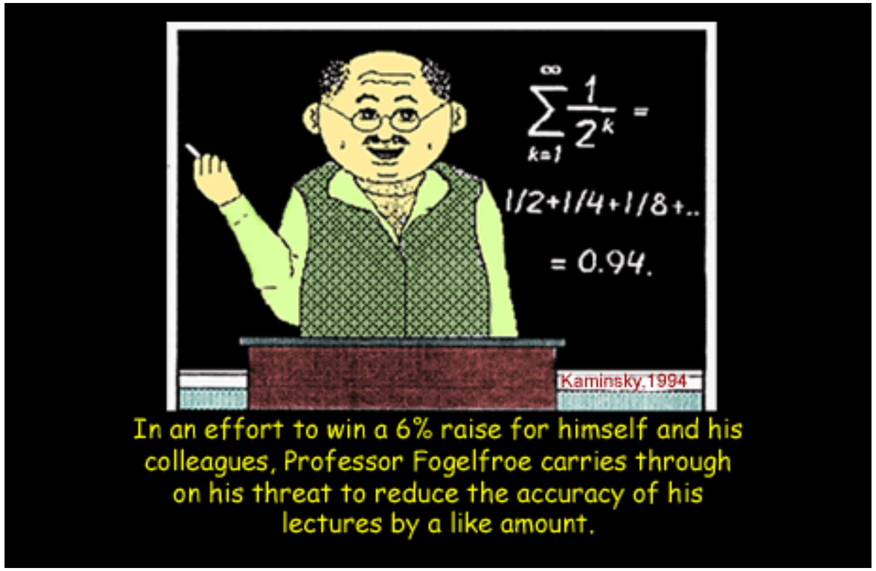
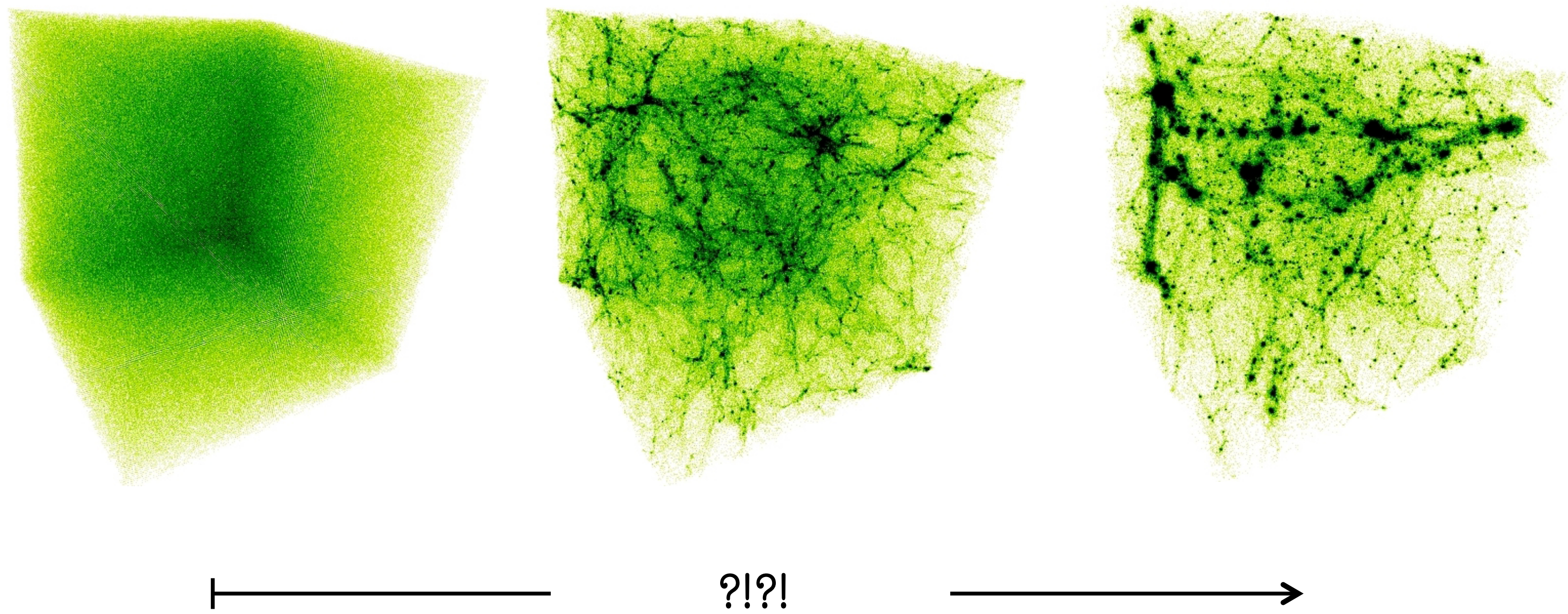
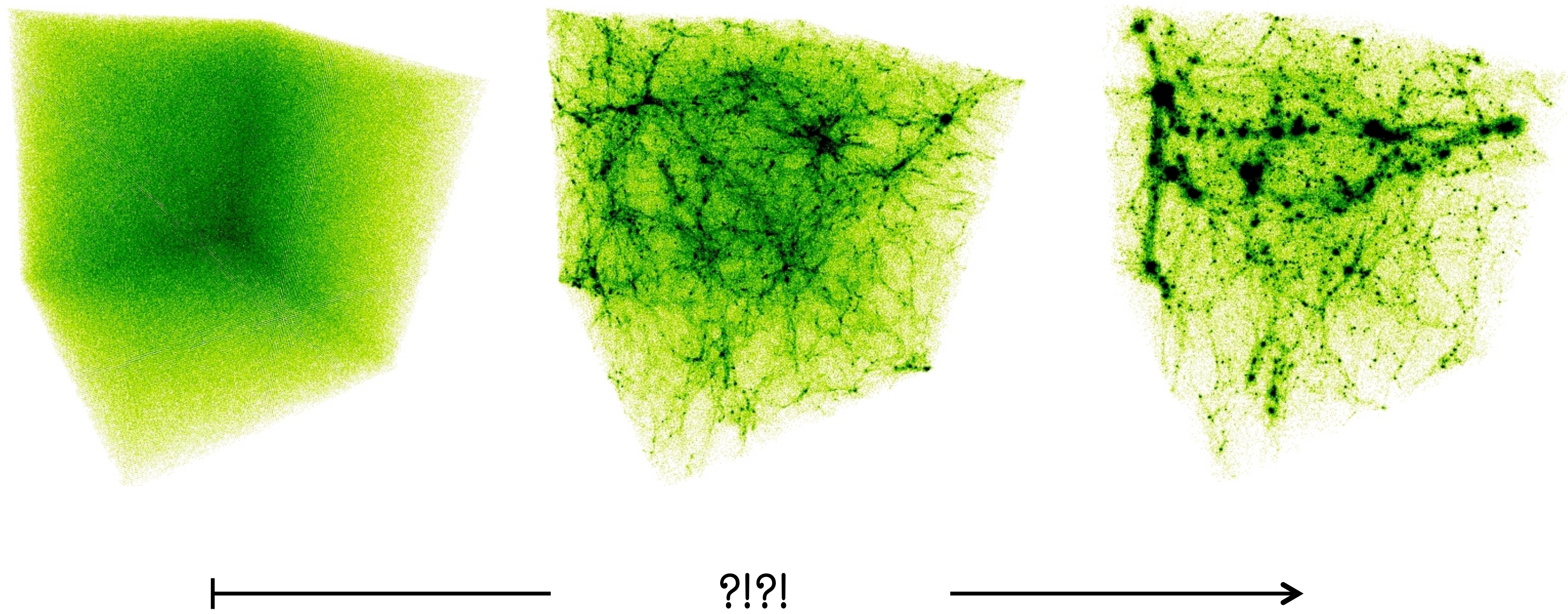
Alexander Knebe, *Universidad Autonoma de Madrid*



Code Testing





how can we be sure to actually model the Universe?

- full set of equations

  - collisionless matter (e.g. dark matter)

$$\frac{d\vec{x}_{DM}}{dt} = \vec{v}_{DM}$$

$$\frac{d\vec{v}_{DM}}{dt} = -\nabla\phi$$

  - Poisson's equation

$$\Delta\phi = 4\pi G\rho_{tot}$$

  - collisional matter (e.g. gas)

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\vec{v}\right) = 0$$

$$\frac{\partial(\rho\vec{v})}{\partial t} + \nabla\cdot\left(\rho\vec{v}\otimes\vec{v} + \left(p + \frac{1}{2\mu}B^2\right)\overleftrightarrow{1} - \frac{1}{\mu}\vec{B}\otimes\vec{B}\right) = \rho\ \left(-\nabla\phi\right)$$

$$\frac{\partial(\rho E)}{\partial t} + \nabla\cdot\left(\left[\rho E + p + \frac{1}{2\mu}B^2\right]\vec{v} - \frac{1}{\mu}\left[\vec{v}\cdot\vec{B}\right]\vec{B}\right) = \rho\vec{v}\cdot\left(-\nabla\phi\right) + \left(\Gamma - L\right)$$

  - ideal gas equations

$$p = \left(\gamma - 1\right)\rho\varepsilon$$

$$\rho\varepsilon = \rho E - \frac{1}{2}\rho v^2$$

  - Maxwell's equation

$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times\left(\vec{v}\times\vec{B}\right)$$

- full set of equations

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$$\frac{d\vec{x}_{DM}}{dt} = \vec{v}_{DM}$$
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time integration lecture

gravity solvers lectures

  - Poisson's equation

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$$\frac{\partial\left(\rho E\right)}{\partial t} + \nabla\cdot\left(\left[\rho E + p + \frac{1}{2\mu}B^2\right]\vec{v} - \frac{1}{\mu}\left[\vec{v}\cdot\vec{B}\right]\vec{B}\right) = \rho\vec{v}\cdot\left(-\nabla\phi\right) + \left(\Gamma - L\right)$$

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time integration **lecture**

gravity solvers **lectures**

- Poisson's equation

$$\Delta\phi = 4\pi G\rho_{tot}$$

2 possible error sources!

  - collisional matter (e.g. gas)

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

$$\frac{\partial(\rho\vec{v})}{\partial t} + \nabla\cdot\left(\rho\vec{v}\otimes\vec{v} + \left(p + \frac{1}{2\mu}B^2\right)\vec{1} - \frac{1}{\mu}\vec{B}\otimes\vec{B}\right) = \rho\ (-\nabla\phi)$$

$$\frac{\partial(\rho E)}{\partial t} + \nabla\cdot\left(\left[\rho E + p + \frac{1}{2\mu}B^2\right]\vec{v} - \frac{1}{\mu}\left[\vec{v}\cdot\vec{B}\right]\vec{B}\right) = \rho\vec{v}\cdot(-\nabla\phi) + (\Gamma - L)$$

- ideal gas equations

$$p = (\gamma - 1)\rho\varepsilon$$

$$\rho\varepsilon = \rho E - \frac{1}{2}\rho v^2$$

- Maxwell's equation

$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times(\vec{v}\times\vec{B})$$

- the time integration
    - how to verify the time integration scheme?
- stationary problems
    - how accurate is the Poisson solver?
- evolutionary problems
    - how do both act together?
- code cross-comparison
- convergence studies

- **the time integration**
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- time step criteria

  - cosmological criterion

$$\Delta t \leq \frac{1}{H}$$

≈ the time step should be smaller than the age of the Universe

  - acceleration/velocity criterion

$$\Delta t \leq \sqrt{\frac{\varepsilon}{a_{\max}}} \qquad \Delta t \leq \frac{\varepsilon}{v_{\max}}$$

≈ particles should not move farther than some preselected threshold ε

ε of order the force resolution

- time step criteria

  - cosmological criterion

$$\Delta t \leq \frac{1}{H} = \frac{a}{\dot{a}} = \frac{a}{\Delta a}\Delta t \Rightarrow 1 \leq \frac{a}{\Delta a}$$

$$\Delta t \leq \frac{1}{H} \quad \Longleftrightarrow \quad \frac{\Delta a}{a} \leq 1$$

≈ the time step should be smaller than the age of the Universe

  - acceleration/velocity criterion

$$\Delta t \leq \sqrt{\frac{\varepsilon}{a_{\max}}} \qquad \Delta t \leq \frac{\varepsilon}{v_{\max}}$$

≈ particles should not move farther than some preselected threshold ε

- the time integration

$$\vec{x}_n = \vec{x} + \vec{e}_n$$

numerically obtained value
(at end of simulation!)

true value

error due to numerics

- the time integration

$$\vec{x}_n = \vec{x} + \vec{e}_n$$

- $\vec{x}_n$: numerically obtained value (at end of simulation!)
- $\vec{x}$: true value
- $\vec{e}_n$: error due to numerics

• second order accurate leap-frog:

$$\vec{e}_n = C(\Delta t_n)^2$$

$$\begin{aligned}\vec{x}_n - \vec{x}_m &= \vec{x} + \vec{e}_n - (\vec{x} + \vec{e}_m)\\ &= \vec{e}_n - \vec{e}_m\\ &= C(\Delta t_n^2 - \Delta t_m^2)\end{aligned}$$

$$\begin{aligned}\vec{x}_m - \vec{x}_l &= \vec{x} + \vec{e}_m - (\vec{x} + \vec{e}_l)\\ &= \vec{e}_m - \vec{e}_l\\ &= C(\Delta t_m^2 - \Delta t_l^2)\end{aligned}$$

- the time integration

$$\vec{x}_n = \vec{x} + \vec{e}_n$$

numerically obtained value (at end of simulation!) — true value — error due to numerics

- second order accurate leap-frog:

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$$\begin{aligned}\vec{x}_m - \vec{x}_l &= \vec{x} + \vec{e}_m - (\vec{x} + \vec{e}_l) \\ &= \vec{e}_m - \vec{e}_l \\ &= C(\Delta t_m^2 - \Delta t_l^2)\end{aligned}$$

$$\text{let}: \quad \Delta t_l = k\Delta t_m = k^2 \Delta t_n$$

- the time integration

$$\vec{x}_n = \vec{x} + \vec{e}_n$$

- $\vec{x}_n$: numerically obtained value (at end of simulation!)
- $\vec{x}$: true value
- $\vec{e}_n$: error due to numerics

• second order accurate leap-frog:

$$\vec{e}_n = C(\Delta t_n)^2$$

$$\frac{\vec{x}_n - \vec{x}_m}{\vec{x}_m - \vec{x}_l} = \ldots = \frac{1}{k^2}$$

$$\text{let :} \quad \Delta t_l = k\Delta t_m = k^2 \Delta t_n$$

- the time integration

$$\vec{x}_n = \vec{x} + \vec{e}_n$$

numerically obtained value (at end of simulation!) — true value — error due to numerics

• second order accurate leap-frog:

$$\vec{e}_n = C(\Delta t_n)^2$$

applicable to test any scheme...

$$\frac{\vec{x}_n - \vec{x}_m}{\vec{x}_m - \vec{x}_l} = \ldots = \frac{1}{k^2}$$

$$\text{let}: \quad \Delta t_l = k\Delta t_m = k^2 \Delta t_n$$

(most obvious choice: k=2)

- the time integration

$$\vec{x}_n = \vec{x} + \vec{e}_n$$

numerically obtained value (at end of simulation!) — true value — error due to numerics

- second order accurate leap-frog – test in practice:

  – run full simulation with three different choices for (constant!) time step:

$$\Delta t,\ 2\Delta t, \text{ and } 4\Delta t$$

  – calculate

$$\frac{\vec{x}_{\Delta t} - \vec{x}_{2\Delta t}}{\vec{x}_{2\Delta t} - \vec{x}_{4\Delta t}} \stackrel{?}{=} \frac{1}{4}$$

  – repeat exercise for $(2\Delta t, 4\Delta t, \text{ and } 8\Delta t)$, $(4\Delta t, 8\Delta t, \text{ and } 16\Delta t)$, …

Code Testing

- the time integration
  - how to verify the time integration scheme?
- **stationary problems**
  - how accurate is the Poisson solver?
- evolutionary problems
  - how do both act together?
- code cross-comparison
- convergence studies

