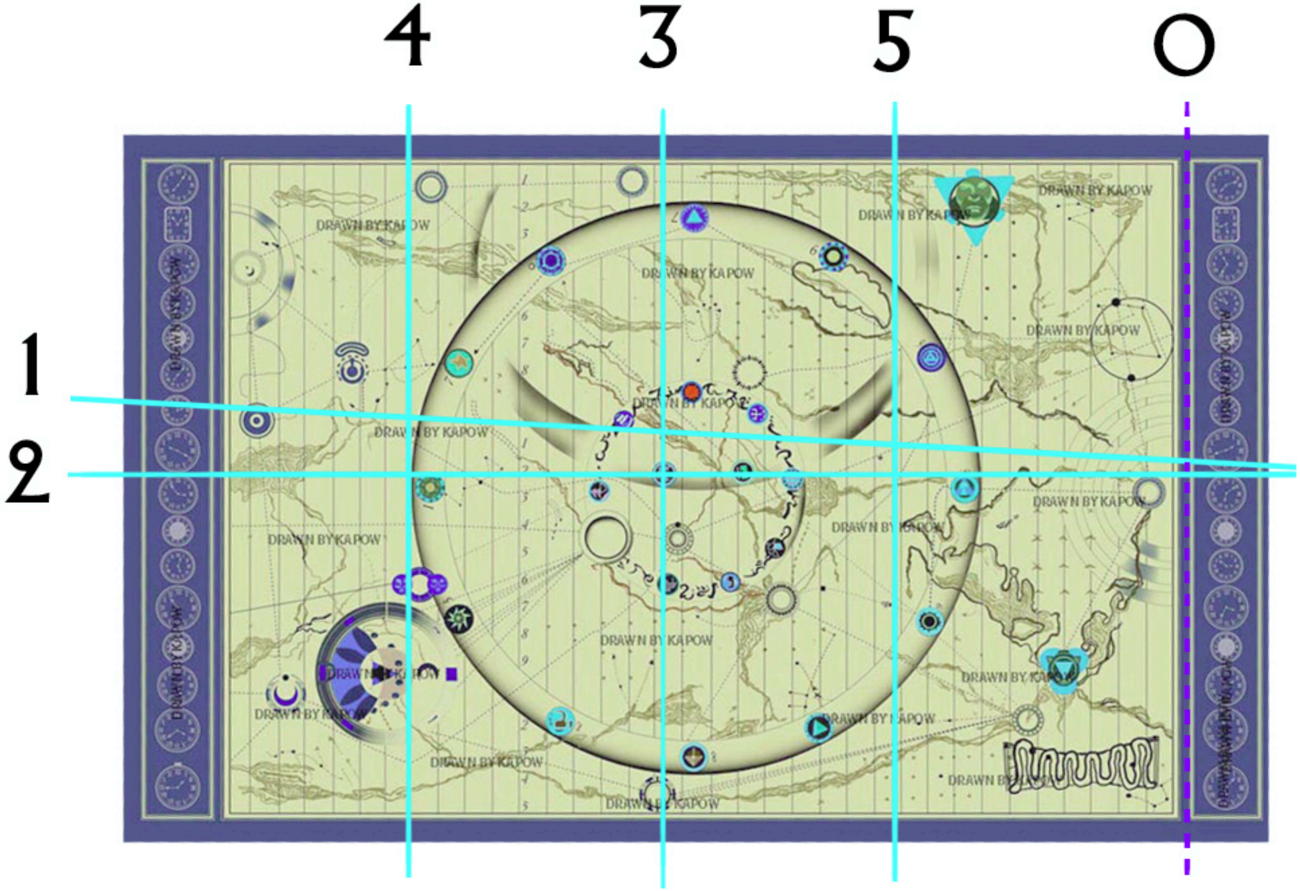
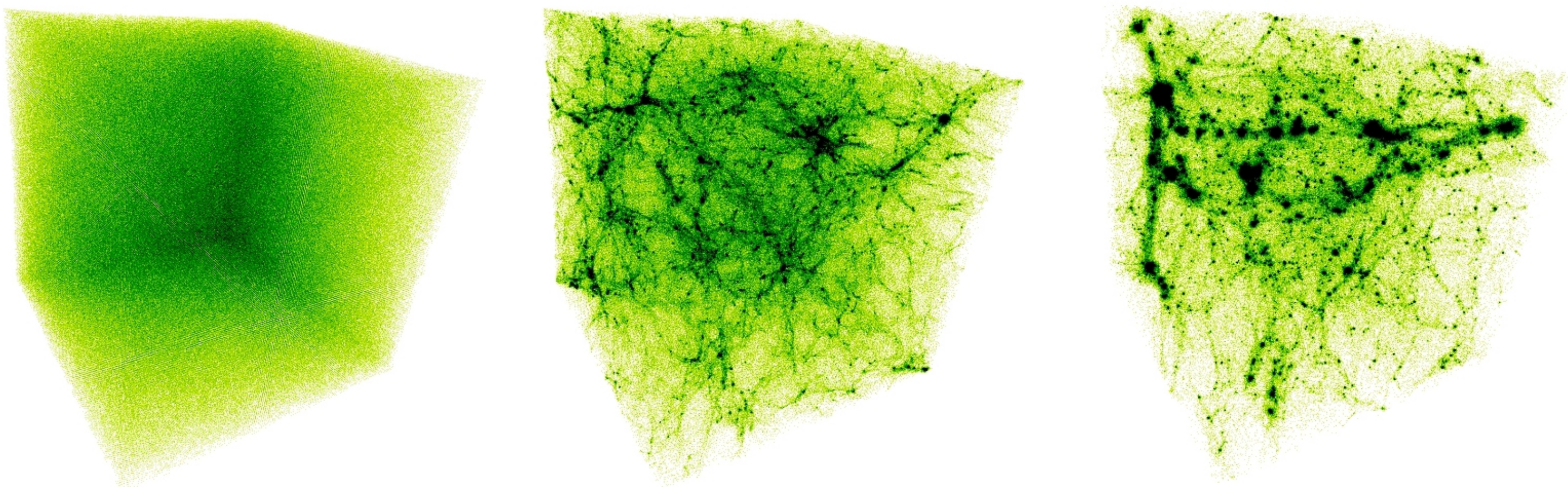
Alexander Knebe, *Universidad Autonoma de Madrid*



initial conditions?!

Generating Initial Conditions



?

- cosmological principle
- perturbations
- limitations
- alternatives
- remarks
- summary

- **cosmological principle**
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1. create an infinite homogenous and isotropic Universe

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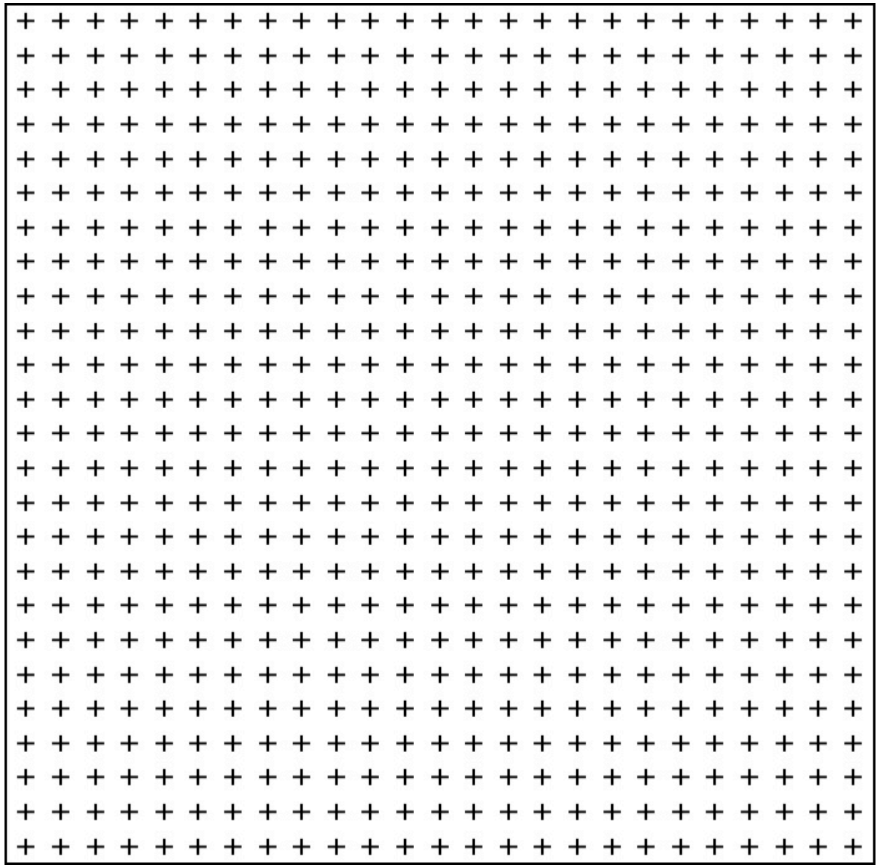
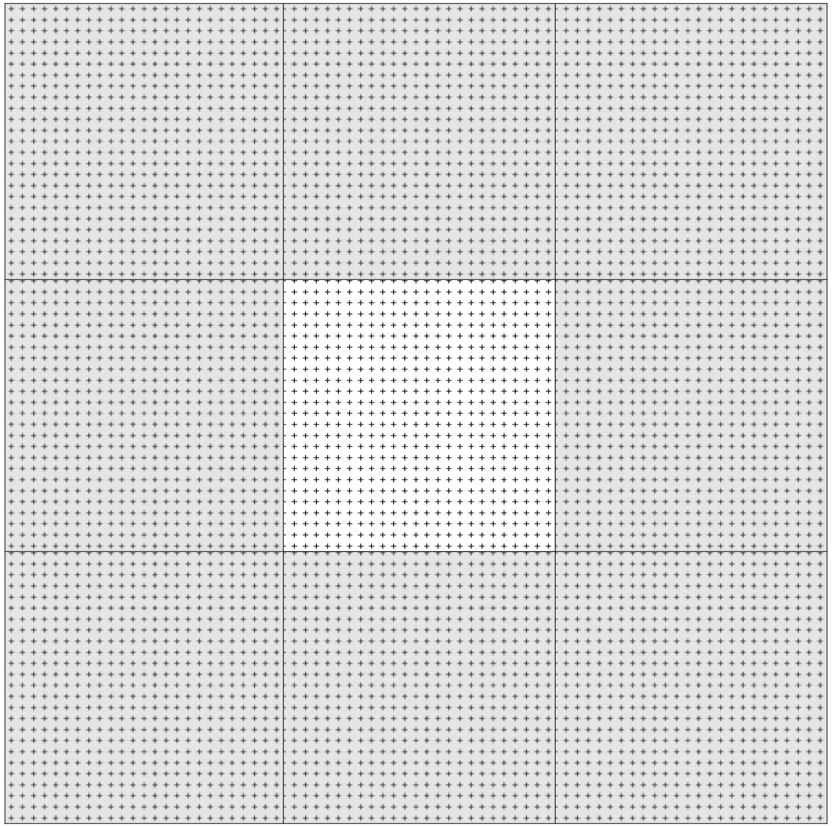
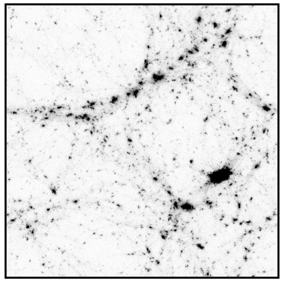
2. superimpose cosmological density perturbations



homogeneous
&
isotropic



infinite
(periodic boundary conditions)



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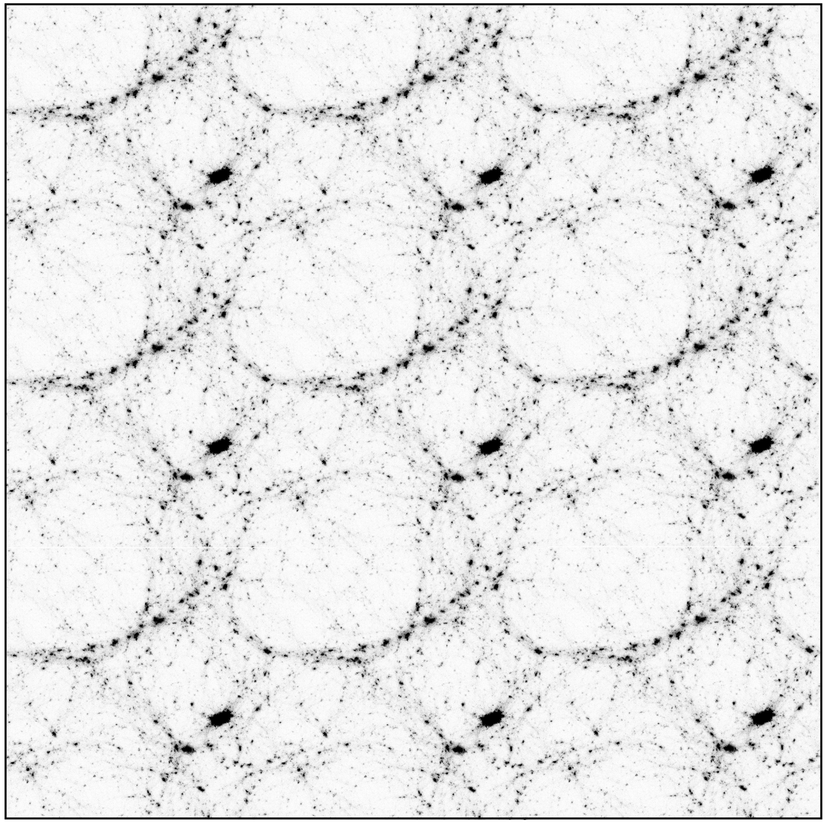

infinite
(periodic boundary conditions)

- cosmological principle
- **perturbations**
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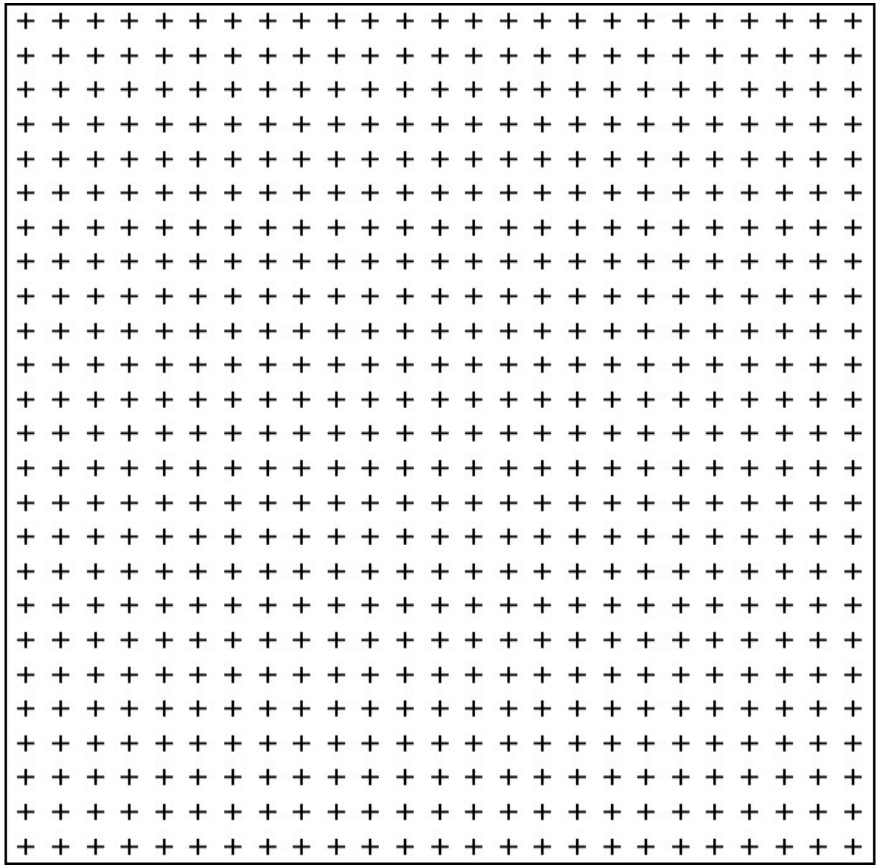

homogeneous
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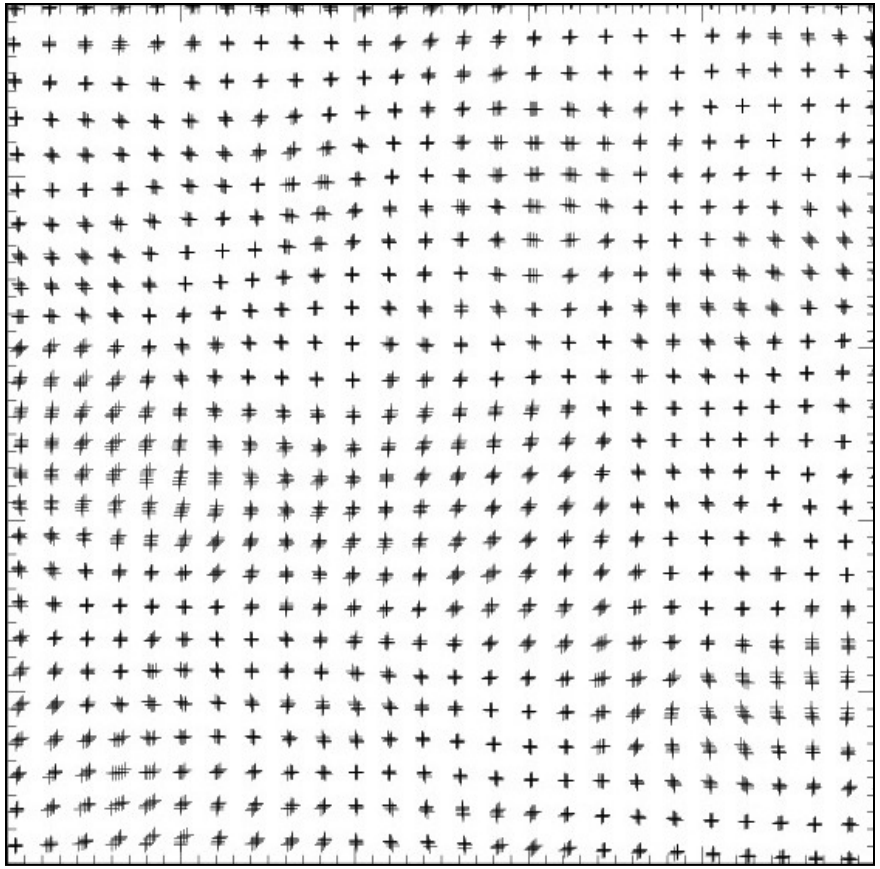
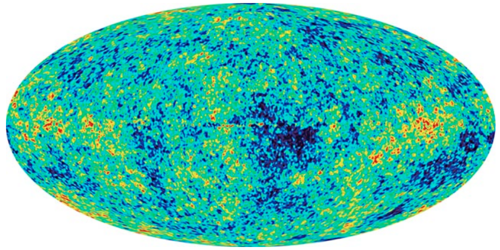
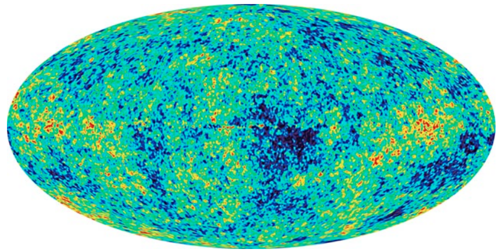




temperature fluctuations
converted into density perturbations



temperature fluctuations
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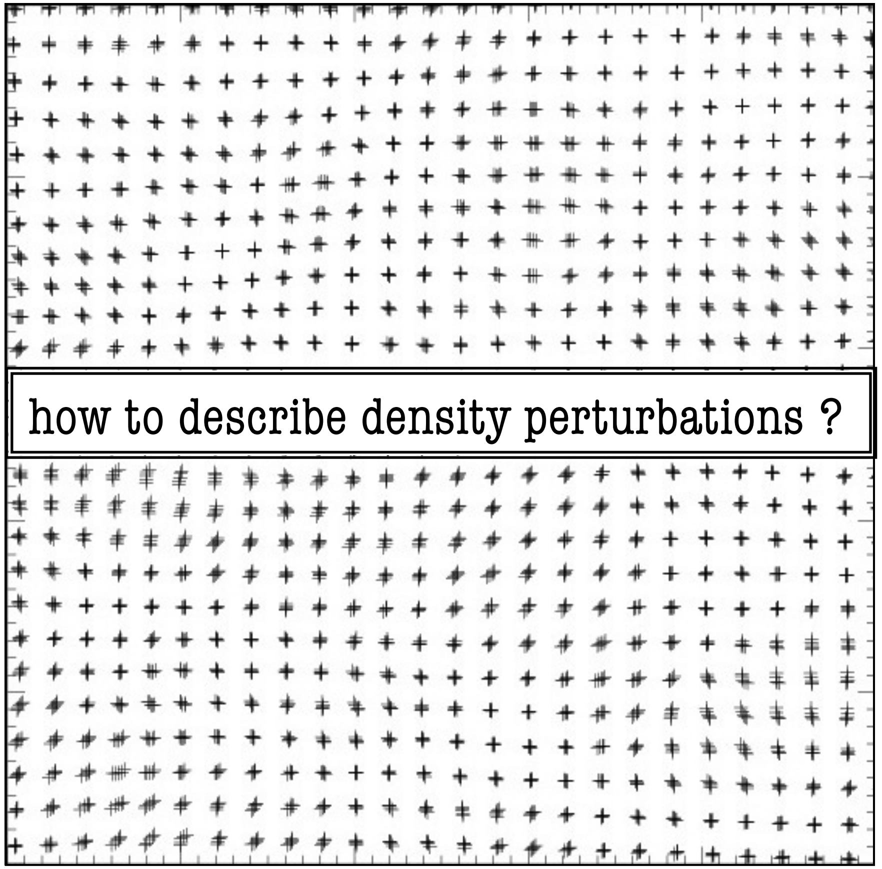


how to describe density perturbations ?

