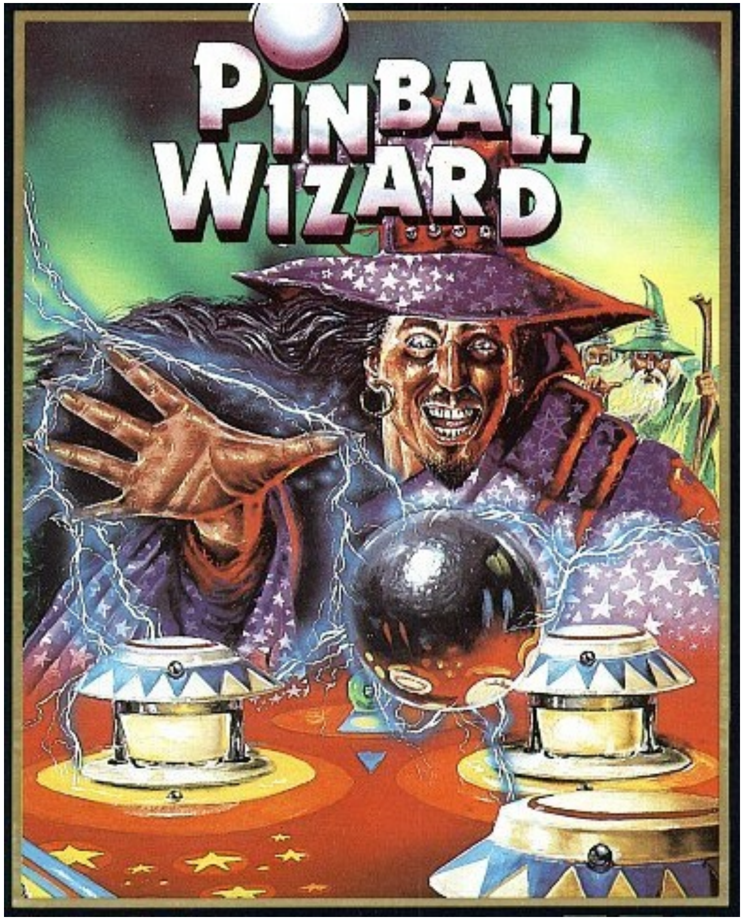
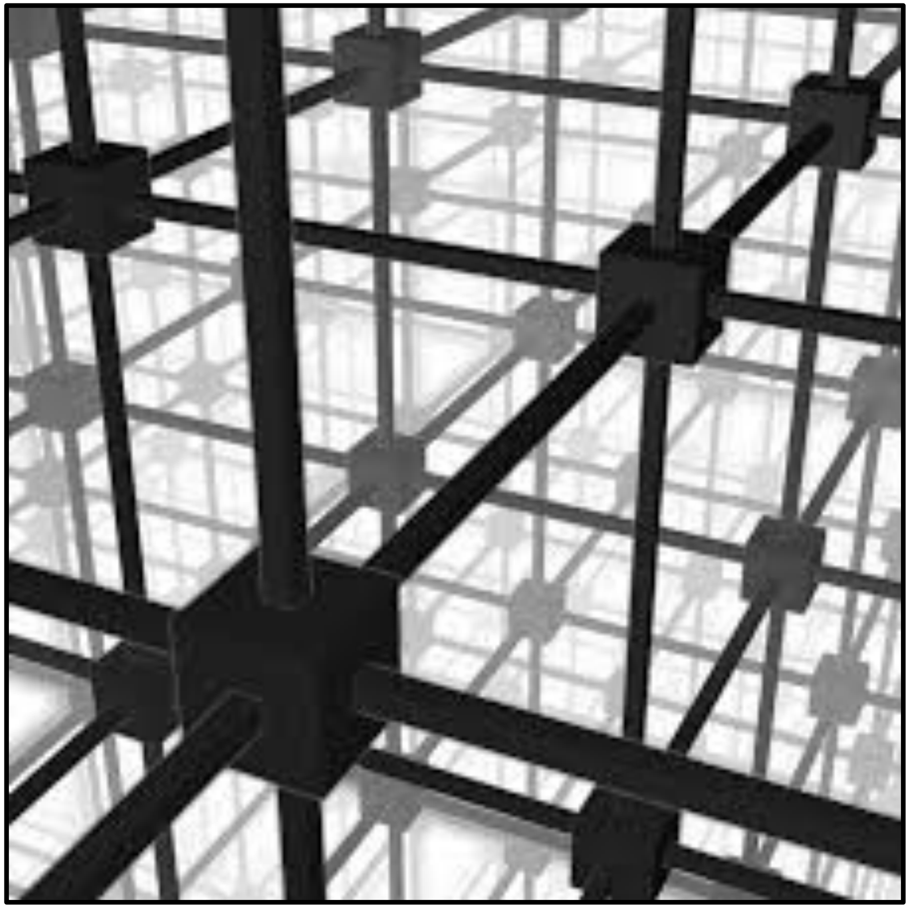
Alexander Knebe, *Universidad Autonoma de Madrid*



VS



Solving for Gravity

- full set of equations

• collisionless matter (e.g. dark matter)

$$\frac{d\vec{x}_{DM}}{dt} = \vec{v}_{DM}$$

$$\frac{d\vec{v}_{DM}}{dt} = -\nabla\phi$$

• Poisson's equation

$$\Delta\phi = 4\pi G\rho_{tot}$$

• collisional matter (e.g. gas)

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\vec{v}\right) = 0$$

$$\frac{\partial\left(\rho\vec{v}\right)}{\partial t} + \nabla\cdot\left(\rho\vec{v}\otimes\vec{v} + \left(p + \frac{1}{2\mu}B^2\right)\overleftrightarrow{1} - \frac{1}{\mu}\vec{B}\otimes\vec{B}\right) = \rho\ \left(-\nabla\phi\right)$$

$$\frac{\partial\left(\rho E\right)}{\partial t} + \nabla\cdot\left(\left[\rho E + p + \frac{1}{2\mu}B^2\right]\vec{v} - \frac{1}{\mu}\left[\vec{v}\cdot\vec{B}\right]\vec{B}\right) = \rho\vec{v}\cdot\left(-\nabla\phi\right) + \left(\Gamma - L\right)$$

• ideal gas equations

$$p = \left(\gamma - 1\right)\rho\varepsilon$$

$$\rho\varepsilon = \rho E - \frac{1}{2}\rho v^2$$

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$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times\left(\vec{v}\times\vec{B}\right)$$

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leap-frog integration

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later...

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hyperbolic partial differential equations:
solutions are wave-like, i.e. perturbations need time to travel...

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electrodynamics lecture...

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physics lecture...

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now!

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elliptical partial differential equation:
solution obtainable via FFT (for constant coefficients)

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- Poisson's equation vs. Poisson's integral
- tree codes
- PM codes

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$$\Phi(\vec{x}) = \iiint \mathcal{G}(\vec{x}-\vec{x}')\,\rho(\vec{x}')d^3x' \quad ; \quad \mathcal{G}(\vec{x}) = \frac{1}{4\pi\,x}$$

proof: see pages 18-35 of http://popia.ft.uam.es/aknebe/page3/files/ComputationalAstrophysics/Gravity01-TreeCodes.pdf

Solving for Gravity

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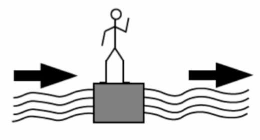
grid approach



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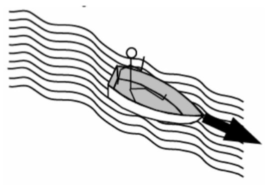
particle approach



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<u>grid approach</u>

$$\Delta\Phi(\vec{r}_{i,j,k}) = 4\pi G\rho(\vec{r}_{i,j,k})$$

($\vec{r}_{i,j,k}$ =position of centre of grid cell (i,j,k))

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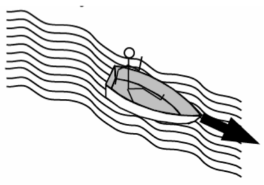

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$$\rho(\vec{r}) = \sum_{i=1}^{N} m_i \delta_{\text{Dirac}}(\vec{r}-\vec{r}_i)$$

<u>particle approach</u>

$$\vec{F}(\vec{r}_i) = -\sum_{i\neq j} \frac{G m_i m_j}{(r_i - r_j)^3}(\vec{r}_i - \vec{r}_j)$$

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weapon of choice: AMR codes

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weapon of choice: tree codes

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- Poisson’s equation vs. Poisson’s integral
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Solving for Gravity

- direct particle-particle summation (PP)

$$\vec{F}_i(\vec{r}_i) = -\sum_{i\neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$

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↯ drawback: extremely time consuming (N^2 operations)

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overcoming the "N^2" issue?!

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organizing particles into a "tree structure" will give $N \log(N)$ operations

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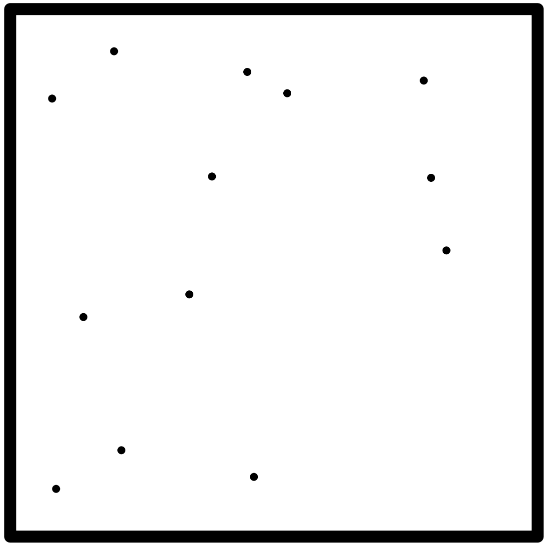

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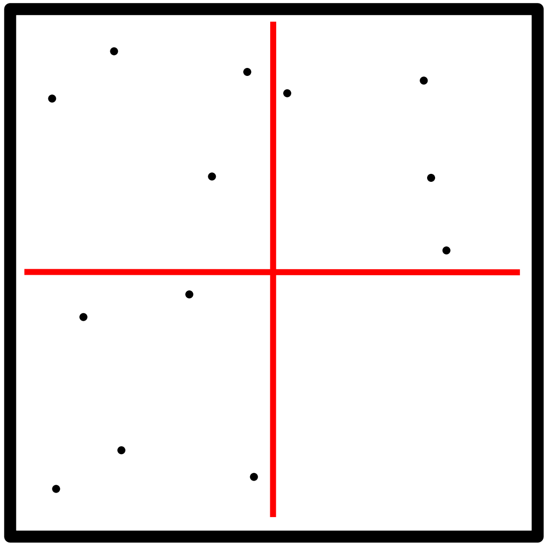

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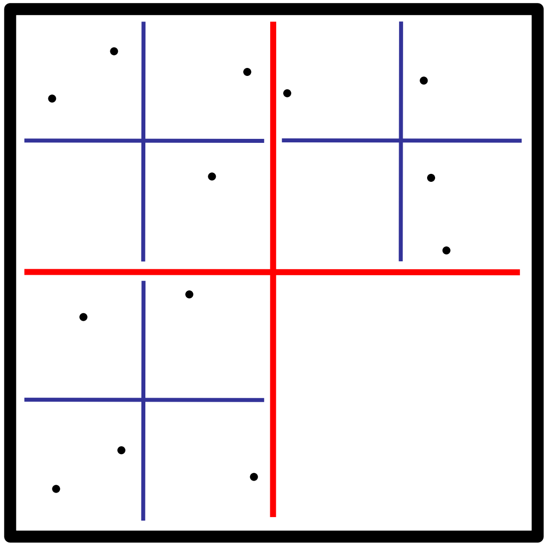

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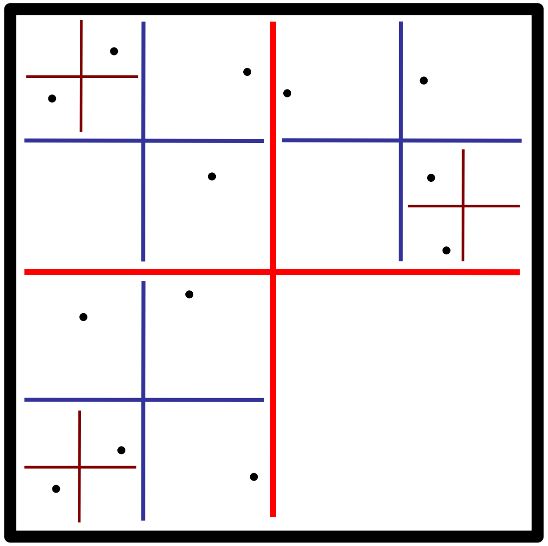

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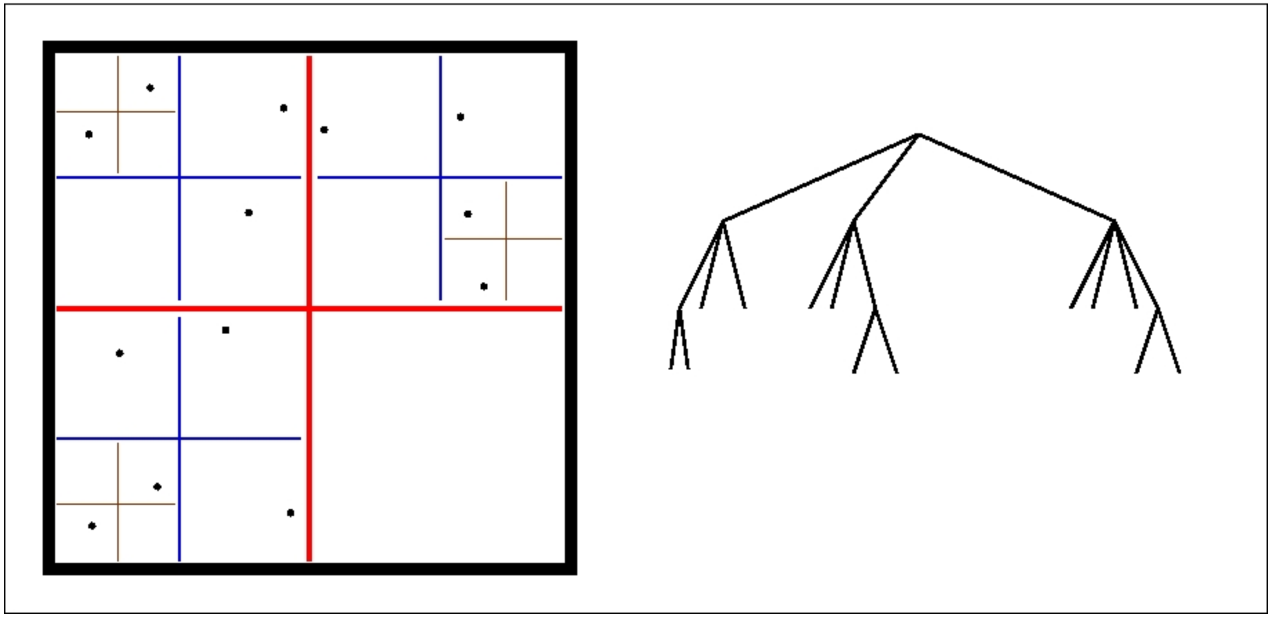

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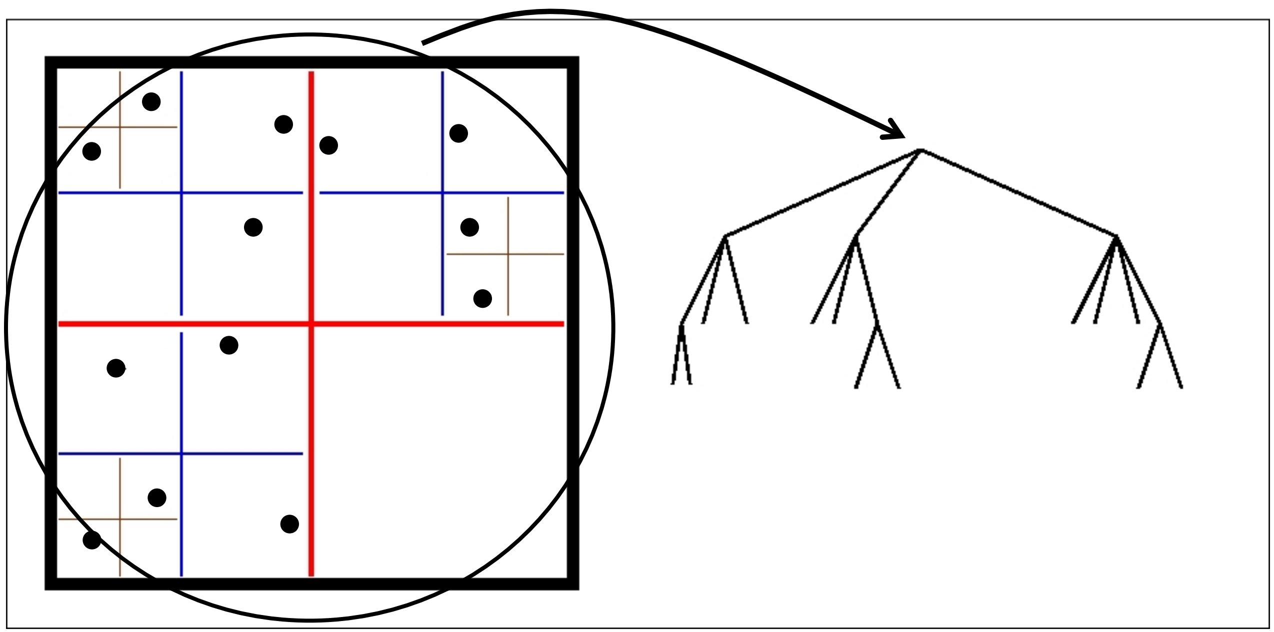

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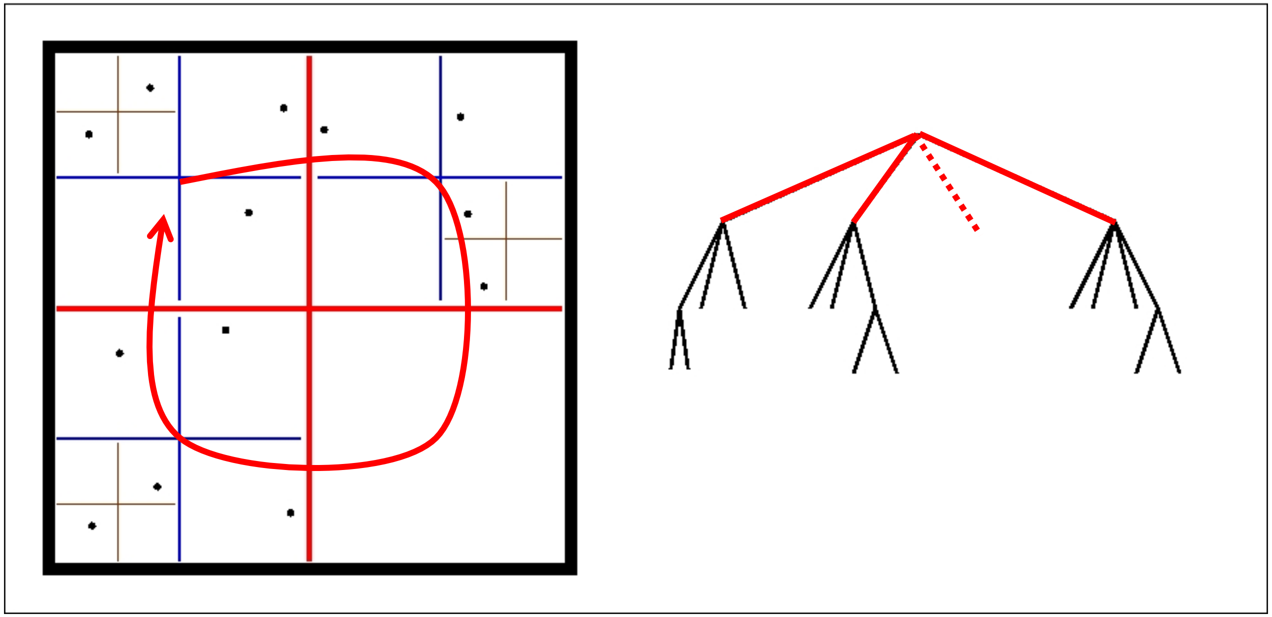

