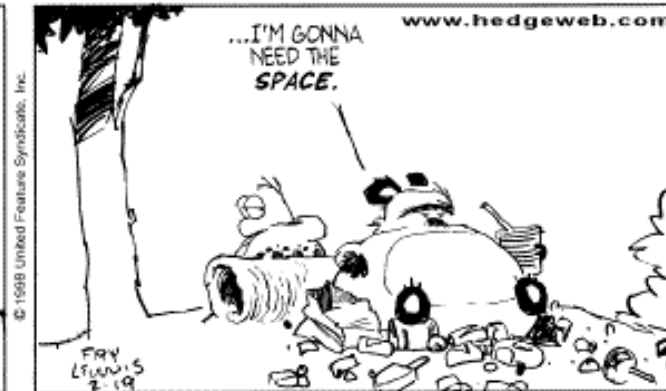
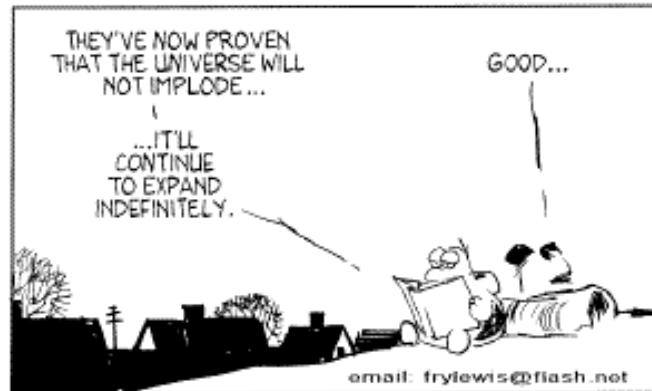


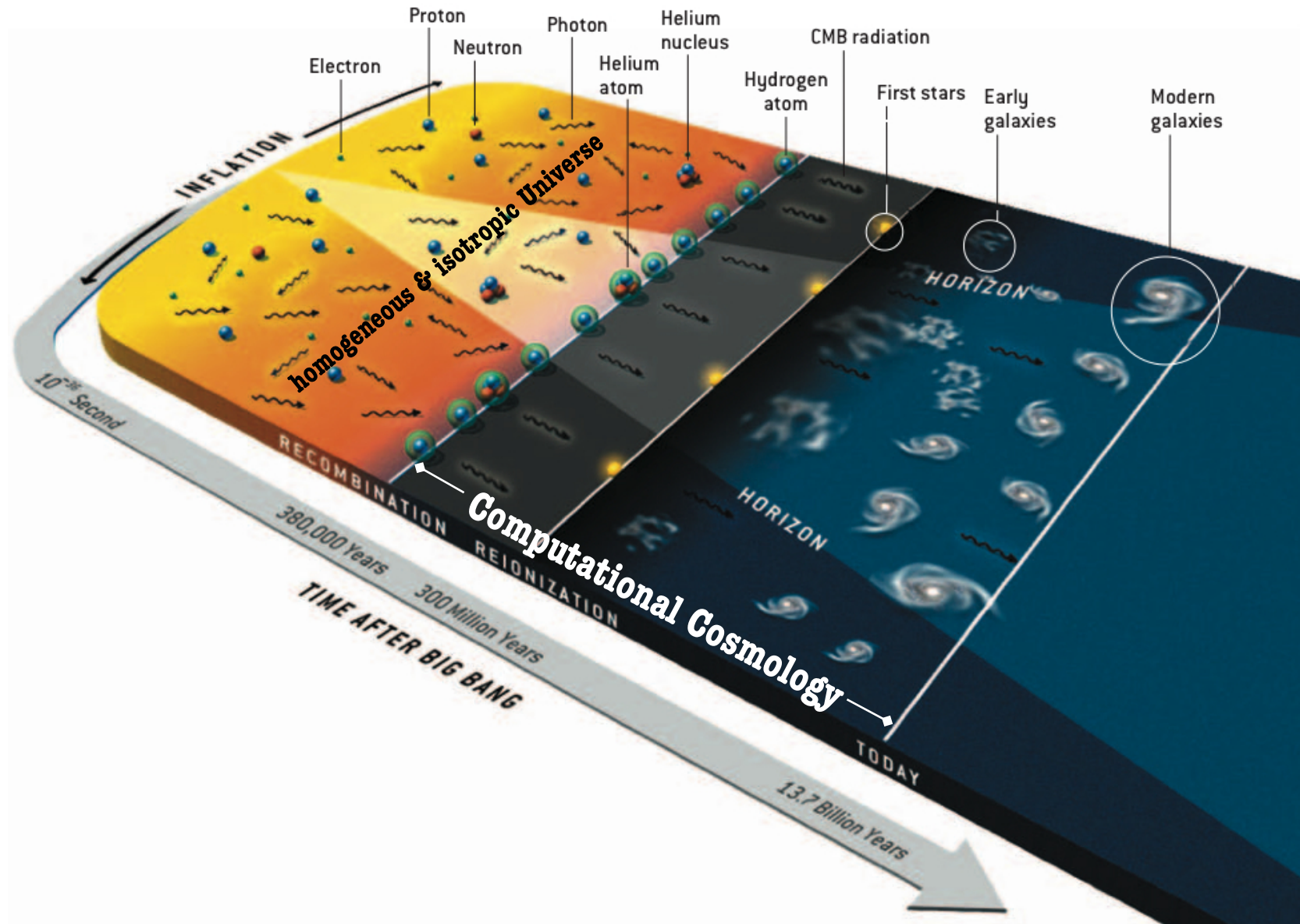
COMPUTATIONAL COSMOLOGY

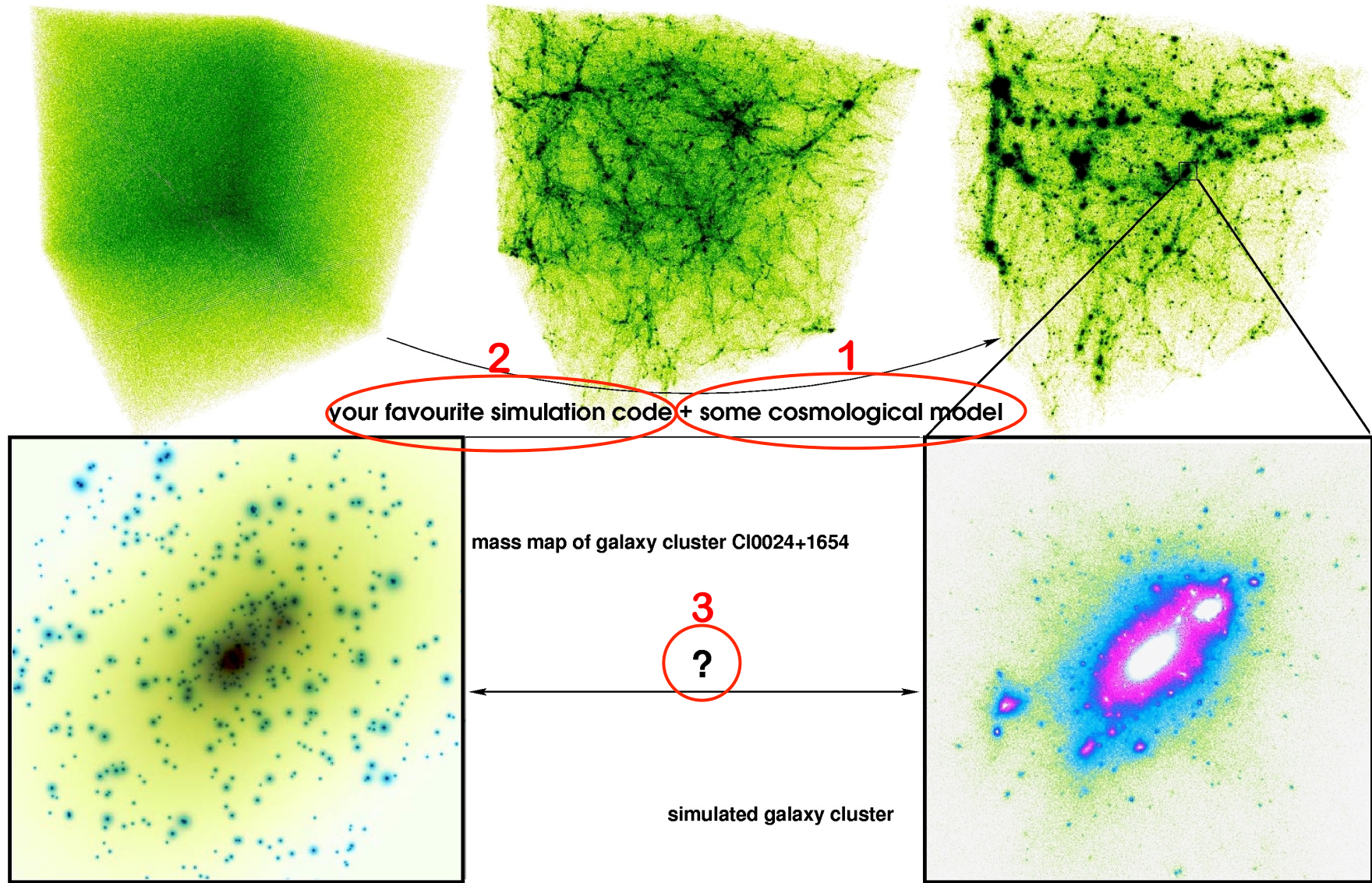
Alexander Knebe, *Universidad Autonoma de Madrid*



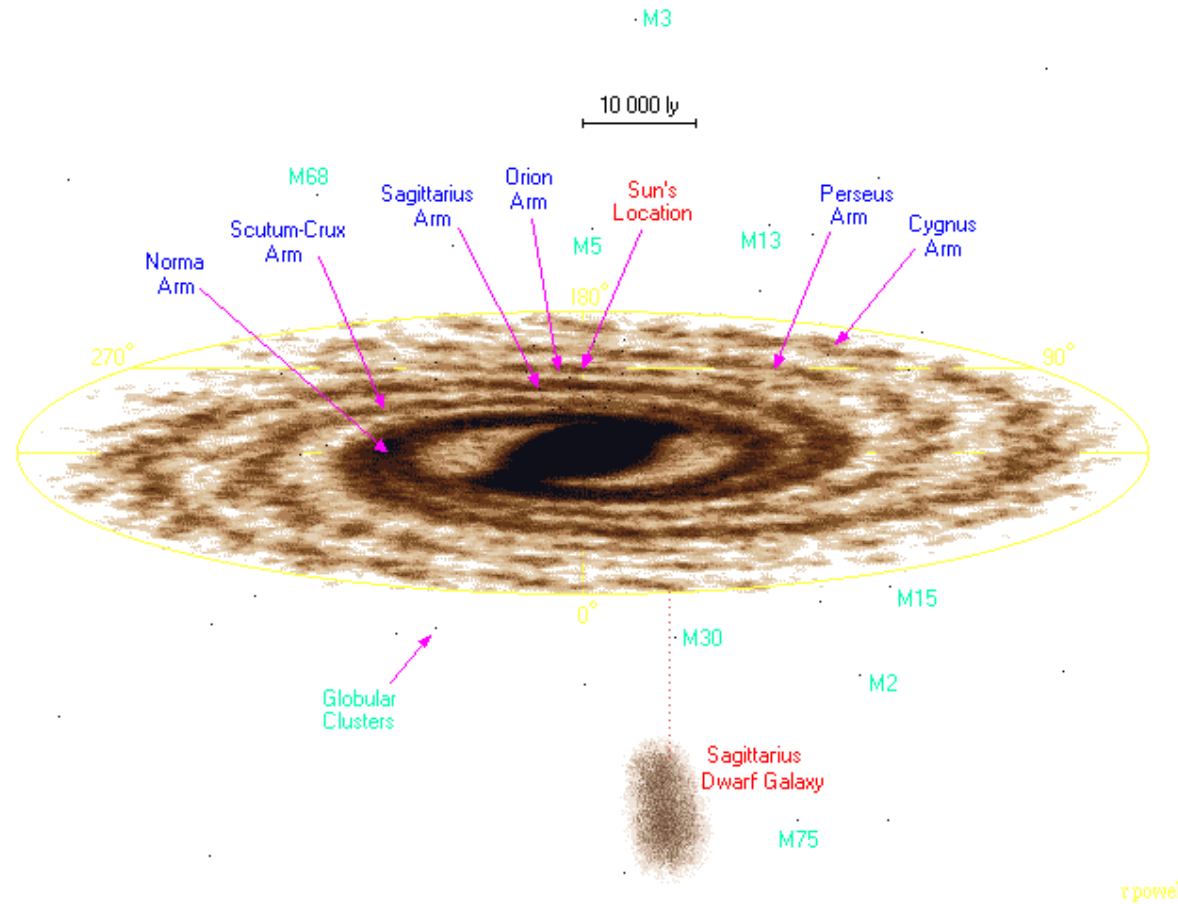
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Cosmology is the study
of the origin, evolution, and future
of our Universe