- density contrast:

$$\delta(\vec{x},t) = \frac{\rho(\vec{x},t) - \overline{\rho}(t)}{\overline{\rho}(t)}$$

- density contrast:

$$\delta(\vec{x},t) = \frac{\rho(\vec{x},t) - \overline{\rho}(t)}{\overline{\rho}(t)}$$

$\Rightarrow$ decomposition of $\delta(x)$ into waves

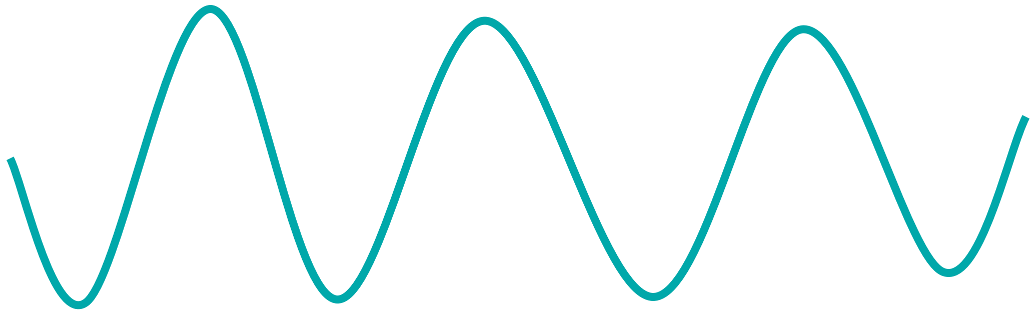
$$\delta(\vec{x}) = \sum \hat{\delta}(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} \qquad P(k) = \left\langle \left| \hat{\delta}(\vec{k}) \right|^2 \right\rangle_{|\vec{k}|=k}$$

Fourier transformation of density contrast

$$\delta(\vec{x}) = \sum \hat{\delta}(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} \qquad P(k) = \left\langle \left| \hat{\delta}(\vec{k}) \right|^2 \right\rangle_{|\vec{k}|=k}$$

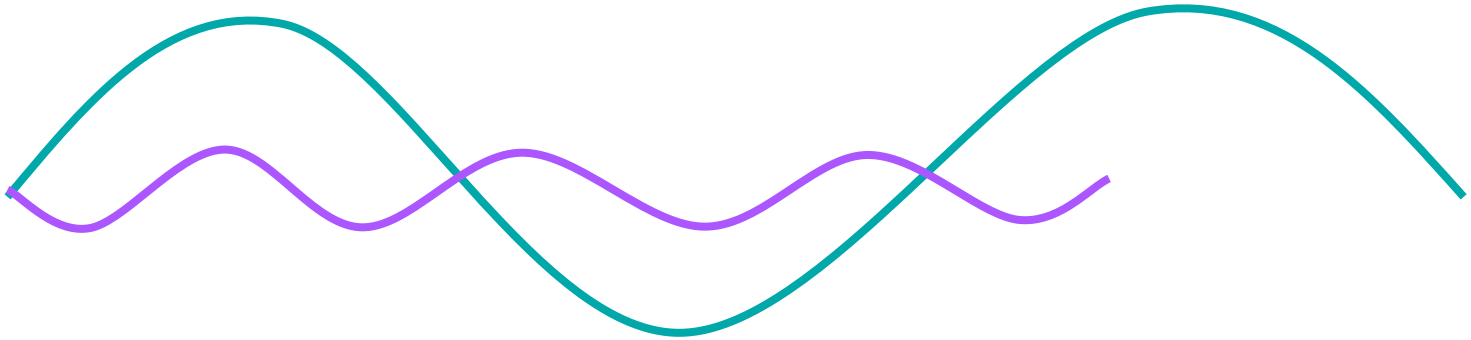
long wavelength

short wavelength,
large amplitude



short wavelength,
small amplitude

$$\delta(\vec{x}) = \sum \hat{\delta}(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} \qquad P(k) = \left\langle \left|\hat{\delta}(\vec{k})\right|^2 \right\rangle_{|\vec{k}|=k}$$



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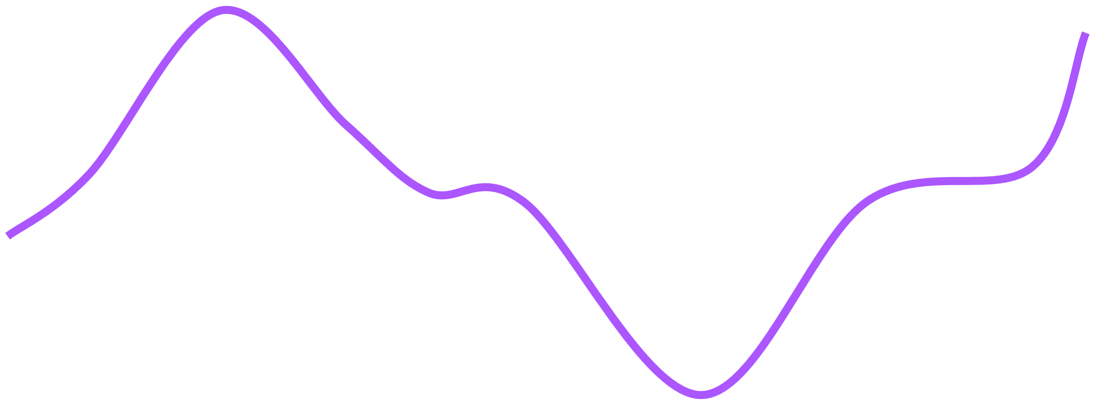

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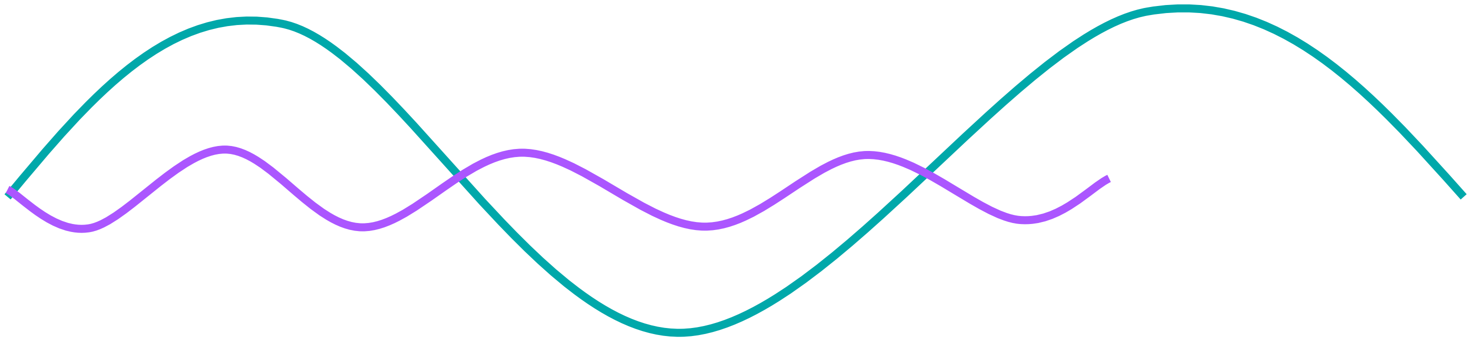
no information about phases!



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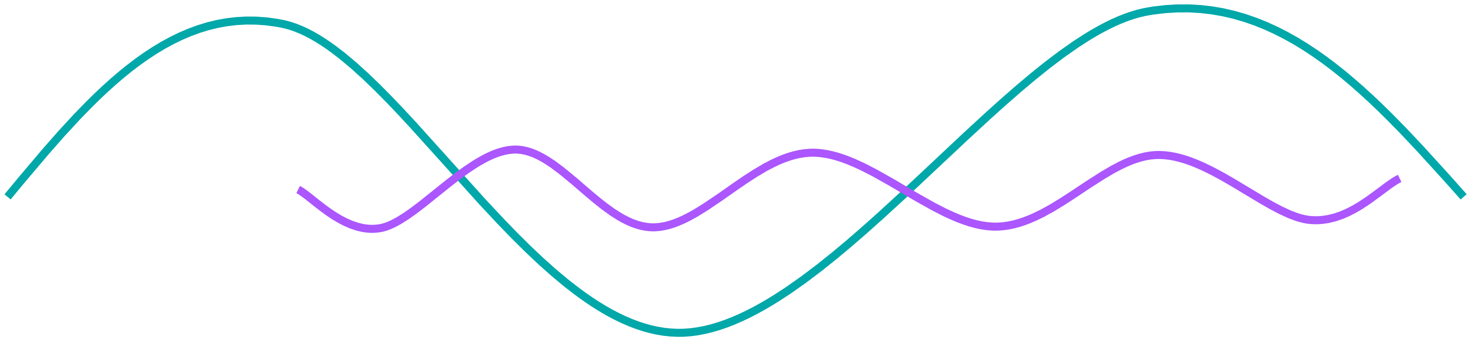

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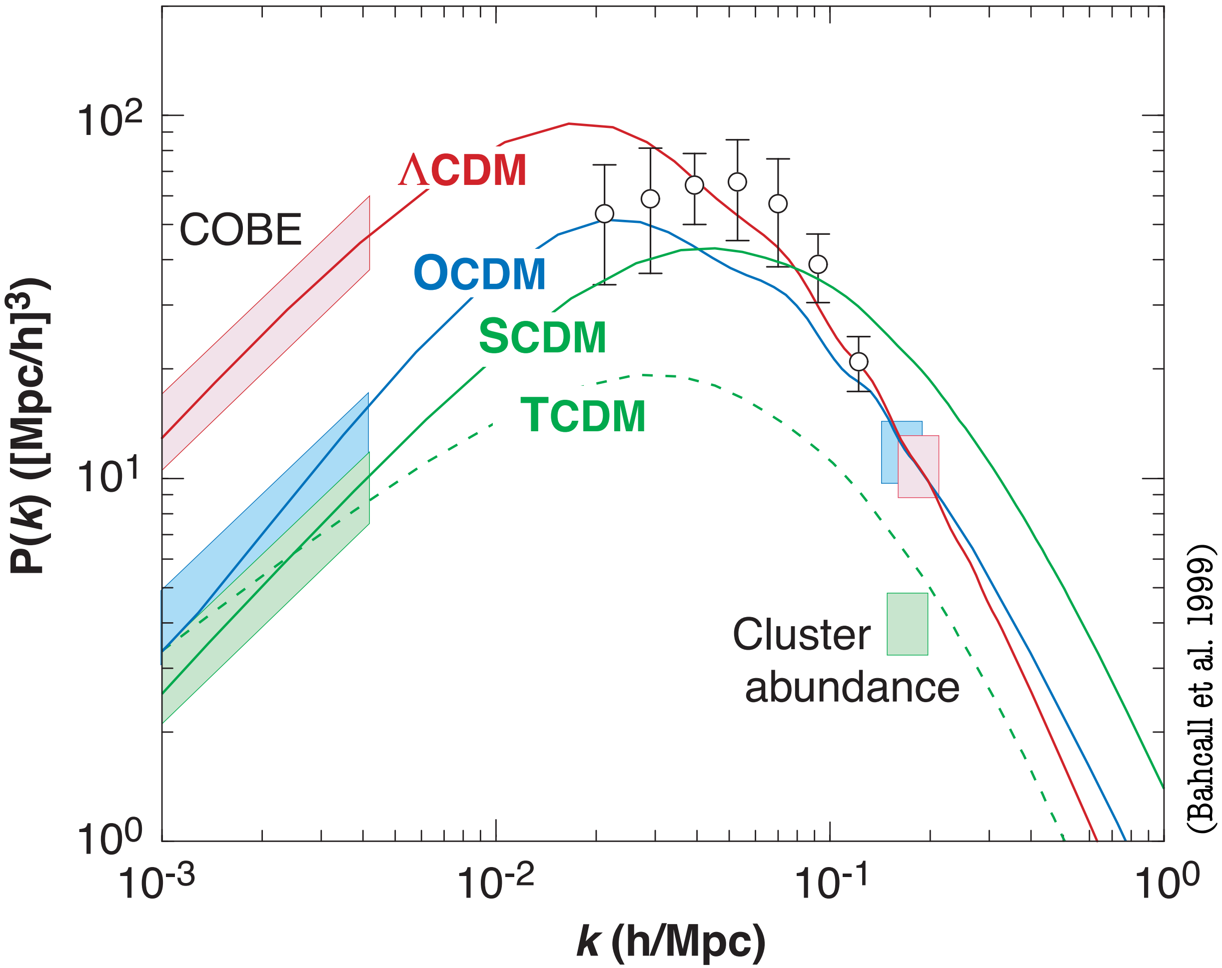
cosmological P(k)'s?

$$\delta(\vec{x}) = \sum \hat{\delta}(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} \qquad P(k) = \left\langle \left| \hat{\delta}(\vec{k}) \right|^2 \right\rangle_{|\vec{k}|=k}$$



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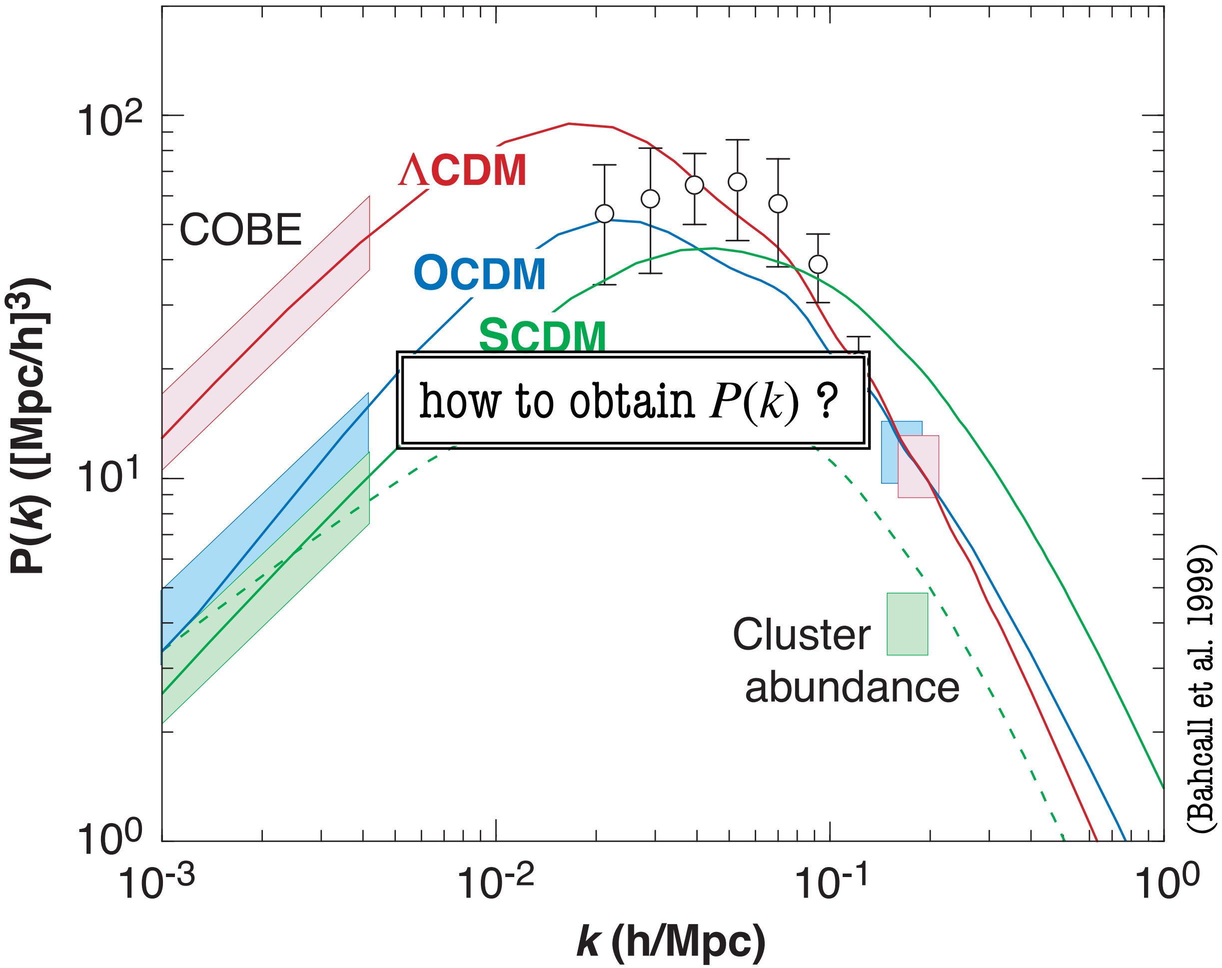

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- inflation theory

$$P_i(k) = Ak^n$$ with $n = 1$ (Harrision - Zeldovich spectrum)

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- inflation theory

$$P_i(k) = Ak^n \qquad \text{with } n = 1 \text{ (Harrision - Zeldovich spectrum)}$$

- transferring $P(k)$ across recombination

$$P(k) = T^2(k) P_i(k)$$

$$\delta(\vec{x}) = \sum \hat{\delta}(\vec{k}) e^{-i\vec{k}\cdot\vec{x}}$$

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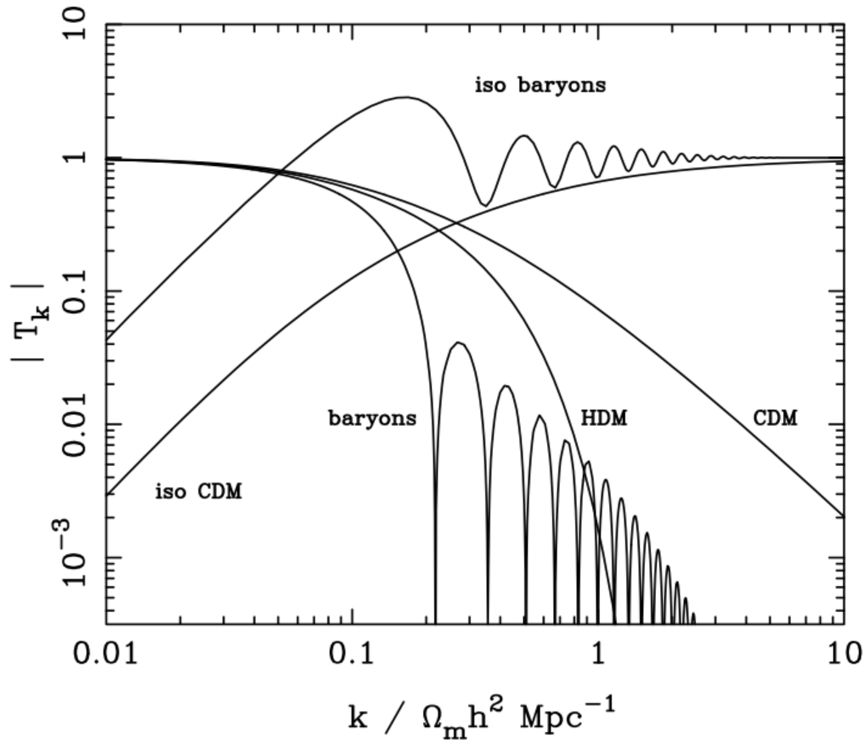

