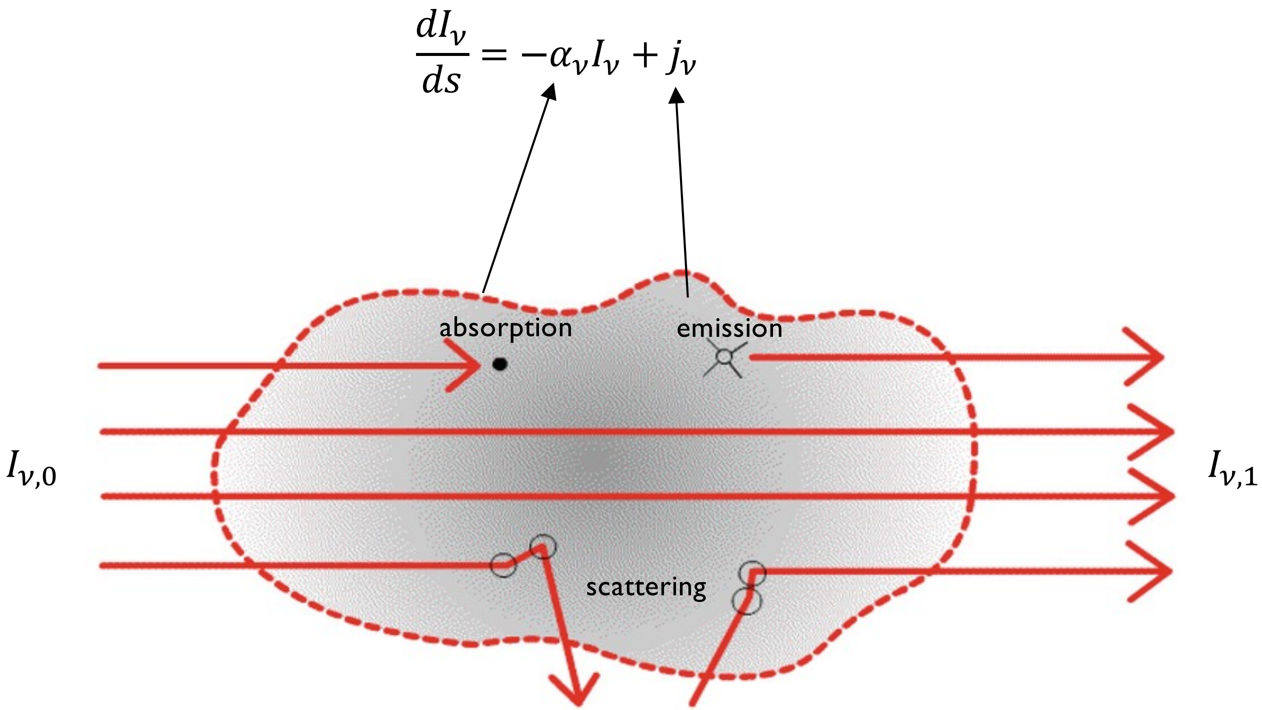
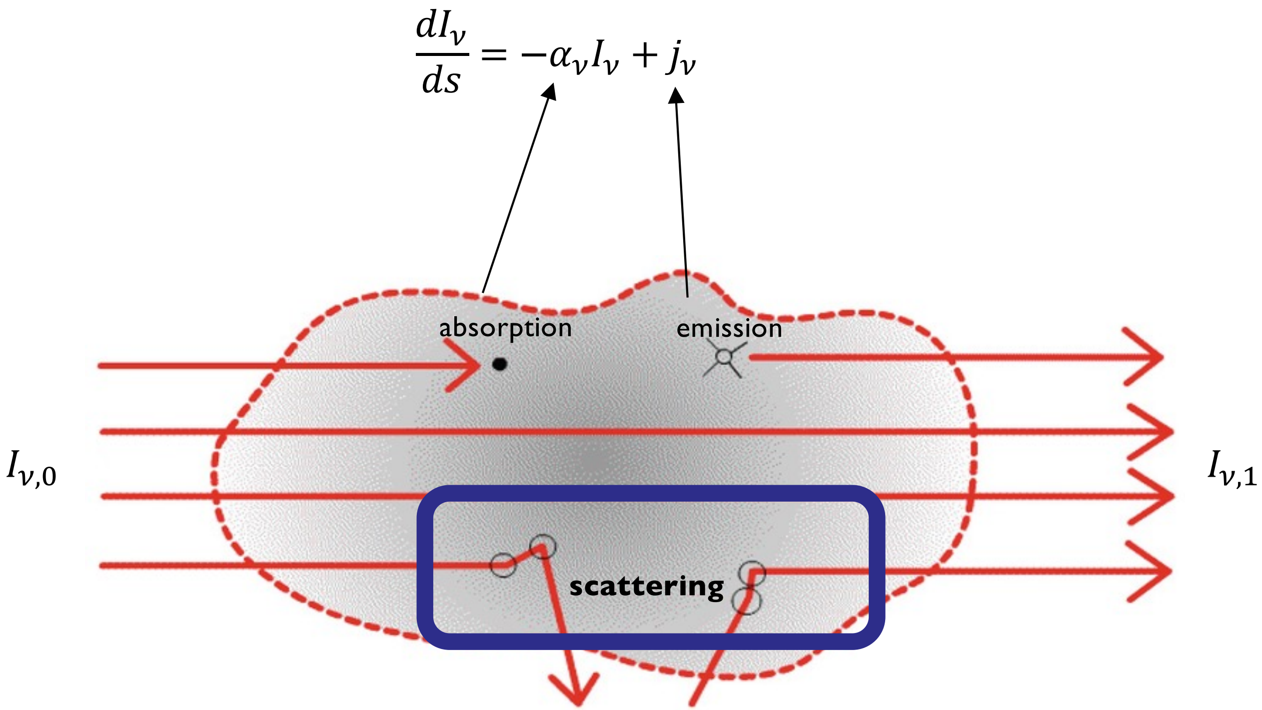
Alexander Knebe (*Universidad Autonoma de Madrid*)

Scattering Effects



Scattering Effects

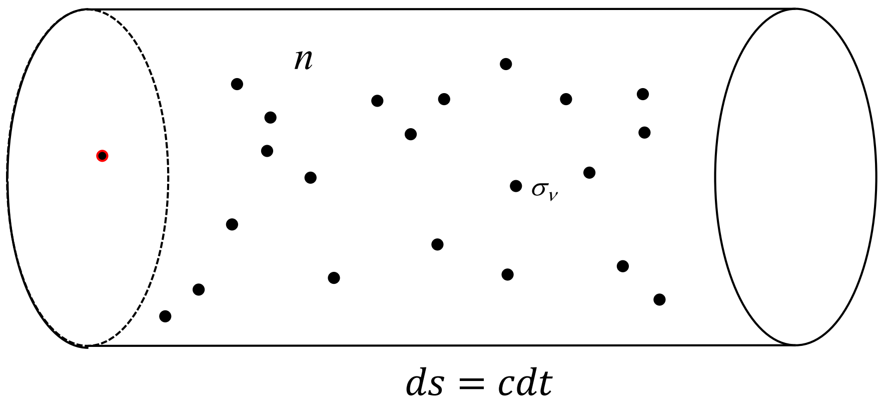


- emission/absorption contributions
- random walk
- photon-electron interactions
- radiative diffusion

- **emission/absorption contributions**
- random walk
- photon-electron interactions
- radiative diffusion

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$



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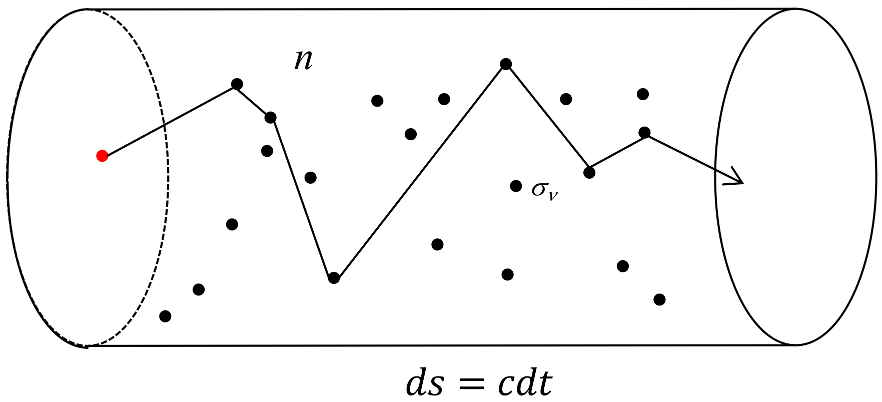
$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- scattering = dispersion of a beam of particles by collisions (or similar interactions)

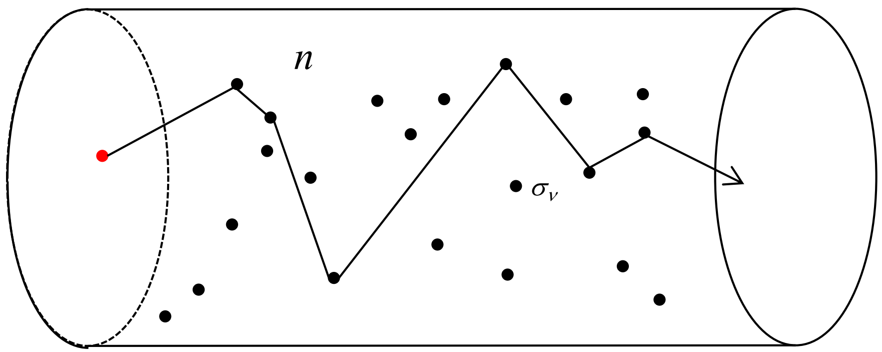


- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- scattering = dispersion of a *beam of particles* by collisions (or similar interactions)

can be photons or other particles…



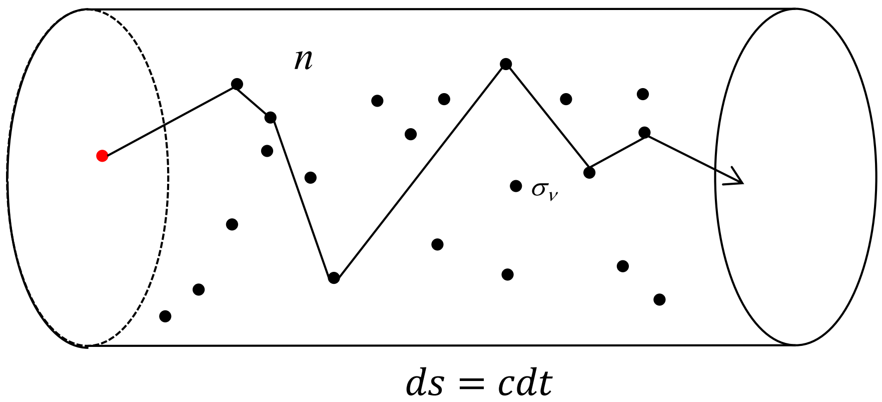
$ds = cdt$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- scattering = dispersion of a beam of particles by collisions (or similar interactions)

- scattering contributes to...
  - emission: dispersion from random directions into the beam
  - absorption: dispersion in random directions away from the beam

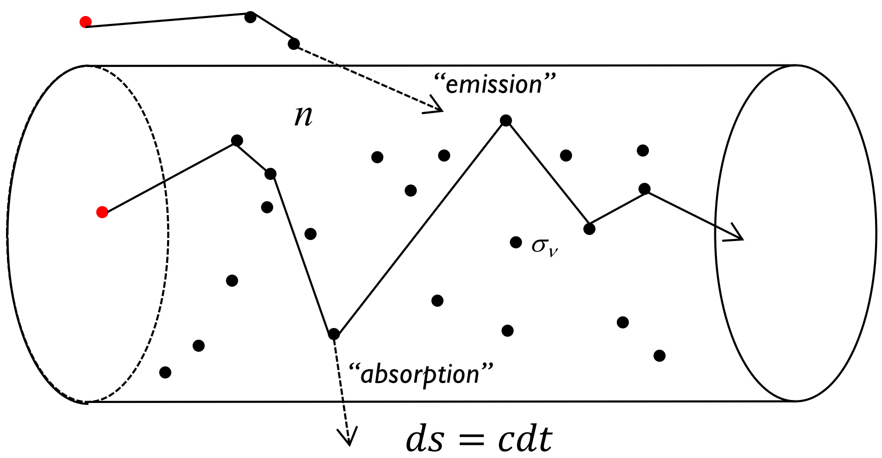


- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \alpha_\nu^s)I_\nu + j_\nu + j_\nu^s$$

- scattering = dispersion of a beam of particles by collisions (or similar interactions)

- scattering contributes to...
  - emission: j_ν^s
  - absorption: α_ν^s



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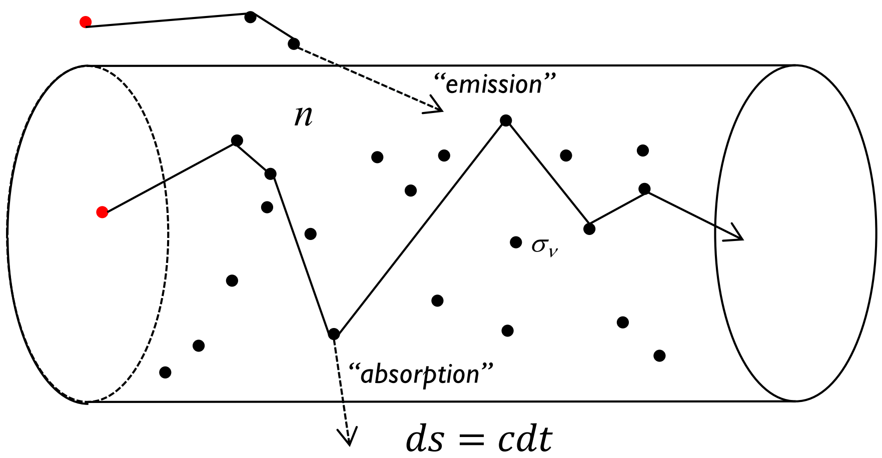
$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \alpha_\nu^s)I_\nu + j_\nu + j_\nu^s$$

- scattering = dispersion of a beam of particles by collisions (or similar interactions)

- scattering contributes to...
  - emission: $j_\nu^s = \int_\Omega \alpha_\nu^s \, I_\nu \, \psi_\nu d\Omega$ photons added to the beam by scattering from other directions
  - absorption: α_ν^s



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$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \alpha_\nu^s)I_\nu + j_\nu + j_\nu^s$$

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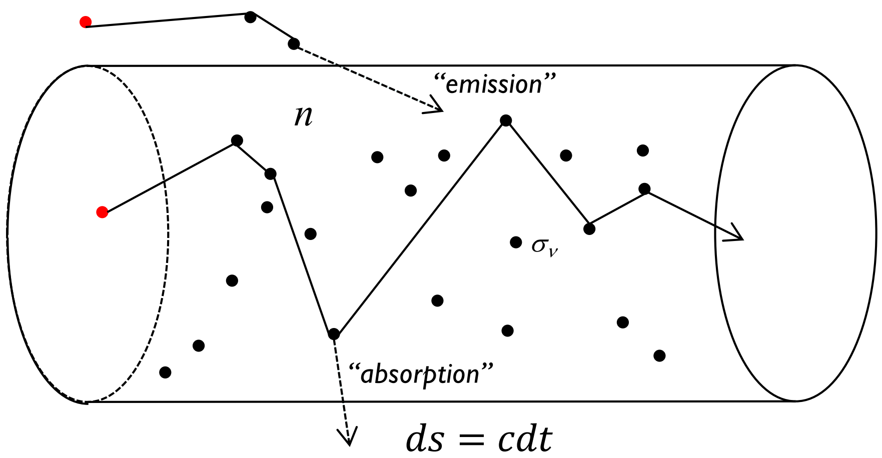
- scattering contributes to...
  - emission: $j_\nu^s = \int_\Omega \alpha_\nu^s \, I_\nu \, \psi_\nu d\Omega$ photons added to the beam by scattering from other directions; ψ_ν: scattering probability
  - absorption: α_ν^s