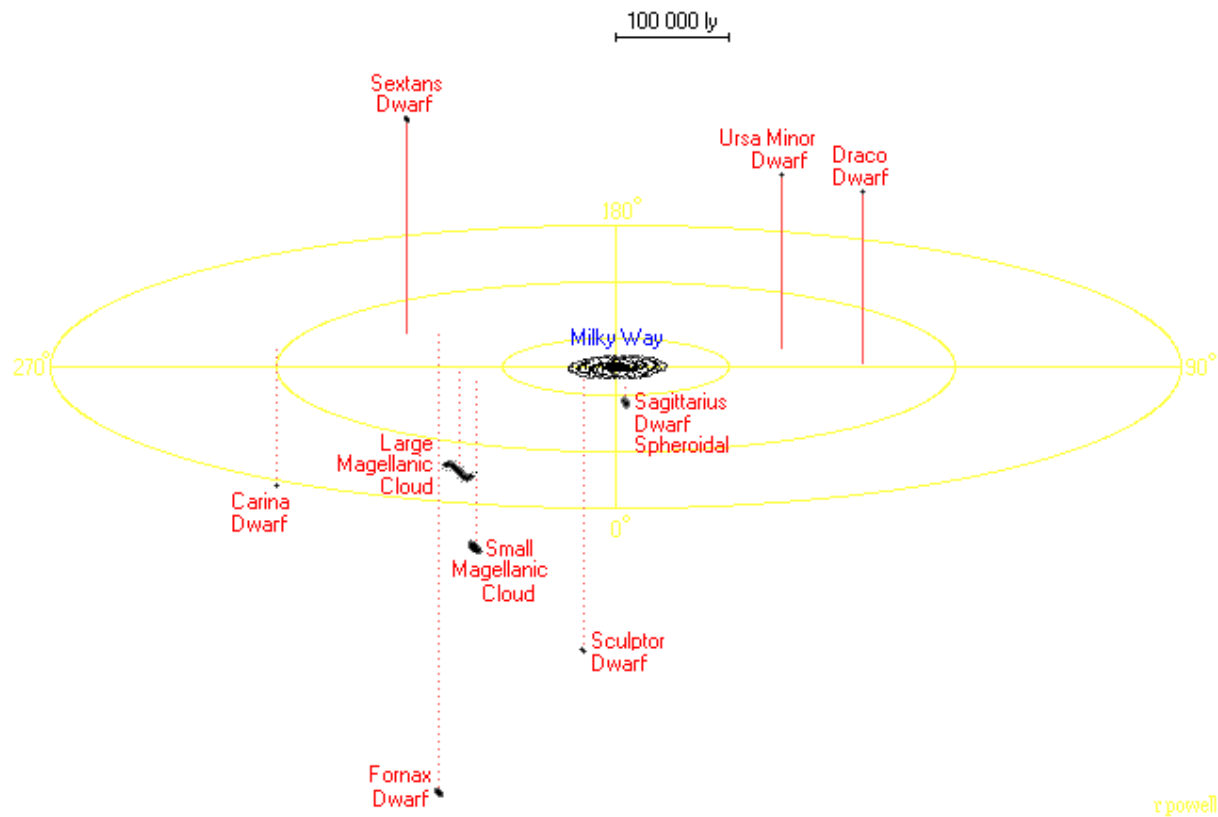


- orders of ten

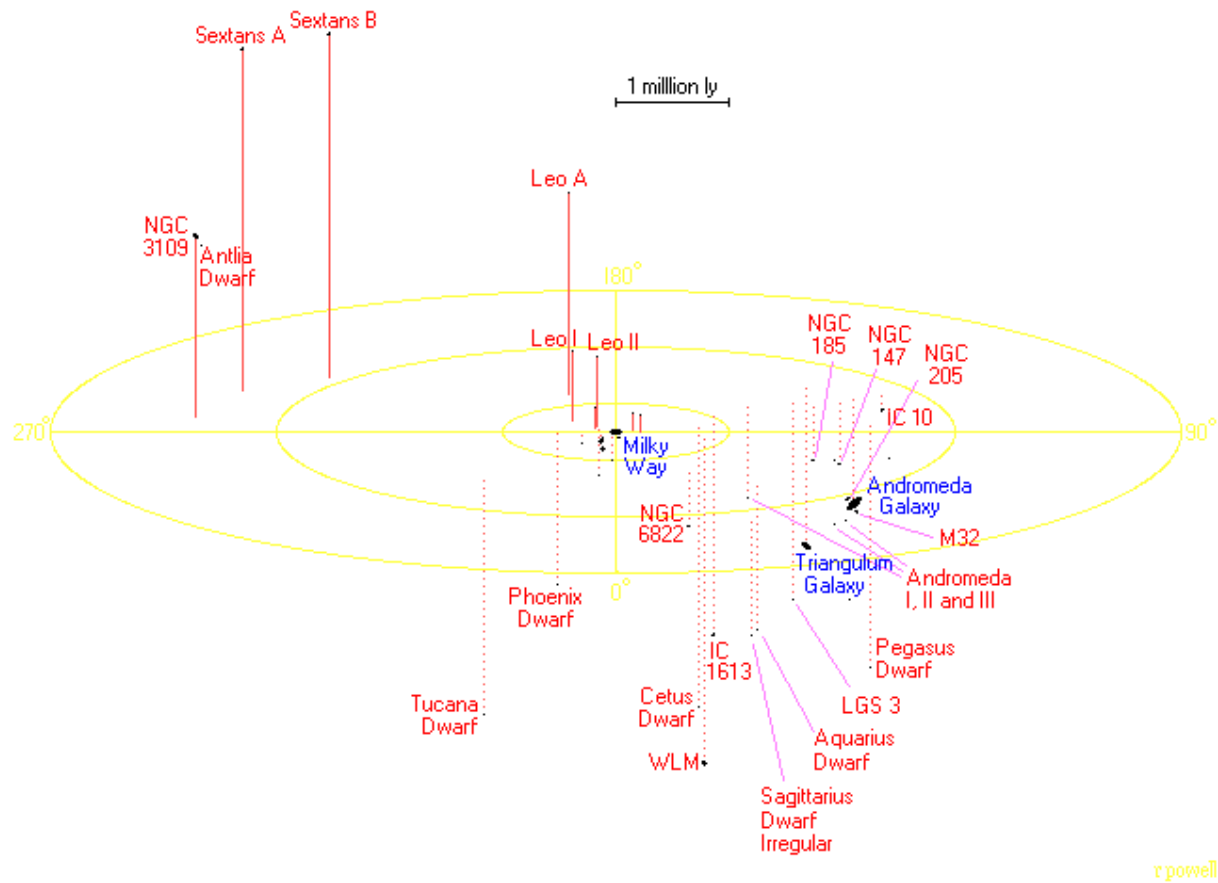


<http://hometown.aol.com/nlpjp/cosmo.htm>

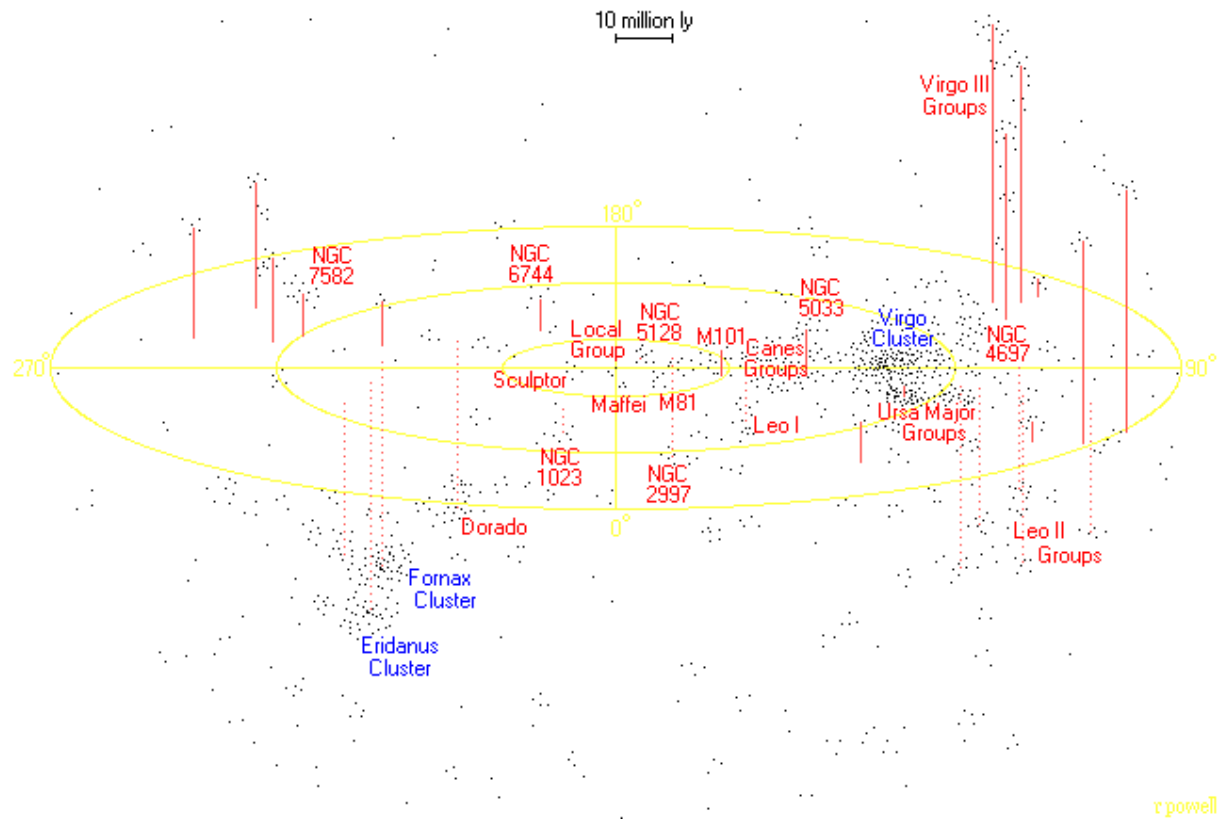
- orders of ten



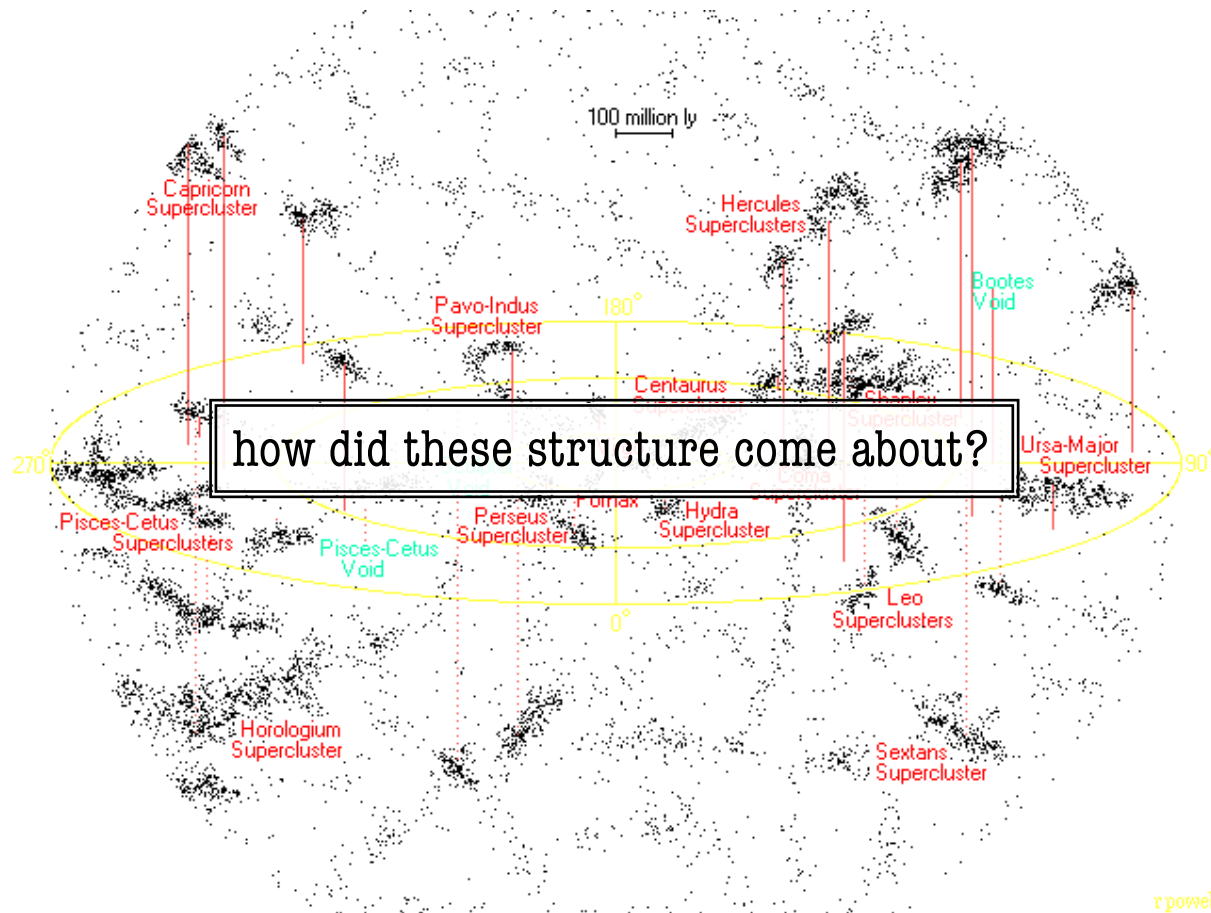
- orders of ten



- orders of ten

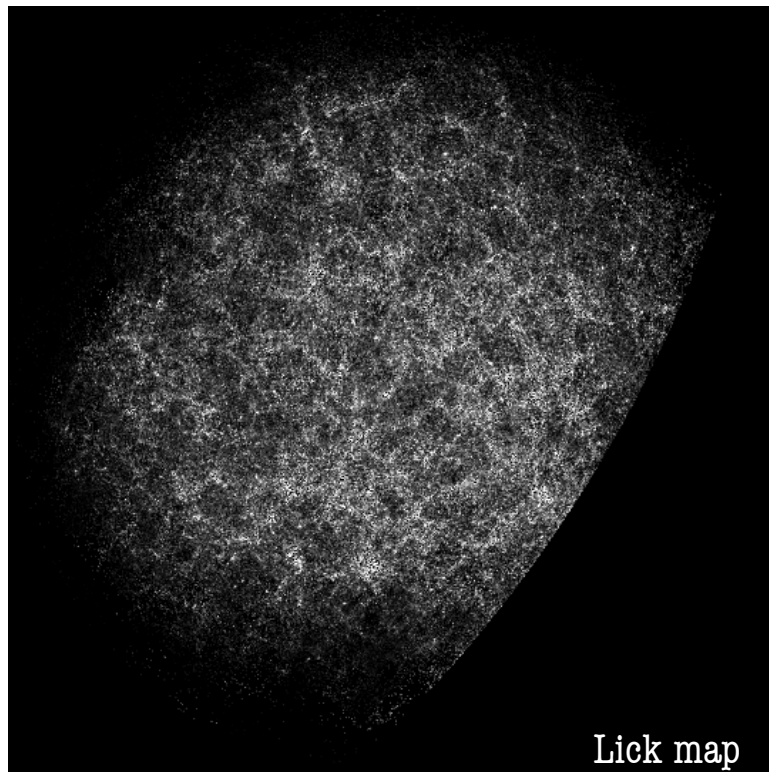


- orders of ten

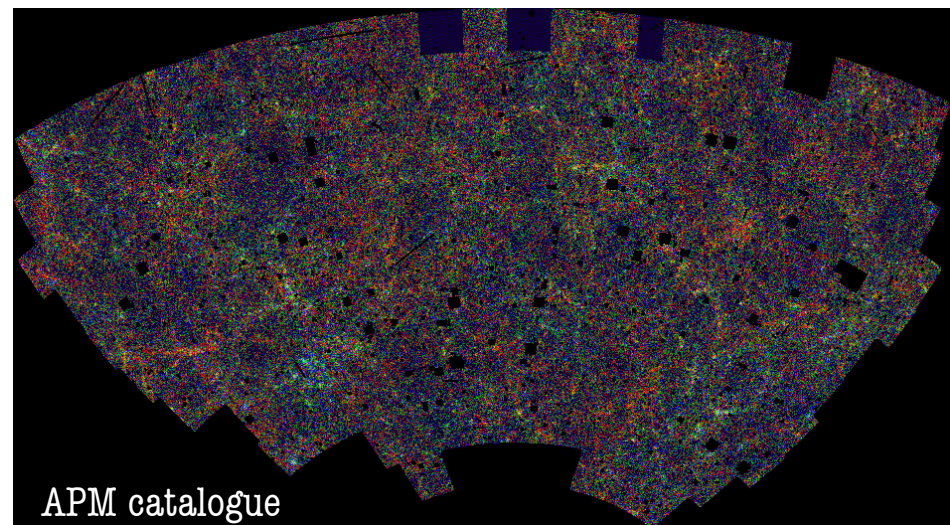


▪ galaxy catalogues (2D)