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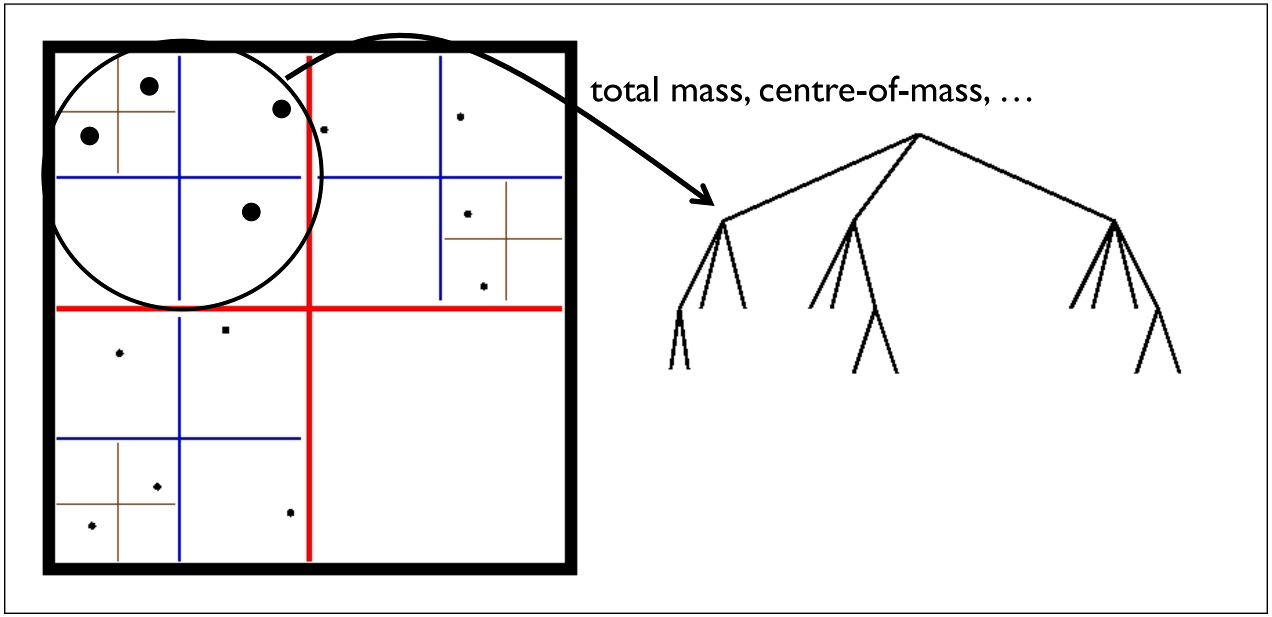

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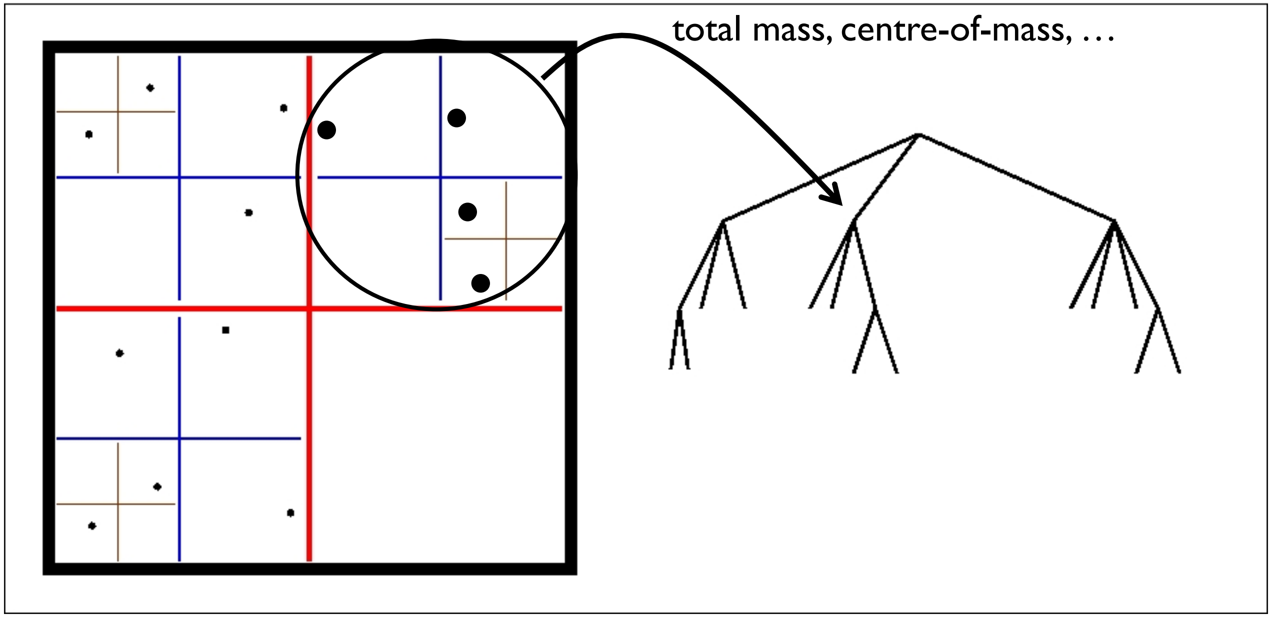

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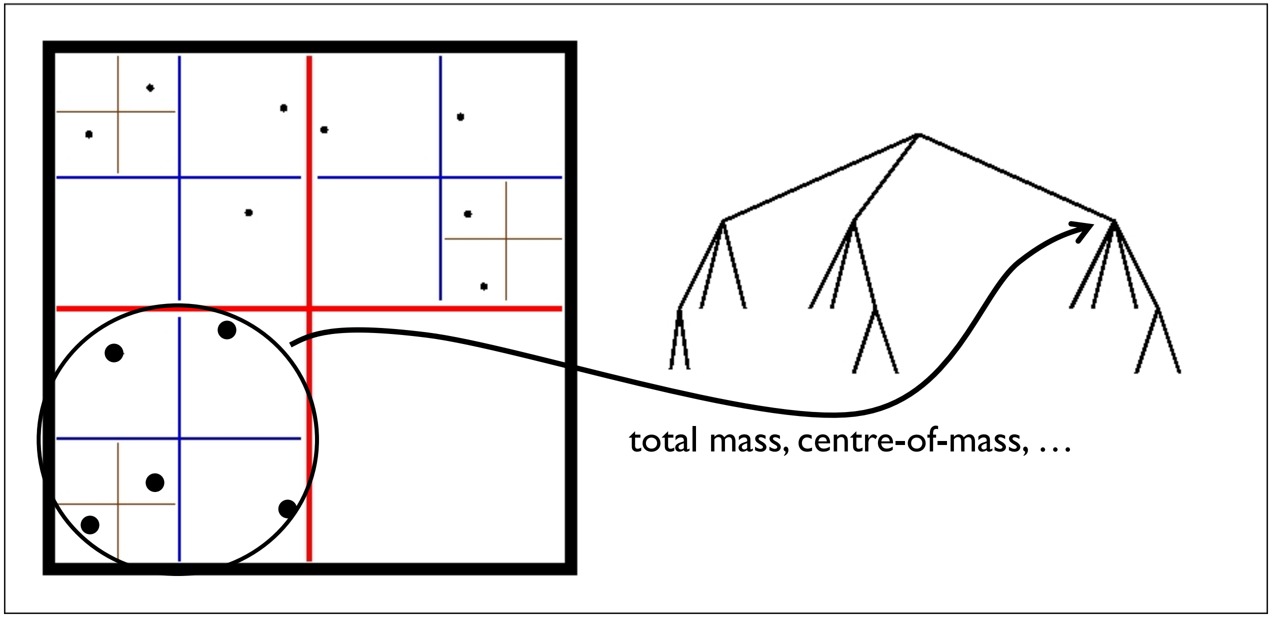

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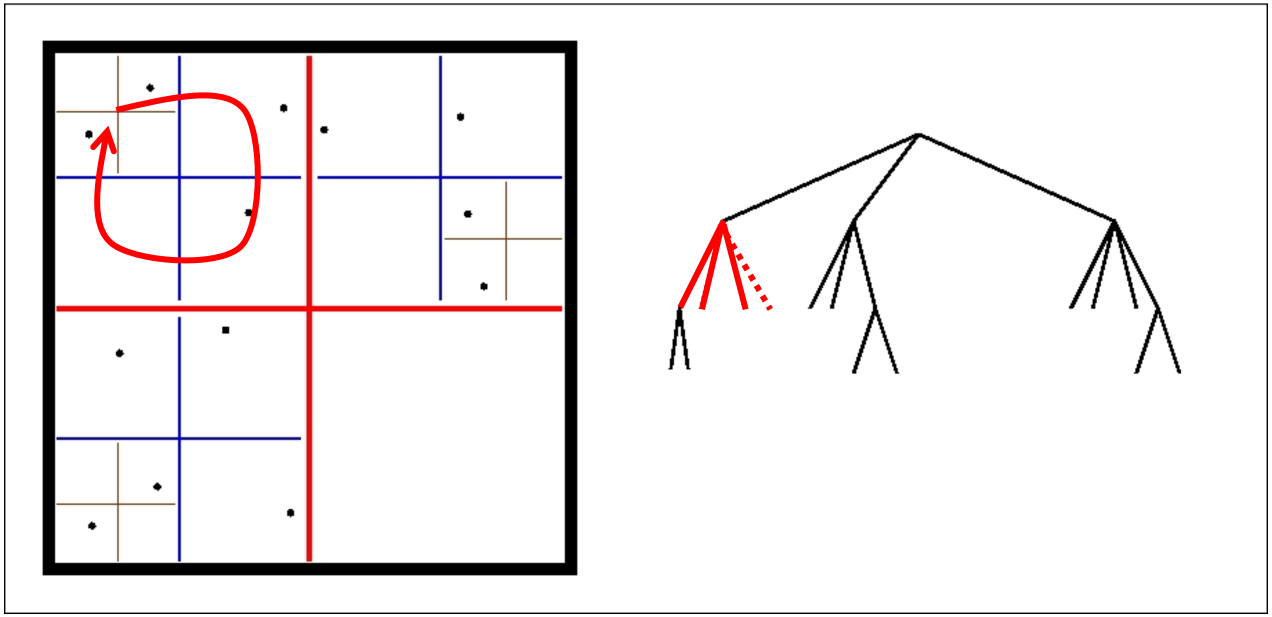

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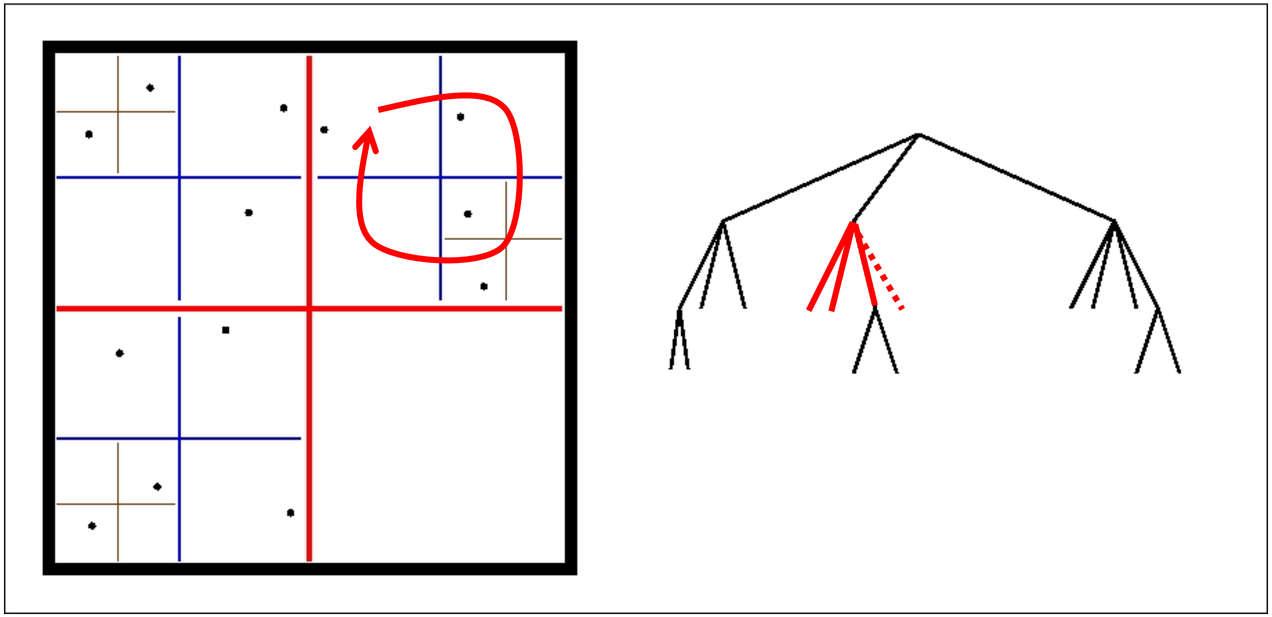

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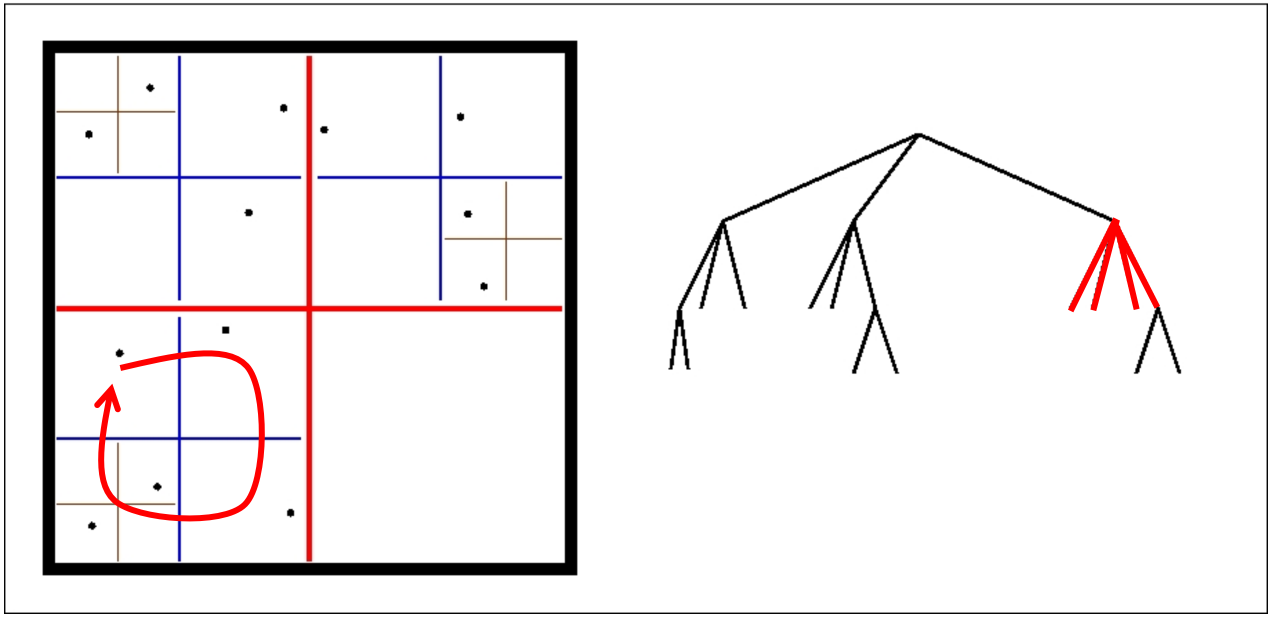

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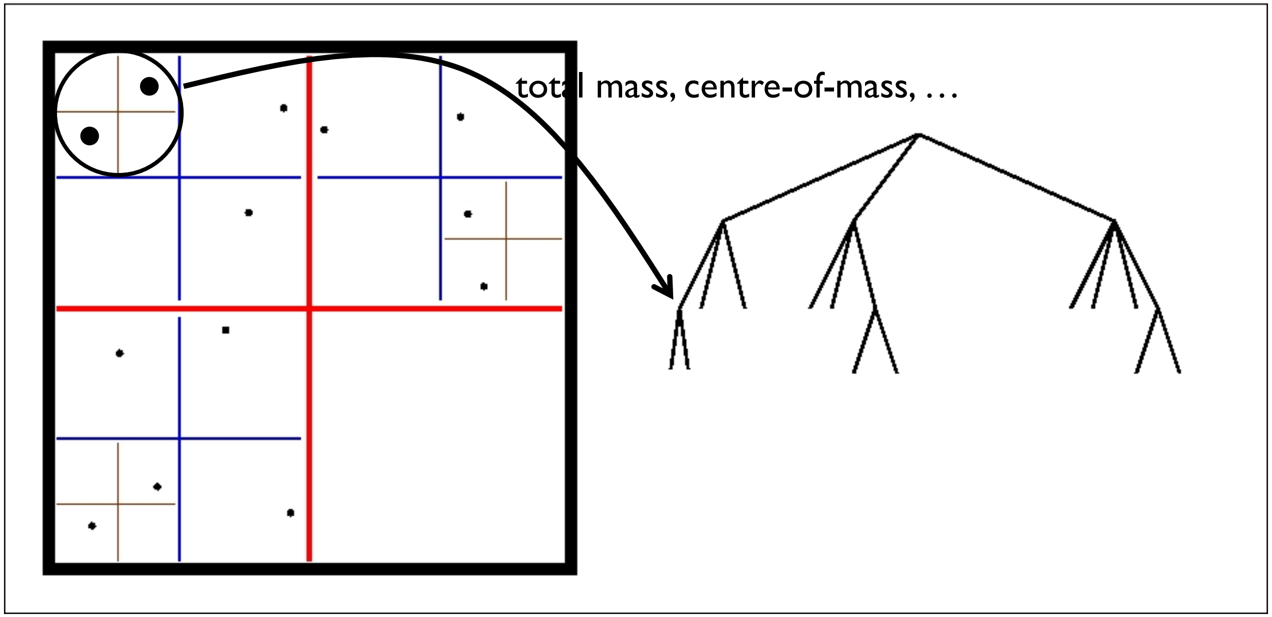