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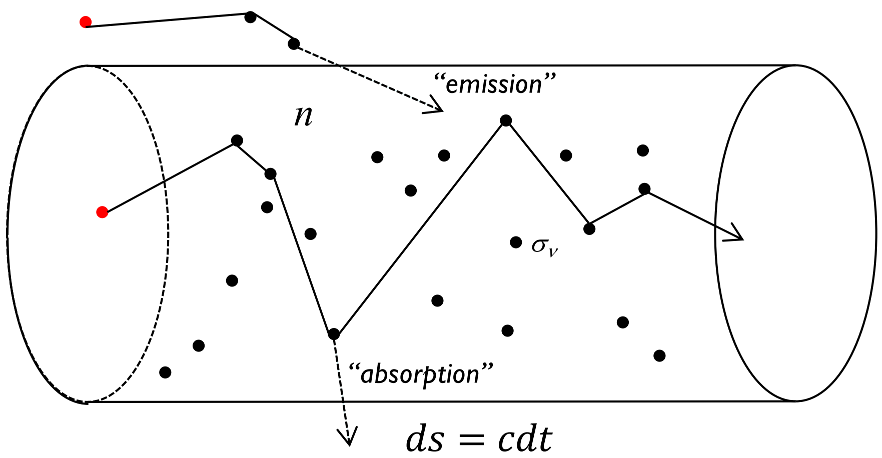
- scattering contributes to...
  - emission: $j_\nu^s = \int_\Omega \alpha_\nu^s \, I_\nu \, \psi_\nu d\Omega = \int_\Omega \alpha_\nu^s \, I_\nu \frac{d\Omega}{4\pi}$
  - absorption: α_ν^s

isotropic scattering



- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \alpha_\nu^s)I_\nu + j_\nu + \int_\Omega \alpha_\nu^s \, I_\nu \frac{d\Omega}{4\pi}$$

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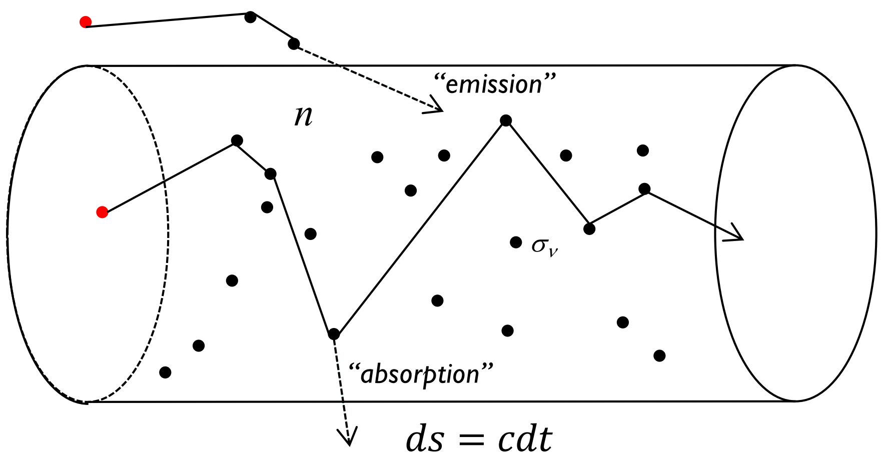

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Material	Sound absorption coefficient								Ref.	Scatt. Coeff. (707 Hz)	Ref.
	Frequency [Hz]										
	63 Hz	125Hz	250Hz	500Hz	1 kHz	2 kHz	4 kHz	8 kHz			
cars	0.1	0.1	0.06	0.04	0.03	0.02	0.02	0.02	(16)	0.1	(15)[2]
roofs	0.03	0.03	0.03	0.03	0.04	0.05	0.07	0.07	(17)	0.5	(20)
asphalt	0.36	0.36	0.44	0.31	0.29	0.39	0.25	0.25	(17)	0.2	(20)
window shutters	0.35	0.35	0.2	0.1	0.05	0.02	0.02	0.02	(18)[1]	0.1	(15)[2]
sidewalks	0.36	0.36	0.44	0.31	0.29	0.39	0.25	0.25	(17)	0.2	(20)
balconies	0.02	0.02	0.03	0.03	0.03	0.04	0.07	0.07	(17)	0.2	(20)
facades	0.05	0.05	0.04	0.04	0.04	0.05	0.05	0.05	(17)	0.1	(15)[2]
tree trunks	0.05	0.05	0.05	0.05	0.1	0.15	0.15	0.15	(18)	0.4	(15)[3]
tree follage	0.21	0.21	0.44	0.52	0.59	0.62	0.54	0.54	(19)	0.16	(21)

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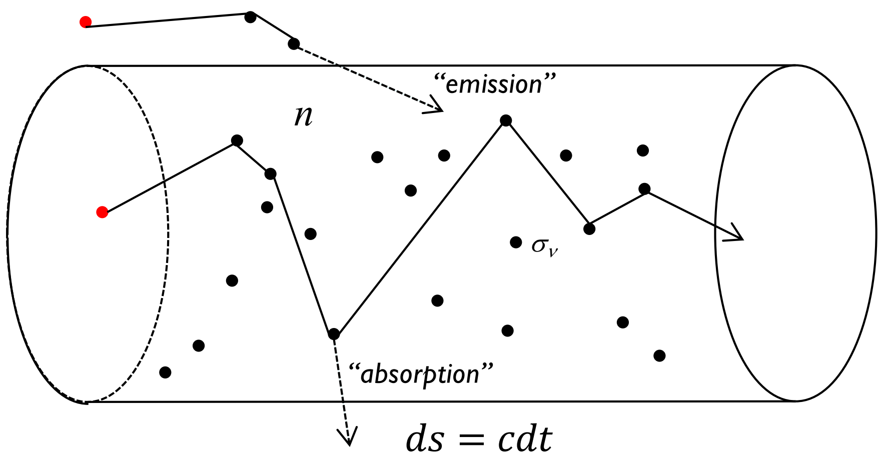
- integro-differential equation

  - is difficult to solve, usually requires numerical integration and iteration



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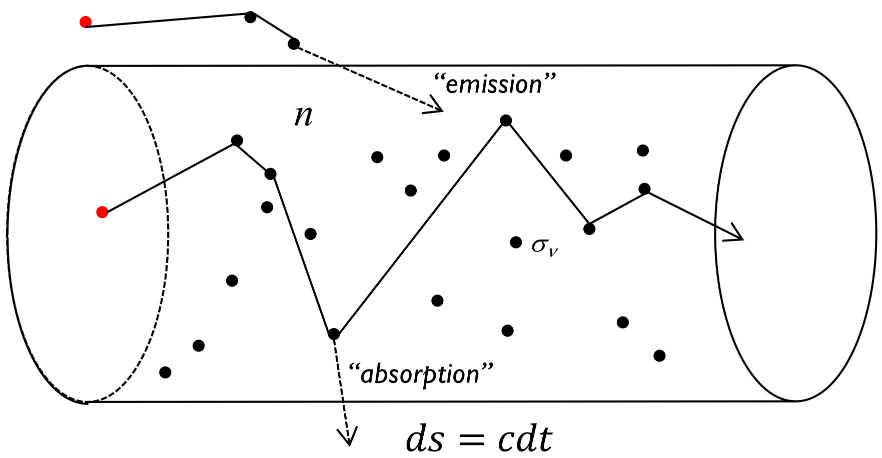
  - approximate methods of scattering have been developed!



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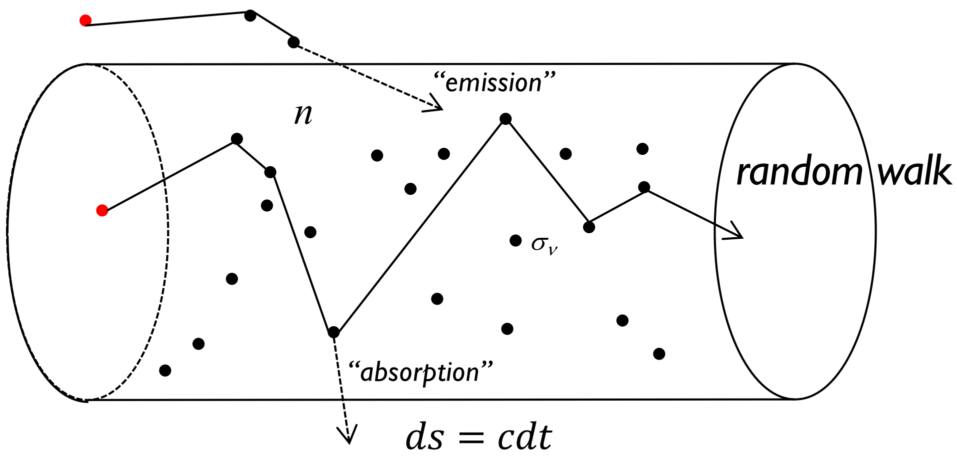

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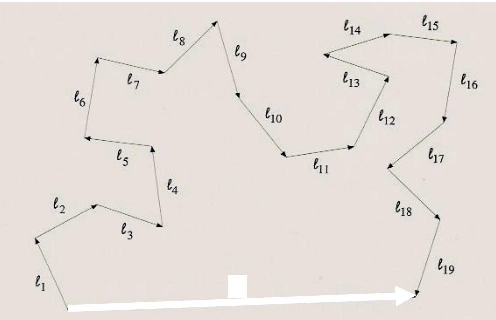

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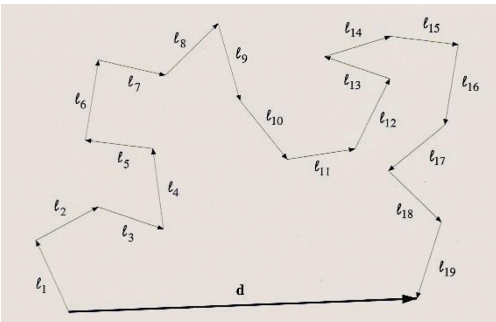

$$\vec{d} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_N$$

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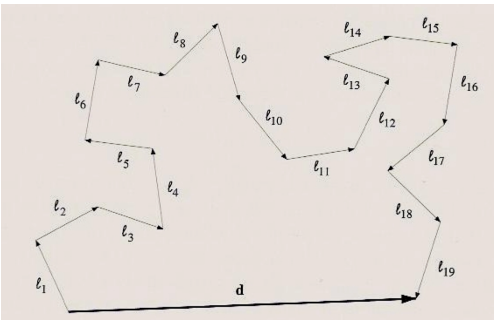

$$\vec{d} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_N$$

$\vec{l}^f = \frac{1}{N}\sum_i^N \vec{l}_i \approx$ mean free path

N: number of random steps

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random walk

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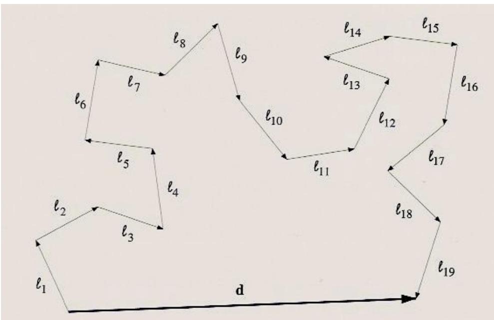
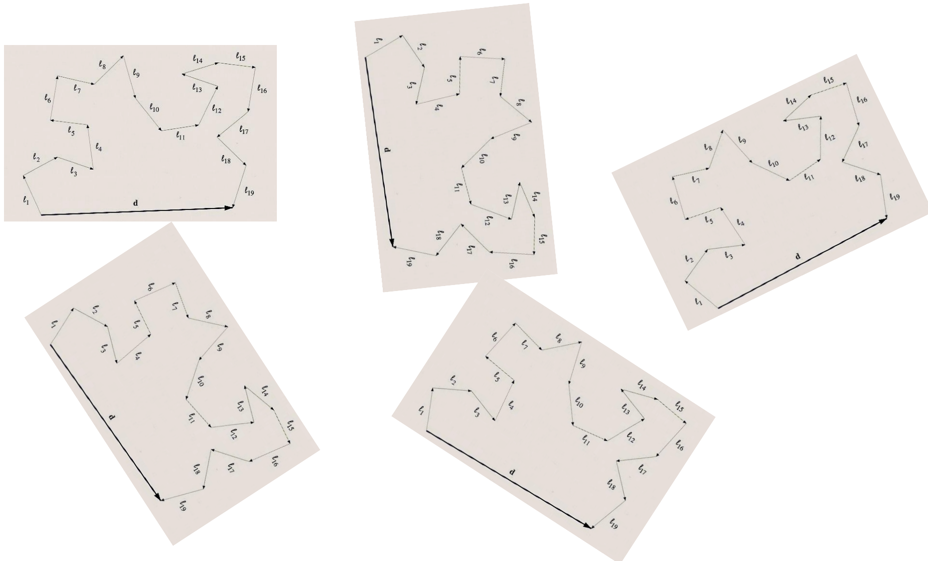


$\langle \vec{d} \rangle_M$

M : number of sample paths



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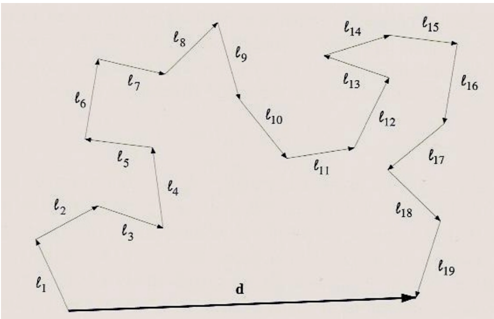

$$\langle \vec{d} \rangle_M = \left\langle \sum_{i=1}^{N} \vec{l}_i \right\rangle_M$$

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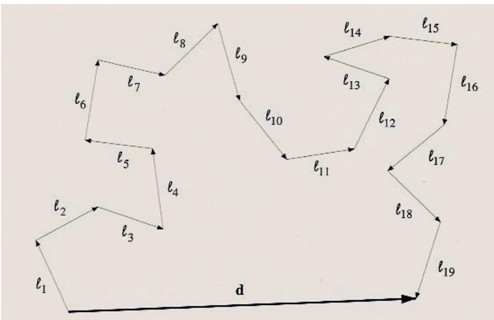

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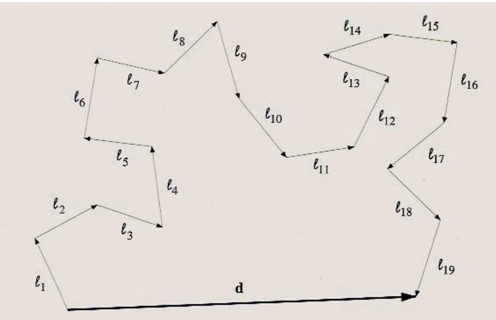

$$\langle \vec{d} \rangle_M = \sum_{i=1}^{N} \langle \vec{l}_i \rangle_M = ?$$

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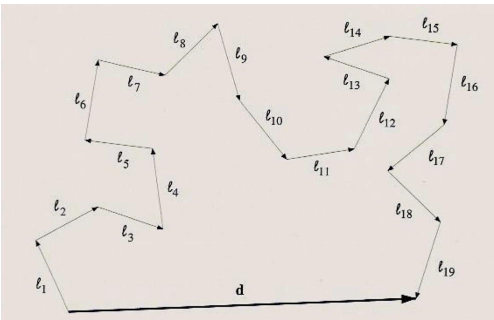

$$\langle \vec{d} \rangle_M = \sum_{i=1}^{N} \langle \vec{l}_i \rangle_M = 0$$

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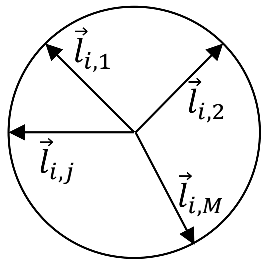

$$\sum_{j=1}^{M} \vec{l}_{i,j} = 0$$

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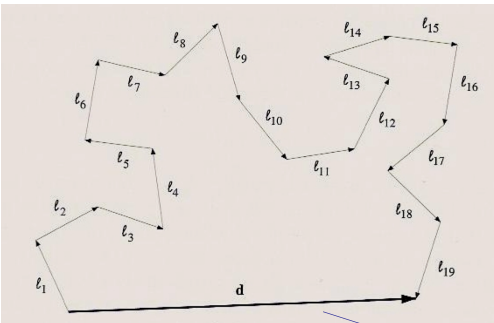

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$$\langle \vec{d} \rangle_M = \sum_{i=1}^{N} \langle \vec{l}_i \rangle_M = 0$$

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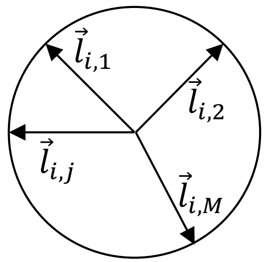

$$\sum_{j=1}^{M} \vec{l}_{i,j} = 0$$

...but the photon will nevertheless have travelled some net distance $d = |\vec{d}|$!