- static test scenarios
    - Hernquist sphere
    - Zel'dovich wave

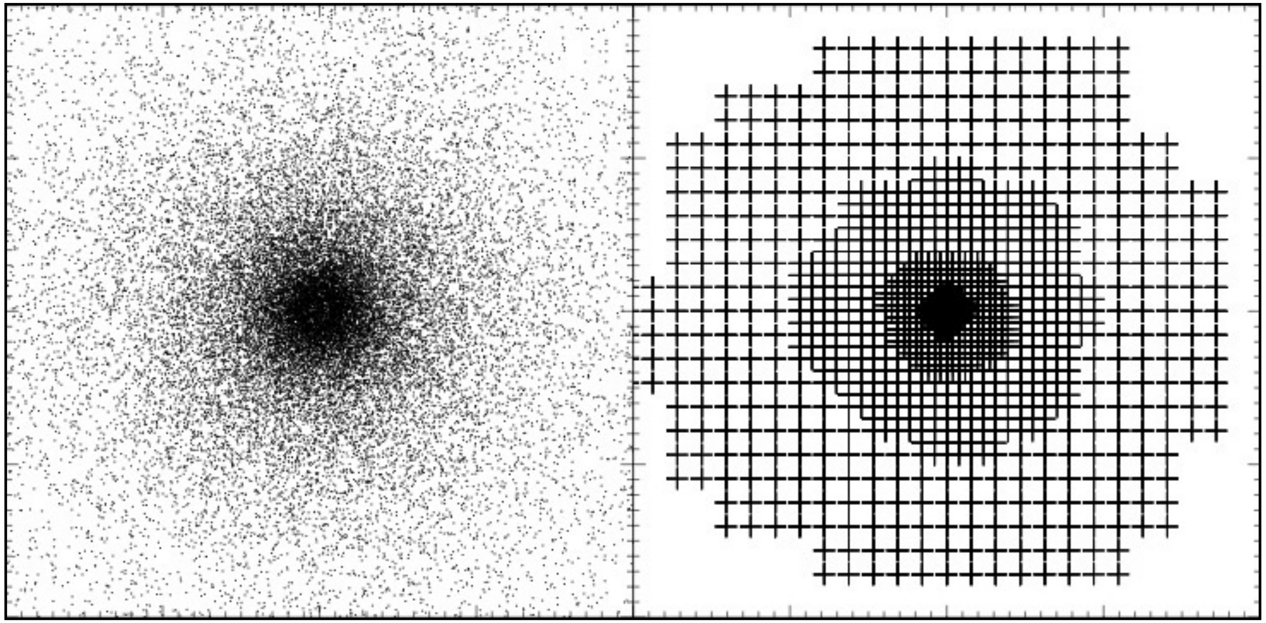
- Hernquist sphere
    - analytic representation

$$\rho(x) = \frac{Mx_0}{2\pi}\frac{1}{x(x_0+x)^3}$$

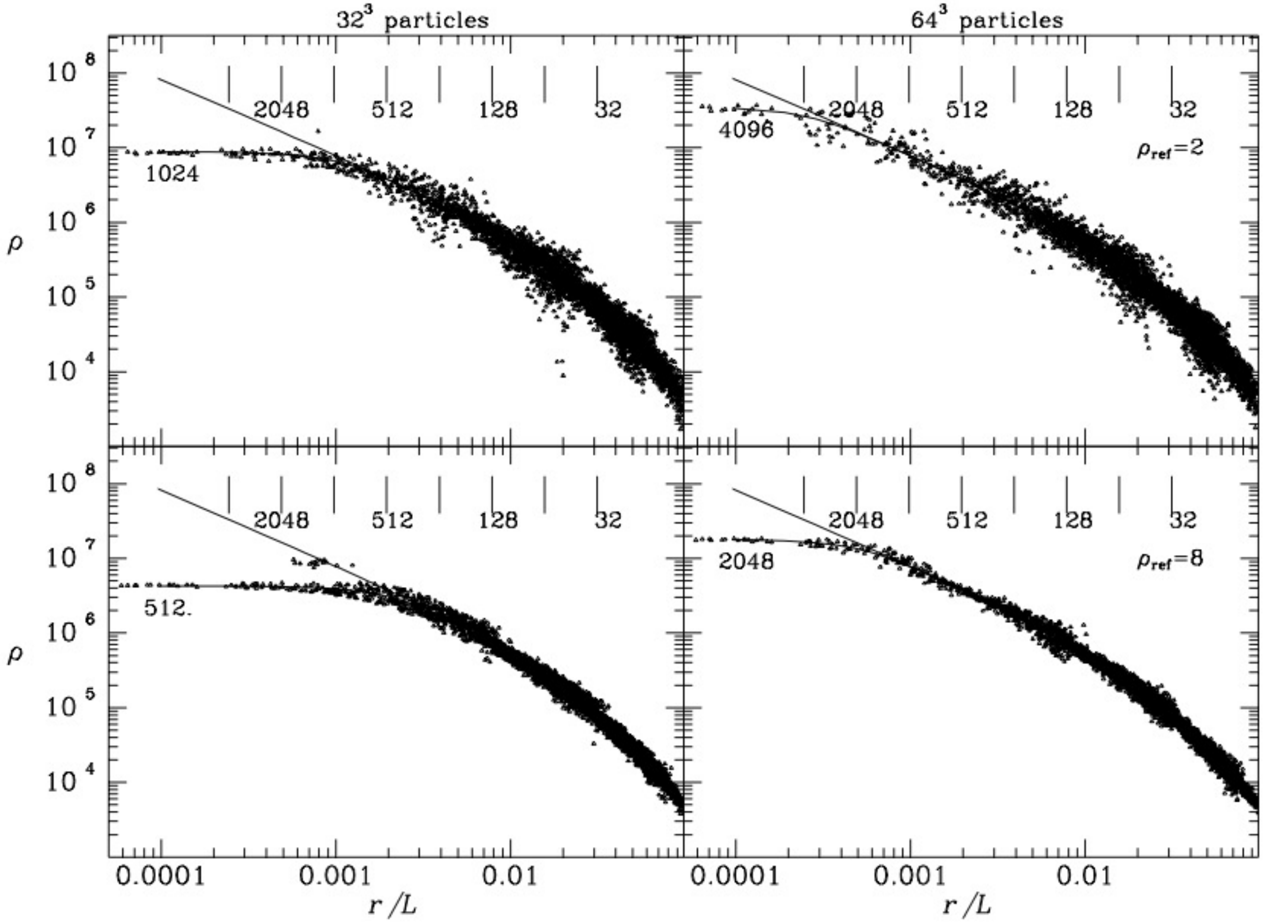
$$\nabla \cdot \vec{F}(\vec{x}) = 4\pi G(\rho - \overline{\rho})$$

$$\vec{F}(\vec{x}) = -\left(\frac{GM}{(x_0+x)^2}\frac{1}{x} + \frac{4\pi G}{3}\overline{\rho}\right)\vec{x}$$

- Hernquist sphere
  - particle representation and AMR refinement hierarchy



- Hernquist sphere
  - recovering densities



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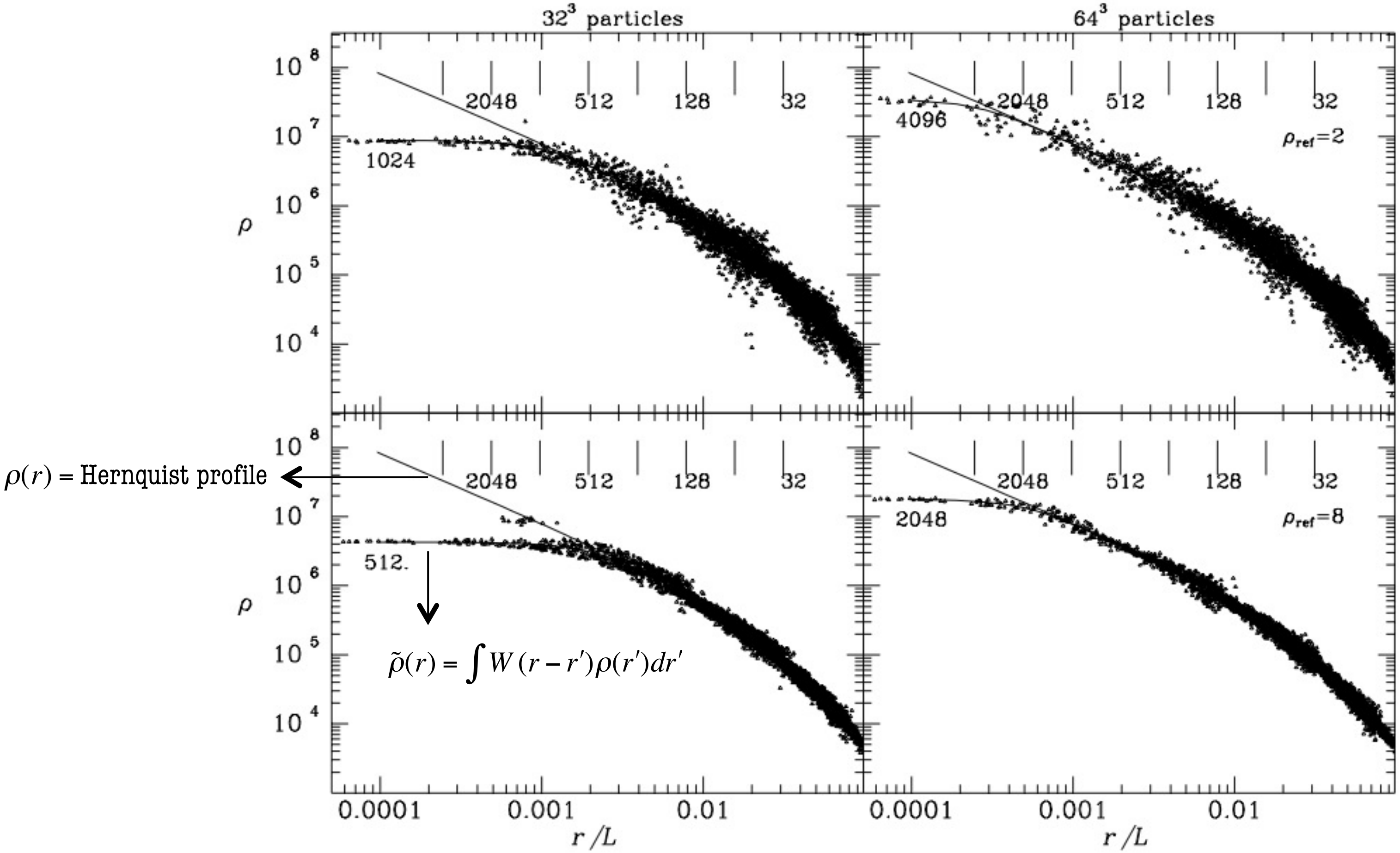
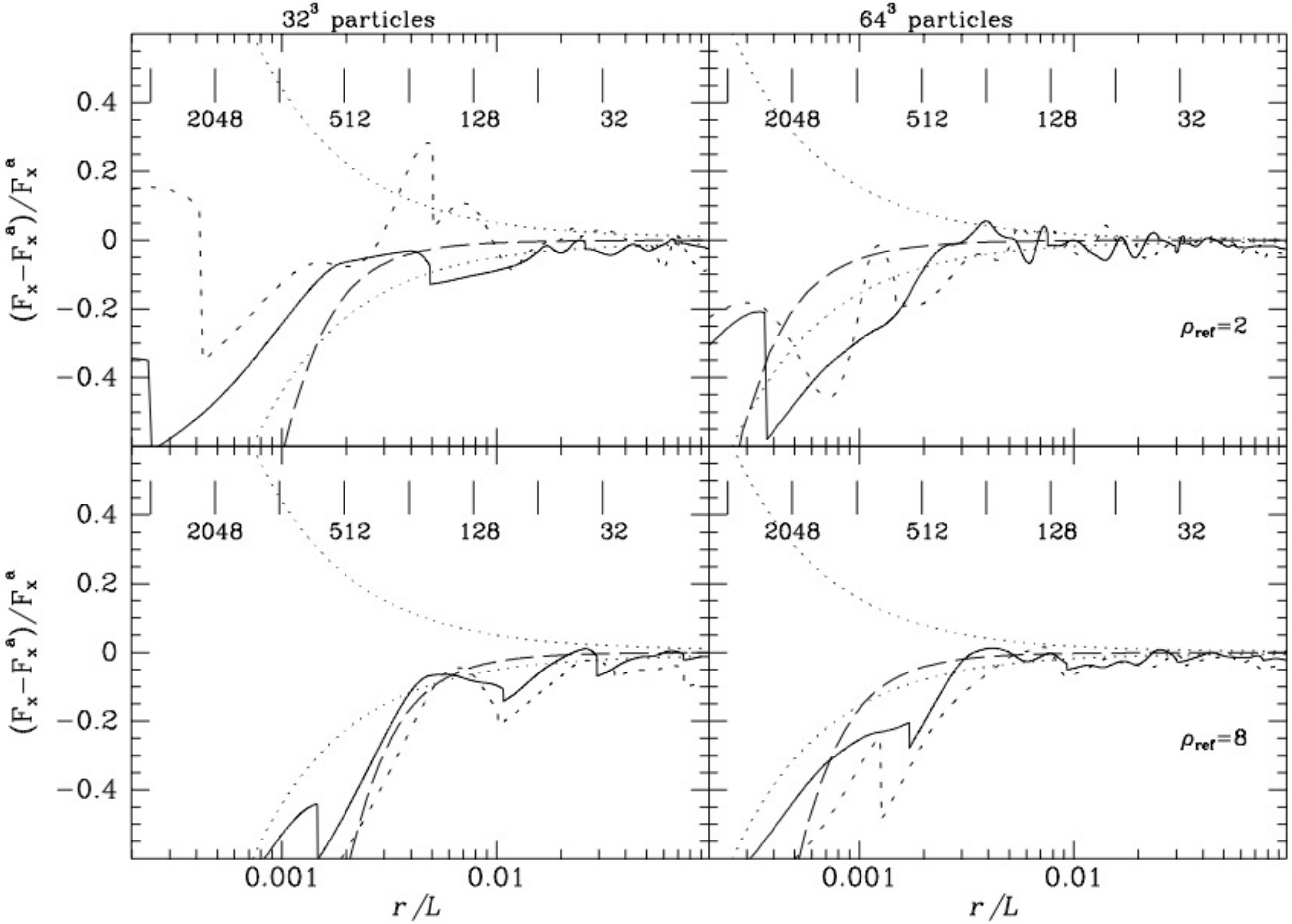
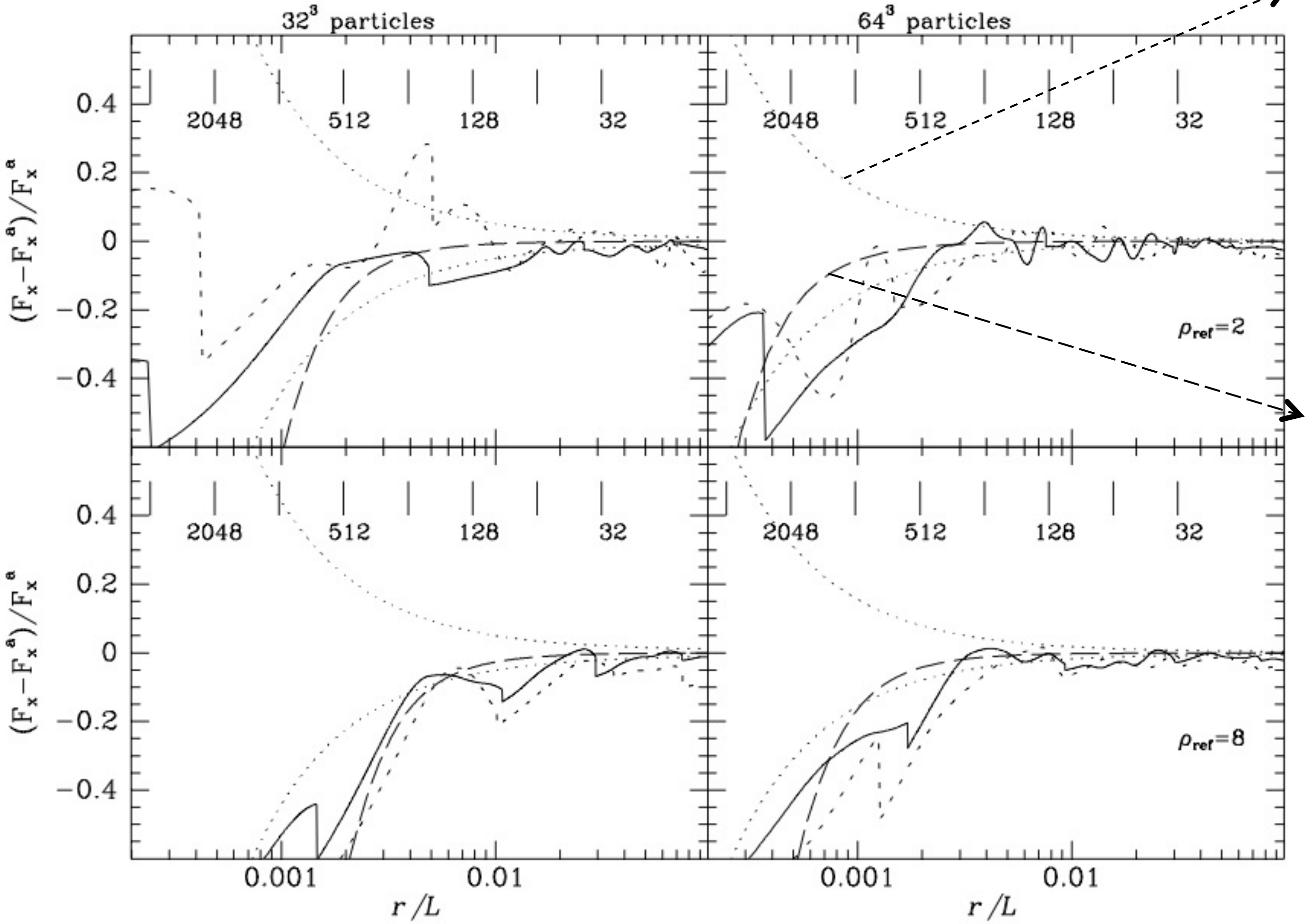


