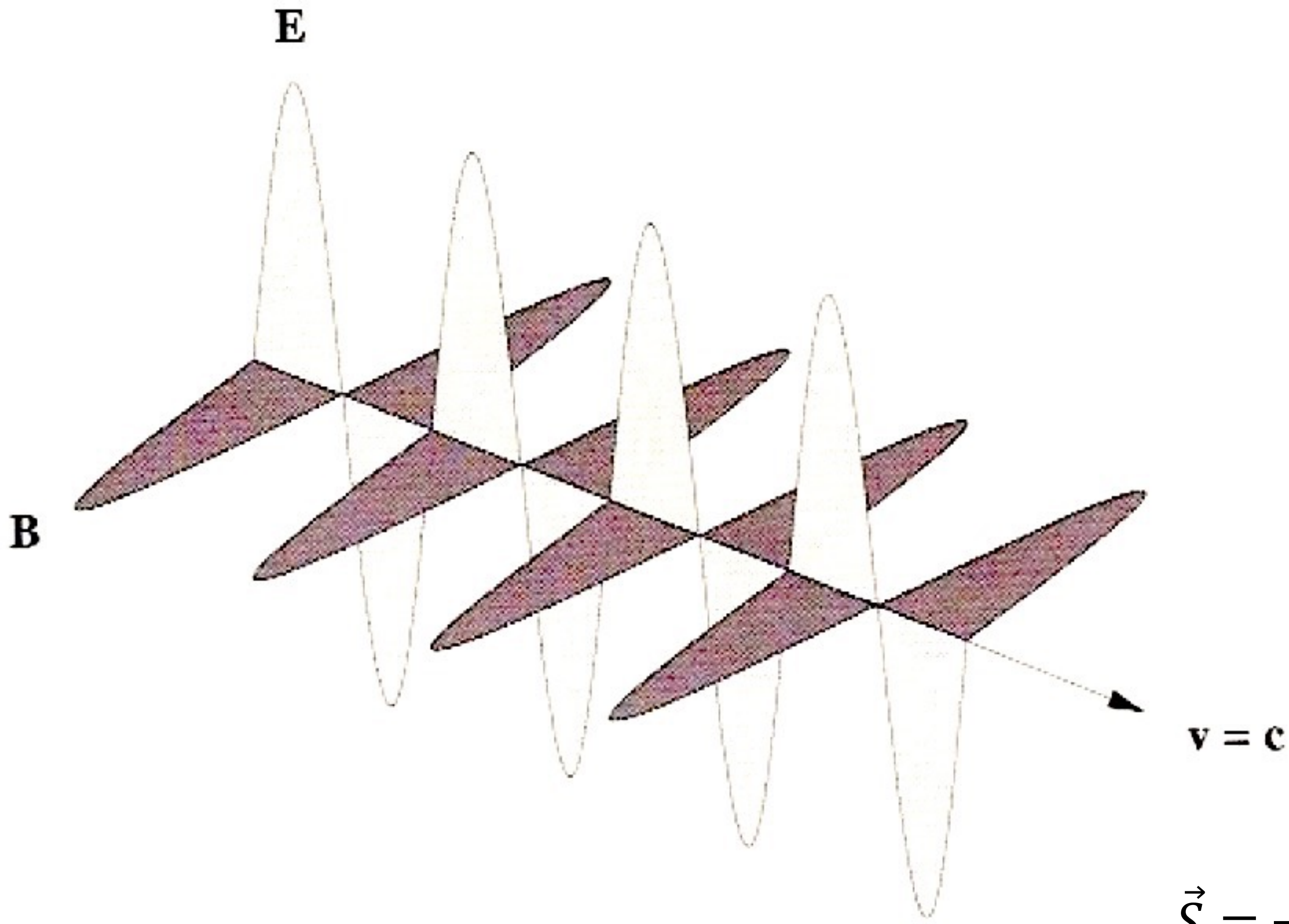
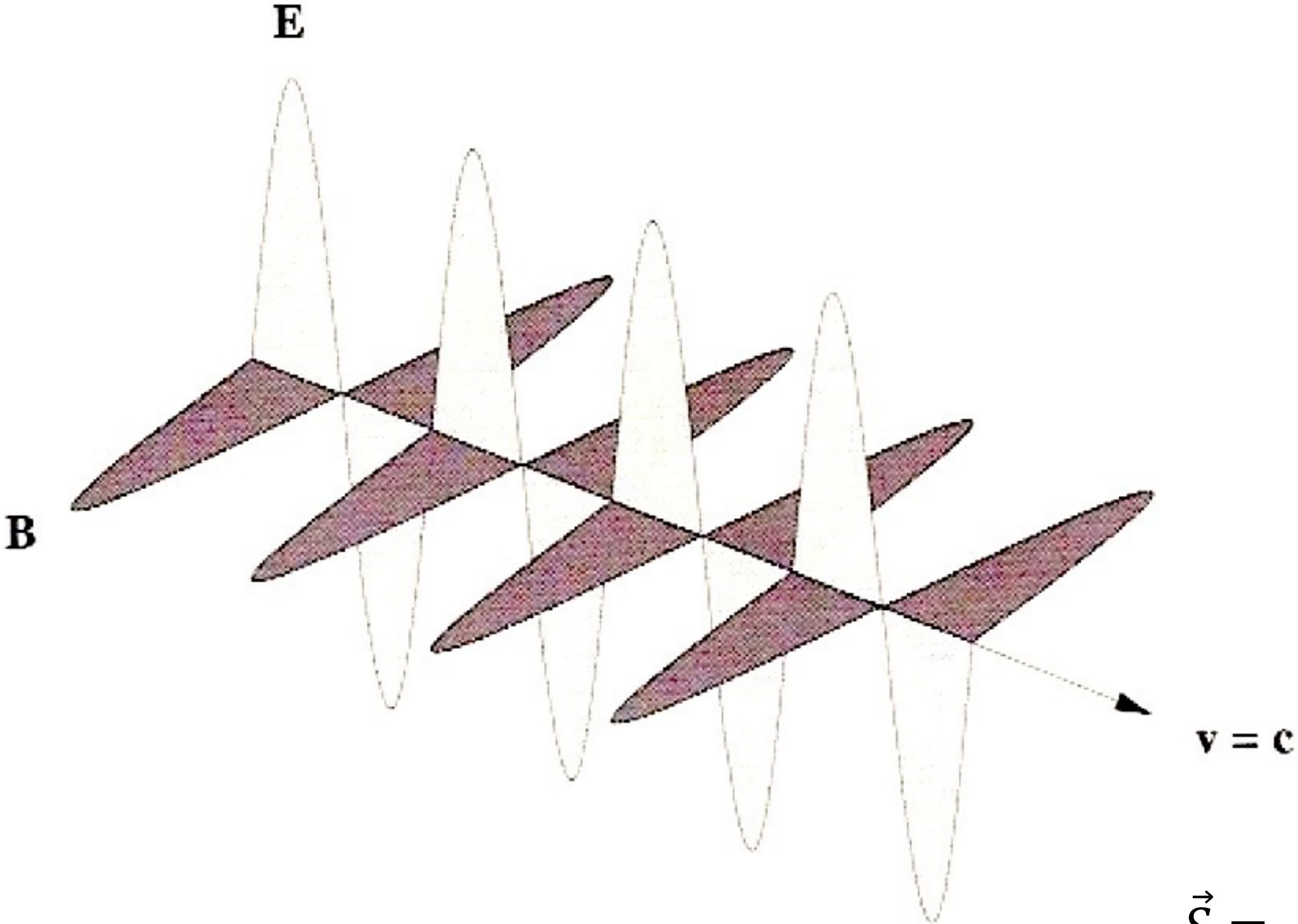
Alexander Knebe (*Universidad Autonoma de Madrid*)



$$\vec{S} = \frac{c}{4\pi}\left(\vec{E}\times\vec{B}\right)$$ (energy/time/area)



$$\vec{S} = \frac{c}{4\pi}\left(\vec{E}\times\vec{B}\right)$$ (energy/time/area)

pure non-relativistic treatment!

- Lorentz force
- Maxwell equations
- Poynting vector
- electromagnetic waves
- radiation spectrum
- electromagnetic potentials

- **Lorentz force**
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- electromagnetic waves
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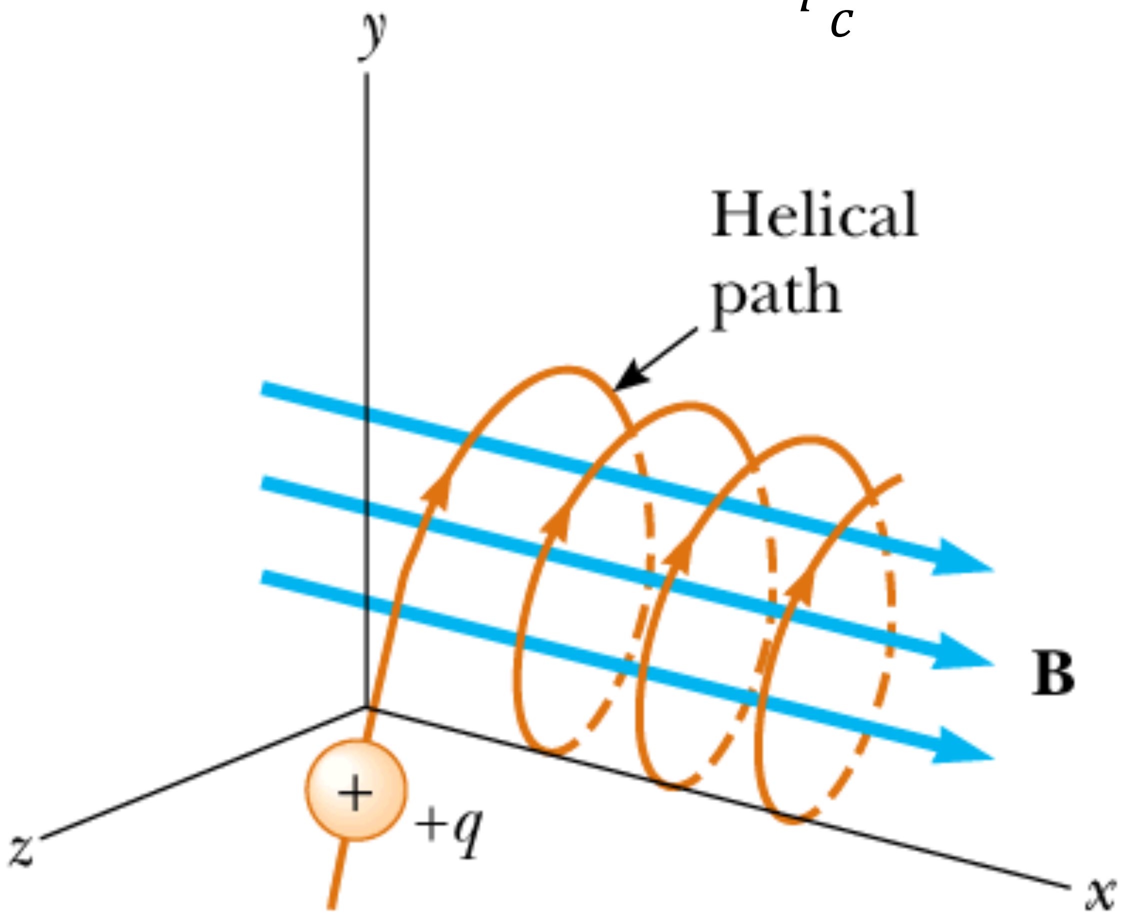
- Lorentz force