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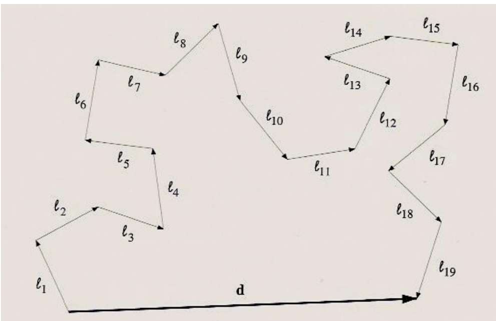

$$\langle \vec{d} \rangle_M = 0$$

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$$\langle d^2 \rangle_M \neq 0$$

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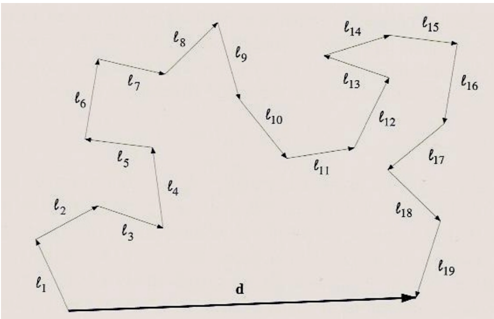

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$$\langle d^2 \rangle_M = \langle l_1^2 \rangle_M + 2 \langle \vec{l}_1 \cdot \vec{l}_2 \rangle_M + \langle l_2^2 \rangle_M + \cdots$$

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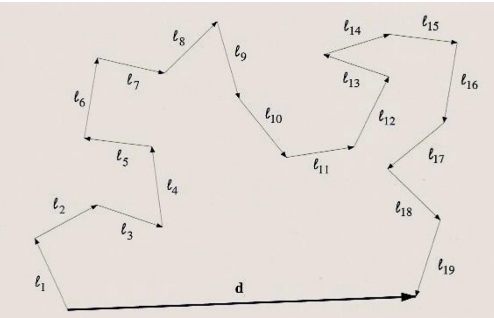

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all $\vec{l}_n \cdot \vec{l}_m$ terms vanish, if they are uncorrelated

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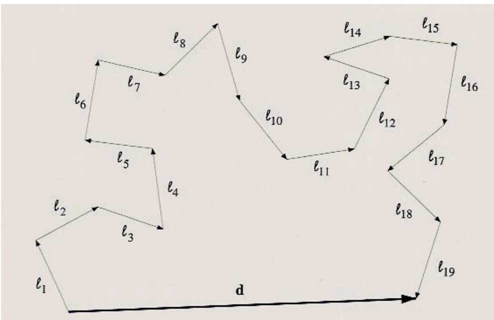

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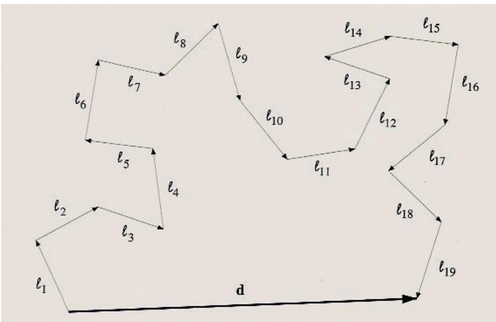

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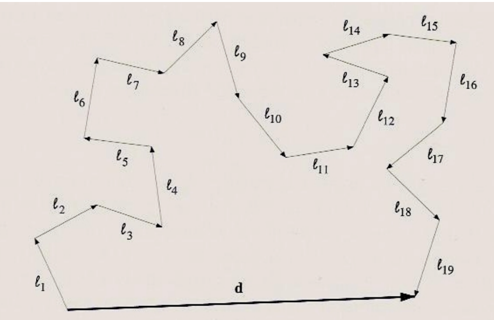

$$\vec{d} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_N$$

$\vec{l}^f = \frac{1}{N} \sum_i^N \vec{l}_i \approx$ mean free path

N : number of random steps

$$\langle \vec{d} \rangle_M = 0$$

M : number of sample paths

$$\langle d^2 \rangle_M \neq 0$$

$$\langle d^2 \rangle_M = \sum_{i=1}^N \langle l_i^2 \rangle_M = \left\langle \sum_{i=1}^N l_i^2 \right\rangle_M$$

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \alpha_\nu^s)I_\nu + j_\nu + \int_\Omega \alpha_\nu^s \, I_\nu \frac{d\Omega}{4\pi}$$

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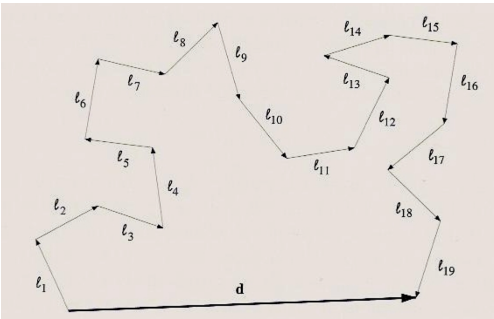

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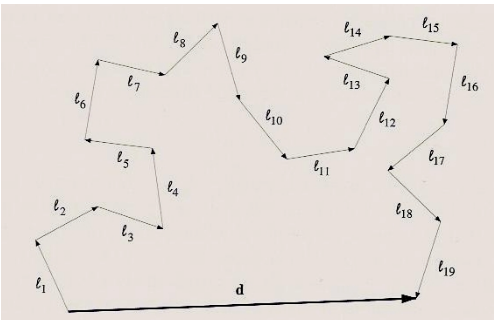

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let's rename l^f to simply l

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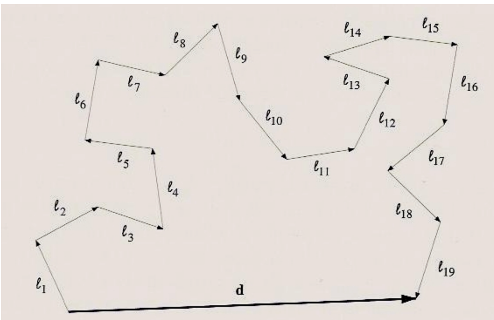

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    - net displacement of photons $d = \sqrt{N}\, l$
        - increases with number of scattering events N
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  } *can be used to calculate N in finite medium!*

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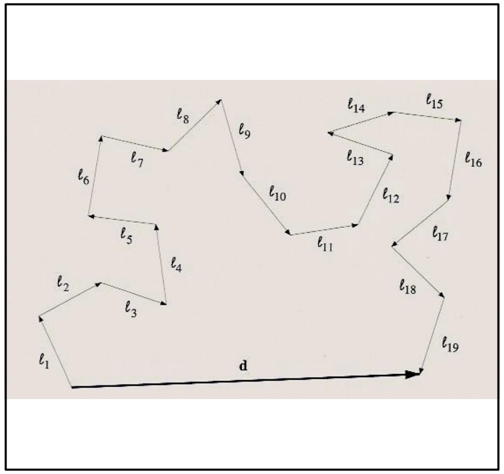

L

photon escapes for $d \geq L$

- equation of radiative transfer

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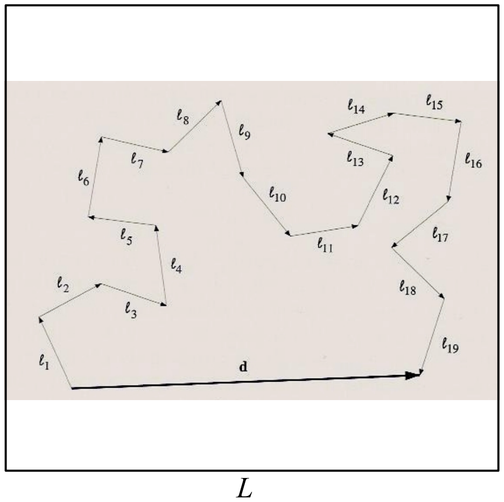

photon escapes for $d \geq L$ $\rightarrow \sqrt{N} l \geq L$

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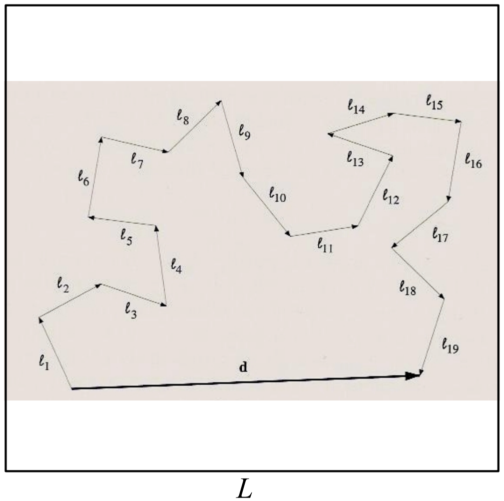

photon escapes for $d \geq L$ $\rightarrow \sqrt{N}l \geq L \rightarrow N \approx \left(\frac{L}{l}\right)^2$ number of collisions

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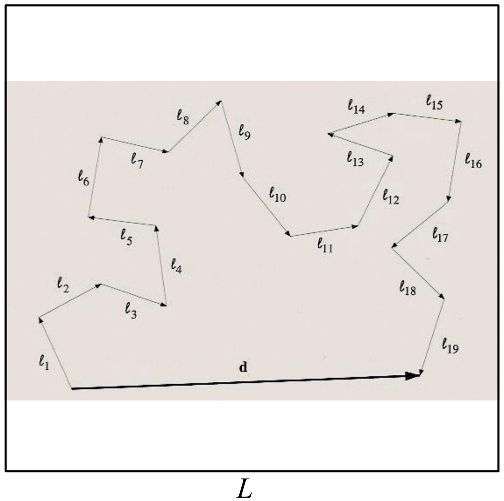

relation to something?

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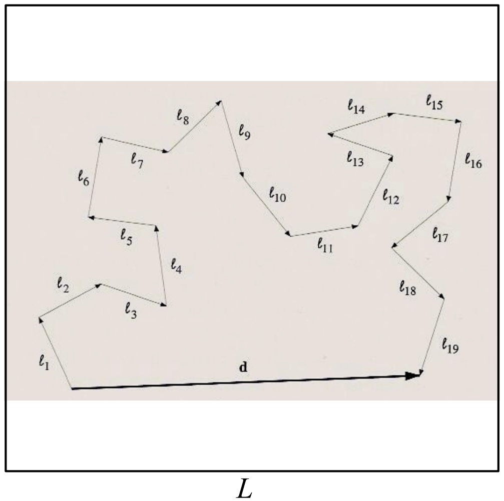

$l = \frac{1}{\alpha}$

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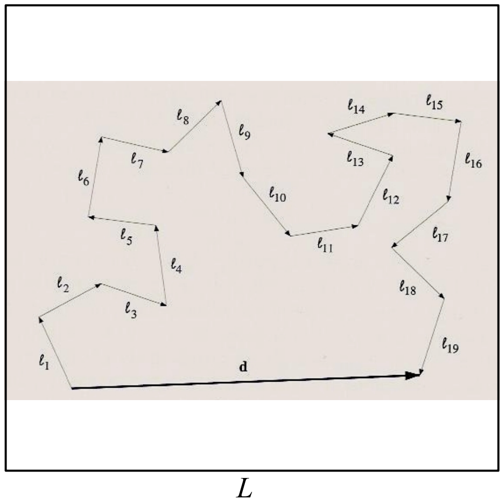

$$l = \frac{1}{\alpha} \qquad \tau = \int_{s_0}^{s} \alpha(s')ds'$$

photon escapes for $d \geq L$ $\rightarrow \sqrt{N}l \geq L$ $\rightarrow$ $N \approx \left(\frac{L}{l}\right)^2$ number of collisions

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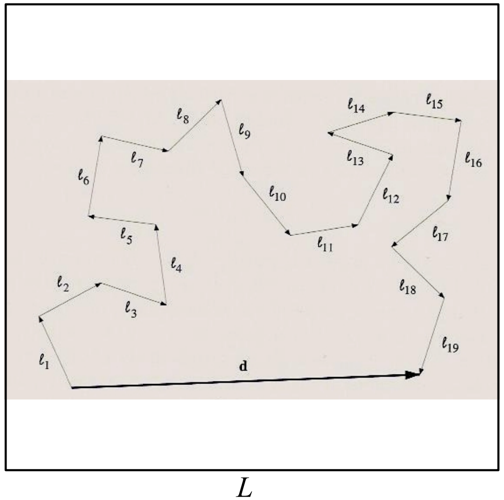

$$\tau = \int_{s_0}^{s} \frac{1}{l} ds' = \frac{s - s_0}{l} = \frac{L}{l} > 1$$

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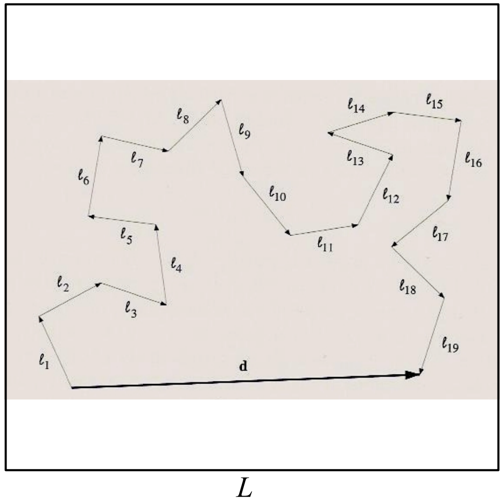