- Shapley & Ames (1932) 1,250 galaxies
- Palomar Sky Survey (1950) ~ 5,000 galaxies
- Lick Survey (1967) ~ 1,000,000 galaxies
- APM catalogue (1990) ~ 2,000,000 galaxies



Lick map

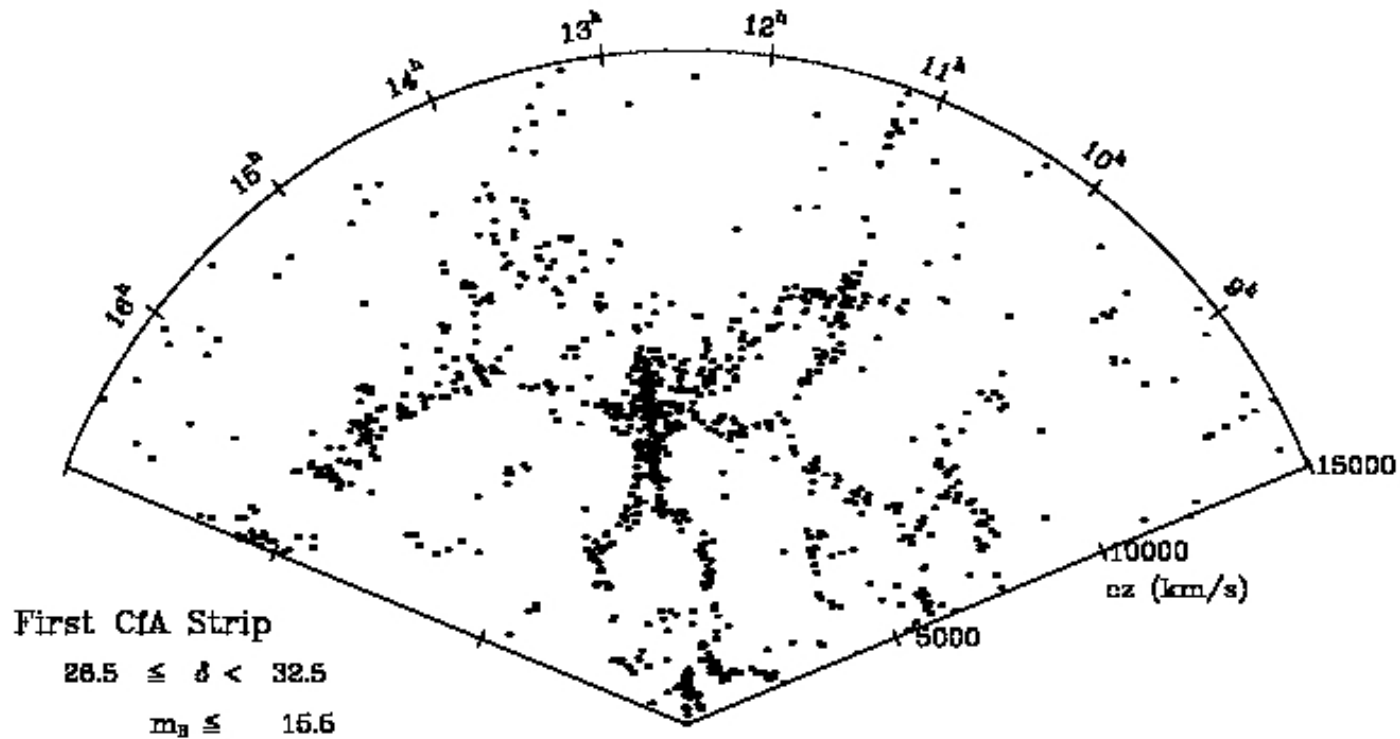


APM catalogue

▪ galaxy catalogues (3D)

- CfA (1986)

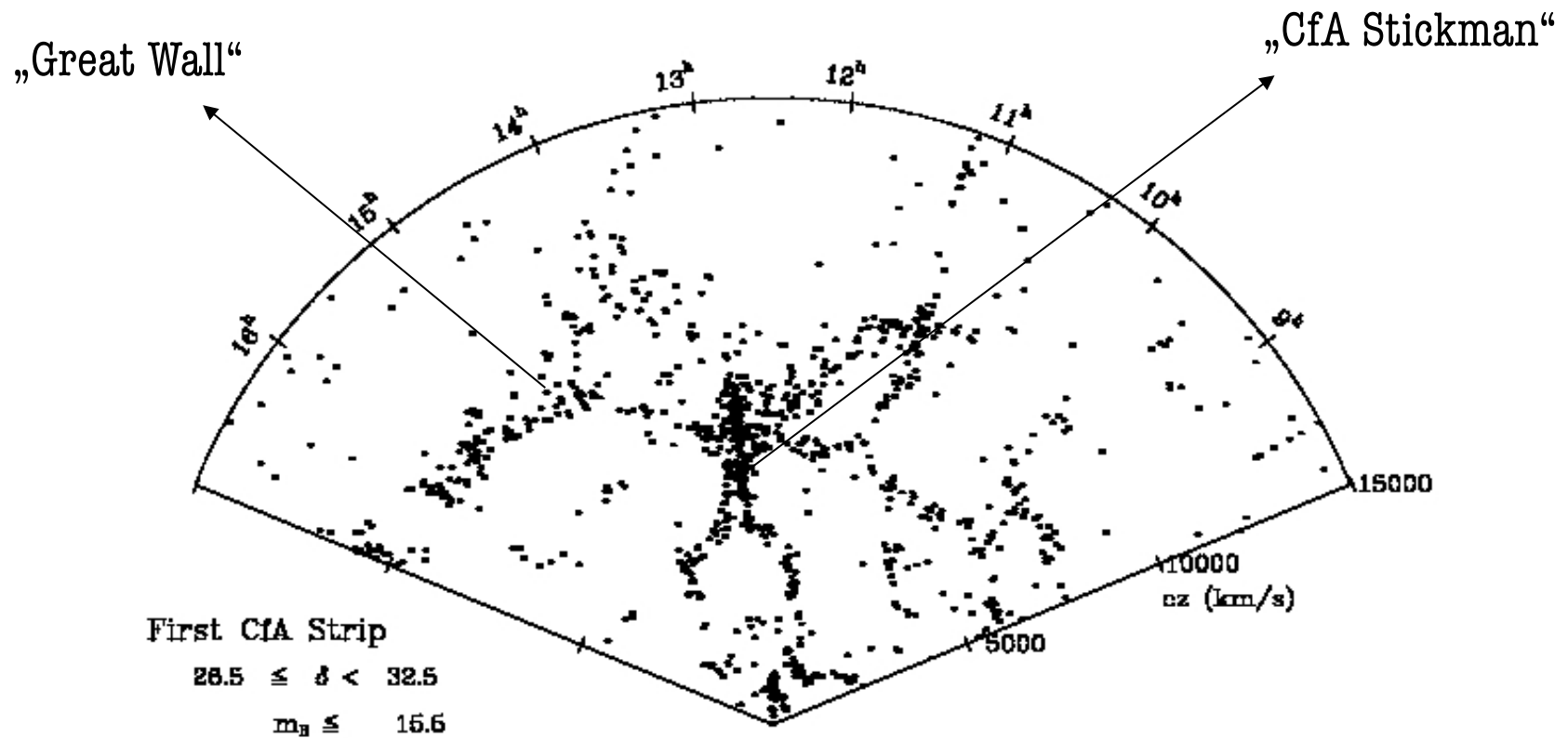
~ 1,000 galaxies



▪ galaxy catalogues (3D)

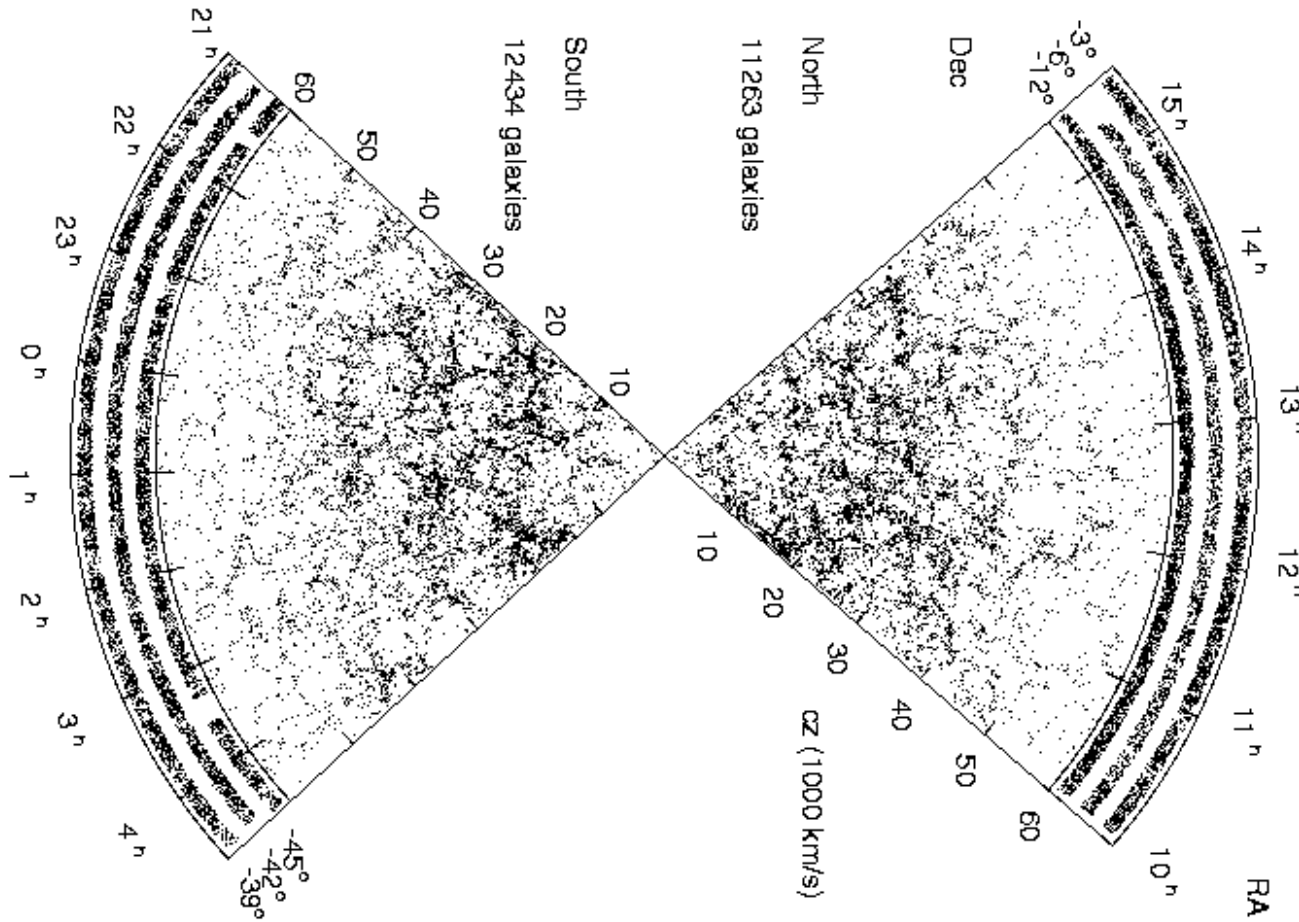
- CfA (1986)