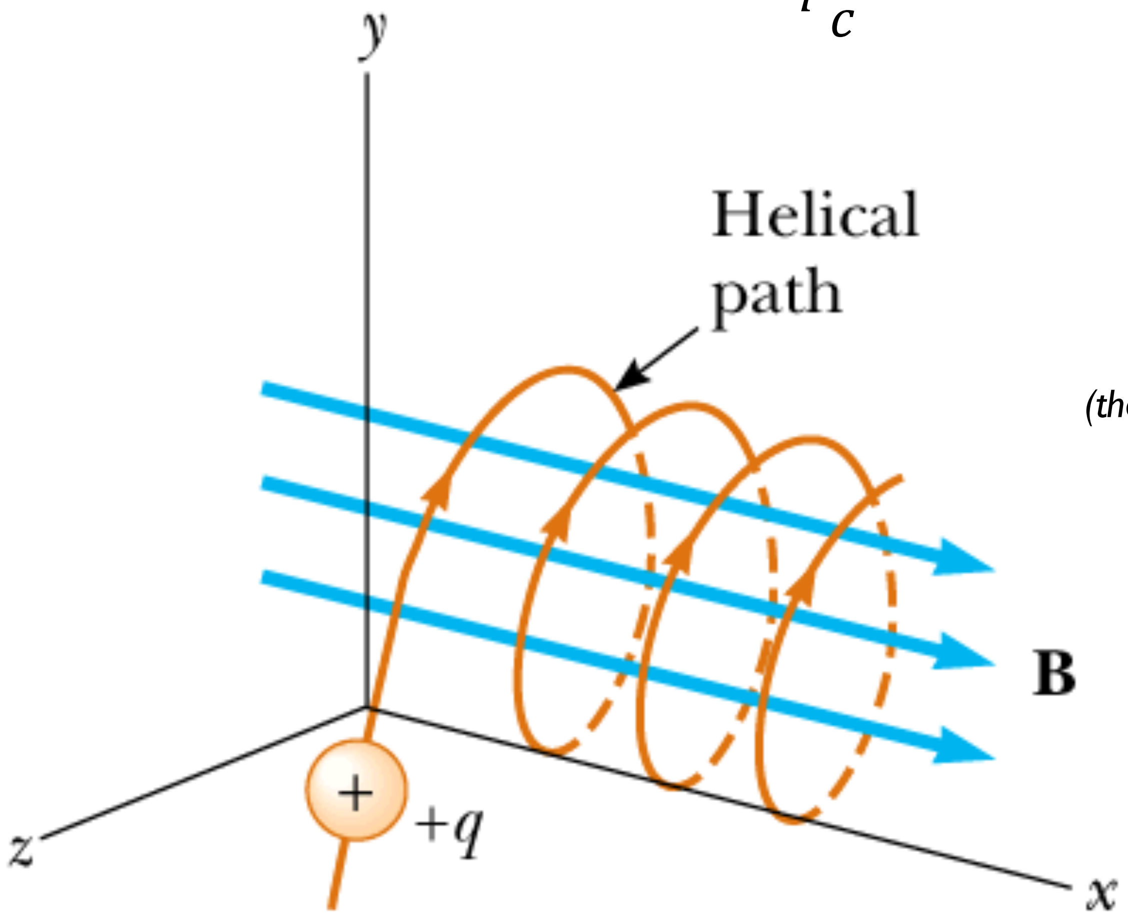
$$\vec{F} = q\frac{1}{c}\vec{v}\times\vec{B}$$



charged particles are forced to spiral around magnetic field lines

- Lorentz force

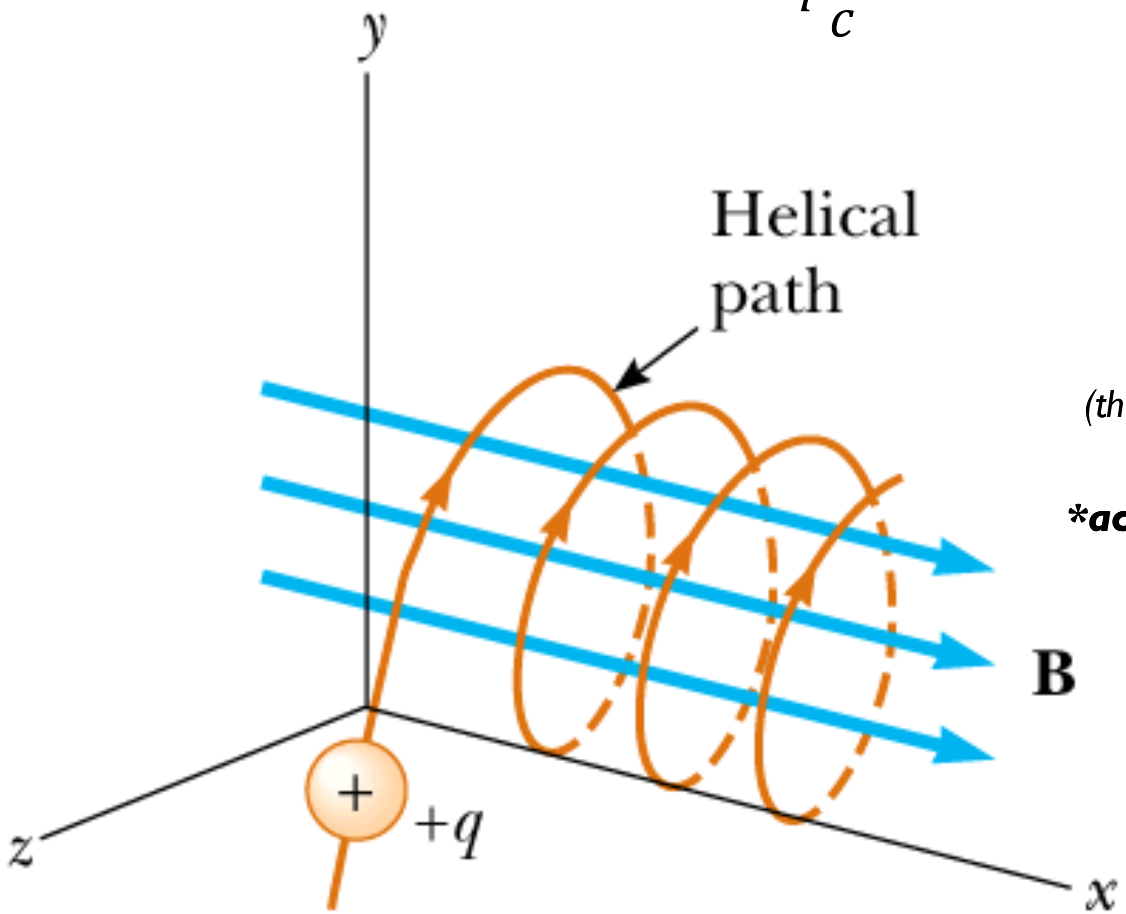
$$\vec{F} = q\frac{1}{c}\vec{v}\times\vec{B}$$



charged particles are forced to spiral around magnetic field lines (they are hence trapped by closed field lines...)

- Lorentz force

$$\vec{F} = q\frac{1}{c}\vec{v}\times\vec{B}$$



charged particles are forced to spiral around magnetic field lines (they are hence trapped by closed field lines...)*

****accelerated charged particles radiate!***

- Lorentz force

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$$

- Lorentz force

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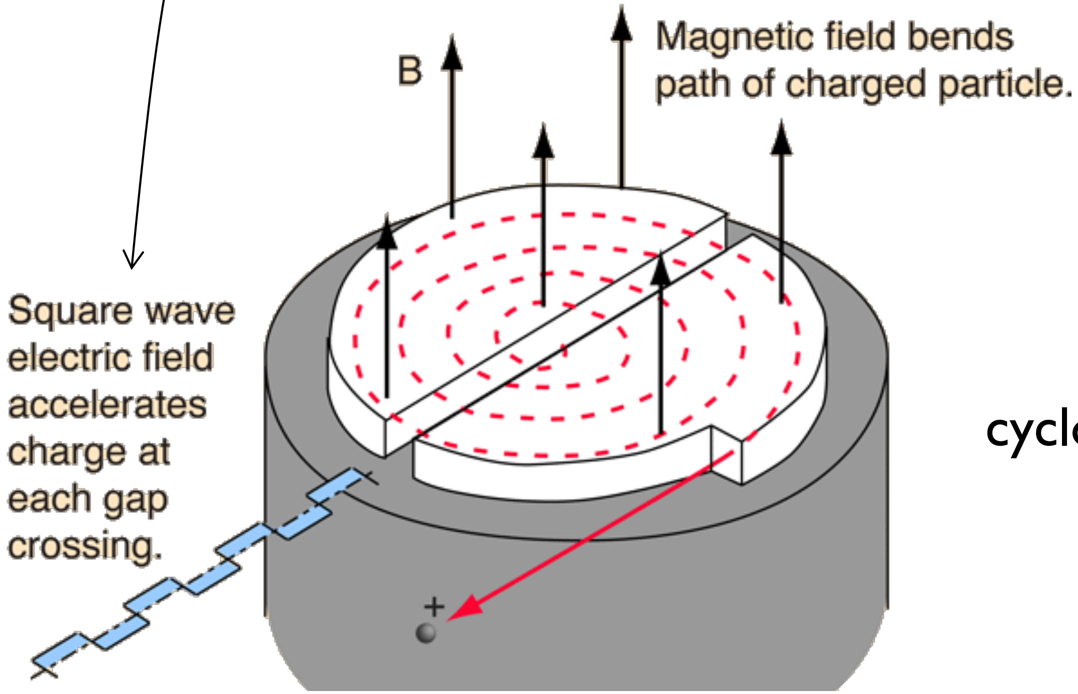
example?

- Lorentz force

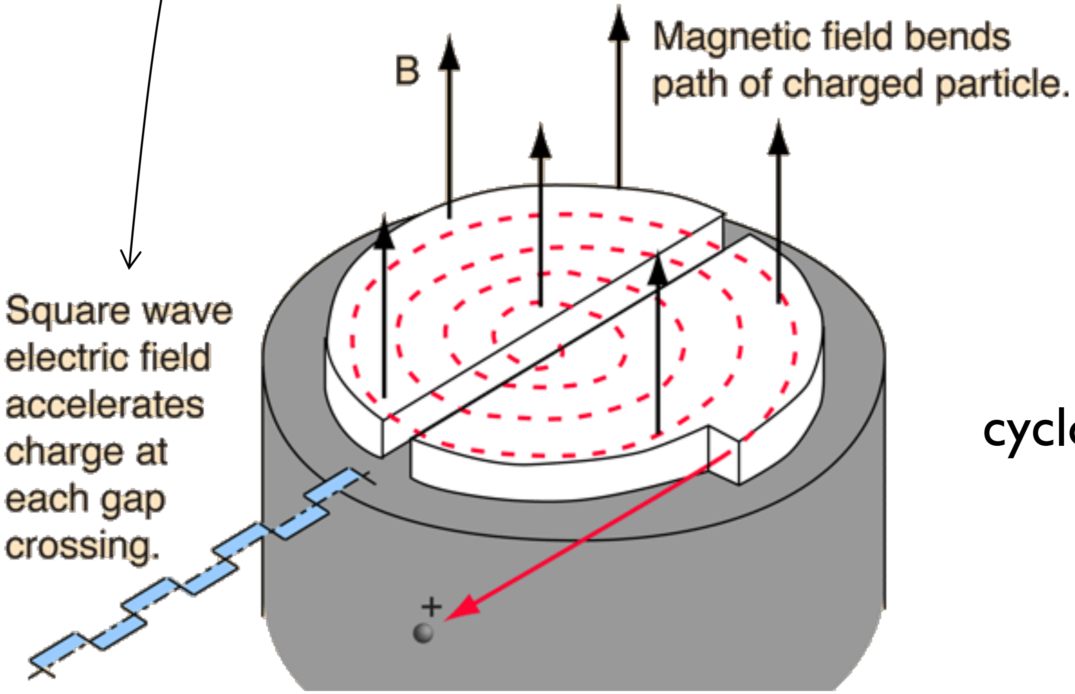
$$\vec{F} = q\left(\overbrace{\vec{E}} + \underbrace{\frac{1}{c}\vec{v}\times\vec{B}}\right)$$

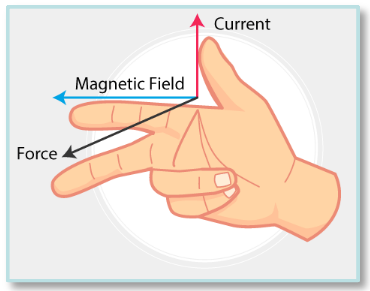


cyclotron

- Lorentz force

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$$





cyclotron

- Lorentz force

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$$

- rate of work done by fields?