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    $N = \tau$ (for $\tau < 1$)

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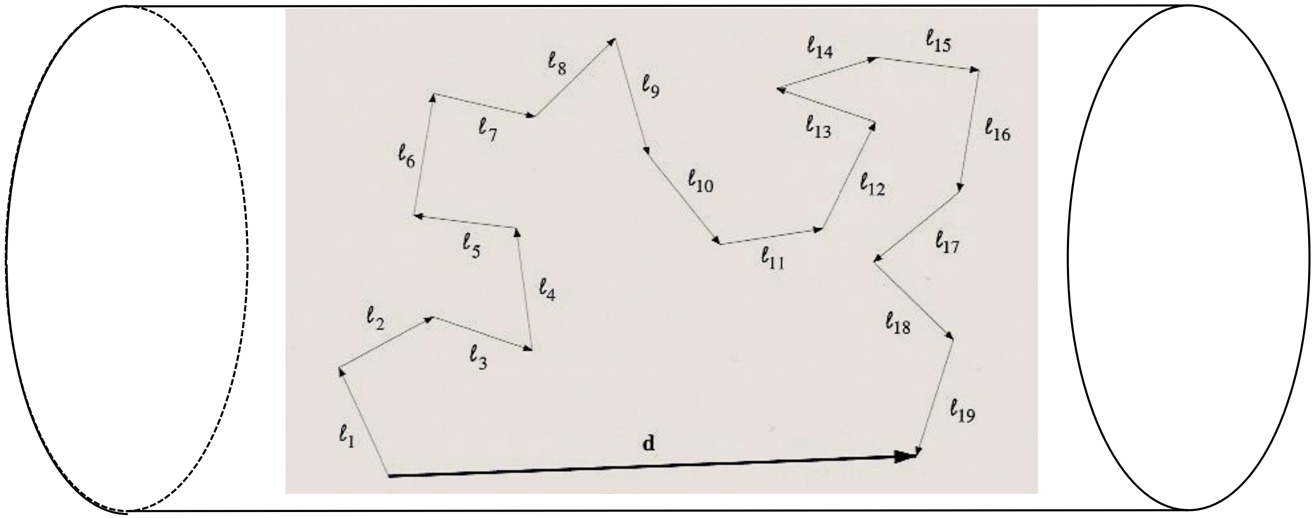
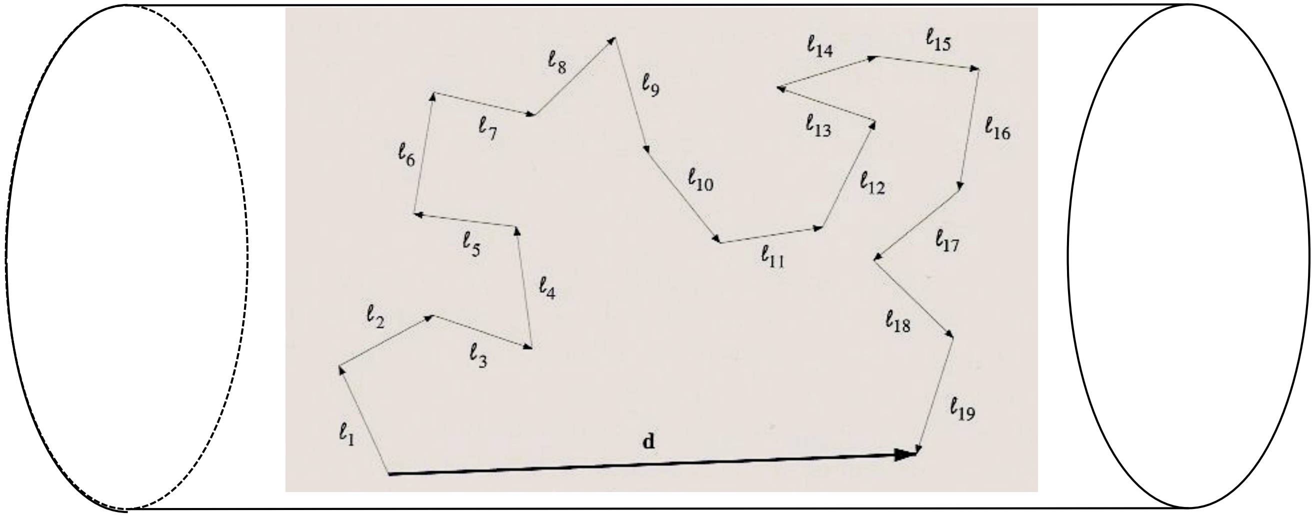
- scattering = dispersion of a beam of particles by collisions (or similar interactions)

- random walk
  - net displacement of photons $d = \sqrt{N}\, l$
  - number of scattering events $N = \max(\tau^2, \tau)$

- emission/absorption contributions
- random walk
- **photon-electron interactions**
- radiative diffusion



no assumption about photons, particles, etc.



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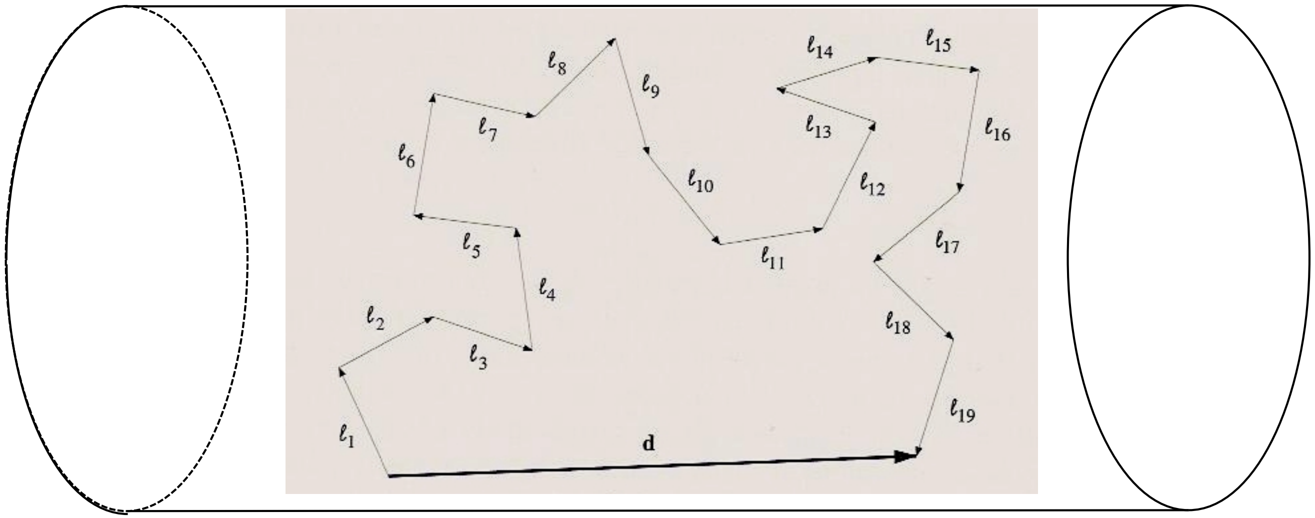
but we are primarily interested in photons!



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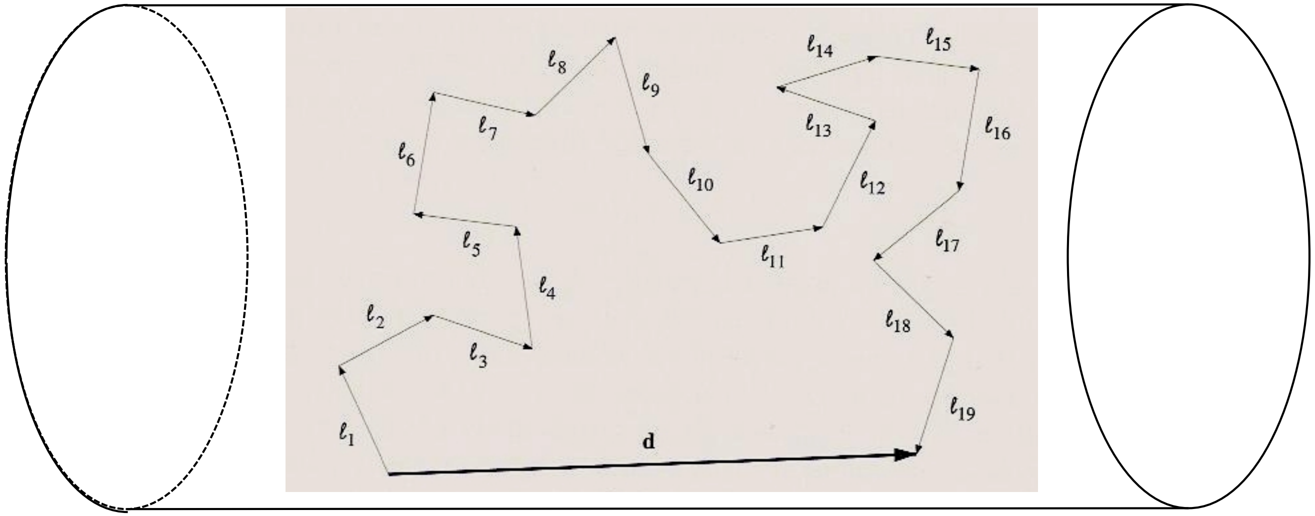
what effects lead to scattering of photons?



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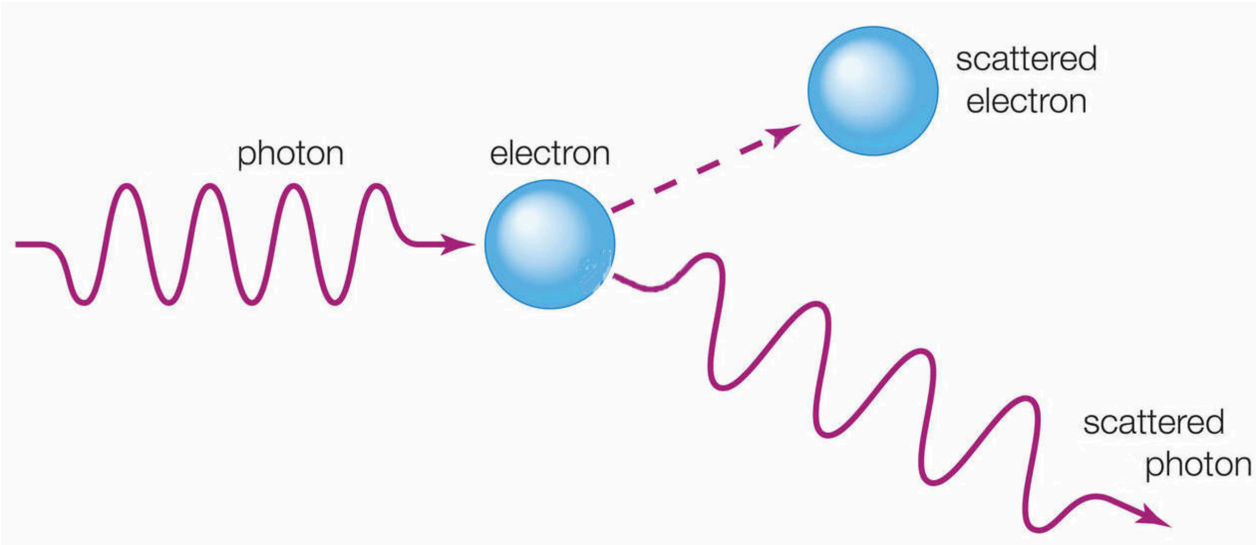
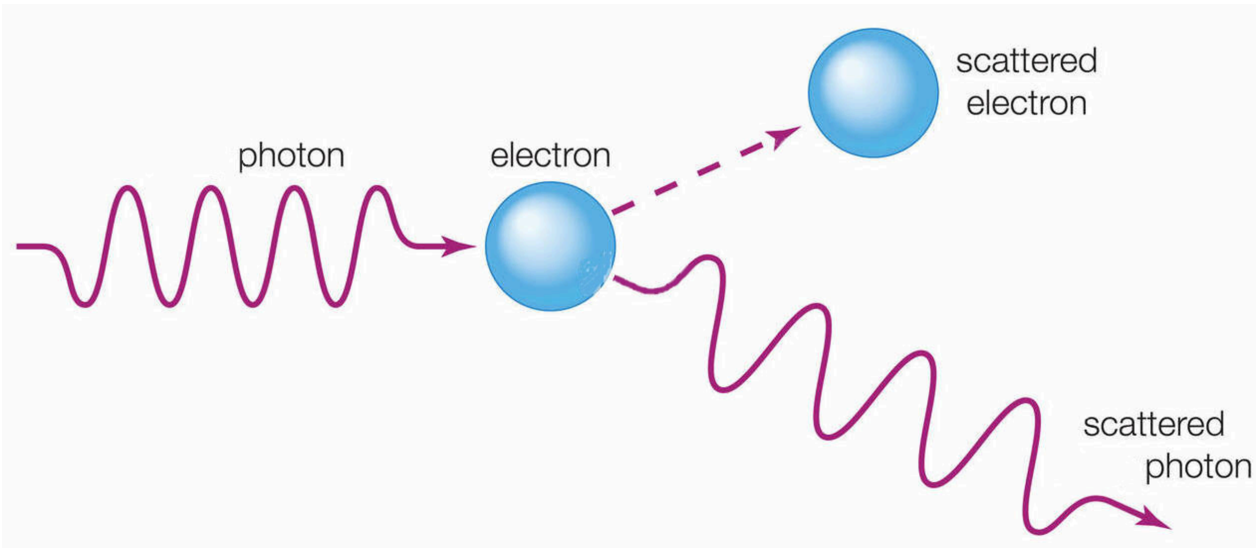
→ ***photon-electron interactions***

