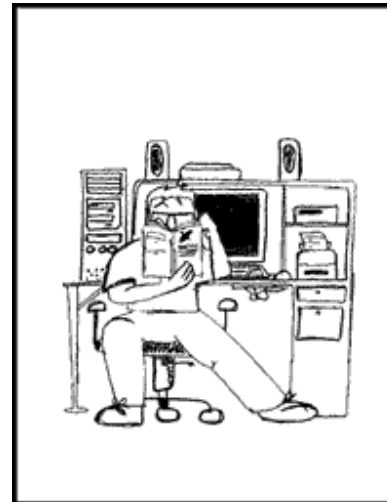
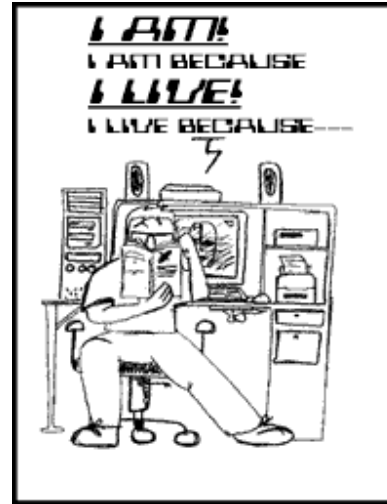


Computational Astrophysics

Solving for Gravity

Alexander Knebe, *Universidad Autonoma de Madrid*



Solving for Gravity

- Poisson's equation

$$\Delta\Phi = 4\pi G\rho$$

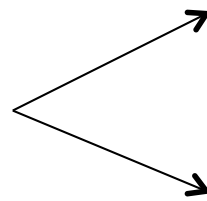
$$\vec{F} = -m\nabla\Phi$$

Solving for Gravity

- Poisson's equation

$$\Delta\Phi(\vec{r}) = 4\pi G\rho(\vec{r})$$

$$\vec{F}(\vec{r}) = -m\nabla\Phi(\vec{r})$$



particle approach

$$\vec{F}(\vec{r}_i) = -\sum_{i \neq j} \frac{Gm_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

grid approach ($\vec{r}_{i,j,k}$ = position of centre of grid cell (i,j,k))

$$\Delta\Phi(\vec{r}_{i,j,k}) = 4\pi G\rho(\vec{r}_{i,j,k})$$

$$\vec{F}(\vec{r}_{i,j,k}) = -m\nabla\Phi(\vec{r}_{i,j,k})$$

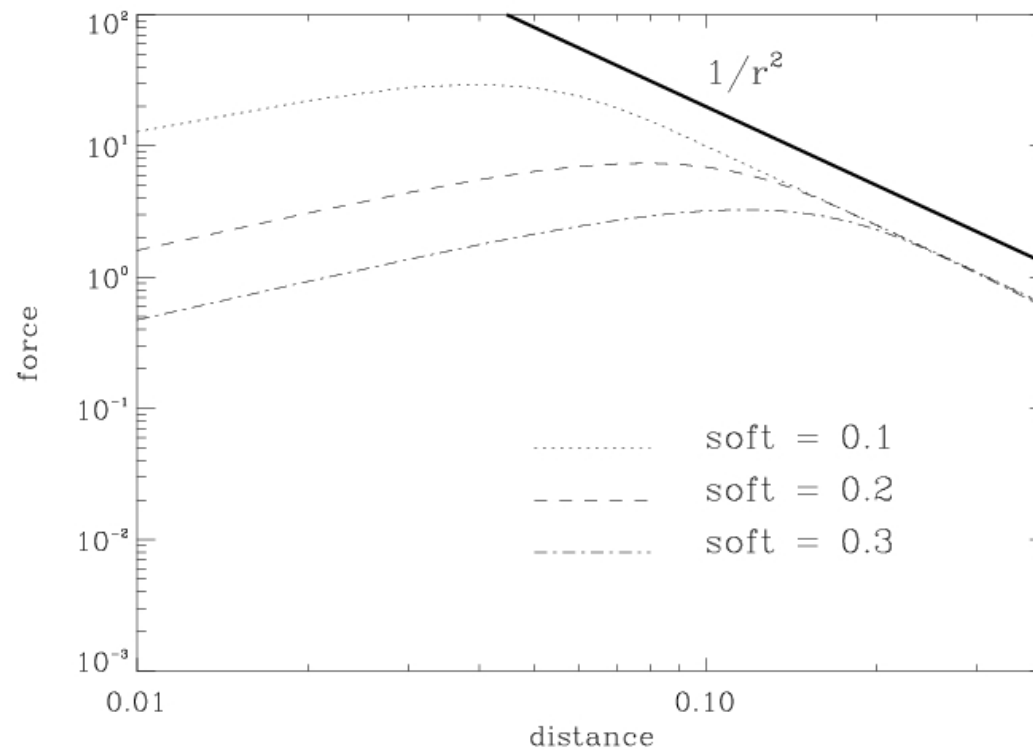
heart and soul of every N -body code

Solving for Gravity

▪ Poisson's equation

- the particle approach

$$\vec{F}(\vec{r}_i) = - \sum_{i \neq j} \frac{Gm_i m_j}{\left(|\vec{r}_i - \vec{x}_j|^2 + \epsilon^2 \right)^{3/2}} (\vec{r}_i - \vec{r}_j)$$