~ 1,000 galaxies



- galaxy catalogues (3D)

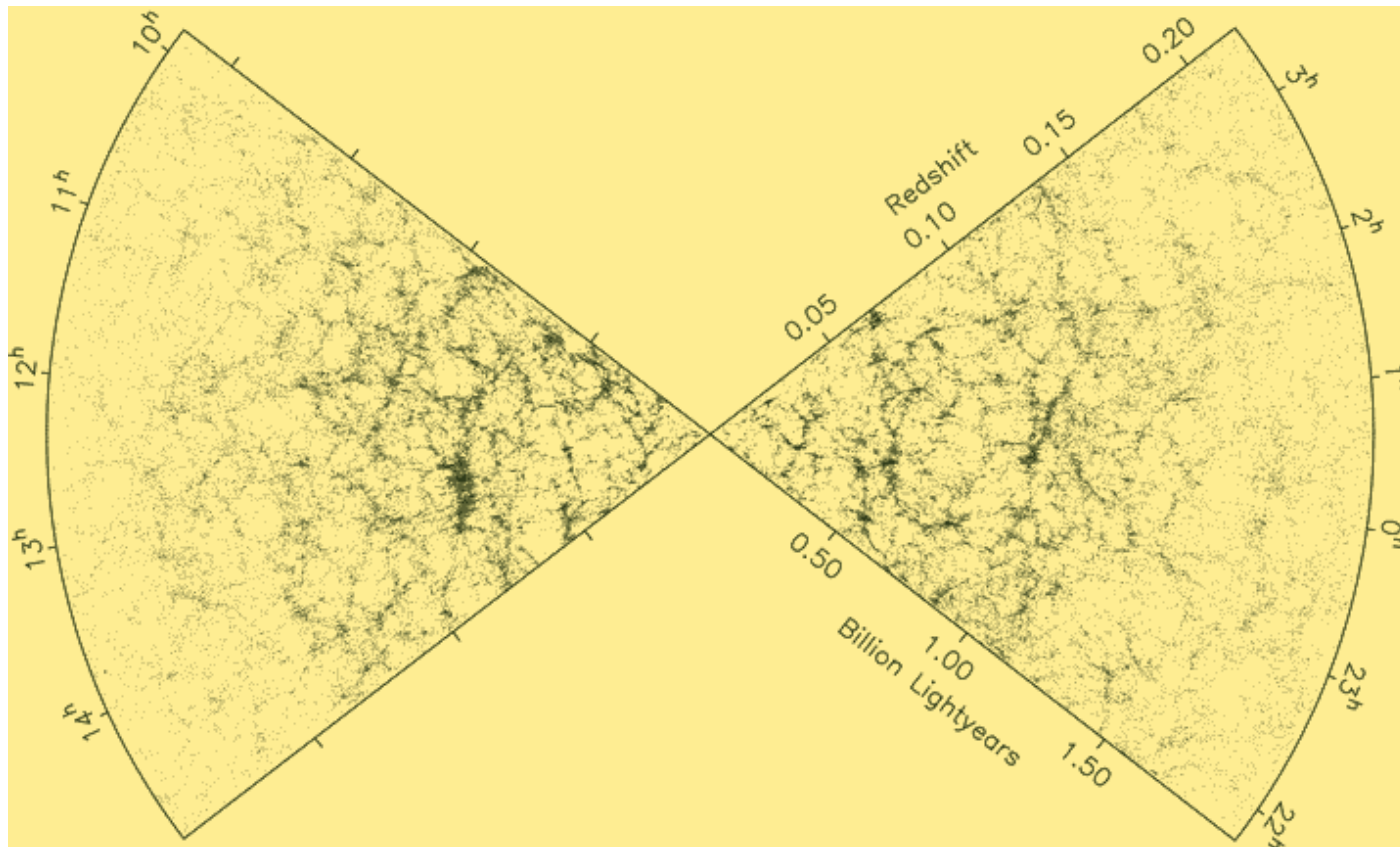
- Las Campanas Redshift Survey (1998) $\sim 20,000$ galaxies



- galaxy catalogues (3D)

- 2dF Survey (2003)

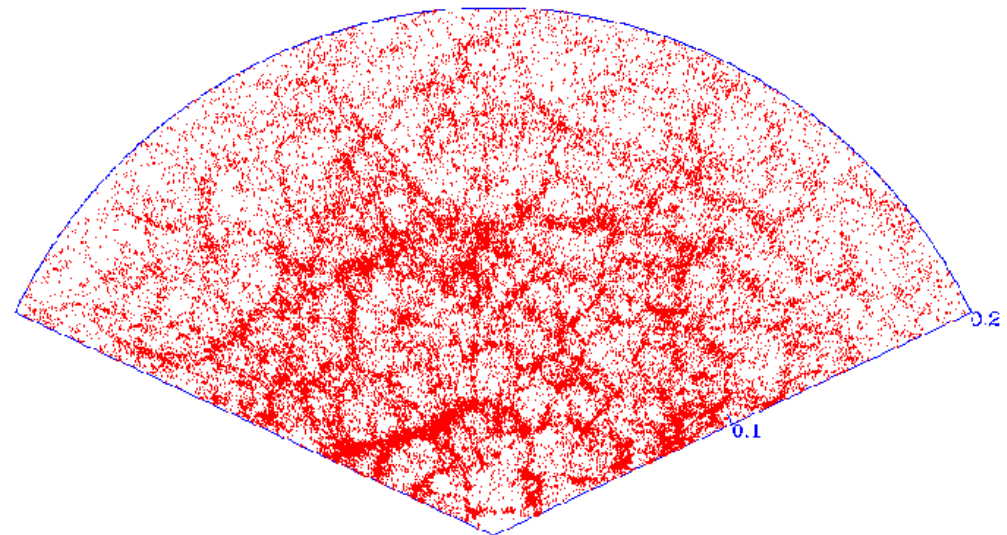
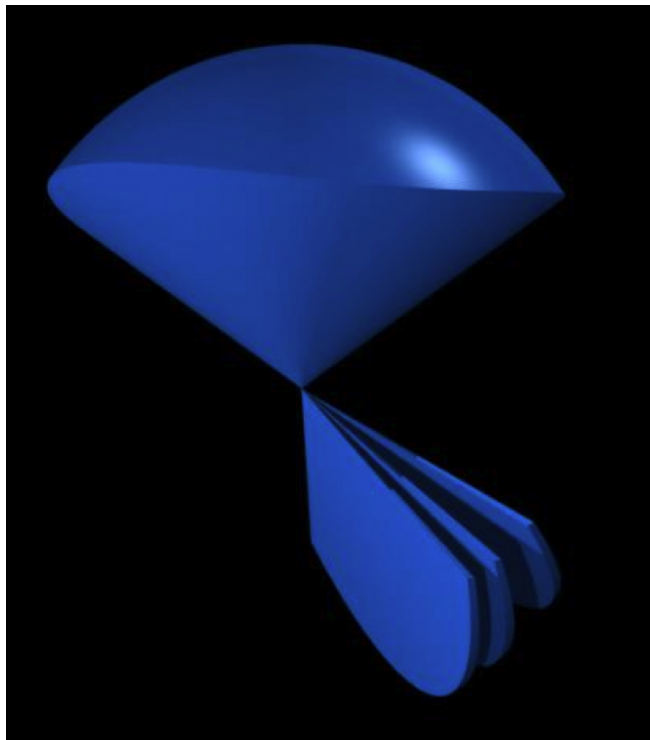
~ 200,000 galaxies



- galaxy catalogues (3D)

- Sloan Digital Sky Survey (2003)

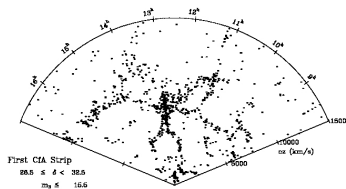
~ 1,000,000 galaxies



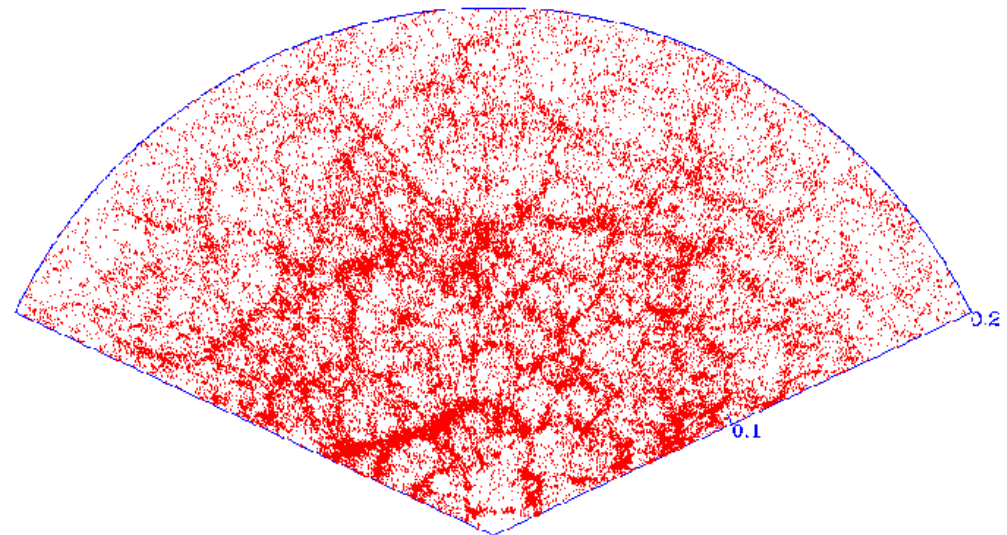
▪ galaxy catalogues (3D)

- Sloan Digital Sky Survey (2003)

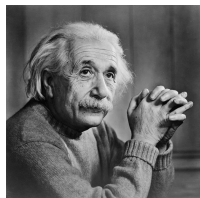
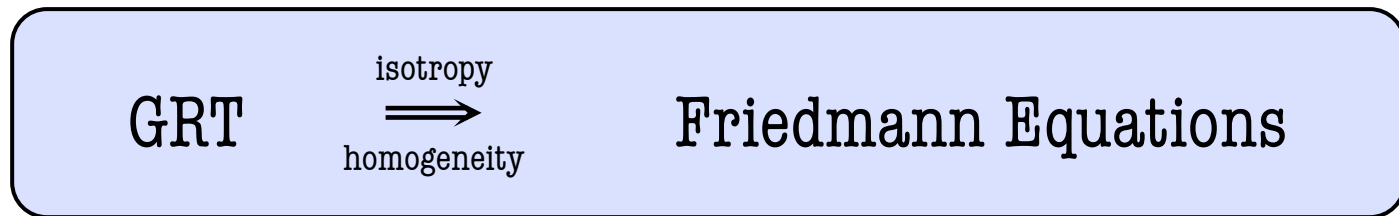
~ 1,000,000 galaxies



CfA (1986)



SDSS (2003)



Albert Einstein



Alexander Friedmann

GRT $\xRightarrow[\text{homogeneity}]{\text{isotropy}}$ Friedmann Equations

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = .H_0^2 (\Omega_0^r a^{-4} + \Omega_0^m a^{-3} + \Omega_0^k a^{-2} + \Omega_0^\Lambda)$$

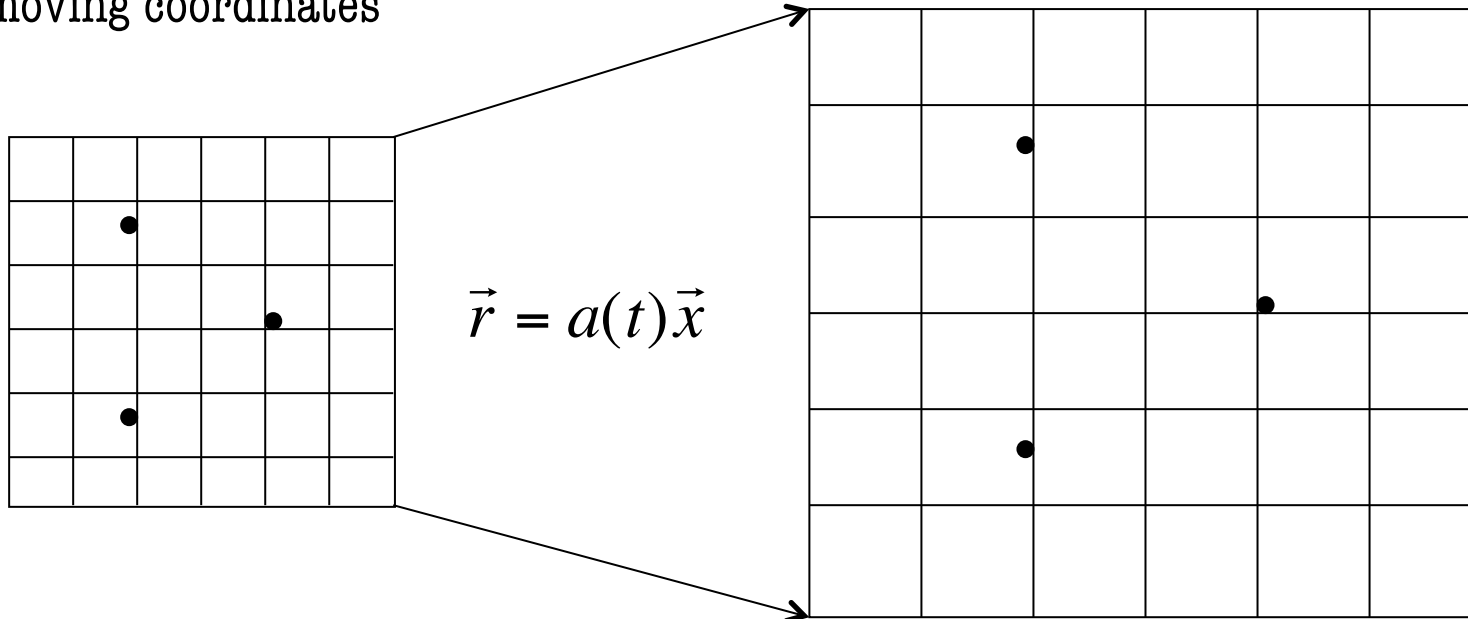
$$q = \frac{1}{2} \Omega_m - \Omega_\Lambda - \Omega_r$$