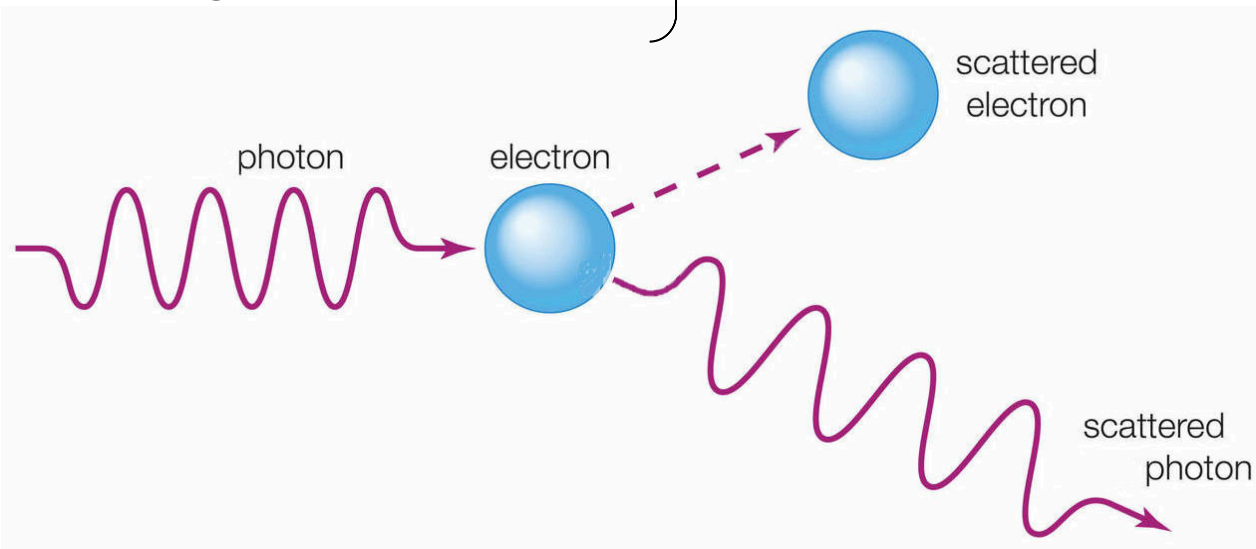

photon
electron
scattered
electron
scattered
photon

collision between charged particle and photon



- Compton scattering
- inverse Compton scattering
- Thomson scattering

collision between charged particle and photon



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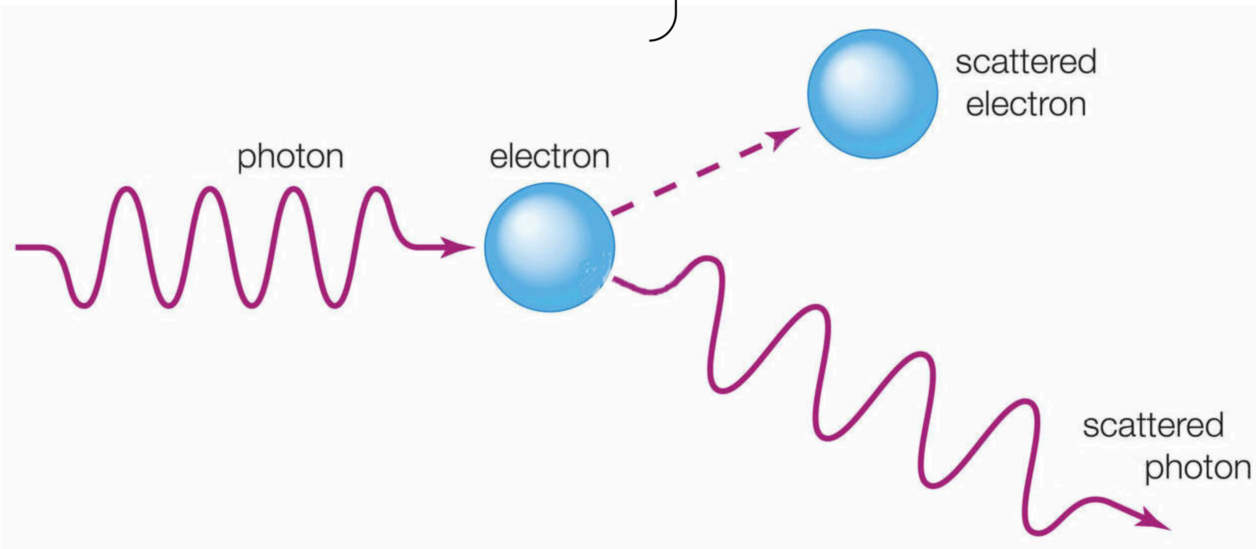
difference?



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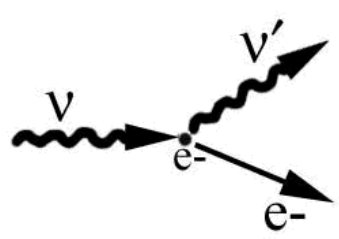

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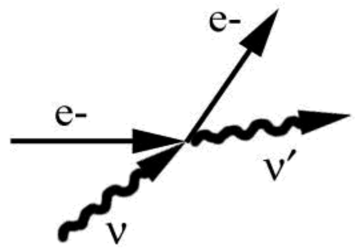

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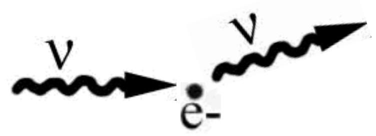
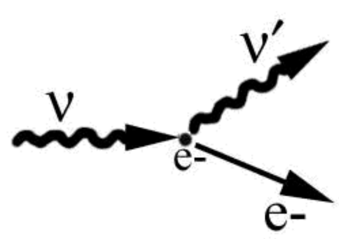


- Compton scattering
  - no energy transfer from electron to photon
  - photon looses energy to kick electron



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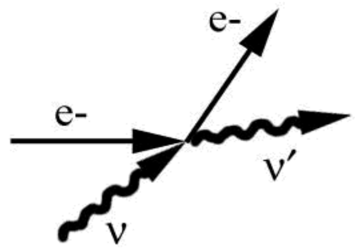

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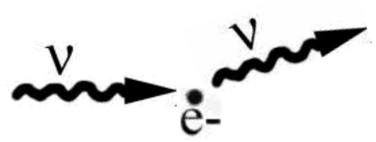

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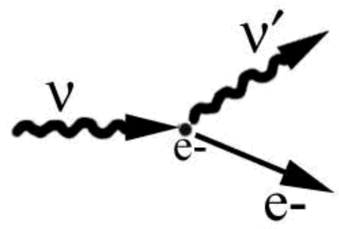

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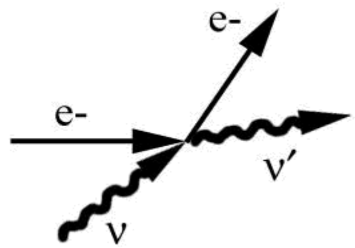

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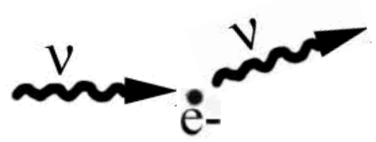

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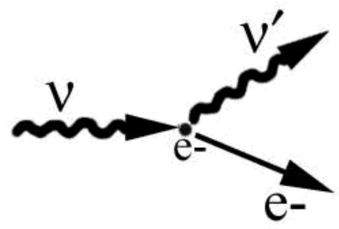

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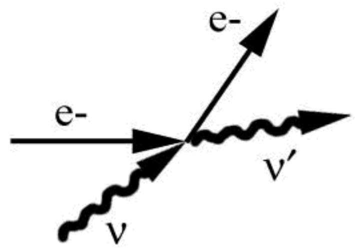
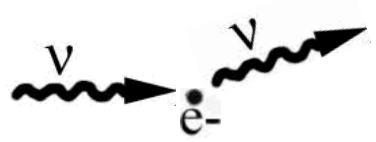


▪ Thomson scattering

- photon just changes direction
- no energy gain or loss
- pure classical (wave) treatment



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necessary conditions?

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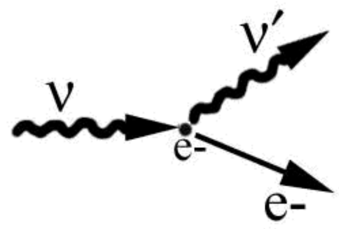

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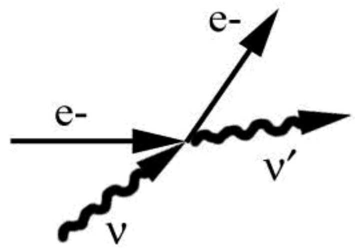

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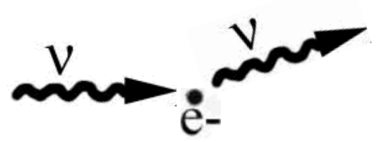

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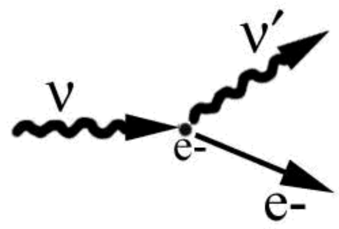
✓ high-energy photons
✓ low-velocity electrons



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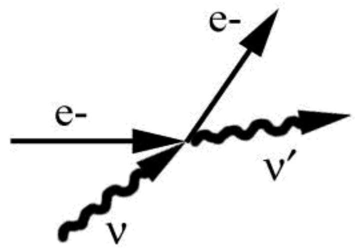

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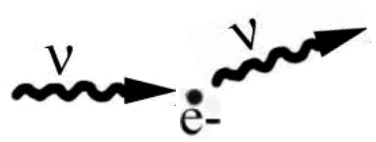

Scattering Effects

photon-electron interactions

necessary conditions:

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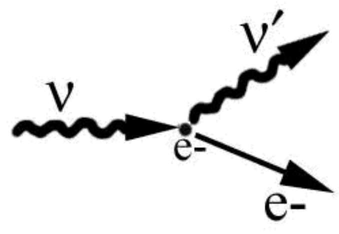

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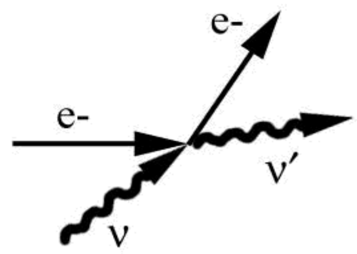

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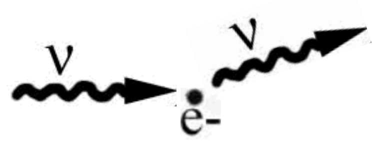


highly relevant in astrophysics...

Scattering Effects

photon-electron interactions

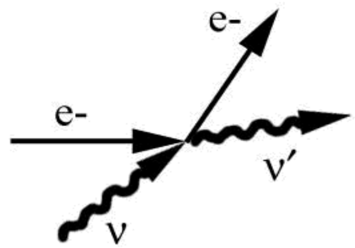
necessary conditions:

- **Compton scattering**
  - no energy transfer from electron to photon
  - photon looses energy to kick electron

✓ high-energy photons
✓ low-velocity electrons

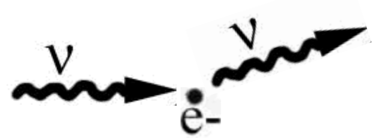
- **inverse Compton scattering takes place in...**
  - stellar interiors
  - active galactic nuclei
  - galaxy clusters

✓ low-energy photons
✓ high-velocity electrons



- **Thomson scattering takes places during...**
  - decoupling of CMB photons

✓ low-energy photons
✓ low-velocity electrons



highly relevant in astrophysics...

Scattering Effects

photon-electron interactions

necessary conditions:

- Compton scattering
  - no energy transfer from electron to photon
  - photon looses energy to kick electron

✓ high-energy photons
✓ low-velocity electrons

- inverse Compton scattering takes place in...
  - stellar interiors
  - active galactic nuclei
  - **galaxy clusters**

✓ low-energy photons
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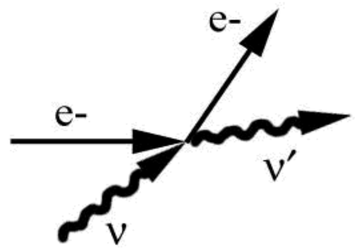
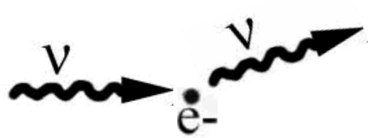


- Thomson scattering takes places during...
  - **decoupling of CMB photons**

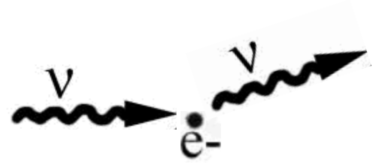
examples

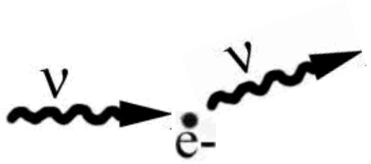
✓ low-energy photons
✓ low-velocity electrons



highly relevant in astrophysics...

- Thomson scattering
  - decoupling of CMB photons

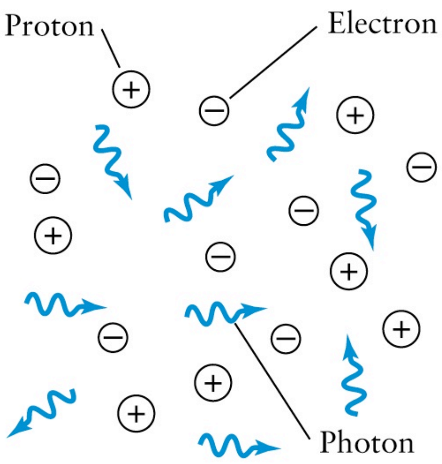




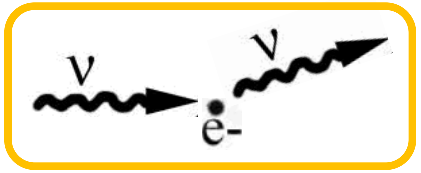
Thomson scattering

- decoupling of CMB photons

early Universe plasma



a Before recombination



- Thomson scattering
  - decoupling of CMB photons

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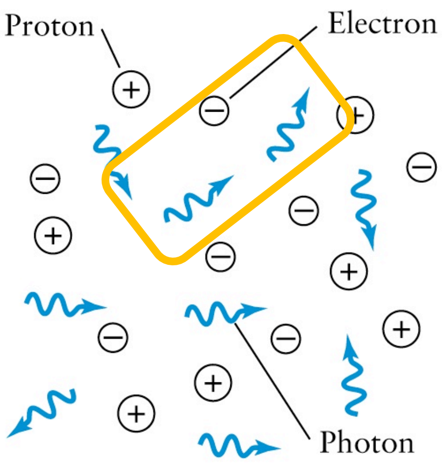

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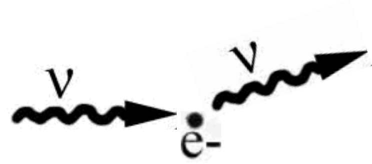
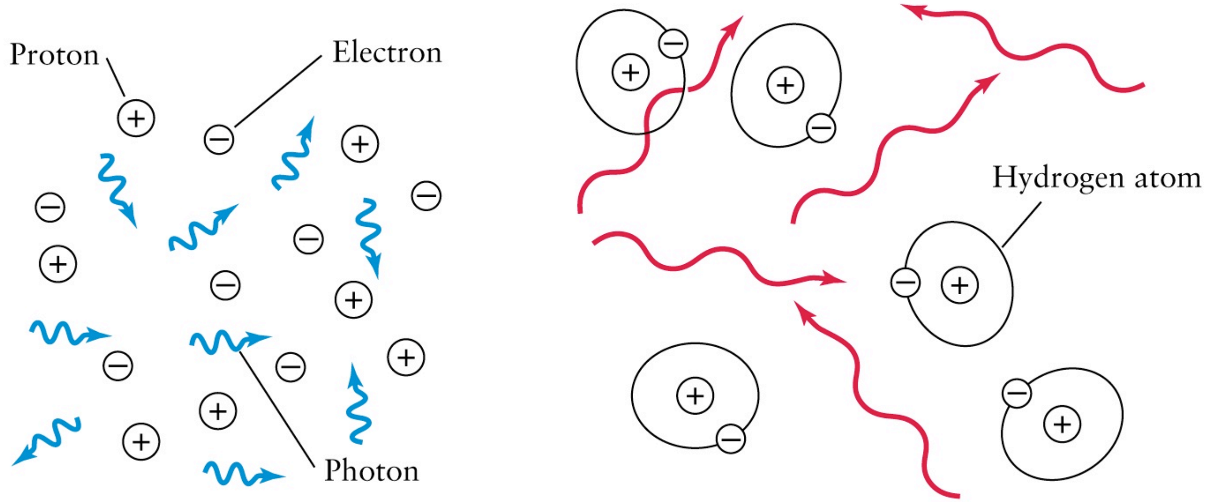