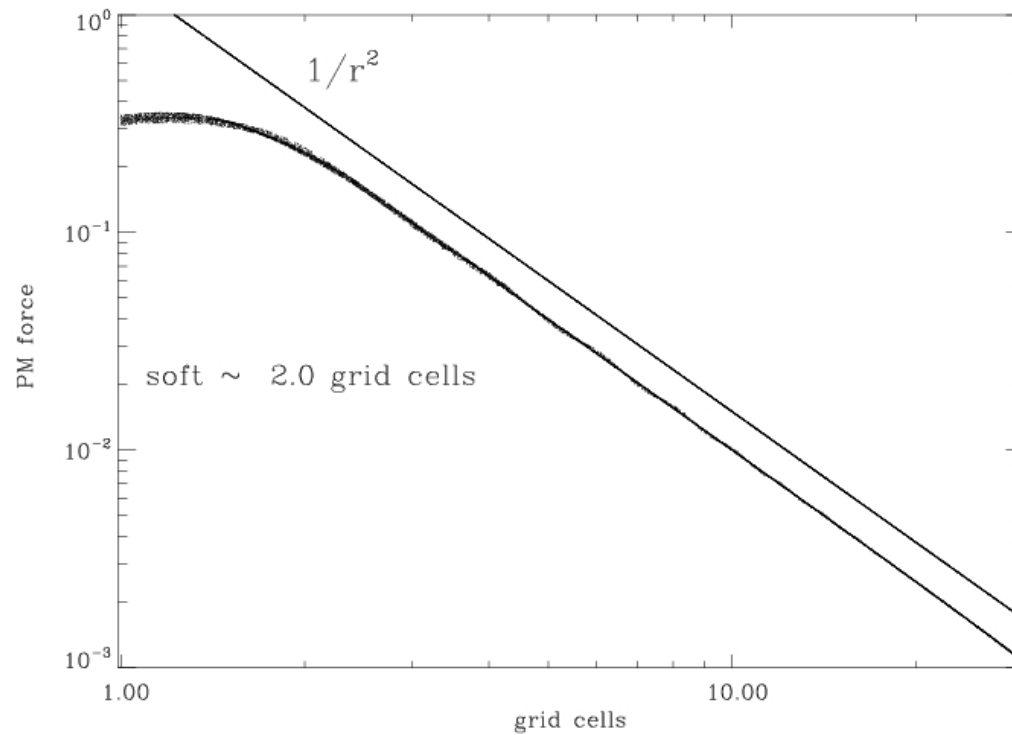


Solving for Gravity

- Poisson's equation
 - the grid approach

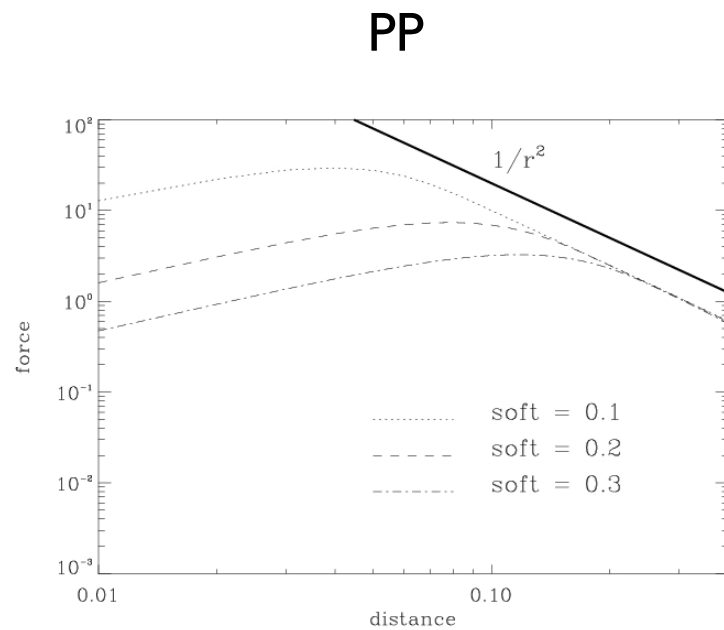
$$\Delta\Phi_{i,j,k} = 4\pi G\rho_{i,j,k}$$

