 H_0{}^2}{a^3}\frac{3}{8\pi\overline{\rho}^{phys}}$$

$$= \frac{\Omega_0 H_0{}^2}{a^3}\frac{3a^3}{8\pi\overline{\rho}}$$

$$= \Omega_0 H_0{}^2 \frac{3}{8\pi\overline{\rho}}$$

$$\mathcal{D} = \frac{\rho_{tot} - \overline{\rho}}{\overline{\rho}}$$

dimensionless

$$a = a(t)$$

$$\mathcal{H} = a\dot{a}$$

$$\Rightarrow \quad 4\pi G = \frac{3}{2}\Omega_0 H_0{}^2 \frac{1}{\overline{\rho}} \quad \Rightarrow \quad \Delta\phi = \frac{3}{2}\Omega_0 H_0^2 a \frac{\rho_{tot} - \overline{\rho}}{\overline{\rho}}$$

2. regular equations

$\mathcal{G} = 4\pi G$ dimension $L^3/M/T^2$

$\mathcal{D} = \rho$ dimension M/L^3

$a = 1$

$\mathcal{H} = 0$

freedom to choose length, time, and mass scale

$$\vec{x} = B_0 \, \vec{x}_c$$
$$t = t_0 \, t_c$$
$$m = m_0 \, m_c$$

freedom to choose length, time, and mass scale

$$\vec{x} = B_0 \, \vec{x}_c$$
$$t = t_0 \, t_c$$
$$m = m_0 \, m_c$$

=>

$$\rho = \rho_0 \, \rho_c \quad , \rho_0 = \frac{m_0}{B_0^3}$$
$$\vec{u} = \frac{B_0}{t_0} \, \vec{u}_c$$
$$p = \left(\frac{B_0}{t_0}\right)^2 \rho_0 \, p_c$$
$$E = \left(\frac{B_0}{t_0}\right)^2 E_c$$
$$\phi = \left(\frac{B_0}{t_0}\right)^2 \phi_c$$

internal units

$$\frac{\partial \rho_c}{\partial t_c} + \nabla_c \cdot \left(\rho_c \vec{v}_c\right) = 0$$

$$\frac{\partial\left(\rho_c \vec{v}_c\right)}{\partial t_c} + \nabla_c \cdot \left(\rho_c \vec{v}_c \otimes \vec{v}_c + p_c \overleftrightarrow{1}\right) = \rho_c \quad \left(-\nabla_c \phi_c\right)$$

$$\frac{\partial\left(\rho_c E_c\right)}{\partial t_c} + \nabla_c \cdot \left(\left[\rho_c E_c + p_c\right] \vec{v}_c\right) = \rho_c \vec{v}_c \cdot \left(-\nabla_c \phi_c\right) - \mathcal{H}_c \rho_c \varepsilon_c \left[3\gamma - 5\right] + \left(\Gamma_c - L_c\right)$$

$$\frac{\partial S_c}{\partial t_c} + \nabla_c \cdot \left(S_c \vec{v}_c\right) = -\mathcal{H}_c S_c \left[3\gamma - 5\right]$$

$$\Delta_c \phi_c = \mathcal{G}_c a \mathcal{D}_c$$

$$\frac{d\vec{x}_{c,DM}}{dt_c} = \vec{v}_{c,DM}$$

$$\frac{d\vec{v}_{c,DM}}{dt_c} = -\nabla_c \phi_c$$

additional/closure equations:

$$E_c = \varepsilon_c + \frac{1}{2} v_c^{\,2}$$

$$p_c = \left(\gamma - 1\right) \rho_c \varepsilon_c$$

$$S_c = \frac{p_c}{\rho_c^{\,\gamma-1}}$$

$$\varepsilon_c = \frac{1}{\left(\gamma - 1\right)} \frac{1}{\mu} \frac{k_B}{m_p} T_c$$

1. cosmological units

B_0 = box size

$$t_0 = \frac{1}{H_0}$$

$$\rho_0 = \overline{\rho} = \Omega_0 \rho_{\mathrm{crit},0}$$

=> ρ_0 defines m_0!

$$\mathcal{G}_c = \frac{3}{2}\Omega_0$$

$$\mathcal{D}_c = \rho_{c,tot} - 1 = \rho_{c,DM} + \rho_c - 1$$

$$\mathcal{H}_c = a\dot{a} \quad , \quad \dot{a} = \sqrt{\Omega_0 a^{-2} + \lambda_0 a^2} \quad , \quad a(t) = \text{ numerical integration...}$$

1. cosmological units

$$B_0 = \text{box size}$$

$$t_0 = \frac{1}{H_0}$$

$$\rho_0 = \overline{\rho} = \Omega_0 \rho_{\mathrm{crit},0}$$

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$$\mathcal{H}_c = a\dot{a} \quad , \quad \dot{a} = \sqrt{\Omega_0 a^{-2} + \lambda_0 a^2} \quad , \quad a(t) = \text{ numerical integration...}$$

Note:

$\overline{\rho}$ is constant as it is the *comoving* mean density!