GRT $\xRightarrow[\text{homogeneity}]{\text{isotropy}}$ Friedmann Equations

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = .H_0^2 (\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda)$$

$$q = \frac{1}{2} \Omega_m - \Omega_\Lambda - \Omega_r$$

- comoving coordinates

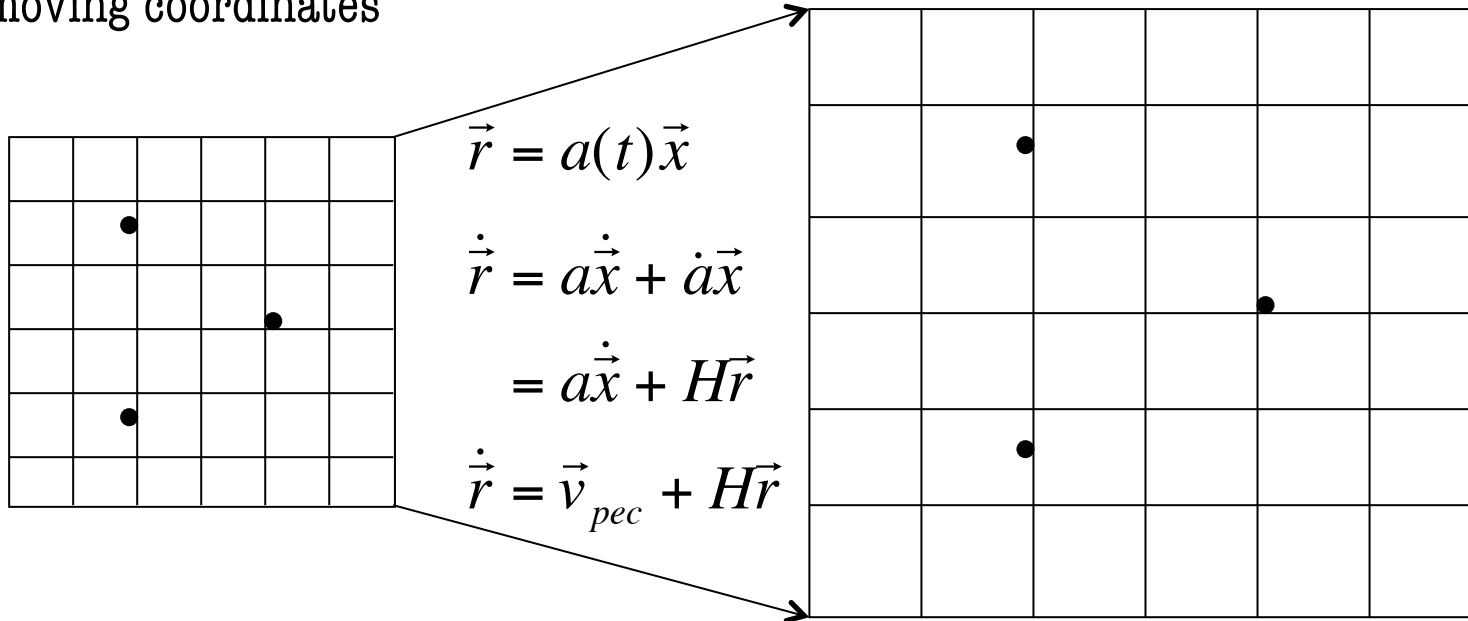


$$a(t_0) = a_0 = 1$$

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = .H_0^2 (\Omega_0^r a^{-4} + \Omega_0^m a^{-3} + \Omega_0^k a^{-2} + \Omega_0^\Lambda)$$

$$q = \frac{1}{2} \Omega_m - \Omega_\Lambda - \Omega_r$$

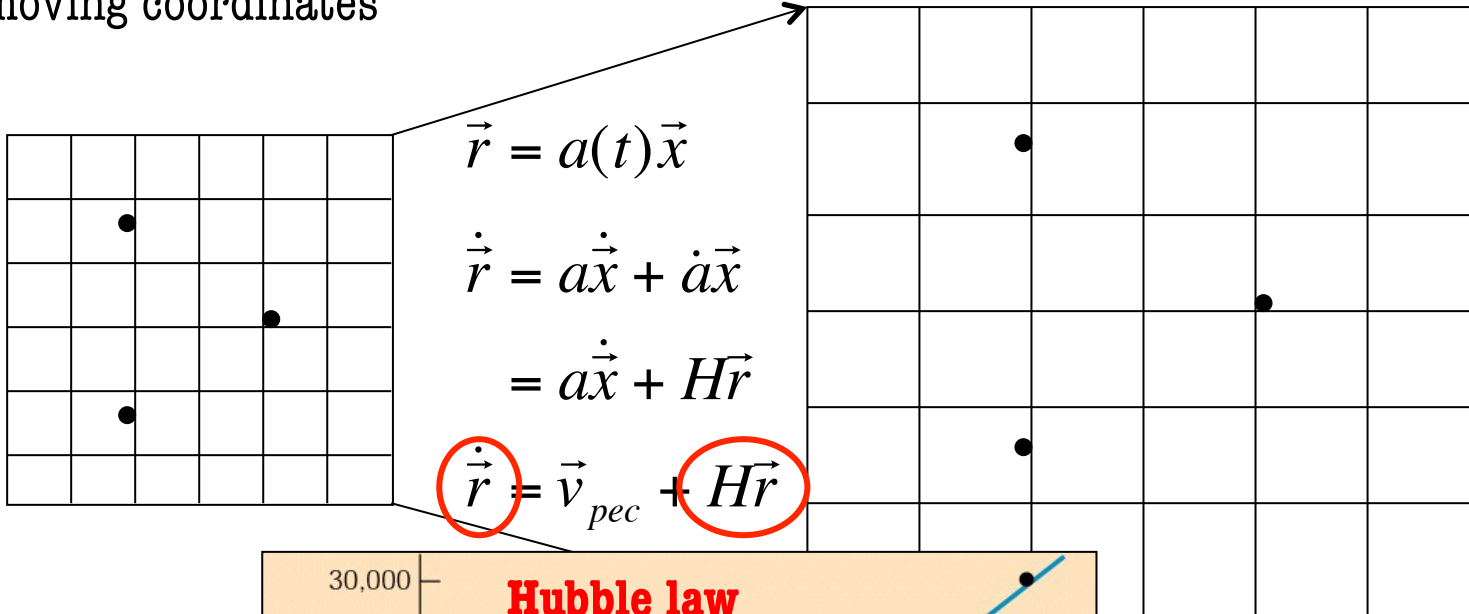
- comoving coordinates



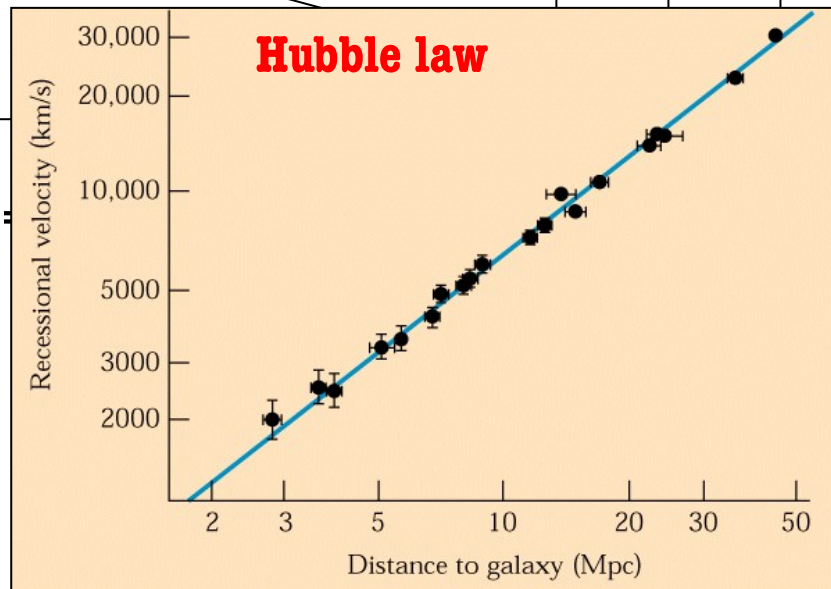
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = .H_0^2 (\Omega_0^r a^{-4} + \Omega_0^m a^{-3} + \Omega_0^k a^{-2} + \Omega_0^\Lambda)$$

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- comoving coordinates

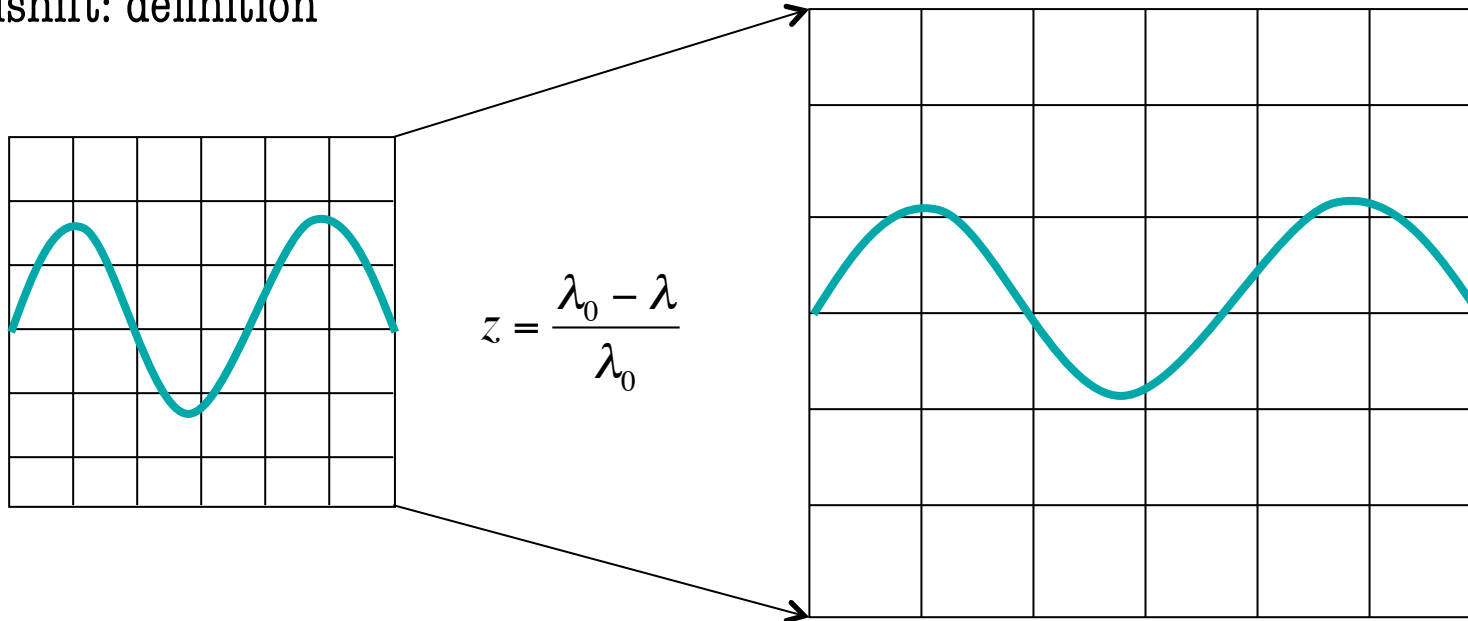


H^2



$\Omega_0^k a^{-2} + \Omega_0^\Lambda$

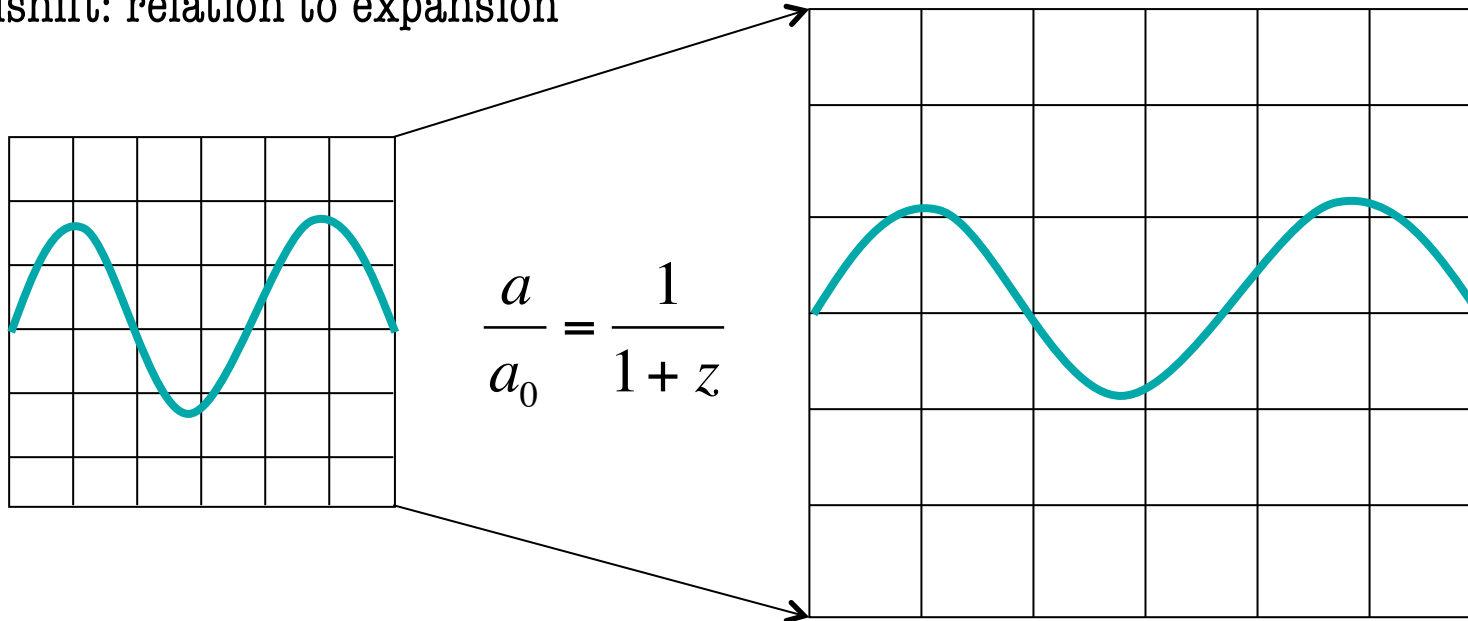
- redshift: definition



$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = .H_0^2 (\Omega_0^r a^{-4} + \Omega_0^m a^{-3} + \Omega_0^k a^{-2} + \Omega_0^\Lambda)$$