$$\vec{F}_{i,j,k} = -m\nabla\Phi_{i,j,k}$$

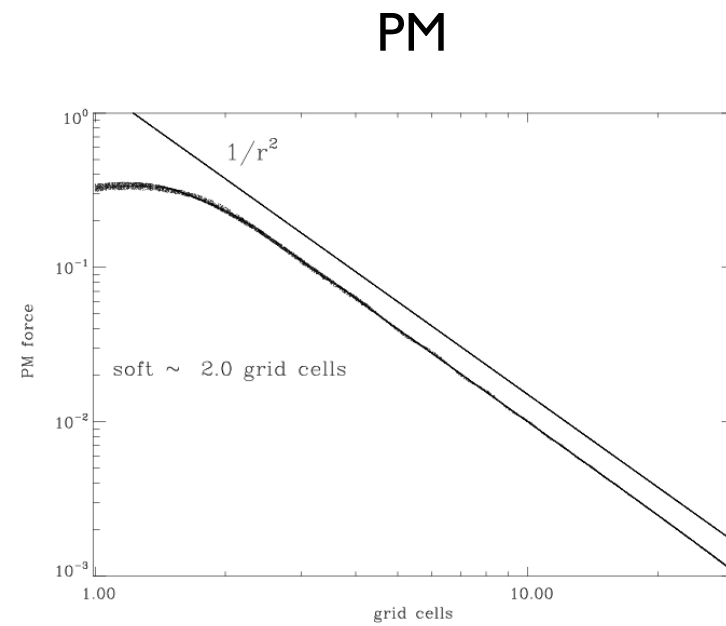


Solving for Gravity

- Poisson's equation
 - the P³M hybrid approach



+

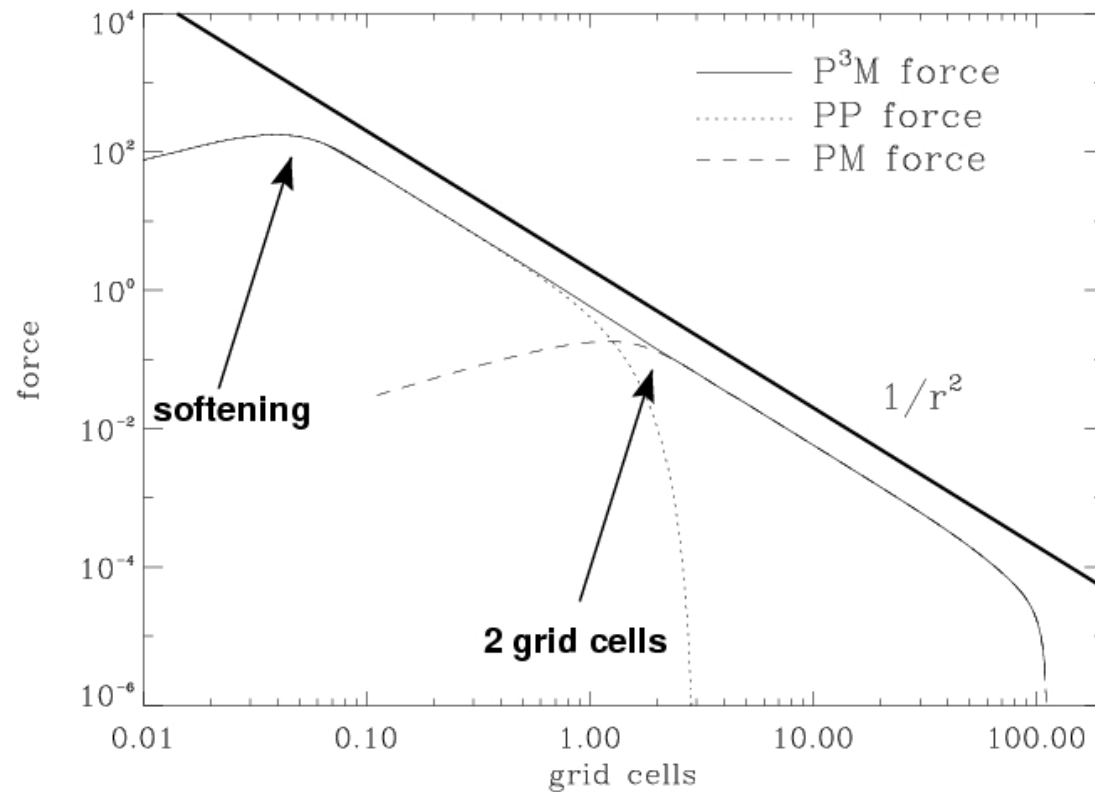


particle-particle-particle-mesh code...