- Lorentz force

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$$

- rate of work done by fields

$$\frac{dQ}{dt} = \frac{\vec{F}\cdot\overrightarrow{ds}}{dt} = \vec{v}\cdot\vec{F} = \vec{v}\cdot q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$$

- Lorentz force

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$$

- rate of work done by fields

$$\vec{v}\cdot\vec{F} = q\vec{v}\cdot\vec{E}$$

- non-relativistic particle

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

- Lorentz force

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$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$$\left.\begin{matrix}\vec{v}\cdot\vec{F} = q\vec{v}\cdot\vec{E} \\ \vec{F} = m\frac{d\vec{v}}{dt}\end{matrix}\right\} \quad m\vec{v}\cdot\frac{d\vec{v}}{dt} = q\vec{v}\cdot\vec{E}$$

- Lorentz force

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$$

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$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$$\left.\begin{array}{l}\vec{v}\cdot\vec{F} = q\vec{v}\cdot\vec{E}\\ \vec{F} = m\dfrac{d\vec{v}}{dt}\end{array}\right\} \underbrace{m\vec{v}\cdot\frac{d\vec{v}}{dt}}_{\frac{d}{dt}\left(\frac{1}{2}mv^2\right)} = q\vec{v}\cdot\vec{E}$$

- Lorentz force

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$$

- rate of work done by fields (on non-relativistic particles)

$$\frac{dE_{kin}}{dt} = q\vec{v}\cdot\vec{E}$$

- Lorentz force

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$$

- rate of work done by fields (on non-relativistic particles)

$$\frac{dE_{kin}}{dt} = \underbrace{q\vec{v}} \cdot \vec{E}$$

current $\vec{j} = \rho_q \vec{v}$; ρ_q= charge density

- Lorentz force

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)$$

- rate of work done by fields (on non-relativistic particles)

$$\frac{de_{kin}}{dt} = \vec{j}\cdot\vec{E}$$

e_{kin}= energy density

- Lorentz force
- **Maxwell equations**
- Poynting vector
- electromagnetic waves
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- Maxwell equations

- Maxwell equations

they describe the behaviour of both electric and magnetic fields
and
their interaction with matter

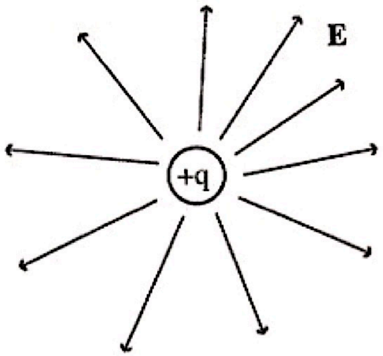
- Maxwell equations

$\vec{D} = \epsilon \vec{E}$

ϵ: dielectric constant

Gauss' law

$\nabla \cdot \mathbf{D} = 4\pi\rho$



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equations are given in Gaussian units

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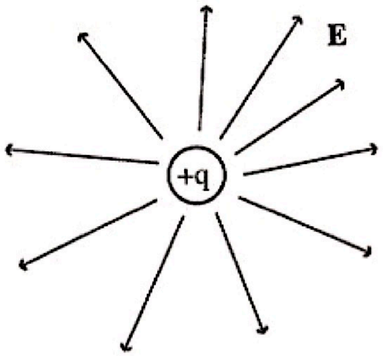
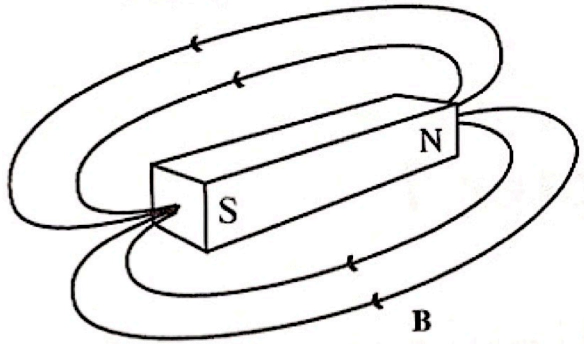


No magnetic monopoles
$\nabla \cdot \mathbf{B} = 0$



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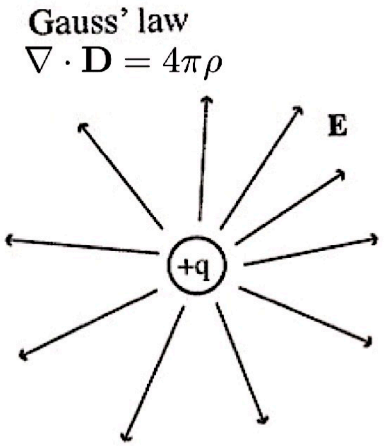

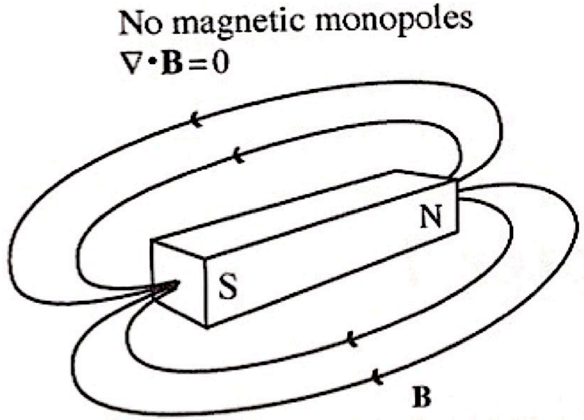

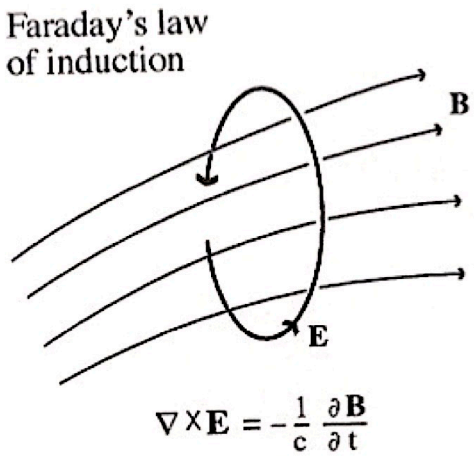

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▪ Maxwell equations

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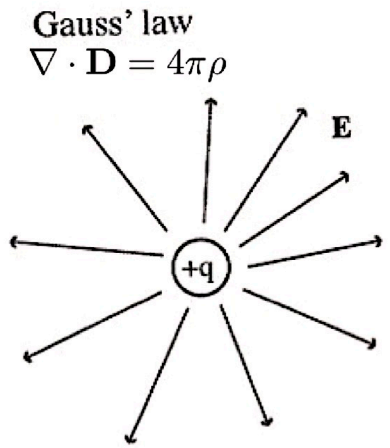

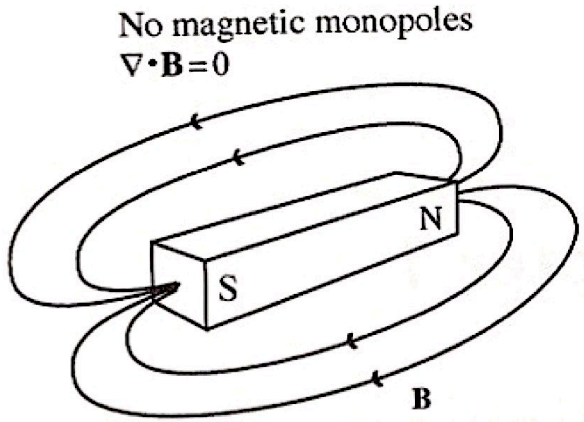

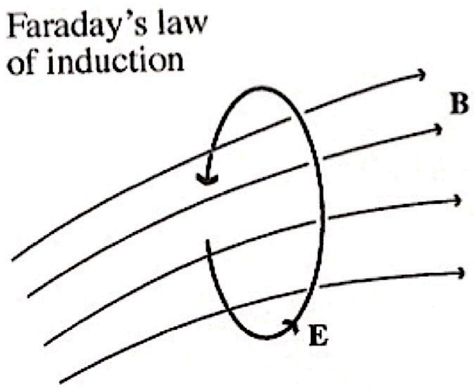

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$ → a time-varying B-field induces a (non-conservative) E-field (which in turn can accelerate charges, leading to a current)

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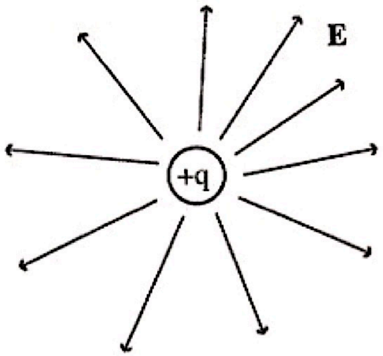

No magnetic monopoles

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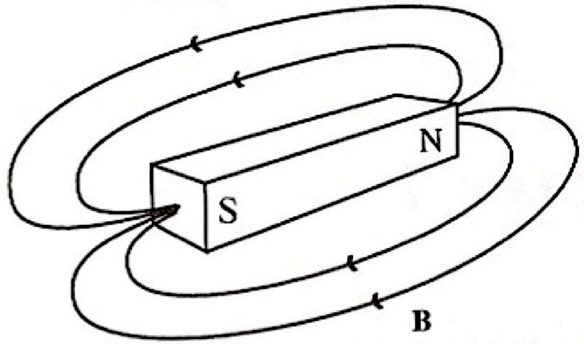
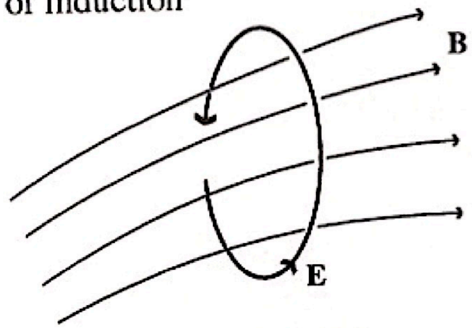
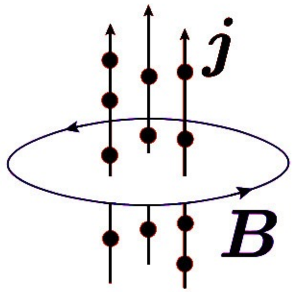