▪ Thomson scattering

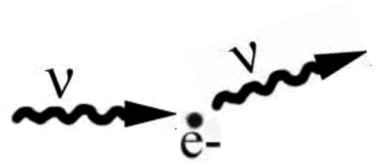
- decoupling of CMB photons

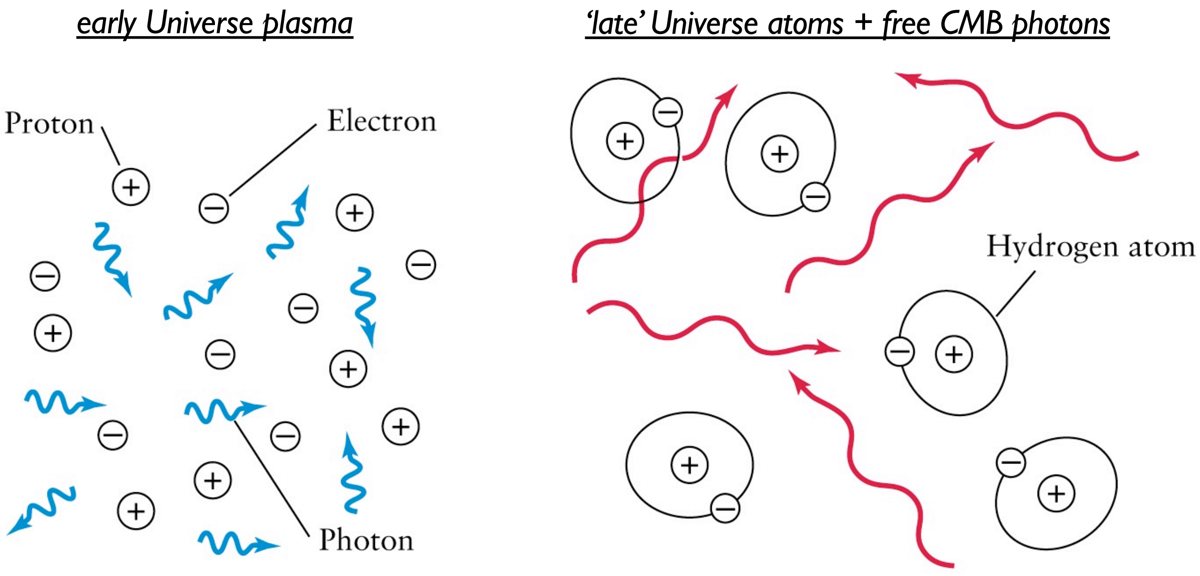
early Universe plasma | *'late' Universe atoms + free CMB photons*



a Before recombination | b After recombination

- Thomson scattering
  - decoupling of CMB photons



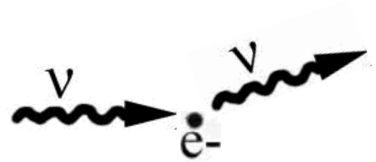


a Before recombination
b After recombination

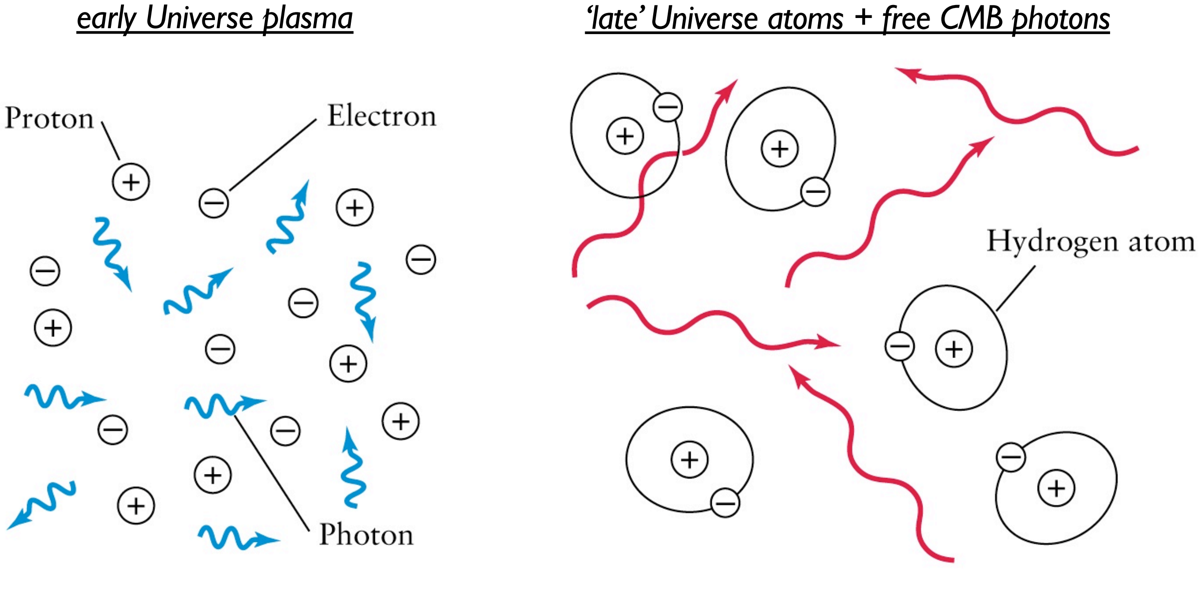
decoupling condition: $\Gamma < H$

Γ: Thomson interaction rate
H: cosmic expansion rate

▪ Thomson scattering



- decoupling of CMB photons



a Before recombination

b After recombination

decoupling condition: $\Gamma < H$

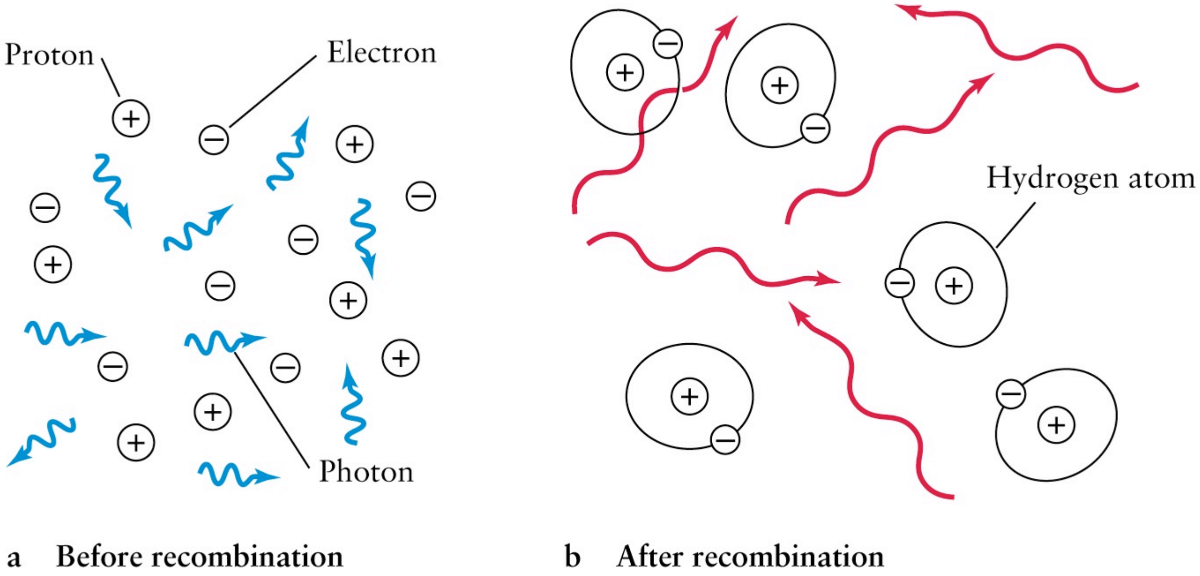
Γ: Thomson interaction rate
H: cosmic expansion rate

$$\Gamma_\gamma \approx n_e \, \sigma_T c$$

$$H = H_0 \sqrt{\Omega_{m,0}} \left(\frac{T}{T_0} \right)^{3/2}$$

- Thomson scattering
  - decoupling of CMB photons

early Universe plasma *'late' Universe atoms + free CMB photons*

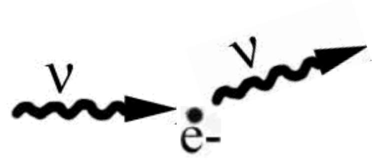


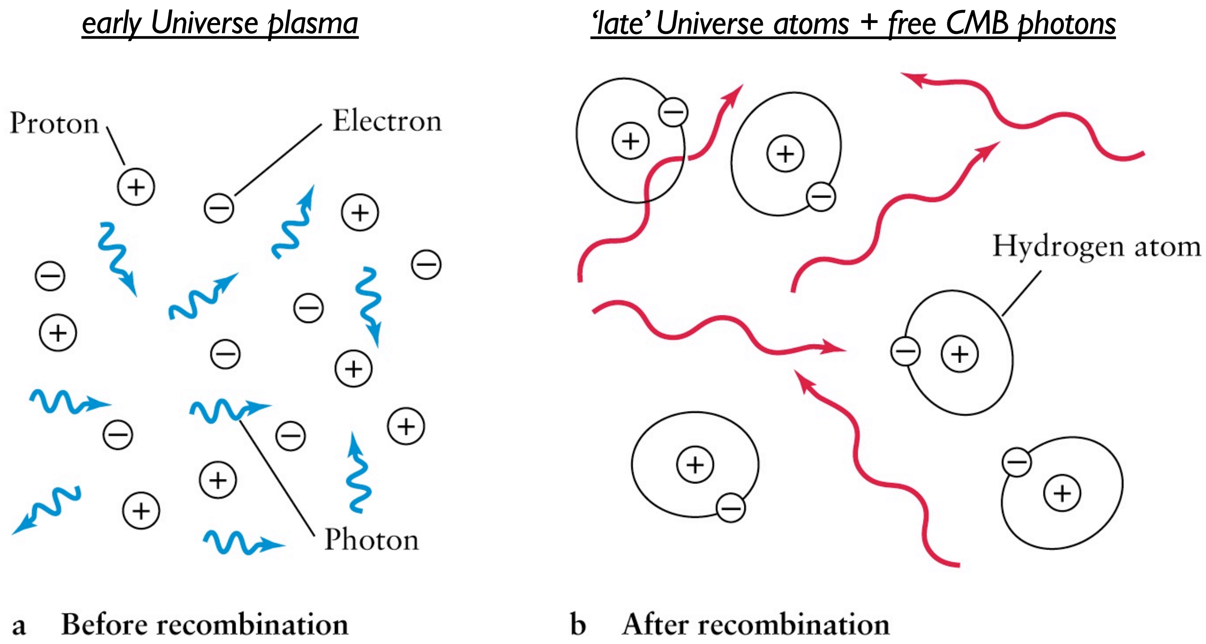
a Before recombination b After recombination

decoupling condition:

$$\eta \frac{2\zeta(3)}{\pi^2}\left(\frac{k}{\hbar c}\right)^3 T_{dec}^3 X_e \sigma_T c \approx H_0 \sqrt{\Omega_{m,0}}\left(\frac{T_{dec}}{T_0}\right)^{3/2}$$

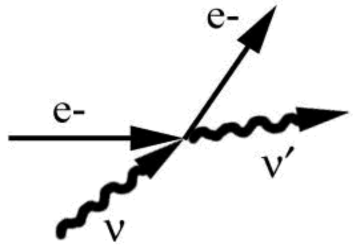
- Thomson scattering
  - decoupling of CMB photons



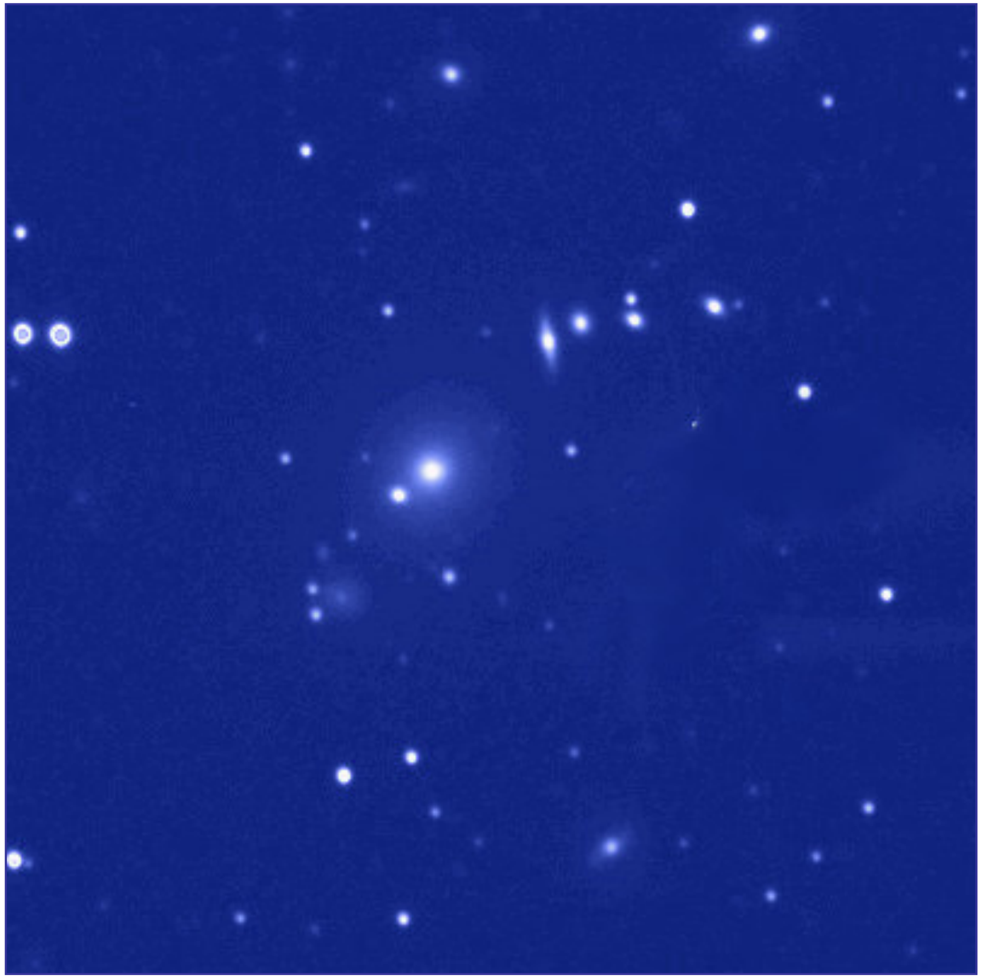


decoupling condition fulfilled for

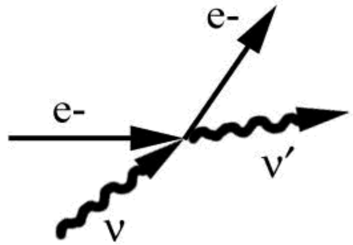
$$T_{\mathrm{dec}} = 0.27\mathrm{eV}$$
$$z_{\mathrm{dec}} = 1090$$