Solving for Gravity

▪ Poisson's equation - **the P³M hybrid approach**

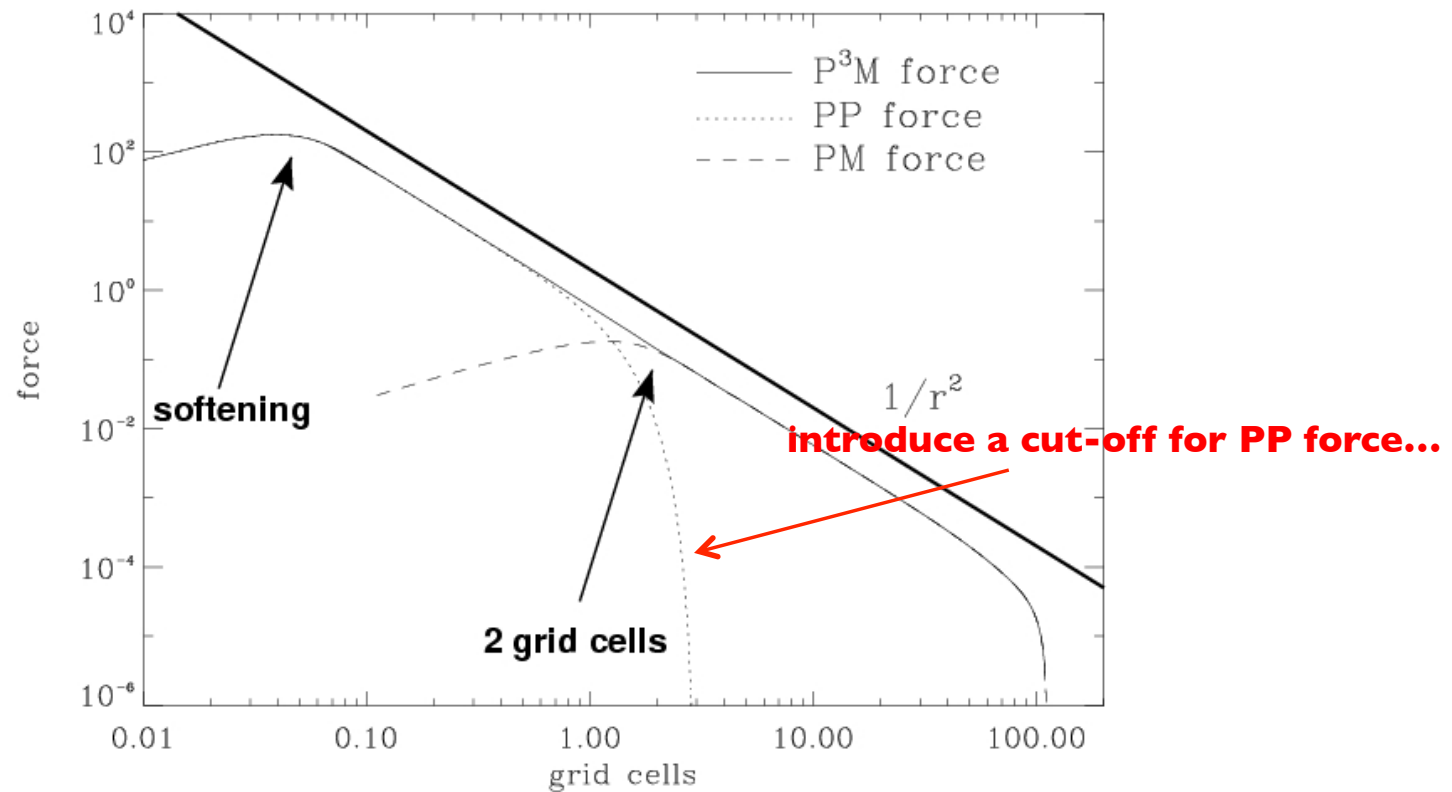
- short range force: pure PP method
- long range force: pure PM method



Solving for Gravity

▪ Poisson's equation - **the P³M hybrid approach**

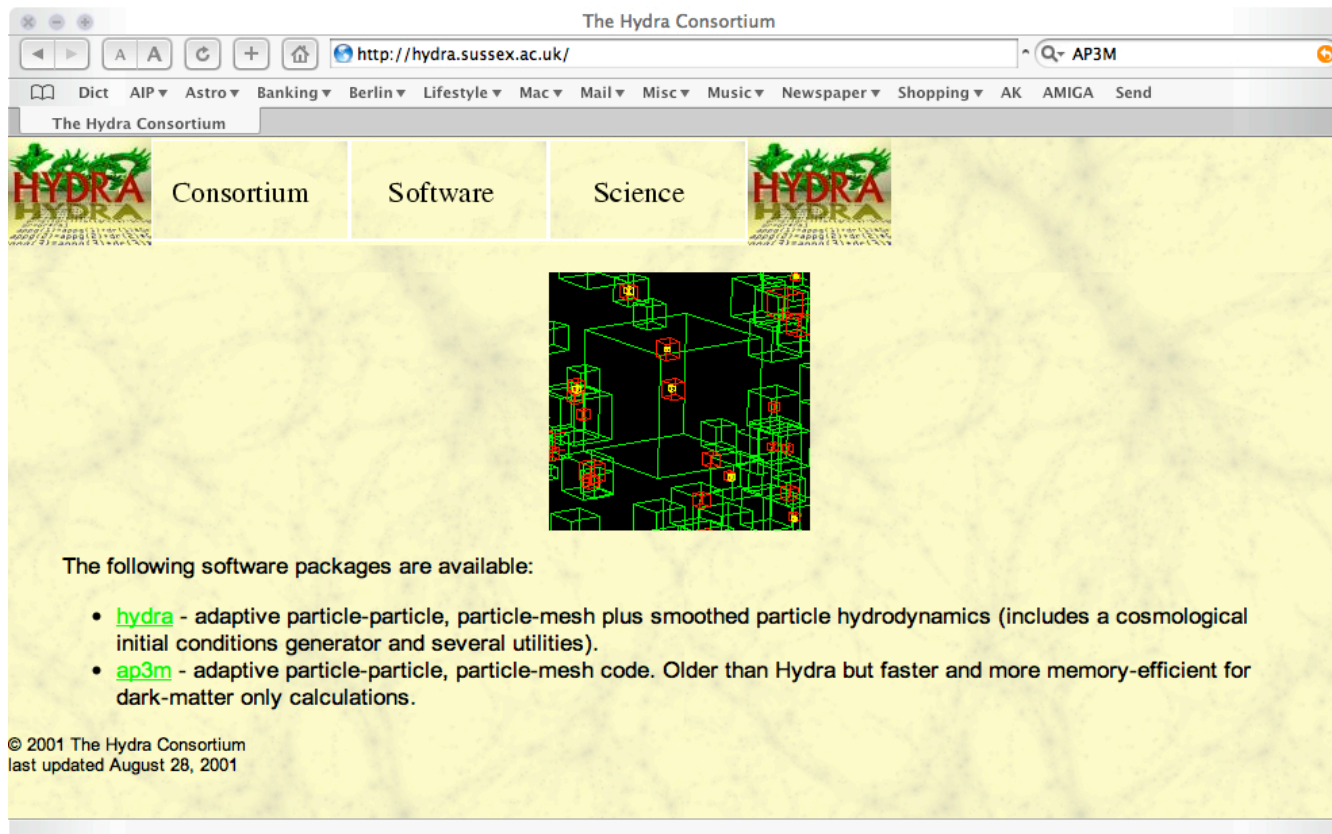
- short range force: pure PP method
- long range force: pure PM method