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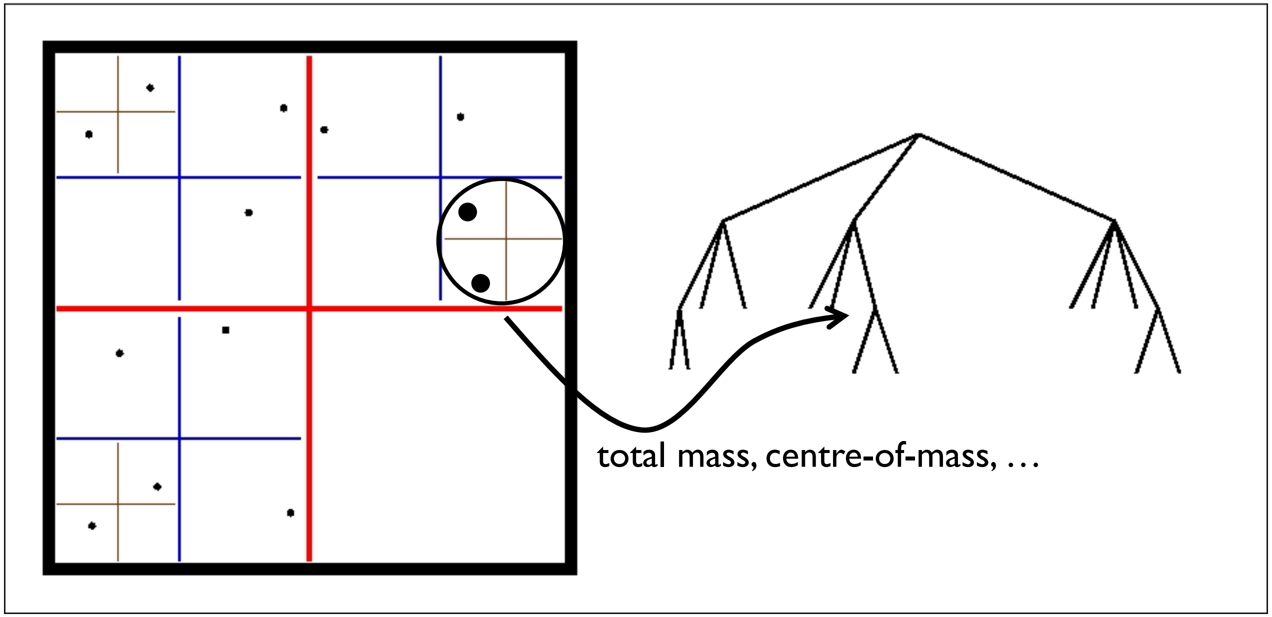

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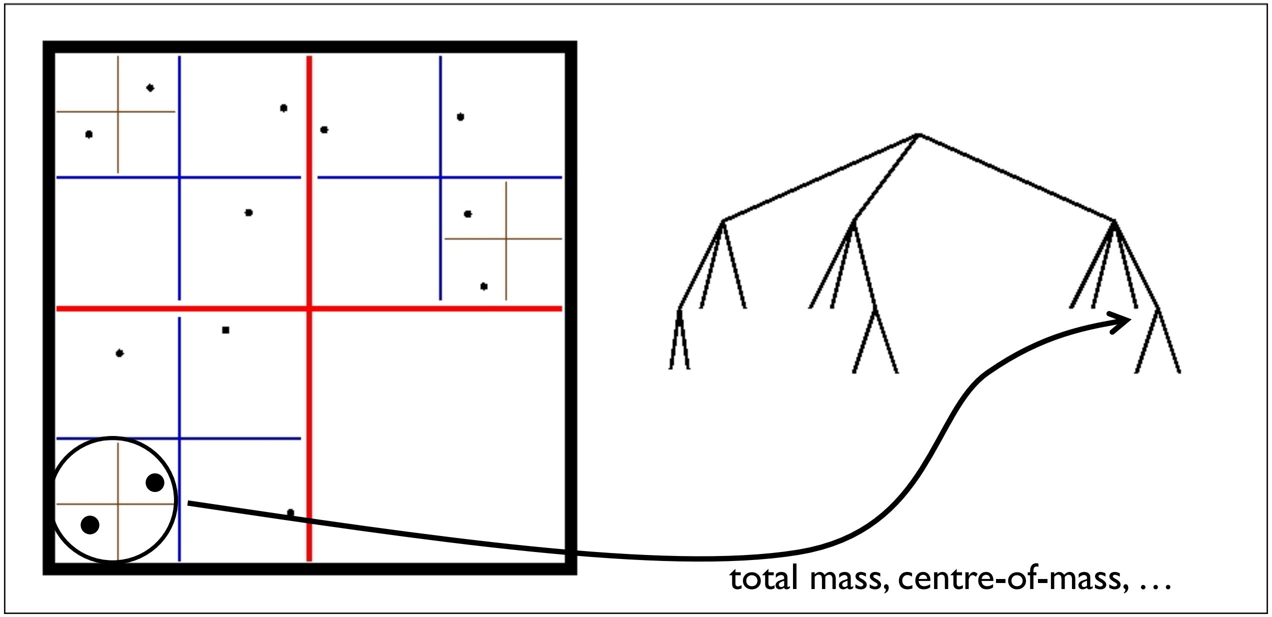

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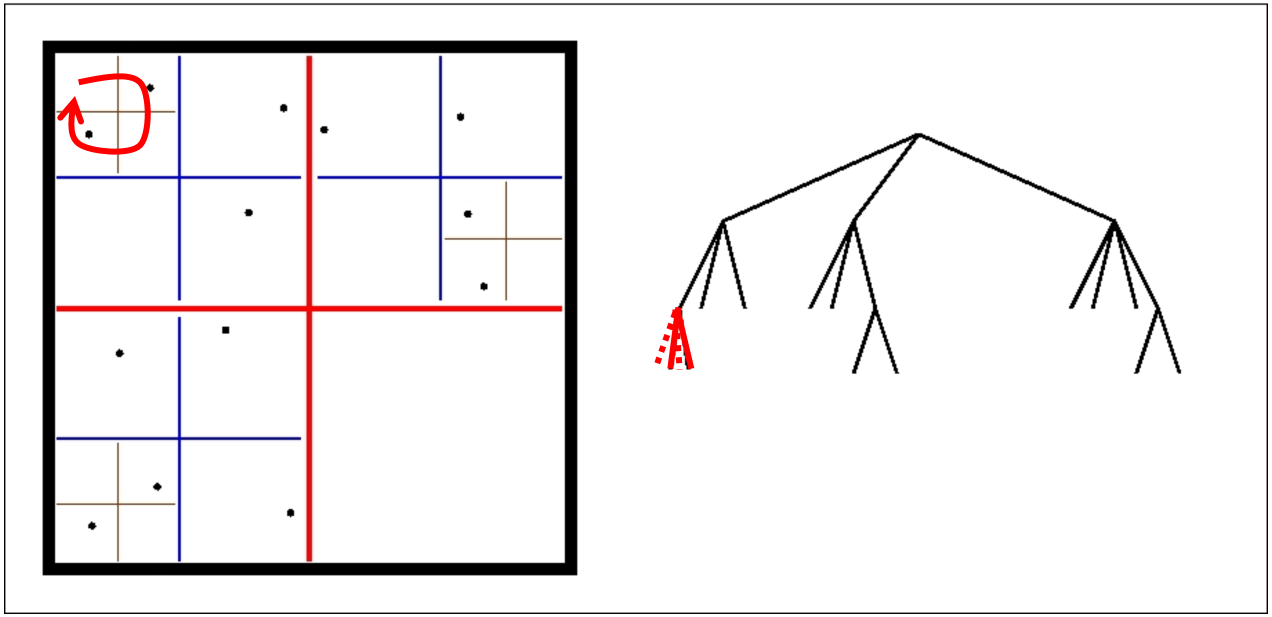

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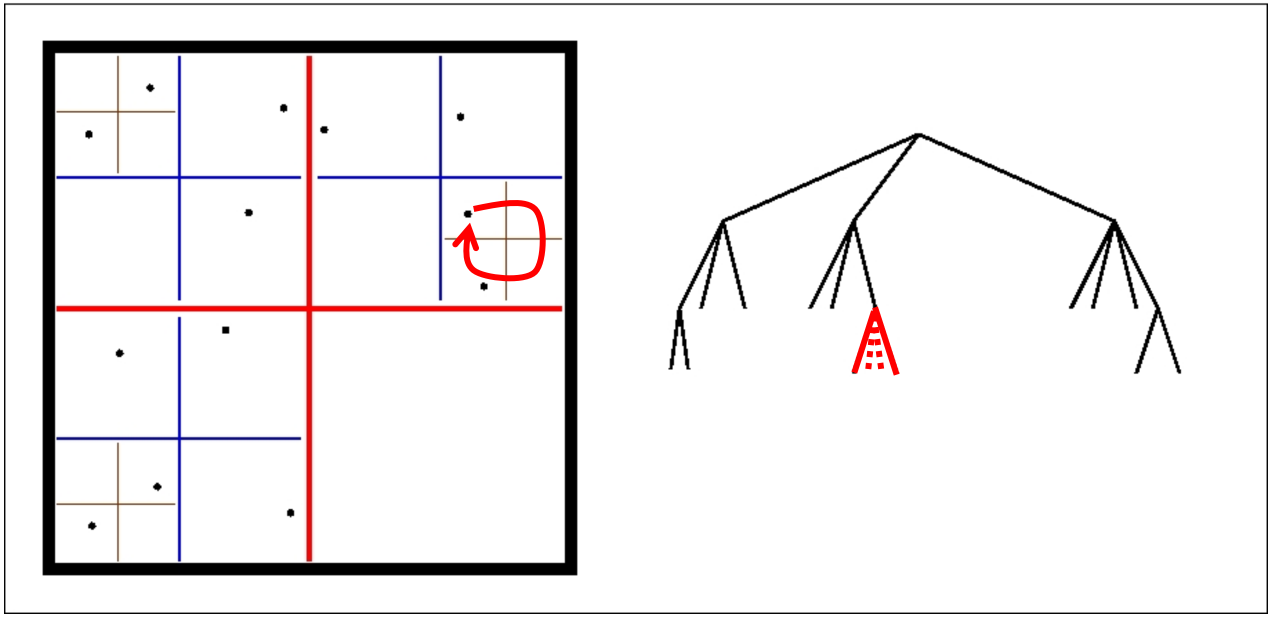
$$\vec{F}_i(\vec{r}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$

- generating the tree:



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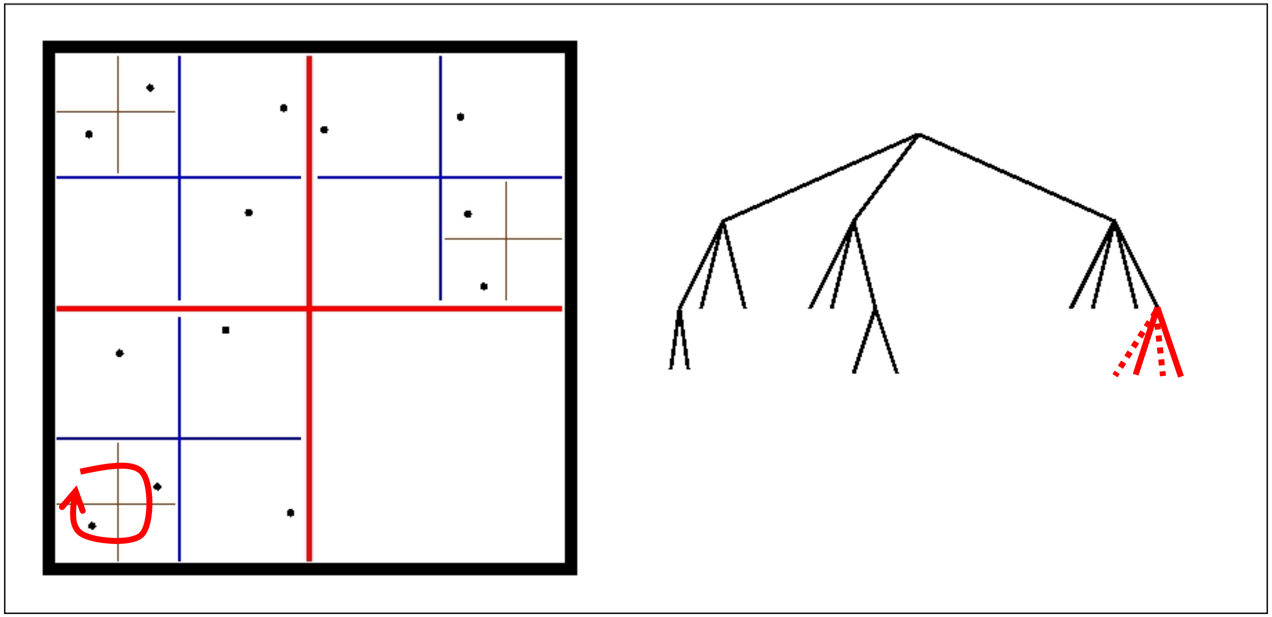

Solving for Gravity

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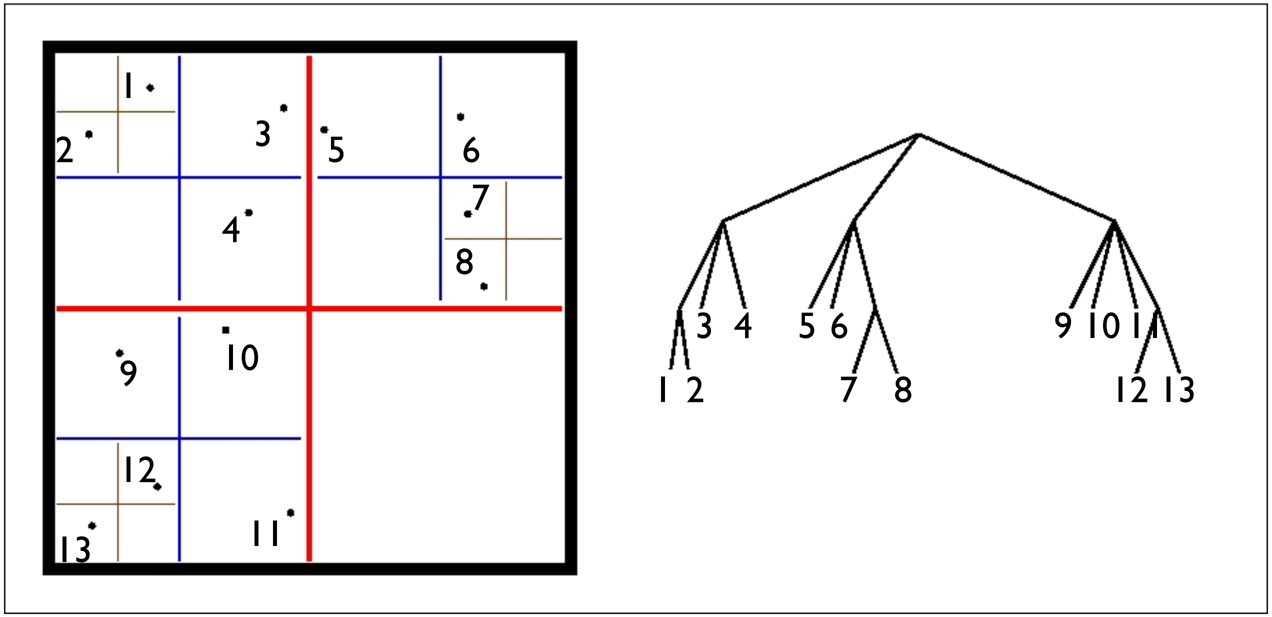

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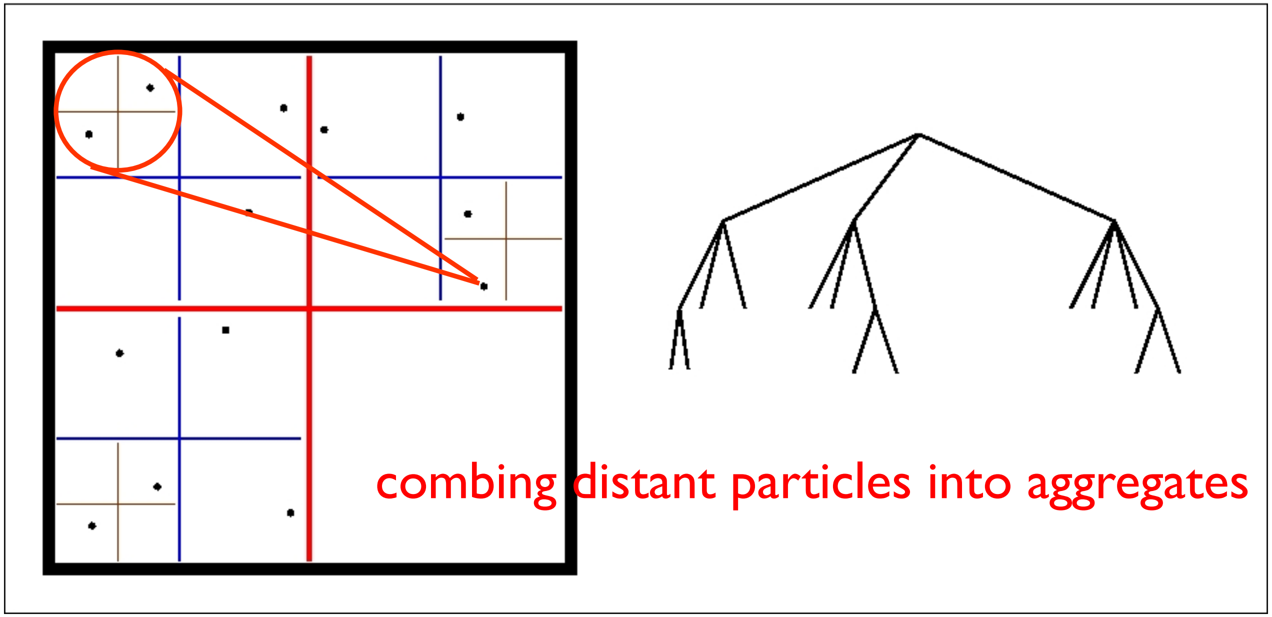
- walking the tree ($\forall i \in N$):



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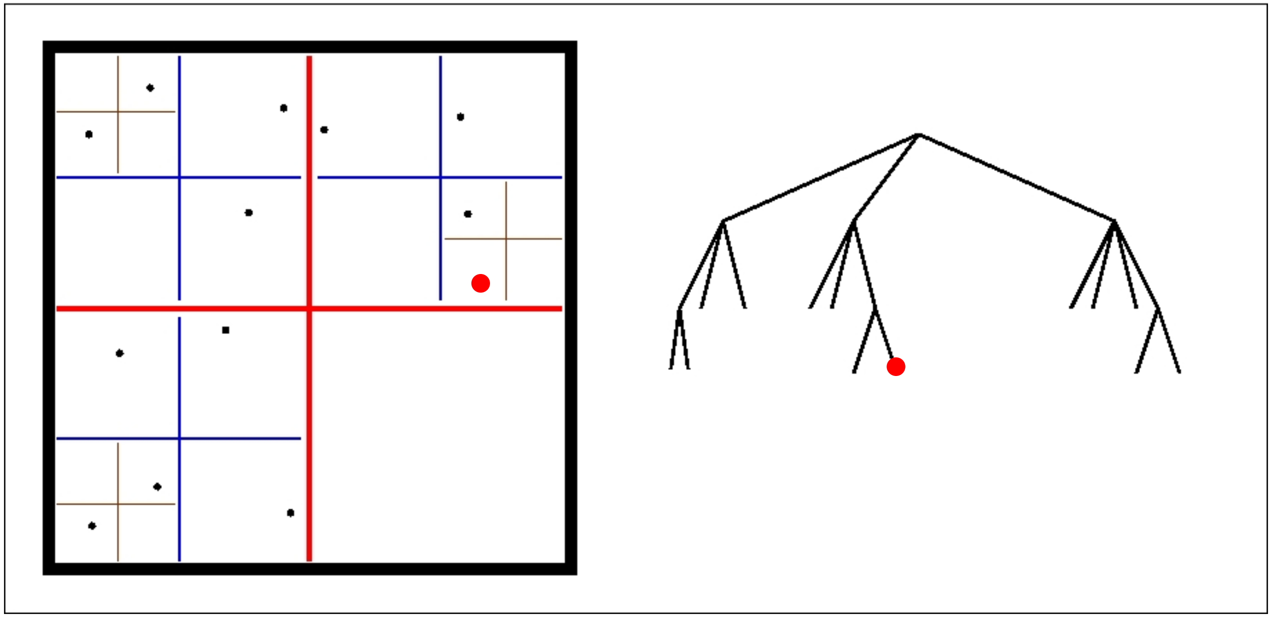