Faraday's law of induction



$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$



$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$$

$\vec{H} = \frac{1}{\mu}\vec{B}$

μ: magnetic permeability

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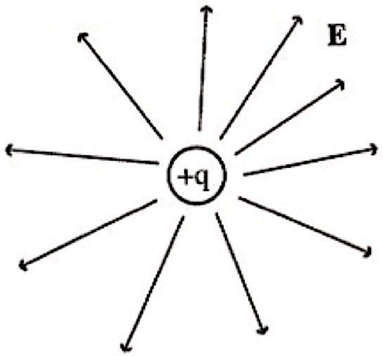

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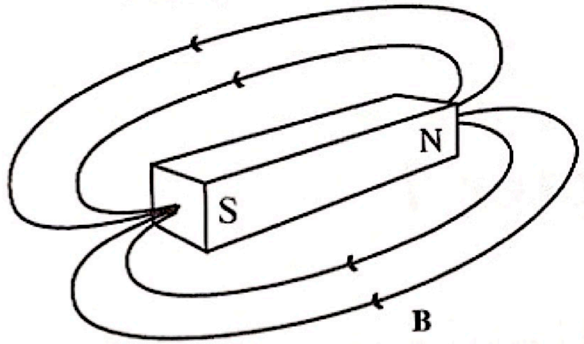

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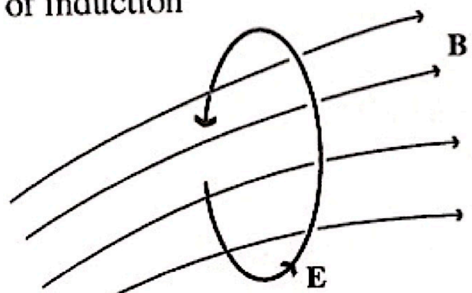
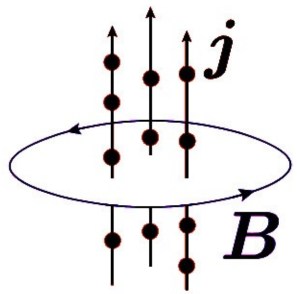


$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$

a current and/or time varying E-field induces a B-field



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$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

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charge conservation: $$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$$

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$\vec{D} = \epsilon\vec{E}$

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- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

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- rate of work done per unit volume:

$$\frac{de_{kin}}{dt} = \vec{j} \cdot \vec{E}$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

- Maxwell equations

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- rate of work done per unit volume:

$$\frac{de_{kin}}{dt} = \vec{j} \cdot \vec{E}$$ → *eventually leading to the Poynting vector...*

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation: $$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \nabla \cdot \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- rate of work done per unit volume:

$$\frac{de_{kin}}{dt} = \vec{j} \cdot \vec{E} \longrightarrow \vec{E} \cdot (\nabla \times \vec{H}) = \frac{4\pi}{c}\vec{E} \cdot \vec{j} + \frac{1}{c}\vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \nabla \cdot \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- rate of work done per unit volume:

$$\frac{de_{kin}}{dt} = \vec{j} \cdot \vec{E} \longrightarrow \vec{E} \cdot \left(\nabla \times \vec{H}\right) = \frac{4\pi}{c}\vec{E} \cdot \vec{j} + \frac{1}{c}\vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

$$\longrightarrow \vec{j} \cdot \vec{E} = \frac{1}{4\pi}\left(c\vec{E} \cdot \left(\nabla \times \vec{H}\right) - \frac{1}{c}\vec{E} \cdot \frac{\partial \vec{D}}{\partial t}\right)$$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

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$\vec{j} = \rho\vec{v}$
$\vec{D} = \epsilon\vec{E}$
$\vec{H} = \frac{1}{\mu}\vec{B}$

$$\longrightarrow \quad \vec{j} \cdot \vec{E} = \frac{1}{4\pi}\left(c\overbrace{\vec{E} \cdot (\nabla \times \vec{H})}^{\vec{E} \cdot (\nabla \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H})} - \frac{1}{c}\vec{E} \cdot \frac{\partial \vec{D}}{\partial t}\right)$$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

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- rate of work done per unit volume:

$$\frac{de_{kin}}{dt} = \vec{j}\cdot\vec{E} \longrightarrow \vec{j}\cdot\vec{E} = \frac{1}{4\pi}\left(-\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} - c\nabla\cdot\left(\vec{E}\times\vec{H}\right) - \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}\right)$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

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$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

charge conservation: $$\nabla\times\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t} \xrightarrow{\nabla\cdot} 0 = \nabla\cdot\vec{j} + \frac{\partial\vec{\rho}}{\partial t} \xrightarrow{\vec{j}=\rho\vec{v}} \frac{\partial\vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- rate of work done per unit volume:

$$\frac{de_{kin}}{dt} = \frac{1}{4\pi}\left(-\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} - c\nabla\cdot\left(\vec{E}\times\vec{H}\right) - \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}\right)$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

Poynting vector...

- Lorentz force
- Maxwell equations
- **Poynting vector**
- electromagnetic waves
- radiation spectrum
- electromagnetic potentials

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

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charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- rate of work done per unit volume:

$$\frac{de_{kin}}{dt} = \frac{1}{4\pi}\left(-\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - c\nabla \cdot \left(\vec{E} \times \vec{H}\right) - \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}\right)$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem:

$$\frac{de_{kin}}{dt} = \frac{1}{4\pi}\left(-\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - c\nabla \cdot \left(\vec{E} \times \vec{H}\right) - \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}\right)$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem:

$$\frac{de_{kin}}{dt} = \frac{1}{4\pi}\left(\boxed{-\vec{H} \cdot \frac{\partial \vec{B}}{\partial t}} - c\nabla \cdot \left(\vec{E} \times \vec{H}\right) \boxed{- \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}}\right)$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

the energy density changes at a rate given by the work done on the charges

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem:

$$\frac{de_{kin}}{dt} = \frac{1}{4\pi}\left(-\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - c\nabla \cdot (\vec{E} \times \vec{H}) - \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}\right)$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

the energy density changes at a rate given by the work done on the charges, minus the rate at which energy is radiated away

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem:

$$\frac{de_{kin}}{dt} = \frac{1}{4\pi}\left(-\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - c\nabla \cdot \left(\vec{E} \times \vec{H}\right) - \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}\right)$$

$\epsilon = const., \mu = const.$

$$= -\frac{1}{8\pi}\frac{\partial}{\partial t}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right) - \frac{c}{4\pi}\nabla \cdot \left(\vec{E} \times \vec{H}\right)$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla\cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j}=\rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem:

$$\frac{de_{kin}}{dt} = -\frac{1}{8\pi}\frac{\partial}{\partial t}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right) - \nabla \cdot \frac{c}{4\pi}\left(\vec{E}\times\vec{H}\right)$$

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem:

$$\frac{de_{kin}}{dt} = -\frac{1}{8\pi}\frac{\partial}{\partial t}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right) - \nabla \cdot \underbrace{\frac{c}{4\pi}\left(\vec{E}\times\vec{H}\right)}$$

Poynting vector: $\vec{S} = \frac{c}{4\pi}\left(\vec{E}\times\vec{H}\right)$ directional energy flux

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem:

$$\frac{de_{kin}}{dt} = -\frac{1}{8\pi}\frac{\partial}{\partial t}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right) - \nabla \cdot \vec{S}$$

$$\vec{S} = \frac{c}{4\pi}(\vec{E} \times \vec{H})$$

$$\vec{j} = \rho\vec{v}$$

$$\vec{D} = \epsilon\vec{E}$$

$$\vec{H} = \frac{1}{\mu}\vec{B}$$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem:

$$\frac{de_{kin}}{dt} = -\frac{1}{8\pi}\frac{\partial}{\partial t}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right) - \nabla \cdot \vec{S}$$

$$\vec{S} = \underbrace{\frac{c}{4\pi}(\vec{E} \times \vec{H})}$$

flux is orthogonal to both $\vec{E}$ and $\vec{B}$!

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

Radiation Fields

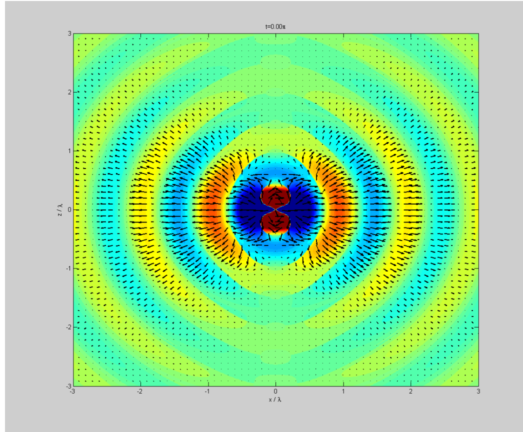
review of electrodynamics

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$



Dipole radiation of a dipole vertically in the page,
showing electric field strength (color) and Poynting vector (arrows) in the plane of the page.

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem:

$$\frac{de_{kin}}{dt} = -\frac{1}{8\pi}\frac{\partial}{\partial t}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right) - \nabla \cdot \vec{S}$$

$$\vec{S} = \underbrace{\frac{c}{4\pi}(\vec{E} \times \vec{H})}$$

flux is orthogonal to both $\vec{E}$ and $\vec{B}$!

$\vec{j} = \rho\vec{v}$

$\vec{D} = \epsilon\vec{E}$

$\vec{H} = \frac{1}{\mu}\vec{B}$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem: *integral form!?*

$$\frac{de_{kin}}{dt} = -\frac{1}{8\pi}\frac{\partial}{\partial t}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right) - \nabla \cdot \vec{S}$$

$$\vec{S} = \frac{c}{4\pi}(\vec{E} \times \vec{H})$$

$$\vec{j} = \rho\vec{v}$$

$$\vec{D} = \epsilon\vec{E}$$

$$\vec{H} = \frac{1}{\mu}\vec{B}$$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem: *integral form*

$$\int_V \frac{de_{kin}}{dt} dV = -\frac{d}{dt}\int_V \frac{1}{8\pi}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right) dV - \int_\Sigma \vec{S} \cdot d\vec{A}$$

$$\vec{S} = \frac{c}{4\pi}(\vec{E} \times \vec{H})$$

$$\vec{j} = \rho\vec{v}$$

$$\vec{D} = \epsilon\vec{E}$$

$$\vec{H} = \frac{1}{\mu}\vec{B}$$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

charge conservation:

$$\nabla\times\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t} \xrightarrow{\nabla\cdot} 0 = \vec{j} + \frac{\partial\vec{\rho}}{\partial t} \xrightarrow{\vec{j}=\rho\vec{v}} \frac{\partial\vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem: *integral form*

$$\underbrace{\int_V \frac{de_{kin}}{dt}dV}_{\text{rate of change of mechanical energy}} = -\underbrace{\frac{d}{dt}\int_V \frac{1}{8\pi}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right)dV}_{\text{rate of change of E- and B-field energy}} - \int_\Sigma \vec{S}\cdot d\vec{A}$$

$$\vec{S} = \frac{c}{4\pi}(\vec{E}\times\vec{H})$$

$$\vec{j} = \rho\vec{v}$$

$$\vec{D} = \epsilon\vec{E}$$

$$\vec{H} = \frac{1}{\mu}\vec{B}$$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem: *integral form*

$$\frac{d}{dt}\left(U_{mech} + U_{field}\right) = -\int_{\Sigma} \vec{S} \cdot d\vec{A}$$

$$\vec{S} = \frac{c}{4\pi}(\vec{E} \times \vec{H})$$

$$\vec{j} = \rho\vec{v}$$

$$\vec{D} = \epsilon\vec{E}$$

$$\vec{H} = \frac{1}{\mu}\vec{B}$$

- Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

charge conservation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \xrightarrow{\nabla \cdot} 0 = \vec{j} + \frac{\partial \vec{\rho}}{\partial t} \xrightarrow{\vec{j} = \rho\vec{v}} \frac{\partial \vec{\rho}}{\partial t} + \rho\vec{v} = 0$$

- Poynting theorem: *integral form*

$$\frac{d}{dt}\left(U_{mech} + U_{field}\right) = -\int_{\Sigma} \vec{S} \cdot d\vec{A}$$

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$$\vec{j} = \rho\vec{v}$$

$$\vec{D} = \epsilon\vec{E}$$

$$\vec{H} = \frac{1}{\mu}\vec{B}$$

the rate of change of total energy within a volume
is equal to the net inward flow of energy through the bounding surface

- Radiation?