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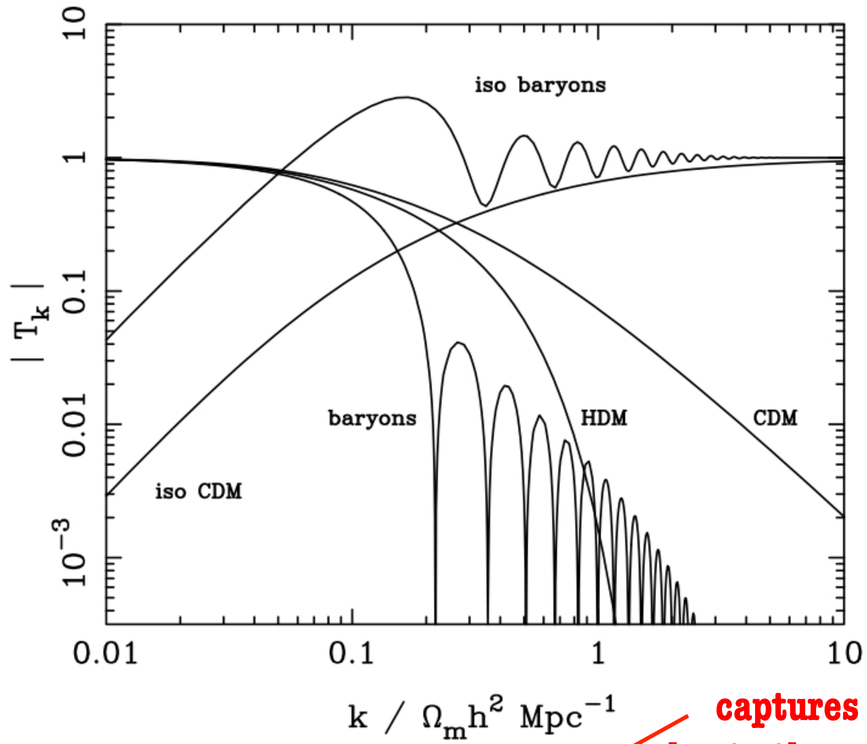

captures all the complicated physics due to the coupling of radiation and matter

$$P(k) = T^2(k) P_i(k)$$

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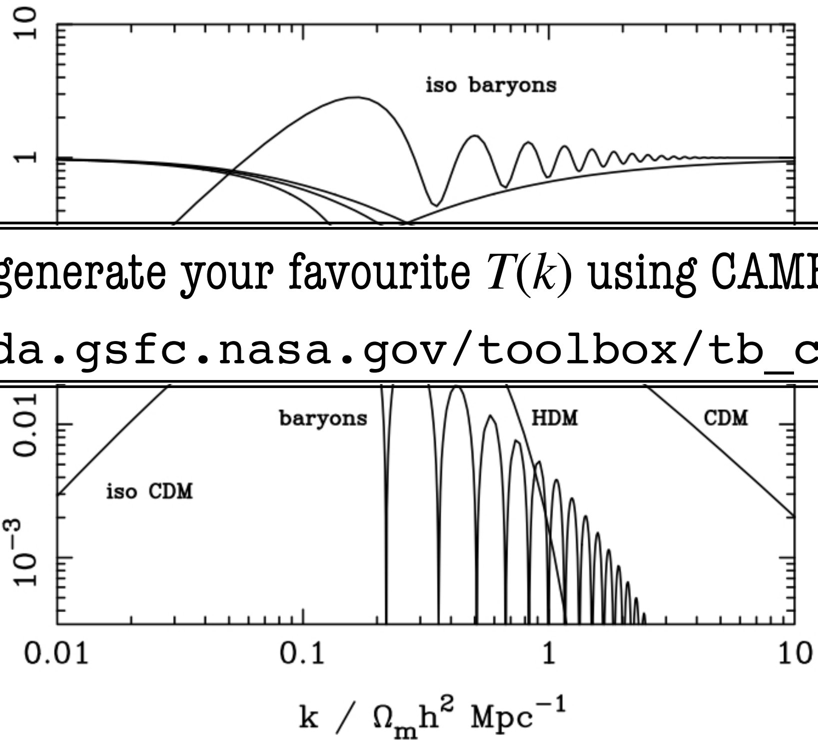

generate your favourite $T(k)$ using CAMB:

http://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm

$$P(k) = T^2(k) P_i(k)$$

$$\delta(\vec{x}) = \sum \hat{\delta}(\vec{k}) e^{-i\vec{k}\cdot\vec{x}}$$

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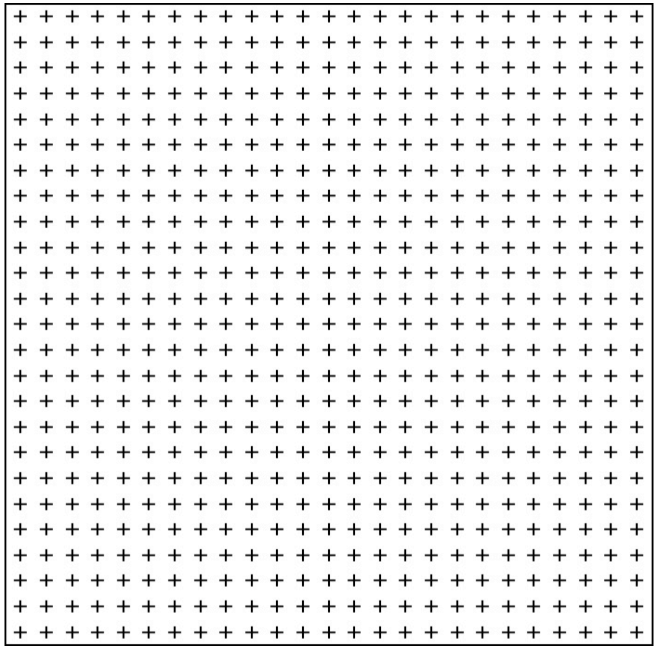
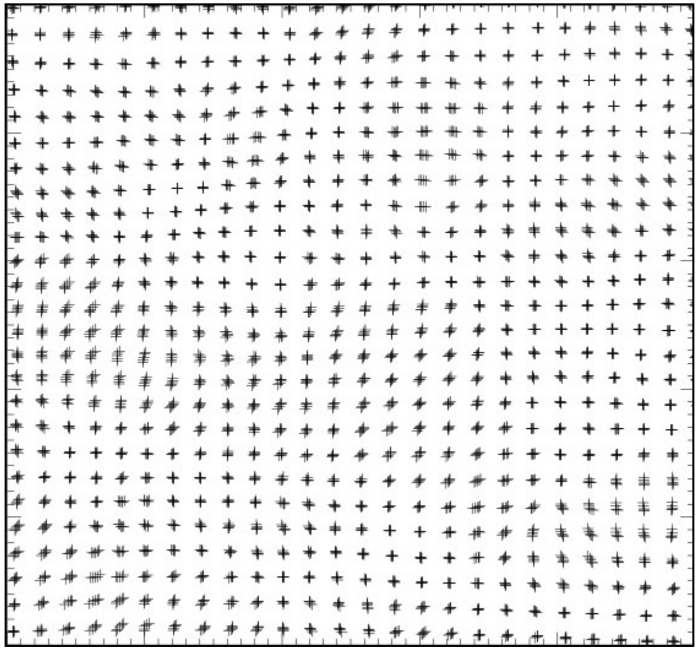
homogeneous & isotropic



?

initial conditions



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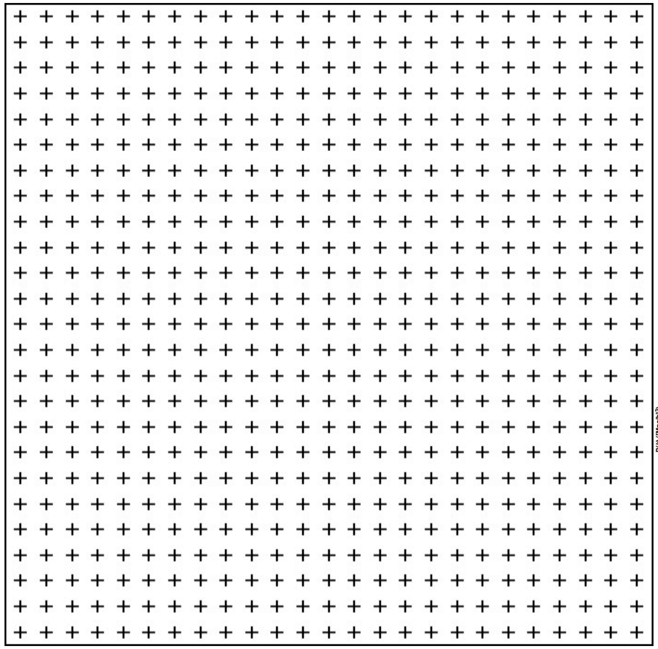
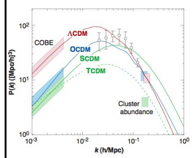


$P(k)$



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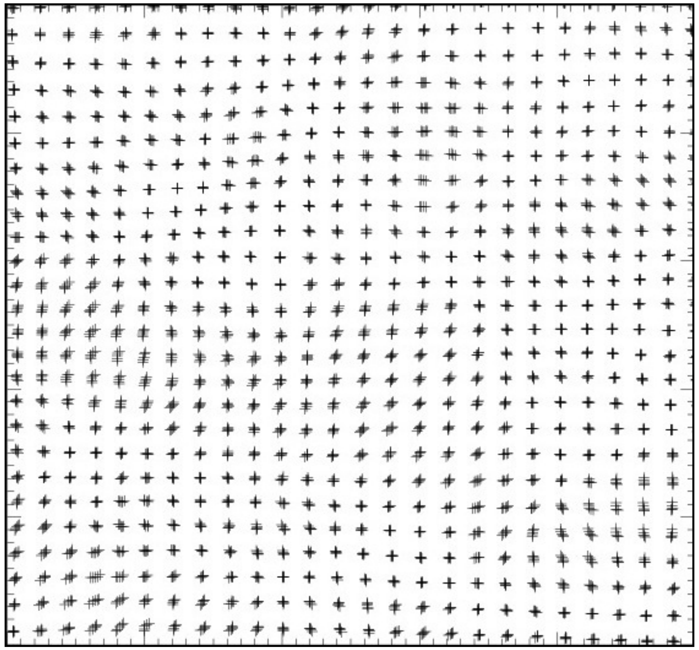

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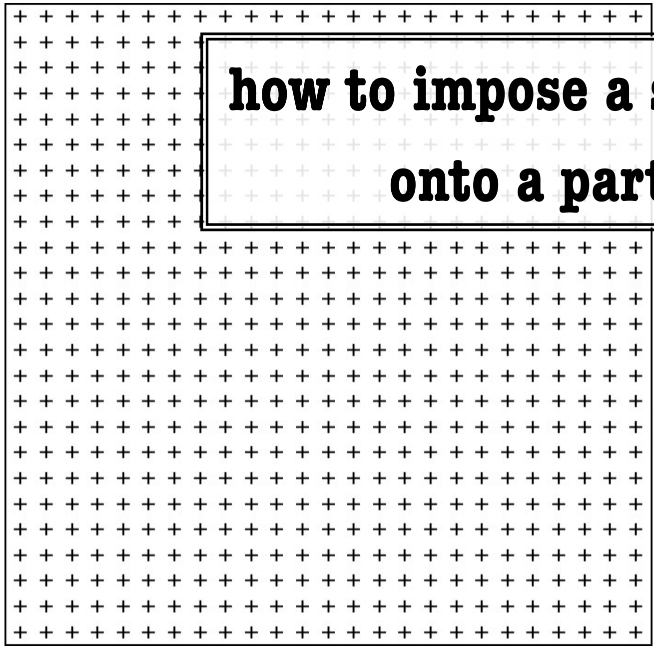
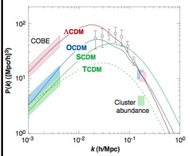
homogeneous & isotropic

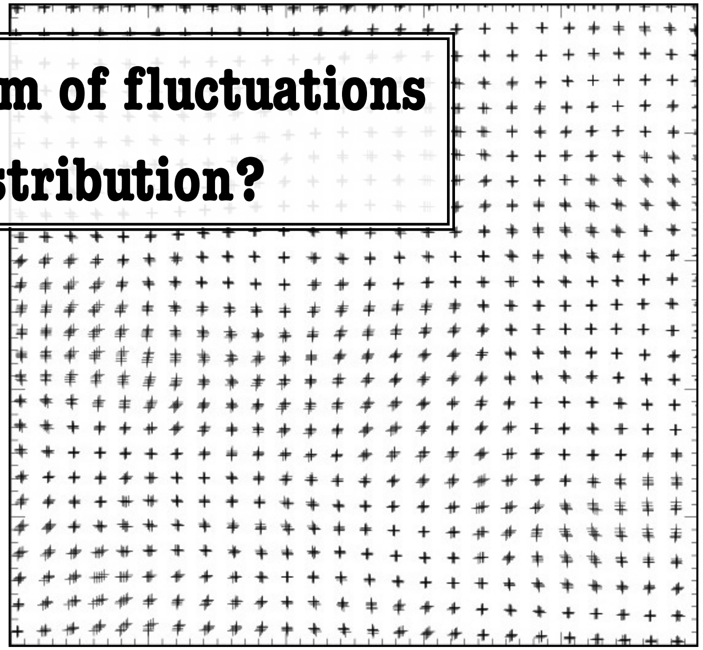


how to impose a spectrum of fluctuations onto a particle distribution?

$P(k)$ →



initial conditions



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1. temporal evolution of P(k)

*cf. "structure formation" lecture at http://popia.ft.uam.es/aknebe

$$\delta(\vec{x}) = \sum \hat{\delta}(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} \qquad P(k) = \left\langle \left| \hat{\delta}(\vec{k}) \right|^2 \right\rangle_{|\vec{k}|=k}$$

- remember linear perturbation theory*:

density contrast: $$\frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \cdot \vec{u} = 0$$ mass conservation

peculiar velocity field: $$\frac{\partial \vec{u}}{\partial t} + \frac{\dot{a}}{a} \vec{u} = -\frac{1}{a} \nabla \Phi$$ momentum conservation

$$\Delta \Phi = 4\pi G a^2 \overline{\rho} \delta$$ Poisson's equation

*cf. "structure formation" lecture at http://popia.ft.uam.es/aknebe

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- remember linear perturbation theory*:

$$\frac{\partial^2\delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial\delta}{\partial t} - 4\pi G\bar{\rho}\delta = 0$$ evolution of matter density

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$$\delta(x,t) = D(t)\delta_0(x)$$

*cf. “structure formation” lecture at http://popia.ft.uam.es/aknebe

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2. displacement of particles

*cf. “structure formation” lecture at http://popia.ft.uam.es/aknebe

- Zel'dovich approximation:

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

Ansatz based upon the idea to

displace particles from their initial positions on a regular mesh

- Zel'dovich approximation:

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

? ? ?

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$\vec{q} =$ Lagrangian position

(i.e. initial positions on regular mesh)

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$$D(t) = \frac{\delta(\vec{x},t)}{\delta_0(\vec{x},t_0)}$$

$$\frac{\partial^2 D}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial D}{\partial t} - 4\pi G\overline{\rho} D = 0$$

general solution:

$$D(t) = \frac{5}{2}\Omega_0 \frac{\dot{a}}{a}\int_{t_0}^{t}\frac{1}{\dot{a}^2}\,dt'$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$$\vec{S}(\vec{q}) \text{ ?}$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$\dot{\vec{x}} = \dot{D}\vec{S}(\vec{q})$ derivative of Ansatz

$\dot{\vec{x}} = \frac{1}{a}\vec{u}$ definition of peculiar velocity field

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$$\left.\begin{aligned} \dot{\vec{x}} &= \dot{D}\vec{S}(\vec{q}) \\ \dot{\vec{x}} &= \frac{1}{a}\vec{u} \end{aligned}\right\} \qquad \vec{u} = a\dot{D}\vec{S}(\vec{q})$$

- Zel'dovich approximation:

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linear perturbation theory

$$\frac{\partial \vec{u}}{\partial t} + \frac{\dot{a}}{a}\vec{u} = -\frac{1}{a}\nabla\Phi$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$$\left.\begin{aligned} \dot{\vec{x}} &= \dot{D}\vec{S}(\vec{q}) \\ \dot{\vec{x}} &= \frac{1}{a}\vec{u} \end{aligned}\right\} \qquad \begin{aligned} \vec{u} &= a\dot{D}\vec{S}(\vec{q}) \\ \frac{\partial \vec{u}}{\partial t} &= \dot{a}\dot{D}\vec{S}(\vec{q}) + a\ddot{D}\vec{S}(\vec{q}) \end{aligned}$$

$$\frac{\partial \vec{u}}{\partial t} + \frac{\dot{a}}{a}\vec{u} = -\frac{1}{a}\nabla\Phi$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

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$$\dot{\vec{x}} = \frac{1}{a}\vec{u} \qquad \frac{\partial \vec{u}}{\partial t} = \dot{a}\dot{D}\vec{S}(\vec{q}) + a\ddot{D}\vec{S}(\vec{q})$$

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$$\dot{a}\dot{D}\vec{S}(\vec{q}) + a\ddot{D}\vec{S}(\vec{q}) + \dot{a}\dot{D}\vec{S}(\vec{q}) = -\frac{1}{a}\nabla\Phi$$

$$(2a\dot{a}\dot{D} + a^2\ddot{D})\vec{S}(\vec{q}) = -\nabla\Phi$$

- Zel'dovich approximation:

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

$$\left.\begin{aligned}\dot{\vec{x}} &= \dot{D}\vec{S}(\vec{q}) \\ \dot{\vec{x}} &= \frac{1}{a}\vec{u}\end{aligned}\right\} \qquad \begin{aligned}\vec{u} &= a\dot{D}\vec{S}(\vec{q}) \\ \frac{\partial\vec{u}}{\partial t} &= \dot{a}\dot{D}\vec{S}(\vec{q}) + a\ddot{D}\vec{S}(\vec{q})\end{aligned}$$

- linear perturbation theory in comoving coordinates

$$\ddot{D} + 2\frac{\dot{a}}{a}\dot{D} - 4\pi G\overline{\rho}D = 0 \qquad \text{mass conservation}$$

$$(2a\dot{a}\dot{D} + a^2\ddot{D})\vec{S}(\vec{q}) = -\nabla\Phi \qquad \text{momentum conservation}$$

$$\Delta\Phi = 4\pi G a^2\overline{\rho}\delta \qquad \text{Poisson's equation}$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

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- linear perturbation theory in comoving coordinates

combine to obtain a relation between S & δ

$$\ddot{D} + 2\frac{\dot{a}}{a}\dot{D} - 4\pi G\overline{\rho}D = 0 \qquad \text{mass conservation}$$

$$(2a\dot{a}\dot{D} + a^2\ddot{D})\vec{S}(\vec{q}) = -\nabla\Phi \qquad \text{momentum conservation}$$

$$\Delta\Phi = 4\pi G a^2\overline{\rho}\delta \qquad \text{Poisson's equation}$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$$\left.\begin{aligned} \dot{\vec{x}} &= \dot{D}\vec{S}(\vec{q}) \\ \dot{\vec{x}} &= \frac{1}{a}\vec{u} \end{aligned}\right\} \qquad \begin{aligned} \vec{u} &= a\dot{D}\vec{S}(\vec{q}) \\ \frac{\partial \vec{u}}{\partial t} &= \dot{a}\dot{D}\vec{S}(\vec{q}) + a\ddot{D}\vec{S}(\vec{q}) \end{aligned}$$

- linear perturbation theory in comoving coordinates

$$\ddot{D} + 2\frac{\dot{a}}{a}\dot{D} - 4\pi G\overline{\rho}D = 0 \qquad \text{mass conservation}$$

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• linear perturbation theory in comoving coordinates

$$4\pi G a^2 \overline{\rho} D\vec{S}(\vec{q}) = -\nabla\Phi$$

$$\Delta\Phi = 4\pi G a^2 \overline{\rho}\delta$$

- Zel'dovich approximation:

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• linear perturbation theory in comoving coordinates

$$\underbrace{4\pi G a^2 \overline{\rho}}D\vec{S}(\vec{q}) = -\nabla\Phi$$

$$\Delta\Phi = \overbrace{4\pi G a^2 \overline{\rho}}\delta$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

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- linear perturbation theory in comoving coordinates

$$D\vec{S}(\vec{q}) = -\nabla\Psi$$

$$\Delta\Psi = \delta$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$$\left.\begin{aligned} \dot{\vec{x}} &= \dot{D}\vec{S}(\vec{q}) \\ \dot{\vec{x}} &= \frac{1}{a}\vec{u} \end{aligned}\right\} \qquad \begin{aligned} \vec{u} &= a\dot{D}\vec{S}(\vec{q}) \\ \frac{\partial \vec{u}}{\partial t} &= \dot{a}\dot{D}\vec{S}(\vec{q}) + a\ddot{D}\vec{S}(\vec{q}) \end{aligned}$$

- linear perturbation theory in comoving coordinates

$$\vec{S}(\vec{q}) = -\nabla\Psi$$

$$\Delta\Psi = \frac{\delta}{D}$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$$\left.\begin{aligned} \dot{\vec{x}} &= \dot{D}\vec{S}(\vec{q}) \\ \dot{\vec{x}} &= \frac{1}{a}\vec{u} \end{aligned}\right\} \qquad \begin{aligned} \vec{u} &= a\dot{D}\vec{S}(\vec{q}) \\ \frac{\partial \vec{u}}{\partial t} &= \dot{a}\dot{D}\vec{S}(\vec{q}) + a\ddot{D}\vec{S}(\vec{q}) \end{aligned}$$

- linear perturbation theory in comoving coordinates

$$\vec{S}(\vec{q}) = -\nabla\Psi$$

$$\Delta\Psi = \delta_0$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

  - linear perturbation theory in comoving coordinates

$\vec{q} =$ Lagrangian position (i.e. the grid)

$$D(t) = \frac{5}{2}\Omega_0 \frac{\dot{a}}{a} \int_{t_0}^{t} \frac{1}{\dot{a}^2} dt'$$

$$\vec{S}(\vec{q}) = -\nabla\Psi$$

$$\Delta\Psi = \delta_0$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

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$$\vec{S}(\vec{q}) = -\nabla\Psi$$

$$\Delta\Psi = \delta_0$$

→ relate δ_0 (and hence S) to $P_0(k)$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$$\vec{S}(\vec{q}) = -\nabla\Psi(\vec{q})$$

$\Delta\Psi = \delta_0$ $\Rightarrow$ potential theory tells us:

(proof follows later!)

$$\Delta\Psi = \rho$$

$$\Delta\mathcal{G} = \delta_{Dirac}$$

$$\Rightarrow \hat{\Psi} = \hat{\rho}\,\hat{\mathcal{G}} \quad \text{with } \hat{\mathcal{G}} = \frac{1}{k^2}$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

$$\vec{S}(\vec{q}) = -\nabla\Psi(\vec{q})$$

$\Delta\Psi = \delta_0$ $\Rightarrow$ potential theory tells us:

$$\Rightarrow \hat{\Psi} = \hat{\delta}_0\,(k)\frac{1}{k^2}$$

- Zel'dovich approximation:

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

$$\vec{S}(\vec{q}) = -\nabla\Psi(\vec{q})$$

$\Delta\Psi = \delta_0$ $\Rightarrow$ potential theory tells us:

$$\Rightarrow \hat{\Psi} = \hat{\delta}_0\,(k)\frac{1}{k^2}$$

$$P_0(k) = \left\langle \left|\hat{\delta}_0(\vec{k})\right|^2 \right\rangle_{|\vec{k}|=k}$$

$$\hat{\delta}_0\,(k) = \sqrt{P_0(k)}R_{\vec{k}}\,e^{i\varphi_{\vec{k}}}$$

$$R_{\vec{k}}e^{i\varphi_{\vec{k}}} = R_1 + iR_2$$

R_1, R_2 = Gaussian random numbers with mean zero and dispersion unity

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

- in practice...