- Hernquist sphere
  - recovering forces



- Hernquist sphere

  - recovering forces: **trade off between bias and variance**



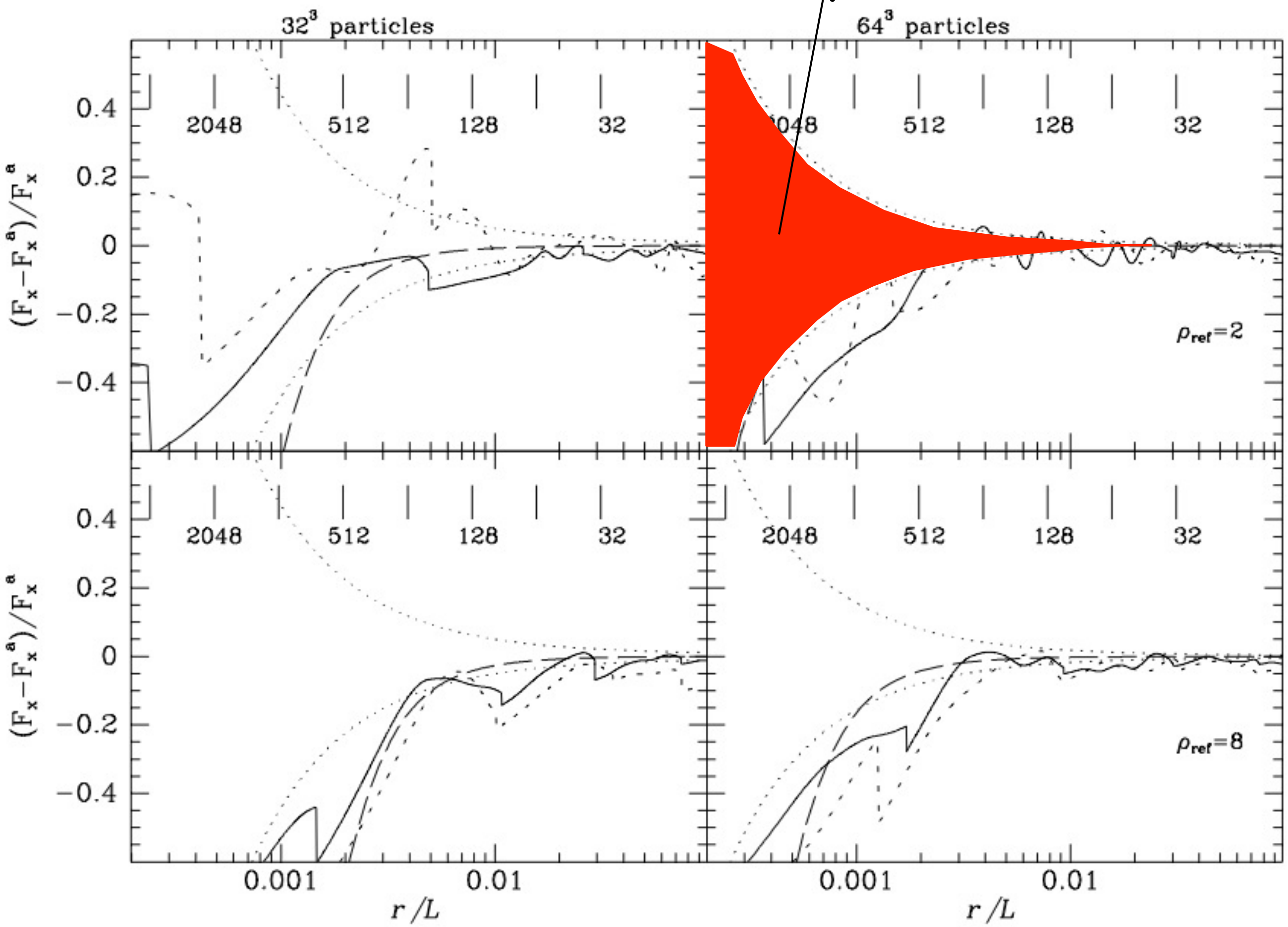
$$\frac{1}{\pm\sqrt{\bar{N}(<r)}}$$

$$\bar{N}(<r) = \frac{1}{m_p}\int_0^r \rho(r')dr'$$

$$\tilde{\vec{F}}(r) = \int W(r-r')\vec{F}(r')dr'$$

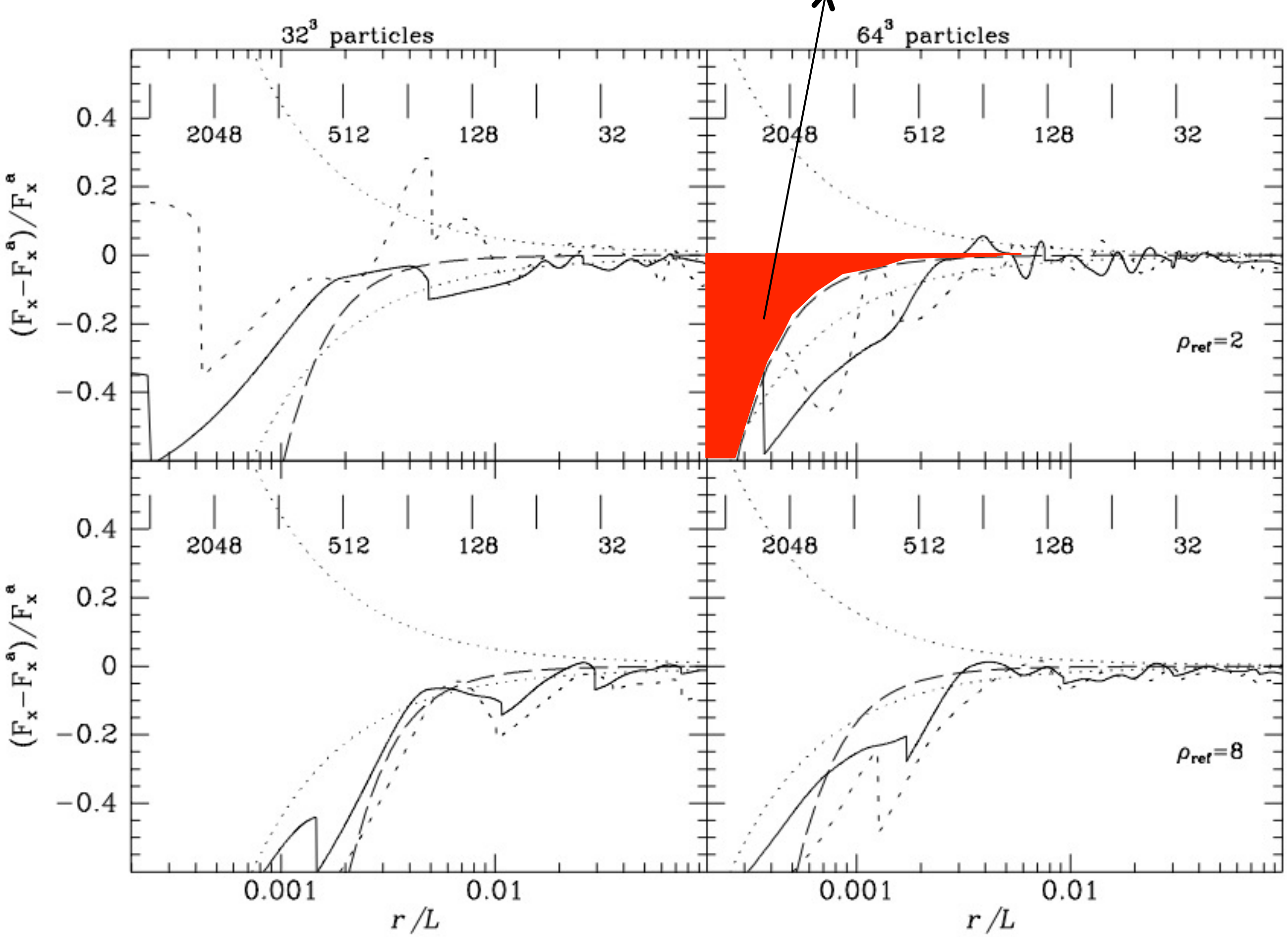
- Hernquist sphere
  - recovering forces

variance due to discrete particles... $\frac{1}{\pm\sqrt{\bar{N}(<r)}}$



- Hernquist sphere
  - recovering forces

unavoidable bias due to smoothing...



- Hernquist sphere
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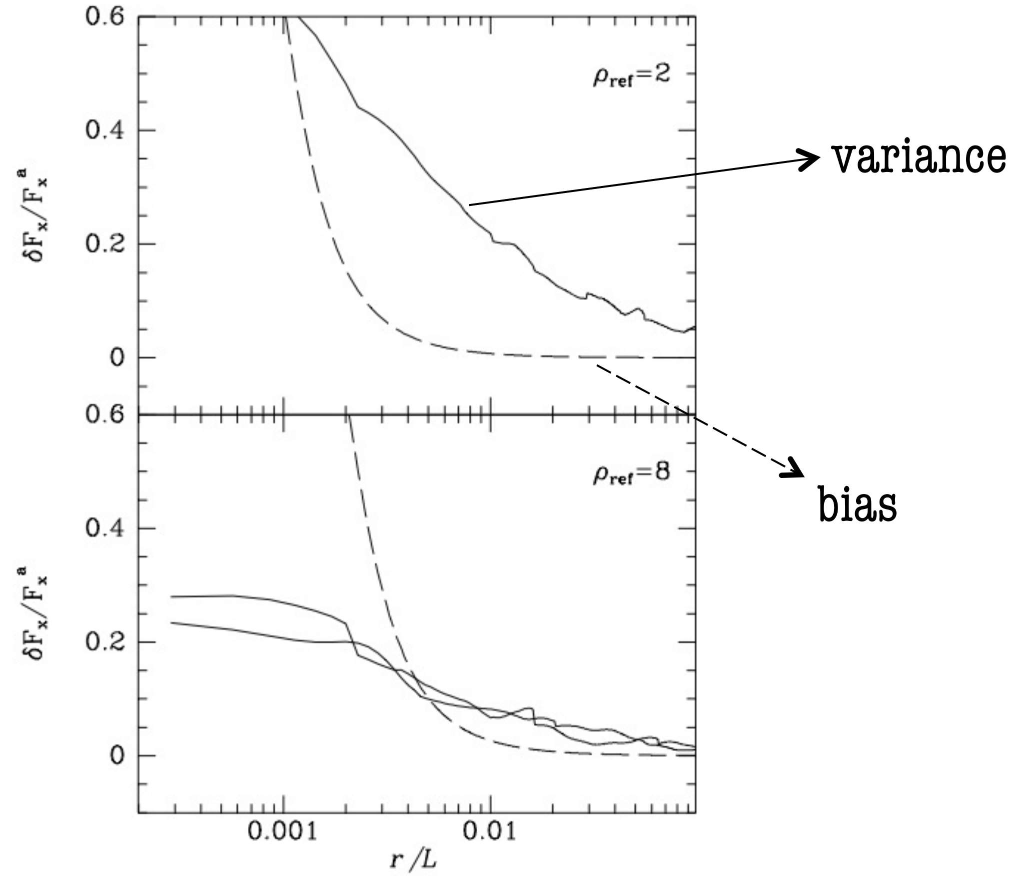

- Hernquist sphere
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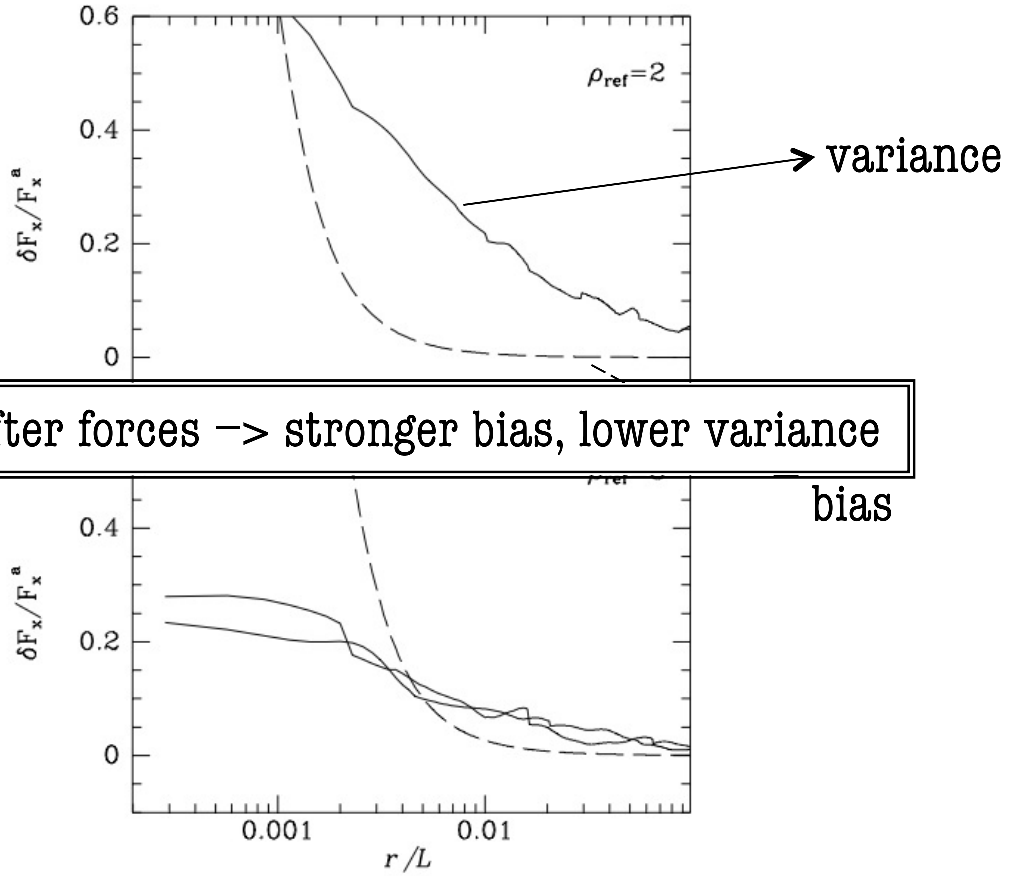

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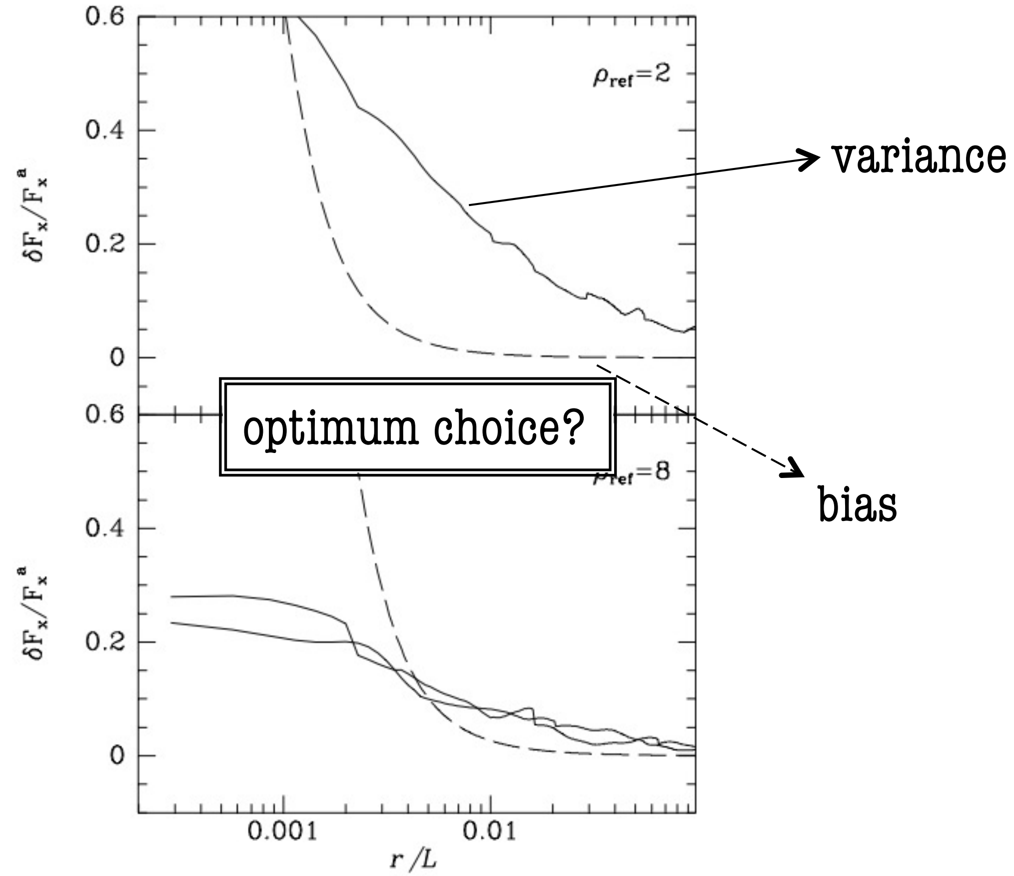

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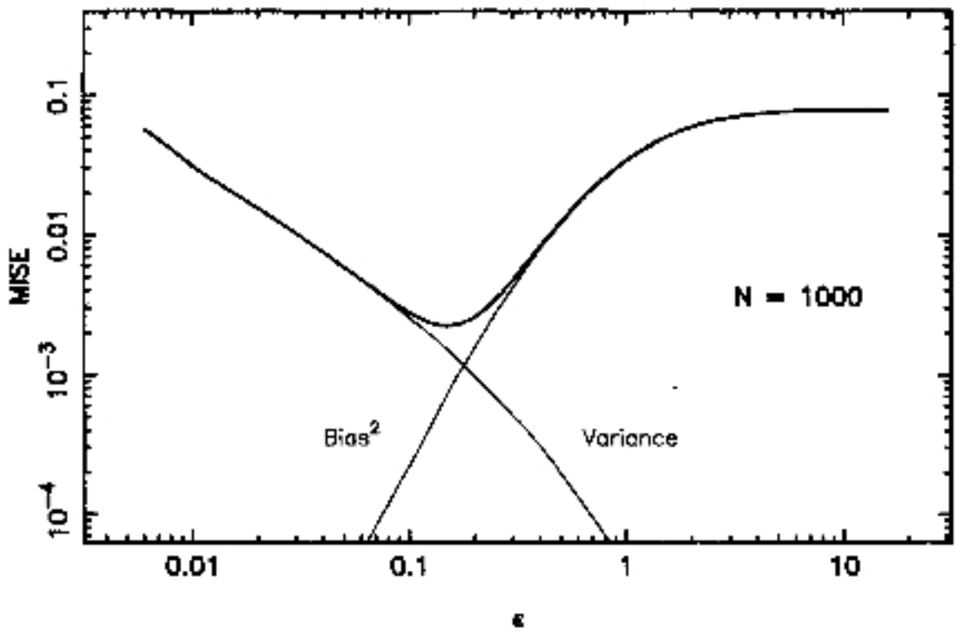

Merritt, ApJ 111, 2462 (1996)

- Zel'dovich wave
  - analytic representation

$$\vec{x} = \vec{q} + a(t)\frac{\vec{k}}{k^2}\cos(\vec{k}\cdot\vec{q})$$

- Zel'dovich wave
  - analytic representation

$$\vec{x} = \vec{q} + a(t)\frac{\vec{k}}{k^2}\cos(\vec{k}\cdot\vec{q})$$

Lagrangian coordinates,
e.g. unperturbed particle positions on a regular grid...