- inverse Compton scattering
  - galaxy clusters



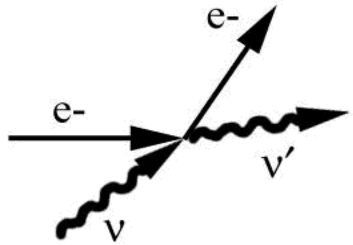
Hydra A – optical

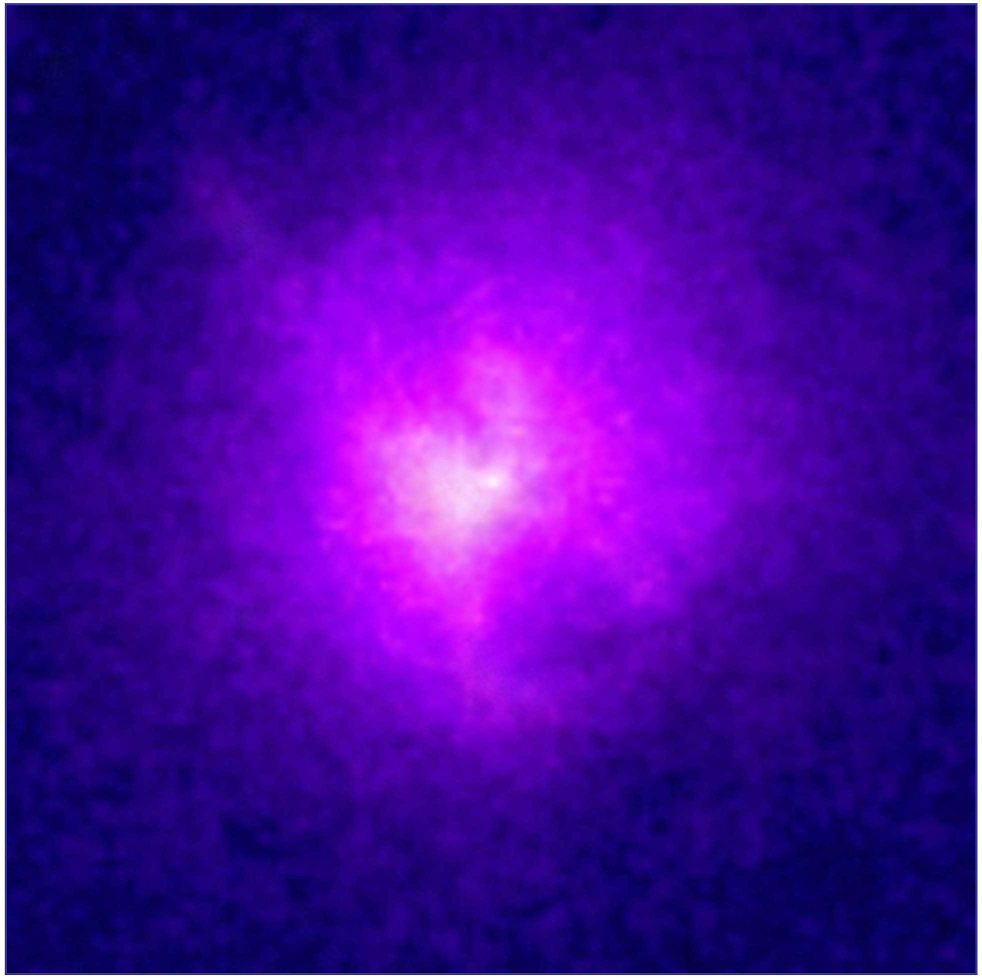


- inverse Compton scattering
  - galaxy clusters

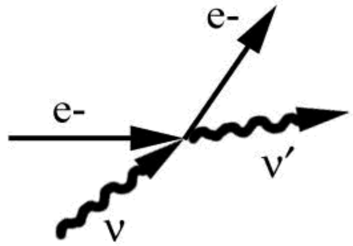
Hydra A – X-rays

- inverse Compton scattering
    - galaxy clusters

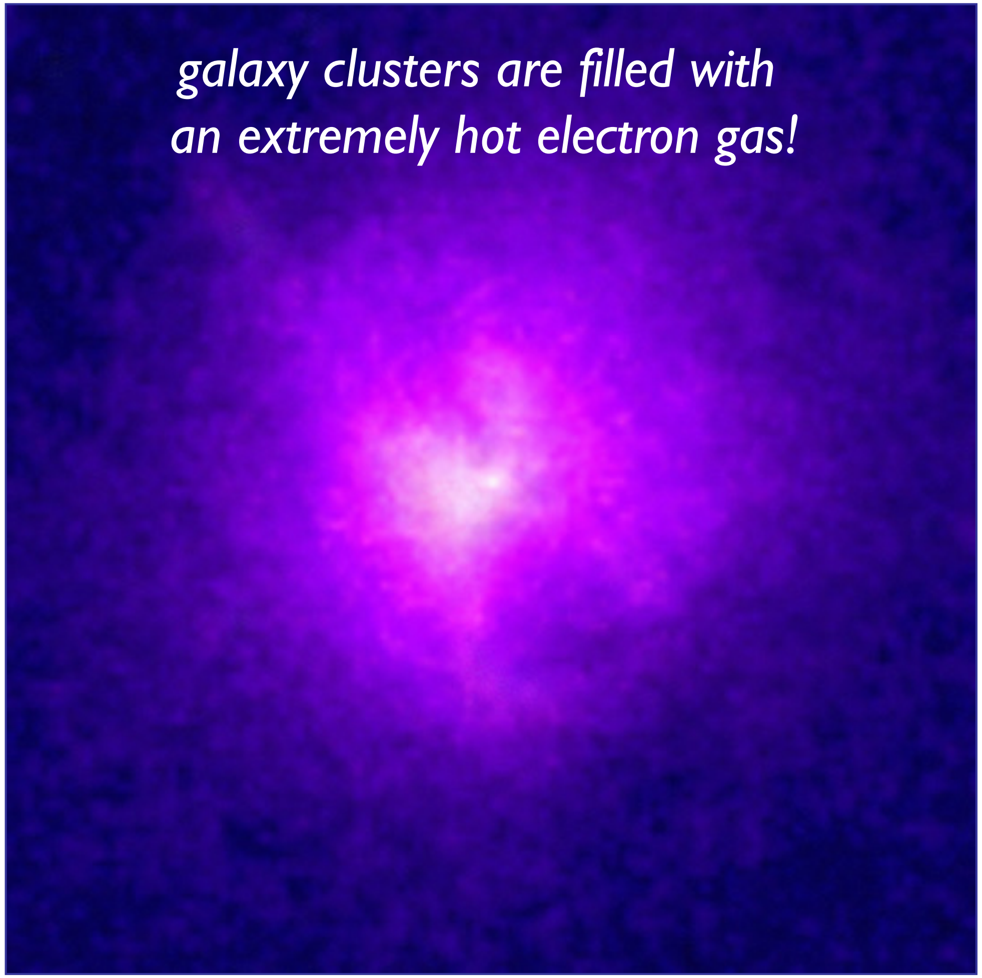




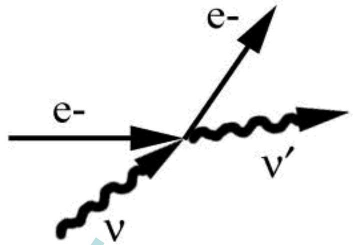
Hydra A – X-rays



- inverse Compton scattering
  - galaxy clusters



Hydra A – X-rays



- inverse Compton scattering
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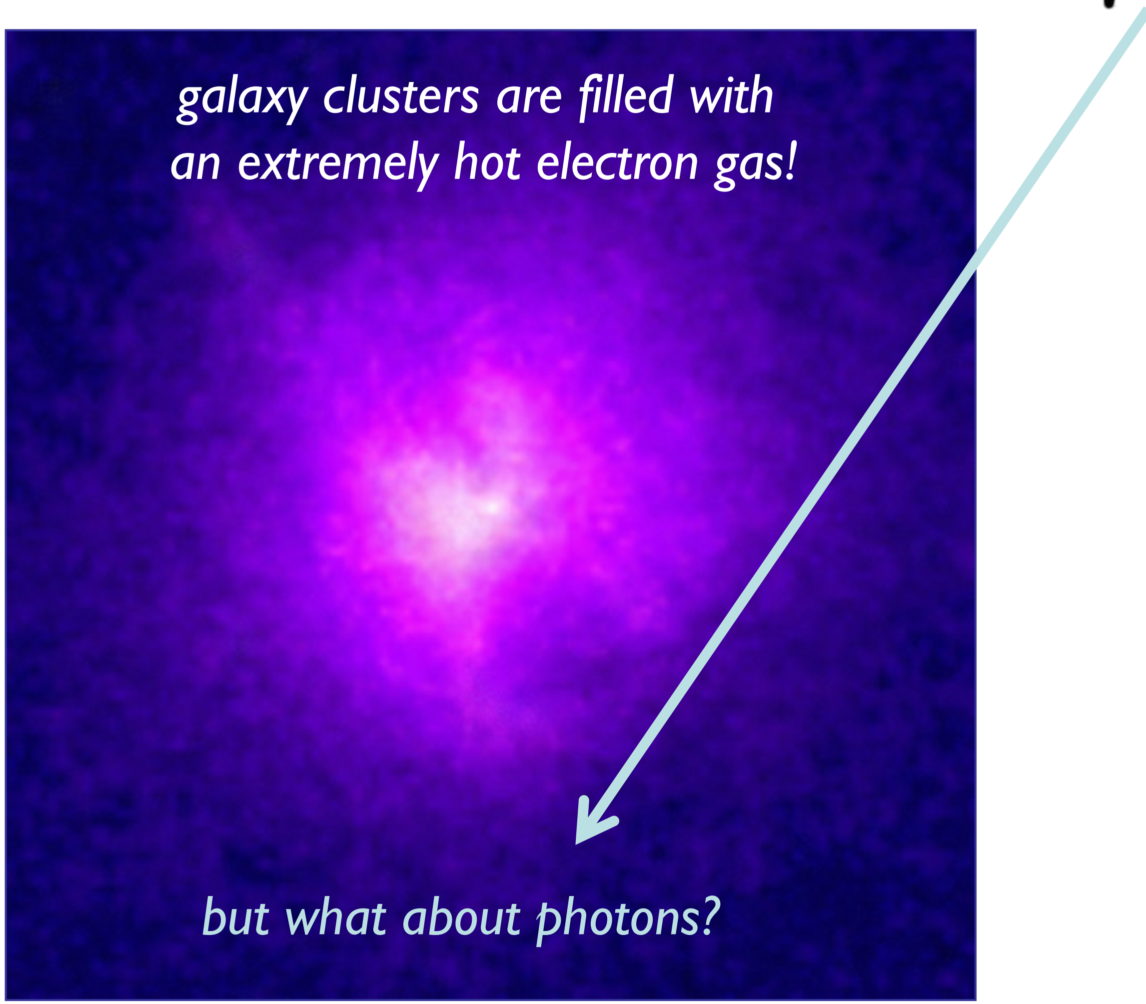
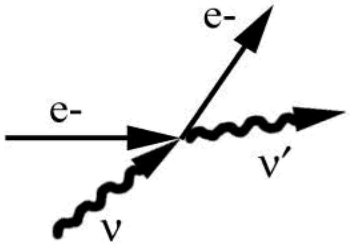
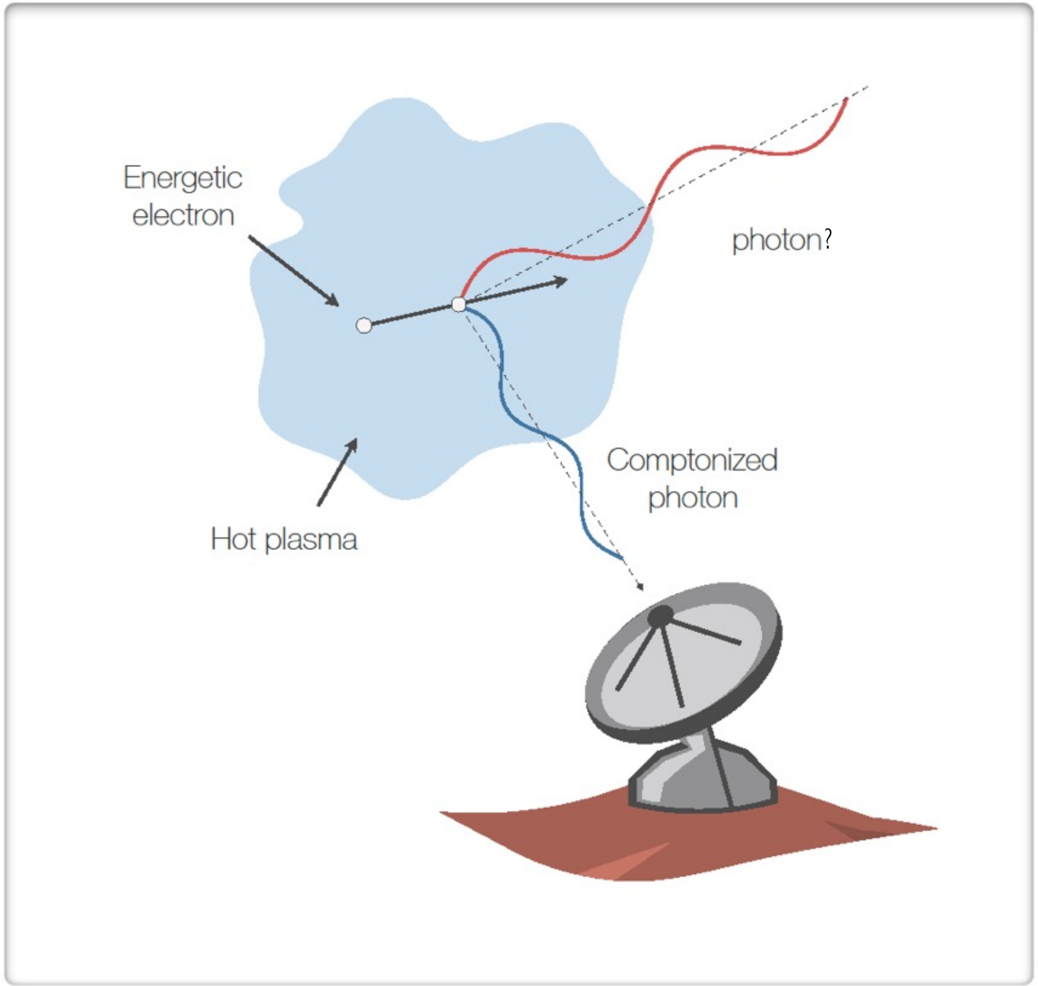
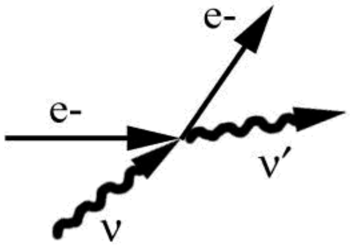