Solving for Gravity

▪ Poisson's equation - **the P³M hybrid approach**

- AP³M code (Couchman 1991)
- HYDRA code (Couchman, Thomas & Pearce 1995)



The screenshot shows a web browser window titled "The Hydra Consortium" with the URL <http://hydra.sussex.ac.uk/>. The browser's search bar contains "AP3M". The website has a navigation menu with links for "Consortium", "Software", and "Science". Below the navigation menu is a 3D visualization of a particle-mesh simulation, showing a grid of particles (red and yellow) and a mesh of green lines. The text "HYDRA" is visible on the left and right sides of the navigation menu.

The following software packages are available:

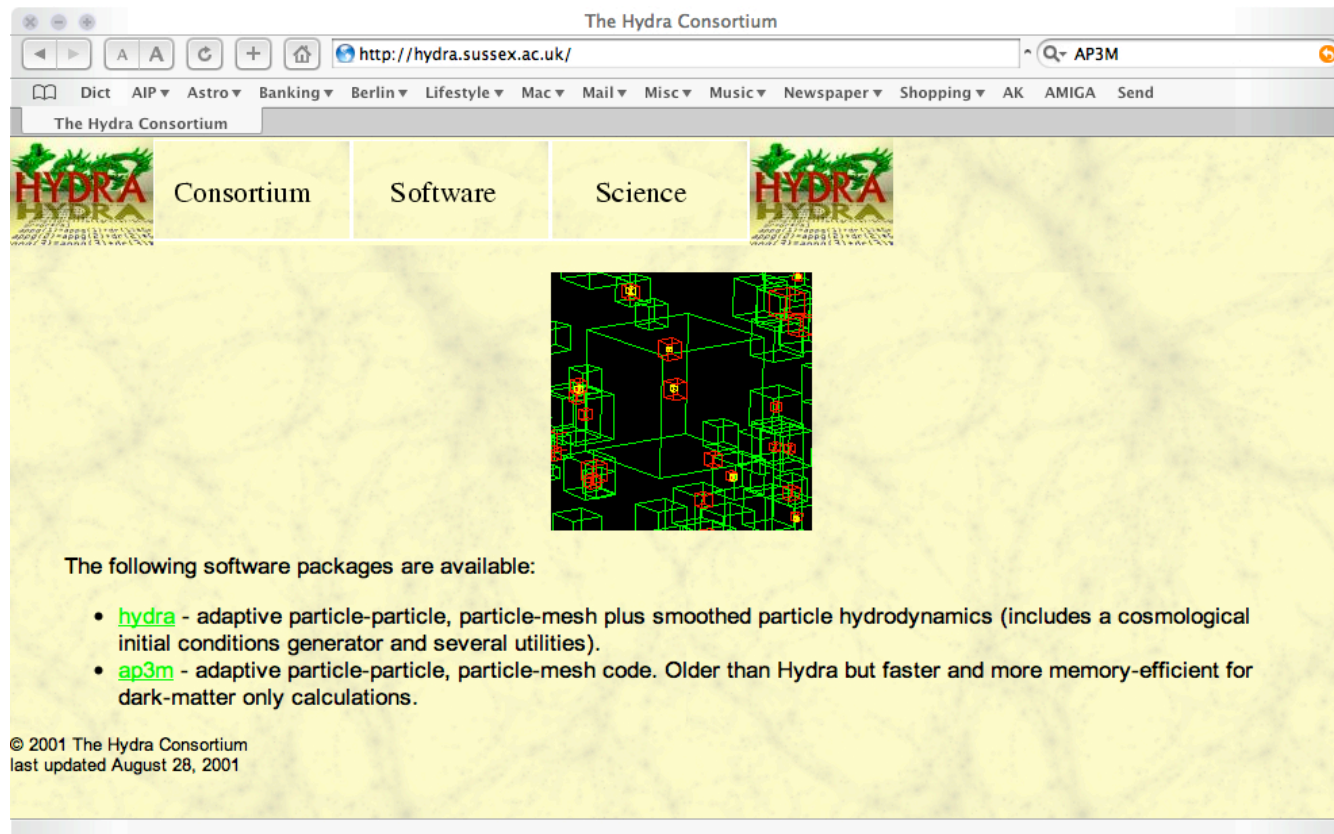
- [hydra](#) - adaptive particle-particle, particle-mesh plus smoothed particle hydrodynamics (includes a cosmological initial conditions generator and several utilities).
- [ap3m](#) - adaptive particle-particle, particle-mesh code. Older than Hydra but faster and more memory-efficient for dark-matter only calculations.