- Zel'dovich wave
  - analytic representation

$$\vec{x} = \vec{q} + a(t)\frac{\vec{k}}{k^2}\cos(\vec{k}\cdot\vec{q})$$

$$\downarrow \quad \vec{F}(\vec{q}) = ?$$

- Zel'dovich wave
  - analytic representation

$$\vec{x} = \vec{q} + a(t)\frac{\vec{k}}{k^2}\cos(\vec{k}\cdot\vec{q})$$

$$\Big\downarrow \quad \vec{F}(\vec{q}) = ?$$

$$\Delta_x \Phi(\vec{x}) = (\rho - \overline{\rho}) \quad \Rightarrow \quad \nabla_x \vec{F}(\vec{x}) = -(\rho - \overline{\rho})$$

- Zel'dovich wave
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$$\nabla_x \vec{F}(\vec{x}) = \frac{1}{\left\| \partial\vec{x} / \partial\vec{q} \right\|} \nabla_q \vec{F}(\vec{q}) \qquad \rho(\vec{x}) = \frac{1}{\left\| \partial\vec{x} / \partial\vec{q} \right\|} \rho(\vec{q}) \qquad \rho(\vec{q}) = \overline{\rho}(\vec{q}) = 1$$

- Zel'dovich wave
    - analytic representation - 1D

$$x = q + \frac{a(t)}{k}\cos(kq)$$

$$\downarrow \quad F(q) = ?$$

$$\frac{d^2\Phi(x)}{dx^2} = (\rho - \overline{\rho}) \quad \Rightarrow \quad \frac{dF(x)}{dx} = -(\rho - \overline{\rho})$$

$$\frac{dF(x)}{dx} = \frac{1}{dx/dq}\frac{dF(q)}{dq} \qquad \rho(x) = \frac{1}{dx/dq}\rho(q) \qquad \rho(q) = \overline{\rho}(q) = 1$$

- Zel'dovich wave
    - analytic representation - 1D

$$x = q + \frac{a(t)}{k}\cos(kq)$$

$$\begin{aligned}\frac{dF(q)}{dq} &= -dx/dq\left(\frac{1}{dq/dq}\rho(q) - \overline{\rho}(q)\right)\\ &= -dx/dq\left(\frac{1}{dx/dq}\overline{\rho}(q) - \rho(q)\right)\\ &= -dx/dq\left(\frac{1}{dx/dq} - 1\right)\overline{\rho}(q)\\ &= dx/dq - 1\end{aligned}$$

$\rho(q) = \overline{\rho}(q) = 1$

- Zel'dovich wave
  - analytic representation - 1D

$$x = q + \frac{a(t)}{k}\cos(kq)$$

$$\begin{aligned}
\frac{dF(q)}{dq} &= -dx/dq\left(\frac{1}{dx/dq}\rho(q) - \overline{\rho}(q)\right) \\
&= -dx/dq\left(\frac{1}{dx/dq}\overline{\rho}(q) - \rho(q)\right) \\
&= -dx/dq\left(\frac{1}{dx/dq} - 1\right)\overline{\rho}(q) \\
&= dx/dq - 1
\end{aligned}$$

$\rho(q) = \overline{\rho}(q) = 1$

$$dx/dq = 1 - a\sin(kq)$$

- Zel'dovich wave
    - analytic representation - 1D

$$x = q + \frac{a(t)}{k}\cos(kq)$$

$$\frac{dF(q)}{dq} = 1 - a\sin(kq) - 1 = -a\sin(kq)$$

- Zel'dovich wave

  - analytic representation - 1D

$$x = q + \frac{a(t)}{k}\cos(kq)$$

$$\frac{dF(q)}{dq} = 1 - a\sin(kq) - 1 = -a\sin(kq)$$

$$\downarrow$$

$$F(q) = \frac{a}{k}\cos(kq)$$

- Zel'dovich wave
  - analytic representation - 1D

$$x = q + \frac{a(t)}{k}\cos(kq)$$

$$\frac{dF(q)}{dq} = 1 - a\sin(kq) - 1 = -a\sin(kq)$$

$$\downarrow$$

$$F(q) = \frac{a}{k}\cos(kq)$$

recall "initial conditions" lecture: $\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$ $\vec{S}(\vec{q}) = -\nabla\Psi$ $\Delta\Psi = \delta_0$

- Zel'dovich wave

  - numerical recovery - 1D

    ∴ put down particles on regular lattice q

    ∴ superimpose Zel'dovich wave x

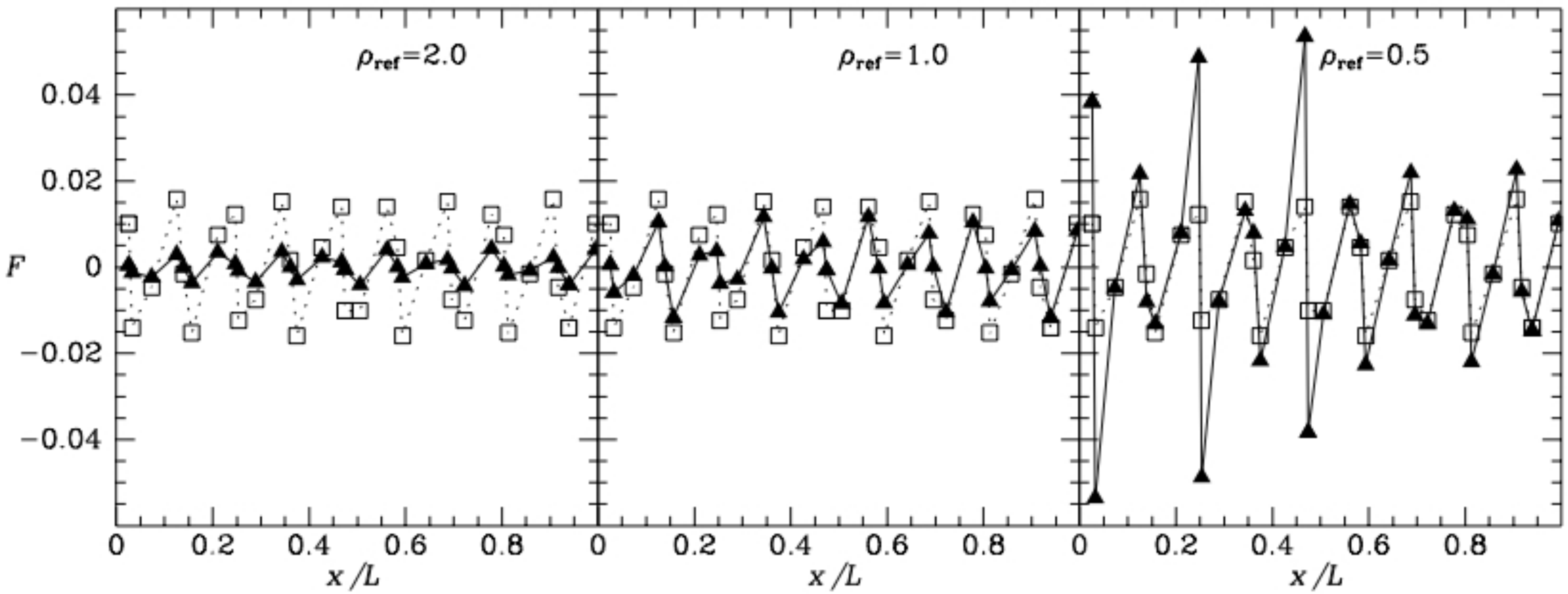
    ∴ numerically calculate forces on lattice $F(q)$