$$q = \frac{1}{2} \Omega_m - \Omega_\Lambda - \Omega_r$$

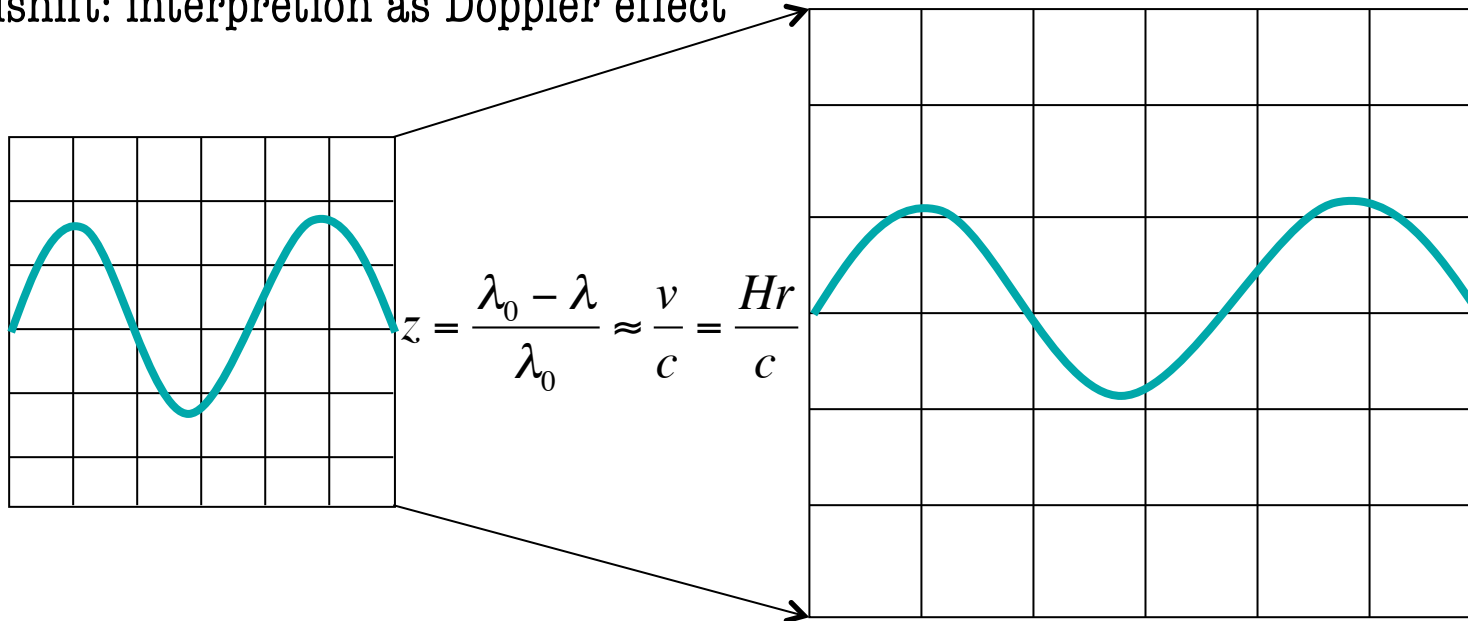
- redshift: relation to expansion



$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = .H_0^2 (\Omega_0^r a^{-4} + \Omega_0^m a^{-3} + \Omega_0^k a^{-2} + \Omega_0^\Lambda)$$

$$q = \frac{1}{2} \Omega_m - \Omega_\Lambda - \Omega_r$$

- redshift: interpretation as Doppler effect



$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = .H_0^2 (\Omega_0^r a^{-4} + \Omega_0^m a^{-3} + \Omega_0^k a^{-2} + \Omega_0^\Lambda)$$

$$q = \frac{1}{2} \Omega_m - \Omega_\Lambda - \Omega_r$$

$a(t)$ expansion factor of the Universe

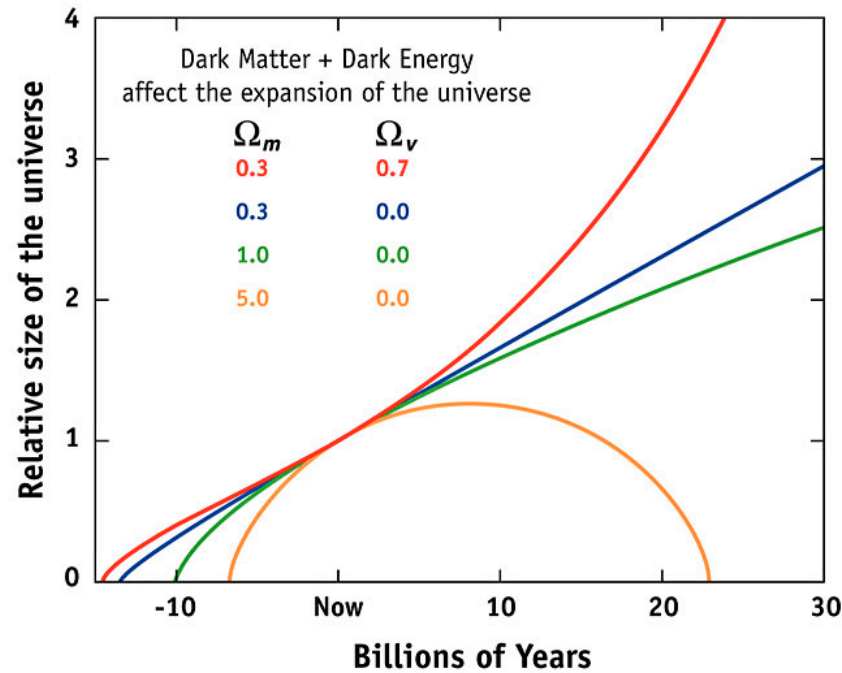
$H(t) = \frac{\dot{a}}{a}$ Hubble parameter

$q(t) = -\frac{\ddot{a}a}{\dot{a}^2}$ deceleration parameter

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = .H_0^2 (\Omega_0^r a^{-4} + \Omega_0^m a^{-3} + \Omega_0^k a^{-2} + \Omega_0^\Lambda)$$

$$q = \frac{1}{2} \Omega_m - \Omega_\Lambda - \Omega_r$$

EXPANSION OF THE UNIVERSE



$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = .H_0^2 (\Omega_0^r a^{-4} + \Omega_0^m a^{-3} + \Omega_0^k a^{-2} + \Omega_0^\Lambda)$$

$$q = \frac{1}{2} \Omega_m - \Omega_\Lambda - \Omega_r$$

$$\Omega_m = \frac{8\pi G}{3H^2} \rho_m \quad \text{matter energy density}$$

$$\Omega_r = \frac{8\pi G}{3H^2} \rho_r \quad \text{radiation energy density}$$

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H^2} \quad \text{cosmological constant}$$

$$\Omega_k = -\frac{kc^2}{H^2} \quad \text{curvature of the Universe}$$

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = .H_0^2 (\Omega_0^r a^{-4} + \Omega_0^m a^{-3} + \Omega_0^k a^{-2} + \Omega_0^\Lambda)$$

$$q = \cdot \frac{1}{2} \Omega_m - \Omega_\Lambda - \Omega_r$$

$$\Omega_m = \frac{8\pi G}{3H^2} \rho_m \quad \text{matter energy density}$$

$$\Omega_r = \frac{8\pi G}{3H^2} \rho_r \quad \text{radiation energy density}$$

$3H^2/8\pi G =$ critical density to “close” the Universe
(ca. 1 H-atom per m^3)

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H^2} \quad \text{cosmological constant}$$

$$\Omega_k = -\frac{kc^2}{H^2} \quad \text{curvature of the Universe}$$

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = .H_0^2 (\Omega_0^r a^{-4} + \Omega_0^m a^{-3} + \Omega_0^k a^{-2} + \Omega_0^\Lambda)$$

$$q = \cdot \frac{1}{2} \Omega_m - \Omega_\Lambda - \Omega_r$$

- Λ CDM - Λ Cold Dark Matter

$$\Omega_m^0 \approx 0.3$$

$$\Omega_\Lambda^0 \approx 0.7$$

$$\Omega_r^0 \approx 2.3 \times 10^{-5} h^{-2}$$

$$\Omega_k^0 \approx 0$$

- Λ CDM - Λ Cold Dark Matter

$$\Omega_m^0 \approx 0.3$$

$$\Omega_\Lambda^0 \approx 0.7$$

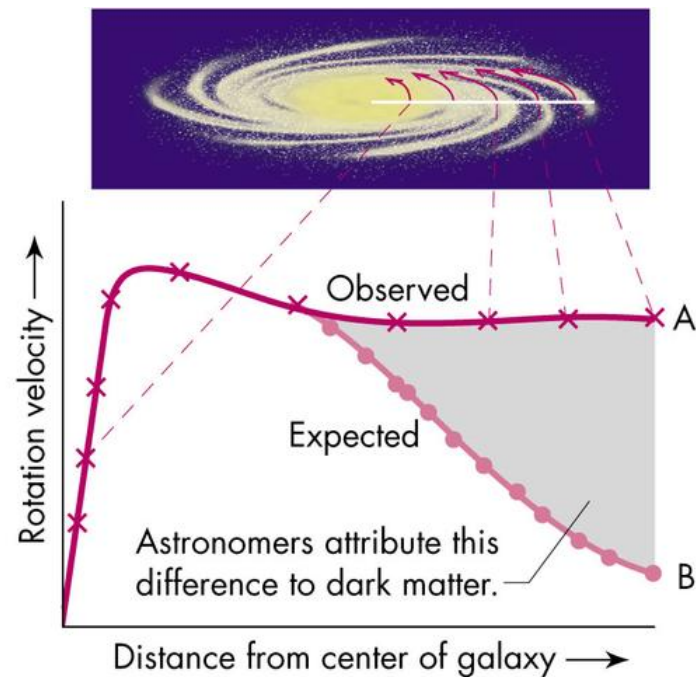
- the Universe is spatially flat
- ca. 30% of the total energy density comes from matter
- ca. 70% of the total energy density is vacuum energy

▪ Λ CDM - Λ Cold Dark Matter

$$\Omega_m^0 \approx 0.3 = 0.26 + 0.04$$

$$\Omega_\Lambda^0 \approx 0.7$$

„dark matter“
luminous matter

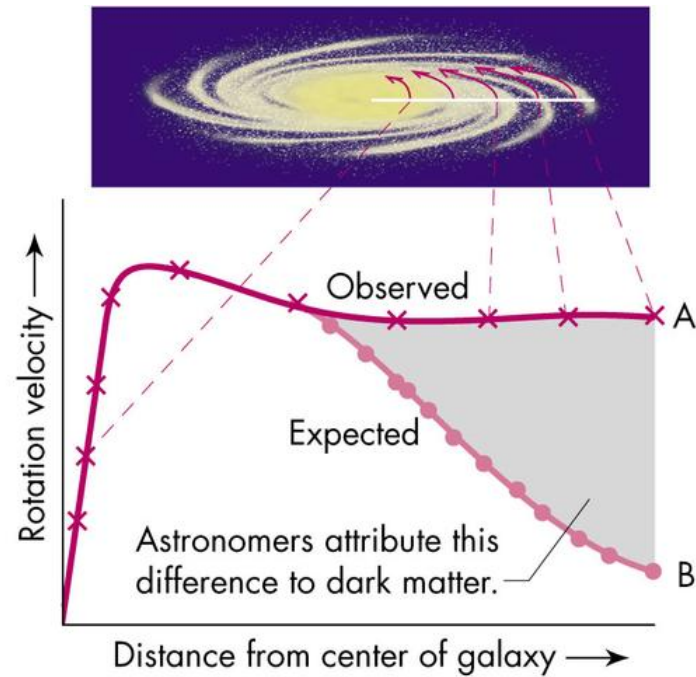


▪ Λ CDM - Λ Cold Dark Matter

$$\Omega_m^0 \approx 0.3 = 0.26 + 0.04$$

$$\Omega_\Lambda^0 \approx 0.7$$

„dark matter“
luminous matter



■ Λ CDM - Λ Cold Dark Matter(non-baryonic) Dark Matter Candidates

axion:	10^{-5} eV
neutrino:	10eV
WIMP:	1- 10^3 GeV
monopoles:	10^{16} GeV
Planck relics:	10^{19} GeV ($h^\alpha c^\beta G^\gamma$)
primordial BHs:	$>10^{15}$ g

???