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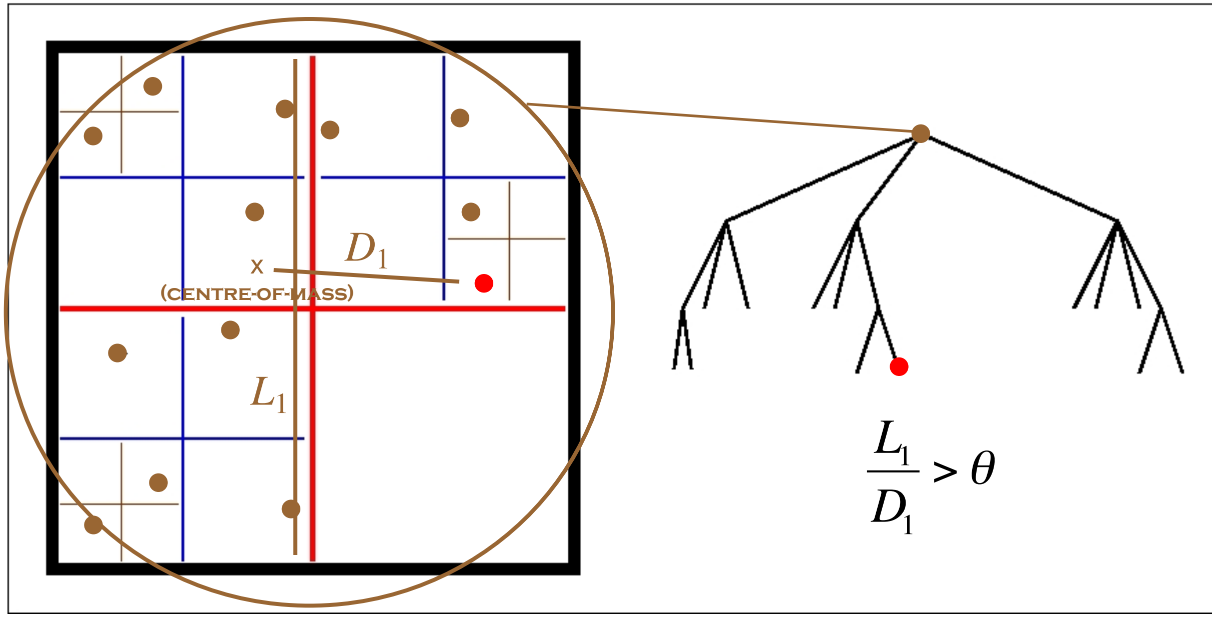

$$\frac{L_1}{D_1} > \theta$$

Solving for Gravity

- direct particle-particle summation (PP)

$$\vec{F}_i(\vec{r}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$

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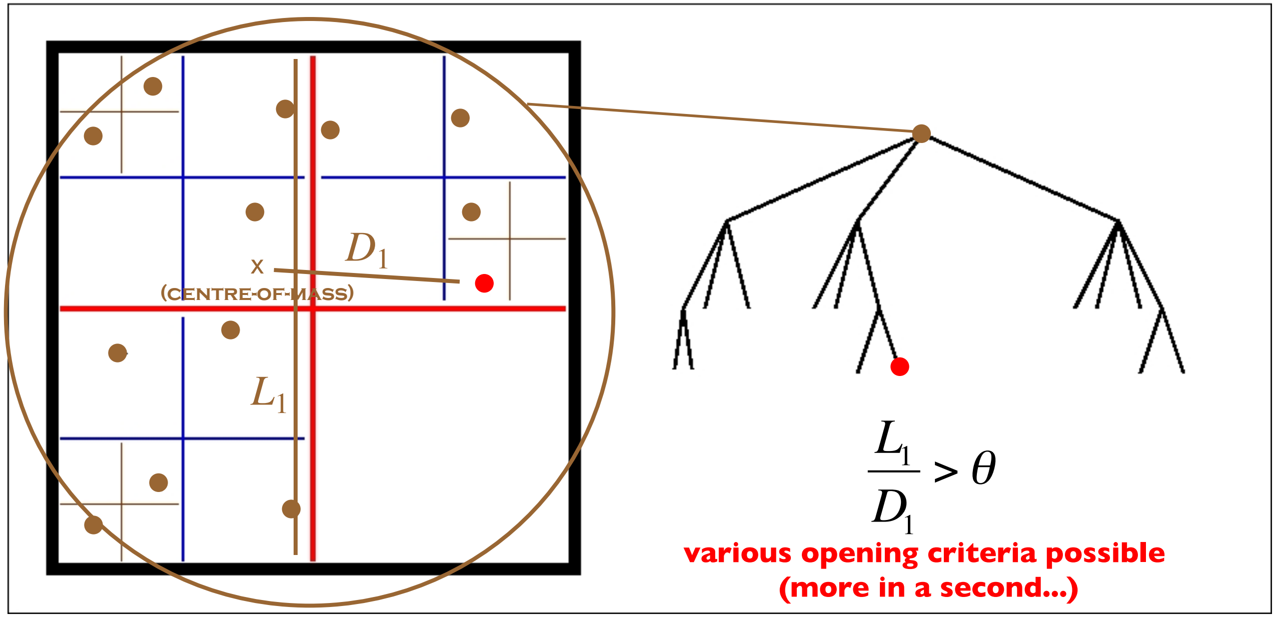

Solving for Gravity

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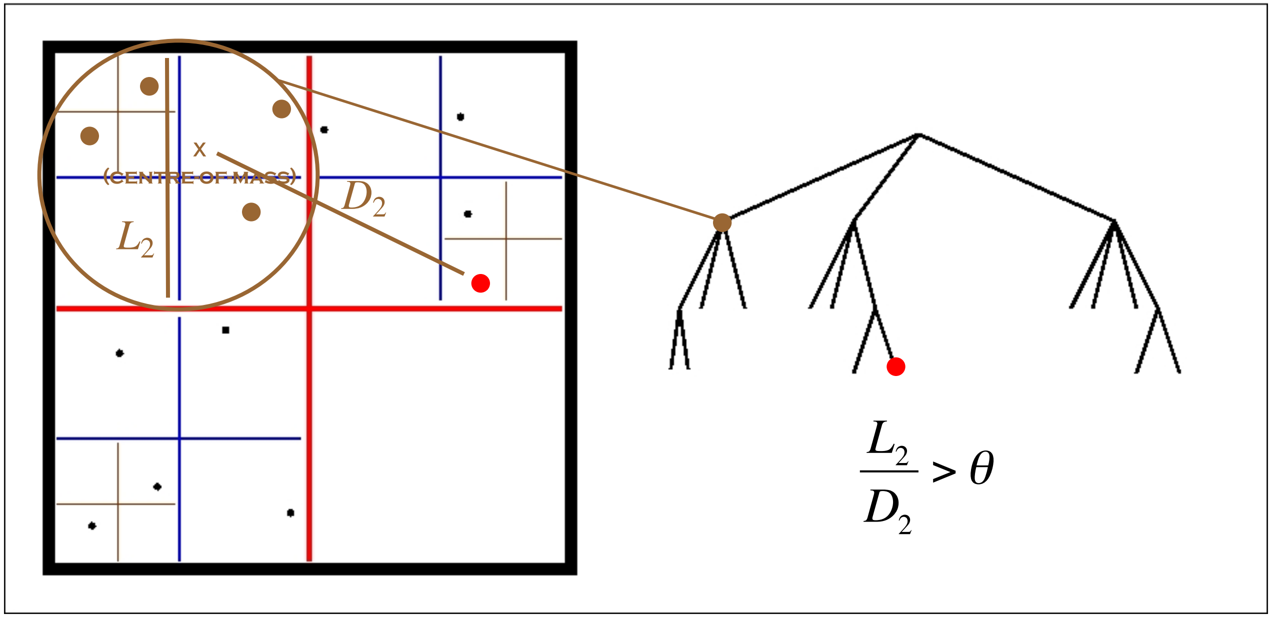

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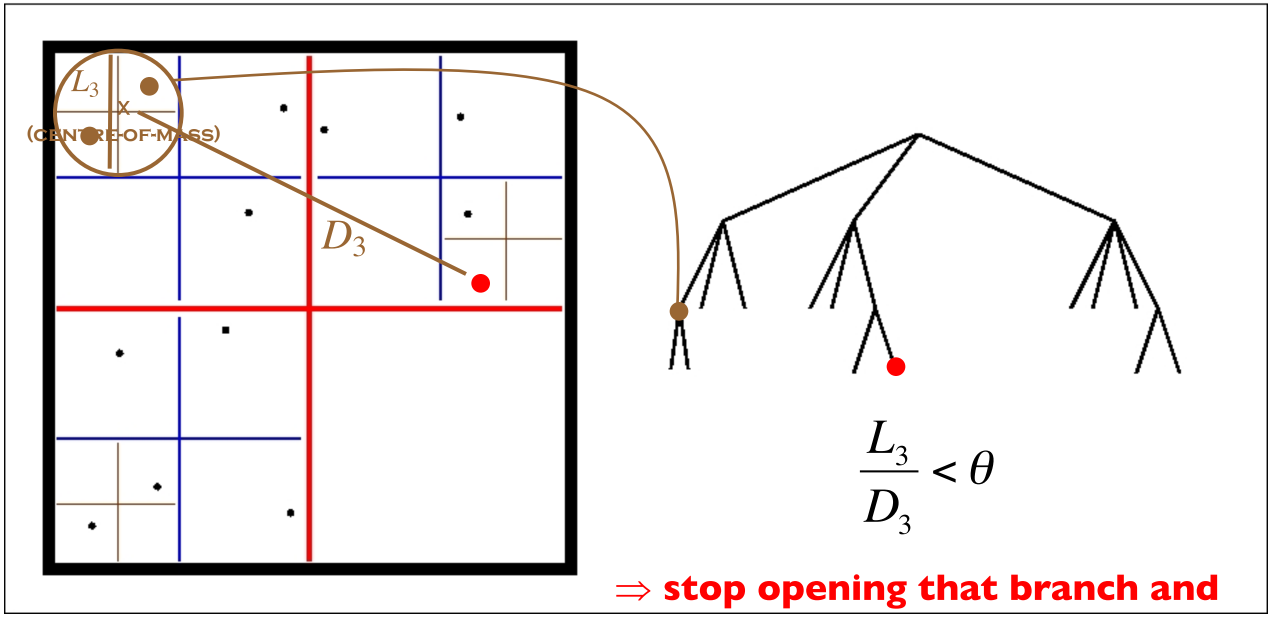

⇒ stop opening that branch and add force contribution from "super-particle"

Solving for Gravity

- direct particle-particle summation (PP)

$$\vec{F}_i(\vec{r}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3}(\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$

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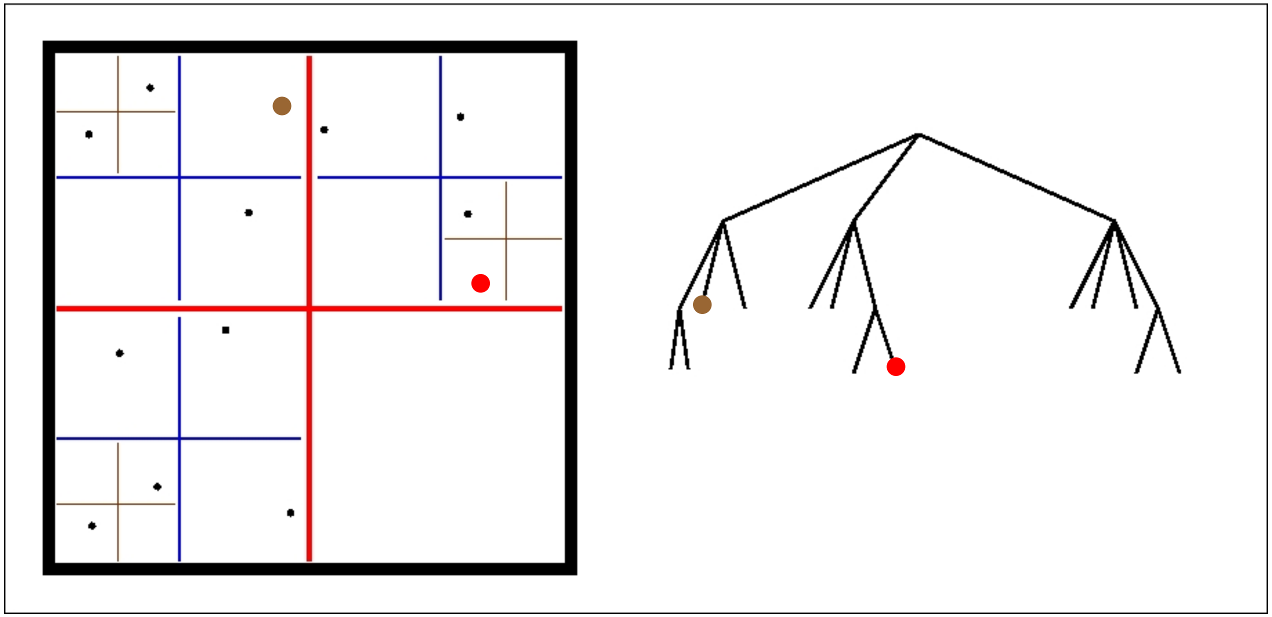

we still need to add the remaining contributions from that branch…

- direct particle-particle summation (PP)

$$\vec{F}_i(\vec{r}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$

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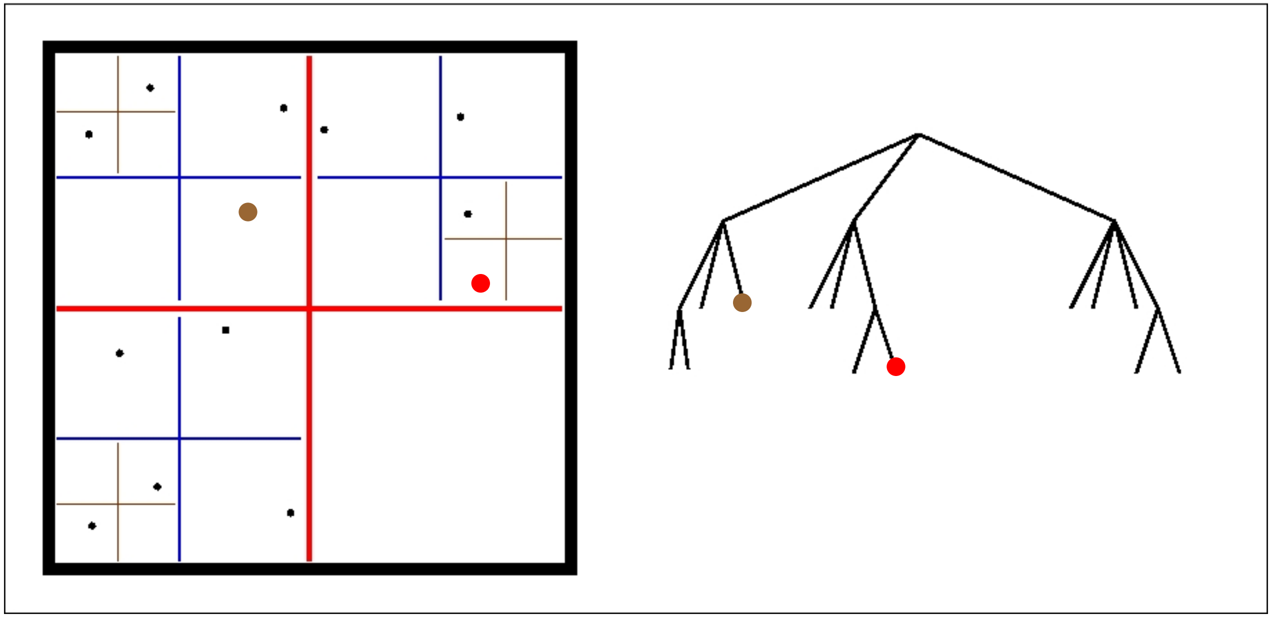

we still need to add the remaining contributions from that branch…

Solving for Gravity

- direct particle-particle summation (PP)

$$\vec{F}_i(\vec{r}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$

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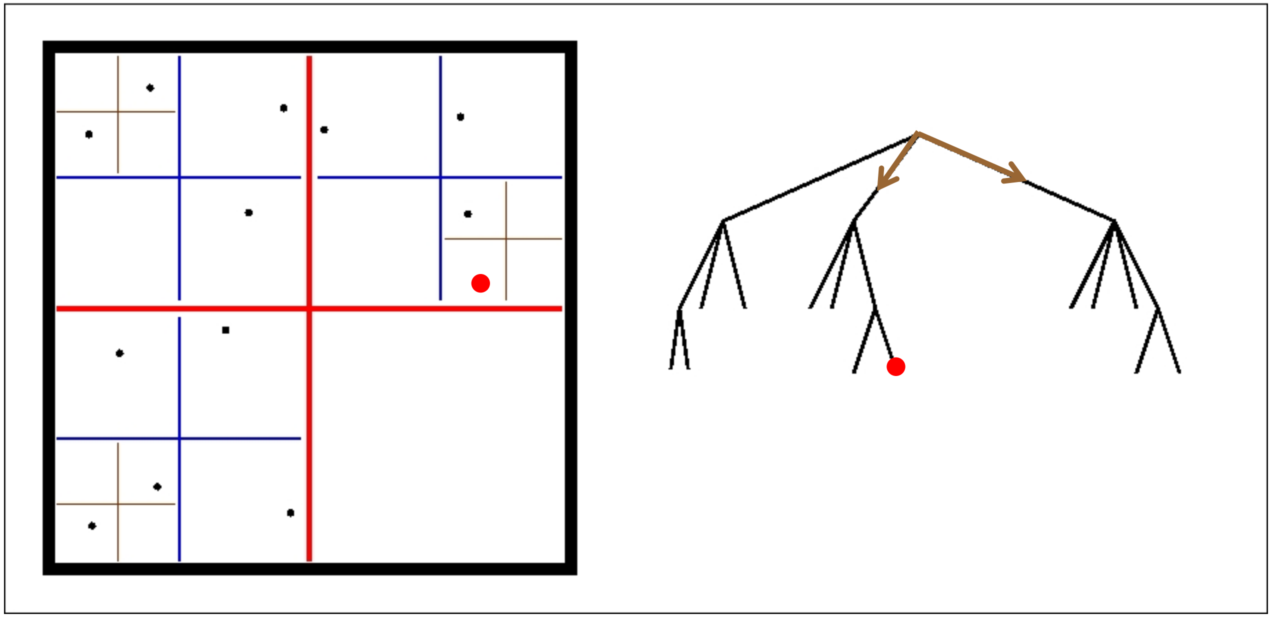


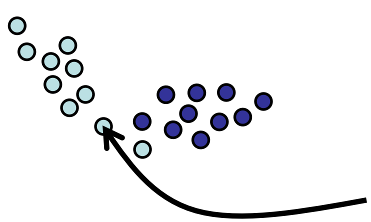
...as well as walking the other branches!