    ∴ compare to analytical forces $F_{\text{true}}(q)$

- Zel'dovich wave
  - numerical recovery - 1D

$$F_{true}(q) = \frac{a}{k}\cos(kq)$$

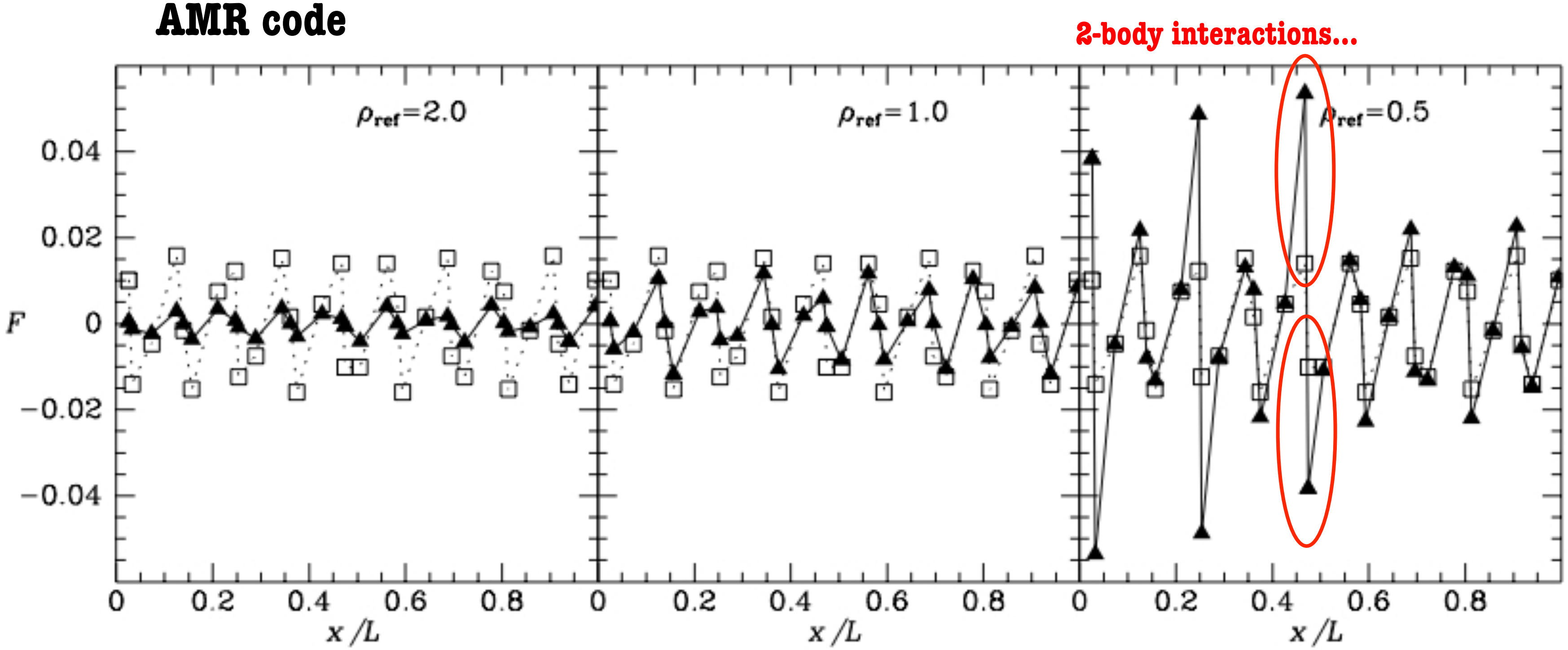
AMR code



- Zel'dovich wave
  - numerical recovery - 1D

$$F_{true}(q) = \frac{a}{k}\cos(kq)$$

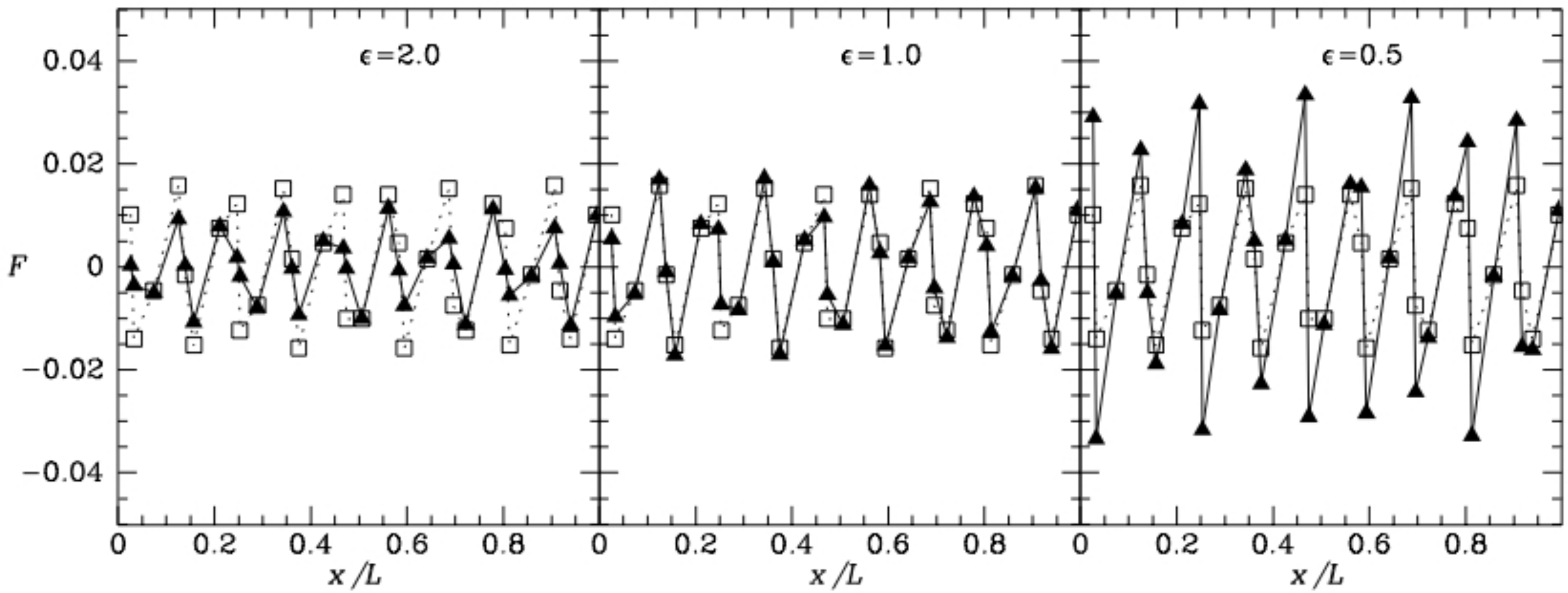
AMR code



- Zel'dovich wave
  - numerical recovery - 1D

$$F_{true}(q) = \frac{a}{k}\cos(kq)$$

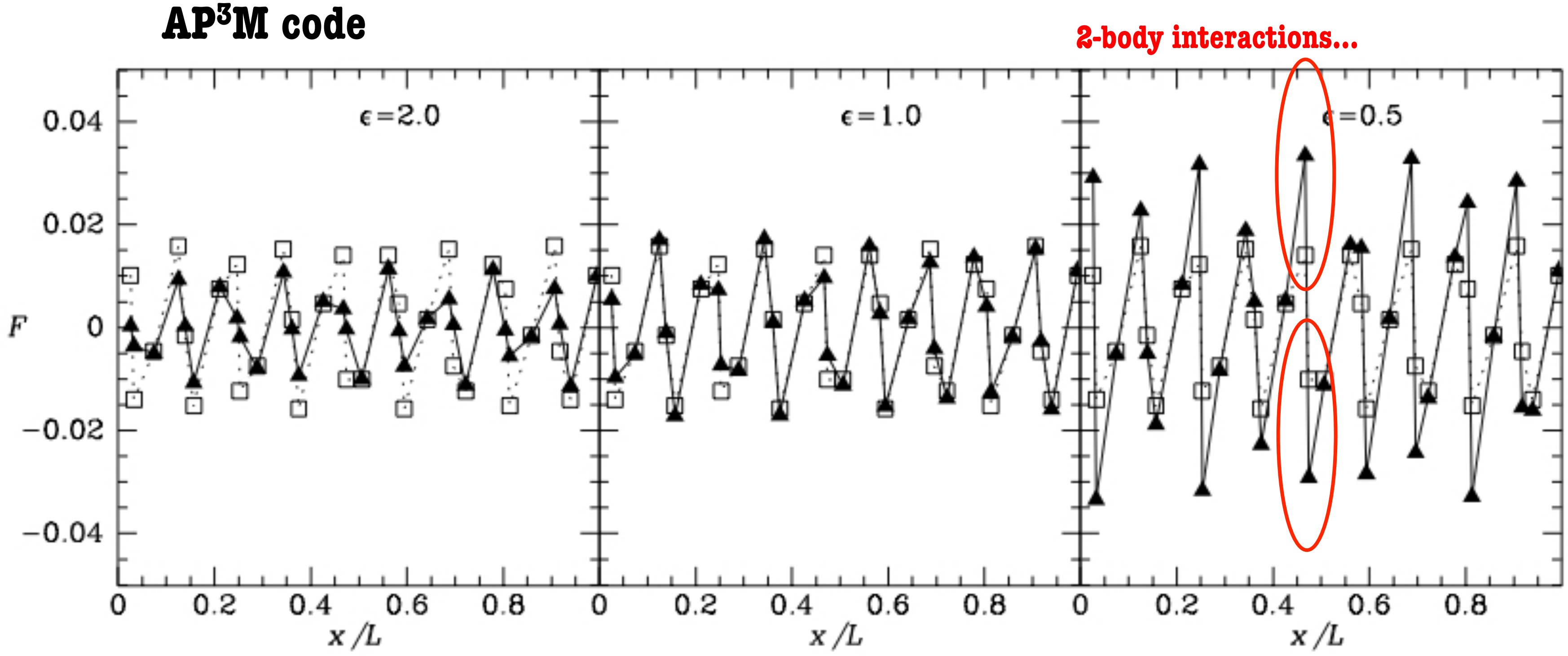
AP³M code



- Zel'dovich wave
  - numerical recovery - 1D

$$F_{true}(q) = \frac{a}{k}\cos(kq)$$

AP[3]M code

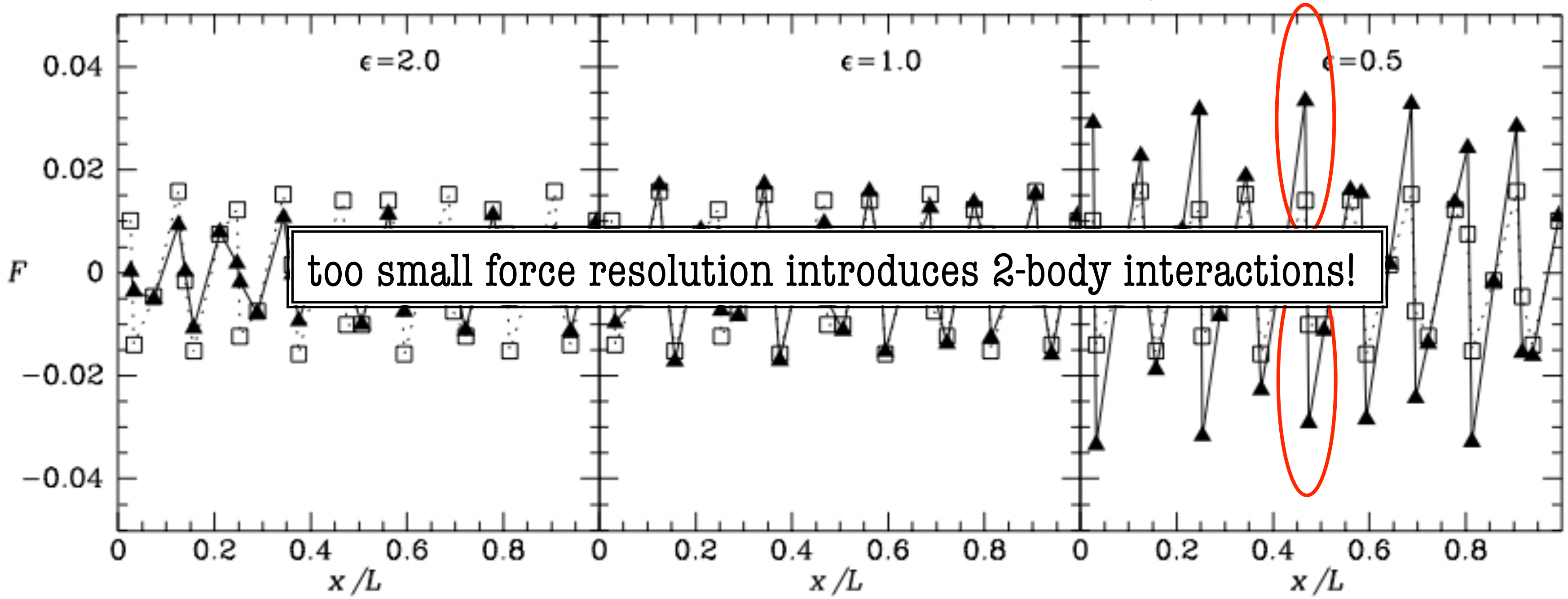


- Zel'dovich wave
  - numerical recovery - 1D

$$F_{true}(q) = \frac{a}{k}\cos(kq)$$

AP³M code

2-body interactions...



- the time integration
  - how to verify the time integration scheme?

- stationary problems
  - how accurate is the Poisson solver?

- **evolutionary problems**
  - how do both act together?

- code cross-comparison

- convergence studies

- evolutionary test scenarios
    - check for momentum conservation
    - Layzer-Irvine Energy conservation
    - Zel'dovich wave, again...

- momentum conservation

$$\left|\sum_{i=1}^{N} \vec{F}_i\right| = 0$$

=> development of net momentum during simulation?

- practical test:

$$\frac{\left|\sum_{i=1}^{N} \vec{F}_i\right|}{\sum_{i=1}^{N} \left|\vec{F}_i\right|} \approx 10^{-4}$$

- Layzer-Irvine energy conservation
  - Hamiltonian (= total energy)

$$\mathcal{H} = \frac{1}{2ma^2} p^2 + m\frac{\Phi(\vec{x})}{a}$$

... with comoving energies as follows: (remember Hamilton formalism...)

$$T = \frac{1}{2}\sum_{i=1}^{N} \frac{p_i^2}{m_i a^2}$$

$$U = -\frac{1}{2a} \iiint_{\text{Box}} (\rho(\vec{x}) - \bar{\rho})\Phi(\vec{x}) d^3 x \qquad \Delta\Phi = 4\pi G(\rho_x - \bar{\rho}_x)$$

- Layzer-Irvine energy conservation
    - Hamiltonian (= total energy)

$$\frac{d\mathcal{H}}{dt} = \frac{\partial\mathcal{H}}{\partial t}$$

$$= -\frac{1}{2}\sum_{i=1}^{N}\frac{p_i^2}{m_i}\frac{2\dot{a}}{a^3} + \frac{1}{2}\frac{2\dot{a}}{a^2}\iiint_{\text{Box}}(\rho(\vec{x}) - \overline{\rho})\Phi(\vec{x})d^3x$$

$$= -\frac{\dot{a}}{a}\left(2T + U\right)$$

$$\Rightarrow \quad 0 = \frac{d(T+U)}{dt} + \frac{\dot{a}}{a}\left(2T + U\right)$$

$$\boxed{C = (T+U)_t - (T+U)_{t_{\text{init}}} + \int_{t_{\text{init}}}^{t}\frac{1}{a}(2T+U)da}$$

- Layzer-Irvine energy conservation

  - Hamiltonian (= total energy)

$$\frac{d\mathcal{H}}{dt} = \frac{\partial\mathcal{H}}{\partial t}$$

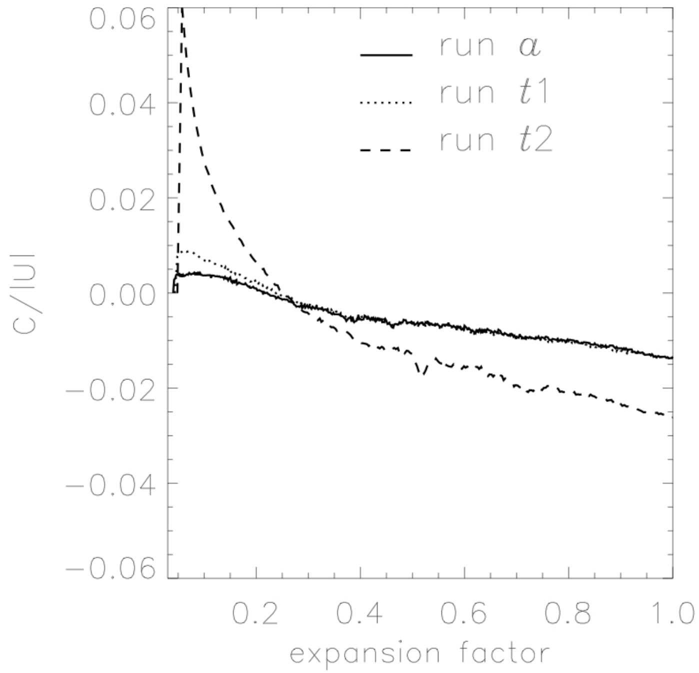
$$= -\frac{1}{2}\sum_{i=1}^{N}\frac{p_i^2}{m_i}\frac{2\dot{a}}{a^3} + \frac{1}{2}\frac{2\dot{a}}{a^2}\iiint_{\text{Box}}(\rho(\vec{x}) - \overline{\rho})\Phi(\vec{x})d^3x$$

$$= -\frac{\dot{a}}{a}\left(2T + U\right)$$

$$\Rightarrow \quad 0 = \frac{d(T+U)}{dt} + \frac{\dot{a}}{a}\underbrace{\left(2T + U\right)}_{=0\ ?!\ \text{(virial theorem)}}$$

$$\boxed{C = (T+U)_t - (T+U)_{t_{\text{init}}} + \int_{t_{\text{init}}}^{t}\frac{1}{a}(2T+U)da}$$

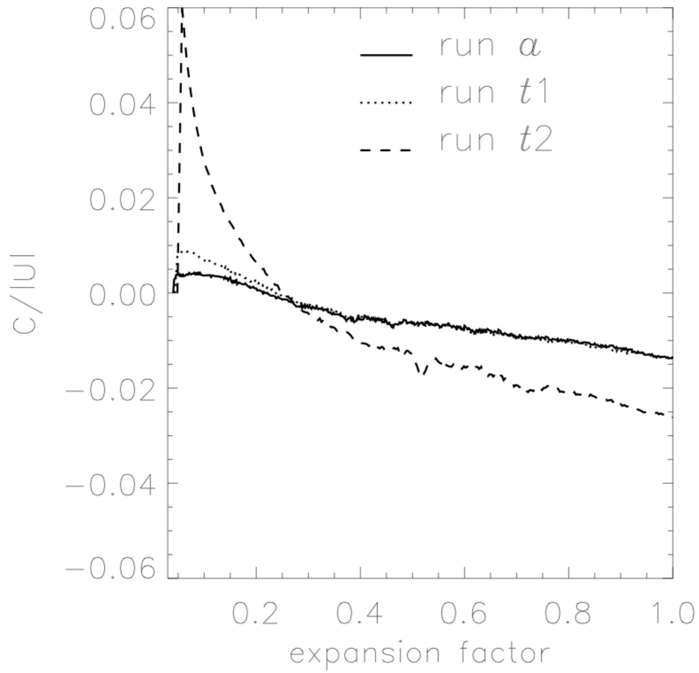
- Layzer-Irvine energy conservation



accuracy at “a few percent” level

$$C = (T+U)_t - (T+U)_{t_{\rm init}} + \int_{t_{\rm init}}^{t} \frac{1}{a}(2T+U)\,da$$

- Layzer-Irvine energy conservation



accuracy at "a few percent" level

errors due to time integration **and** Poisson solver...

- Zel'dovich wave
  - analytic representation - 1D

$$x = q + \frac{a(t)}{k}\cos(kq)$$

- Zel'dovich wave
    - analytic representation - 1D

$$x = q + \frac{a(t)}{k}\cos(kq)$$

$$\dot{x} = \frac{\dot{a}(t)}{k}\cos(kq)$$

- Zel'dovich wave

  - analytic representation - 1D

$$x = q + \frac{a(t)}{k}\cos(kq)$$

$$\dot{x} = \frac{\dot{a}(t)}{k}\cos(kq)$$

$$\sigma_x^2 = \frac{\sum_i \left(x^i - x_{true}\right)^2}{\sum_i \left(x_{true} - q\right)^2}$$

$$\sigma_v^2 = \frac{\sum_i \left(\dot{x}^i - \dot{x}_{true}\right)^2}{\sum_i \left(\dot{x}_{true}\right)^2}$$

$\longrightarrow$ check rms errors as function of k and a

- the time integration
    - how to verify the time integration scheme?
- stationary problems
    - how accurate is the Poisson solver?
- evolutionary problems
    - how do both act together?
- **code cross-comparison**
- convergence studies

▪ GADGET-1 http://www.mpa-garching.mpg.de/gadget

- **fully particle based** force derivation
- combining distant particles into aggregates (i.e. tree code)

▪ *AMIGA* http://www.aip.de/People/Aknebe/AMIGA

- **fully grid based** force derivation
- places finer and finer grids of arbitrary shape in high density regions (i.e. AMR code)

▪ HYDRA/AP^{3}M http://hydra.mcmaster.ca/hydra

- **combination** of particle and grid based force derivation
- no refined grids, no tree structures

- ΛCDM simulation run with various codes
  - identical initial conditions
  - comparable parameter setup

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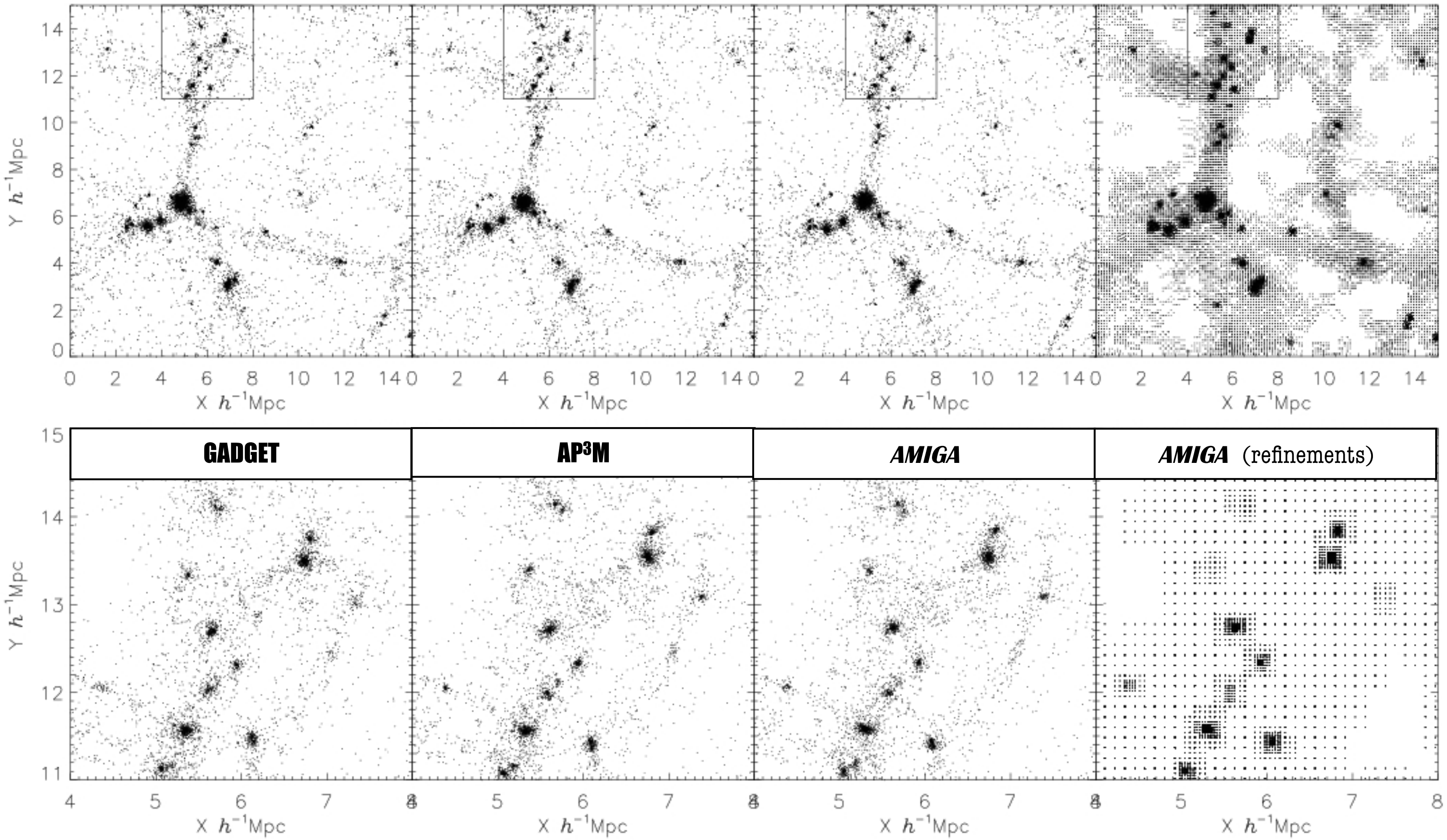
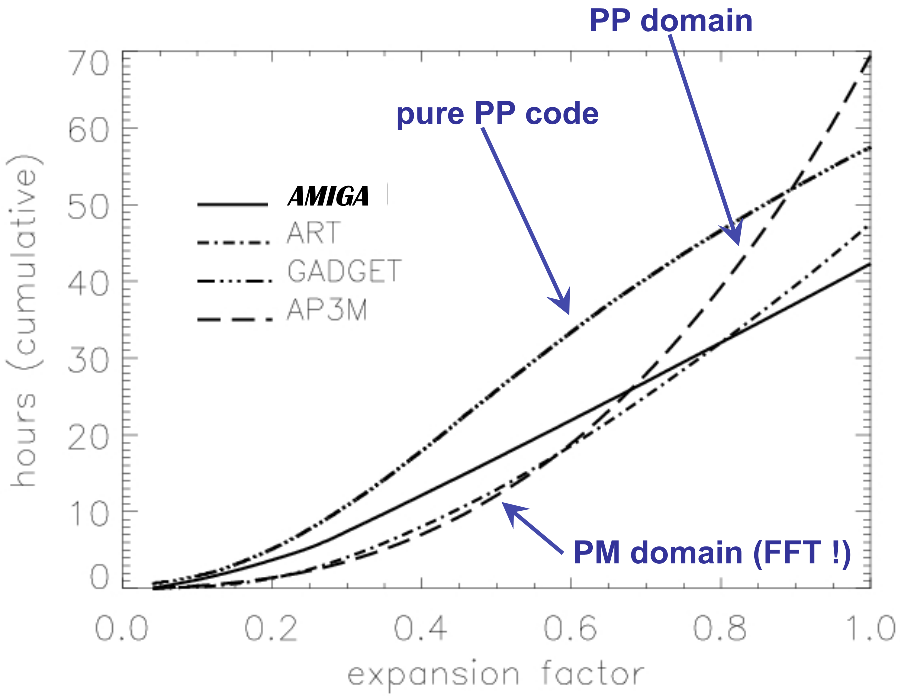
    - identical initial conditions
    - comparable parameter setup