- Poynting theorem:

$$\frac{d}{dt}\left(U_{mech} + U_{field}\right) = -\int_{\Sigma} \vec{S} \cdot d\vec{A}$$

$$\vec{S} = \frac{c}{4\pi}\left(\vec{E}\times\vec{H}\right)$$

the rate of change of total energy within a volume
is equal to the net inward flow of energy through the bounding surface

▪ Radiation

In **electrostatics** both $\vec{E}$ and $\vec{B}$ decrease like r^{-2}

$\rightarrow \vec{S}$ decreases like r^{-4} and thus the integral goes to zero since the surface area increases only as r^{-2}.

▪ Poynting theorem:

$$\frac{d}{dt}\left(U_{mech} + U_{field}\right) = -\int_{\Sigma} \vec{S}\cdot d\vec{A}$$

$$\vec{S} = \frac{c}{4\pi}(\vec{E}\times\vec{H})$$

the rate of change of total energy within a volume
is equal to the net inward flow of energy through the bounding surface

▪ Radiation

In **electrostatics** both $\vec{E}$ and $\vec{B}$ decrease like r^{-2}

→ $\vec{S}$ decreases like r^{-4} and thus the integral goes to zero since the surface area increases only as r^{-2}.

For **time varying** fields $\vec{E}$ and $\vec{B}$ decrease like r^{-1}

→ the integral can contribute a finite amount to the rate of change of energy of the system.

▪ Poynting theorem:

$$\frac{d}{dt}\left(U_{mech} + U_{field}\right) = -\int_{\Sigma} \vec{S} \cdot d\vec{A}$$

$$\vec{S} = \frac{c}{4\pi}\left(\vec{E}\times\vec{H}\right)$$

the rate of change of total energy within a volume
is equal to the net inward flow of energy through the bounding surface

▪ Radiation

In electrostatics both $\vec{E}$ and $\vec{B}$ decrease like r^{-2}

→ $\vec{S}$ decreases like r^{-4} and thus the integral goes to zero since the surface area increases only as r^{-2}.

For **time varying** fields $\vec{E}$ and $\vec{B}$ decrease like r^{-1}

→ the integral can contribute a finite amount to the rate of change of energy of the system.

This energy flowing in (or out) at large distances is called **radiation**.

▪ Poynting theorem:

$$\frac{d}{dt}\left(U_{mech} + U_{field}\right) = -\int_{\Sigma} \vec{S} \cdot d\vec{A}$$

$$\vec{S} = \frac{c}{4\pi}\left(\vec{E} \times \vec{H}\right)$$

the rate of change of total energy within a volume
is equal to the net inward flow of energy through the bounding surface

- Lorentz force
- Maxwell equations
- Poynting vector
- **electromagnetic waves**
- radiation spectrum
- electromagnetic potentials

- Electromagnetic Waves

- Electromagnetic Waves – Maxwell equations

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \quad \xrightarrow{\nabla\times} \quad \nabla \times \left(\nabla \times \vec{E}\right) = -\frac{1}{c}\frac{\partial \nabla \times \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \xrightarrow{\nabla\times} \nabla \times \left(\nabla \times \vec{E}\right) = -\frac{1}{c}\frac{\partial \nabla \times \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \xrightarrow{\nabla\times} \nabla \times \left(\nabla \times \vec{E}\right) = -\frac{1}{c}\frac{\partial \nabla \times \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \longrightarrow \qquad = -\frac{1}{c}\frac{\partial \nabla \times \vec{B}}{\partial t} = -\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \xrightarrow{\nabla\times} \nabla \times (\nabla \times \vec{E}) = -\frac{1}{c}\frac{\partial \nabla \times \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \longrightarrow \quad = -\frac{1}{c}\frac{\partial \nabla \times \vec{B}}{\partial t} = -\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\downarrow$$

$$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \xrightarrow{\nabla\times} \nabla\times\left(\nabla\times\vec{E}\right) = -\frac{1}{c}\frac{\partial \nabla\times\vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \longrightarrow = -\frac{1}{c}\frac{\partial \nabla\times\vec{B}}{\partial t} = -\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla\times\left(\nabla\times\vec{E}\right) = \nabla\left(\nabla\cdot\vec{E}\right) - \boxed{\nabla^2\vec{E}}$$

reminder: $\nabla^2 \vec{A} = \begin{pmatrix} \nabla^2 A_x \\ \nabla^2 A_y \\ \nabla^2 A_z \end{pmatrix}$

(it is a vectorial Laplace operator...)

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \xrightarrow{\nabla\times} \nabla \times (\nabla \times \vec{E}) = -\frac{1}{c}\frac{\partial \nabla \times \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \longrightarrow \quad = -\frac{1}{c}\frac{\partial \nabla \times \vec{B}}{\partial t} = -\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla \times (\nabla \times \vec{E}) = \underbrace{\nabla(\nabla \cdot \vec{E})}_{=0} - \nabla^2 \vec{E}$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \quad \xrightarrow{\nabla\times} \quad \nabla\times\left(\nabla\times\vec{E}\right) = -\frac{1}{c}\frac{\partial\nabla\times\vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t} \quad \longrightarrow \quad = -\frac{1}{c}\frac{\partial\nabla\times\vec{B}}{\partial t} = -\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\downarrow$$

$$\nabla\times\left(\nabla\times\vec{E}\right) = -\nabla^2\vec{E}$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\underline{\underline{\nabla \cdot \vec{E} = 0}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \xrightarrow{\nabla \times} \nabla \times \left(\nabla \times \vec{E}\right) = -\frac{1}{c}\frac{\partial \nabla \times \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \longrightarrow \quad = -\frac{1}{c}\frac{\partial \nabla \times \vec{B}}{\partial t} = \underline{\underline{-\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}}}$$

$$\downarrow$$

$$\nabla \times \left(\nabla \times \vec{E}\right) = \underline{\underline{-\nabla^2 \vec{E}}}$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \quad \nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

analogy for $\vec{B}$...

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\left.\right\} \quad \begin{aligned} \nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} &= 0 \\ \nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} &= 0 \end{aligned}$$

- Electromagnetic Waves – Maxwell equations in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

well known wave-equations for $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$

- Electromagnetic Waves – propagation in vacuum

Ansatz:

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

$$\nabla^2\vec{B} - \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} = 0$$

- Electromagnetic Waves – propagation in vacuum

Ansatz:

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$\vec{k} = k\vec{n}$, $\vec{n}$: unit vector in direction of wave propagation

- Electromagnetic Waves – propagation in vacuum

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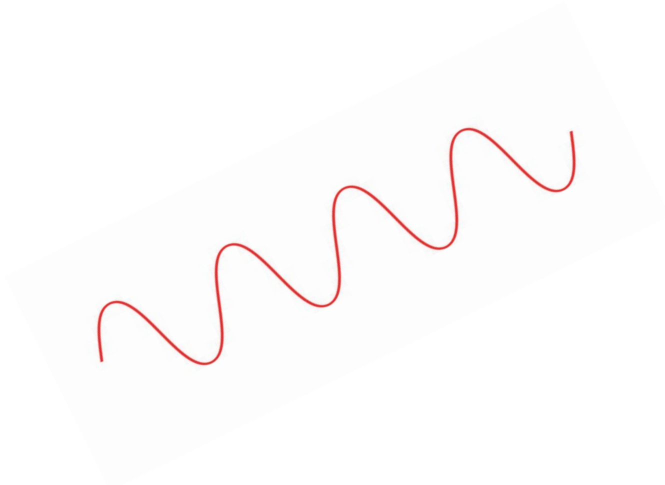

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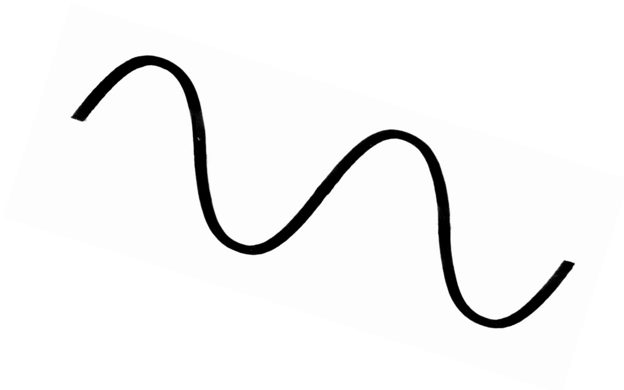


- Electromagnetic Waves – propagation in vacuum

Ansatz:

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

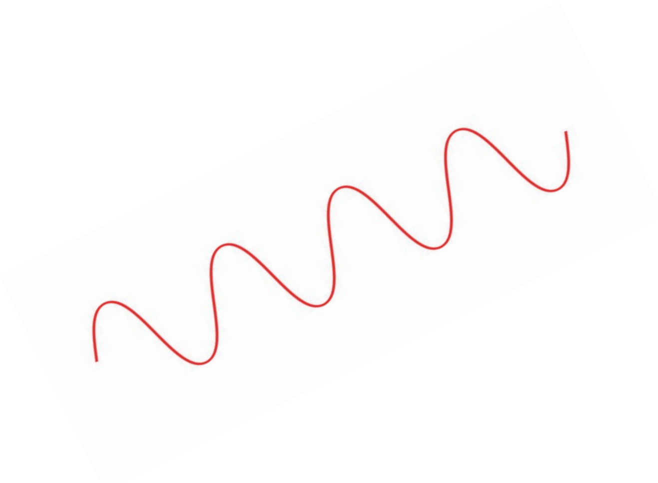
$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$\vec{a}_1$: unit vector, direction **to be determined**
E_0: complex constant, value **to be determined**

?