$\vec{q}$ = regular grid, i.e. $q_{k,l,m}$

$$D = \frac{5}{2}\Omega_0 \frac{\dot{a}}{a}\int_{t_0}^{t} \frac{1}{\dot{a}^2}\,dt'$$

$$\vec{S}(\vec{q}) = -\nabla\Psi(\vec{q})$$

$$\Psi(\vec{q}) = FFT^{-1}(\hat{\Psi}(\vec{k}))$$

$$\hat{\Psi} = \hat{\delta}_0\,(k)\frac{1}{k^2}$$

$$\hat{\delta}_0\,(k) = \sqrt{P_0(k)}R_{\vec{k}}\,e^{i\varphi_{\vec{k}}}$$

$$R_{\vec{k}}e^{i\varphi_{\vec{k}}} = R_1 + iR_2$$

R_1, R_2 = Gauss(0,1)

***D*(*t*): determines the initial redshift of the simulation**
***S*(*q*): determines the direction of displacement**

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

  - in practice... convenient re-shuffling of terms

$\vec{q}$ = regular grid, i.e. $q_{k,l,m}$

$$D = \frac{5}{2}\Omega_0 \frac{\dot{a}}{a}\int_{t_0}^{t} \frac{1}{\dot{a}^2}\,dt'$$

$$\vec{S}(\vec{q}) = -\nabla\Psi(\vec{q})$$

$$P(k) = D^2(t)P_0(k)$$

$$\Psi(\vec{q}) = FFT^{-1}(\hat{\Psi}(\vec{k}))$$

$$\hat{\Psi} = \hat{\delta}_0\,(k)\frac{1}{k^2}$$

$$\hat{\delta}_0\,(k) = \sqrt{P_0(k)}R_{\vec{k}}\,e^{i\varphi_{\vec{k}}}$$

$$R_{\vec{k}}e^{i\varphi_{\vec{k}}} = R_1 + iR_2$$

$$R_1, R_2 = \text{Gauss}(0,1)$$

- Zel'dovich approximation:

$$\boxed{\vec{x}(t) = \vec{q} + \vec{s}(\vec{q})}$$

• in practice…convenient re-shuffling of terms

$\vec{q}$ = regular grid, i.e. $q_{k,l,m}$

$$\vec{s}(\vec{q}) = -\nabla\psi(\vec{q})$$

$$\psi(\vec{q}) = FFT^{-1}(\hat{\psi}(\vec{k}))$$

$$\hat{\psi} = \hat{\delta}\,(k)\frac{1}{k^2}$$

$$\hat{\delta}(k) = \sqrt{P(k)}R_{\vec{k}}\,e^{i\varphi_{\vec{k}}}$$

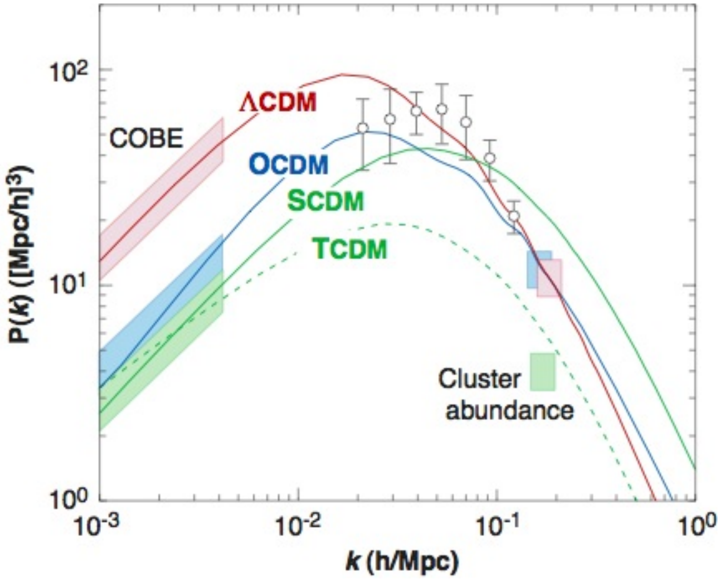
$$R_{\vec{k}}e^{i\varphi_{\vec{k}}} = R_1 + iR_2$$

$P(k)$ = power spectrum at initial redshift of simulation

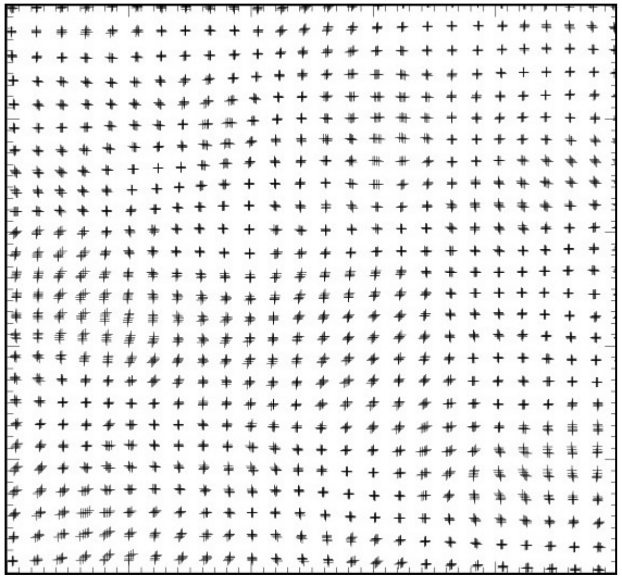
$$R_1, R_2 = \text{Gauss}(0,1)$$

- Zel'dovich approximation:

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

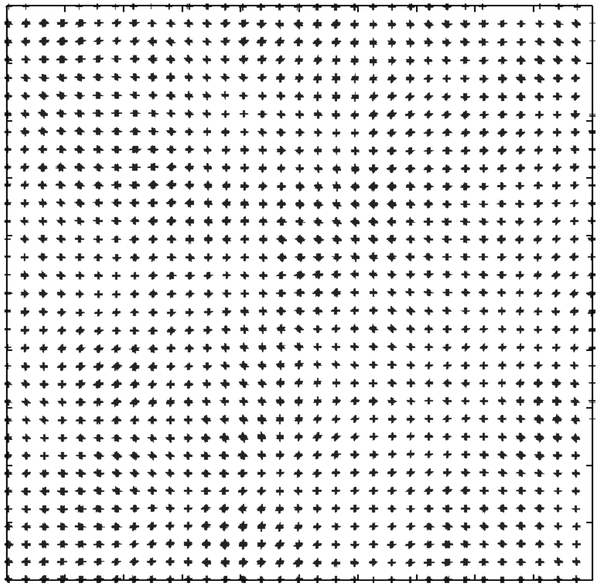


→



- Zel'dovich approximation:

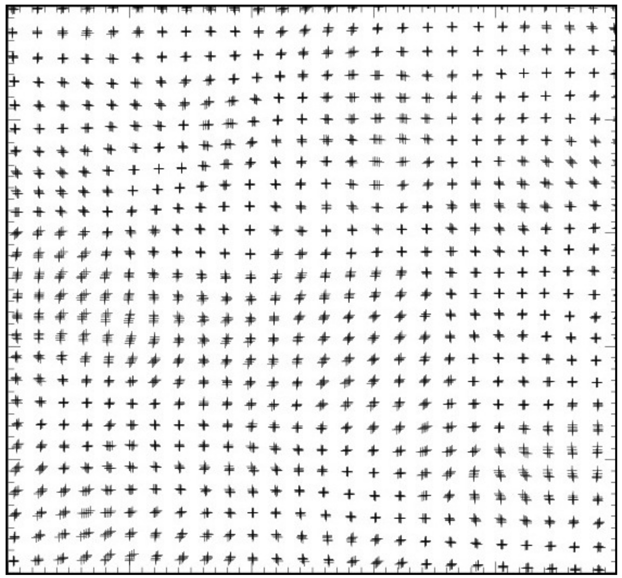
$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$



$$\hat{\delta}(k) = \sqrt{P(k)}R_{\vec{k}}\, e^{i\varphi_{\vec{k}}}$$

$$R_{\vec{k}}e^{i\varphi_{\vec{k}}} = R_1 + iR_2$$

$$R_1, R_2 = \text{Gauss}(0,1)$$



- Zel'dovich approximation:
  - positions

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$

  - velocities

$$\boxed{\dot{\vec{x}}(t) = \dot{D}(t)\vec{S}(\vec{q})}$$

- 2nd order Lagrangian perturbation theory

$$\vec{x}(t) = \vec{q} + D(a)S(\vec{q}) - D^{(2)}S^{(2)}(\vec{q})$$

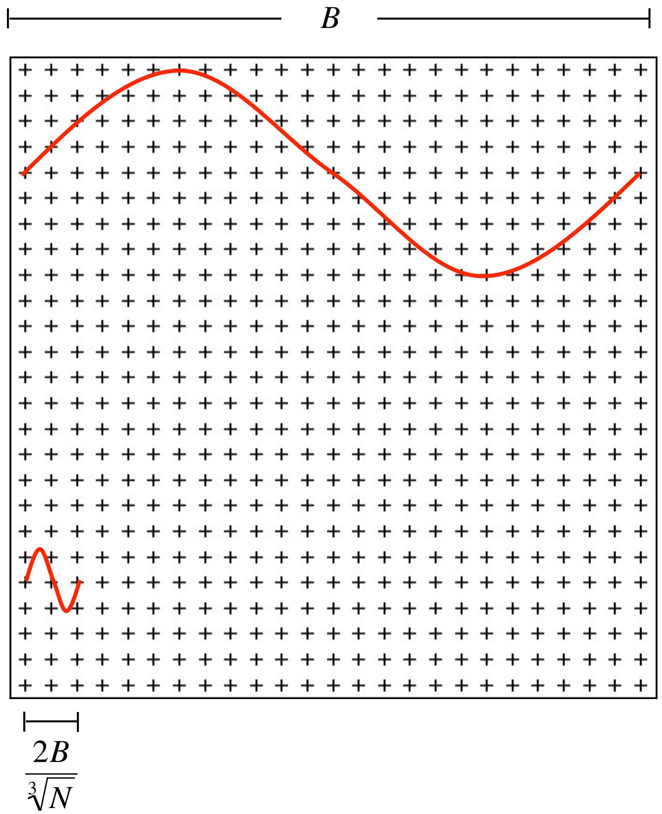
- cosmological principle
- perturbations
- **limitations**
- alternatives
- remarks
- summary

- generating IC's in practice

  - choose cosmological model — ΛCDM?!
  - choose box size — B
  - choose number of particles — N
  - choose starting redshift — z_i

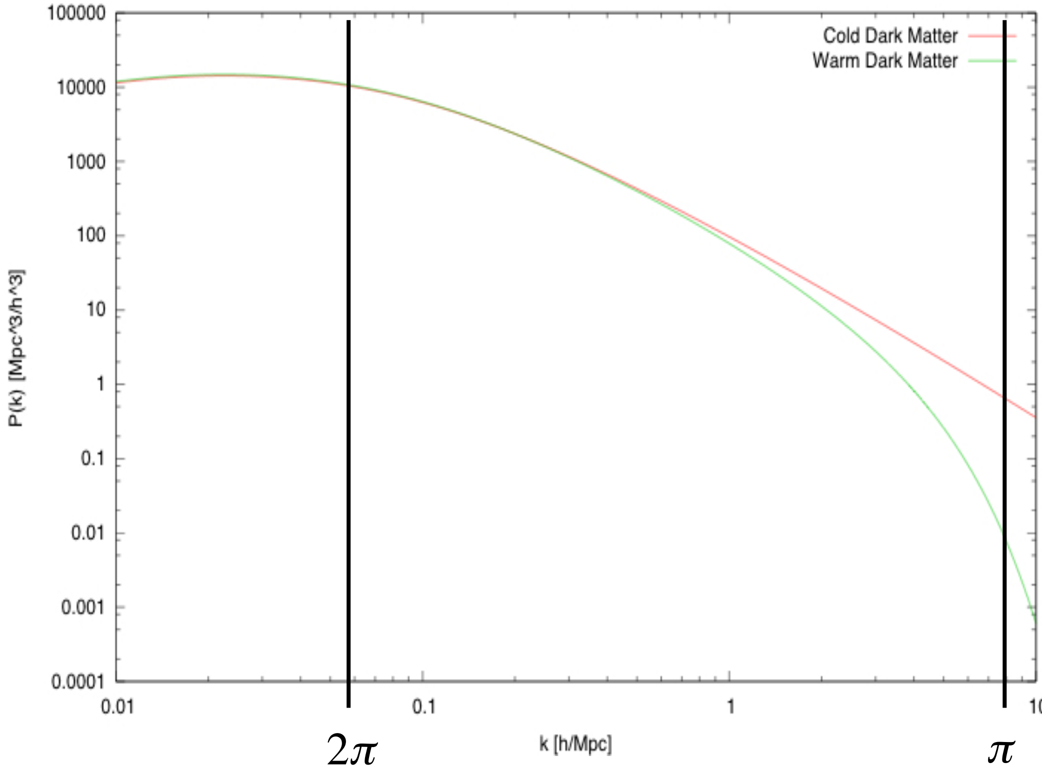
these choices are not free but interwoven...

- generating IC's in practice
  - wavenumber limitation



ΛCDM vs. ΛWDM
B=100 h^{-1}Mpc
N=256^3

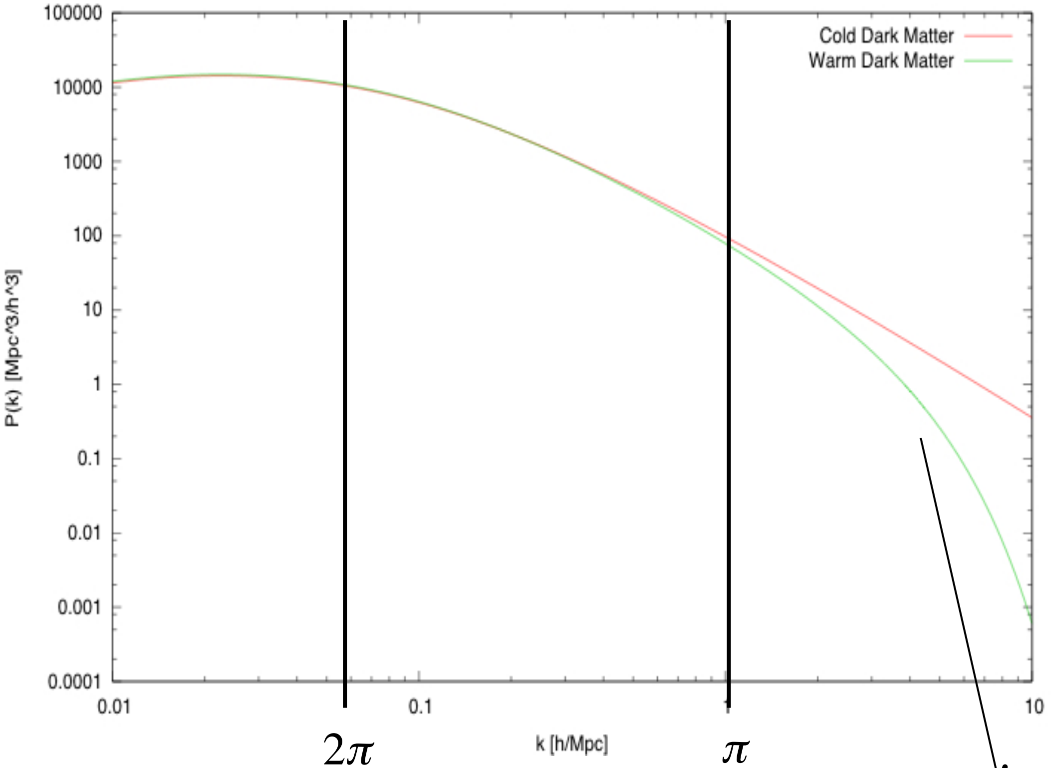
- generating IC's in practice
  - wavenumber limitation



$$\frac{2\pi}{B} \qquad \frac{\pi}{B/\sqrt[3]{N}}$$

ΛCDM vs. ΛWDM
B=100 h^{-1}Mpc
N=32^3

- generating IC's in practice
  - wavenumber limitation

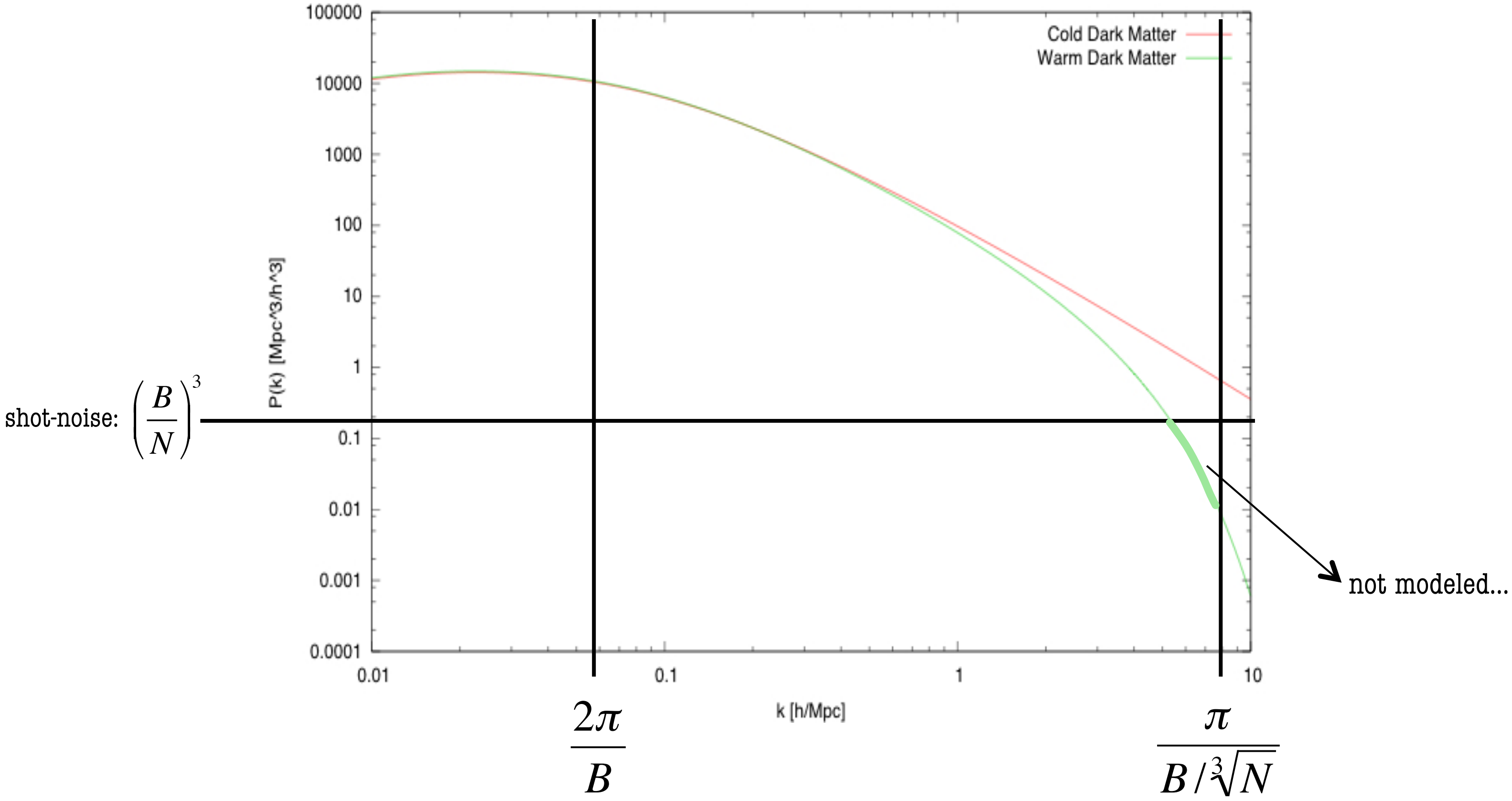


$$\frac{2\pi}{B} \qquad \frac{\pi}{B/\sqrt[3]{N}}$$

interesting features not modeled...