AIP

1. cosmological units

$$B_0 = \text{box size}$$

$$t_0 = \frac{1}{H_0}$$

$$\rho_0 = \overline{\rho} = \Omega_0 \rho_{\text{crit},0}$$

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$$\mathcal{H}_c = a\dot{a} \quad , \quad \dot{a} = \sqrt{\Omega_0 a^{-2} + \lambda_0 a^2} \quad , \quad a(t) = \text{ numerical integration...}$$

Note:

we still have the freedom to measure B_0, H_0, and ρ_0 in whatever units we fancy...

1. cosmological units

$$B_0 = \text{box size}$$

$$t_0 = \frac{1}{H_0}$$

$$\rho_0 = \overline{\rho} = \Omega_0 \rho_{\text{crit},0}$$

=> ρ_0 defines m_0!

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$$\mathcal{H}_c = a\dot{a} \quad , \quad \dot{a} = \sqrt{\Omega_0 a^{-2} + \lambda_0 a^2} \quad , \quad a(t) = \text{ numerical integration...}$$

Note:

we still have the freedom to measure B_0, H_0, and ρ_0 in whatever units we fancy...

convenient choice:

$$[B_0] = h^{-1}\,\text{Mpc}$$

$$[H_0] = h\,\frac{\text{km/s}}{\text{Mpc}}$$

$$[\rho_0] = ?$$

1. cosmological units

$$B_0 = \text{box size}$$

$$t_0 = \frac{1}{H_0}$$

$$\rho_0 = \overline{\rho} = \Omega_0 \rho_{\text{crit},0}$$

=> ρ_0 defines m_0!

$$\mathcal{G}_c = \frac{3}{2}\Omega_0$$

$$\mathcal{D}_c = \rho_{c,tot} - 1 = \rho_{c,DM} + \rho_c - 1$$

$$\mathcal{H}_c = a\dot{a} \quad , \quad \dot{a} = \sqrt{\Omega_0 a^{-2} + \lambda_0 a^2} \quad , \quad a(t) = \text{ numerical integration...}$$

Note:

we still have the freedom to measure B_0, H_0, and ρ_0 in whatever units we fancy...

convenient choice:

$$[B_0] = h^{-1}\,\text{Mpc}$$

$$[H_0] = h\,\frac{\text{km/s}}{\text{Mpc}}$$

$$[\rho_0] = \;?$$ ⟶ #parts/cellvolume !?

AIP

1. cosmological units

B_0 = box size

$t_0 = \frac{1}{H_0}$

$\rho_0 = \overline{\rho} = \Omega_0 \rho_{\mathrm{crit},0}$

$$\frac{\partial \rho_c}{\partial t_c} + \nabla_c \cdot \left(\rho_c \vec{v}_c\right) = 0$$

$$\frac{\partial(\rho_c \vec{v}_c)}{\partial t_c} + \nabla_c \cdot \left(\rho_c \vec{v}_c \otimes \vec{v}_c + p_c \overleftrightarrow{1}\right) = \rho_c \left(-\nabla_c \phi_c\right)$$

$$\frac{\partial(\rho_c E_c)}{\partial t_c} + \nabla_c \cdot \left(\left[\rho_c E_c + p_c\right]\vec{v}_c\right) = \rho_c \vec{v}_c \cdot \left(-\nabla_c \phi_c\right) - \mathcal{H}_c \rho_c \varepsilon_c \left[3\gamma - 5\right] + \left(\Gamma_c - L_c\right)$$

$$\frac{\partial S_c}{\partial t_c} + \nabla_c \cdot \left(S_c \vec{v}_c\right) = -\mathcal{H}_c S_c \left[3\gamma - 5\right]$$

$$\Delta_c \phi_c = \frac{3}{2}\Omega_0 a \left(\rho_{c,DM} + \rho_c - 1\right)$$

$$\frac{d\vec{x}_{c,DM}}{dt_c} = \vec{v}_{c,DM}$$

$$\frac{d\vec{v}_{c,DM}}{dt_c} = -\nabla_c \phi_c$$

additional/closure equations:

$$E_c = \varepsilon_c + \frac{1}{2} v_c^{\,2}$$

$$p_c = (\gamma - 1)\rho_c \varepsilon_c$$

$$S_c = \frac{p_c}{\rho_c^{\,\gamma-1}}$$

$$\varepsilon_c = \frac{1}{(\gamma-1)} \frac{1}{\mu} \frac{k_B}{m_p} T_c$$

2. regular equations

$$B_0 = ?$$
$$t_0 = ?$$
$$m_0 = ?$$

$$\mathcal{G}_c = 4\pi G \rho_0 t_0^2$$

$$\mathcal{D}_c = \rho_c$$

$$\mathcal{H}_c = 0 \quad , \quad a = 1$$