$\vec{a}_2$: unit vector, direction **to be determined**
B: complex constant, value **to be determined**

$\vec{k} = k\vec{n}$, $\vec{n}$: unit vector in direction of wave propagation



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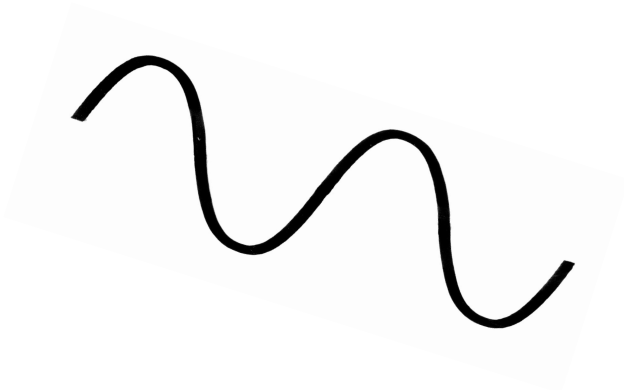

- Electromagnetic Waves – propagation in vacuum

Ansatz:

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

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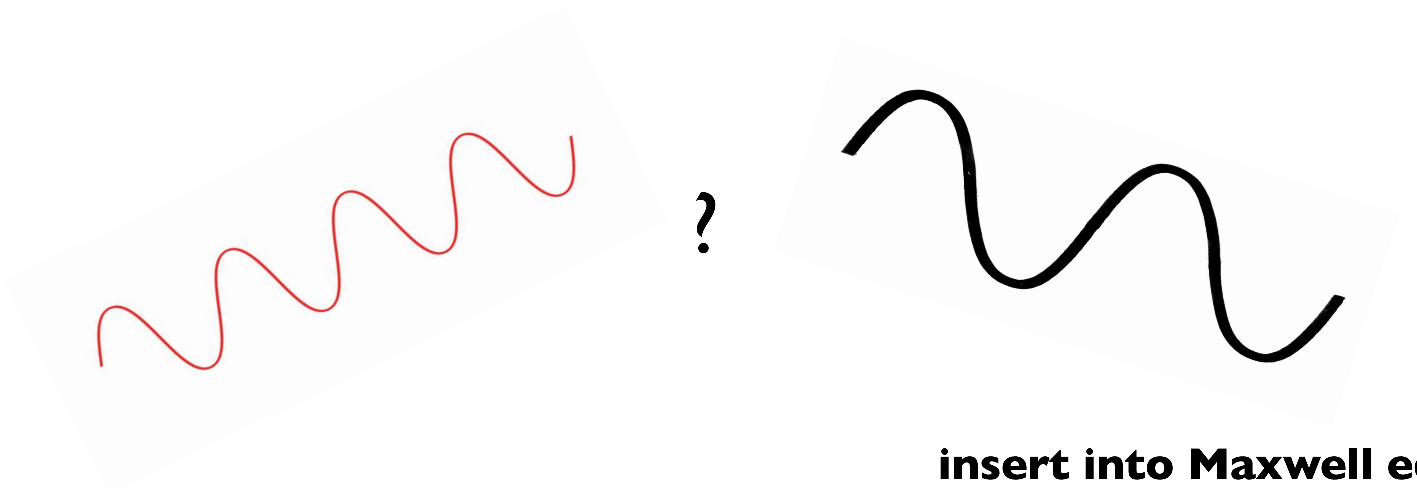
$\vec{k} = k\vec{n}$, $\vec{n}$: unit vector in direction of wave propagation



insert into Maxwell equations →

- Electromagnetic Waves – propagation in vacuum

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- Electromagnetic Waves – propagation in vacuum

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$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\longrightarrow \quad \vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

- Electromagnetic Waves – propagation in vacuum

$$\nabla \cdot \vec{E} = 0 \qquad\qquad i\vec{k} \cdot \vec{a}_1 E_0 = 0$$

$$\nabla \cdot \vec{B} = 0 \qquad\qquad i\vec{k} \cdot \vec{a}_2 B_0 = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \quad \xrightarrow[\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}]{\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}} \quad i\vec{k} \times \vec{a}_1 E_0 = \frac{i\omega}{c}\vec{a}_2 B_0$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \qquad\qquad i\vec{k} \times \vec{a}_2 B_0 = -\frac{i\omega}{c}\vec{a}_1 E_0$$

- Electromagnetic Waves – propagation in vacuum

both $\vec{a}_1$ and $\vec{a}_2$ are orthogonal to the direction of the wave propagation $\vec{k}$

$$\begin{cases} i\vec{k}\cdot\vec{a}_1 E_0 = 0 \\ i\vec{k}\cdot\vec{a}_2 B_0 = 0 \end{cases}$$

$$i\vec{k}\times\vec{a}_1 E_0 = \frac{i\omega}{c}\vec{a}_2 B_0$$

$$i\vec{k}\times\vec{a}_2 B_0 = -\frac{i\omega}{c}\vec{a}_1 E_0$$

▪ Electromagnetic Waves – propagation in vacuum

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⇓

$\vec{a}_1$ and $\vec{a}_2$ are orthogonal to each other

$$\begin{cases} i\vec{k}\times\vec{a}_1 E_0 = \frac{i\omega}{c}\vec{a}_2 B_0 \\ i\vec{k}\times\vec{a}_2 B_0 = -\frac{i\omega}{c}\vec{a}_1 E_0 \end{cases}$$

- Electromagnetic Waves – propagation in vacuum

$\vec{k}, \vec{a}_1$ and $\vec{a}_2$ form a right-handed set

$$i\vec{k}\cdot\vec{a}_1 E_0 = 0$$

$$i\vec{k}\cdot\vec{a}_2 B_0 = 0$$

$$i\vec{k}\times\vec{a}_1 E_0 = \frac{i\omega}{c}\vec{a}_2 B_0$$

$$i\vec{k}\times\vec{a}_2 B_0 = -\frac{i\omega}{c}\vec{a}_1 E_0$$

- Electromagnetic Waves – propagation in vacuum

$\vec{k}, \vec{a}_1$ and $\vec{a}_2$ form a right-handed set:

$$\vec{a}_1 = \vec{k}\times\vec{a}_2$$
$$\vec{a}_2 = -\vec{k}\times\vec{a}_1$$

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▪ Electromagnetic Waves – propagation in vacuum

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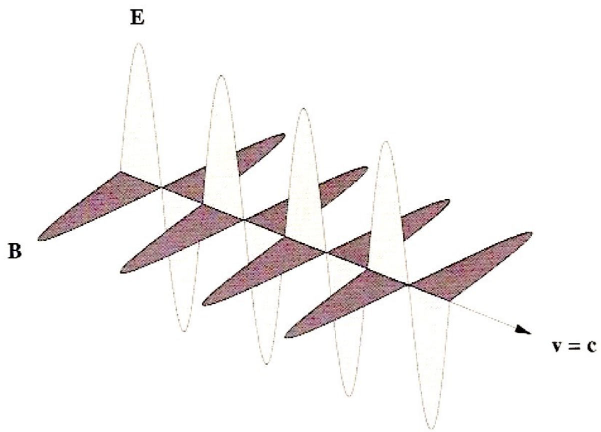
$$i\vec{k} \cdot \vec{a}_2 B_0 = 0$$

$$i\vec{k} \times \vec{a}_1 E_0 = \frac{i\omega}{c} \vec{a}_2 B_0$$

$$i\vec{k} \times \vec{a}_2 B_0 = -\frac{i\omega}{c} \vec{a}_1 E_0$$



$$\vec{E}(\vec{r}, t) = \vec{a}_1 E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$
$$\vec{B}(\vec{r}, t) = \vec{a}_2 B_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

▪ Electromagnetic Waves – propagation in vacuum

$\vec{k}, \vec{a}_1$ and $\vec{a}_2$ form a right-handed set:

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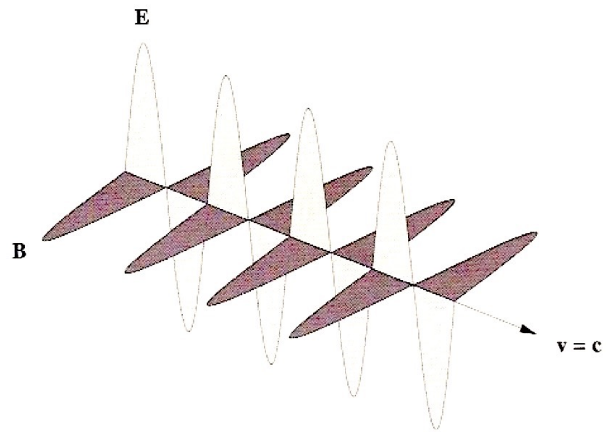
$$\begin{cases} i\vec{k}\times\vec{a}_1 E_0 = \dfrac{i\omega}{c}\vec{a}_2 B_0 \\ i\vec{k}\times\vec{a}_2 B_0 = -\dfrac{i\omega}{c}\vec{a}_1 E_0 \end{cases}$$

$$E_0 = \frac{\omega}{kc} B_0$$

$$B_0 = \frac{\omega}{kc} E_0$$



$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$
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- Electromagnetic Waves – propagation in vacuum

$\vec{k}, \vec{a}_1$ and $\vec{a}_2$ form a right-handed set: $\vec{a}_1 = \vec{k}\times\vec{a}_2$, $\vec{a}_2 = -\vec{k}\times\vec{a}_1$

$$i\vec{k}\cdot\vec{a}_1 E_0 = 0$$

$$i\vec{k}\cdot\vec{a}_2 B_0 = 0$$

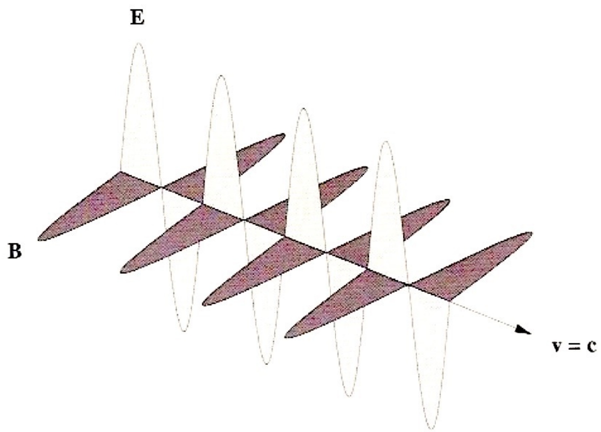
$$i\vec{k}\times\vec{a}_1 E_0 = \frac{i\omega}{c}\vec{a}_2 B_0$$

$$i\vec{k}\times\vec{a}_2 B_0 = -\frac{i\omega}{c}\vec{a}_1 E_0$$

$$E_0 = \frac{\omega}{kc}B_0 \qquad B_0 = \frac{\omega}{kc}E_0$$

$$E_0 = B_0$$

$$\omega = kc$$



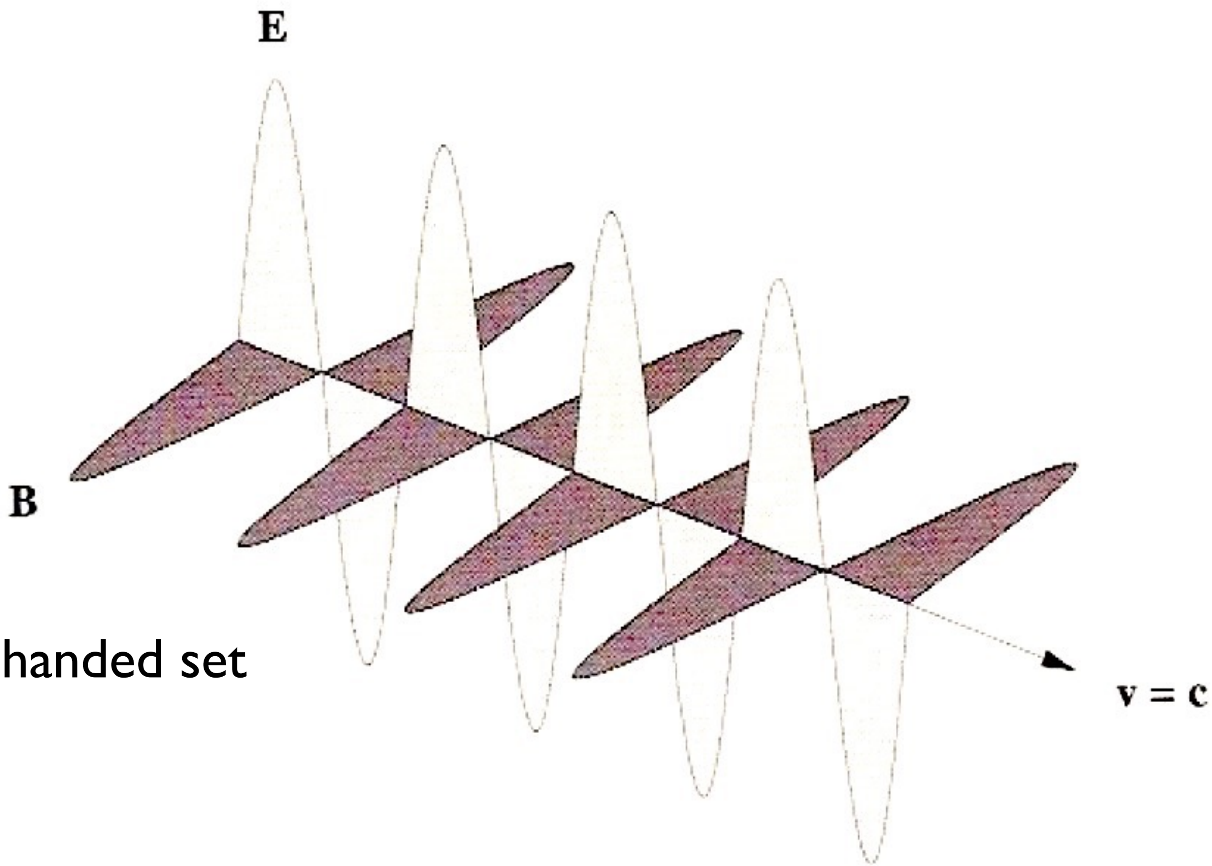
$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