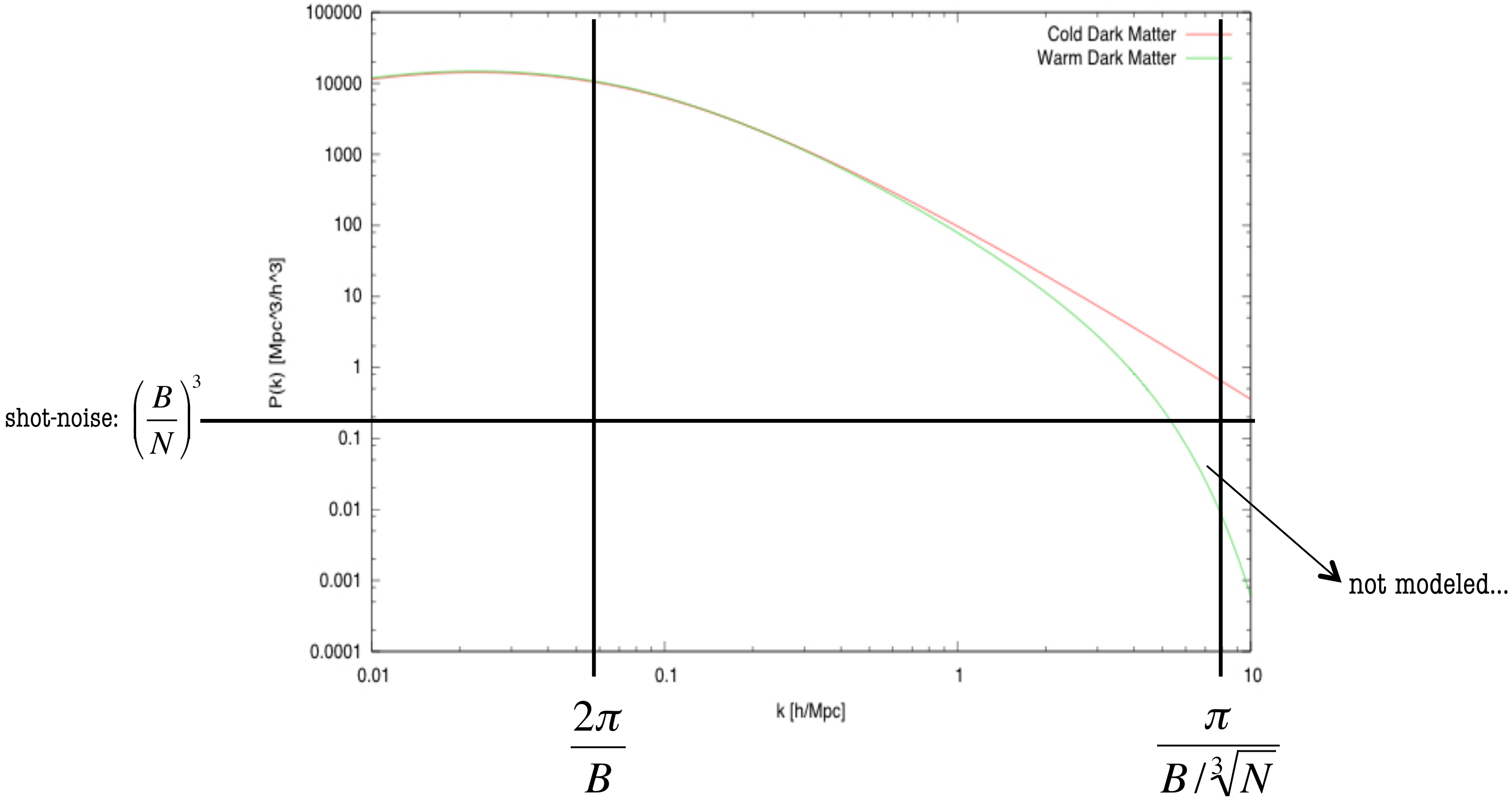
ΛCDM vs. ΛWDM
B=100 h^{-1}Mpc
N=256^3

- generating IC's in practice
  - wavenumber limitation
  - amplitude limitation due to shot-noise



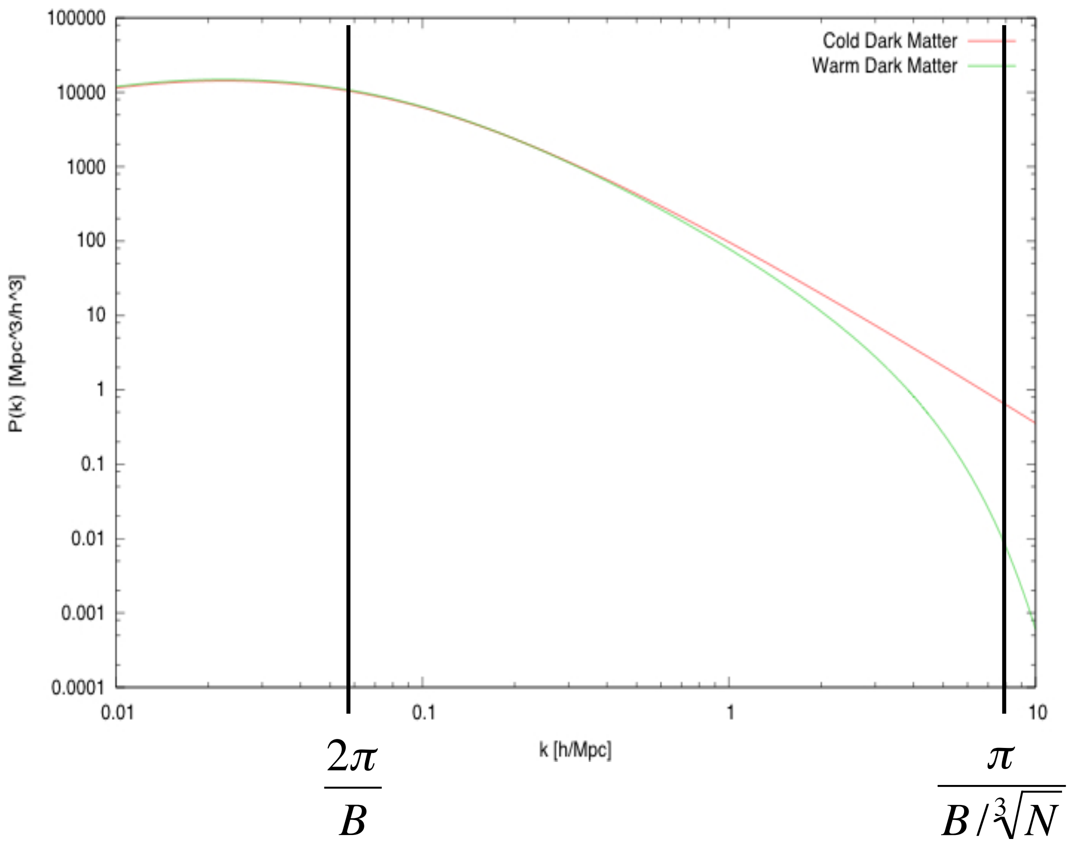
ΛCDM vs. ΛWDM
B=100 h^{-1}Mpc ?
N=256^3

- generating IC's in practice
  - wavenumber limitation
  - amplitude limitation due to shot-noise



ΛCDM vs. ΛWDM
B=100 h^{-1}Mpc ?
N=256^3

- generating IC's in practice
  - wavenumber limitation



ΛCDM vs. ΛWDM
B=100 h^{-1}Mpc ?
N=32^3

- why h^{-1}Mpc ?

$$\rho = \frac{N m_{simu}}{B^3} = \Omega_0 \rho_{crit,0} = \Omega_0 \frac{3H_0^2}{8\pi G} = \Omega_0 \frac{3\cdot(100h^2)}{8\pi G}$$

$$\Rightarrow \quad m_{simu} = \Omega_0 \frac{300h^2}{8\pi G}\frac{B^3}{N}$$

$$\Rightarrow \quad h m_{simu} = \Omega_0 \frac{300}{8\pi G}\frac{(hB)^3}{N}$$

$$\Rightarrow \quad \tilde{m}_{simu} = \Omega_0 \frac{300}{8\pi G}\frac{\tilde{B}^3}{N}$$

distances and masses have to be divided by h to get physical values...

- generating IC's in practice
  - linearity constraint on box size

$$\delta(\vec{x},t) = \frac{\rho(\vec{x},t) - \overline{\rho}(t)}{\overline{\rho}(t)} \quad \propto \quad D(t) \qquad \Rightarrow \qquad \boxed{P(k,t) \propto D^2(t)}$$

– linear perturbation theory (again...)

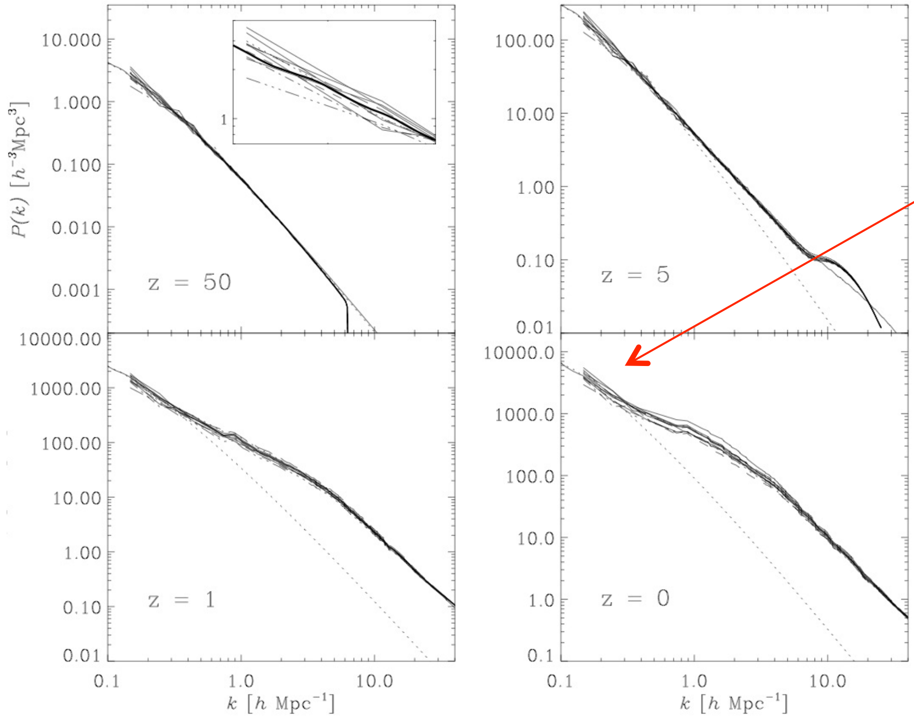
$$D(t) = \frac{5}{2}\Omega_0 \frac{\dot{a}}{a} \int_{t_0}^{t} \frac{1}{\dot{a}^2} dt'$$

$$D(a) \approx \frac{5a}{2}\Omega_m \left[\Omega_m^{4/7} - \Omega_\Lambda + \left(1 + \frac{\Omega_m}{2}\right)\left(1 + \frac{\Omega_\Lambda}{70}\right) \right]^{-1}$$

($D(a) = a$ for SCDM)

- generating IC's in practice
  - linearity constraint on box size

$$\delta(\vec{x},t) = \frac{\rho(\vec{x},t) - \overline{\rho}(t)}{\overline{\rho}(t)} \quad \propto \quad D(t) \qquad \Rightarrow \qquad \boxed{P(k,t) \propto D^2(t)}$$



the largest mode
has to stay
"linear"

- generating IC's in practice

  - linearity constraint on box size

$$\delta(\vec{x},t) = \frac{\rho(\vec{x},t) - \overline{\rho}(t)}{\overline{\rho}(t)} \quad \propto \quad D(t) \qquad \Rightarrow \qquad \boxed{P(k,t) \propto D^2(t)}$$

  - mass variance

$$\langle \delta(\vec{x}) \rangle = \left\langle \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}} \right\rangle = \frac{\langle \rho(\vec{x}) \rangle - \overline{\rho}}{\overline{\rho}} = 0$$

$$\sigma_M^2 = \sum_{\vec{k}} \left\langle \left| \hat{\delta}(\vec{k}) \right|^2 \right\rangle_{|\vec{k}|=k} \quad \rightarrow \quad \frac{1}{(2\pi)^3} \iiint P(k) d^3k = \frac{1}{2\pi^2} \int_0^\infty P(k) k^2 dk$$

- generating IC's in practice
  - linearity constraint on box size

$$\delta(\vec{x},t) = \frac{\rho(\vec{x},t) - \overline{\rho}(t)}{\overline{\rho}(t)} \quad \propto \quad D(t) \qquad \Rightarrow \qquad \boxed{P(k,t) \propto D^2(t)}$$

– linearly extrapolate $P(k)$ to $z = 0$ => $P(k, z=0)$

– iteratively determine k_{nl} via $1 = \frac{1}{2\pi^2} \int_0^{k_{nl}} P(k, z=0) k^2 dk$

– set $B \geq 2\pi/k_{\mathrm{nl}}$

- generating IC's in practice
  - linearity constraint on box size

$$\delta(\vec{x},t) = \frac{\rho(\vec{x},t) - \overline{\rho}(t)}{\overline{\rho}(t)} \quad \propto \quad D(t) \qquad \Rightarrow \qquad \boxed{P(k,t) \propto D^2(t)}$$

– linearly extrapolate $P(k)$ to $z = 0$ => $P(k, z{=}0)$

note: $P(k, z{=}0)$ is tabulated in the provided files

– iteratively determine $k_{\rm nl}$ via $1 = \frac{1}{2\pi^2} \int_0^{k_{nl}} P(k, z=0) k^2 dk$

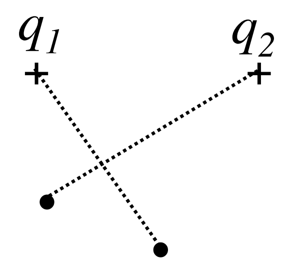
– set $B \geq 2\pi / k_{\rm nl}$

$B \geq 20\ h^{-1}$Mpc for ΛCDM

- generating IC's in practice
  - initial redshift - not too late, not too early

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$

- – avoid shell crossing



- – avoid integration of numerical noise

- generating IC's in practice
    - initial redshift - not too late, not too early

$$\sigma^2_{\text{Box}}(a_i) = \frac{1}{2\pi^2} \int_{2\pi/B}^{\pi/(B/\sqrt[3]{N})} P_i(k) k^2 dk$$

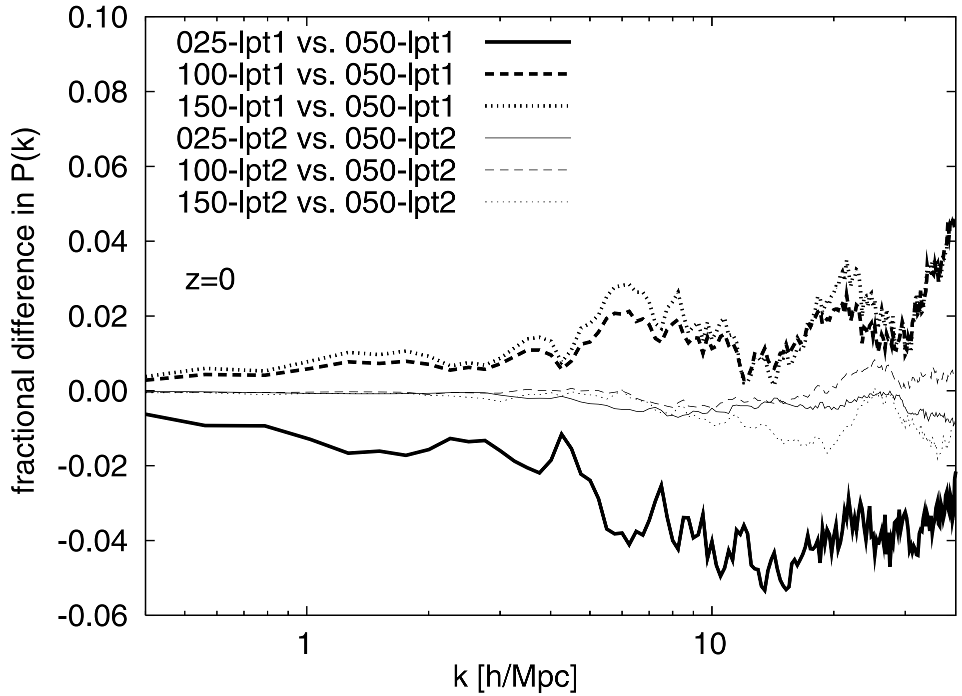
$$P_i(k) = \frac{D(a_i)}{D(a=1)} P(k, z=0)$$

$$\boxed{\sigma_{\text{Box}}(a_i) \leq 0.1 - 0.2}$$

Note: PMmodels.f returns σ_{Box} for the initial (trial) redshift –> iteratively change z_i until criterion satisfied...

- generating IC's in practice
  - initial redshift - not too late, not too early

$$\sigma^2_{\mathrm{Box}}(a_i) = \frac{1}{2\pi^2} \int_{2\pi/B}^{\pi/(B/\sqrt[3]{N})} P_i(k) k^2 dk$$

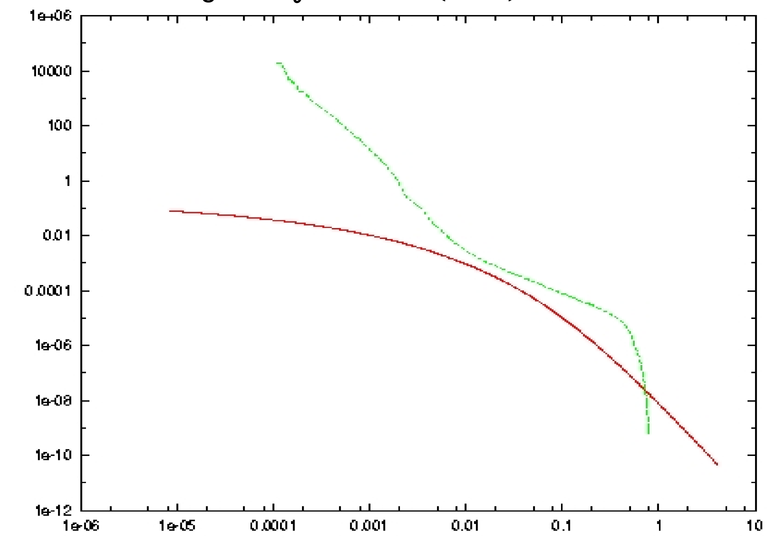


- problems with this method of generating IC's?