Knebe et al. (2000)

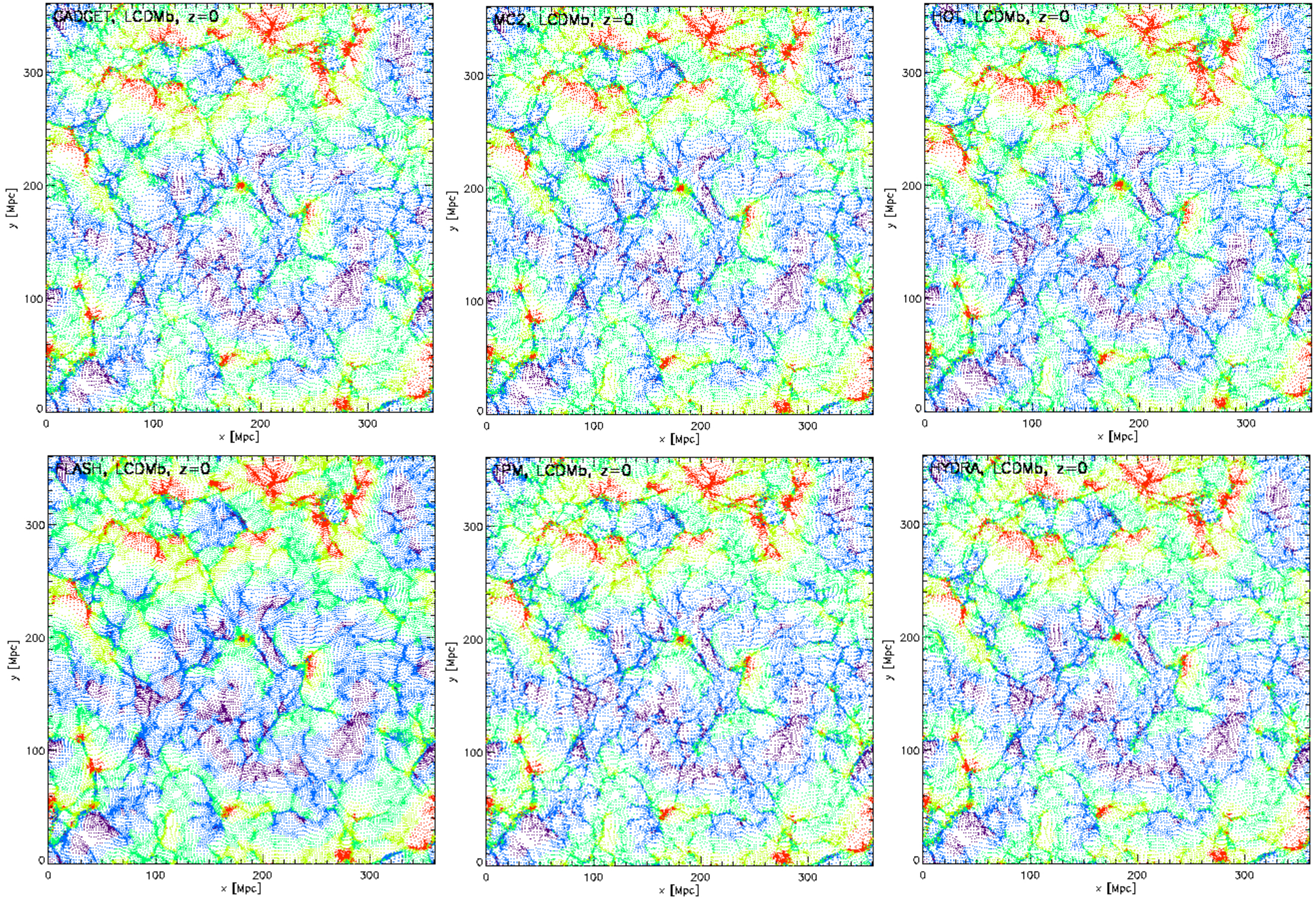
- ΛCDM simulation run with various codes
  - identical initial conditions
  - comparable parameter setup

code	time
AMIGA	42.3 hours
ART	47.4 hours
GADGET	57.5 hours
AP³M	69.4 hours



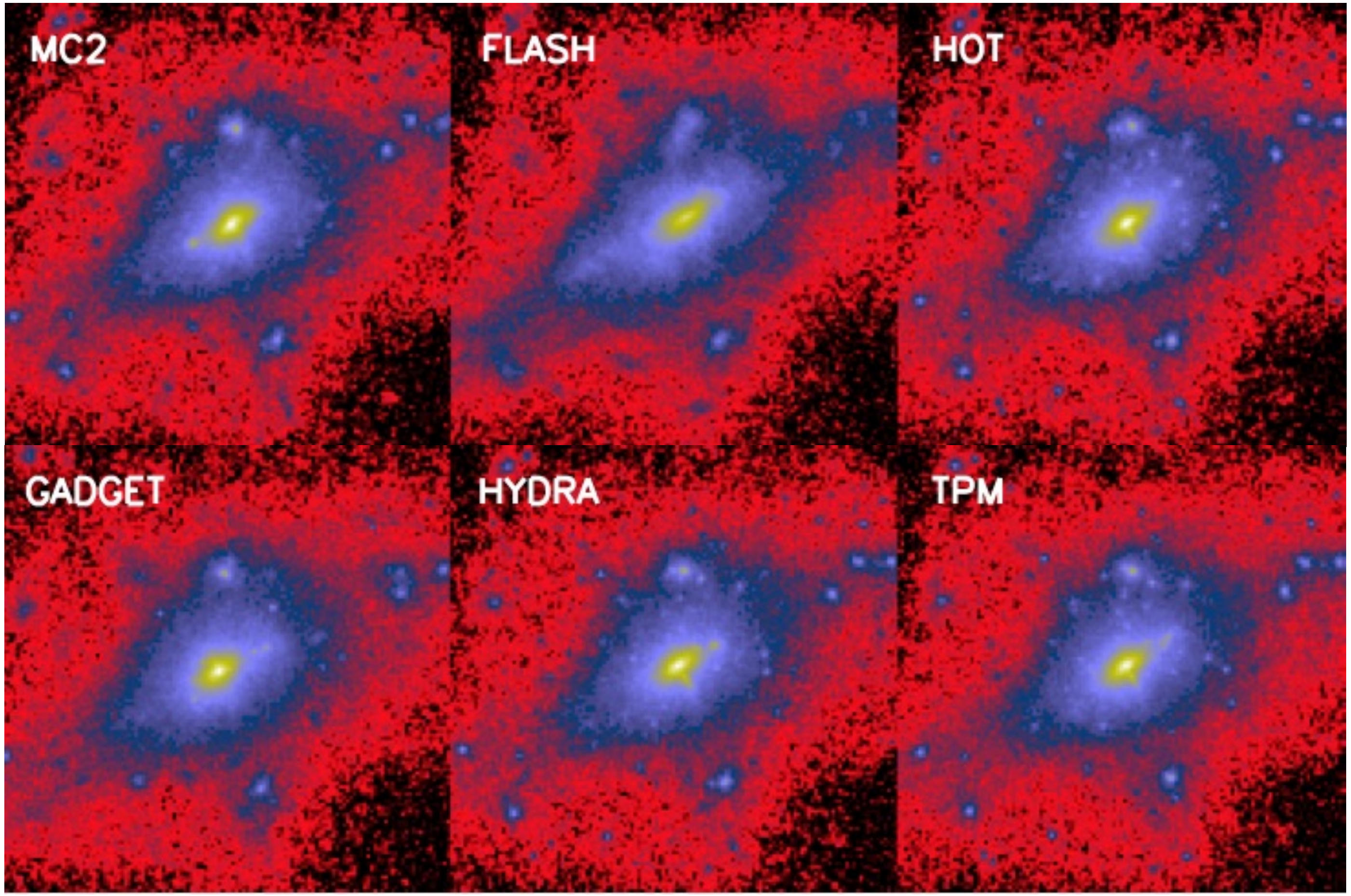
- ΛCDM simulation run with various codes
  - identical initial conditions
  - comparable parameter setup

more recent comparison by Heitmann et al. (2005)...



Heitmann et al. (2005)

- "Santa Barbara Cluster" (Frenck et al. 1999)



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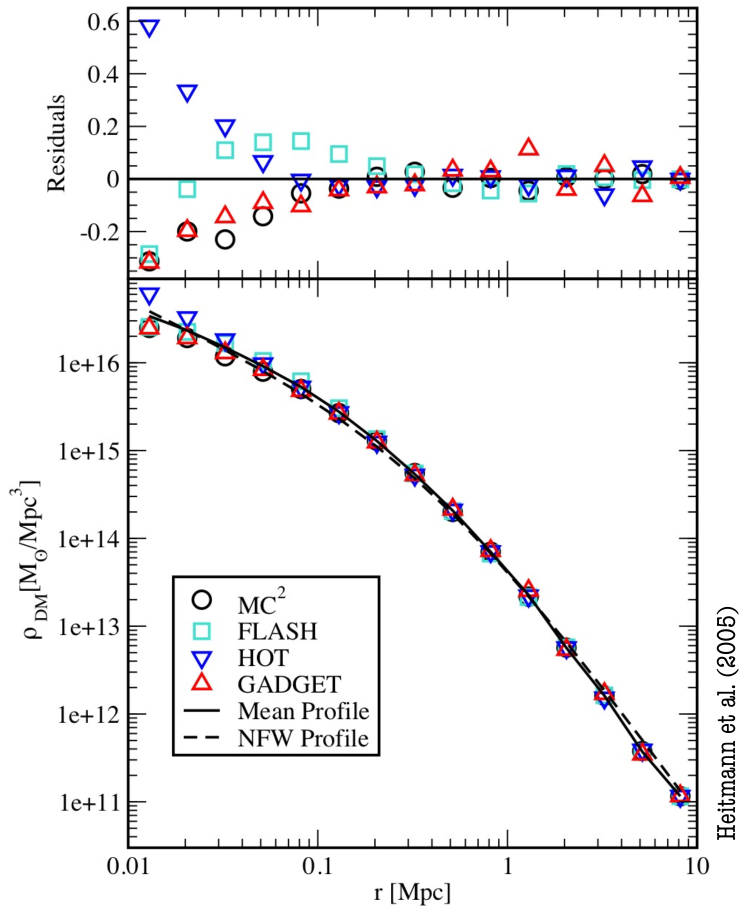
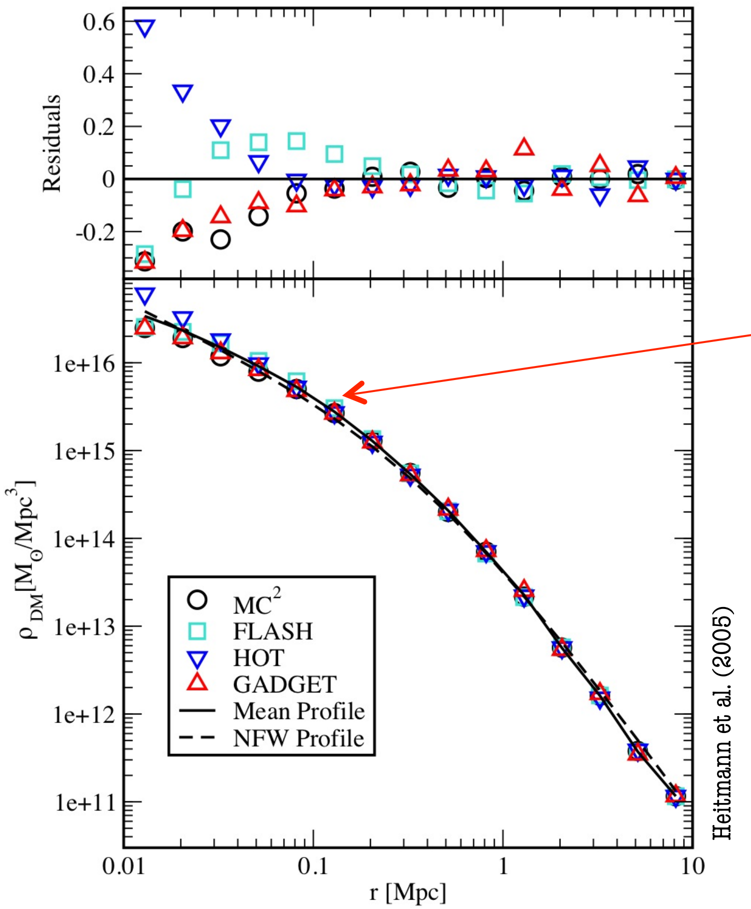


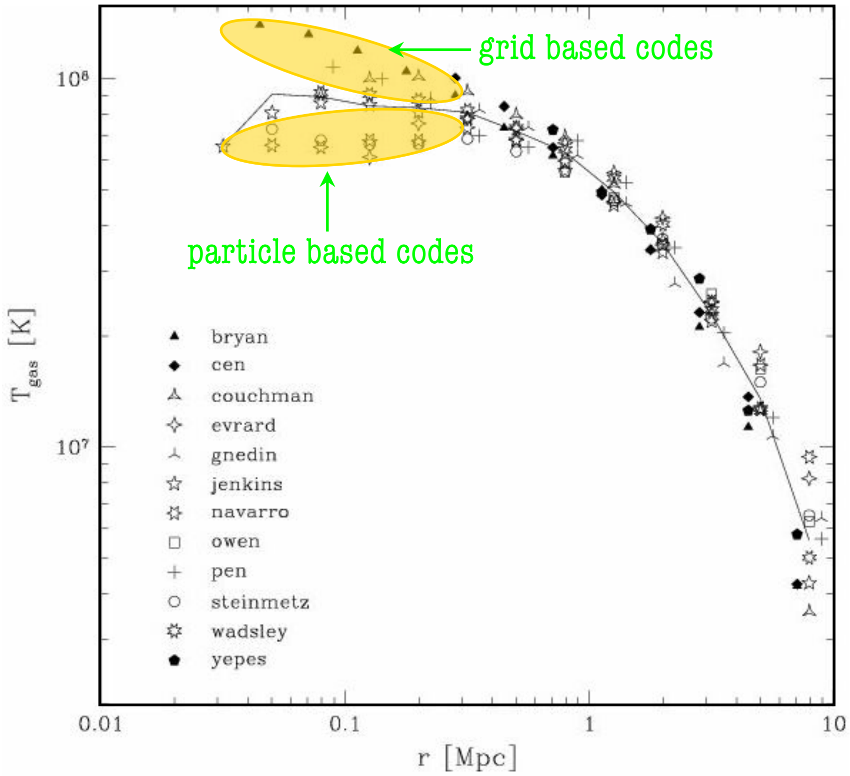
- "Santa Barbara Cluster" (Frenck et al. 1999)



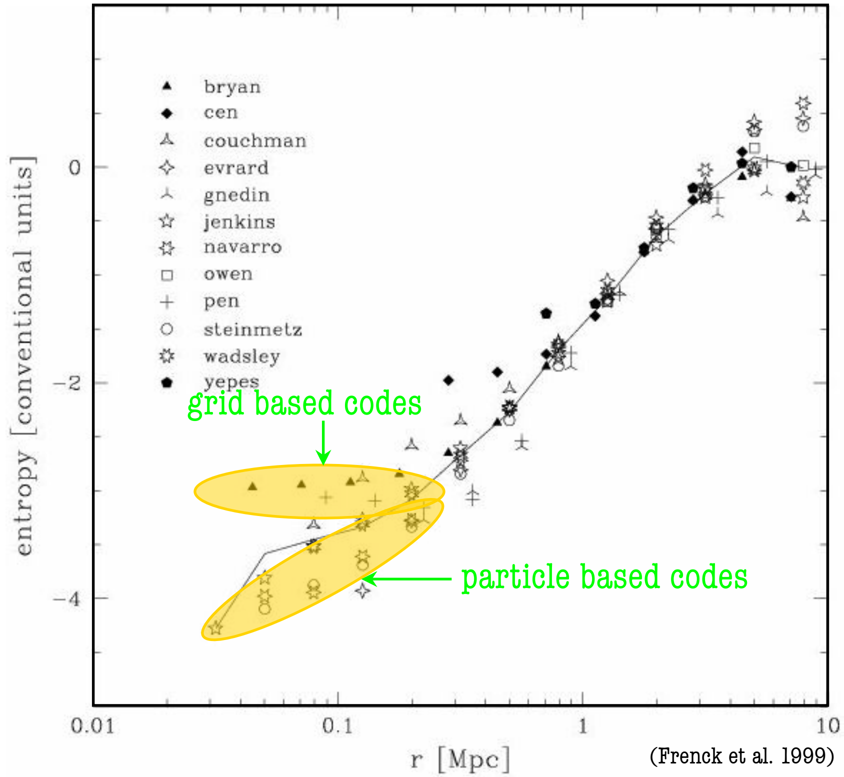
they all seem to agree more or less...