- Electromagnetic Waves – propagation in vacuum

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$



$$E_0 = B_0$$

$$\omega = kc$$

$\vec{k}, \vec{a}_1$ and $\vec{a}_2$ form a right-handed set

- Electromagnetic Waves – energy flux and density

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$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

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- Electromagnetic Waves – energy flux and density

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$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{S} = \frac{c}{4\pi}\left(\vec{E}\times\vec{B}\right)$$

Poynting vector = directional energy flux

- Electromagnetic Waves – energy flux and density

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\langle\vec{S}\rangle = \frac{c}{4\pi}\langle\vec{E}\times\vec{B}\rangle$$

time-averging to eliminate ωt part

- Electromagnetic Waves – energy flux and density

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\langle \vec{S} \rangle = \frac{c}{8\pi}|E_0|^2 = \frac{c}{8\pi}|B_0|^2$$ time-averging to eliminate ωt part

- Electromagnetic Waves – energy flux and density

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\langle\vec{S}\rangle = \frac{c}{8\pi}|E_0|^2 = \frac{c}{8\pi}|B_0|^2$$ time-averging to eliminate ωt part

$$U_{field} = \frac{1}{8\pi}(E^2 + B^2)$$

- Electromagnetic Waves – energy flux and density

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\langle \vec{S} \rangle = \frac{c}{8\pi}|E_0|^2 = \frac{c}{8\pi}|B_0|^2$$ time-averging to eliminate ωt part

$$\langle U_{field} \rangle = \frac{1}{8\pi}|E_0|^2 = \frac{1}{8\pi}|B_0|^2$$

- Electromagnetic Waves – energy flux and density

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\langle\vec{S}\rangle = \frac{c}{8\pi}|E_0|^2 = \frac{c}{8\pi}|B_0|^2$$ time-averging to eliminate ωt part

$$\langle U_{field}\rangle = \frac{1}{8\pi}|E_0|^2 = \frac{1}{8\pi}|B_0|^2$$

→ velocity of energy flow: $\frac{\langle\vec{S}\rangle}{\langle U_{field}\rangle} = c$

- Lorentz force
- Maxwell equations
- Poynting vector
- Electromagnetic waves
- **radiation spectrum**
- electromagnetic potentials

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

- electric and magnetic field

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$$\vec{B}(\vec{r},t) = \vec{a}_2 B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

remember: $E_0 = B_0$

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{a}_2 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

sufficient to focus on electric field alone...

$$\vec{B}(\vec{r},t) = \vec{a}_2 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

distribution of all possible ω?

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

distribution of all possible ω ?

- frequency distribution (Fourier transformation of $E(t)$)

$$\hat{E}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(t) e^{i\omega t} dt$$

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

distribution of all possible ω?

- frequency distribution

$$\hat{E}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(t)e^{i\omega t}dt$$

- radiation spectrum
  - energy per unit area per unit time

$$\frac{dW}{dA\,dt} = ?$$

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

distribution of all possible ω?

- frequency distribution

$$\hat{E}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(t) e^{i\omega t} dt$$

- radiation spectrum
  - energy per unit area per unit time

$$\frac{dW}{dA\,dt} = \text{Poynting vector}$$

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

distribution of all possible ω?

- frequency distribution

$$\hat{E}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(t) e^{i\omega t} dt$$

- radiation spectrum
  - energy per unit area per unit time

$$\frac{dW}{dA\,dt} = \frac{c}{4\pi}|E(t)|^2$$

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

distribution of all possible ω?

- frequency distribution

$$\hat{E}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(t) e^{i\omega t} dt$$

- radiation spectrum
  - energy per unit area per unit time

$$\frac{dW}{dA\,dt} = \frac{c}{4\pi}|E(t)|^2$$

  - energy per unit area per unit *frequency*

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$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

distribution of all possible ω?

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  - energy per unit area per unit time

$$\frac{dW}{dA\,dt} = \frac{c}{4\pi}|E(t)|^2 \longrightarrow \frac{dW}{dA} = \frac{c}{4\pi}\int_{-\infty}^{\infty}|E(t)|^2 dt$$

  - energy per unit area per unit *frequency*

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$$\frac{dW}{dA\,dt} = \frac{c}{4\pi}|E(t)|^2 \longrightarrow \frac{dW}{dA} = \frac{c}{4\pi}\int_{-\infty}^{\infty}|E(t)|^2 dt$$

$$= \frac{c}{4\pi}2\pi\int_{-\infty}^{\infty}|E(\omega)|^2 d\omega$$

  - energy per unit area per unit *frequency*

$$\frac{dW}{dA\,d\omega} = ?$$

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

distribution of all possible ω ?

- frequency distribution

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  - energy per unit area per unit time

$$\frac{dW}{dA\,dt} = \frac{c}{4\pi}|E(t)|^2 \longrightarrow \frac{dW}{dA} = \frac{c}{4\pi}\int_{-\infty}^{\infty}|E(t)|^2 dt$$

$$= \frac{c}{4\pi} 2\pi \int_{-\infty}^{\infty}|E(\omega)|^2 d\omega$$

$$= \frac{c}{4\pi} 2\pi \;\; 2\int_{0}^{\infty}|E(\omega)|^2 d\omega$$

  - energy per unit area per unit *frequency*

$$\frac{dW}{dA\,d\omega} = ?$$

- electric and magnetic field

$$\vec{E}(\vec{r},t) = \vec{a}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

distribution of all possible ω ?

- frequency distribution

$$\hat{E}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(t) e^{i\omega t} dt$$

- radiation spectrum
  - energy per unit area per unit time

$$\frac{dW}{dA\,dt} = \frac{c}{4\pi}|E(t)|^2 \longrightarrow \frac{dW}{dA} = c\int_0^{\infty}|E(\omega)|^2 d\omega$$

  - energy per unit area per unit *frequency*

$$\frac{dW}{dA\,d\omega} = ?$$

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  - energy per unit area per unit *frequency*

$$\frac{dW}{dA\,d\omega} = c|E(\omega)|^2$$

- radiation spectrum – energy per unit area and unit frequency

$$\frac{dW}{dA\,d\omega} = c|E(\omega)|^2$$

- radiation spectrum – energy per unit area and unit frequency

$$\frac{dW}{dA\,d\omega} = c|E(\omega)|^2$$

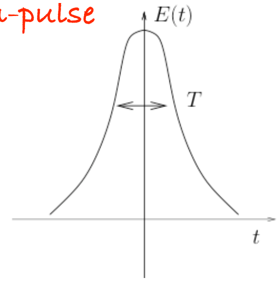
- examples

- radiation spectrum – energy per unit area and unit frequency

$$\frac{dW}{dA\,d\omega} = c|E(\omega)|^2$$

- examples
    - finite pulse

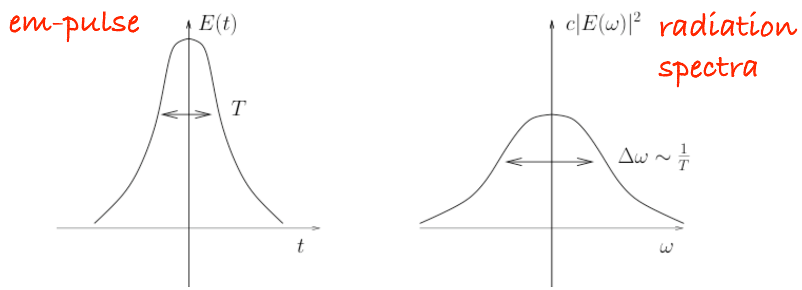
em-pulse



- radiation spectrum – energy per unit area and unit frequency

$$\frac{dW}{dA\,d\omega} = c|E(\omega)|^2$$

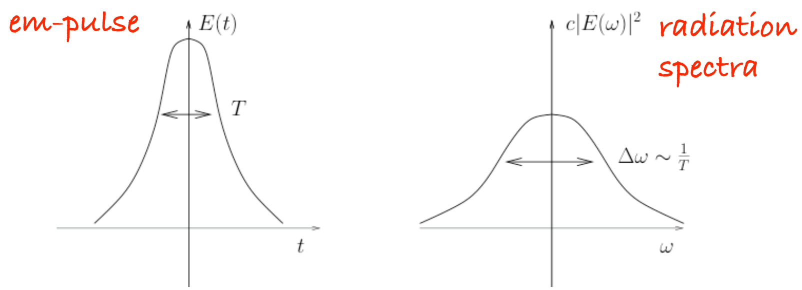
- examples
  - finite pulse



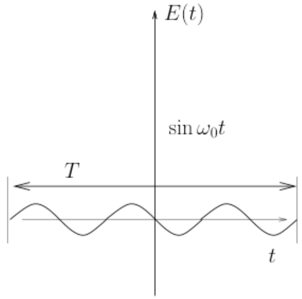
- radiation spectrum – energy per unit area and unit frequency

$$\frac{dW}{dA\,d\omega} = c|E(\omega)|^2$$

- examples
  - finite pulse



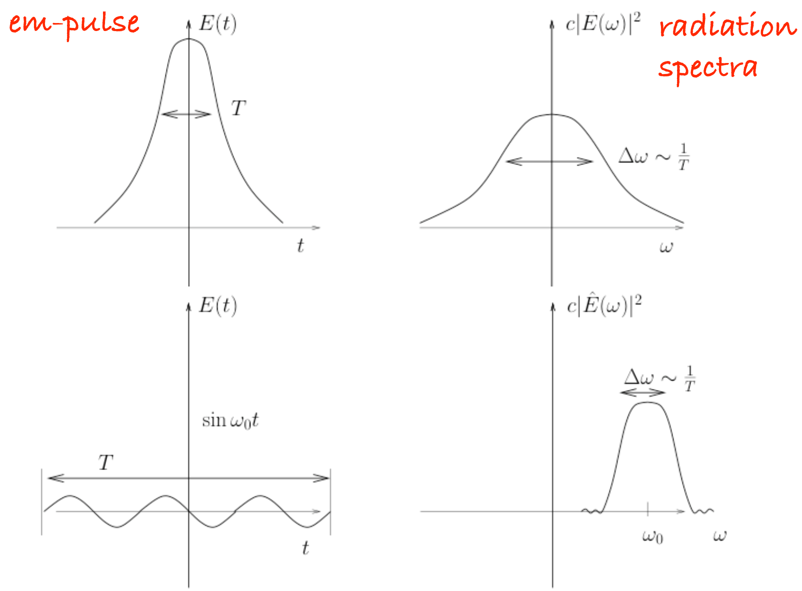
  - periodic signal of finite time



- radiation spectrum – energy per unit area and unit frequency

$$\frac{dW}{dA\,d\omega} = c|E(\omega)|^2$$

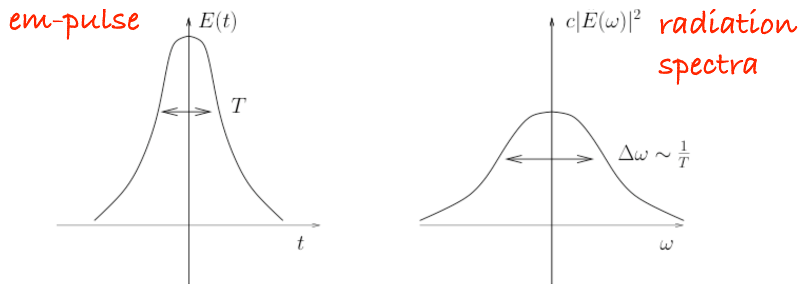
- examples
    - finite pulse
    - periodic signal of finite time