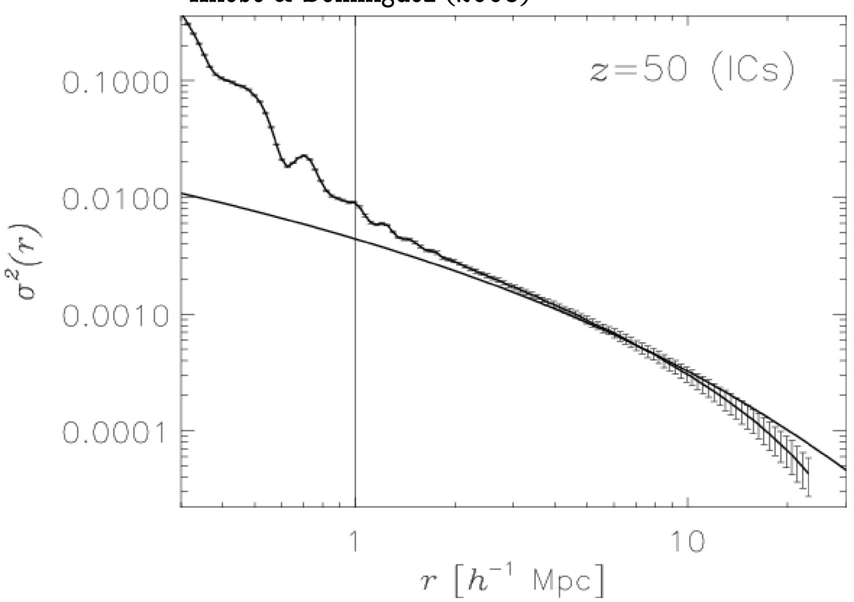
$$\sigma_M^2(r) = \frac{1}{2\pi^2}\int_0^\infty P_i(k)\hat{W}(kr)k^2dk$$

$$\hat{W}(x) = \frac{3}{x^3}(\sin x - x\cos x)$$

Baertschiger & Sylos Labini (2001)



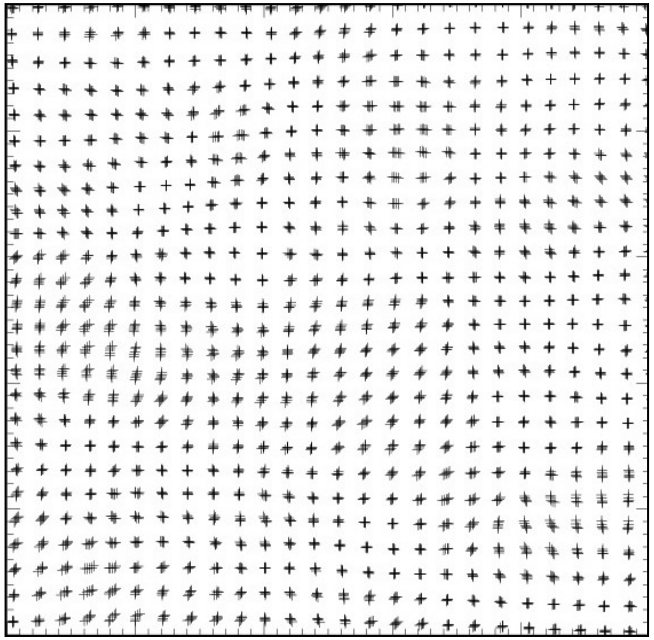
Knebe & Dominguez (2003)



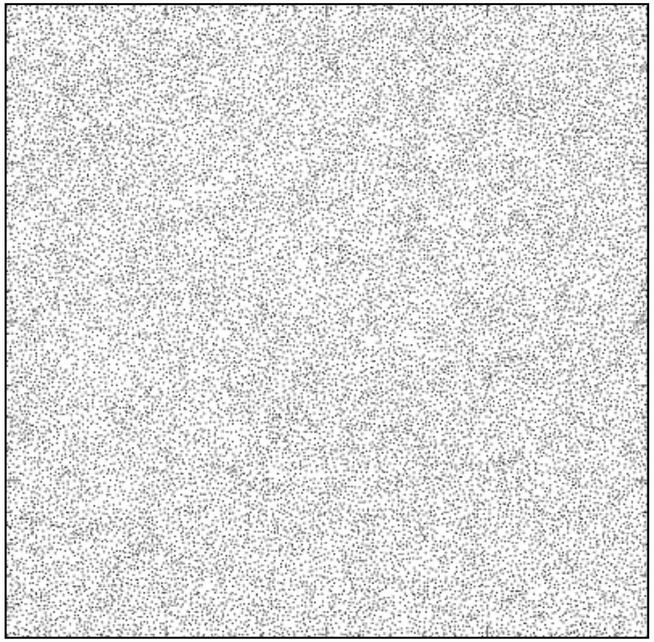
- cosmological principle
- perturbations
- limitations
- **alternatives**
- remarks
- summary

- alternative - Glass IC's

"Grid" IC's



"Glass" IC's

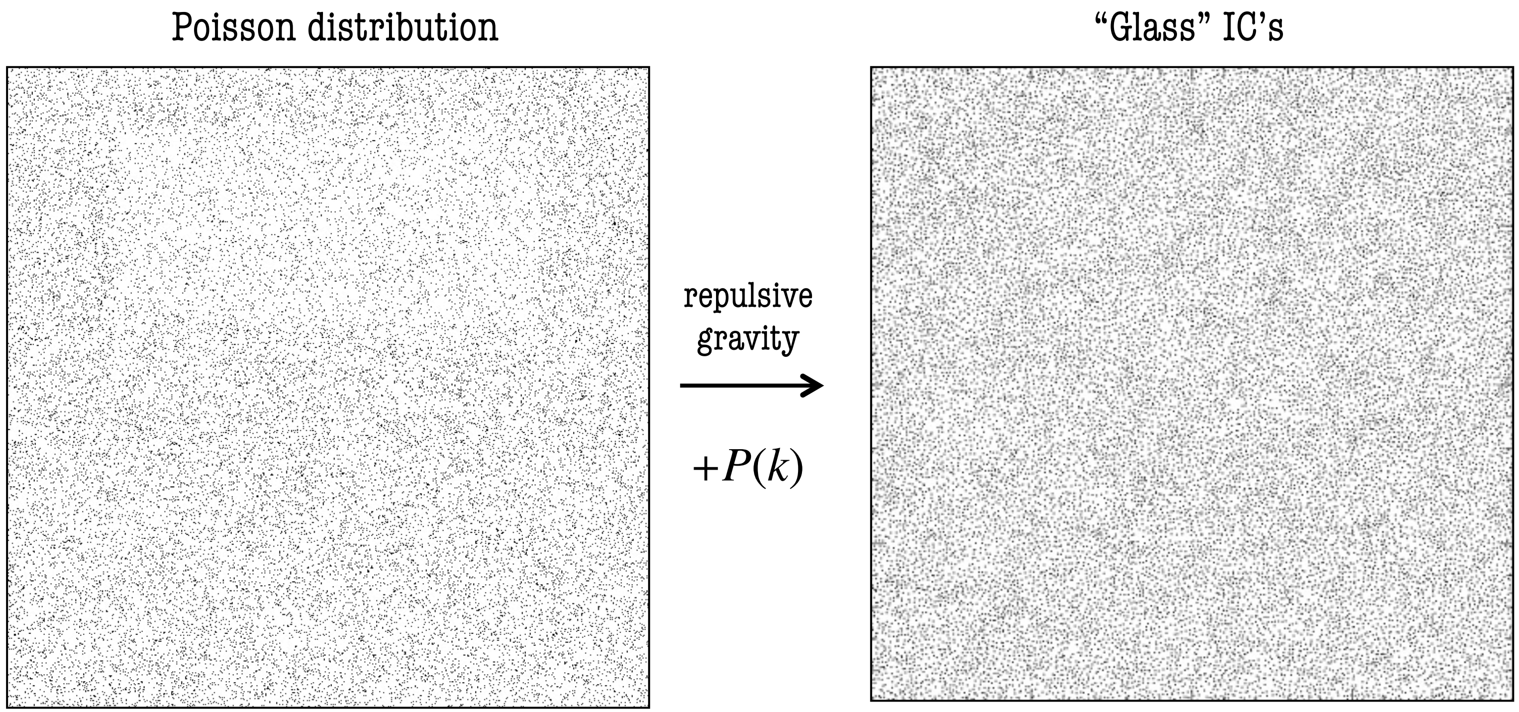


- alternative - Glass IC's
  - random positions for N particles
  - evolve them forward in time under their mutual gravity (i.e. N-body code), but:

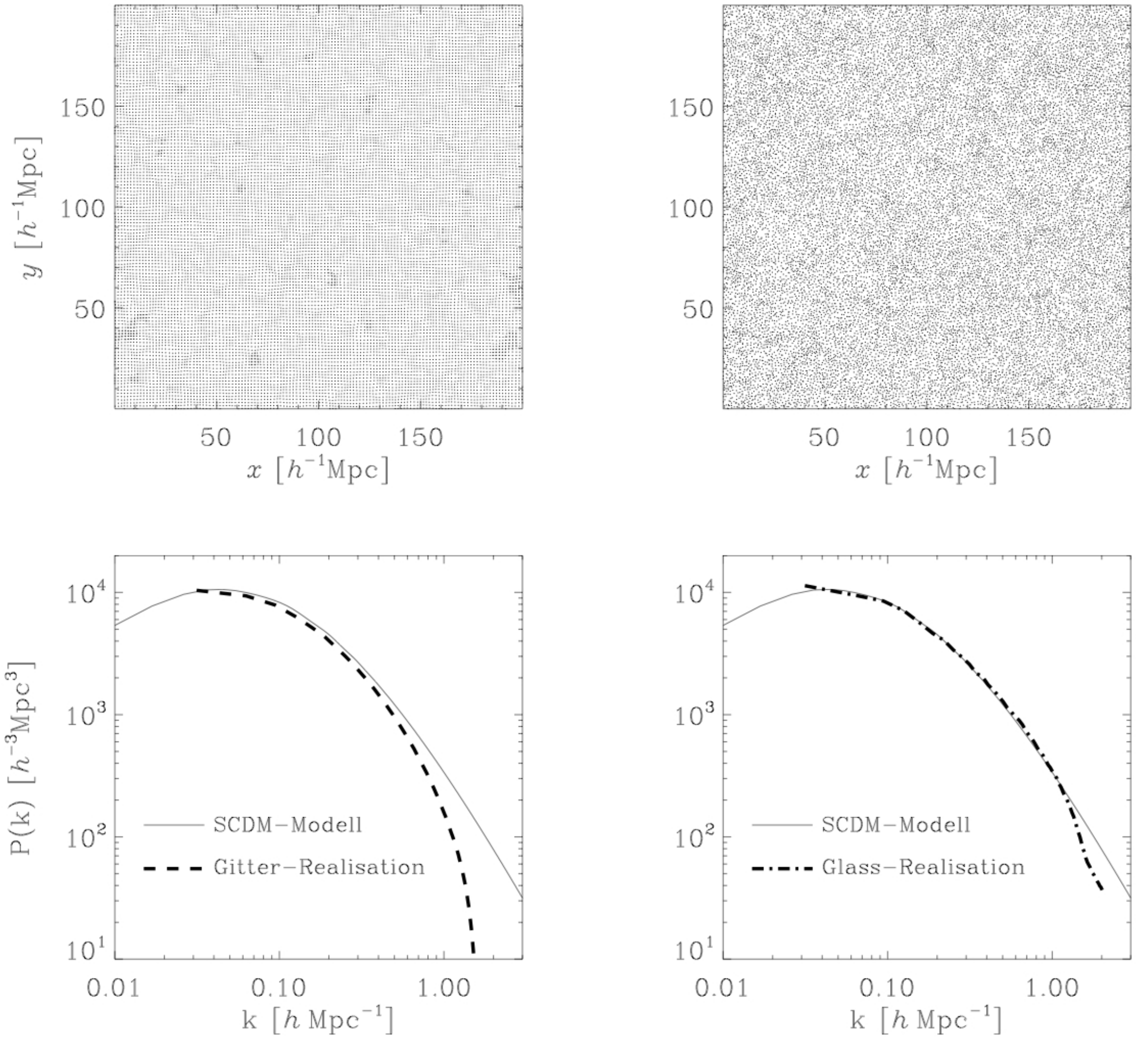
    reverse the sign of gravity!

  - use this "Glass" as Lagrangian positions q for Zel'dovich approximation

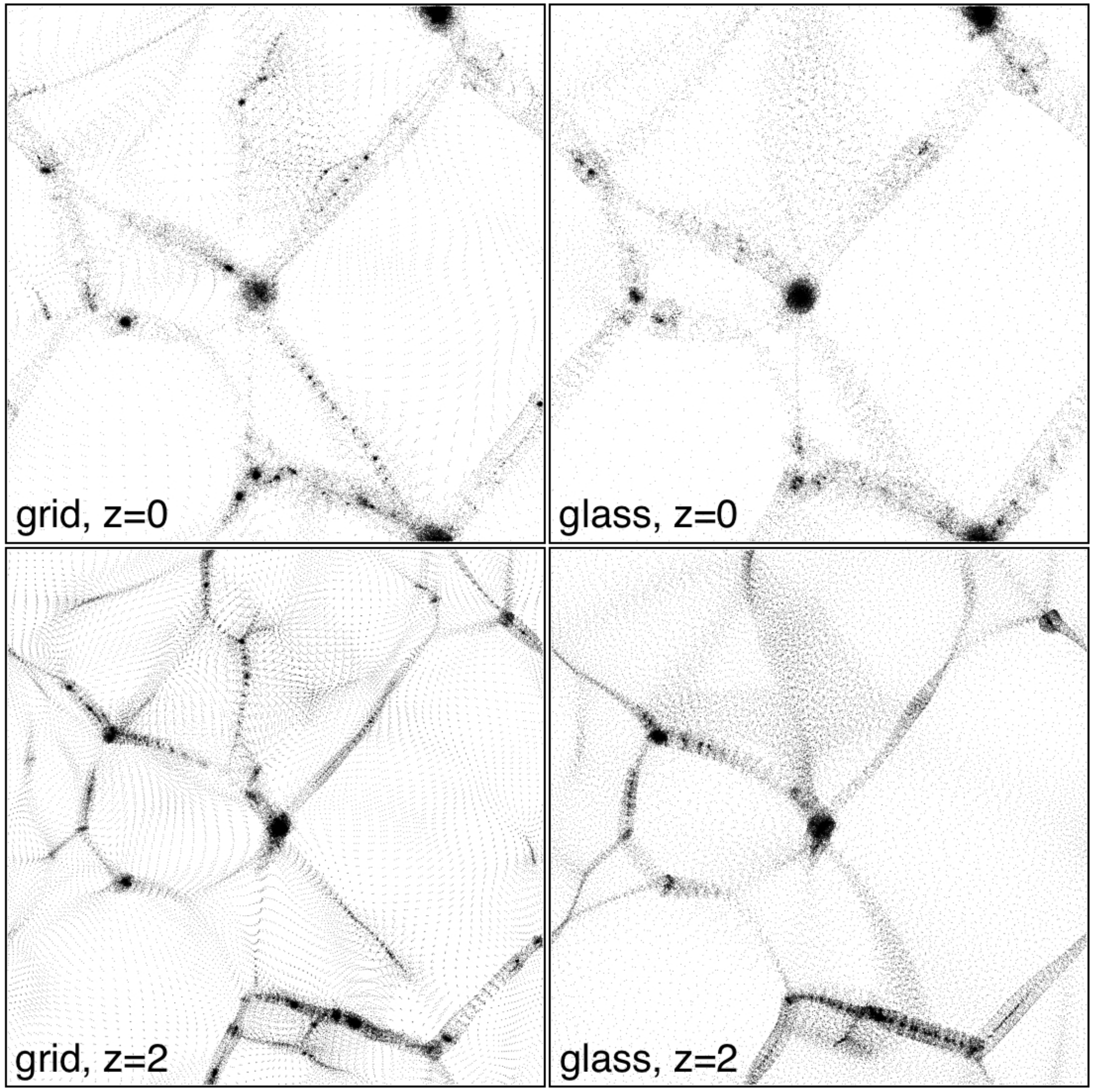
- alternative - Glass IC's



- alternative - Glass IC's

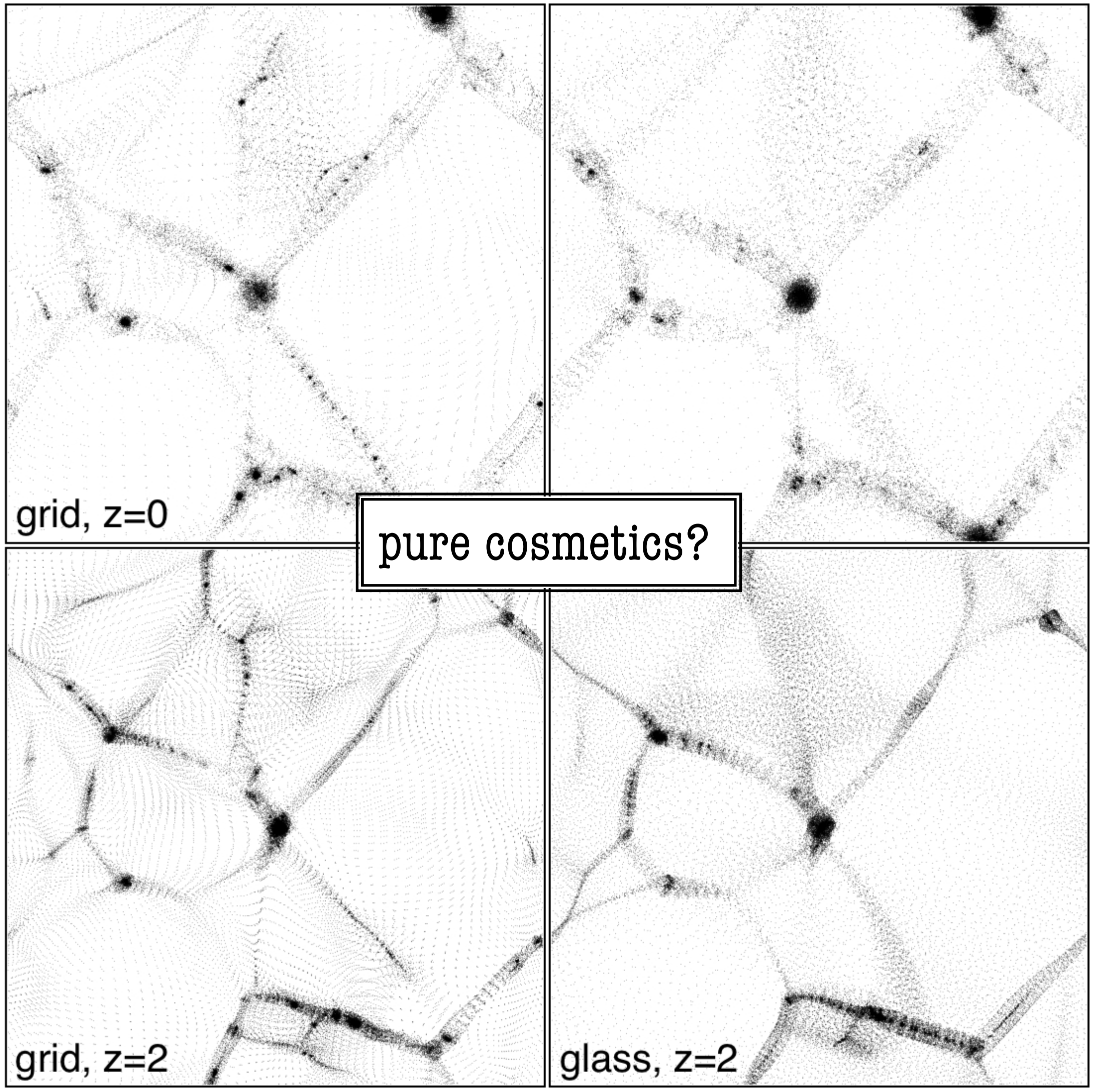


- alternative - Glass IC's



Goetz & Sommer-Larson (2003)

- alternative - Glass IC's



Goetz & Sommer-Larson (2003)

- cosmological principle
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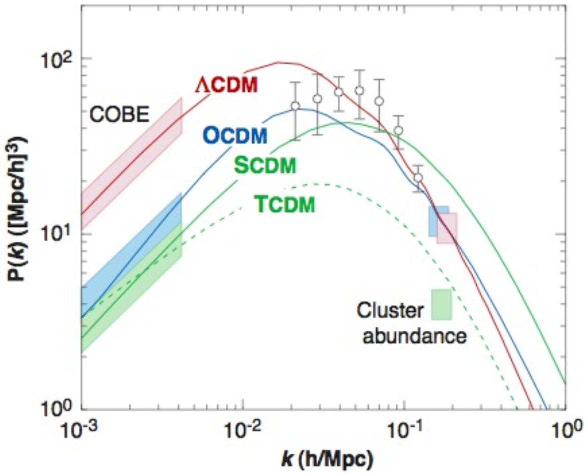
- sampling variance

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$



$R_1, R_2 \equiv$ Gaussian random numbers with mean zero, dispersion unity

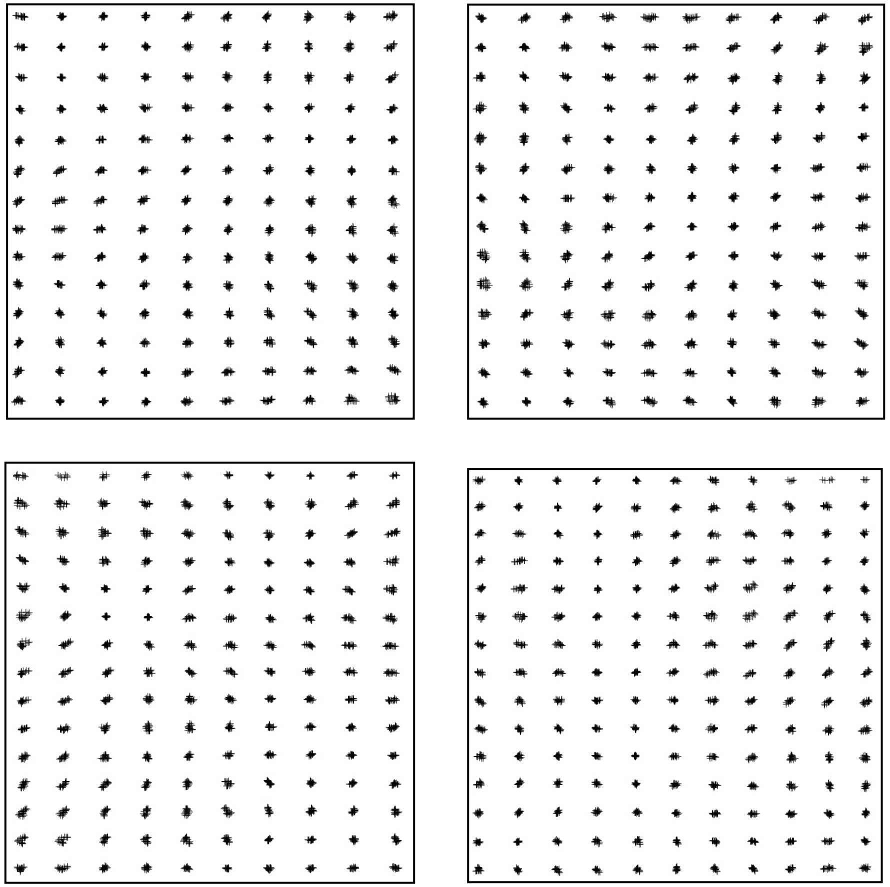
$$\hat{\delta}(k) = \sqrt{P(k)} R_{\vec{k}}\, e^{i\varphi_{\vec{k}}}$$

$$R_{\vec{k}} e^{i\varphi_{\vec{k}}} = R_1 + iR_2$$

sampling variance!