Solving for Gravity

- direct particle-particle summation (PP)

$$\vec{F}_i(\vec{r}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$

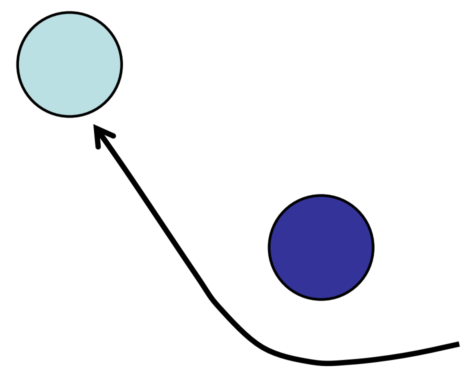
we use (collisionless) particles to sample $f(x,v,t)$



the particles sampling the field adjust

well sampled system

vs.



the particle sampling the field bounces off

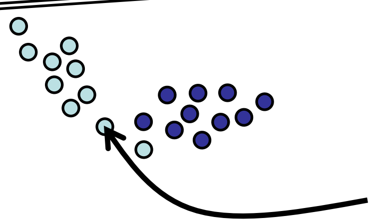
undersampled system

- direct particle-particle summation (PP)

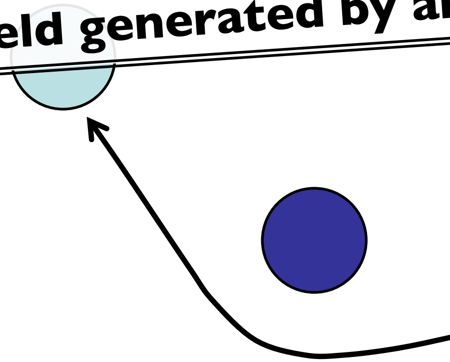
$$\vec{F}_i(\vec{r}_i) = -\sum_{i\neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$

we use (collisionless) particles to sample $f(x,v,t)$

each particle should only feel the mean field generated by all particles!



vs.



the particles sampling the field adjust

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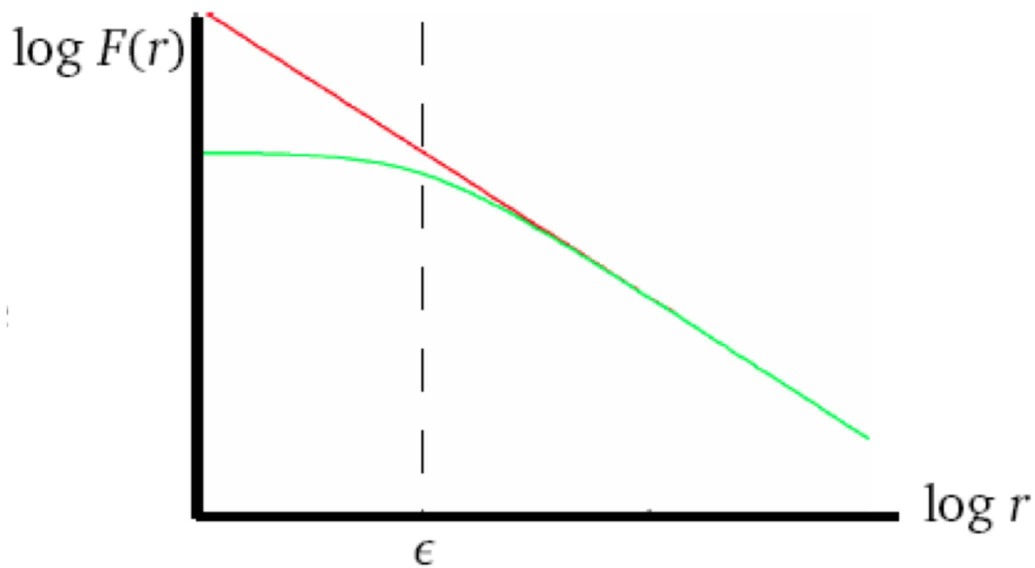
well sampled system

undersampled system

Solving for Gravity

- direct particle-particle summation (PP)

$$\vec{F}_i(\vec{r}_i) = -\sum_{i\neq j} \frac{G m_i m_j}{\left((r_i - r_j)^2 + \varepsilon^2\right)^{3/2}} (\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$



Solving for Gravity

- direct particle-particle summation (PP)

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"soften" the force to…

1. avoid the singularity for $r_i = r_j$
2. smooth mass density on small scales

Solving for Gravity

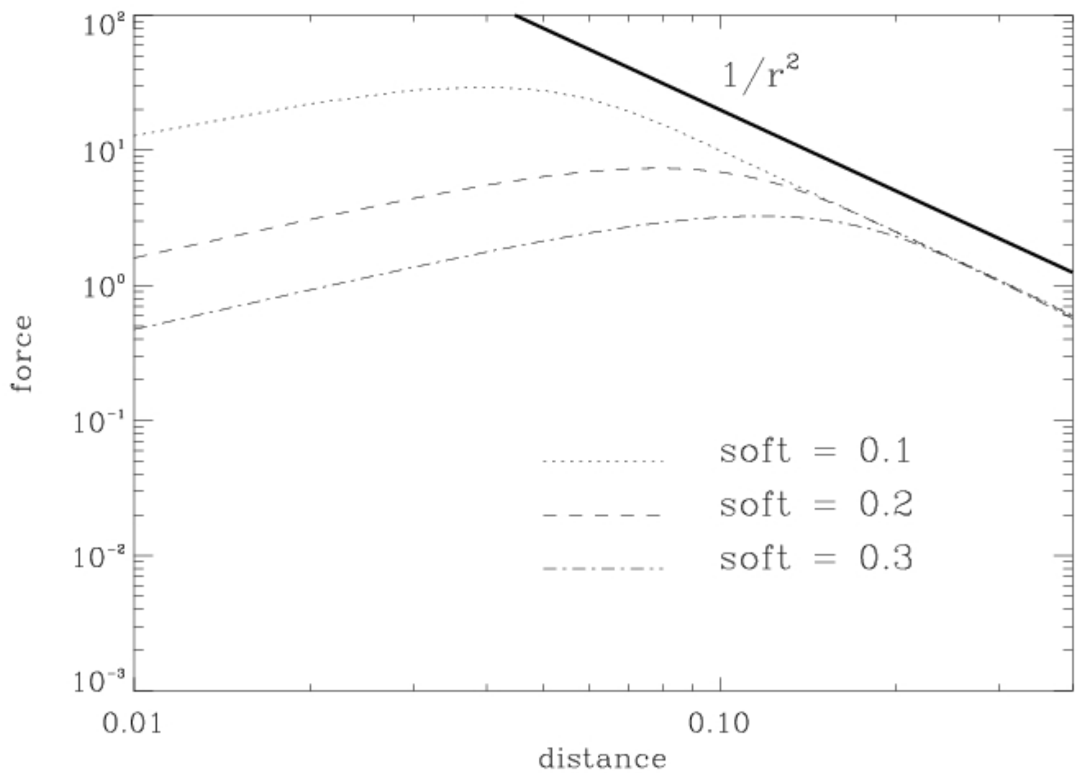
- direct particle-particle summation (PP)

$$\vec{F}_i(\vec{r}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{\left((r_i - r_j)^2 + \varepsilon^2\right)^{3/2}} (\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$

ε determines the overall force resolution of the simulation

- direct particle-particle summation (PP)

$$\vec{F}_i(\vec{r}_i) = -\sum_{i \neq j} \frac{G m_i m_j}{\left((r_i - r_j)^2 + \varepsilon^2\right)^{3/2}} (\vec{r}_i - \vec{r}_j) \quad \forall i \in N$$



Solving for Gravity

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error budget?

Solving for Gravity

- direct particle-particle summation (PP)

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  - error estimate:

$$MISE = \left\langle \iiint \rho(\vec{x}) \left|\vec{F}(\vec{x}) - \vec{F}_{true}(\vec{x})\right|^2 d^3x \right\rangle$$

MISE = mean integrated square error

Solving for Gravity

- direct particle-particle summation (PP)

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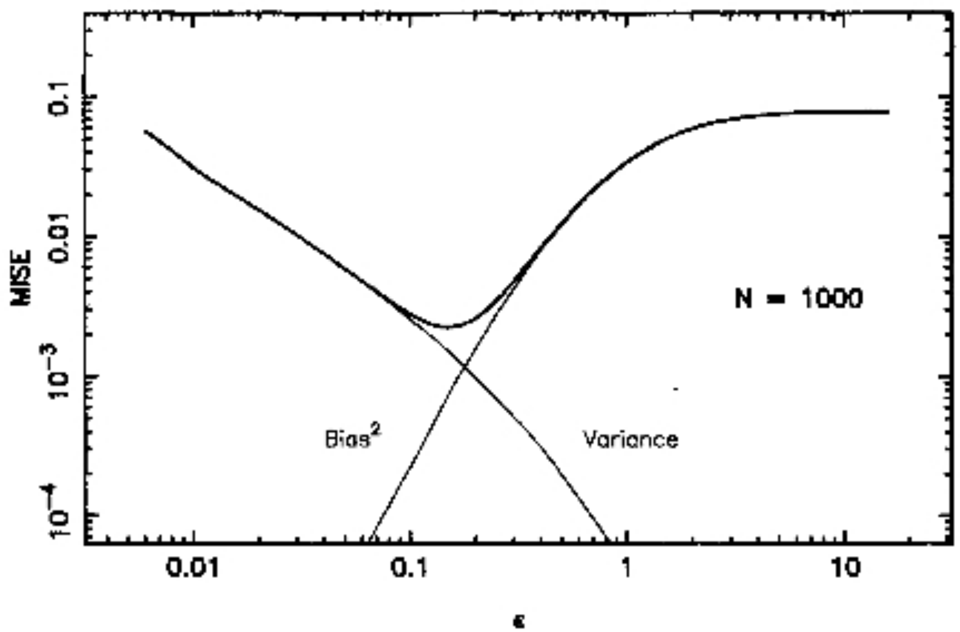

Merritt, ApJ 111, 2462 (1996)

interplay between N and ε: $N\varepsilon^3$=const.

MISE = mean integrated square error

const.=$\left(\frac{B}{30}\right)^3$ for cosmological simulations (where B is the size of the cubical domain in 1D)

Solving for Gravity

- direct particle-particle summation (PP)

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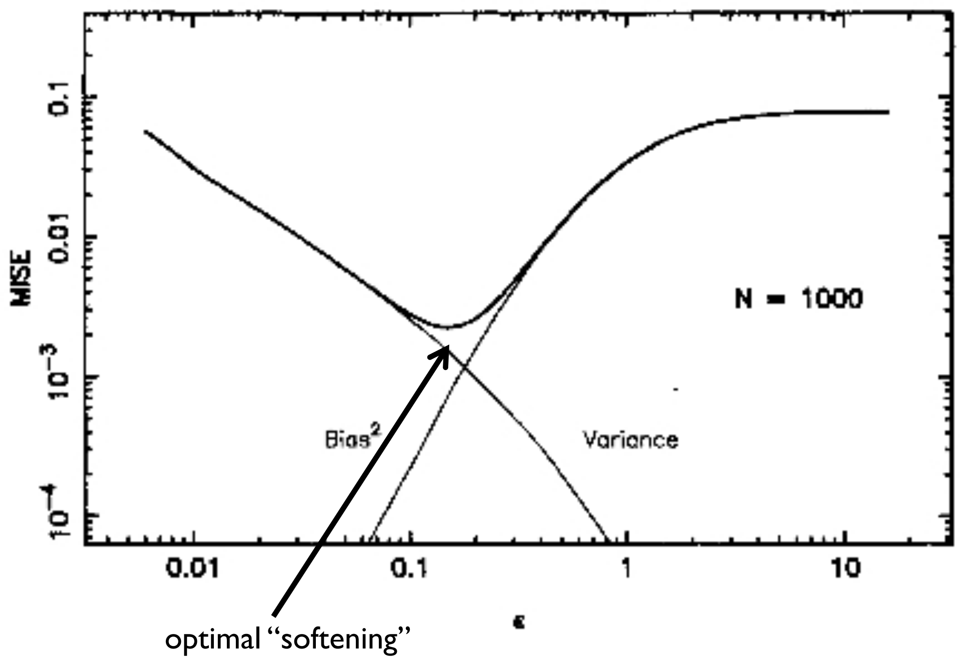

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MISE = mean integrated square error

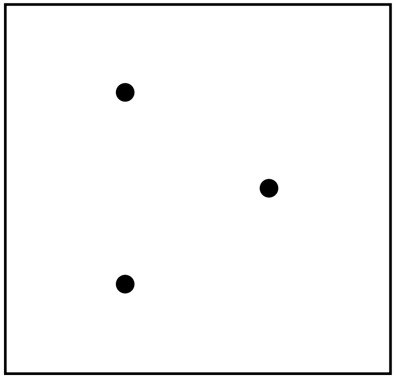
- Poisson’s equation vs. Poisson’s integral
- tree codes
- **PM codes**

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi(\vec{g}_{k,l,m}) = 4\pi G\rho(\vec{g}_{k,l,m})$$

$$\vec{F}(\vec{g}_{k,l,m}) = -m\nabla\Phi(\vec{g}_{k,l,m})$$



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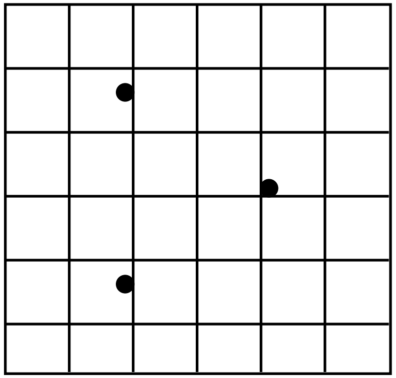

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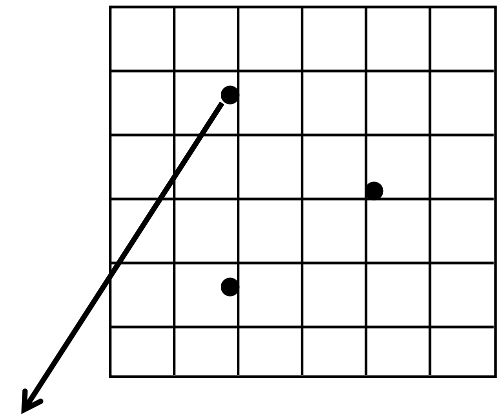

1. calculate mass density on grid

$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m}) \quad \forall i \in N$

Solving for Gravity

- numerically integrate Poisson's equation

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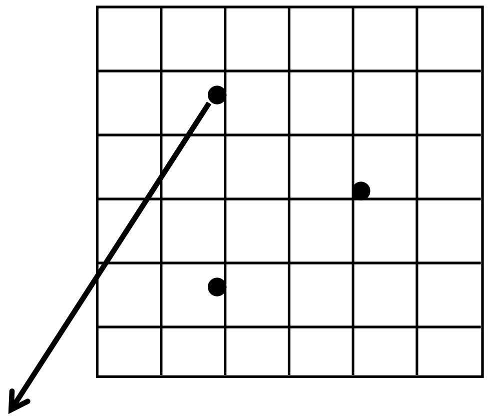


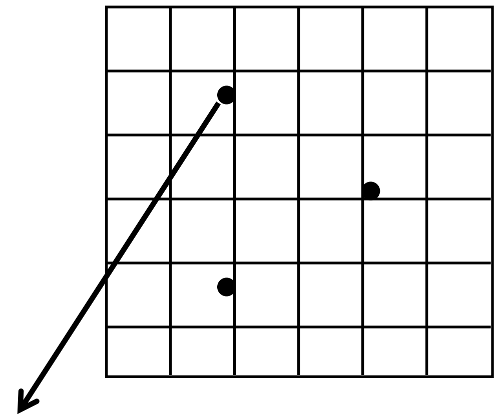
1. calculate mass density on grid $\vec{x}_i \to \rho(\vec{g}_{k,l,m}) \quad \forall i \in N$
2. solve Poisson's equation on grid $\Phi(\vec{g}_{k,l,m})$

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi(\vec{g}_{k,l,m}) = 4\pi G\rho(\vec{g}_{k,l,m})$$

$$\vec{F}(\vec{g}_{k,l,m}) = -m\nabla\Phi(\vec{g}_{k,l,m})$$



1. calculate mass density on grid $\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$ $\forall i \in N$
2. solve Poisson's equation on grid $\Phi(\vec{g}_{k,l,m})$
3. differentiate potential to get forces $\vec{F}(\vec{g}_{k,l,m})$

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi(\vec{g}_{k,l,m}) = 4\pi G\rho(\vec{g}_{k,l,m})$$

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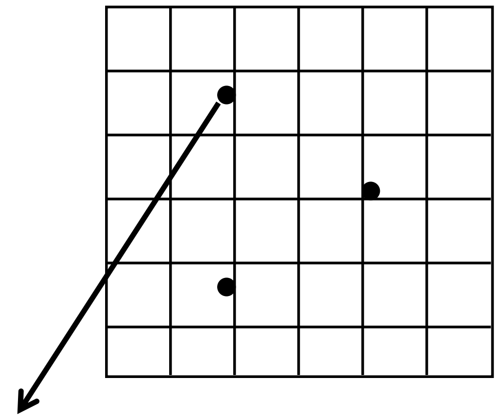

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3. differentiate potential to get forces $\vec{F}(\vec{g}_{k,l,m})$
4. interpolate forces back to particles $\vec{F}(\vec{g}_{k,l,m}) \rightarrow \vec{F}(\vec{x}_i) \quad \forall i \in N$

Solving for Gravity

- numerically integrate Poisson's equation

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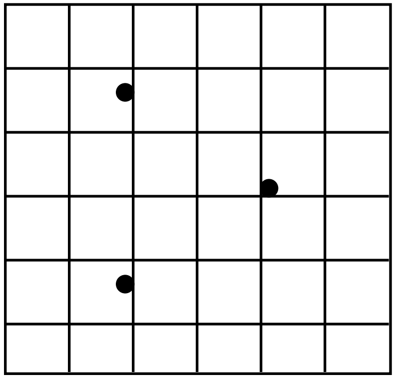

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3. differentiate potential to get forces $\vec{F}(\vec{g}_{k,l,m})$
4. interpolate forces back to particles $\vec{F}(\vec{g}_{k,l,m}) \rightarrow \vec{F}(\vec{x}_i)$

sounds like a waste of time and computer resources, but **exceptionally fast** in practice

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi(\vec{g}_{k,l,m}) = 4\pi G\rho(\vec{g}_{k,l,m})$$