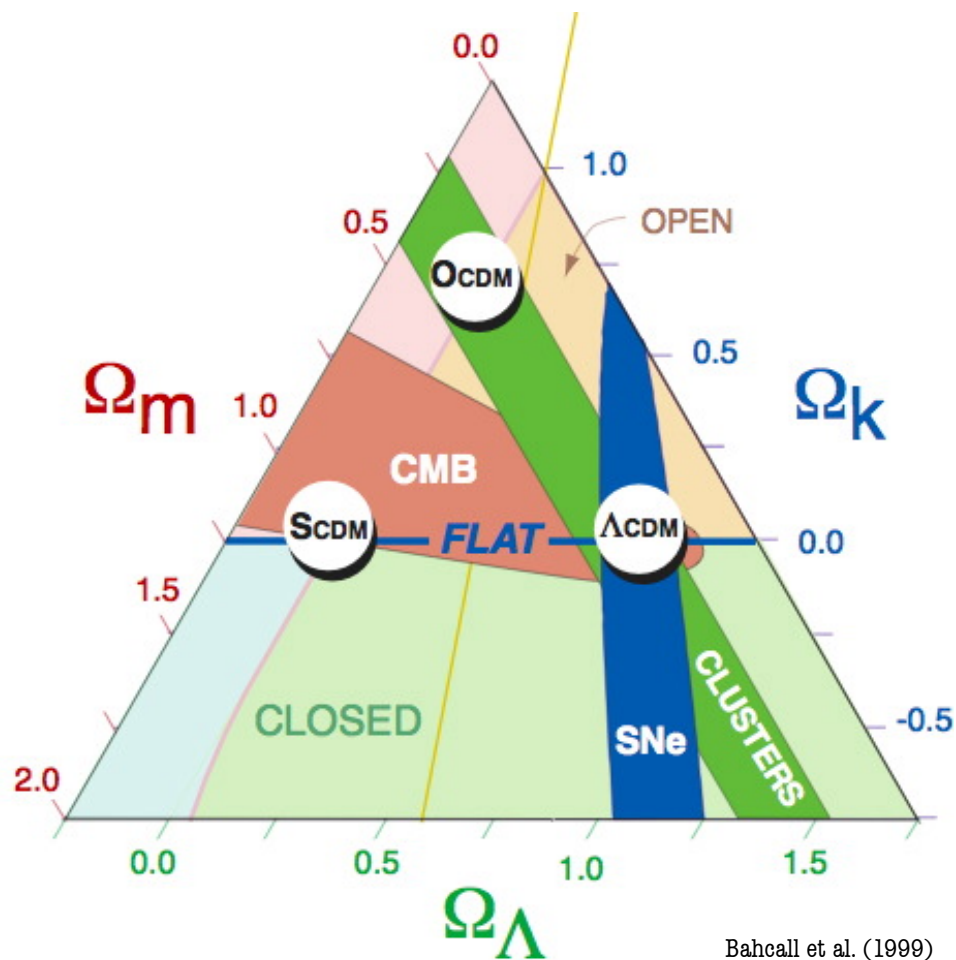
- Λ CDM - Λ Cold Dark Matter

(non-baryonic) Dark Matter Candidates

hot dark matter	{	axion:	10^{-5} eV	
warm dark matter		neutrino:	10eV	
cold dark matter	{	WIMP:	1- 10^3 GeV	
		monopoles:	10^{16} GeV	
		Planck relics:	10^{19} GeV	$(h^\alpha c^\beta G^\gamma)$
		primordial BHs:	$>10^{15}$ g	

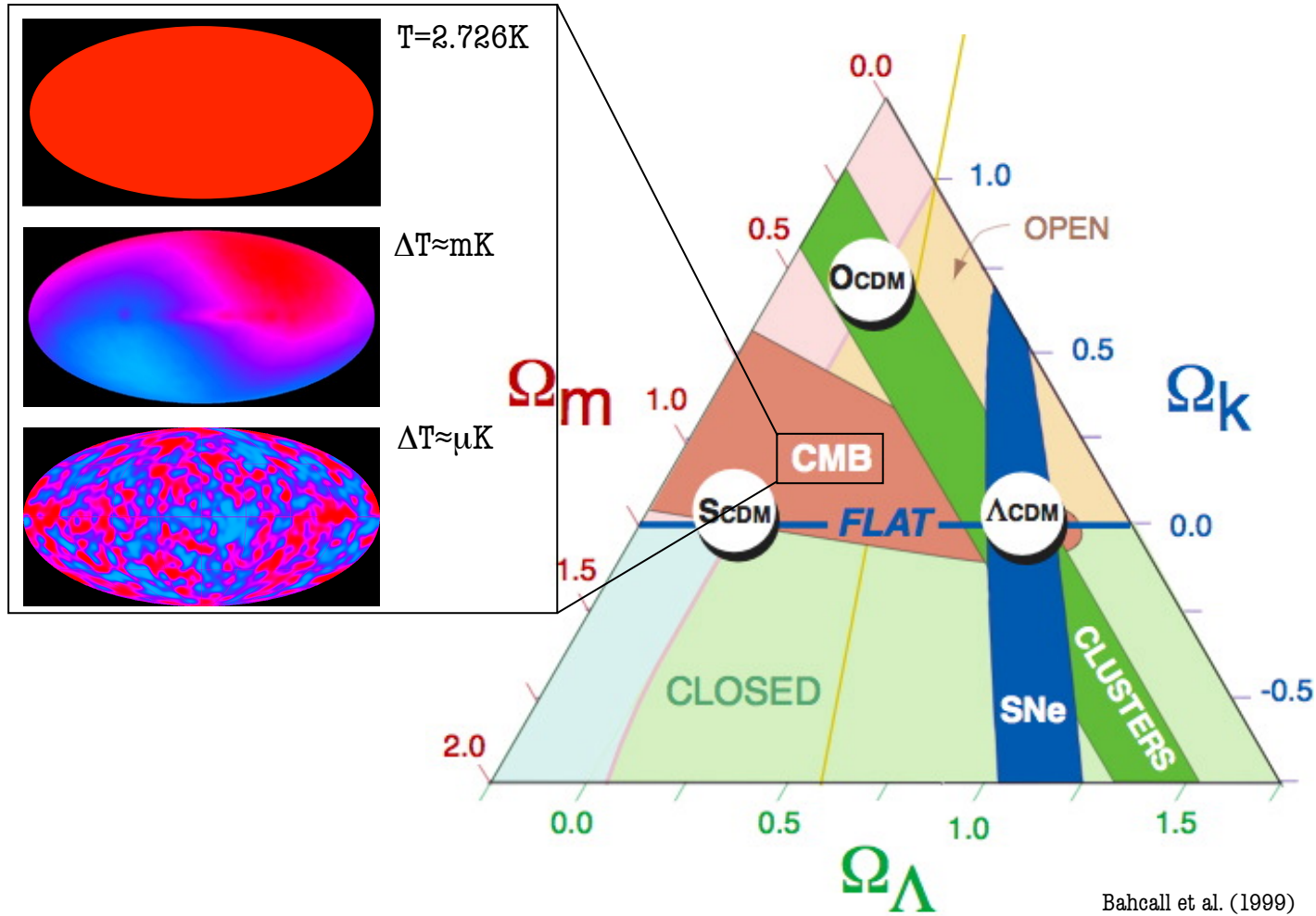
???

- Λ CDM - Λ Cold Dark Matter

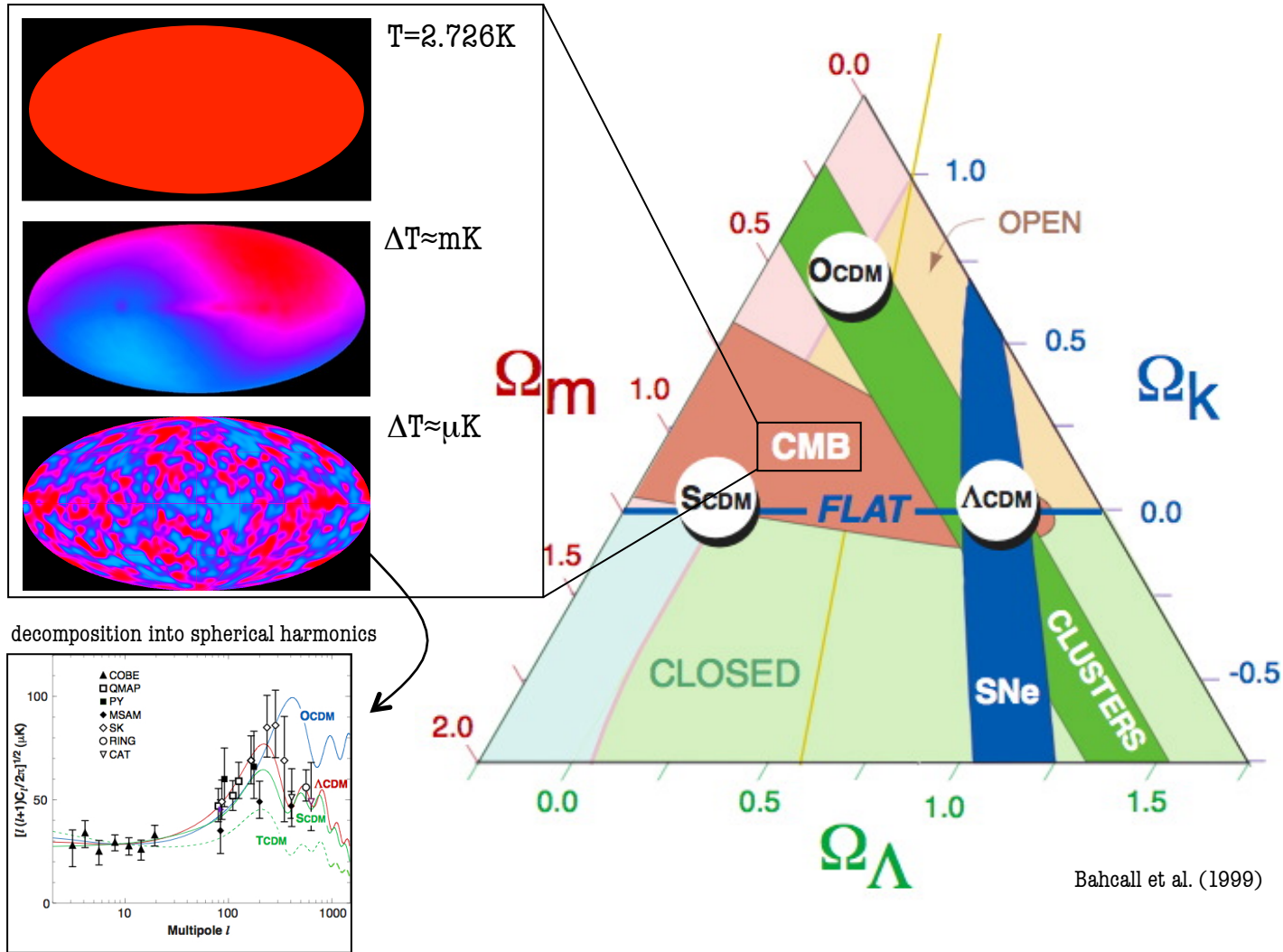


Bahcall et al. (1999)