- sampling variance

$$\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})$$



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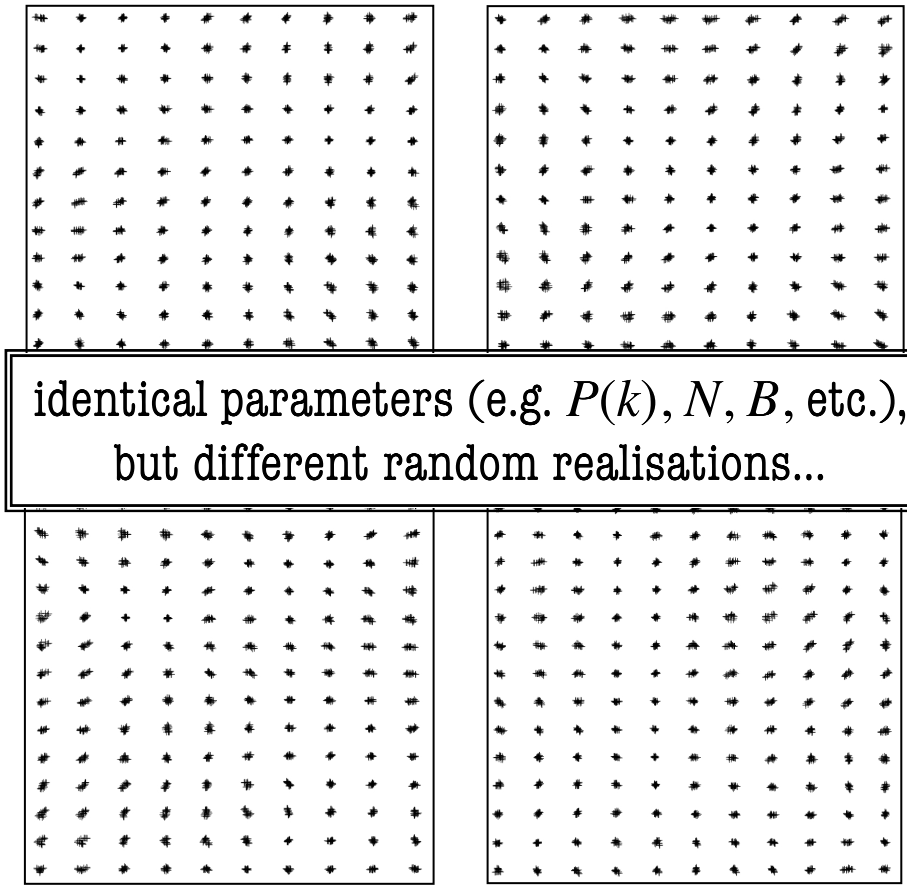


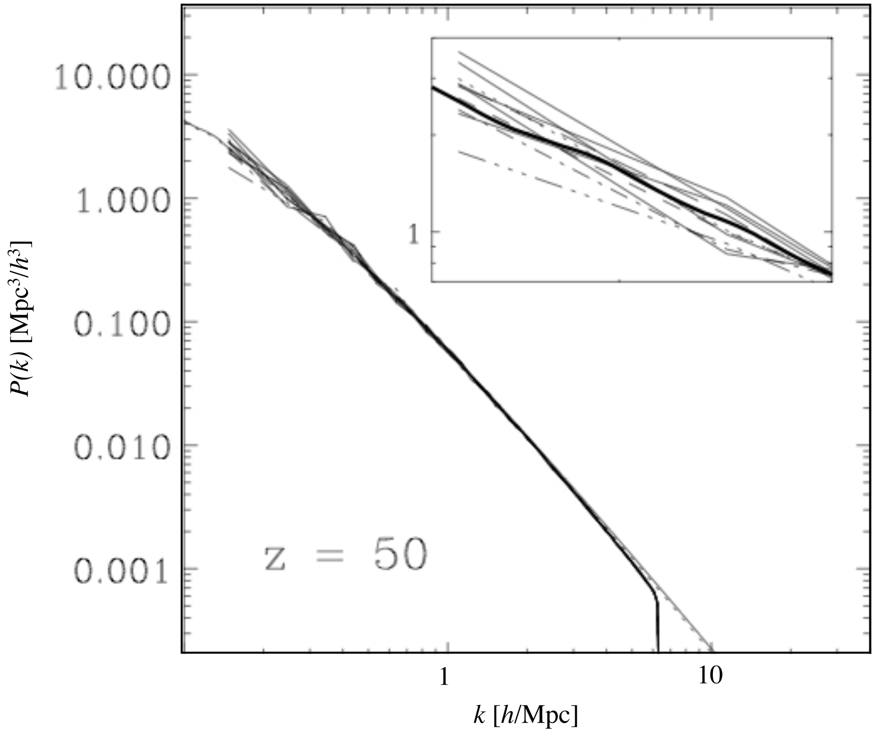
identical parameters (e.g. $P(k)$, N, B, etc.), but different random realisations...

$$\hat{\delta}(k) = \sqrt{P(k)}R_{\vec{k}}\, e^{i\varphi_{\vec{k}}}$$

$$R_{\vec{k}}e^{i\varphi_{\vec{k}}} = R_1 + iR_2$$

- sampling variance

$$\boxed{\vec{x}(t) = \vec{q} + D(t)\vec{S}(\vec{q})}$$



$$\hat{\delta}(k) = \sqrt{P(k)} R_{\vec{k}}\, e^{i\varphi_{\vec{k}}}$$

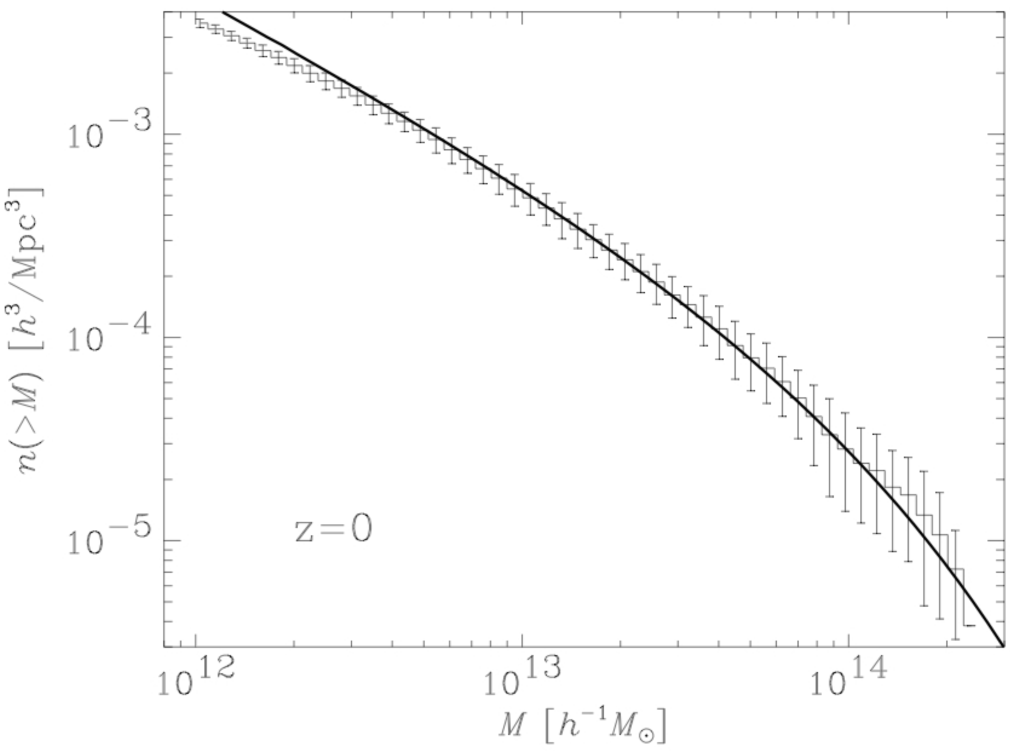
$$R_{\vec{k}} e^{i\varphi_{\vec{k}}} = R_1 + iR_2$$

- sampling variance
  - effects of the sampling variance…

generate a suite of IC's
with
different random realisations of R_1 and R_2

- sampling variance

  - effects of the sampling variance...

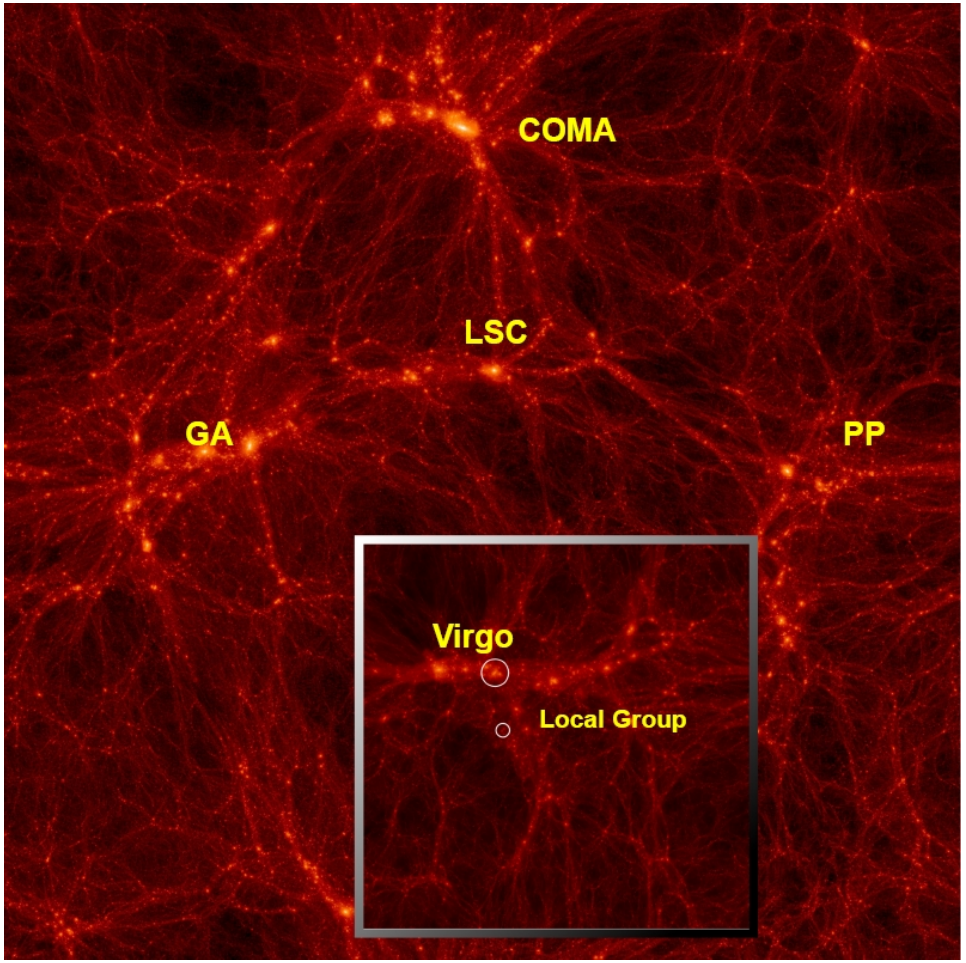


...scatter can be as large as 20% for properties of individual objects (Knebe & Dominguez 2003)

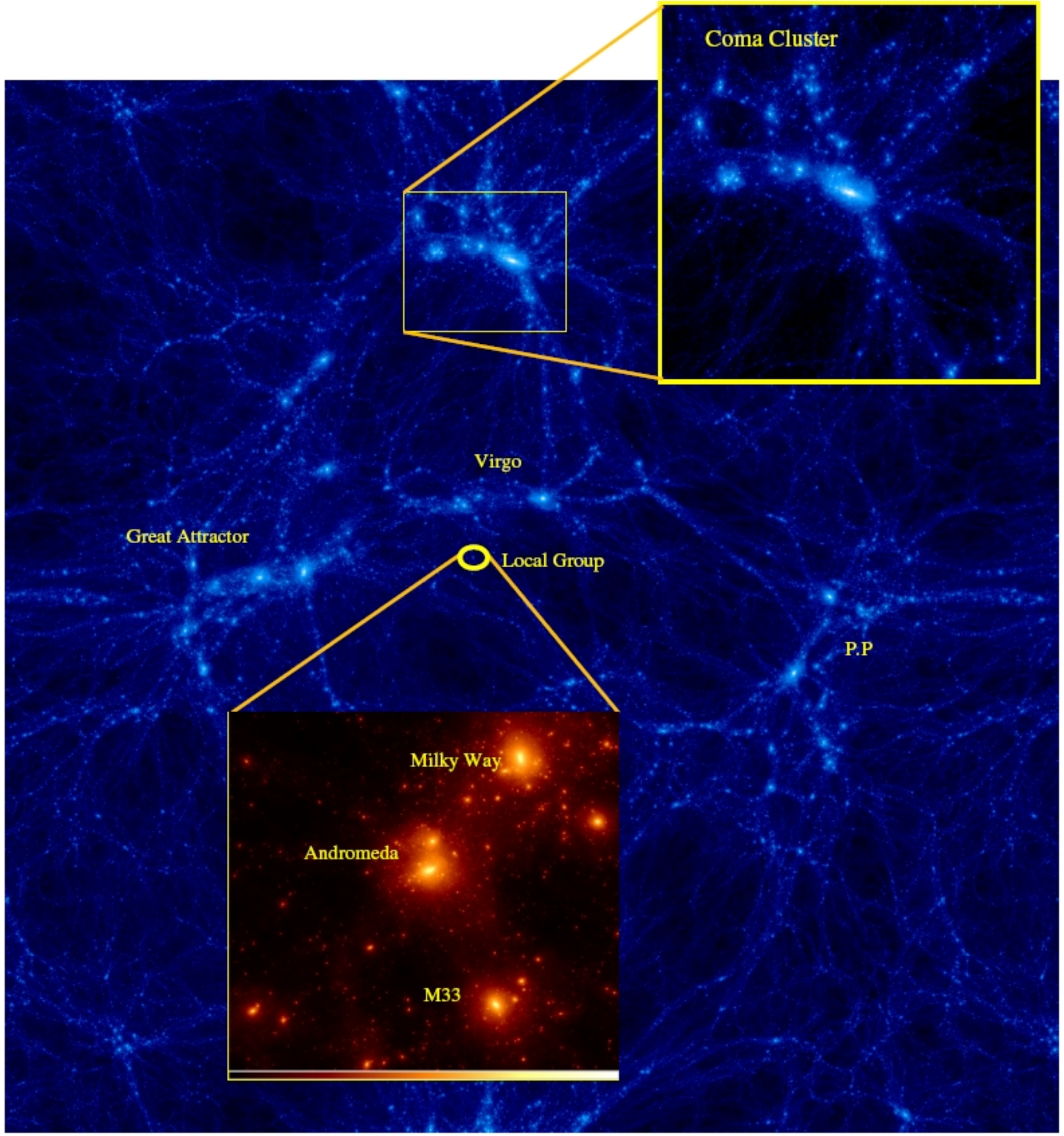
- zoom simulations

  - run a low resolution simulation
  - identify an interesting object
  - trace back particles of that object to Lagrangian positions in IC's
  - re-sample waves in that area with more particles
  - re-run the whole simulation