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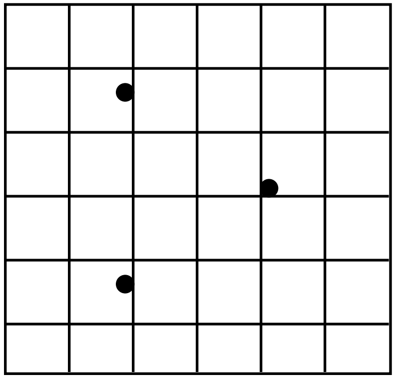

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Solving for Gravity

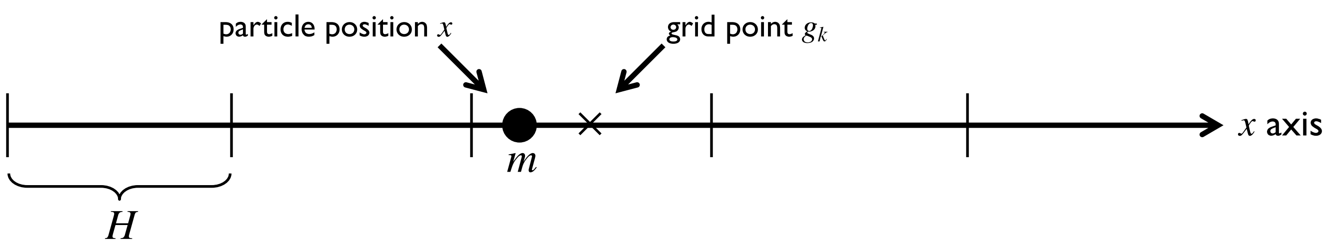
- density assignment schemes

$$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$$

<u>example:</u> 1 particle on 1 dimensional grid

$$M(g_k) = mW(d) \qquad d = |x - g_k|$$

$$\rho(g_k) = \frac{M(g_k)}{H}$$



Solving for Gravity

- density assignment schemes

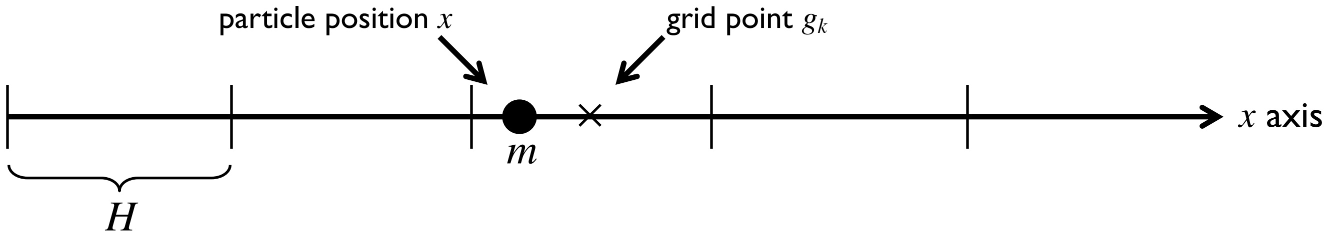
$$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$$

example: 1 particle on 1 dimensional grid

$$M(g_k) = mW(d) \qquad d = |x - g_k|$$

mass assignment function

$$\rho(g_k) = \frac{M(g_k)}{H}$$



Solving for Gravity

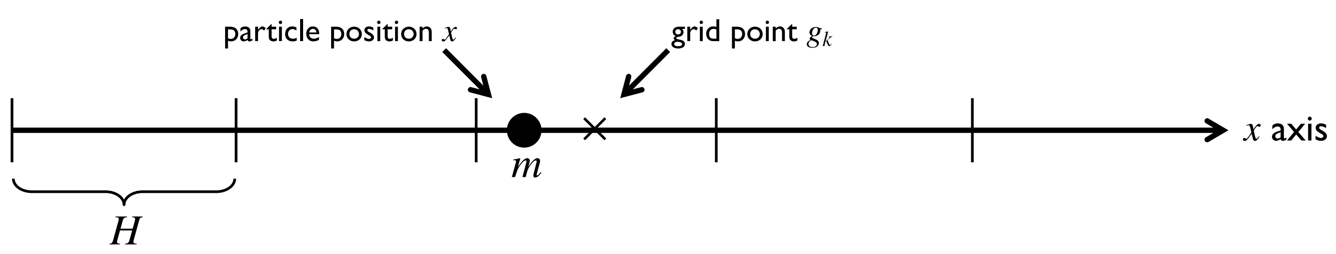
- density assignment schemes

$$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$$

<u>example:</u> 1 particle on 1 dimensional grid

• hierarchy of mass assignment schemes:

– Nearest-Grid-Point	NGP
– Could-In-Cell	CIC
– Triangular-Shaped Cloud	TSC
– …	



Solving for Gravity

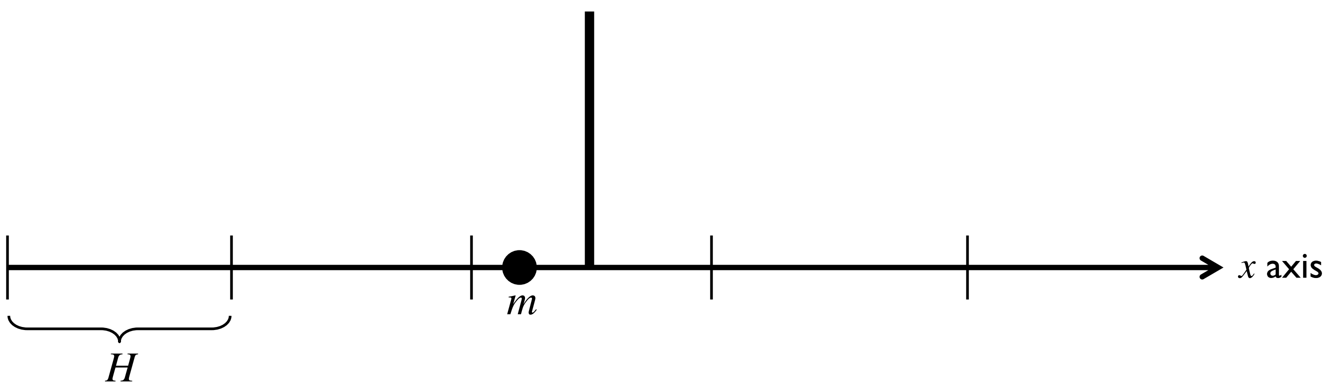
- density assignment schemes

$$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$$

<u>Nearest-Grid-Point (NGP):</u>

mass assignment function:

$$W(d) = \begin{cases} 1 & d \leq H/2 \\ 0 & \text{otherwise} \end{cases}$$



- density assignment schemes

$$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$$

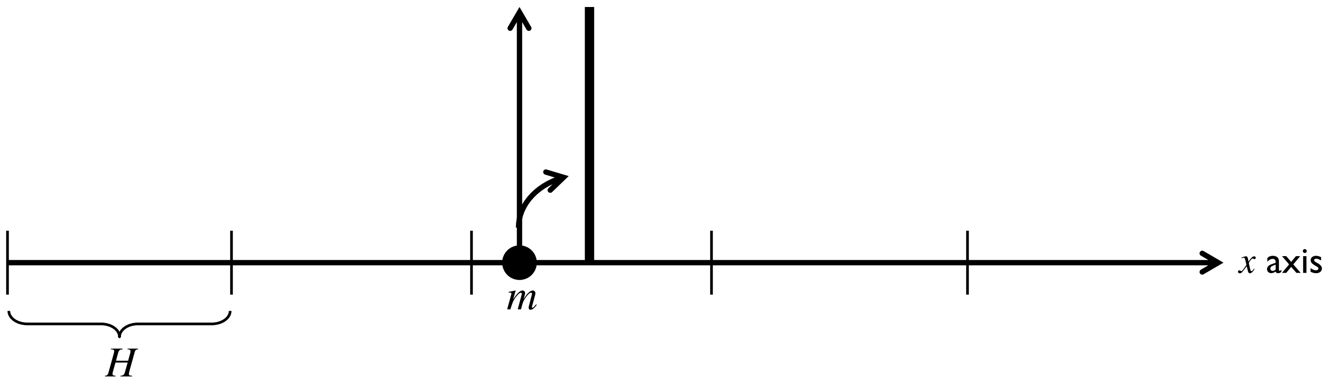
Nearest-Grid-Point (NGP):

particle shape:

$$S(x) = \delta(x)$$

mass assignment function:

$$W(d) = \begin{cases} 1 & d \leq H/2 \\ 0 & \text{otherwise} \end{cases}$$



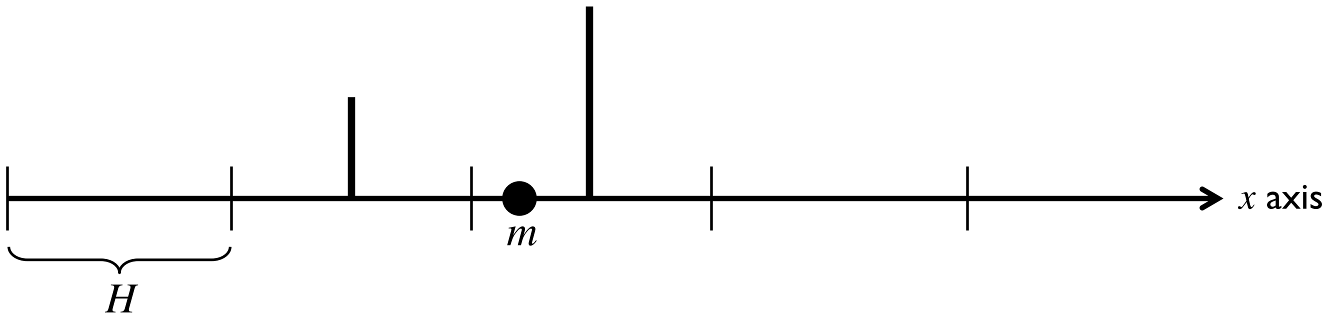
- density assignment schemes

$$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$$

<u>Cloud-In-Cell (CIC):</u>

mass assignment function:

$$W(d) = \begin{cases} 1 - \dfrac{d}{H} & d \leq H \\ 0 & \text{otherwise} \end{cases}$$



- density assignment schemes

$$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$$

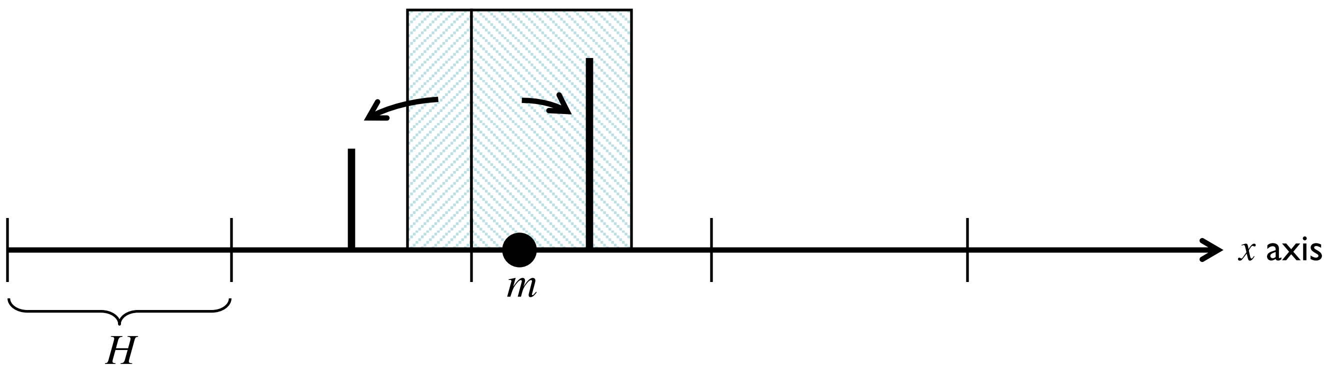
Cloud-In-Cell (CIC):

particle shape:

$$S(x) = \begin{cases} 1 & |x| \le H/2 \\ 0 & \text{otherwise} \end{cases}$$

mass assignment function:

$$W(d) = \begin{cases} 1 - \dfrac{d}{H} & d \le H \\ 0 & \text{otherwise} \end{cases}$$



Solving for Gravity

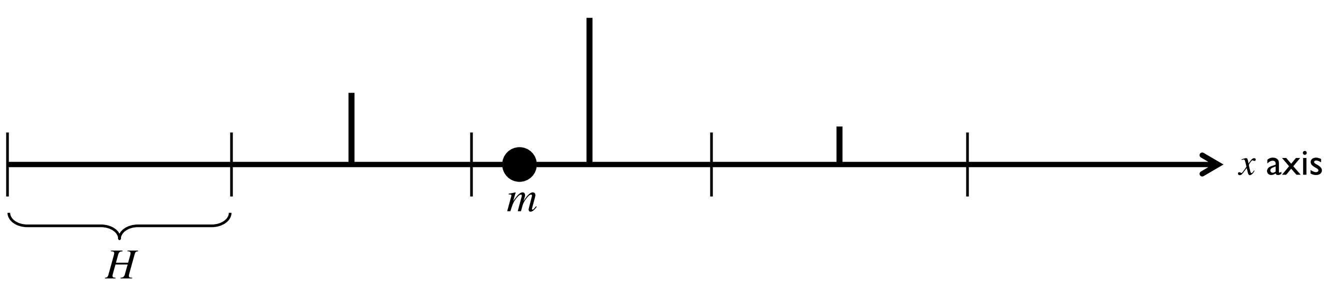
- density assignment schemes

$$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$$

Triangular-Shaped-Cloud (TSC):

mass assignment function:

$$W(d) = \begin{cases} \frac{3}{4} - \left(\frac{d}{H}\right)^2 & d \leq \frac{H}{2} \\ \frac{1}{2}\left(\frac{3}{2} - \frac{d}{H}\right)^2 & \frac{H}{2} \leq d \leq \frac{3H}{2} \\ 0 & \text{otherwise} \end{cases}$$



Solving for Gravity

- density assignment schemes

$$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$$

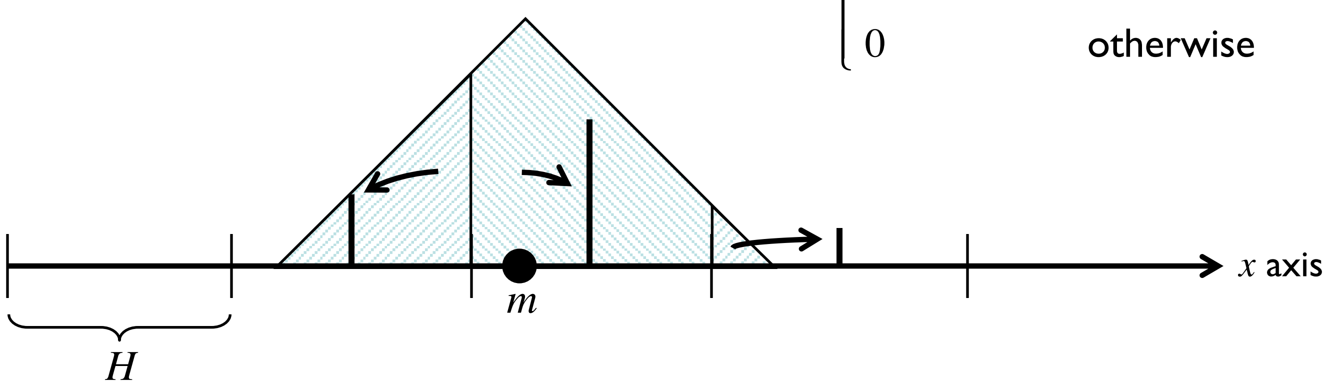
Triangular-Shaped-Cloud (TSC):

particle shape:

$$S(x) = \begin{cases} 1 - \frac{|x|}{H} & |x| \le H \\ 0 & \text{otherwise} \end{cases}$$

mass assignment function:

$$W(d) = \begin{cases} \frac{3}{4} - \left(\frac{d}{H}\right)^2 & d \le \frac{H}{2} \\ \frac{1}{2}\left(\frac{3}{2} - \frac{d}{H}\right)^2 & \frac{H}{2} \le d \le \frac{3H}{2} \\ 0 & \text{otherwise} \end{cases}$$



Solving for Gravity

- density assignment schemes

$$\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$$

<u>N particles on 3 dimensional grid</u>

$$\vec{d} = \vec{x}_i - \vec{g}_{k,l,m}$$

$$M(\vec{g}_{k,l,m}) = \sum_{i=1}^{N} m_i W(|d_x|) W(|d_y|) W(|d_z|)$$

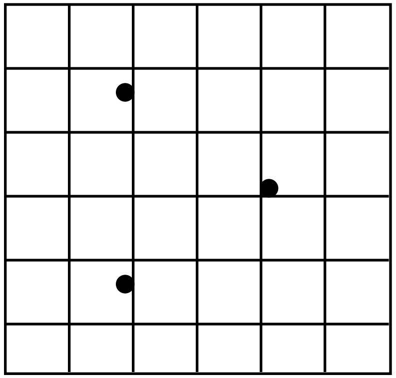
$$\rho(\vec{g}_{k,l,m}) = \frac{M(\vec{g}_{k,l,m})}{H^3}$$

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi(\vec{g}_{k,l,m}) = 4\pi G\rho(\vec{g}_{k,l,m})$$

$$\vec{F}(\vec{g}_{k,l,m}) = -m\nabla\Phi(\vec{g}_{k,l,m})$$



1. calculate mass density on grid $\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$
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3. differentiate potential to get forces $\vec{F}(\vec{g}_{k,l,m})$
4. interpolate forces back to particles $\vec{F}(\vec{g}_{k,l,m}) \rightarrow \vec{F}(\vec{x}_i)$

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi_{k,l,m} = \rho_{k,l,m}$$

- relaxation technique: applicable and usable for ***any*** differential equation
- FTT technique: only applicable and usable for ***linear*** differential equation

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi_{k,l,m} = \rho_{k,l,m}$$

- relaxation technique: applicable and usable for ***any*** differential equation
- **FTT technique:** only applicable and usable for ***linear*** differential equation

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi_{k,l,m} = \rho_{k,l,m}$$

- Green's function method:
  - solve differential equation by Fourier transformation
  - applicable and usable for ***linear*** differential equations

Solving for Gravity

- numerically integrate Poisson's equation *fast fourier transform method*
  - Green's function method

$$\Delta\Phi = \rho \qquad \Phi(\vec{x}) = \iiint \mathcal{G}(\vec{x}-\vec{x}')\,\rho(\vec{x}')d^3x' \;\; ; \;\; \mathcal{G}(\vec{x}) = \frac{1}{4\pi\, x}$$

Solving for Gravity

- numerically integrate Poisson's equation *fast fourier transform method*
  - Green's function method

$$\Delta\Phi = \rho \qquad \Phi(\vec{x}) = \iiint \mathcal{G}(\vec{x} - \vec{x}')\, \rho(\vec{x}')\, d^3x' \;\; ; \quad \mathcal{G}(\vec{x}) = \frac{1}{4\pi\, x}$$

$$\Phi = \rho \otimes \mathcal{G}$$

FFT ➔ convolution becomes multiplication

$$\hat{\Phi} = \hat{\rho} \quad \hat{\mathcal{G}}$$

(with $\hat{\mathcal{G}} = -\frac{1}{k^2}$ for Poisson's equation)

Solving for Gravity

- numerically integrate Poisson's equation *fast fourier transform method*
  - Green's function method

$$\Delta\Phi = \rho \qquad \Phi(\vec{x}) = \iiint \mathcal{G}(\vec{x}-\vec{x}')\,\rho(\vec{x}')d^3x' \;\; ; \;\; \mathcal{G}(\vec{x}) = \frac{1}{4\pi x}$$

$$\Phi = \rho \otimes \mathcal{G}$$

FFT → convolution becomes multiplication

$$\hat{\Phi} = \hat{\rho} \quad \hat{\mathcal{G}}$$

(with $\hat{\mathcal{G}} = -\frac{1}{k^2}$ for Poisson's equation)

FFT^{-1}

$$\Phi$$

(FFT demands a *regular* grid though!)