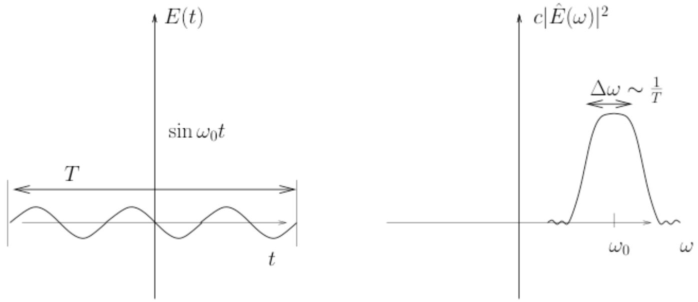
- radiation spectrum – energy per unit area and unit frequency

$$\frac{dW}{dA\,d\omega} = c|E(\omega)|^2$$

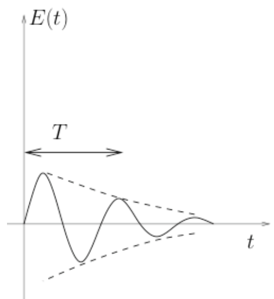
- examples
  - finite pulse



  - periodic signal of finite time



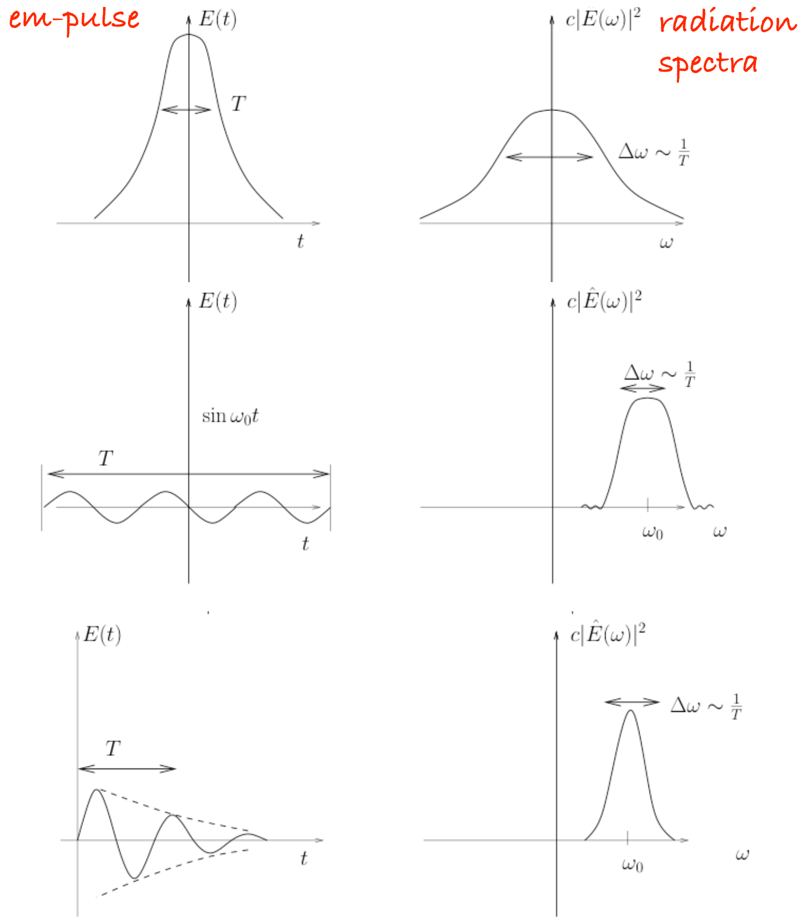
  - decaying periodic signal of finite time



- radiation spectrum – energy per unit area and unit frequency

$$\frac{dW}{dA\,d\omega} = c|E(\omega)|^2$$

- examples
  - finite pulse
  - periodic signal of finite time
  - decaying periodic signal of finite time



- Lorentz force
- Maxwell equations
- Poynting vector
- electromagnetic waves
- radiation spectrum
- **electromagnetic potentials**

Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

▪ Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- vector fields

$$\vec{E}(\vec{r}, t) \qquad \vec{B}(\vec{r}, t)$$

▪ Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

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- vector fields

$$\vec{E}(\vec{r}, t) \qquad \vec{B}(\vec{r}, t)$$

- potential $\vec{A}$

→ *a divergence-free field can be written as a rotation of a vector field*

Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- vector fields

$\vec{E}(\vec{r},t) \qquad \vec{B}(\vec{r},t)$

- potential $\vec{A}$

$$\longrightarrow \quad \vec{B} = \nabla \times \vec{A}$$

▪ Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0 \longrightarrow \vec{B} = \nabla\times\vec{A}$$

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \longrightarrow \nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\nabla\times\vec{A}}{\partial t} = -\nabla\times\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

- potential $\vec{A}$

- vector fields

$$\vec{E}(\vec{r},t) \qquad \vec{B}(\vec{r},t)$$

▪ Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

- potential $\vec{A}$

$$\longrightarrow \quad \vec{B} = \nabla\times\vec{A}$$

$$\longrightarrow \quad \underline{\underline{\nabla\times}}\vec{E} = -\frac{1}{c}\frac{\partial\nabla\times\vec{A}}{\partial t} = -\underline{\underline{\nabla\times}}\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

- vector fields

$$\vec{E}(\vec{r},t) \qquad \vec{B}(\vec{r},t)$$

Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- potential $\vec{A}$

$$\nabla \cdot \vec{B} = 0 \longrightarrow \vec{B} = \nabla \times \vec{A}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \longrightarrow 0 = \nabla \times \left(\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right)$$

- vector fields

$$\vec{E}(\vec{r}, t) \qquad \vec{B}(\vec{r}, t)$$

Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- vector fields

$$\vec{E}(\vec{r},t) \qquad \vec{B}(\vec{r},t)$$

- potential $\vec{A}$

$$\longrightarrow \quad \vec{B} = \nabla\times\vec{A}$$

$$\longrightarrow \quad 0 = \nabla\times\underbrace{\left(\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right)}$$

a curl-free field can be written as a gradient of a potential

Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- vector fields

$$\vec{E}(\vec{r}, t) \qquad \vec{B}(\vec{r}, t)$$

- potential $\vec{A}$

$$\longrightarrow \quad \vec{B} = \nabla \times \vec{A}$$

$$\longrightarrow \quad 0 = \nabla \times \underbrace{\left(\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right)}$$

$$\left(\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right) = -\nabla\phi$$

a curl-free field can be written as a gradient of a potential

Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

- Electromagnetic Potentials
  - Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

satisfied by construction

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

  - potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla\times\vec{A}$$

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Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

turn equation around!?

Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$4\pi\rho = \nabla\cdot\vec{E} = -\nabla\cdot\nabla\phi - \frac{1}{c}\frac{\partial\nabla\cdot\vec{A}}{\partial t} = -\nabla\cdot\nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} - \frac{1}{c}\frac{\partial\phi}{\partial t}\right) = -\nabla\cdot\nabla\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

▪ Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$4\pi\rho = \nabla \cdot \vec{E} = -\nabla \cdot \nabla\phi - \frac{1}{c}\frac{\partial \nabla \cdot \vec{A}}{\partial t} = -\nabla \cdot \nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla \cdot \vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t} - \frac{1}{c}\frac{\partial \phi}{\partial t}\right) = -\nabla \cdot \nabla\phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla \cdot \vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t}\right)$$

$$\longrightarrow \quad \nabla^2\phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla \cdot \vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t}\right)$$

Electromagnetic Potentials

- Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$4\pi\rho = \nabla \cdot \vec{E} = -\nabla \cdot \nabla\phi - \frac{1}{c}\frac{\partial \nabla \cdot \vec{A}}{\partial t} = -\nabla \cdot \nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla \cdot \vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t} - \frac{1}{c}\frac{\partial \phi}{\partial t}\right) = -\nabla \cdot \nabla\phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla \cdot \vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t}\right)$$

$$\longrightarrow \quad \nabla^2\phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla \cdot \vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t}\right)$$ *inhomogeneous wave-equation*

Electromagnetic Potentials

- Maxwell equations

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

Electromagnetic Potentials

- Maxwell equations
- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

▪ Electromagnetic Potentials

- Maxwell equations
- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

$$\nabla\times\vec{B} = \nabla\times\nabla\times\vec{A} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial}{\partial t}\left(-\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right)$$

▪ Electromagnetic Potentials

- Maxwell equations
- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

$$\nabla\times\vec{B} = \nabla\times\nabla\times\vec{A} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial}{\partial t}\left(-\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right) \xrightarrow{\nabla\times\nabla\times\vec{A} = -\nabla^2\vec{A} + \nabla(\nabla\vec{A})} \nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

Electromagnetic Potentials

- Maxwell equations
- potentials $\vec{A}$ and ϕ

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

$$\nabla\times\vec{B} = \nabla\times\nabla\times\vec{A} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial}{\partial t}\left(-\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right) \xrightarrow{\nabla\times\nabla\times\vec{A} = -\nabla^2\vec{A} + \nabla(\nabla\vec{A})} \nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

inhomogeneous wave-equation

▪ Electromagnetic Potentials

- electromagnetic fields $\vec{E}$ and $\vec{B}$ equations

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

- expressed via potentials $\vec{A}$ and ϕ

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

▪ Electromagnetic Potentials

- electromagnetic fields $\vec{E}$ and $\vec{B}$ equations

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

- expressed via potentials $\vec{A}$ and ϕ

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

how to solve them?

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

▪ Electromagnetic Potentials

• electromagnetic fields $\vec{E}$ and $\vec{B}$ equations

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

• expressed via potentials $\vec{A}$ and ϕ

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

how to solve them?

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

$\vec{A}$ and ϕ are not uniquely determined!

▪ Electromagnetic Potentials

- electromagnetic fields $\vec{E}$ and $\vec{B}$ equations

Gauge transformation

$$\vec{B} = \nabla\times\vec{A} \qquad = \nabla\times\left(\vec{A} + \nabla\psi\right)$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

- expressed via potentials $\vec{A}$ and ϕ

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi\rho - \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

how to solve them?

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right)$$

$\vec{A}$ and ϕ are not uniquely determined!

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Gauge transformation

$$= \nabla\times\left(\vec{A} + \nabla\psi\right)$$

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can be used as constraint…

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$$\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0$$

(condition to determine ψ)

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→ time duration taken for a signal to reach its destination (propagation delay)

- Lorentz force
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- Poynting vector
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summary!?

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a) we have known charge and current distributions

$$\rho(\vec{r}, t)$$

$$\vec{j}(\vec{r}, t)$$

▪ summary

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