- zoom simulations

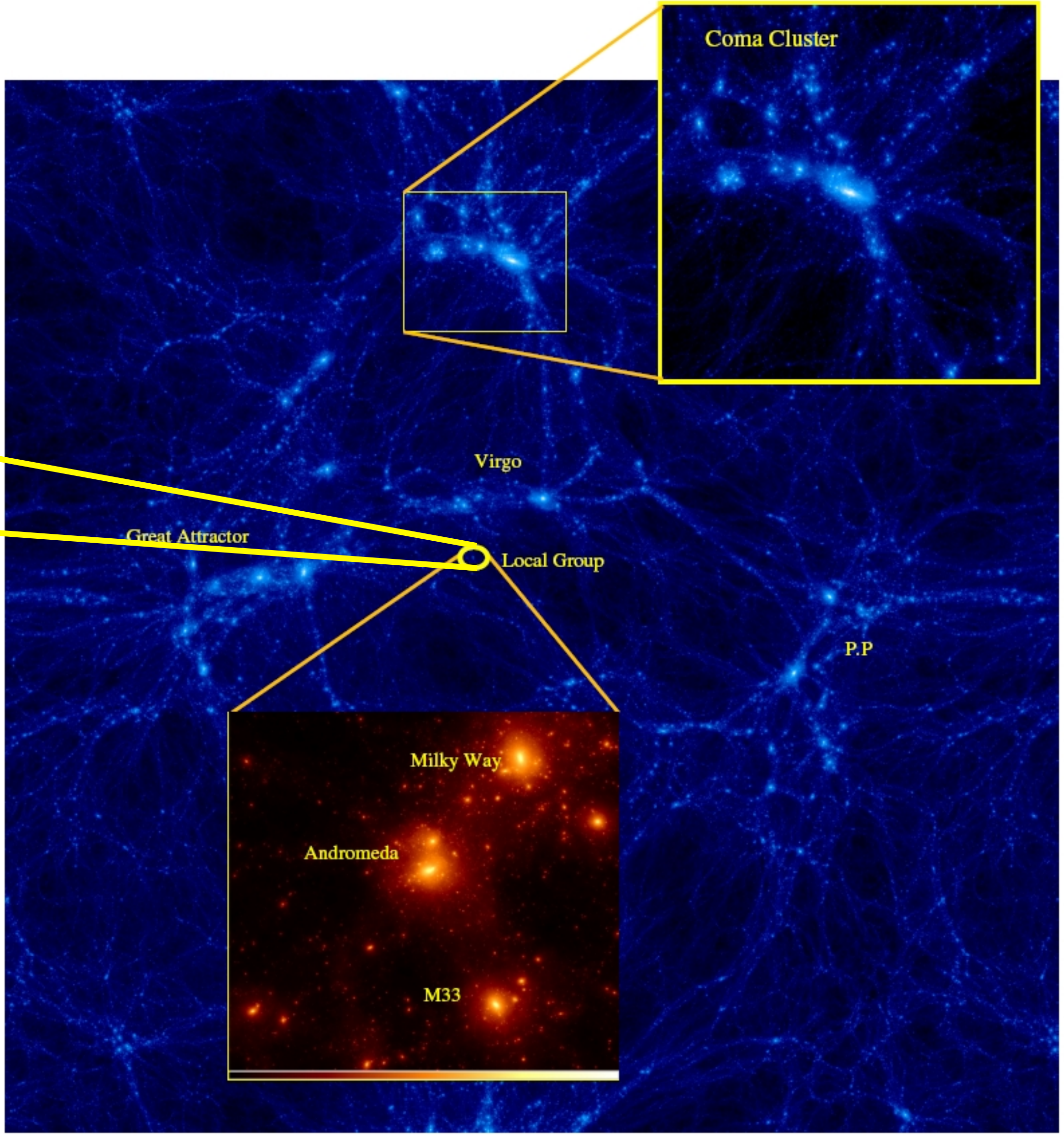


- zoom simulations

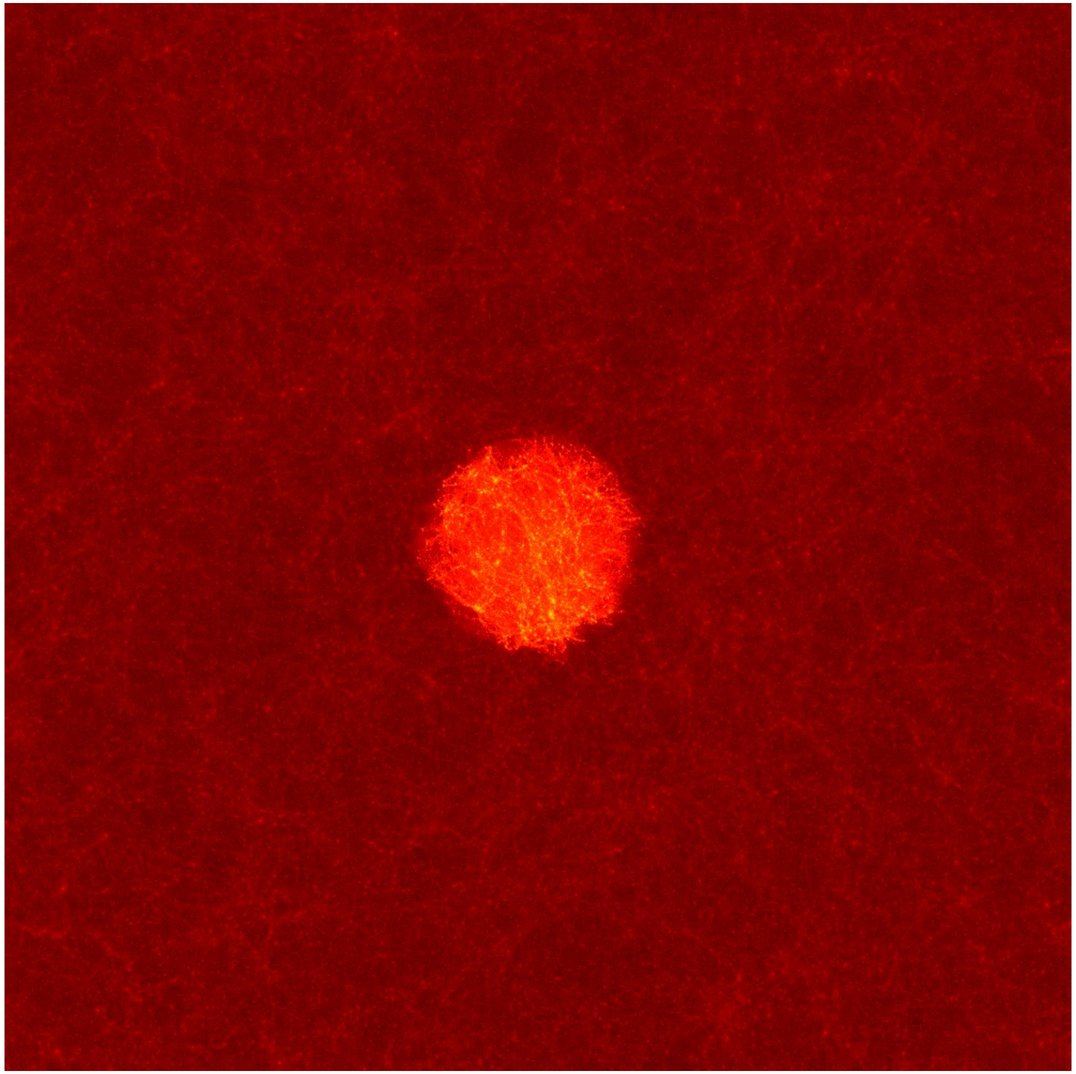


- zoom simulations

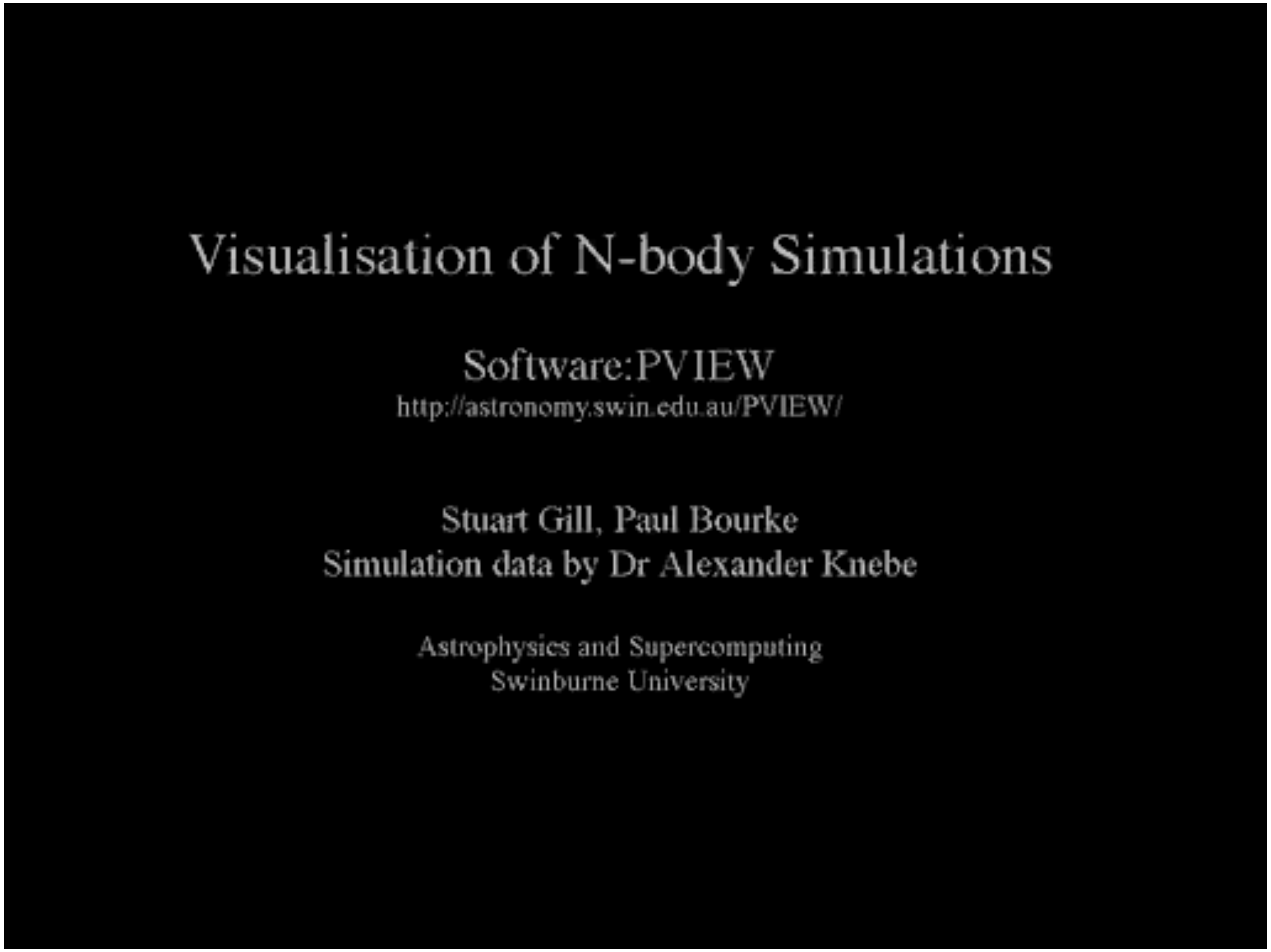
find all these particles
back to the initial conditions



- zoom simulations



- zoom simulations



- Constrained Simulations

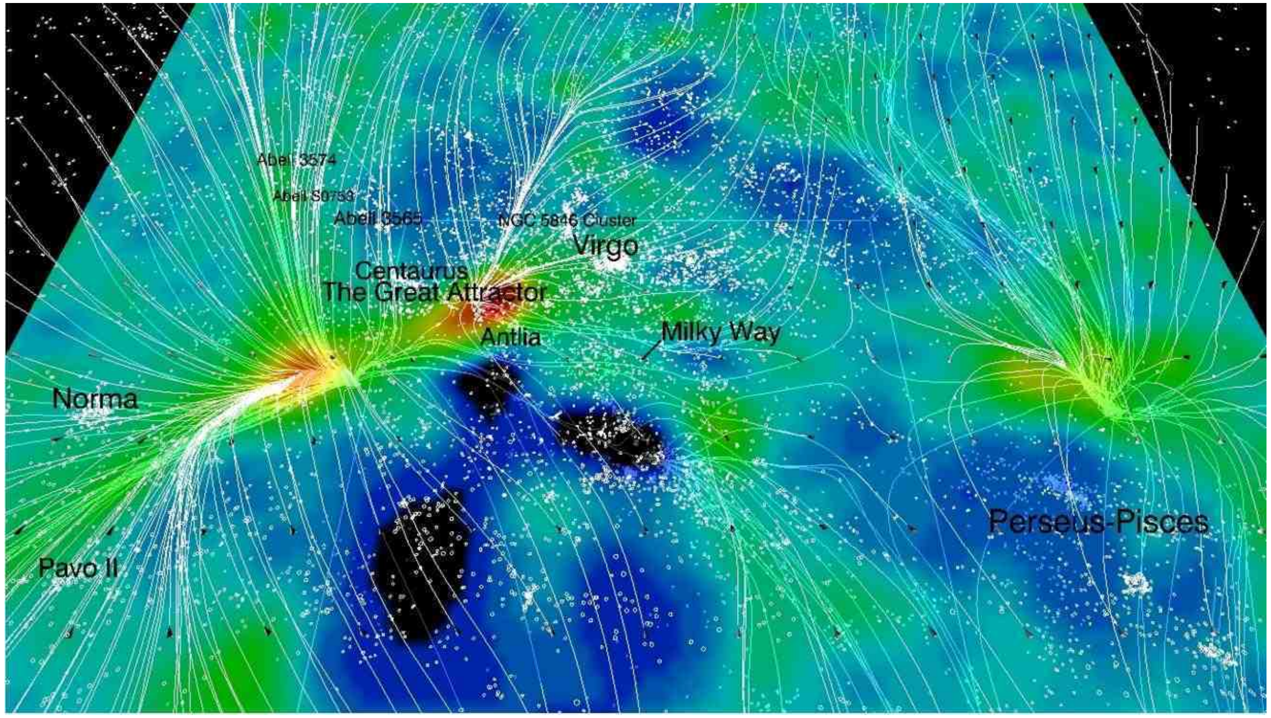
$$\dot{\vec{x}}(t) = \dot{D}(t)\vec{S}(\vec{q})$$

(Knebe et al. 2009)

- Constrained Simulations

$$\dot{\vec{x}}(t) = \dot{D}(t)\vec{S}(\vec{q})$$

observed



(Cosmic Flow 2 team, http://www.ipnl.in2p3.fr/projet/cosmicflows)

- Constrained Simulations

$$\dot{\vec{x}}(t) = \dot{D}(t)\vec{S}(\vec{q})$$

cosmology

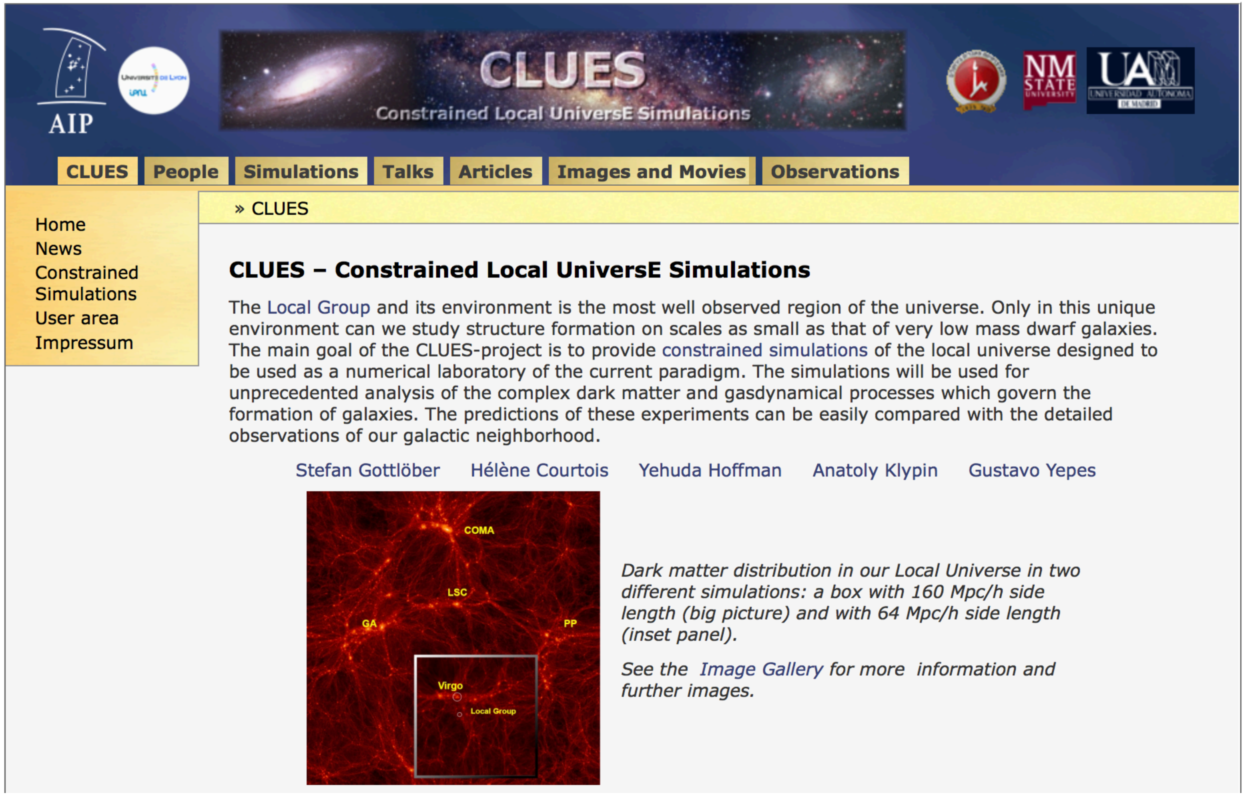
$$D = \frac{5}{2}\Omega_0 \frac{\dot{a}}{a}\int_{t_0}^{t} \frac{1}{\dot{a}^2} dt'$$

- Constrained Simulations

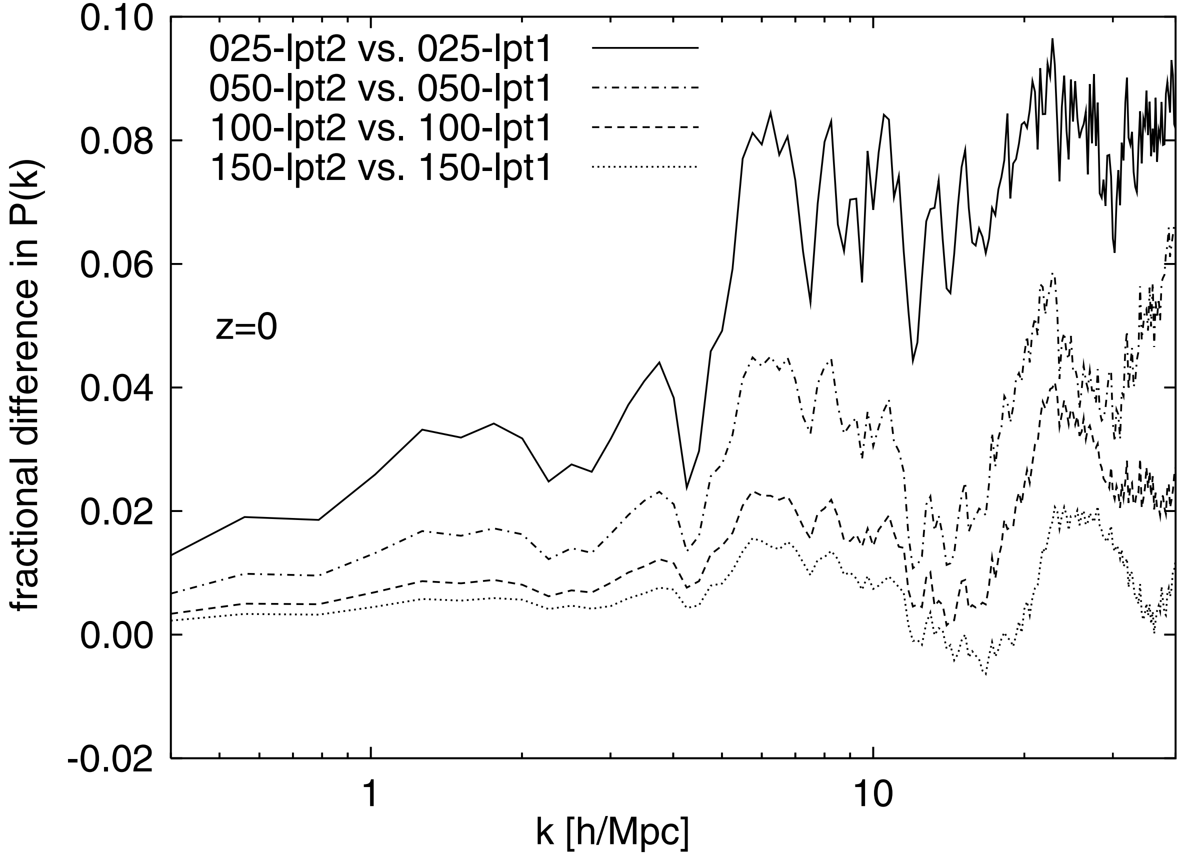
$$\dot{\vec{x}}(t) = \dot{D}(t)\vec{S}(\vec{q})$$

calculate!

http://www.clues-project.org

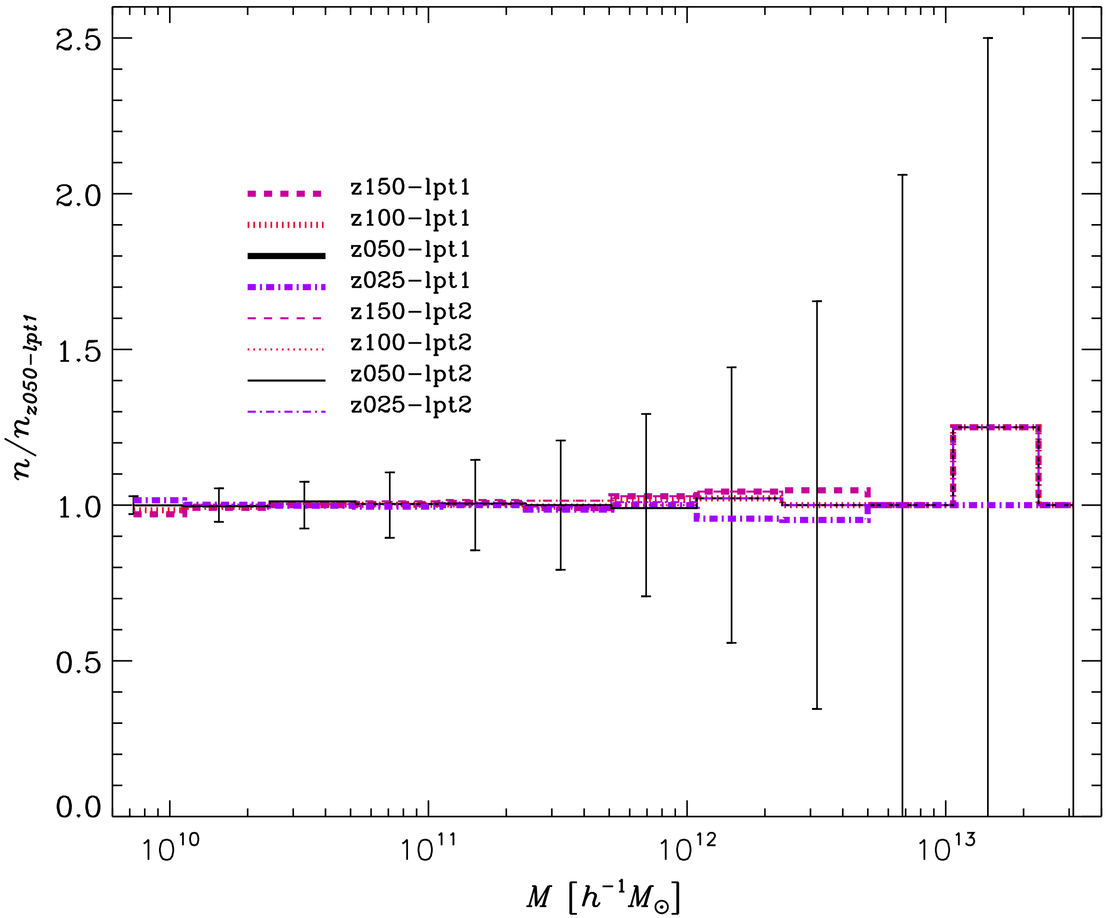


- Zel'dovich vs. 2^{nd} order LPT



(Knebe et al. 2009)

- Zel'dovich vs. 2nd order LPT



(Knebe et al. 2009)

- cosmological principle
- perturbations
- limitations
- alternatives
- remarks
- **summary**

- choose cosmological model ΛCDM?!

  => cosmological power spectrum of density perturbations $P(k)$

- choose box size B
- choose number of particles N^3

  => put them down on regular NxNxN grid

- choose starting redshift z_i

  => use Zel'dovich approximation to displace particles according to $P(k)$

- available codes:

  - N-genIC http://www.mpa-garching.mpg.de/gadget
  - 2LPTic http://cosmo.nyu.edu/roman/2LPT
  - Panphasia http://icc.dur.ac.uk/Panphasia.php
  - ginnungagap http://code.google.com/p/ginnungagap
  - MPgrafic http://www2.iap.fr/users/pichon/mpgrafic.html
  - PMstartM http://astro.nmsu.edu/~aklypin/PM/pmcode