▪ Λ CDM - Λ Cold Dark Matter



■ Λ CDM - Λ Cold Dark Matter

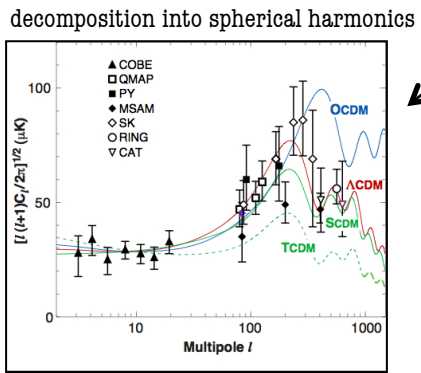
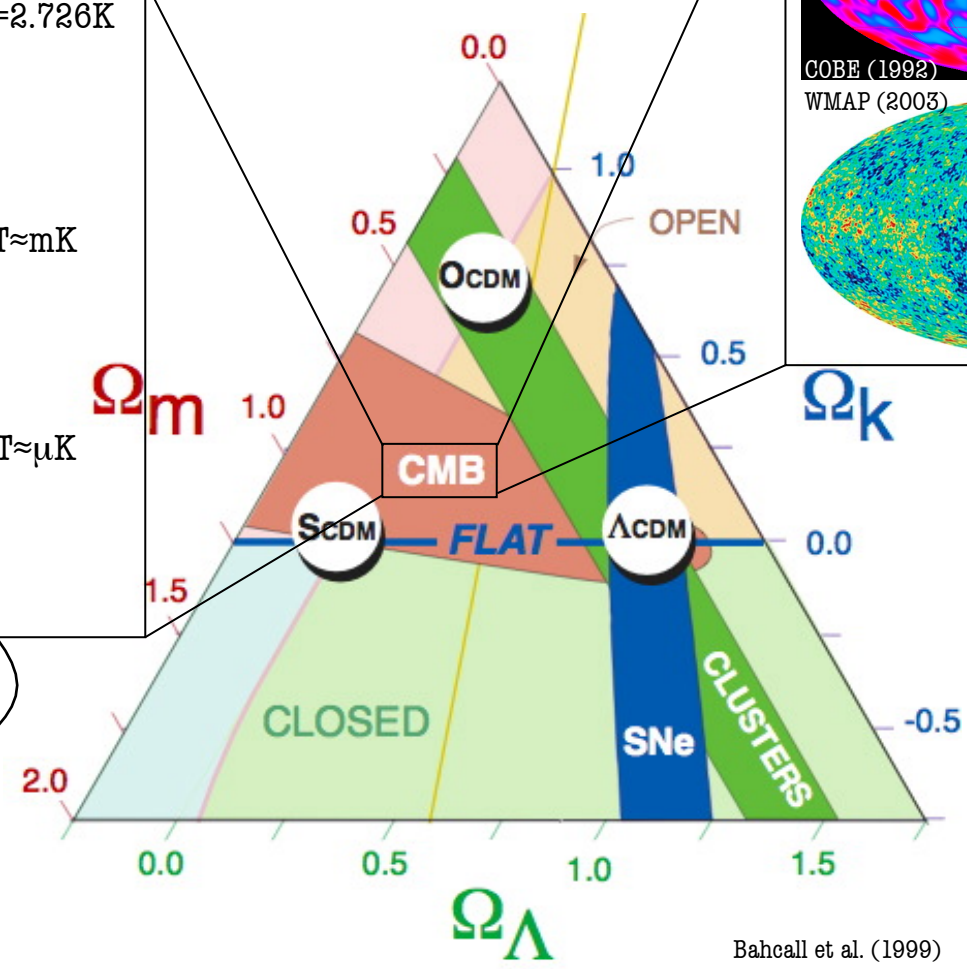
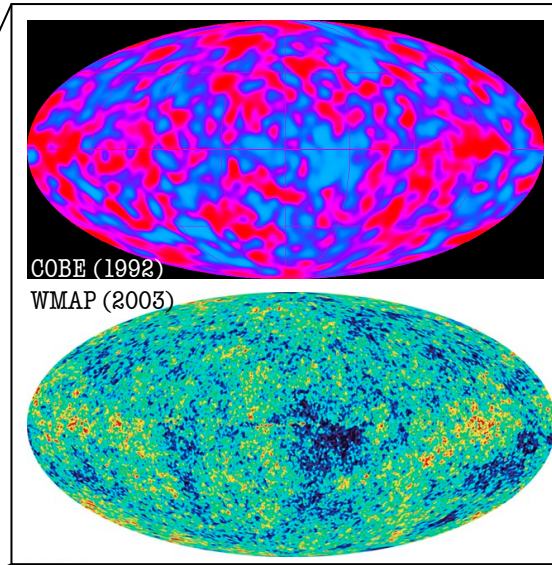
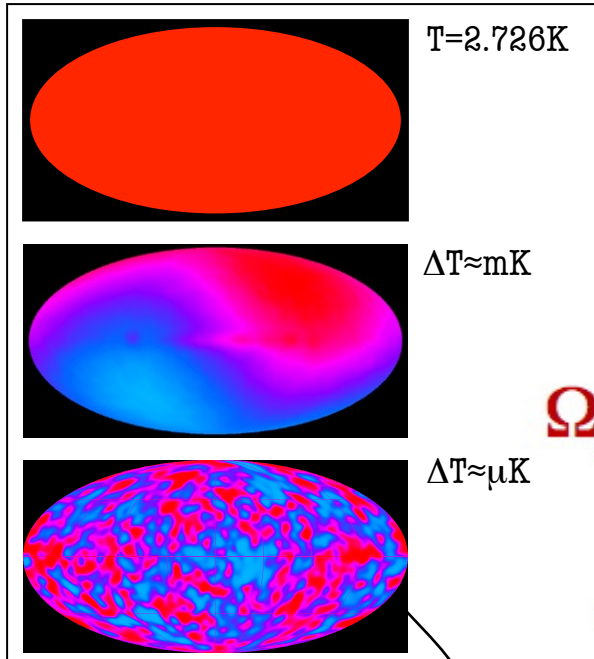


Bahcall et al. (1999)

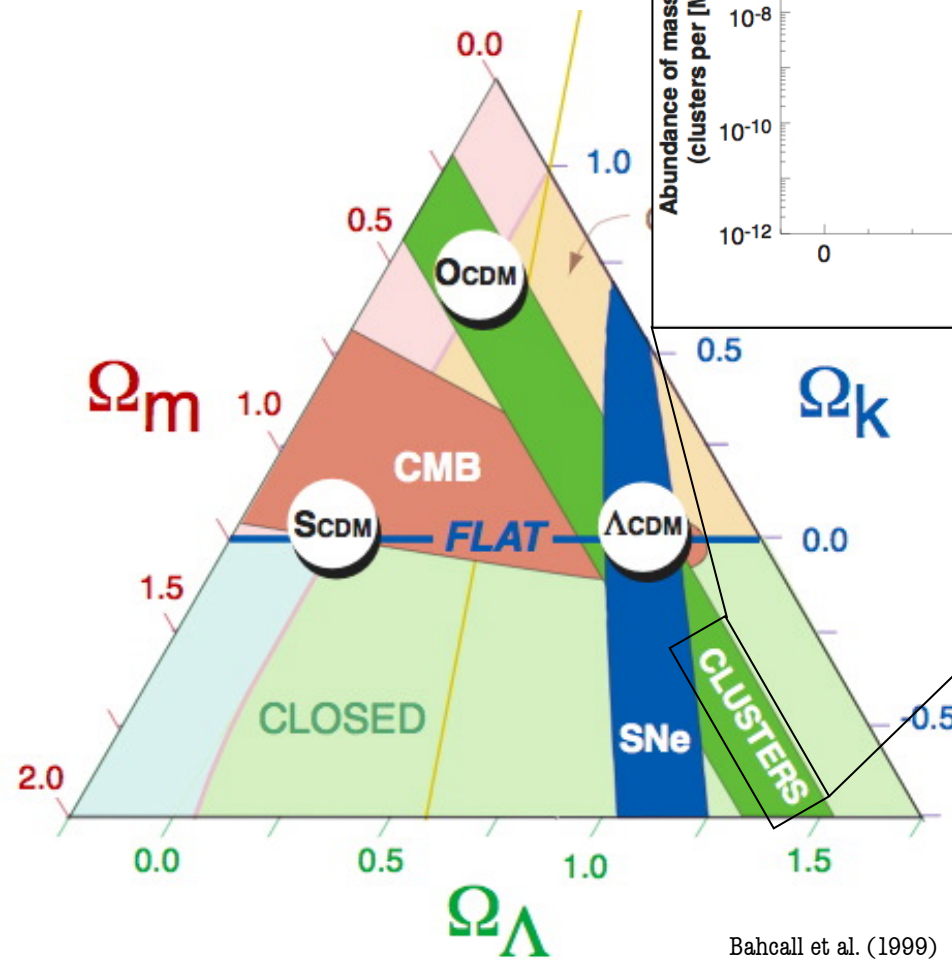
COSMOLOGY

Λ CDM - THE STANDARD MODEL

■ Λ CDM - Λ Cold Dark Matter

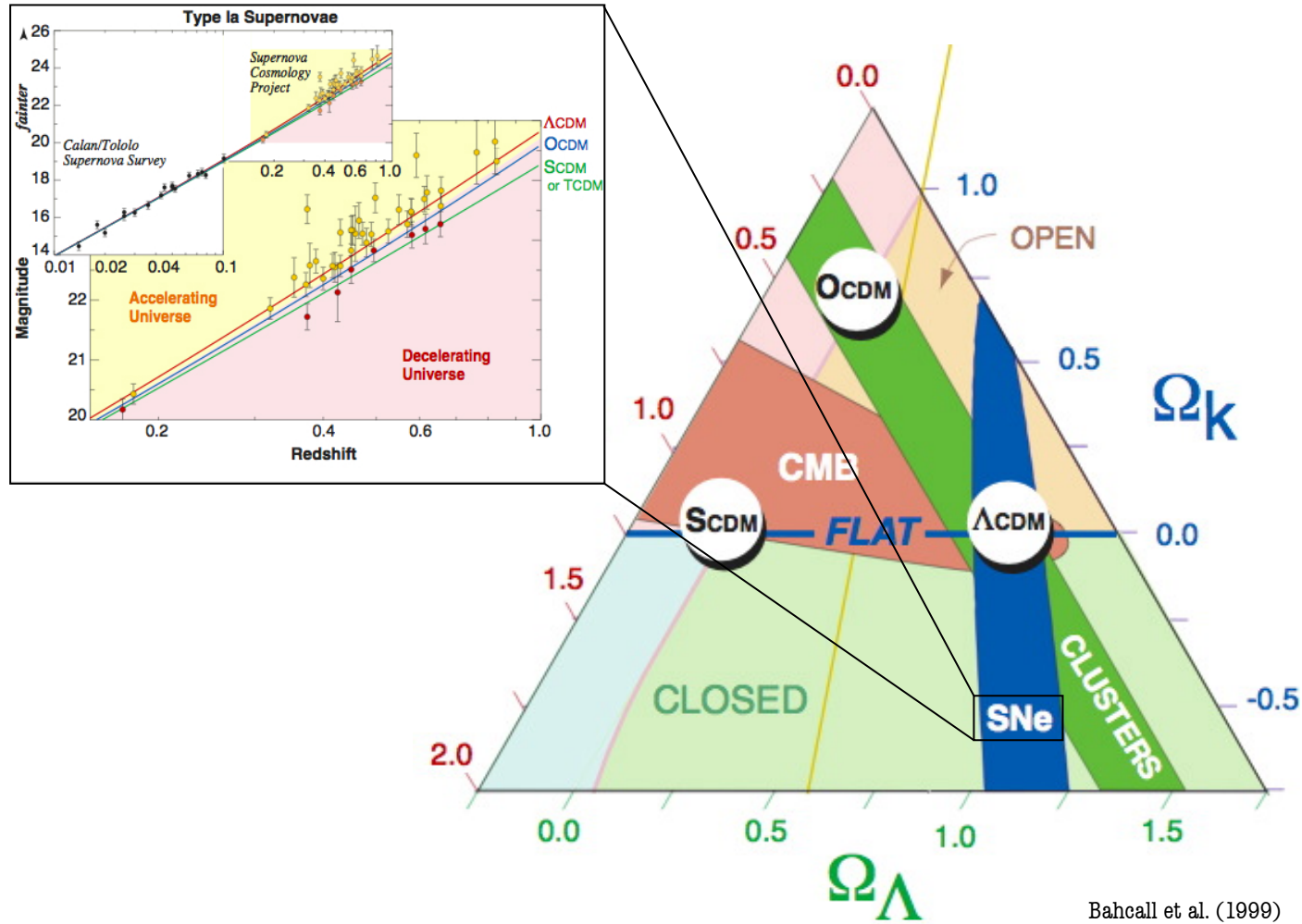


- Λ CDM - Λ Cold Dark Matter



Bahcall et al. (1999)

■ Λ CDM - Λ Cold Dark Matter



▪ background cosmology

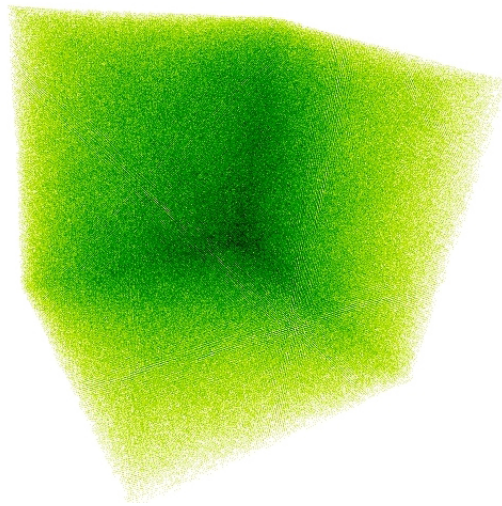
$$H^2 = H_0^2 (\Omega_m^0 a^{-3} + \Omega_\Lambda^0)$$

$$q = \frac{1}{2} \Omega_m - \Omega_\Lambda$$

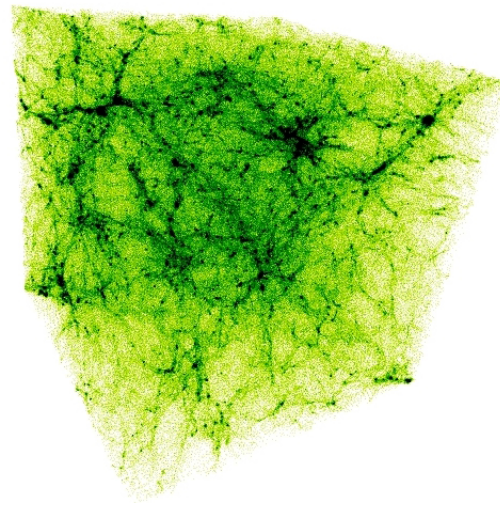
$$\Omega_m^0 \approx 0.3 = 0.26 + 0.04$$

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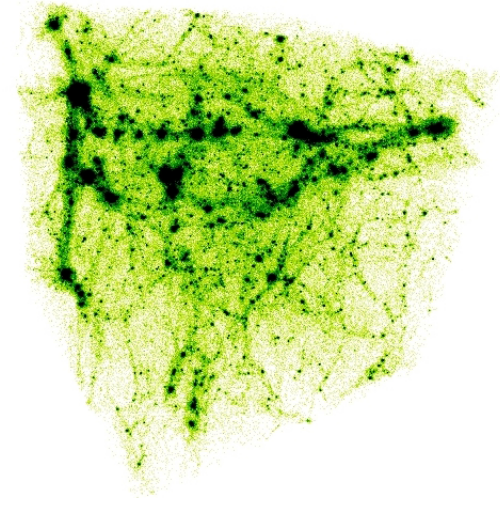
- the Universe is spatially flat
- ca. 30% of the total energy density comes from matter
 - about 85% of that matter is “dark”
 - **only 15% of all matter in the Universe is luminous**
- ca. 70% of the total energy density is vacuum energy



generating the initial conditions



running the simulation



analysing the data