Hydra A – X-rays

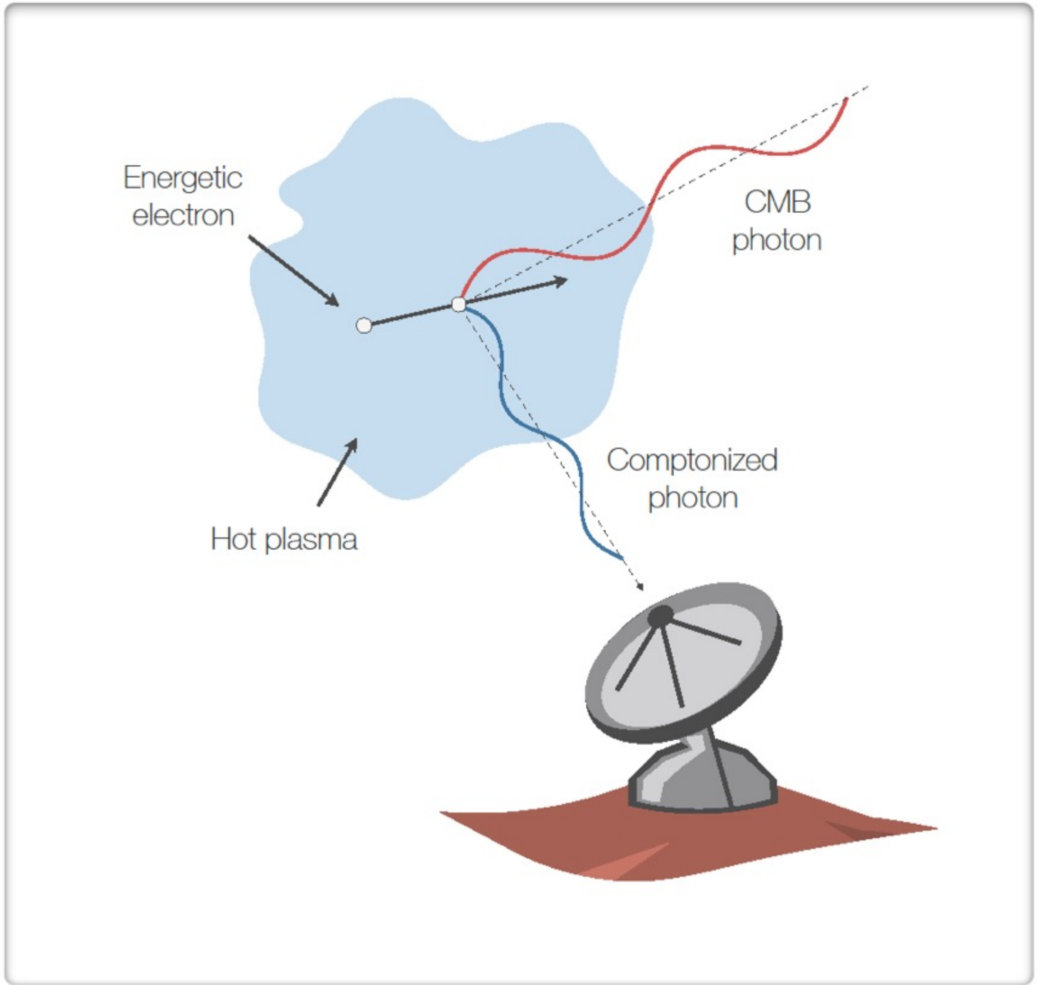


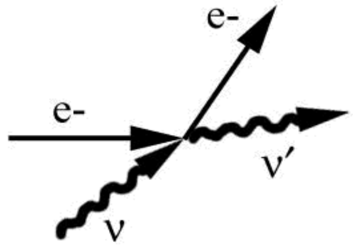
- inverse Compton scattering
  - galaxy clusters



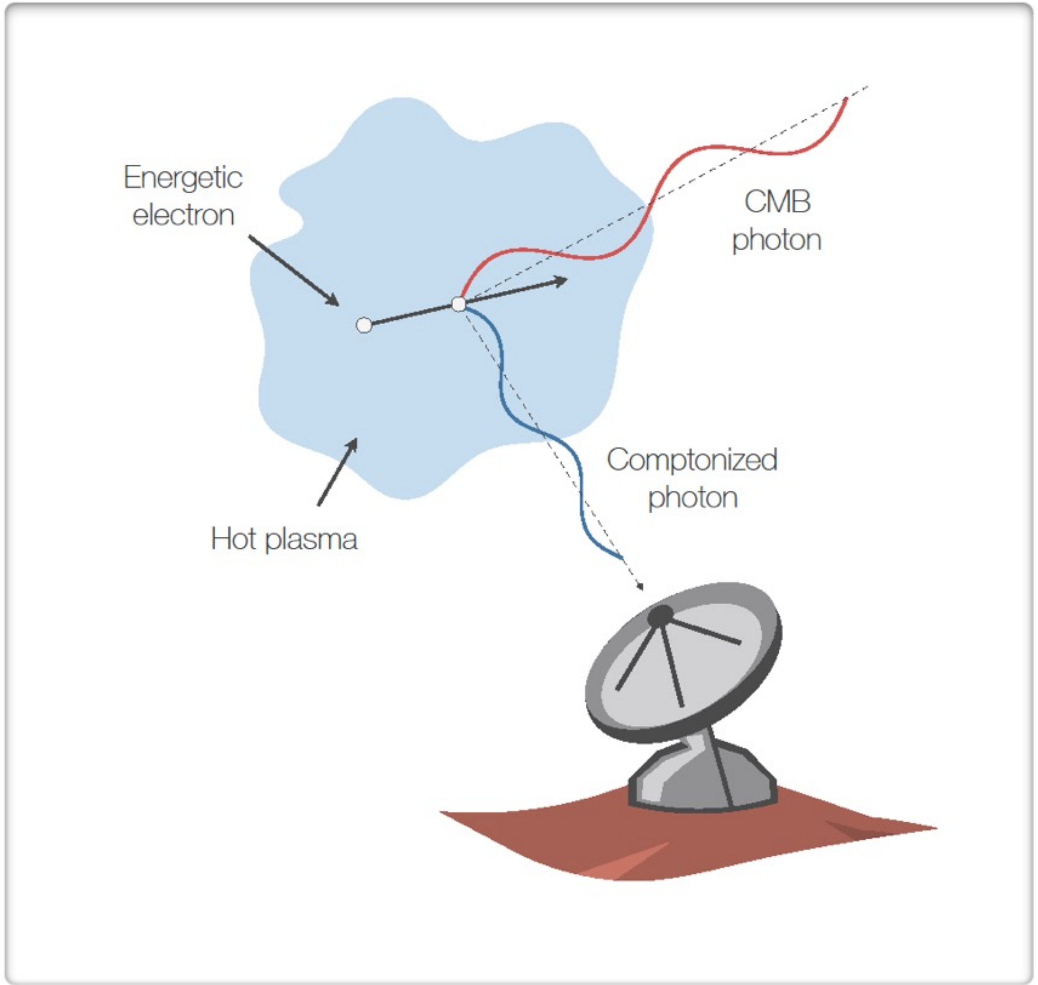


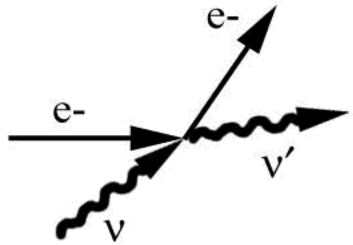
- inverse Compton scattering
  - galaxy clusters



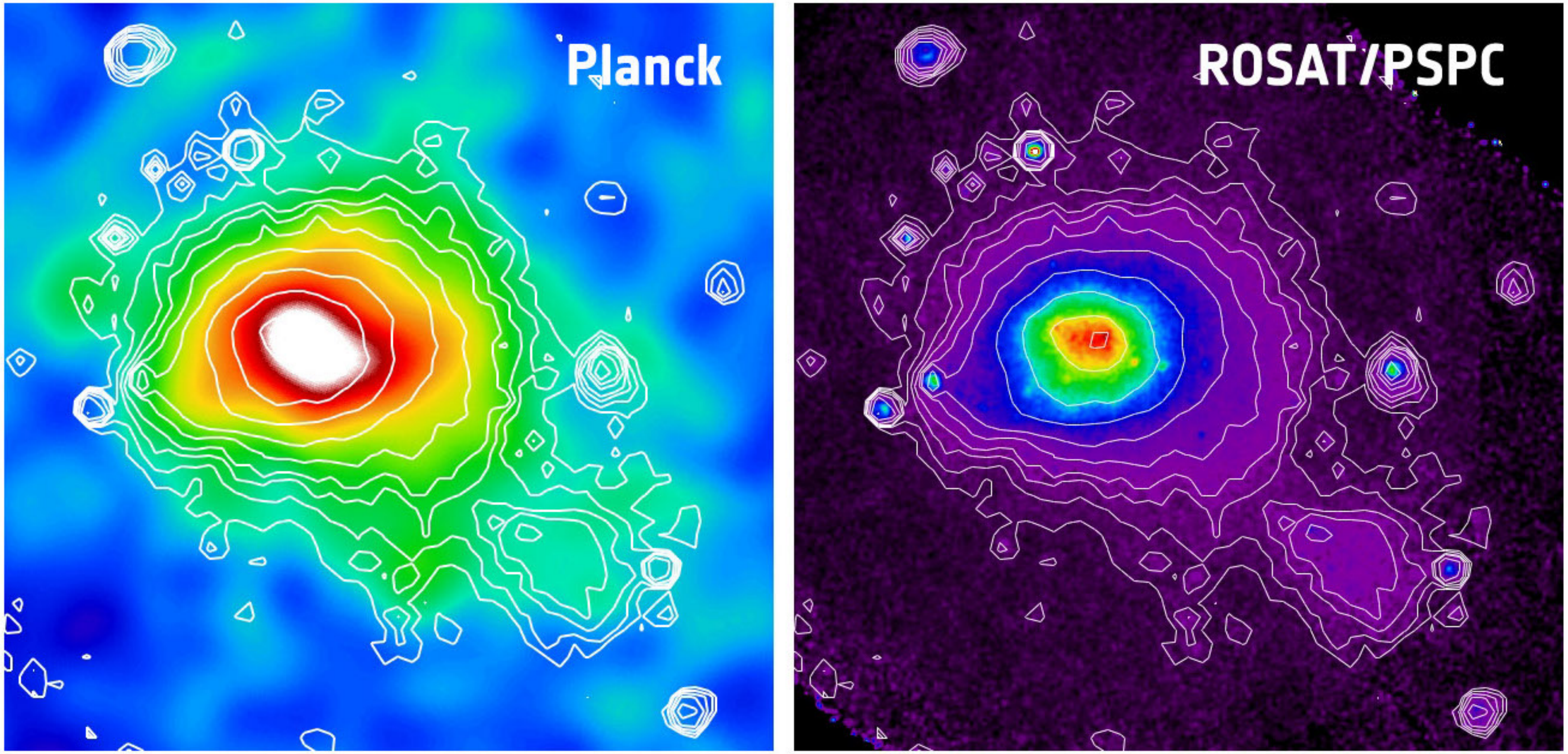


- inverse Compton scattering
  - galaxy clusters: **Sunyaev-Zel'dovich effect**





- inverse Compton scattering
  - galaxy clusters: **Sunyaev-Zel'dovich effect**



Coma cluster of galaxies: SZ map (left, incl. X-ray contours) vs. X-ray map (right), ESA

- emission/absorption contributions
- random walk
- photon-electron interactions
- **radiative diffusion**

- Rosseland approximation
- Eddington approximation

- Rosseland approximation

expression for the energy flux, relating it to the local temperature gradient

- Rosseland approximation

expression for the energy flux, relating it to the local temperature gradient

$$F(z) = -\frac{16\sigma_B T^3}{3\alpha_R}\frac{\partial T}{\partial z}$$

$$\frac{1}{\alpha_R} = \frac{\int \frac{1}{\alpha_\nu}\frac{dI_\nu}{dT}d\nu}{\int \frac{dI_\nu}{dT}d\nu}$$

Rosseland mean opacity

- Rosseland approximation

expression for the energy flux, relating it to the local temperature gradient

$$F(z) = -\frac{16\sigma_B T^3}{3\alpha_R}\frac{\partial T}{\partial z}$$

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Rosseland mean opacity

- Eddington approximation

approximations developed to make the modelling of stars practical...

- Eddington approximation (approximations developed to make the modelling of stars practical...)

general solution: $$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

star*: $$I_\nu(\tau_\nu) = \int_0^{\infty} e^{-(\tau'_\nu - \tau_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

*going all the way from the centre ($\tau = \infty$) to the surface ($\tau = 0$)

- ## Eddington approximation (approximations developed to make the modelling of stars practical...)

general solution: $$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

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$$S_\nu = const.$$

- Eddington approximation (approximations developed to make the modelling of stars practical...)

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→ exercise #3

- Eddington approximation (approximations developed to make the modelling of stars practical...)

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star: $$I_\nu(\tau_\nu) = \int_0^{\infty} e^{-(\tau_\nu-\tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

Taylor expanding S_ν*: $$S_\nu = a_\nu + b_\nu\tau_\nu$$

*assumption: S increases (linearly) with optical depth

E.g. isotropy would fail to realistically transfer energy toward the surface where the photons are emitted.
The Eddington approximation assumes there is a general transfer of energy away from the middle of the star ("up")

- **Eddington approximation** (approximations developed to make the modelling of stars practical...)

general solution: $$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu-\tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

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Taylor expanding S_ν: $$S_\nu = a_\nu + b_\nu\tau_\nu$$

$$I_\nu(\tau_\nu) = \int_0^{\infty} e^{-(\tau_\nu-\tau'_\nu)}(a_\nu + b_\nu\tau'_\nu)\, d\tau'_\nu$$

$$= \int_0^{\infty} e^{-(\tau_\nu-\tau'_\nu)}\, a_\nu\, d\tau'_\nu + \int_0^{\infty} e^{-(\tau_\nu-\tau'_\nu)}\, b_\nu\tau'_\nu d\tau'_\nu$$

$$= a_\nu + b_\nu\tau_\nu$$

- Eddington approximation (approximations developed to make the modelling of stars practical...)

general solution: $$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$$

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$$= a_\nu + b_\nu \tau_\nu$$

$I_\nu(\tau_\nu) = S_\nu$ Eddington-Barbier relation

- Eddington approximation (approximations developed to make the modelling of stars practical...)

general solution: $I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu-\tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$

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$$= a_\nu + b_\nu\tau_\nu$$

$I_\nu(\tau_\nu) = S_\nu$ Eddington-Barbier relation

same solution as for an optical thick medium (cf. Fundamentals)

- Eddington approximation (approximations developed to make the modelling of stars practical...)

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frequency-independent: $S_\nu(\tau_\nu) = S(\tau)$

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frequency-independent: $S_\nu(\tau_\nu) = S(\tau)$

"grey atmosphere" approximation

- Eddington approximation (approximations developed to make the modelling of stars practical...)

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star: $I_\nu(\tau_\nu) = \int_0^{\infty} e^{-(\tau_\nu - \tau'_\nu)}\, S_\nu(\tau'_\nu)\, d\tau'_\nu$

frequency-independent: $S_\nu(\tau_\nu) = S(\tau) = ?$

what is a reasonable relation?

- Eddington approximation (approximations developed to make the modelling of stars practical...)

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frequency-independent: $$S_\nu(\tau_\nu) = S(\tau) = a + b\tau$$

- Eddington approximation (approximations developed to make the modelling of stars practical...)

general solution: $$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau_\nu')}\, S_\nu(\tau_\nu')\, d\tau_\nu'$$

star: $$I_\nu(\tau_\nu) = \int_0^{\infty} e^{-(\tau_\nu - \tau_\nu')}\, S_\nu(\tau_\nu')\, d\tau_\nu'$$

frequency-independent: $$S_\nu(\tau_\nu) = S(\tau) = a + b\tau$$

$$I_\nu(\tau_\nu) = \int_0^{\infty} e^{-(\tau_\nu - \tau_\nu')} S(\tau') d\tau_\nu'$$

...

- equation of radiative transfer

$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \alpha_\nu^s)I_\nu + j_\nu + \int_\Omega \alpha_\nu^s \, I_\nu \frac{d\Omega}{4\pi}$$

- scattering (in Astrophysics) = dispersion of a beam of photons by...
    - inverse Compton scattering
    - Thomson scattering

- random walk
    - net displacement of photons $d = \sqrt{N}\, l$
    - number of scattering events $N = \max(\tau^2, \tau)$