© 2001 The Hydra Consortium
last updated August 28, 2001

Solving for Gravity

- Poisson's equation - **the P³M hybrid approach**

- **AP³M** code (Couchman 1991)
Adaptive P³M
- HYDRA code (Couchman, Thomas & Pearce 1995)



Solving for Gravity

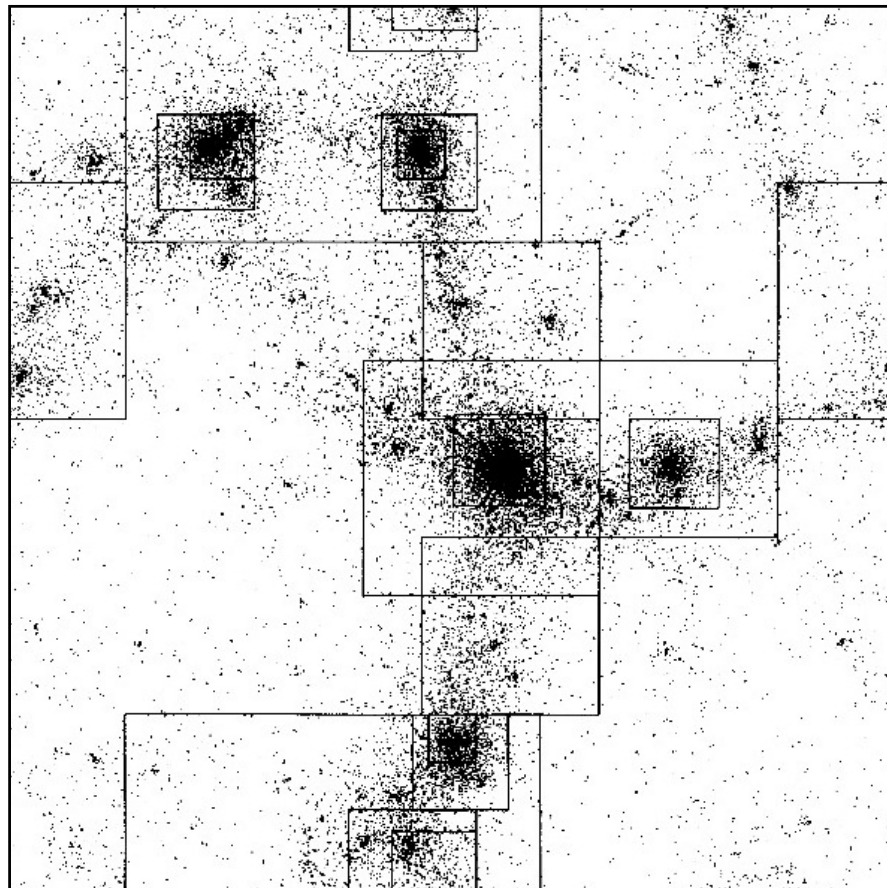
▪ Poisson's equation - **the P³M hybrid approach**

- AP³M code
Adaptive P³M

(Couchman 1991)

- HYDRA code

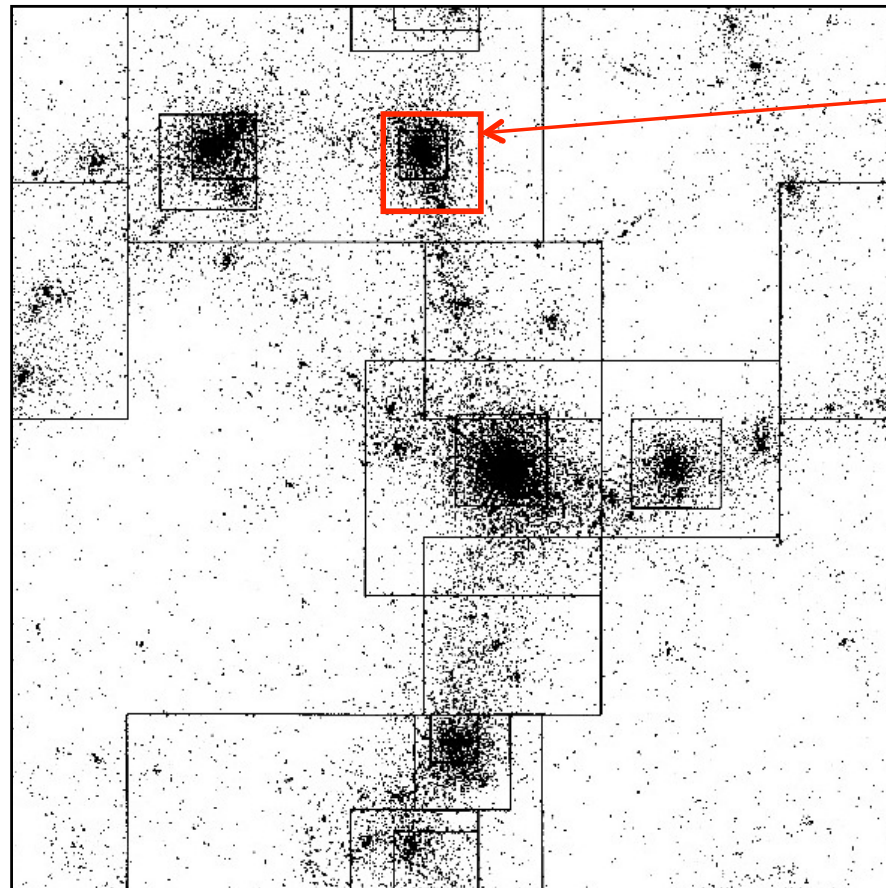
(Couchman, Thomas & Pearce 1995)