- "Santa Barbara Cluster" (incl. gas physics...)

temperature profile

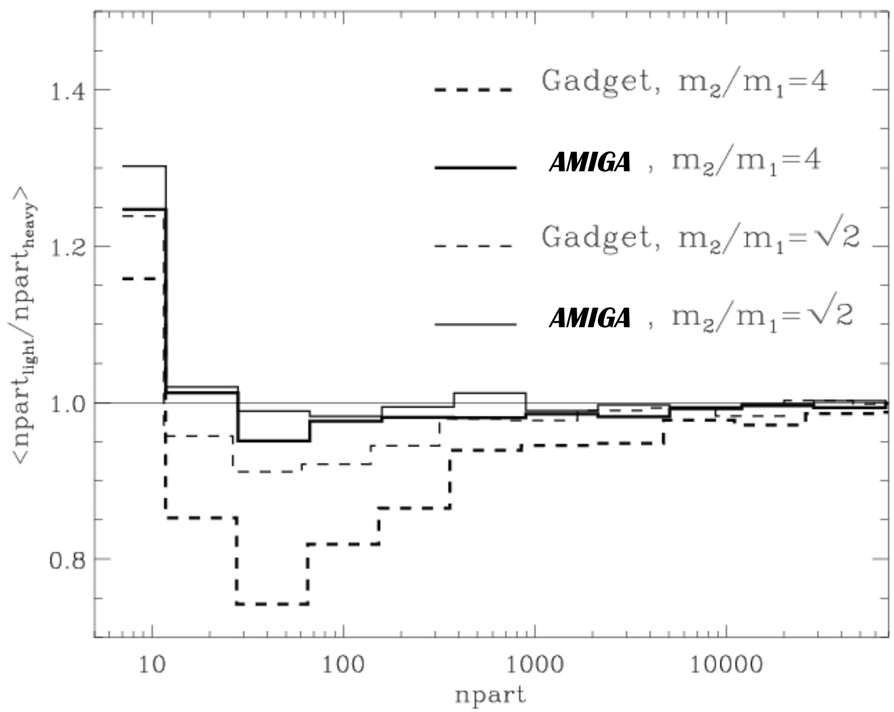


entropy profile



(Frenck et al. 1999)

- mass segregation
  - run simulation with 2 mass species and check for segregation



☠ **GADGET** (tree code) => expels lighter particles from halos

✓ ***AMIGA*** (AMR code) => ratio always about unity

- major differences

  - tree codes: spatially fixed force resolution
  - AMR codes spatially adaptive force resolution

✓ resolve the local inter-particle separation at all times and at all places

... nor more, no less!

✓ particles are "phase-space" markers rather than interacting "billiard balls"

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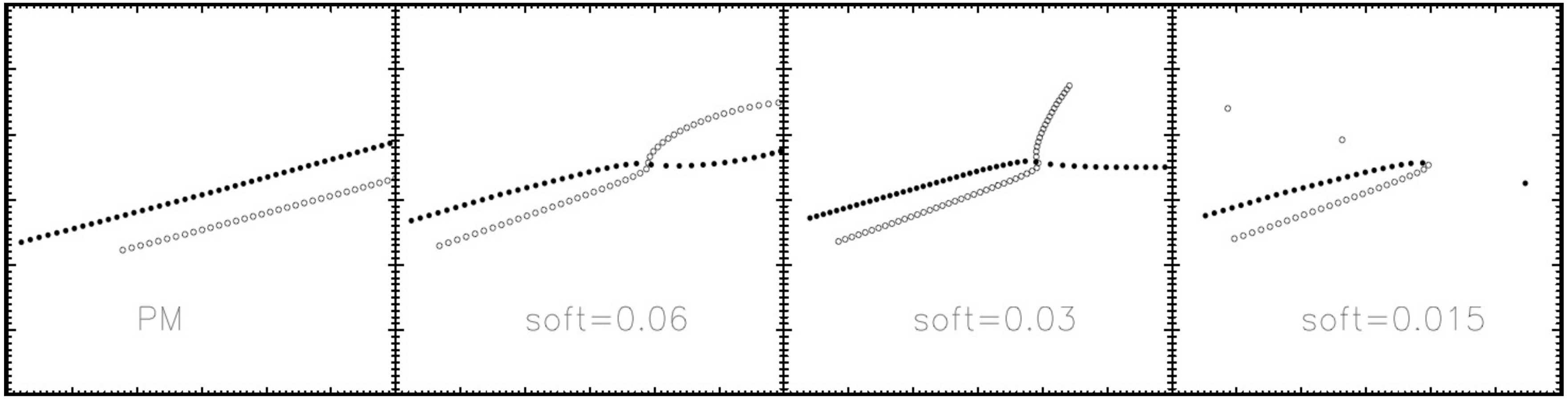
stability and credibility of (scientific) results...

- the relation between...

$\rightarrow$ *particle number, time step and softening?*

in-depth study by Power et al., MNRAS 338, 14 (2003)

- time step and softening

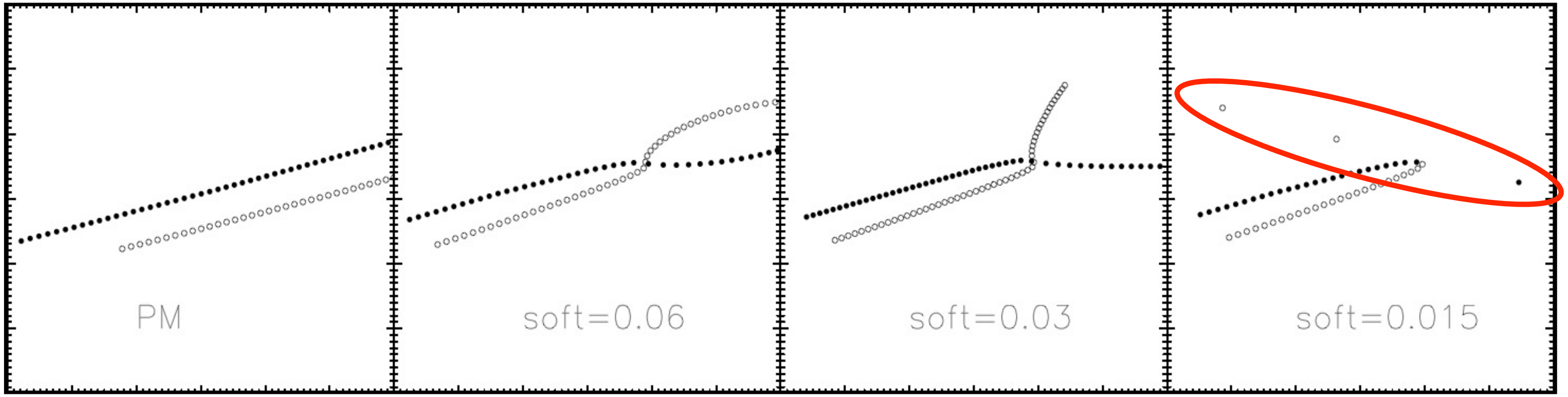


(Knebe et al. 2000)

increasing the force resolution
(without adjusting the time step...)

- time step and softening

unphysical two-body scattering



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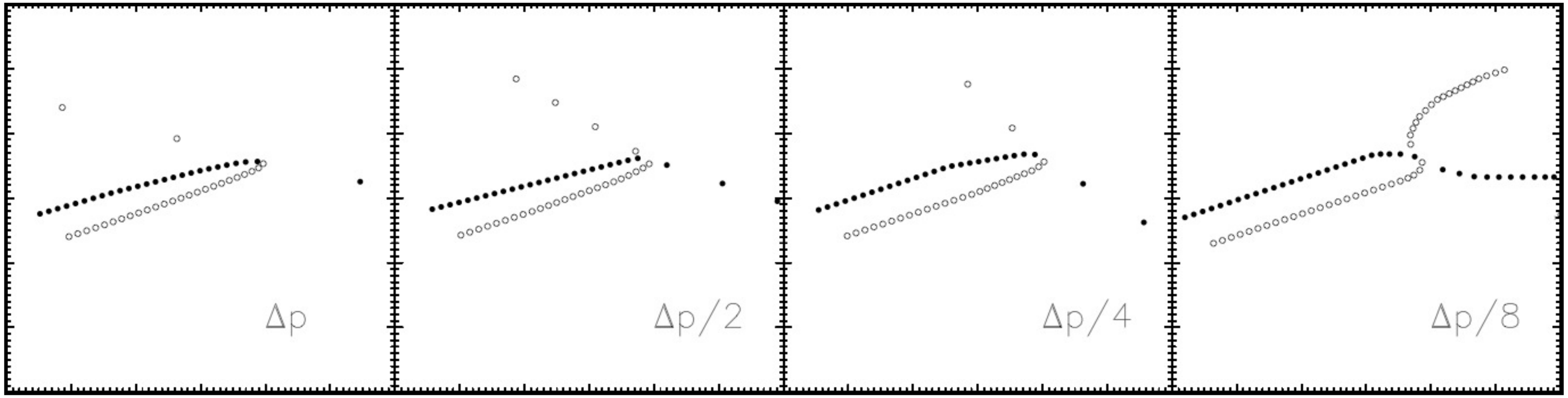
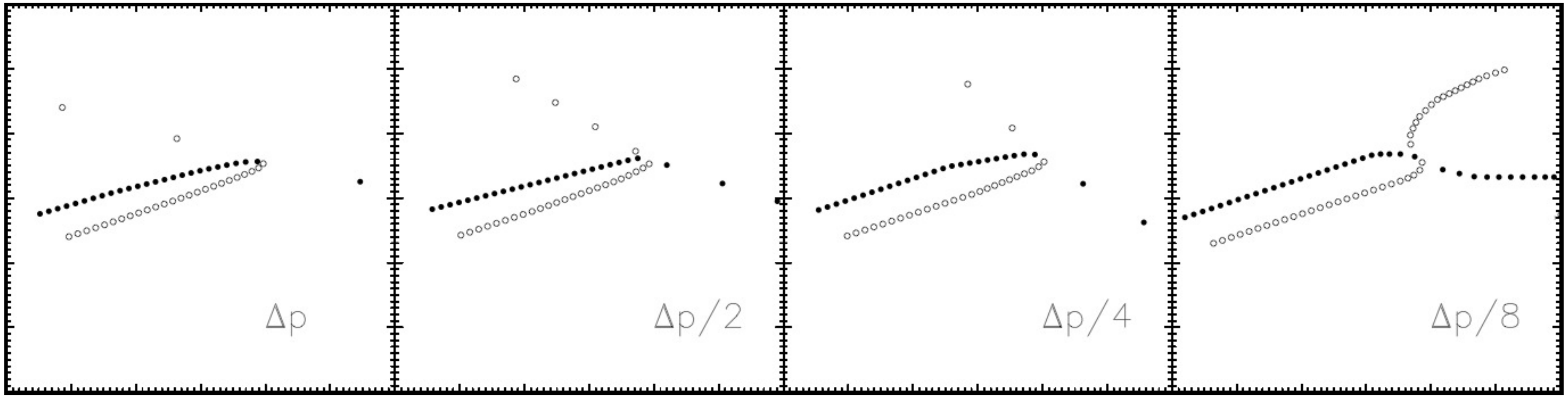


(Knebe et al. 2000)

simultaneously decreasing the time step remedies the problem

- time step and softening



(Knebe et al. 2000)

simultaneously decreasing the time step
remedies the problem

choose time step and softening wisely...

- the relation between...

→ *particle number, time step and softening?*

- convergence study:

run the same simulation again and again

gradually varying one of the technical parameters...

in-depth study by Power et al., MNRAS 338, 14 (2003)

- particle number

we aim at solving the collisionless Boltzmann equation
using particles as phase-space markers[1]...

...and hence their dynamics should be determined
by the mean field and not two-body interactions!

[1] cf. "The N-Body Approach" lecture...

- particle number

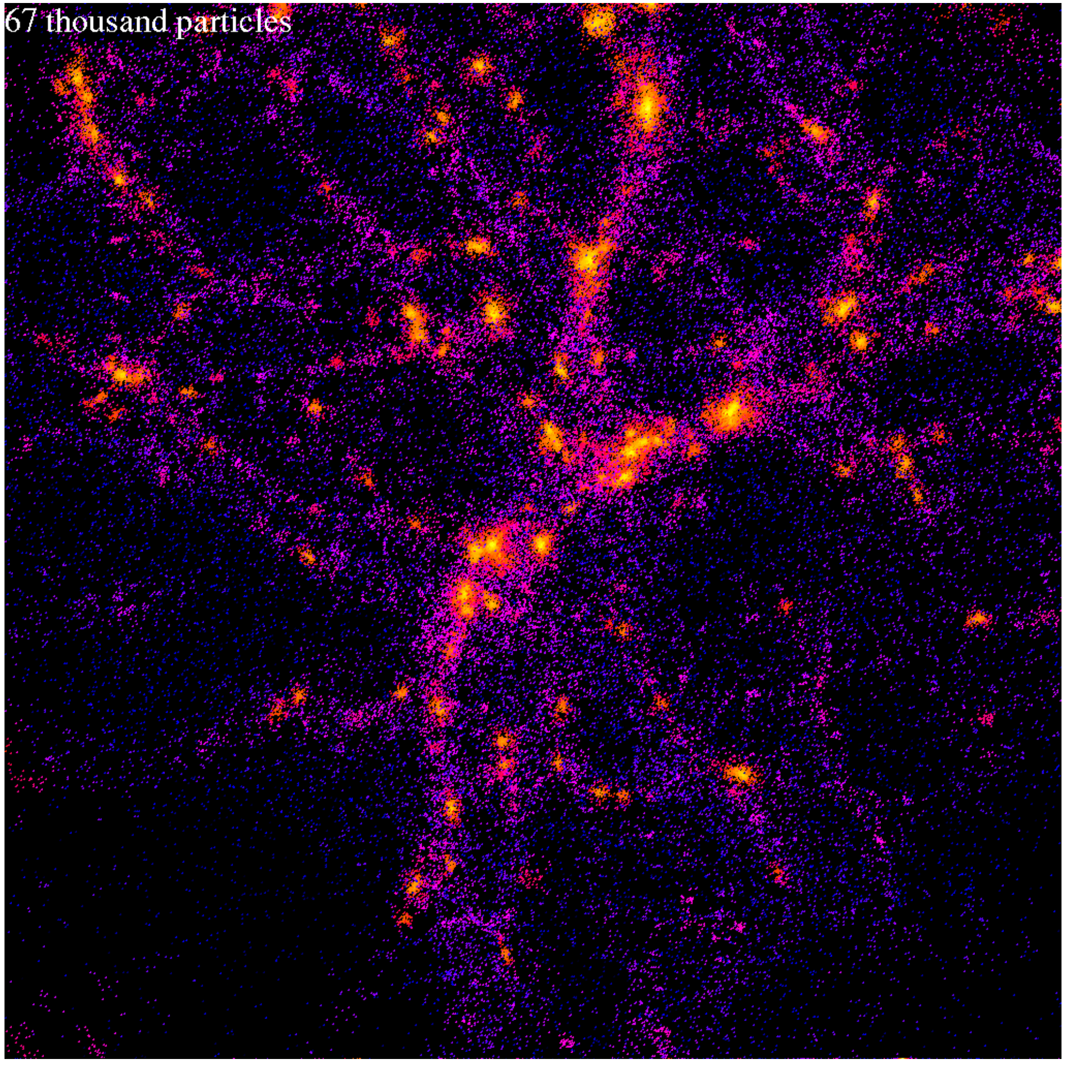
we aim at solving the collisionless Boltzmann equation
using particles as phase-space markers[1]...

...and hence their dynamics should be determined
by the mean field and not two-body interactions!