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi_{k,l,m} = \rho_{k,l,m}$$

- **relaxation technique**: applicable and usable for ***any*** differential equation
- FTT technique: only applicable and usable for ***linear*** differential equation

Solving for Gravity

- numerically integrate Poisson's equation *relaxation technique*

obtain iterative solver by discretizing differential equation

$$\Delta\Phi_{k,l,m} = \rho_{k,l,m}$$

Solving for Gravity

- numerically integrate Poisson's equation *relaxation technique*

obtain iterative solver by discretizing differential equation

$$\Delta\Phi_{k,l,m} = \rho_{k,l,m}$$

$$\begin{aligned}
\Delta\Phi_{k,l,m} &= \nabla\cdot\nabla\Phi_{k,l,m} \\
&= \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial\Phi_{k,l,m}}{\partial x} \\ \frac{\partial\Phi_{k,l,m}}{\partial y} \\ \frac{\partial\Phi_{k,l,m}}{\partial z} \end{pmatrix} \\
&= \frac{1}{H}\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} \Phi_{k+1/2,l,m} - \Phi_{k-1/2,l,m} \\ \Phi_{k,l+1/2,m} - \Phi_{k,l-1/2,m} \\ \Phi_{k,l,m+1/2} - \Phi_{k,l,m-1/2} \end{pmatrix} \\
&= \frac{1}{H}\left(\frac{\partial\Phi_{k+1/2,l,m}}{\partial x} - \frac{\partial\Phi_{k-1/2,l,m}}{\partial x} + \frac{\partial\Phi_{k,l+1/2,m}}{\partial y} - \frac{\partial\Phi_{k,l-1/2,m}}{\partial y} + \frac{\partial\Phi_{k,l,m+1/2}}{\partial z} - \frac{\partial\Phi_{k,l,m-1/2}}{\partial z} \right) \\
&= \frac{1}{H^2}\left(\Phi_{k+1,l,m} - 2\Phi_{k,l,m} + \Phi_{k-1,l,m} + \Phi_{k,l+1,m} - 2\Phi_{k,l,m} + \Phi_{k,l-1,m} + \Phi_{k,l,m+1} - 2\Phi_{k,l,m} + \Phi_{k,l,m-1} \right)
\end{aligned}$$

Solving for Gravity

- numerically integrate Poisson's equation *relaxation technique*

obtain iterative solver by discretizing differential equation

$$\Delta\Phi_{k,l,m} = \rho_{k,l,m}$$

discretized Poisson's equation

$$\Phi_{k,l,m} = \frac{1}{6}(\Phi_{k+1,l,m} + \Phi_{k-1,l,m} + \Phi_{k,l+1,m} + \Phi_{k,l-1,m} + \Phi_{k,l,m+1} + \Phi_{k,l,m-1} - \rho_{k,l,m}H^2)$$

Solving for Gravity

- numerically integrate Poisson's equation *relaxation technique*

obtain iterative solver by discretizing differential equation

$$\Delta\Phi_{k,l,m} = \rho_{k,l,m}$$

iterative solution: $\Phi^{i}_{k,l,m} \rightarrow \Phi^{i+1}_{k,l,m}$

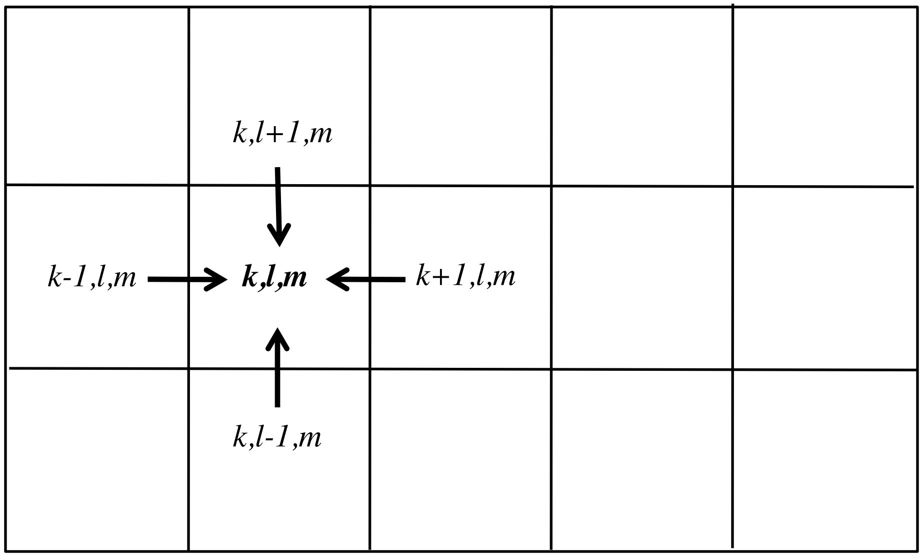
discretized Poisson's equation

$$\Phi^{i+1}_{k,l,m} = \frac{1}{6}(\Phi^{i}_{k+1,l,m} + \Phi^{i}_{k-1,l,m} + \Phi^{i}_{k,l+1,m} + \Phi^{i}_{k,l-1,m} + \Phi^{i}_{k,l,m+1} + \Phi^{i}_{k,l,m-1} - \rho_{k,l,m}H^2)$$

Solving for Gravity

- numerically integrate Poisson's equation *relaxation technique*

obtain iterative solver by discretizing differential equation

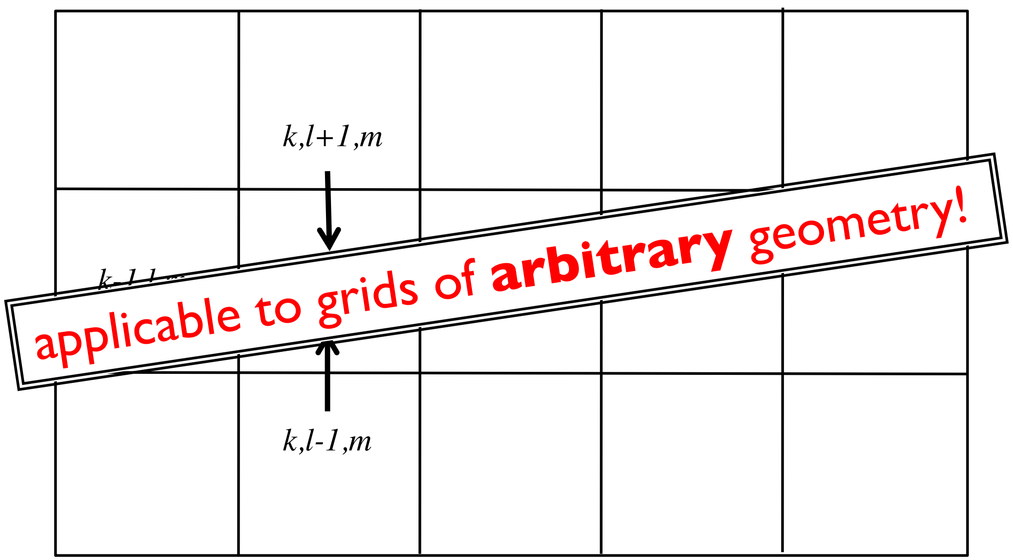


discretized Poisson's equation

$$\Phi^{i+1}_{k,l,m} = \frac{1}{6}(\Phi^{i}_{k+1,l,m} + \Phi^{i}_{k-1,l,m} + \Phi^{i}_{k,l+1,m} + \Phi^{i}_{k,l-1,m} + \Phi^{i}_{k,l,m+1} + \Phi^{i}_{k,l,m-1} - \rho_{k,l,m} H^2)$$

- numerically integrate Poisson's equation *relaxation technique*

obtain iterative solver by discretizing differential equation



discretized Poisson's equation

$$\Phi^{i+1}_{k,l,m} = \frac{1}{6}(\Phi^{i}_{k+1,l,m} + \Phi^{i}_{k-1,l,m} + \Phi^{i}_{k,l+1,m} + \Phi^{i}_{k,l-1,m} + \Phi^{i}_{k,l,m+1} + \Phi^{i}_{k,l,m-1} - \rho_{k,l,m} H^2)$$

Solving for Gravity

- numerically integrate Poisson's equation

accuracy of either

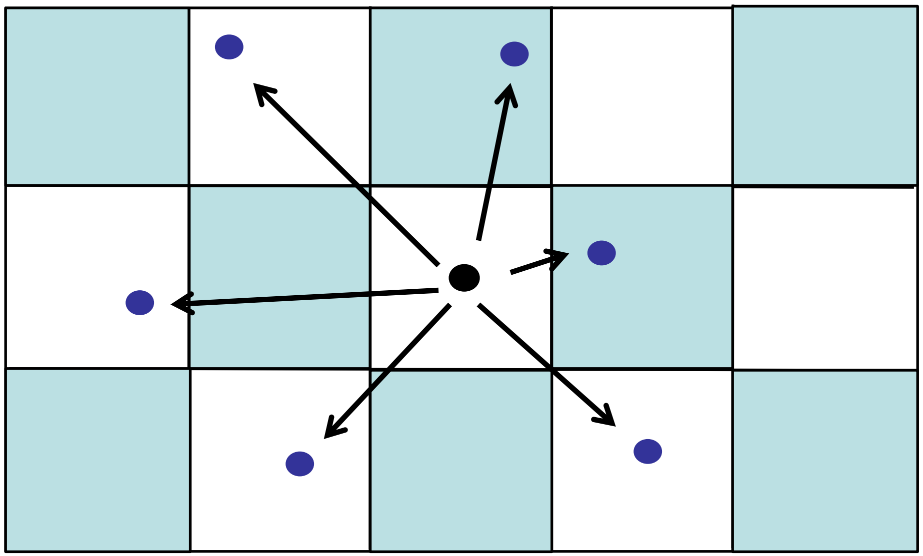
relaxation
or
FFT method

to solve Poisson's equation?

Solving for Gravity

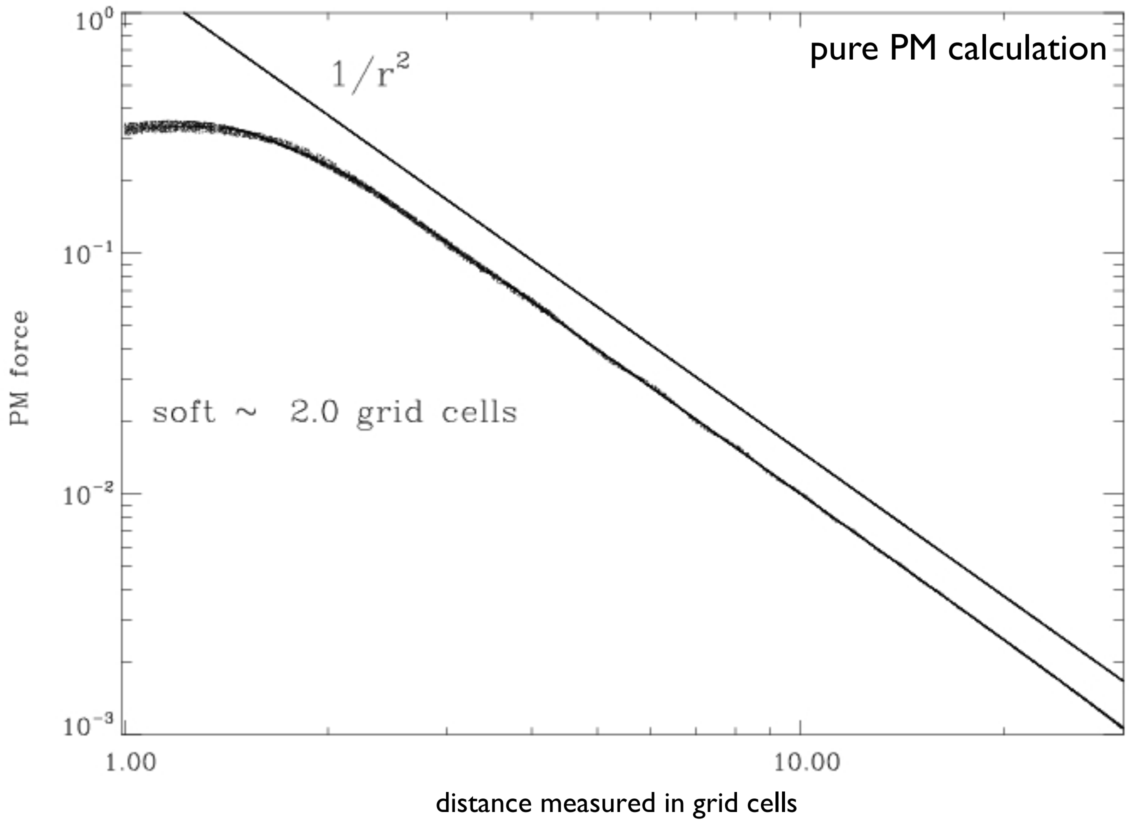
- numerically integrate Poisson's equation

pure PM calculation



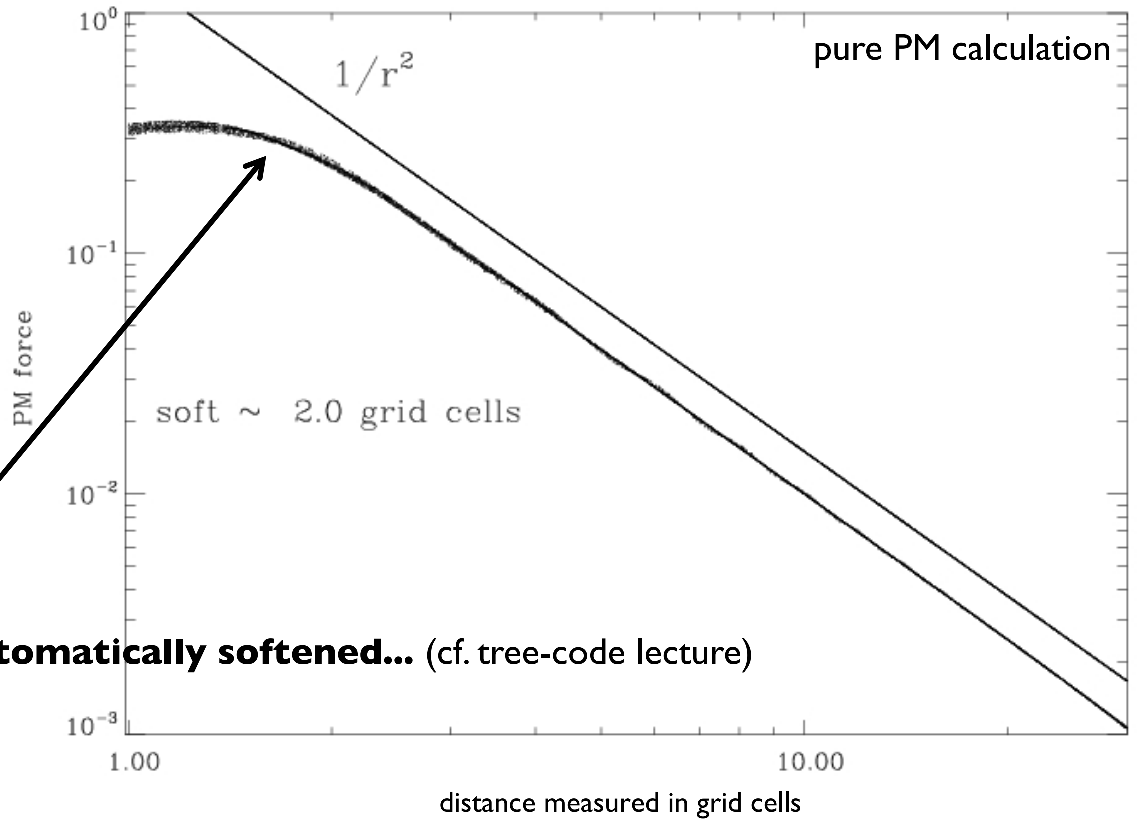
Solving for Gravity

- numerically integrate Poisson's equation



Solving for Gravity

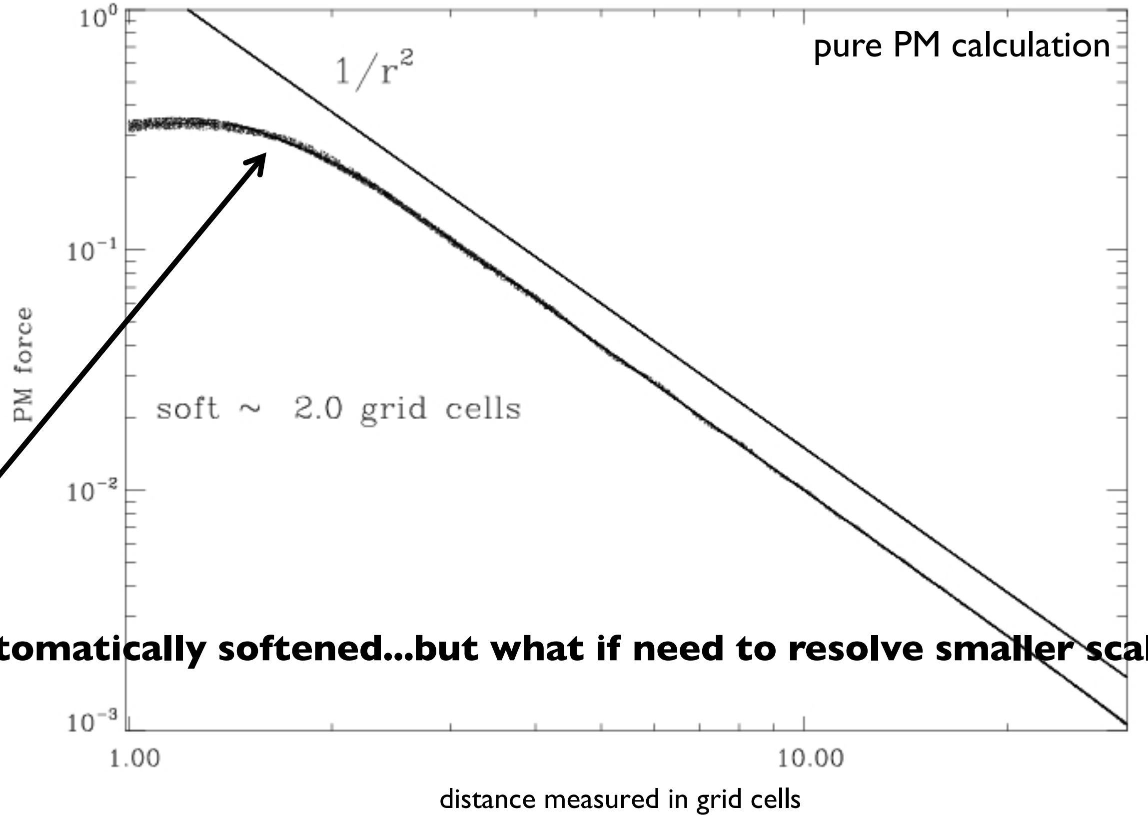
- numerically integrate Poisson's equation



the force is automatically softened... (cf. tree-code lecture)