Solving for Gravity

▪ Poisson's equation - **the P³M hybrid approach**

- AP³M code (Couchman 1991)
Adaptive P³M
- HYDRA code (Couchman, Thomas & Pearce 1995)



**individual P³M calculation
with
isolated boundaries**

Solving for Gravity

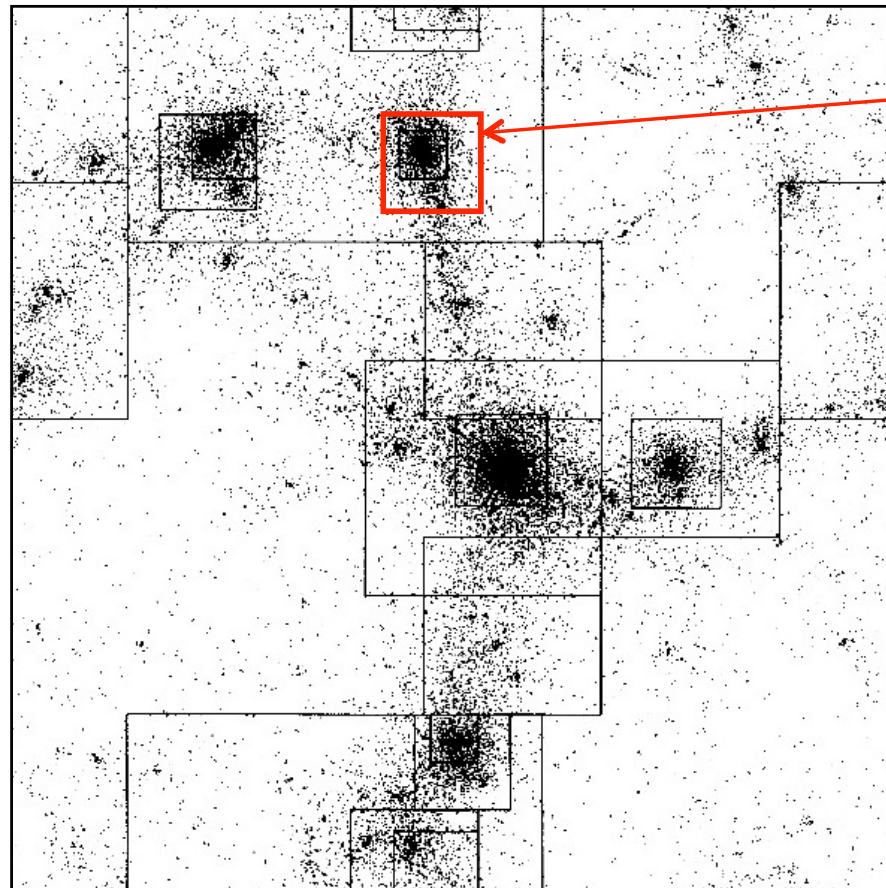
▪ Poisson's equation - **the P³M hybrid approach**

- AP³M code
Adaptive P³M

(Couchman 1991)

- HYDRA code

(Couchman, Thomas & Pearce 1995)



individual P³M calculation
with
isolated boundaries



no gain in accuracy,
“just” speed-up...

Solving for Gravity

- Poisson's equation - **the hybrid approach**

- (A)P³M

(Couchman 1991)

- Tree-PM

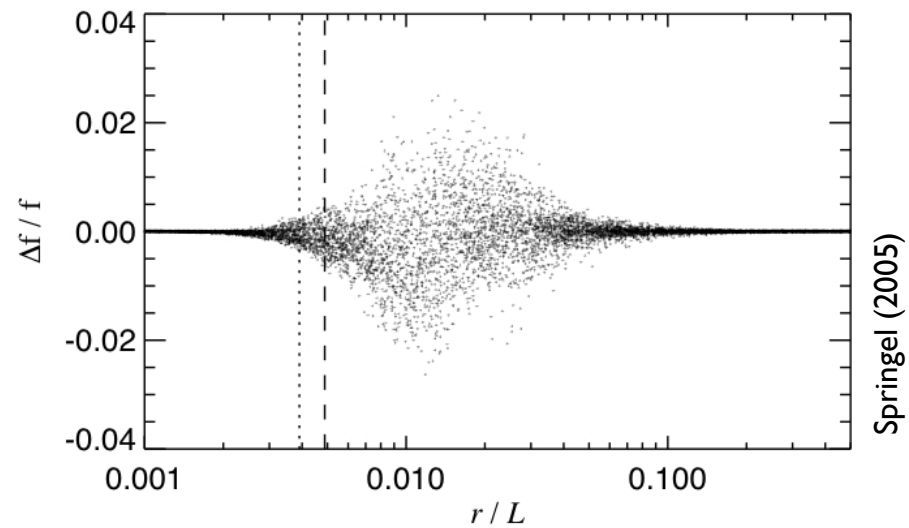
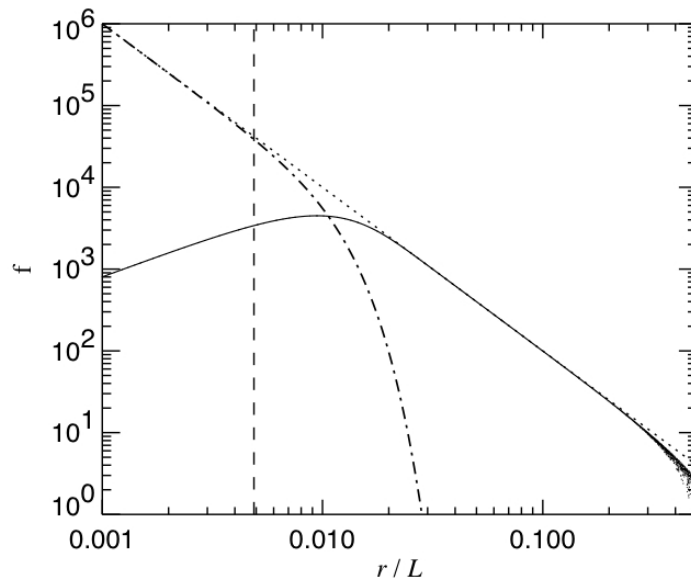
(Xu 1995; Bode & Ostriker 2003; Dubinski et al. 2004; Springel 2005)