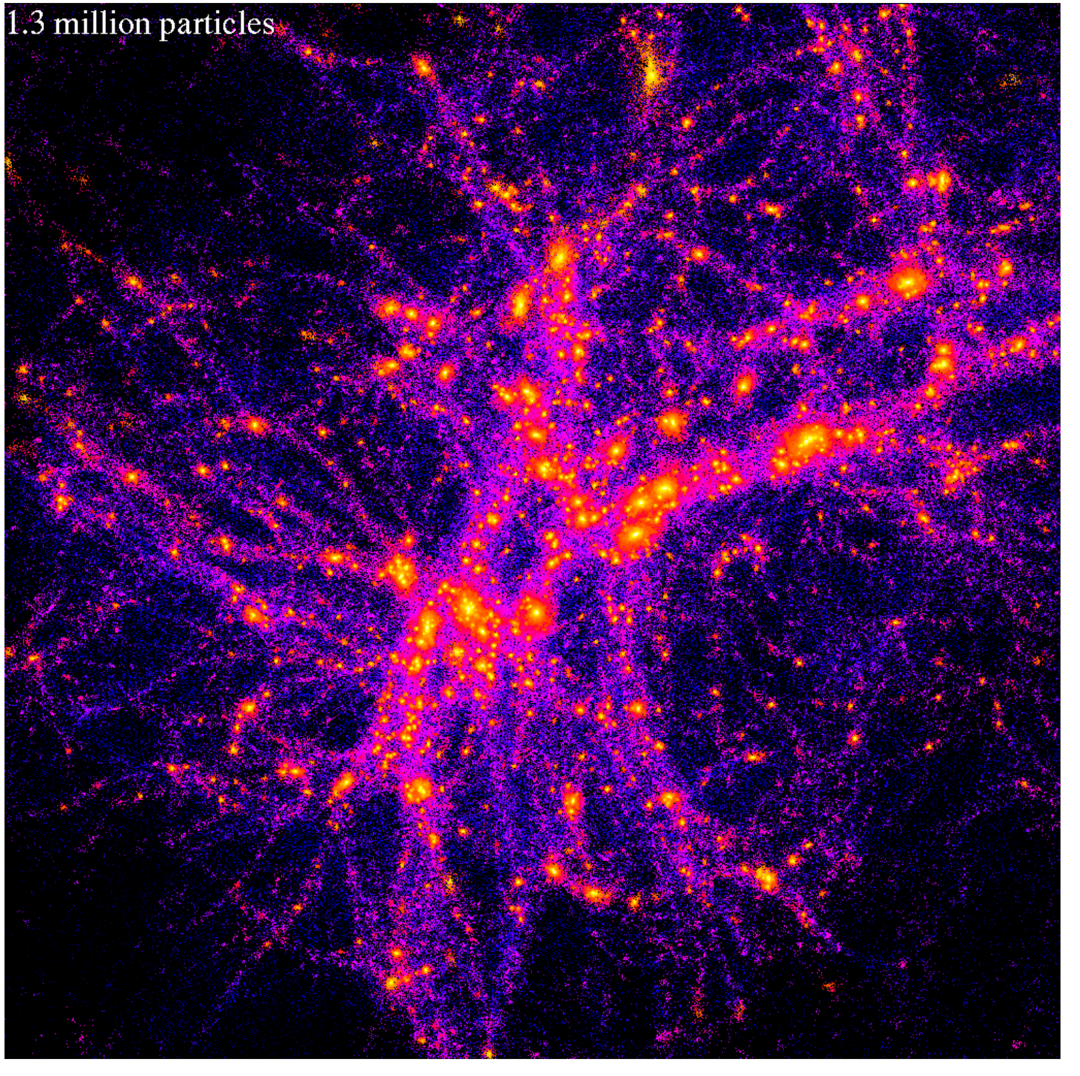
how can we be sure to use enough particles in this approach?

[1] cf. "The N-Body Approach" lecture...

- particle number

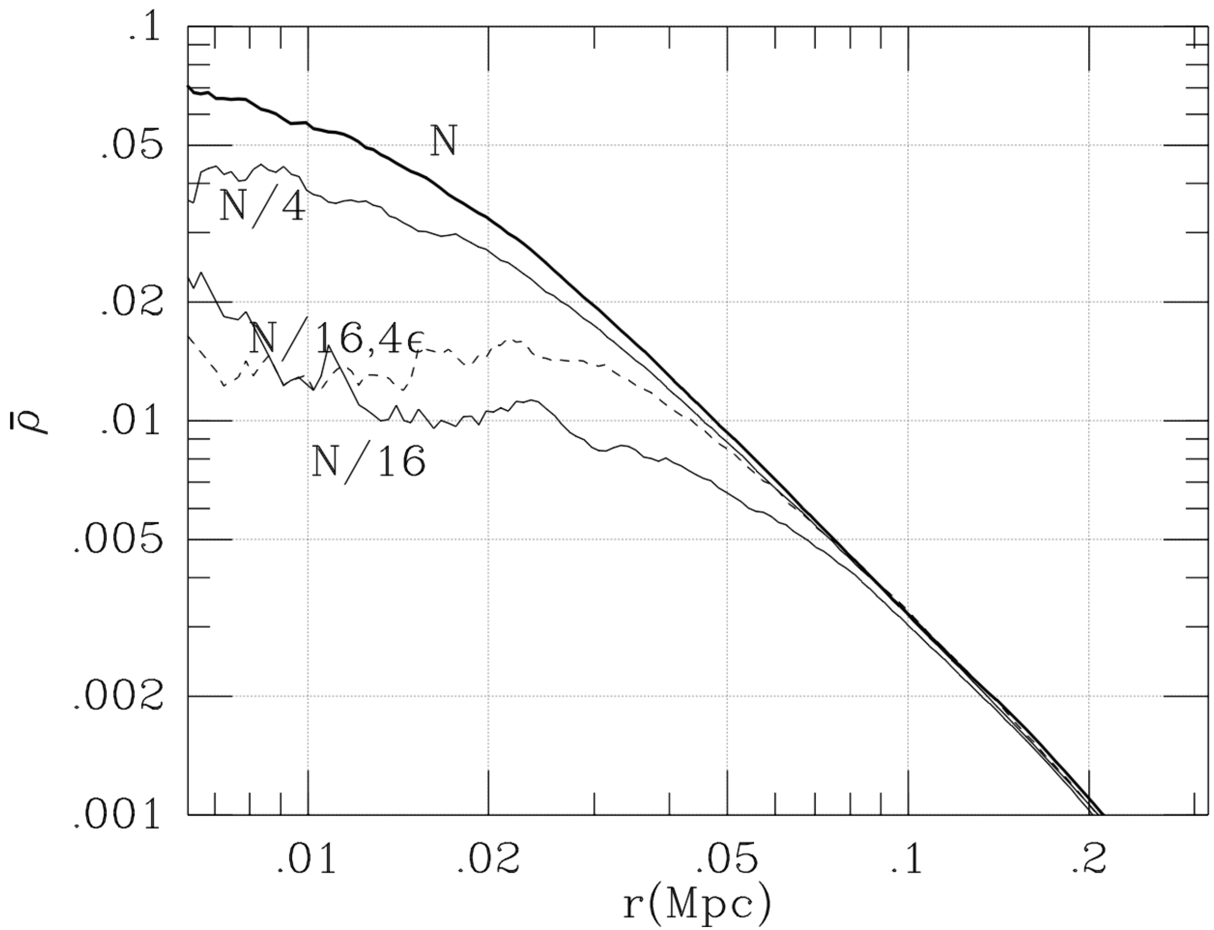


- particle number



- particle number

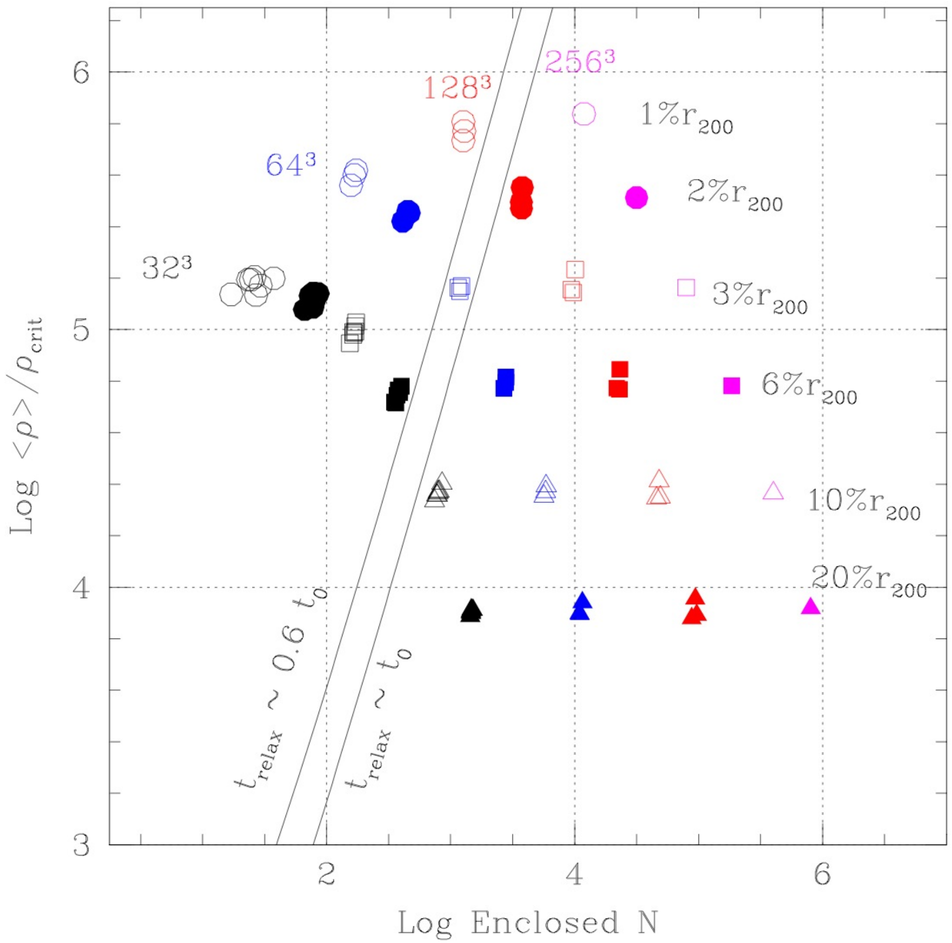
density profile of individual dark matter halo



(Fukushige et al. 2001)

- particle number

density profile of individual dark matter halo

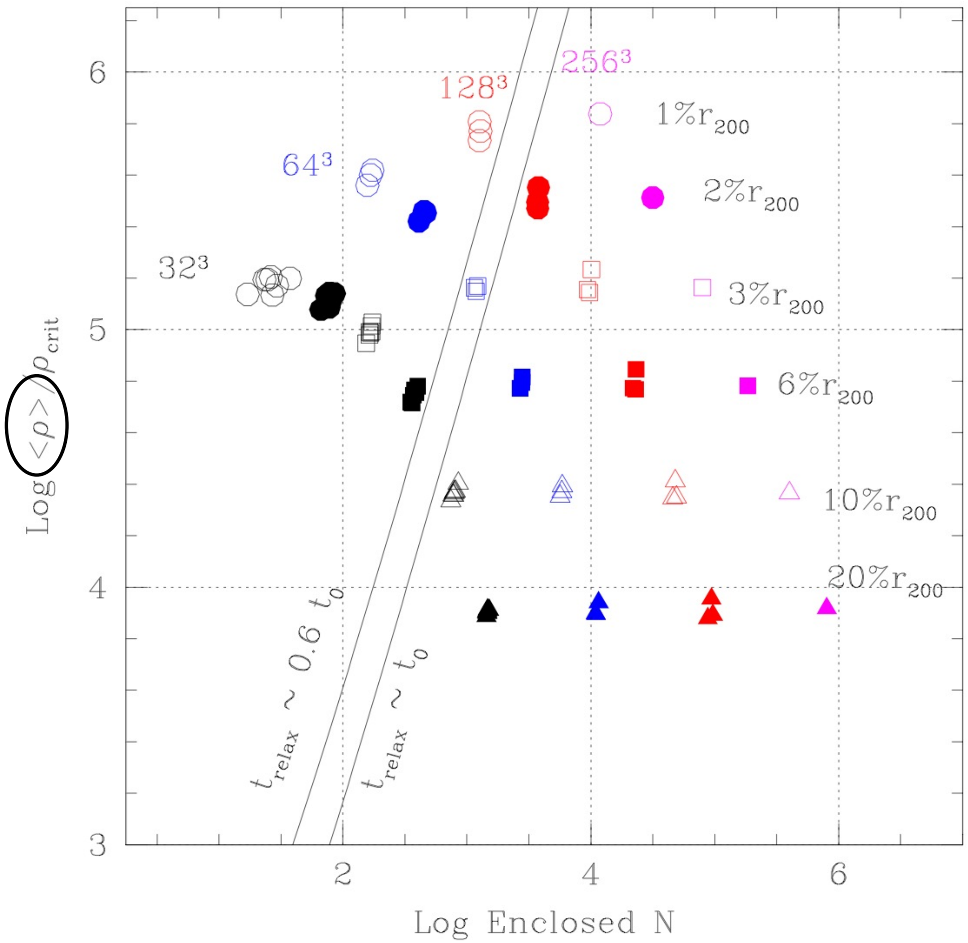


(Power et al. 2003)

- particle number

density profile of individual dark matter halo

$$\langle \rho \rangle = \frac{N(<r)}{\frac{4\pi}{3} r^3}$$



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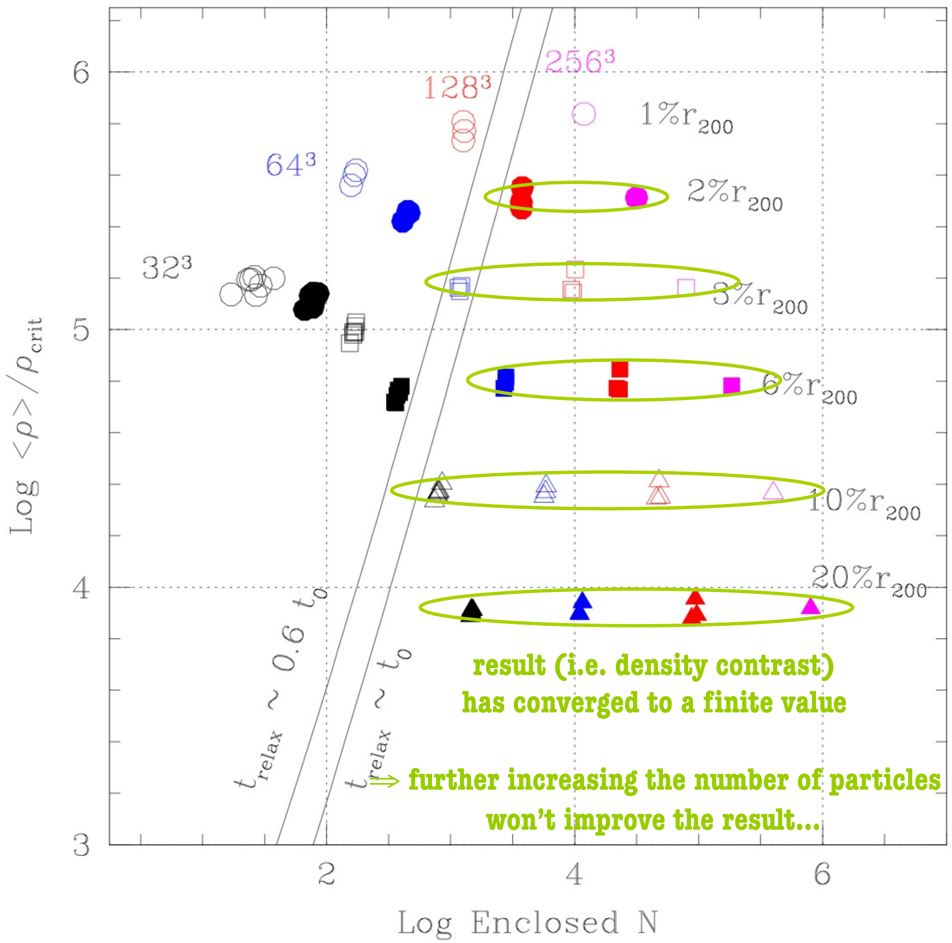

(Power et al. 2003)

- collisional relaxation
    - relaxation time

“When a finite number of particles is used to represent a system, individual particle accelerations will inevitably deviate from the mean-field value when particles pass close each other.”
(Power et al. 2003)

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$$\frac{t_{relax}}{t_{cross}} \approx \frac{N(<r)}{8\ln(r/\varepsilon)} \approx \frac{N}{8\ln N}$$

$t_{cross} = r/v$

number of encounters required to change a particle’s velocity by of order itself...
(cf. Binney & Tremaine 1987)

- collisional relaxation

  - relaxation time

“When a finite number of particles is used to represent a system, individual particle accelerations will inevitably deviate from the mean-field value when particles pass close each other.”
(Power et al. 2003)

$$\frac{t_{relax}}{t_{cross}} \approx \frac{N(<r)}{8\ln(r/\varepsilon)} \approx \frac{N}{8\ln N}$$

$$\frac{r}{\varepsilon} \approx \frac{rv^2}{Gm} \approx \frac{r\frac{GNm}{r}}{Gm} \approx N$$

a close encounter ε entails $\Delta v \approx v$

- collisional relaxation

  - relaxation time

"When a finite number of particles is used to represent a system, individual particle accelerations will inevitably deviate from the mean-field value when particles pass close each other."
(Power et al. 2003)

$$\frac{t_{relax}}{t_{cross}} \approx \frac{N(<r)}{8\ln(r/\varepsilon)} \approx \frac{N}{8\ln N}$$

$\Rightarrow$ the relaxation time t_{relax} should exceed the age of the Universe t_0:

$$t_{relax}(r,N,\varepsilon) \geq 0.6 t_0$$

- empirically derived relations meeting this requirement:

  - choose gravitational softening to ensure $a_{\mathrm{2body}} < a_{\mathrm{meanfield}}$

$$\varepsilon \approx 4 \times \frac{R_{vir}}{\sqrt{N_{vir}}}$$

  - regard those regions as converged where the circular orbit time-scales exceeds

$$t_{circ}(r_{converged}) > 15 \times \left(\frac{\Delta t}{t_0}\right)^{5/6} t_{circ}(R_{vir})$$

$$\frac{\Delta t}{t_0} = \frac{1}{N_{steps}}$$

- do trustworthy science...