Solving for Gravity

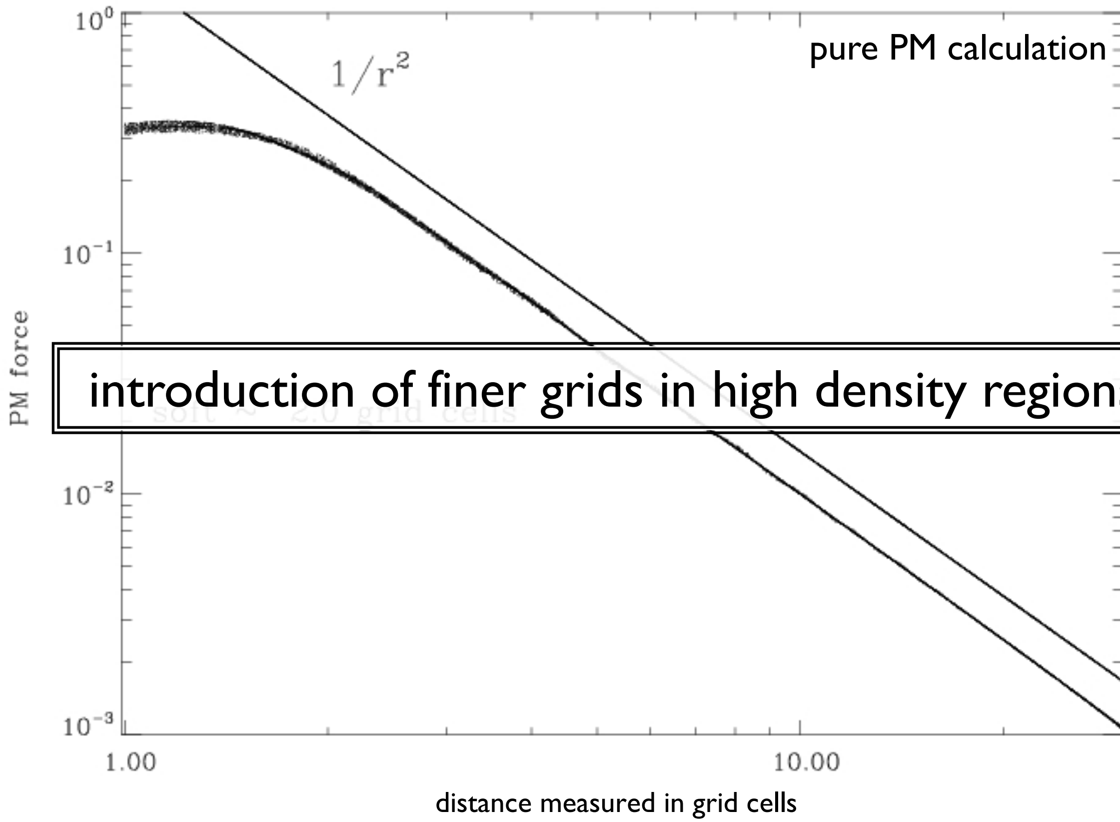
- numerically integrate Poisson's equation



the force is automatically softened...but what if need to resolve smaller scales?

Solving for Gravity

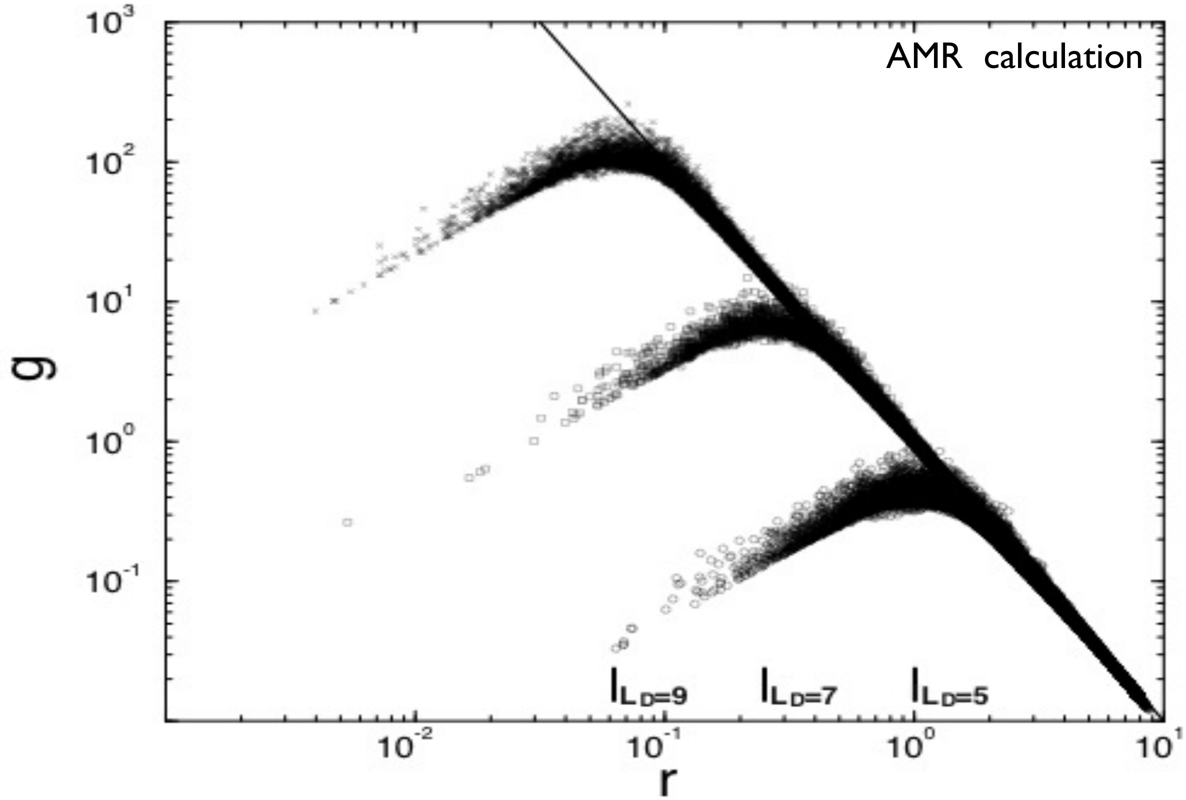
- numerically integrate Poisson's equation



introduction of finer grids in high density regions

Solving for Gravity

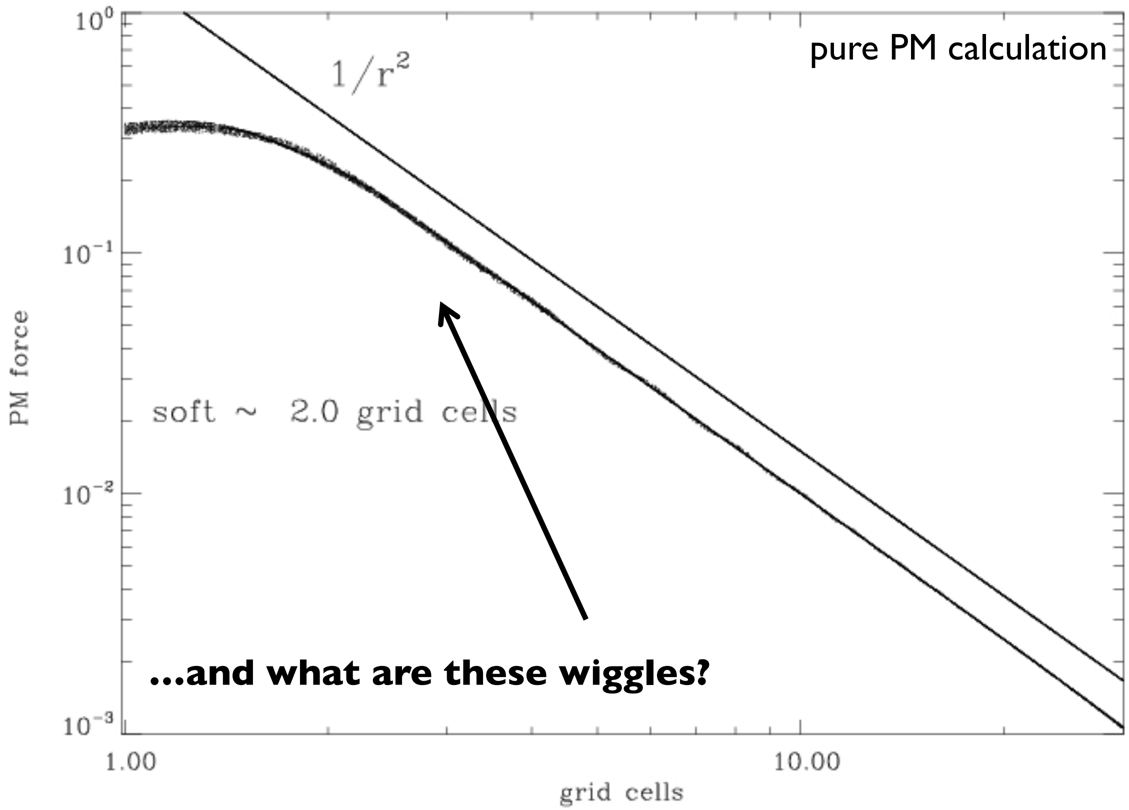
- numerically integrate Poisson's equation



Yahagi & Yoshi (2001)

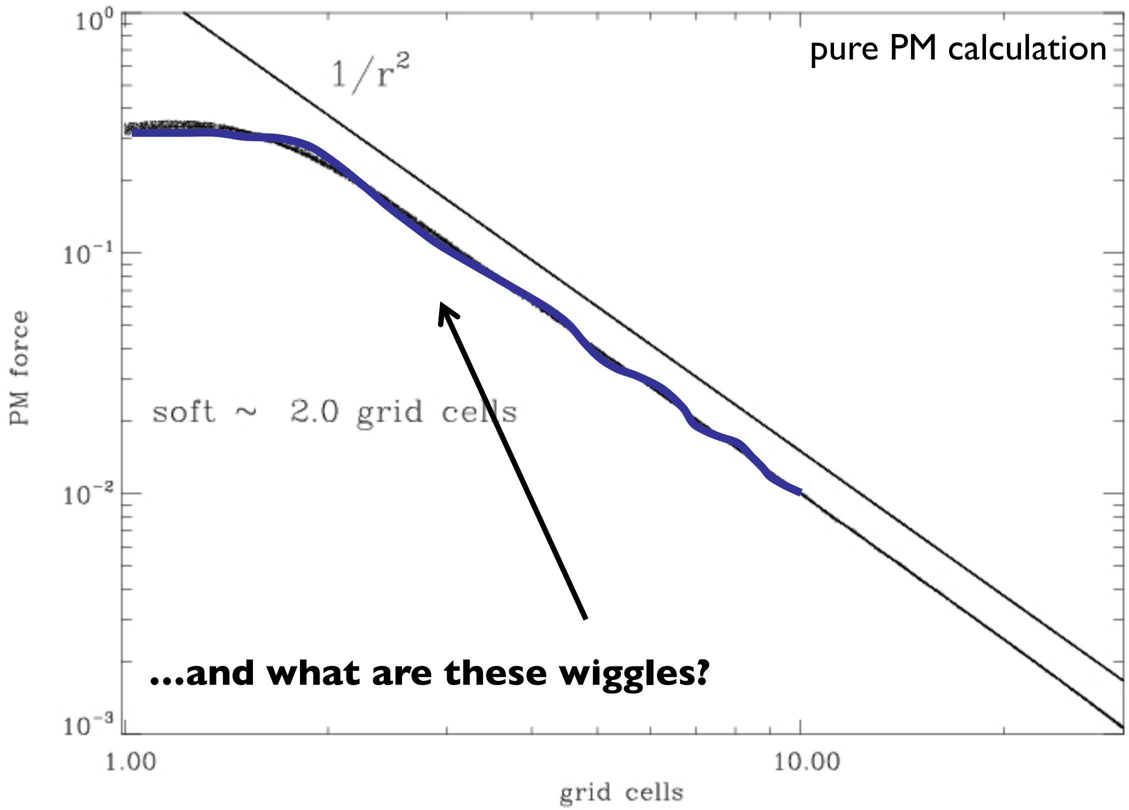
Solving for Gravity

- numerically integrate Poisson's equation



Solving for Gravity

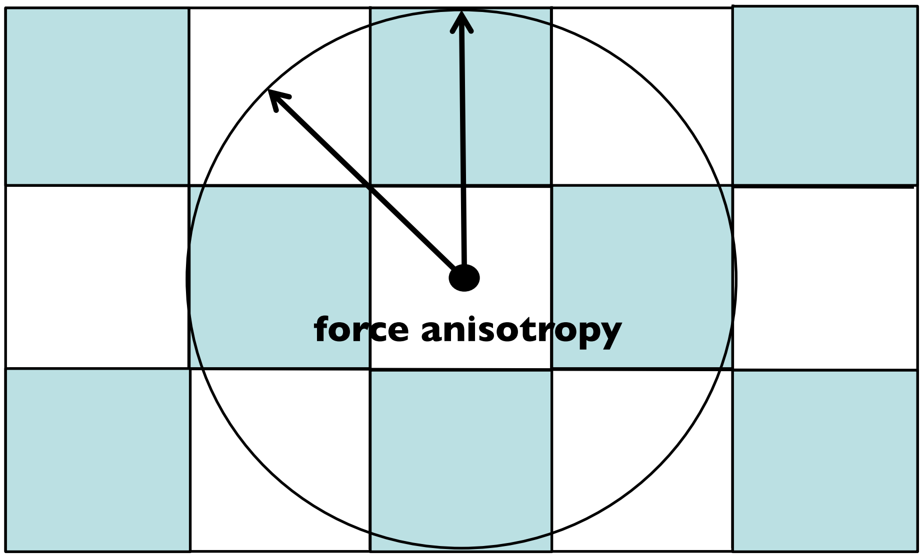
- numerically integrate Poisson's equation



Solving for Gravity

- numerically integrate Poisson's equation

pure PM calculation



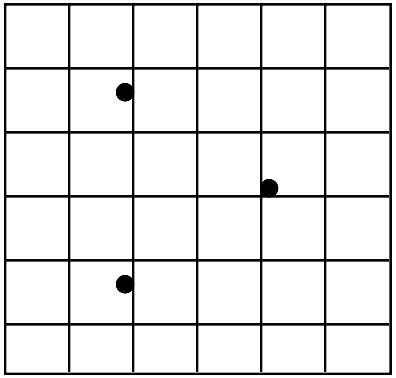
...and what are these wiggles?

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi(\vec{g}_{k,l,m}) = 4\pi G\rho(\vec{g}_{k,l,m})$$

$$\vec{F}(\vec{g}_{k,l,m}) = -m\nabla\Phi(\vec{g}_{k,l,m})$$



1. calculate mass density on grid $\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$
2. solve Poisson's equation on grid $\Phi(\vec{g}_{k,l,m})$
3. differentiate potential to get forces $\vec{F}(\vec{g}_{k,l,m})$
4. interpolate forces back to particles $\vec{F}(\vec{g}_{k,l,m}) \rightarrow \vec{F}(\vec{x}_i)$

Solving for Gravity

- obtaining the forces

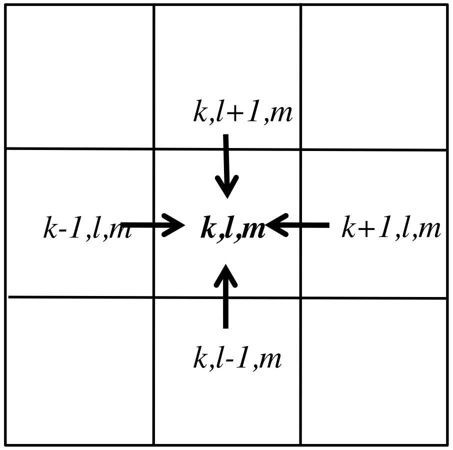
$$\vec{F}(\vec{g}_{k,l,m}) = -m\nabla\Phi(\vec{g}_{k,l,m})$$

$$\downarrow$$

$$F_x(\vec{g}_{k,l,m}) = -m\frac{\Phi(\vec{g}_{k+1,l,m}) - \Phi(\vec{g}_{k-1,l,m})}{2H}$$

$$F_y(\vec{g}_{k,l,m}) = -m\frac{\Phi(\vec{g}_{k,l+1,m}) - \Phi(\vec{g}_{k,l-1,m})}{2H}$$

$$F_z(\vec{g}_{k,l,m}) = -m\frac{\Phi(\vec{g}_{k,l,m+1}) - \Phi(\vec{g}_{k,l,m-1})}{2H}$$



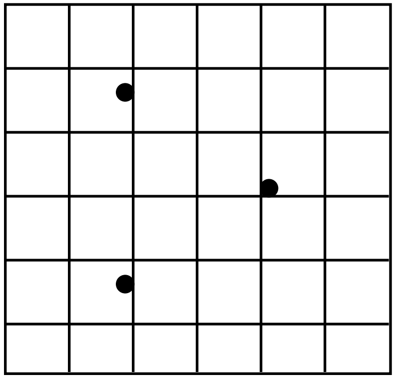
H = (current) grid spacing

Solving for Gravity

- numerically integrate Poisson's equation

$$\Delta\Phi(\vec{g}_{k,l,m}) = 4\pi G\rho(\vec{g}_{k,l,m})$$

$$\vec{F}(\vec{g}_{k,l,m}) = -m\nabla\Phi(\vec{g}_{k,l,m})$$



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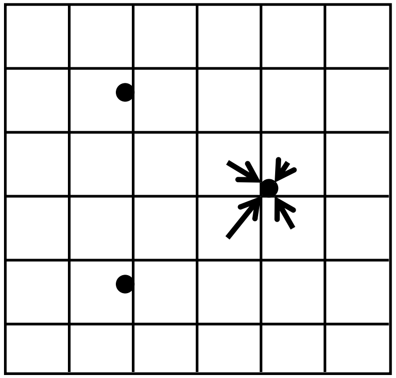
Solving for Gravity

- interpolating the forces

$$\vec{F}(\vec{g}_{k,l,m}) \rightarrow \vec{F}(\vec{r}_i)$$

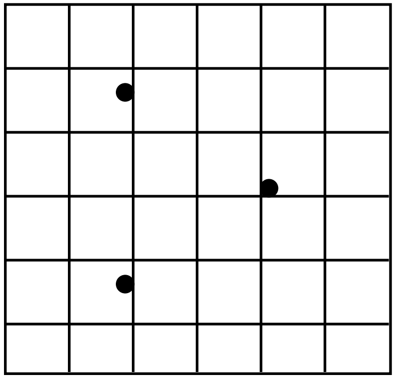


Solving for Gravity

- Particle-Mesh (PM) method

$$\Delta\Phi(\vec{g}_{k,l,m}) = 4\pi G\rho(\vec{g}_{k,l,m})$$

$$\vec{F}(\vec{g}_{k,l,m}) = -m\nabla\Phi(\vec{g}_{k,l,m})$$



1. calculate mass density on grid $\vec{x}_i \rightarrow \rho(\vec{g}_{k,l,m})$
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4. interpolate forces back to particles $\vec{F}(\vec{g}_{k,l,m}) \rightarrow \vec{F}(\vec{x}_i)$

anyone fancies to write a PM code as the project?