- ...

Solving for Gravity

▪ Poisson's equation - **the hybrid approach**

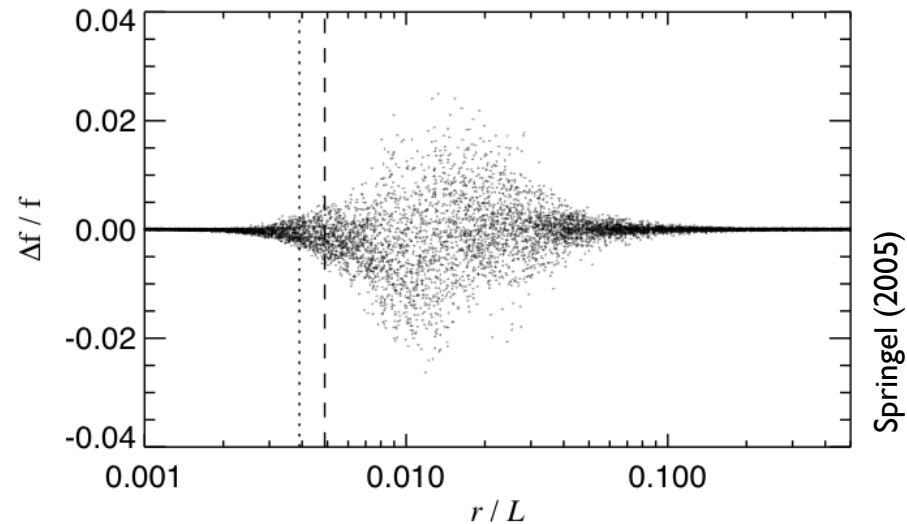
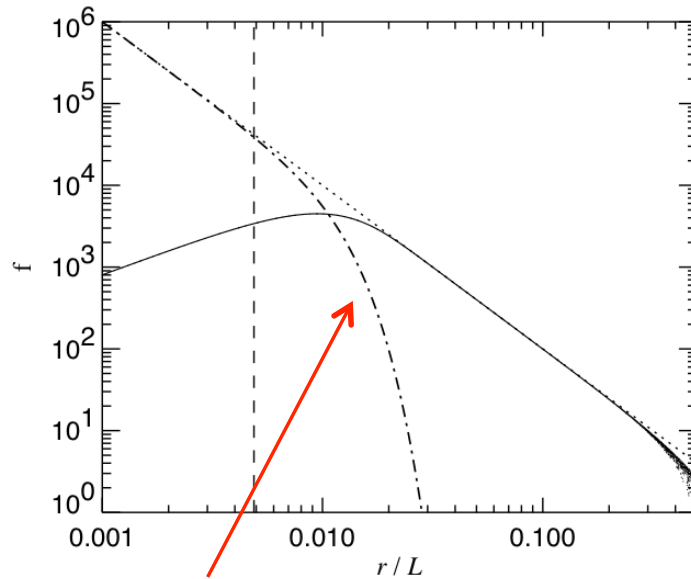
- (A)P³M (Couchman 1991)
- **Tree-PM** (Xu 1995; Bode & Ostriker 2003; Dubinski et al. 2004; Springel 2005)
- ...



Solving for Gravity

▪ Poisson's equation - **the hybrid approach**

- (A)P³M (Couchman 1991)
- **Tree-PM** (Xu 1995; Bode & Ostriker 2003; Dubinski et al. 2004; Springel 2005)
- ...



modified via short-range cut-off factor again...

Solving for Gravity

▪ Poisson's equation - **the hybrid approach**

- (A)P³M (Couchman 1991)
- **Tree-PM** (Xu 1995; Bode & Ostriker 2003; Dubinski et al. 2004; Springel 2005)
- ...

- tree walk only in spatial vicinity of target particle
- no periodic boundaries

=> substantial performance improvement!

Solving for Gravity

- Poisson's equation - **the hybrid approach**

- (A)P³M (Couchman 1991)
- Tree-PM (Xu 1995; Bode & Ostriker 2003; Dubinski et al. 2004; Springel 2005)
- ...